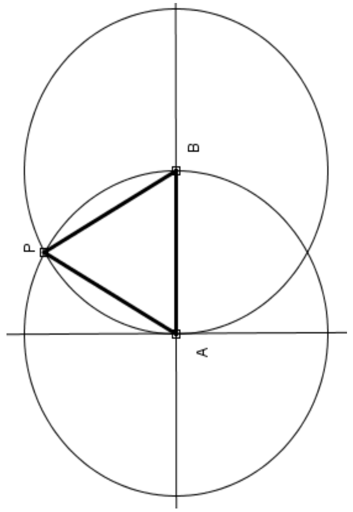
GEOMETRÍA

Capítulo 11

5th
SECONDARY

POLÍGONOS REGULARES

SACO OLIVEROS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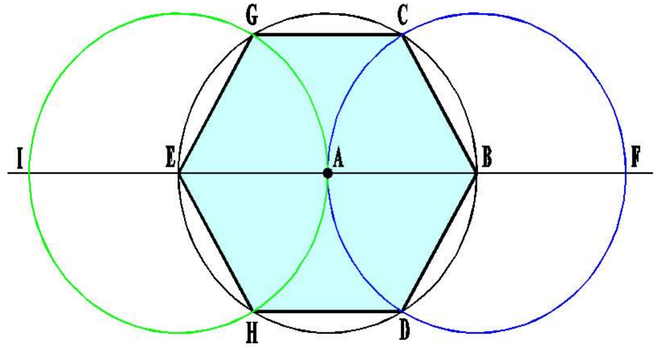
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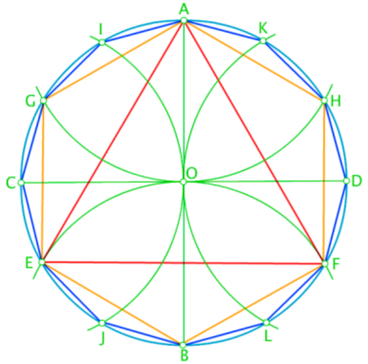
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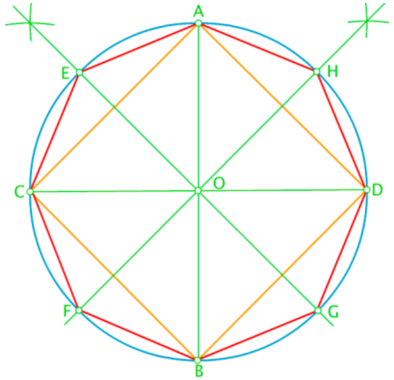
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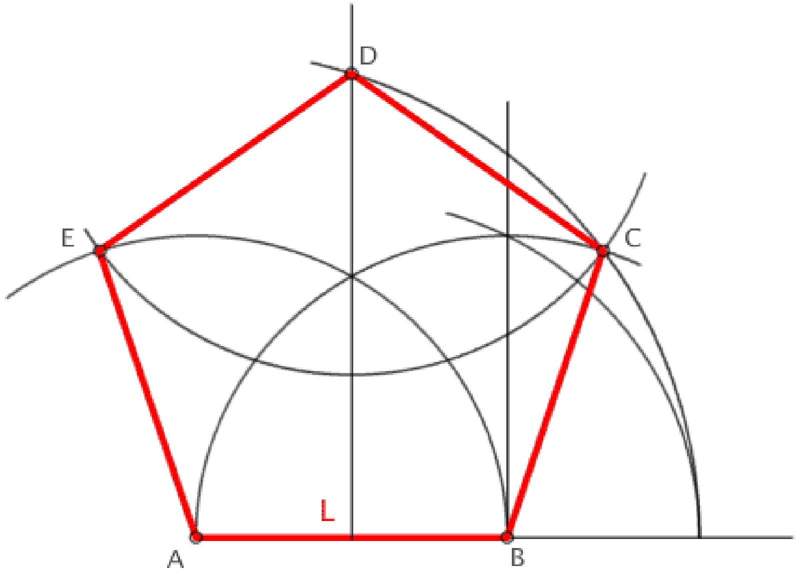
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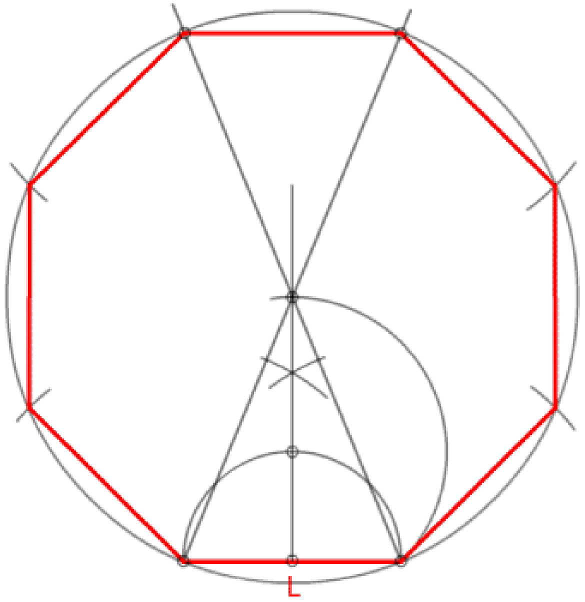
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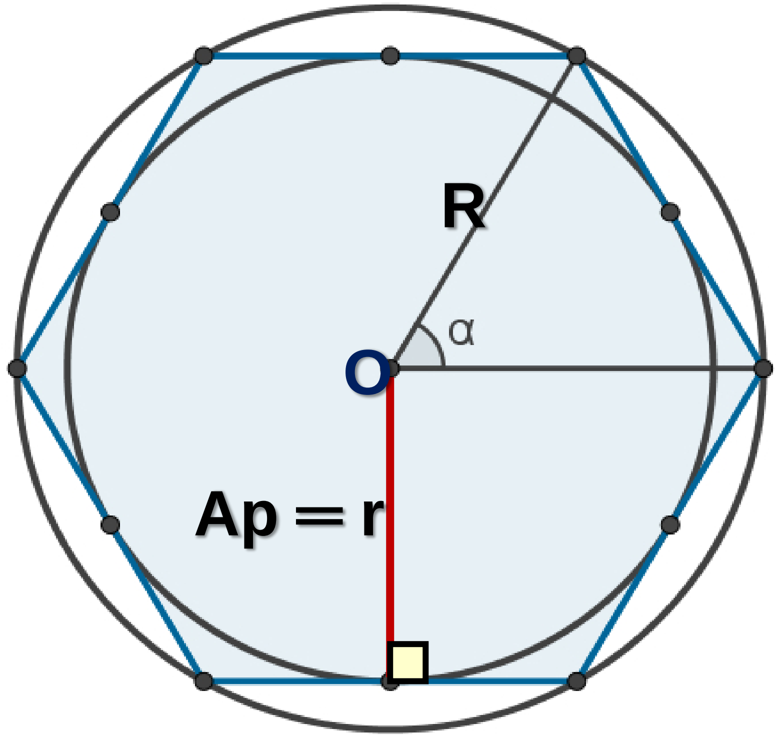
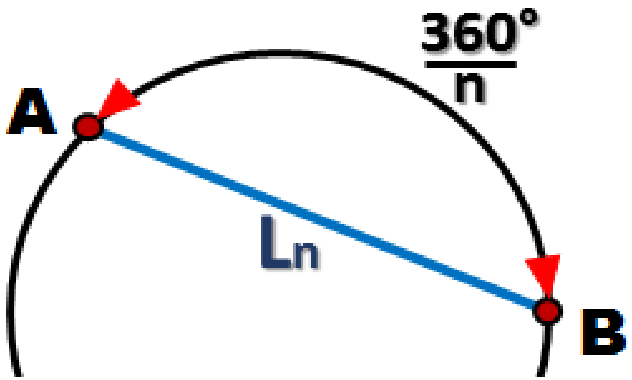
Polígonos Regulares

