GEOMETRÍA

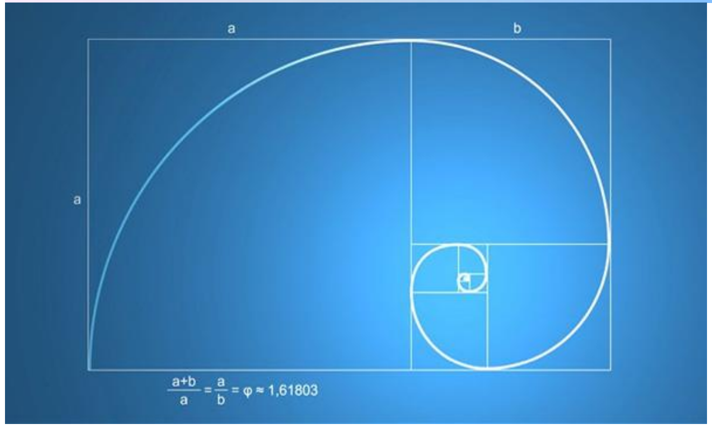
Capítulo 7

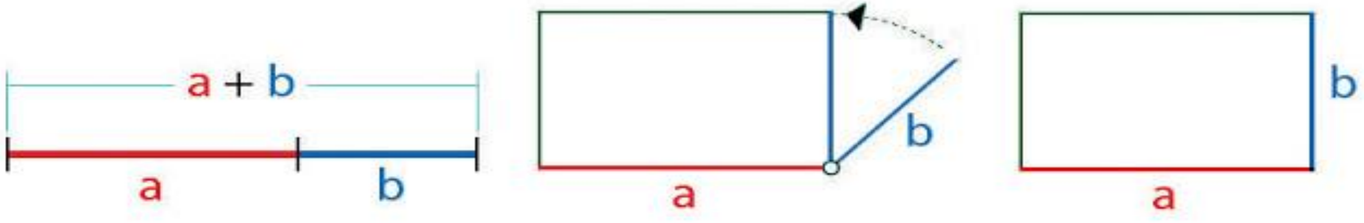
5th
SECONDARY

SEGMENTOS PROPORCIONALES

SACO OLIVEROS

MOTIVATING | STRATEGY





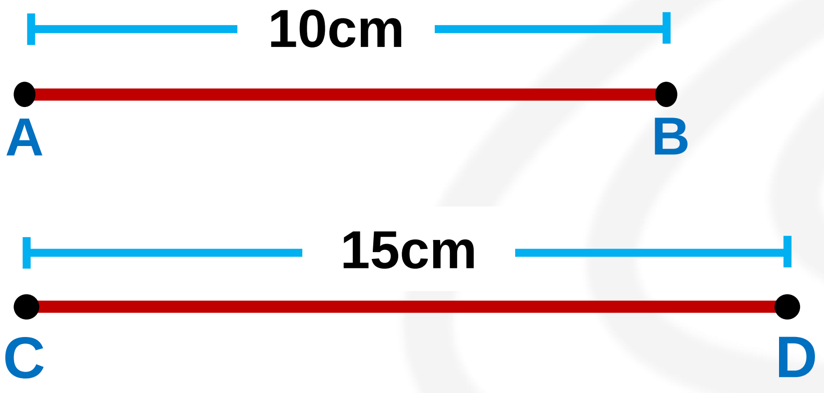
$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\ldots$$

SEGMENTOS PROPORCIONALES

Razón geométrica de dos segmentos

Es el cociente que se obtiene al dividir las longitudes de dos segmentos que tienen la misma unidad de medida.

Ejemplo:

