ALGEBRA

Chapter 09

4th

FACTORIAL Y NÚMERO COMBINATORIO

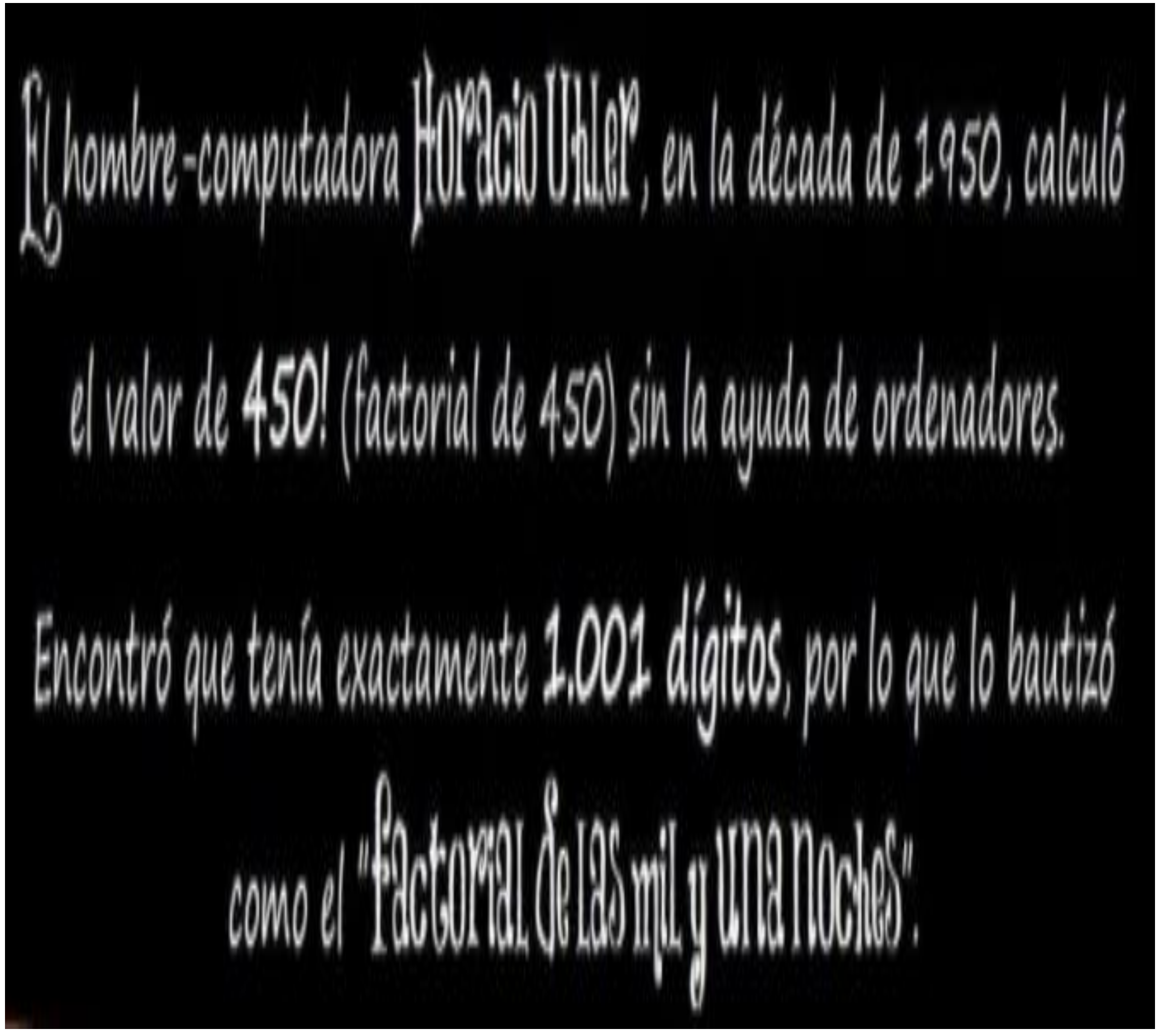
SACO OLIVEROS

HELICO

MOTIVATING

SACO OLIVEROS

SABIAS QUE



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.510.155.692.075.095.553.330.016.834.367.506.046.750.882.904.38
7.106.145.811.284.518.424.097.858.618.583.806.301.650.208.347.29
6.181.351.667.570.171.918.700.422.280.962.237.272.230.663.528.08
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HELICO THEORY