Se denomina polígono regular al polígono convexo que es equiángulo y equilátero a la vez. Todo polígono regular se puede inscribir y circunscribir a dos circunferencias concéntricas, siendo el centro de estas el centro del polígono regular.



- **Para todo polígono regular, el cálculo de la medida del ángulo central es:**

$$m\text{<}central = \frac{360^\circ}{n}$$

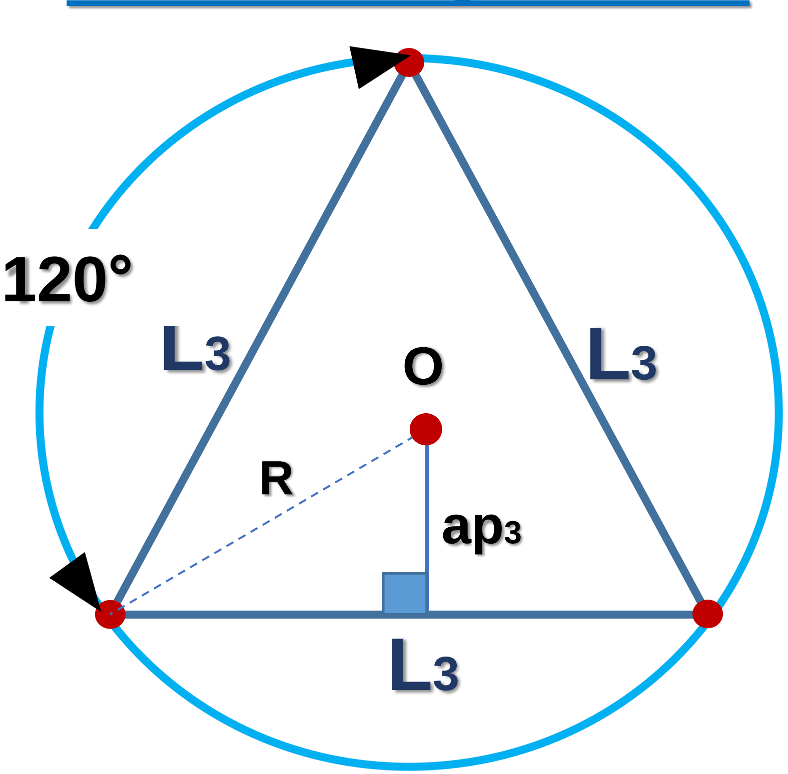


- **Si: AB = Ln**

$$m\overset{\frown}{AB} = \frac{360^\circ}{n}$$

Principales Polígonos Regulares

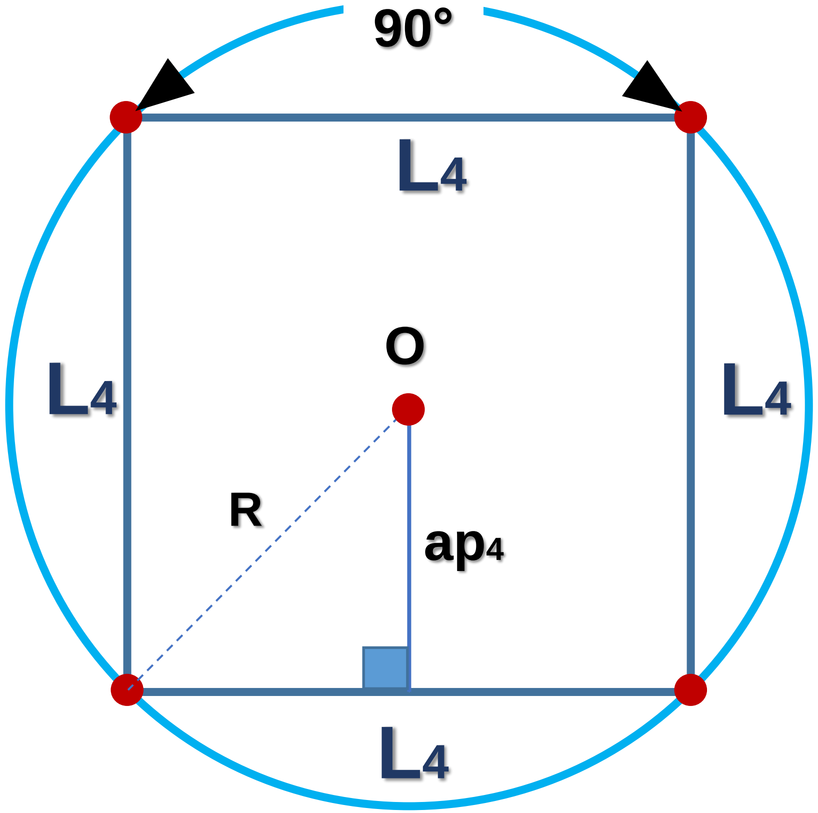
TRIÁNGULO EQUILÁTERO



$L_3 = R\sqrt{3}$

$ap_3 = R/2$

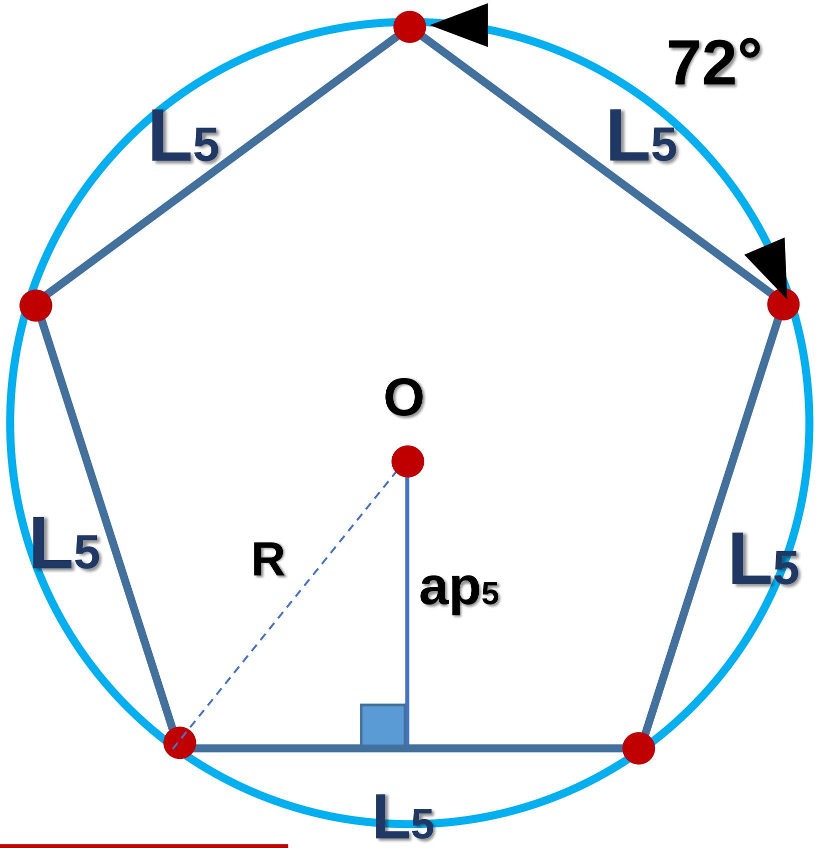
CUADRADO