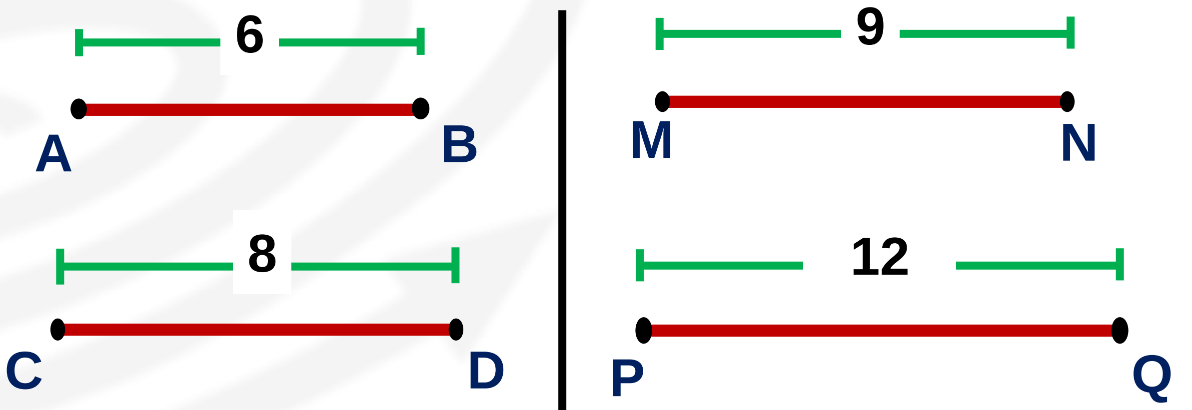


$$\frac{AB}{CD} = \frac{\cancel{10}cm}{\cancel{15}cm} \Rightarrow \frac{AB}{CD} = \frac{2}{3}$$

$\frac{2}{3}$: razón geométrica de $\overline{AB}$ y $\overline{CD}$

Segmentos proporcionales

Si la razón geométrica de 2 segmentos es igual a la de otros dos, dichos pares de segmentos son proporcionales.

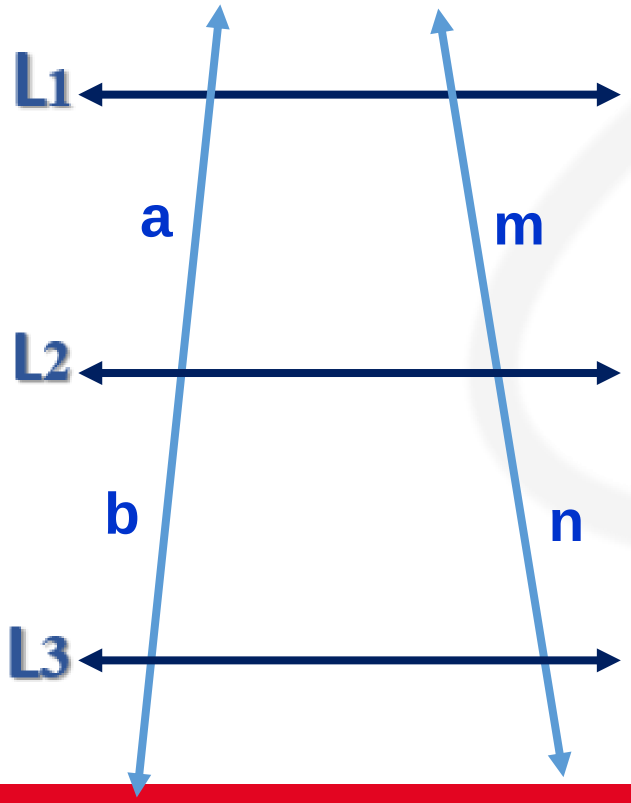


$$\frac{AB}{CD} = \frac{\cancel{6}}{\cancel{8}} = \boxed{\frac{3}{4}} \qquad \frac{MN}{PQ} = \frac{\cancel{9}}{\cancel{12}} = \boxed{\frac{3}{4}}$$

$$\boxed{\frac{AB}{CD} = \frac{MN}{PQ}} \Rightarrow$$ Son proporcionales

Teorema de Tales

Tres o más rectas paralelas determinan sobre dos rectas transversales o secantes a ellas segmentos cuyas longitudes son, respectivamente, proporcionales.

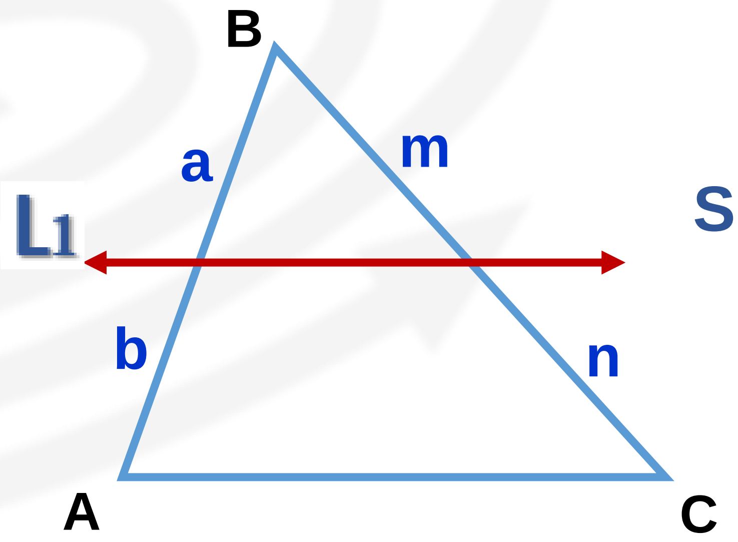


Si: $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2} \parallel \overleftrightarrow{L_3}$