CHAPTHER 09

SACO OLIVEROS

FACTORIAL

DEFINICIÓN

Sea **n ∈ N (además del cero)**, denotado por n! ; se define como:

$$n! = \begin{cases} 1, & si\ n = 0 \ \vee \ n = 1 \\ 1 \times 2 \times 3 \ldots\ldots n, & si\ n \in \mathbb{N} \ \wedge \ n \geq 2 \end{cases}$$

Ejemplos:

$$5! = 5(4)(3)(2)(1) = 120$$

$$3! = 3(2)(1) = 6$$

$$14! = \underbrace{14(13)(12)(11)(10)}\, 9! = 240240.\, 9!$$

Degradación de factorial

Propiedades

$$\boxed{n! + (n+1)! = n!(n+2)}$$

$$5! + 6! = 5!(5+2) = 5!(7)$$

$$\boxed{n! + (n+1)! + (n+2)! = n!(n+2)^2}$$

$$4! + 5! + 6! = 4!(4+2)^2 = 4!(36)$$

$$\boxed{(n+1)! - n! = n!(n)}$$

$$5! - 4! = 4!(4)$$

NÚMERO COMBINATORIO

DEFINICIÓN

El número combinatorio denotado por C_k^n representa el número total de combinaciones que se pueden realizar con *n* elementos tomados de *k* en *k*.

$$\boxed{C_k^n = \frac{n!}{k!.(n-k)!}} \quad (n, k \in \mathbb{N} \wedge n \geq k)$$

Ejemplo:

$$C_2^7 = \frac{7!}{2!.(7-2)!} = \frac{7(6).5!}{2(1).(5)!} = \boxed{21}$$

Caso Práctico:

$$C_2^7 = \frac{7(6)}{2(1)} = \boxed{21}$$

Propiedades

$$\boxed{C_k^n = C_{n-k}^n}$$

Ejemplo: $C_2^7 = C_{7-2}^7 = C_5^7$

Si: $\boxed{C_k^n = C_p^n} \Rightarrow \boxed{k = p \quad \vee \quad n = k + p}$

Ejemplo: **Si:** $C_{10}^{15} = C_p^{15}$

$\Rightarrow p = 10 \quad \vee \quad 15 = p + 10$

$$\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}$$

Ejemplo: $C_4^{12} + C_5^{12} = C_5^{12+1} = C_5^{13}$

$$\boxed{C_k^n = \frac{n}{k} C_{k-1}^{n-1}}$$

Ejemplo:

$$C_9^{15} = \frac{\cancel{15}}{\cancel{9}} C_{9-1}^{15-1} = \frac{5}{3} C_8^{14}$$

HELICO PRACTICE

CHAPTHER 09

1. Reduzca

$$P=\left(\frac{32!+33!}{34!}\right)\left(\frac{67!}{66!+65!}\right)$$

RESOLUCIÓN

$$\boxed{n!+(n+1)!=n!(n+2)}$$

$$P=\left(\frac{\overbrace{32!+33!}}{34!}\right)\left(\frac{67!}{\underbrace{66!+65!}}\right)$$

$$\boxed{n!+(n+1)!=n!(n+2)}$$

$$P=\left(\frac{\cancel{32!}\,(\cancel{34})}{\cancel{34}\,(33)\,\cancel{32!}}\right)\left(\frac{\cancel{67}\,(66)\,\cancel{65!}}{\cancel{65!}\,(\cancel{67})}\right)$$

$$P=\left(\frac{\cancel{66}}{\cancel{33}}\right)$$

$$\boxed{P=2}$$

2. Halle el valor de “x” en:

$$\frac{(x+4)!\,(x+2)!}{(x+3)!+(x+2)!}=720$$

RESOLUCIÓN

Degradación de factorial

$$\frac{\overbrace{(x+4)!}\,(x+2)!}{\underbrace{(x+3)!+(x+2)!}}=720$$

$$\boxed{n!+(n+1)!=n!(n+2)}$$

$$\frac{\cancel{(x+4)}(x+3)!.\cancel{(x+2)!}}{\cancel{(x+2)!}\cancel{(x+4)}}=720$$

$$(x+3)!=\underbrace{720}_{6!}$$

$$(x+3)!=6!$$

$$\boxed{x=3}$$

3. Halle el valor de x, si se cumple:

$$\frac{(x+2)!+(x+3)!+(x+4)!}{(x+3)!-(x+2)!}=\frac{25}{x+2}$$

RESOLUCIÓN

$$\boxed{n!+(n+1)!+(n+2)!=n!(n+2)^2}$$

$$\frac{\overbrace{(x+2)!+(x+3)!+(x+4)!}}{\underbrace{(x+3)!-(x+2)!}}=\frac{25}{x+2}$$