$L_4 = R\ \sqrt{2}$

$ap_4 = R\sqrt{2}/2$

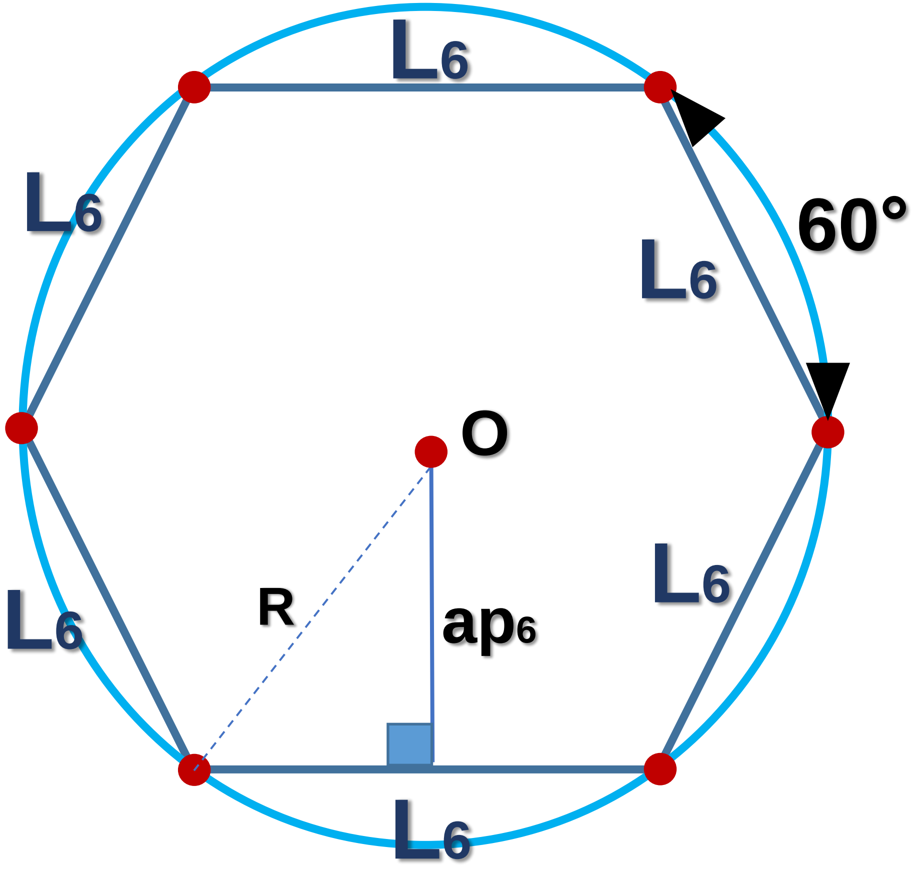
PENTÁGONO REGULAR



$L_5 = \frac{R}{2}\sqrt{10-2\sqrt{5}}$

$ap_5 = \frac{R}{2}(\sqrt{5}+1)$

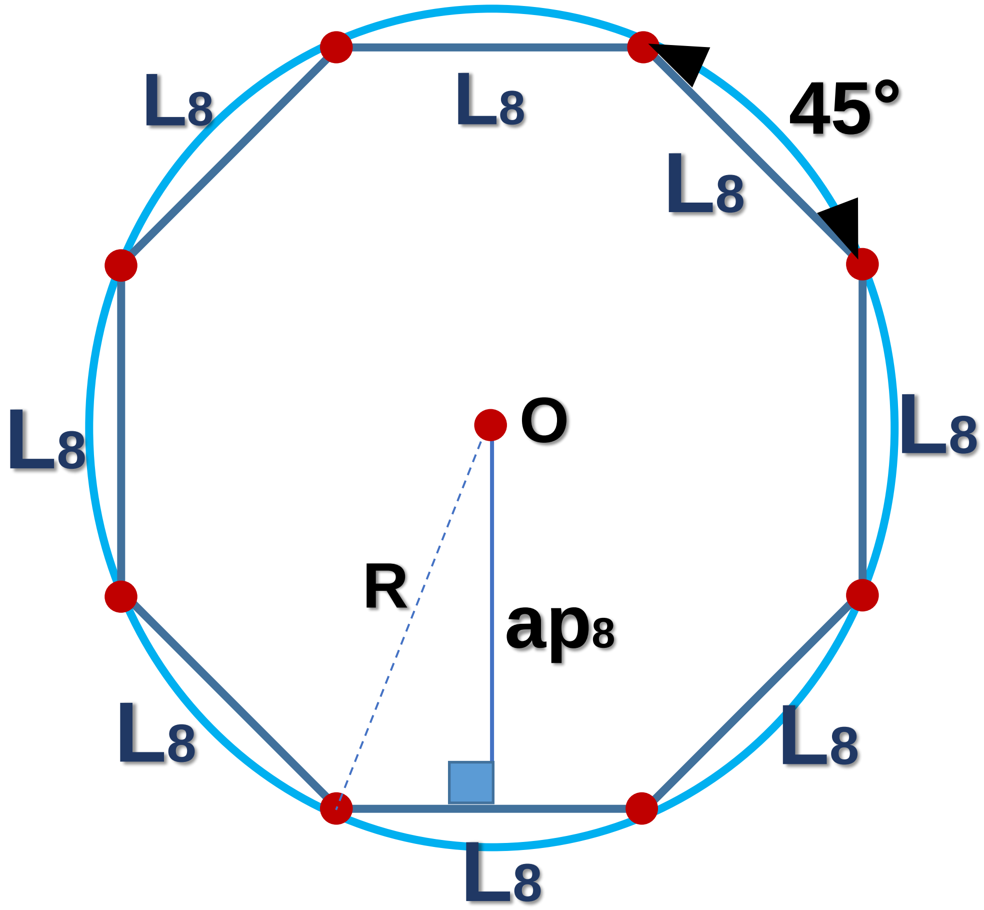
HEXÁGONO REGULAR



$\boxed{L_6 = R}$

$$\boxed{ap_6 = \frac{R\sqrt{3}}{2}}$$

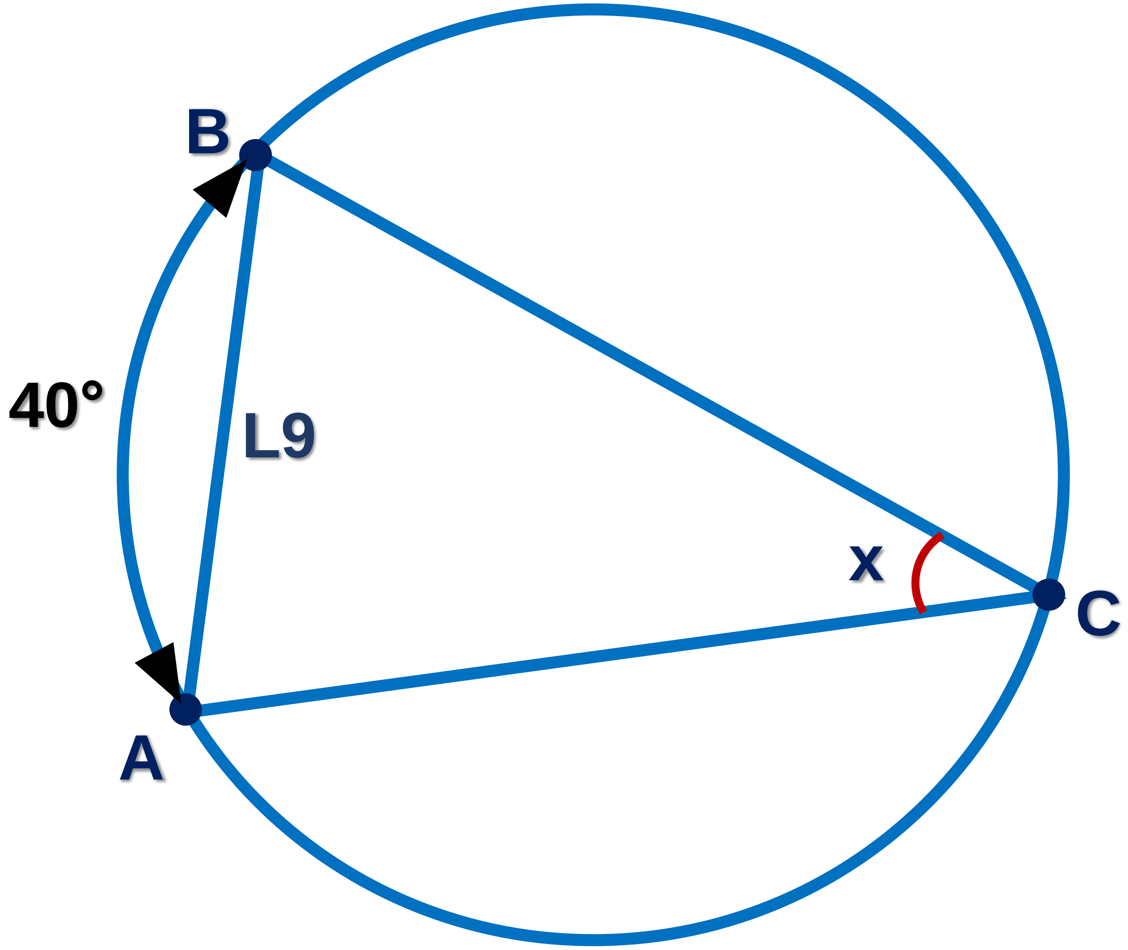
OCTÁGONO REGULAR

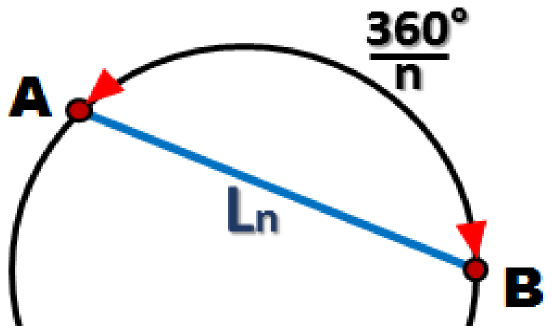


$\boxed{L_8 = R\sqrt{2-\sqrt{2}}}$

$$\boxed{ap_8 = \frac{R}{2}\sqrt{2+\sqrt{2}}}$$

1. Se tiene un triángulo ABC inscrito en una circunferencia, tal que AB = L9. Halle la m∡ACB.





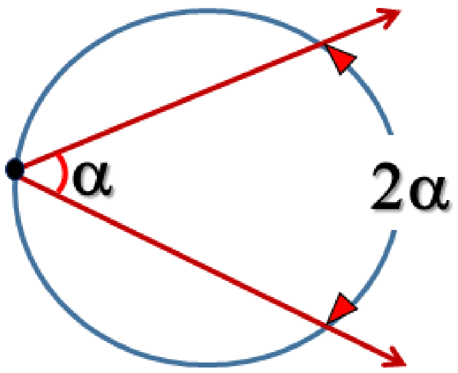
$$\boxed{m\overset{\frown}{AB} = \frac{360°}{n}}$$