$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

Corolario de Tales

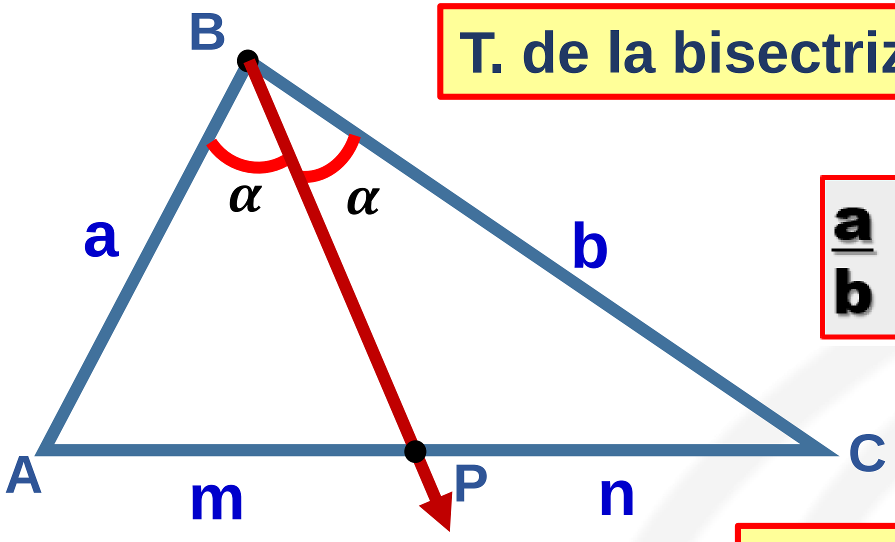
Toda recta paralela a un lado de un triángulo divide interna o externamente a los otros lados en segmentos proporcionales.



Si: $\overleftrightarrow{L_1} \parallel \overline{AC}$

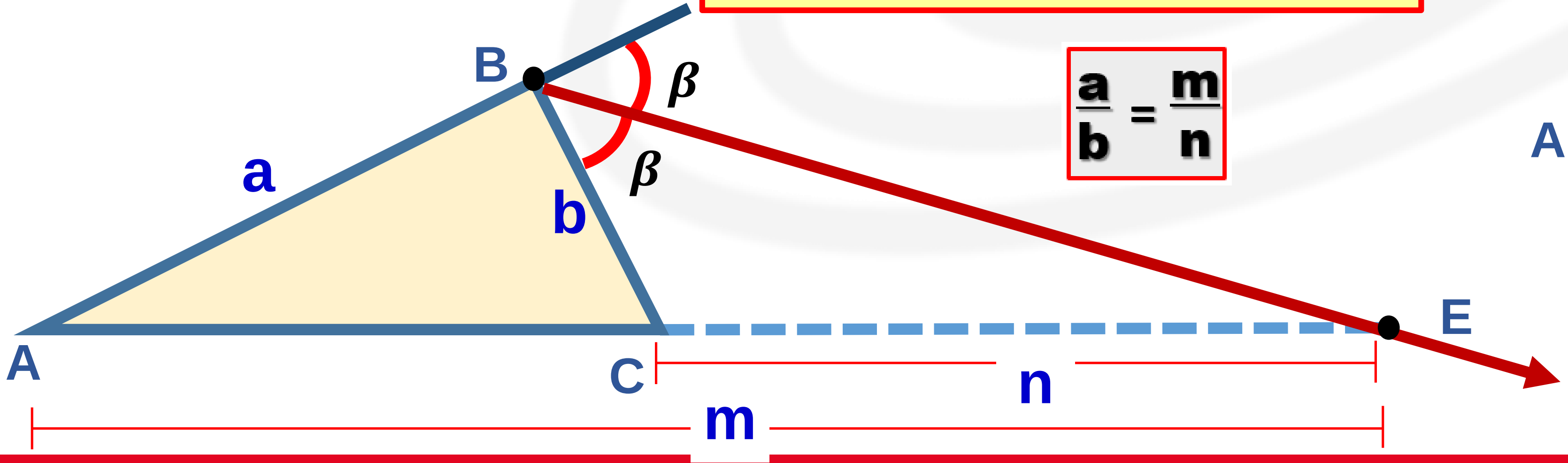
$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