$$\boxed{(n+1)!-n!=n!\,(n)}$$

$$\Rightarrow \frac{\cancel{(x+2)!}\,(x+4)^2}{\cancel{(x+2)!}\,\cancel{(x+2)}}=\frac{25}{\cancel{x+2}}$$

$$\Rightarrow (x+4)^2=25$$

$$\boxed{x=1}$$

4. Halle el valor de "n" en:

$$3C_3^{2n} = 44C_2^{n}$$

$$\boxed{n = 6}$$

RESOLUCIÓN

Caso Práctico:

$$\cancel{3}\left\{\frac{\cancel{2}n(2n-1)(2n-2)}{(\cancel{3})(\cancel{2})(1)}\right\} = \overset{22}{\cancel{44}}\left\{\frac{n(n-1)}{(\cancel{2})(1)}\right\}$$

$$\cancel{n}(2n-1)\cancel{2}\cancel{(n-1)} = \overset{11}{\cancel{22}}\cancel{n}\cancel{(n-1)}$$

$$(2n-1) = 11$$

5. Calcule

$$P=\frac{C_4^{12}+C_5^{12}+C_7^{13}}{C_6^{14}+C_7^{14}}$$

RESOLUCIÓN

$$\boxed{C_k^n+C_{k+1}^n=C_{k+1}^{n+1}}$$

$$P=\frac{\underbrace{C_4^{12}+C_5^{12}}+C_7^{13}}{\underbrace{C_6^{14}+C_7^{14}}}$$

$$\boxed{C_k^n+C_{k+1}^n=C_{k+1}^{n+1}}$$

$$P=\frac{C_5^{13}+C_7^{13}}{C_7^{15}}$$

$$\boxed{C_7^{13}=C_6^{13}}$$

$$\boxed{C_k^n+C_{k+1}^n=C_{k+1}^{n+1}}$$

$$P=\frac{C_5^{13}+C_6^{13}}{C_7^{15}}=\frac{C_6^{14}}{C_7^{15}}$$

Por Degradación

$$P=\frac{C_6^{14}}{\frac{15}{7}C_{7-1}^{15-1}}=\frac{7\cancel{C_6^{14}}}{15\cancel{C_6^{14}}}=\frac{7}{15}$$

$$\boxed{P=\frac{7}{15}}$$

6. Pedro le regala a su esposa una licuadora marca OSTER, cuyo precio fue el valor de 2T soles, donde T está dado por:

$$T = C_5^8 + C_6^8 + C_7^9 + C_8^{10} + C_2^{11}$$

¿Cuánto le costó la licuadora a Pedro?

RESOLUCIÓN

$$T = \underbrace{C_5^8 + C_6^8}_{\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}} + C_7^9 + C_8^{10} + C_2^{11}$$

$$T = \underbrace{C_6^9 + C_7^9}_{\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}} + C_8^{10} + C_2^{11}$$

$$T = \underbrace{C_7^{10} + C_8^{10}}_{\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}} + C_2^{11}$$

$$T = C_8^{11} + \underbrace{C_2^{11}}_{\boxed{C_k^n = C_{n-k}^n}}$$

$$T = \underbrace{C_8^{11} + C_9^{11}}_{\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}}$$

$$T = C_9^{12} \qquad \boxed{C_k^n = \frac{n!}{k!.(n-k)!}}$$

$$T = \frac{12!}{9!.(3)!} = \frac{\overset{2}{\cancel{12}}(11)(10)\cancel{9!}}{\cancel{9!}.(\cancel{3})(\cancel{2})(1)} = 220$$

$$\boxed{El\ costo\ de\ la\ licuadora = S/.440}$$

7. Halle el valor de M en:

$$M = \frac{3C_2^{11} - 5C_9^{11} + 7C_2^{11}}{C_9^{11}}$$

Si 3M representa la edad de Arturito. ¿Cuál será su edad dentro de 5 años?

RESOLUCIÓN

$$\boxed{C_k^n = C_{n-k}^n}$$

$$M = \frac{3C_2^{11} - 5\overbrace{C_9^{11}}^{} + 7C_2^{11}}{\underbrace{C_9^{11}}_{}}$$

$$\boxed{C_k^n = C_{n-k}^n}$$

$$\Rightarrow M = \frac{3C_2^{11} - 5C_2^{11} + 7C_2^{11}}{C_2^{11}}$$

$$\Rightarrow M = \frac{5\cancel{C_2^{11}}}{\cancel{C_2^{11}}} \Rightarrow M = 5$$

Su edad dentro de 5 años será: 20 años