$$\boxed{n = 9}$$

- $m\overset{\frown}{AB} = \frac{360°}{9}$

$$\boxed{m\overset{\frown}{AB} = 40°}$$

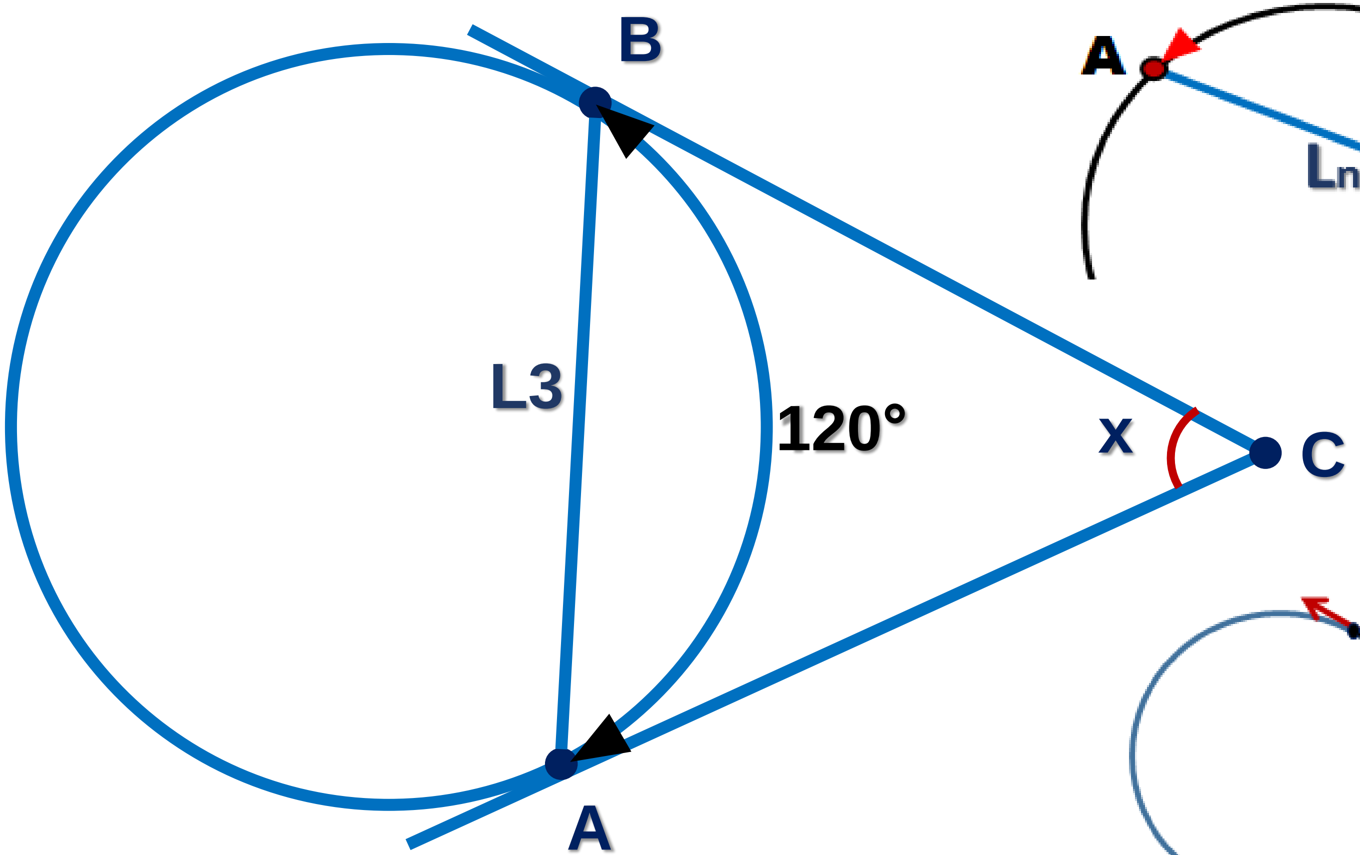
Ángulo inscrito

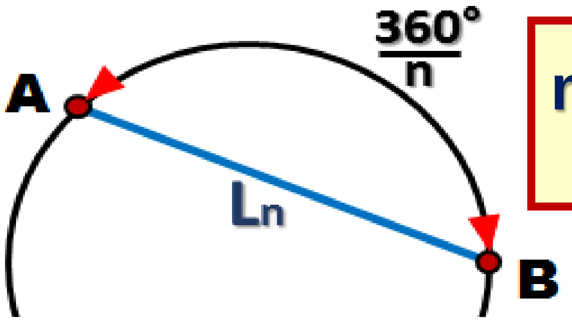


$$x = \frac{40°}{2}$$

$$\boxed{x = 20°}$$

2. Desde un punto C exterior a una circunferencia, se trazan los segmentos tangentes CA y CB. Si AB = L3, halle la m∡ACB.