Teorema de la bisectriz



T. de la bisectriz interior

$$\frac{a}{b} = \frac{m}{n}$$

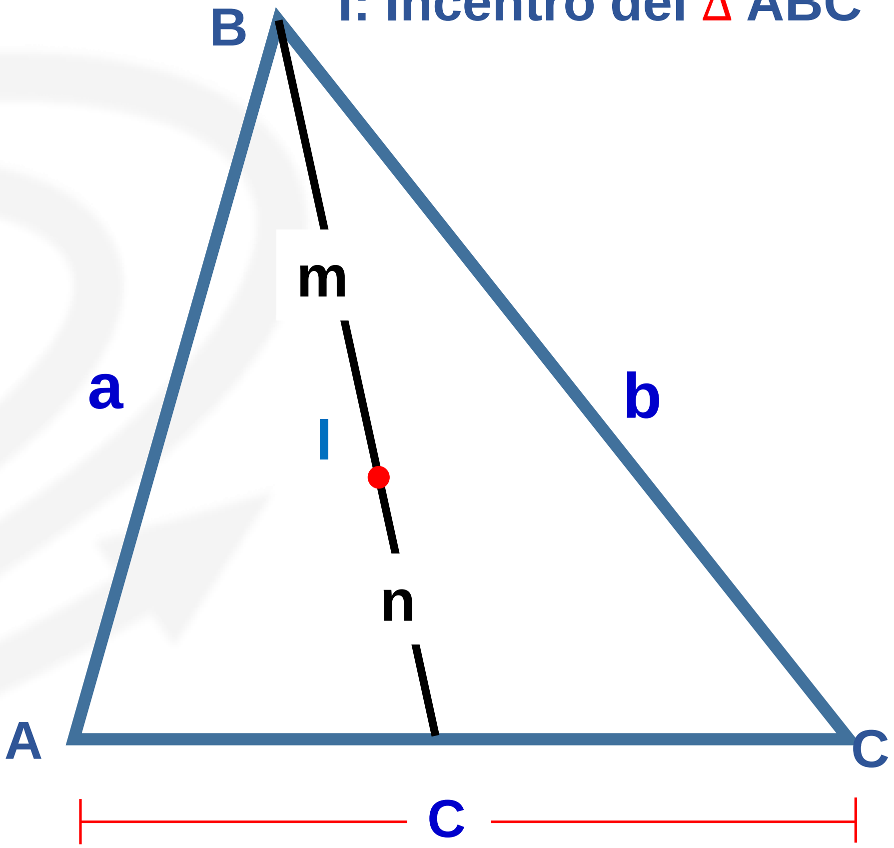


T. de la bisectriz exterior

$$\frac{a}{b} = \frac{m}{n}$$