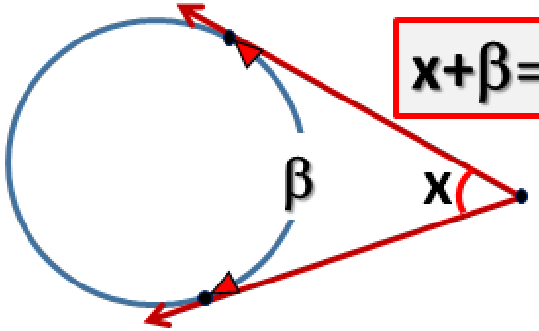


$$m\overset{\frown}{AB} = \frac{360°}{n}$$

$$n = 3$$

- $m\overset{\frown}{AB} = \dfrac{360°}{3}$

$$m\overset{\frown}{AB} = 120°$$

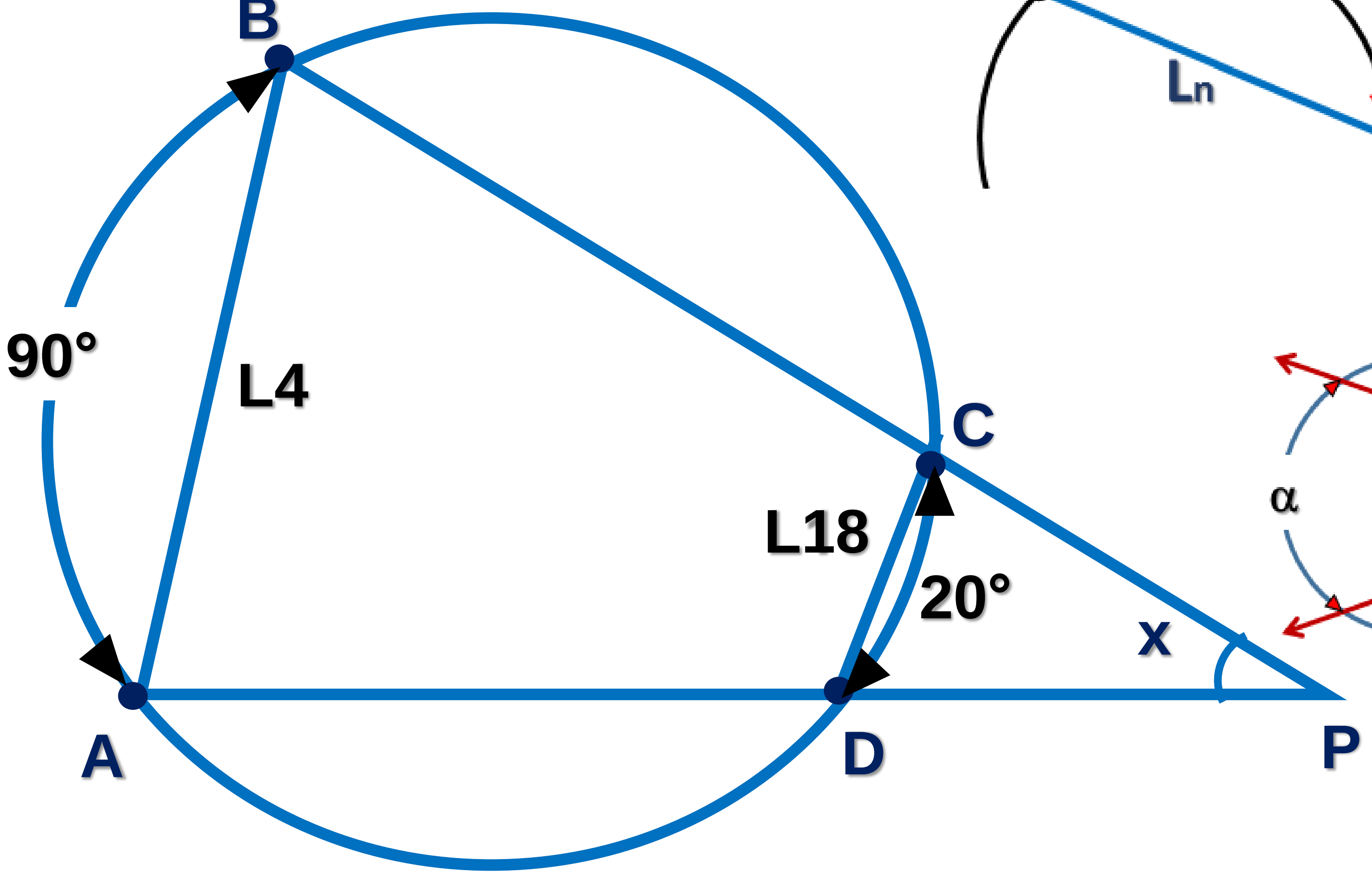


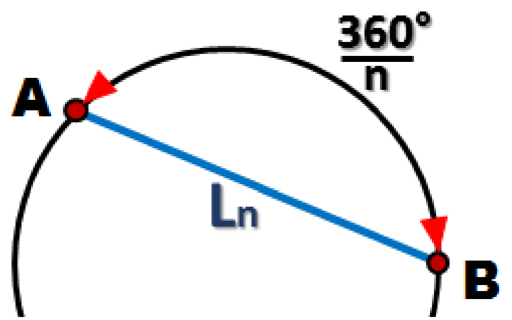
$$x + \beta = 180°$$

$$x + 120° = 180°$$

$$x = 60°$$

3. Halle el valor de x, si $AB = L_4$ y $CD = L_{18}$.





$n = 4$

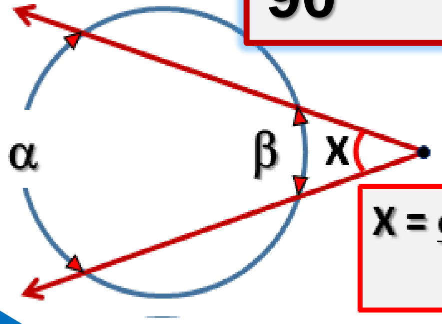
$m\overset{\frown}{AB} = \dfrac{360°}{4}$

$\boxed{m\overset{\frown}{AB} = 90°}$

$n = 18$

$m\overset{\frown}{CD} = \dfrac{360°}{18}$

$\boxed{m\overset{\frown}{CD} = 20°}$



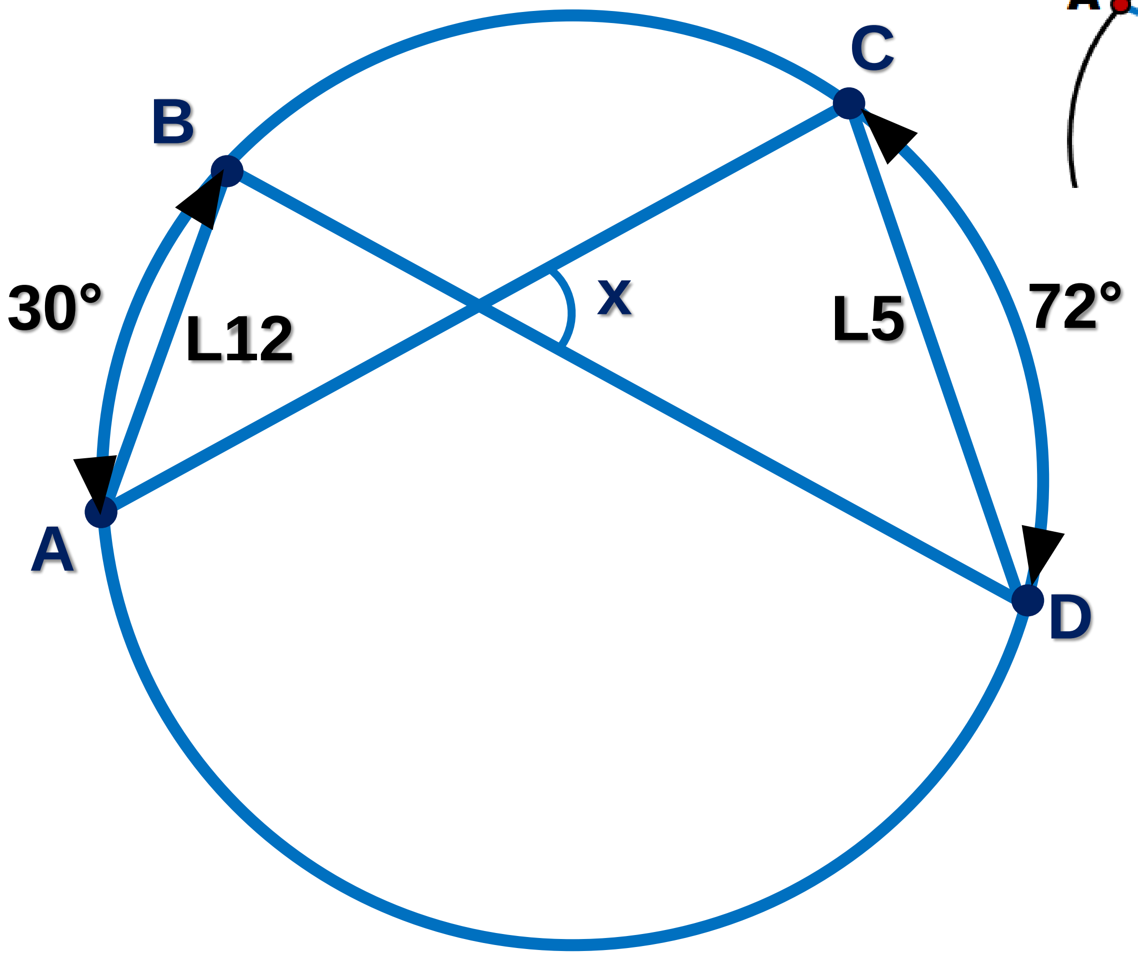
$\boxed{x = \dfrac{\alpha - \beta}{2}}$

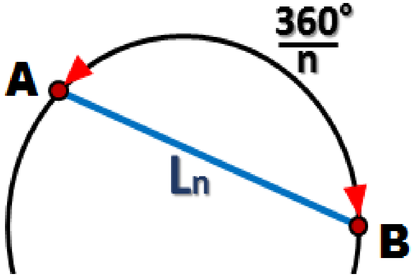
Ángulo exterior

$x = \dfrac{90° - 20°}{2}$

$\boxed{x = 35°}$

4. Halle el valor de x, si AB = L12 y CD = L5.





$\boxed{n = 12}$

$m\overset{\frown}{AB} = \frac{360°}{12}$

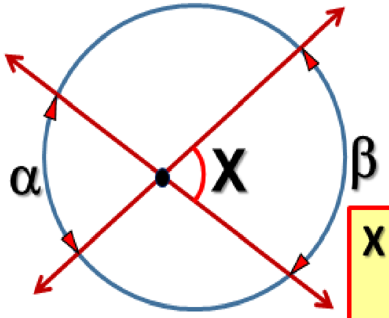
$\boxed{m\overset{\frown}{AB} = 30°}$

$\boxed{n = 5}$

$m\overset{\frown}{CD} = \frac{360°}{5}$

$\boxed{m\overset{\frown}{CD} = 72°}$

Ángulo interior

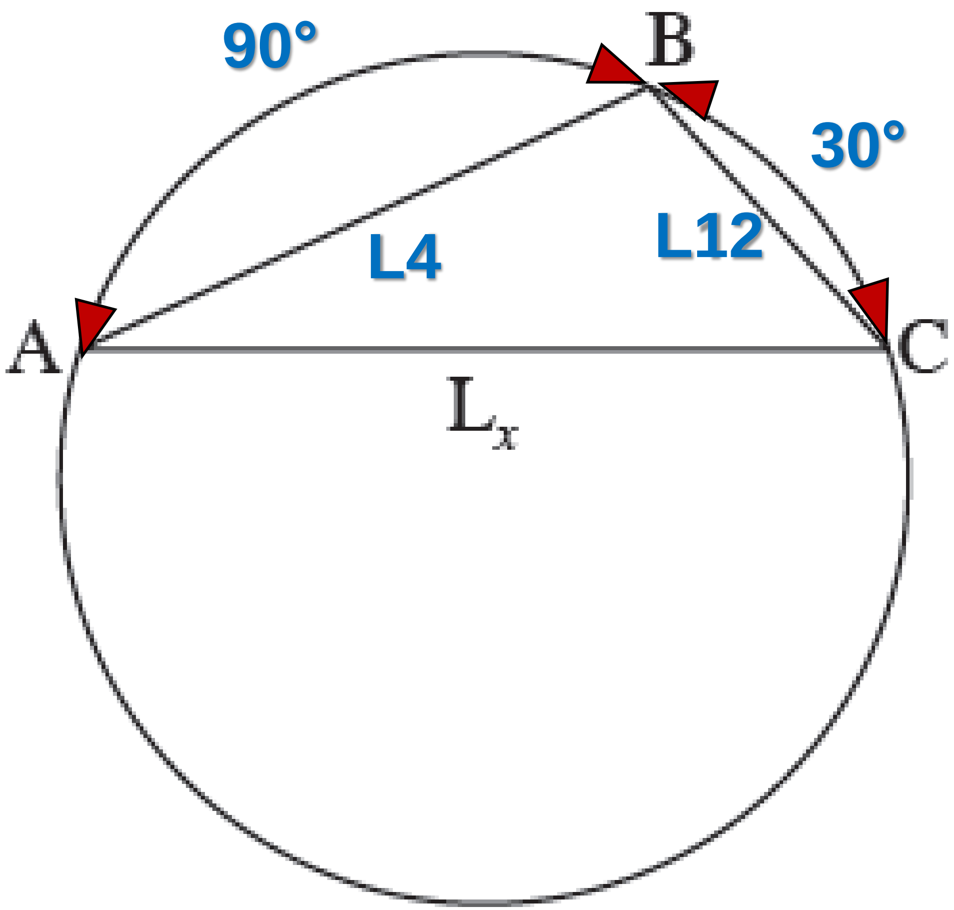


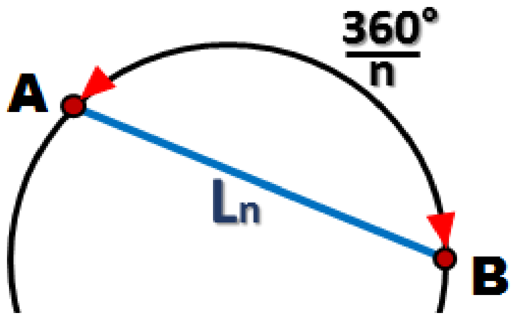
$X = \frac{\alpha+\beta}{2}$

$x = \frac{30° + 72°}{2}$

$\boxed{x = 51°}$

5. Halle el valor de x, si AB = L4 y BC = L12.





$$m\overset{\frown}{AB} = \frac{360°}{n}$$

n = 4

$$m\overset{\frown}{AB} = \frac{360°}{4}$$

$$m\overset{\frown}{AB} = 90°$$

n = 12

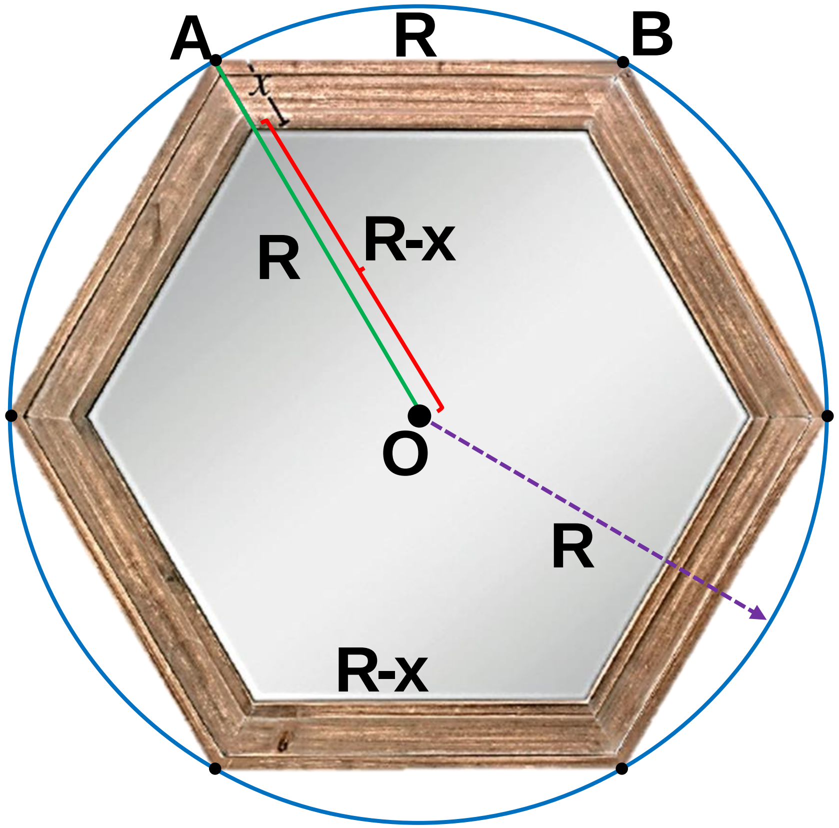
$$m\overset{\frown}{BC} = \frac{360°}{12}$$

$$m\overset{\frown}{BC} = 30°$$

$$120° = \frac{360°}{x}$$

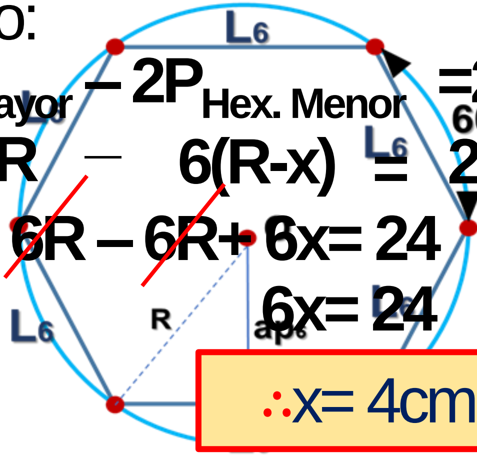
$$x = 3$$

6. **En la figura se muestra el marco de un espejo, el cual tiene en sus bordes interior y exterior a dos hexágonos regulares de lados paralelos. Si la diferencia entre los perímetros de dichos hexágonos es 24 cm; calcule el valor de x.**



RESOLUCIÓN

- Sea **O** el centro de los hexágonos regulares.
  R: longitud del circunradio del hexágono mayor
  R-x: longitud del circunradio del hexágono menor
- Por propiedad del hexágono regular.
- Por dato:



$$2P_{\text{Hex. Mayor}} - 2P_{\text{Hex. Menor}} = 24$$

$$6R - 6(R-x) = 24$$

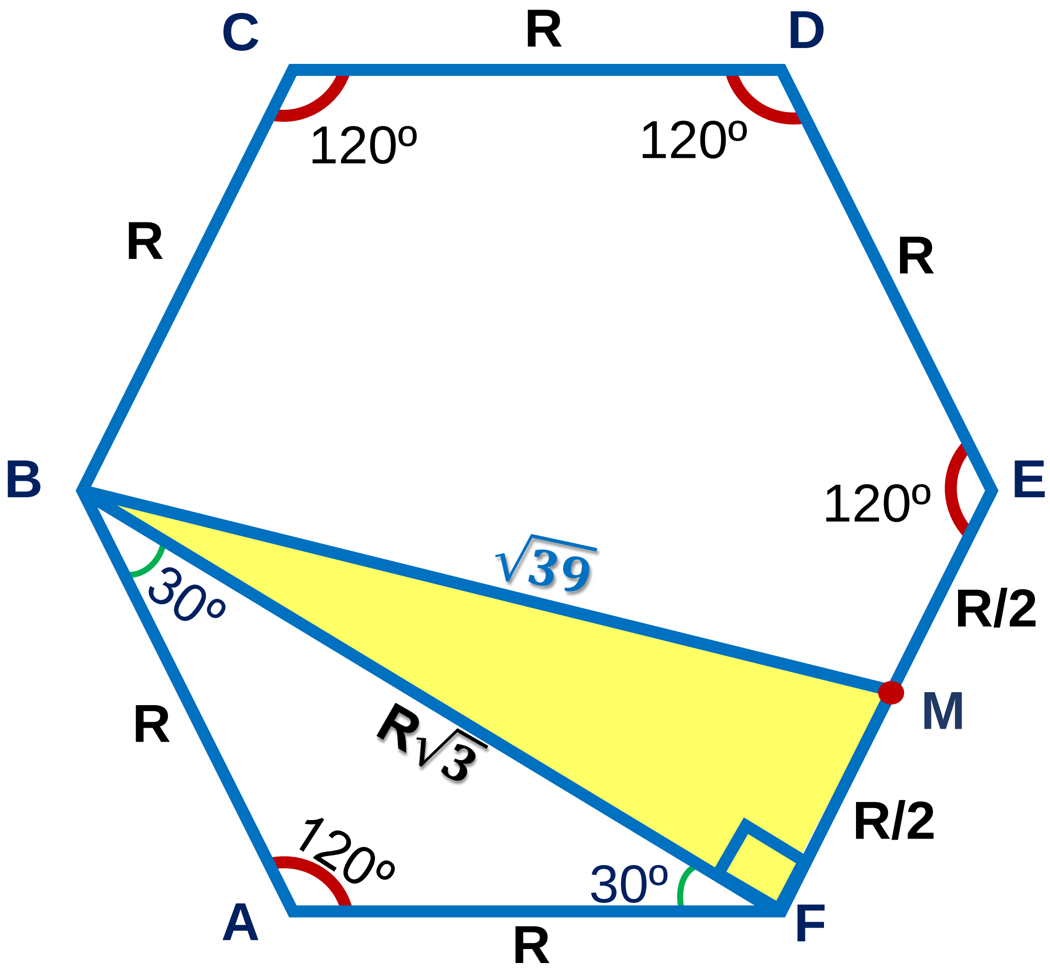
$$\cancel{6R} - \cancel{6R} + 6x = 24$$

$$6x = 24$$

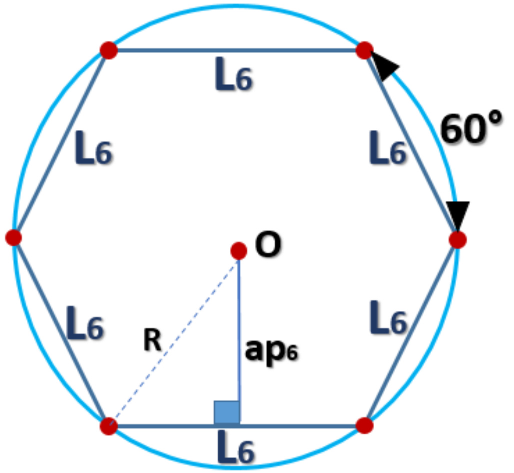
$$\boxed{L_6 = R}$$

$$\boxed{\therefore x = 4cm}$$

7. Una de las plazas de la ciudad de Chiclayo tiene en su contorno a un hexágono regular ABCDEF. Si M es punto medio de $\overline{FE}$ y $BM = \sqrt{39}$. Calcular la longitud de la apotema de dicho hexágono.

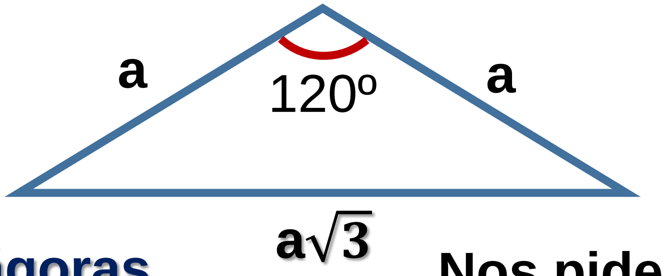


HEXÁGONO REGULAR



$L6 = R$

$Ap6 = \dfrac{R\sqrt{3}}{2}$



BFM : T. Pitágoras

$(\sqrt{39})^2 = (R\sqrt{3})^2 + (R/2)^2$

$39 = 3R^2 + R^2/4$

$39 = 13R^2/4$

$12 = R^2$

$2\sqrt{3} = R$

Nos piden

$Ap6 = \dfrac{R\sqrt{3}}{2}$

$Ap6 = \dfrac{(\cancel{2}\sqrt{3})\sqrt{3}}{\cancel{2}}$

$\boxed{Ap6 = 3}$