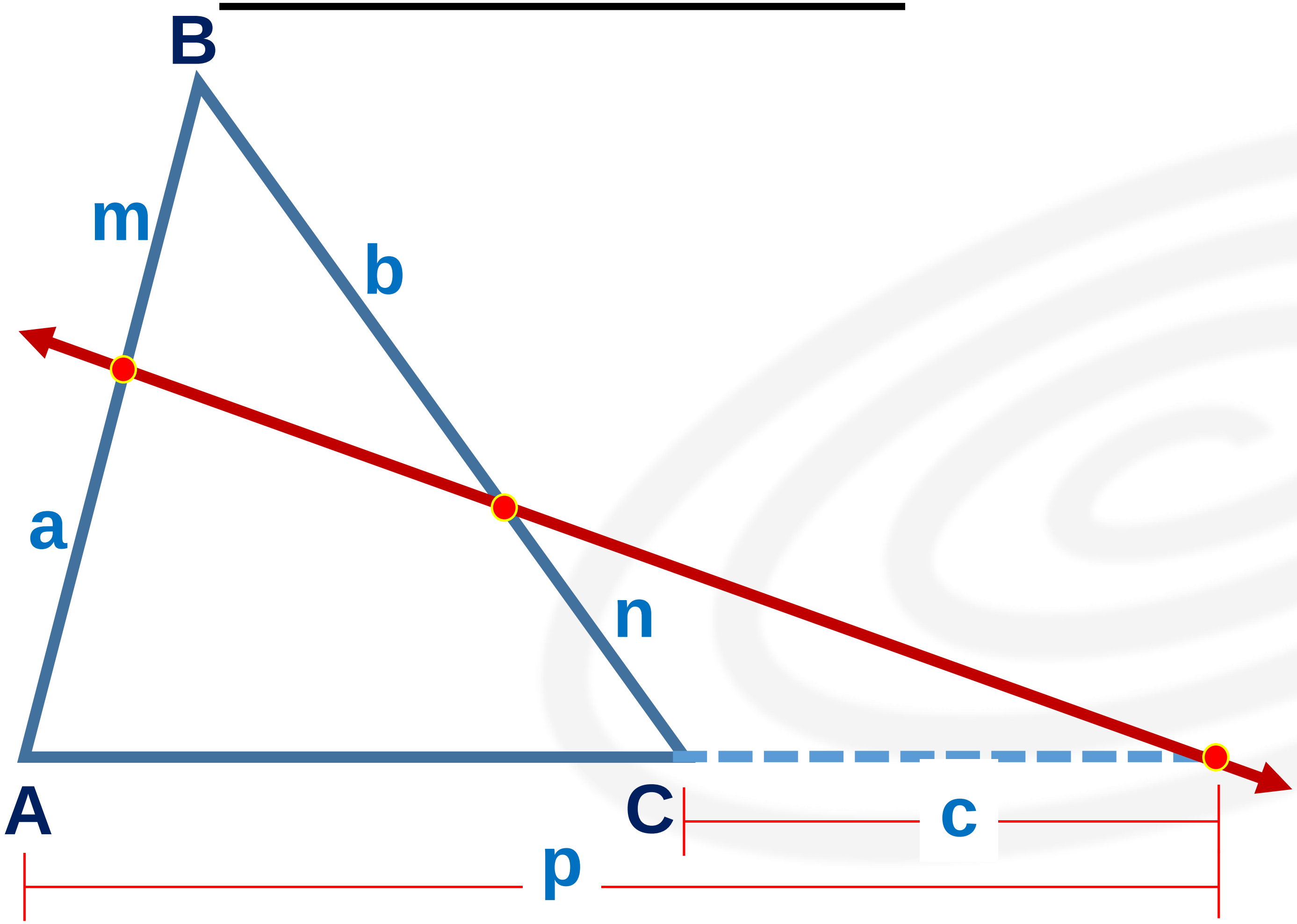
Teorema del incentro

I: Incentro del Δ ABC



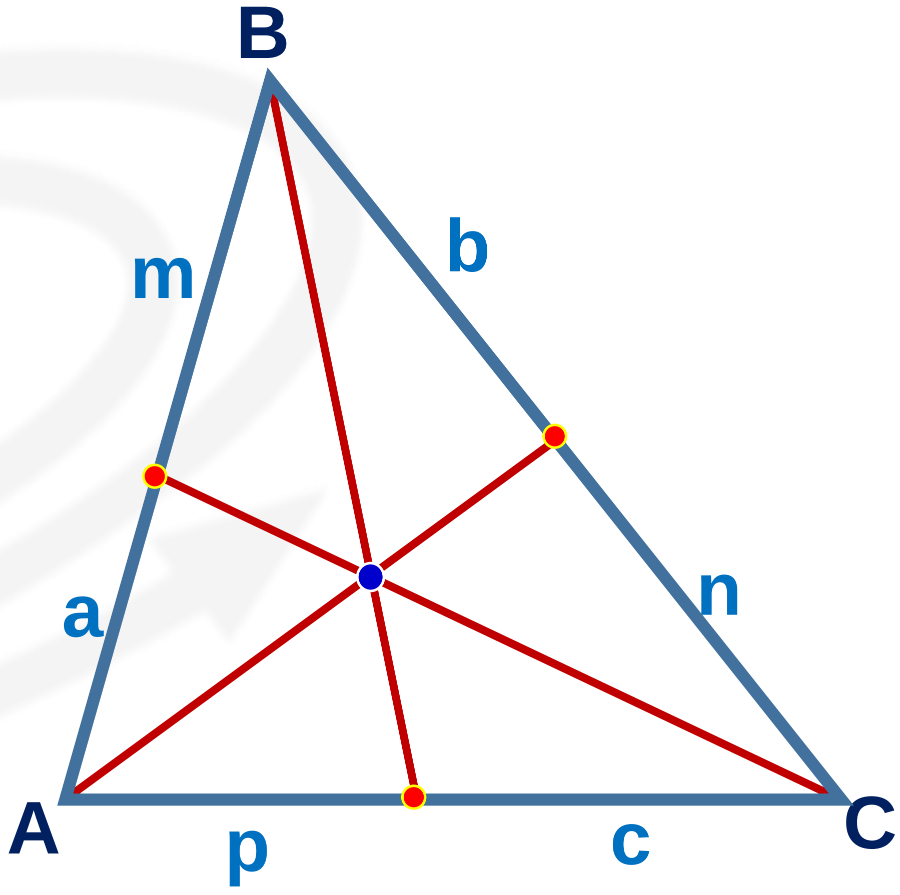
$$\frac{m}{n} = \frac{a+b}{c}$$

Teorema de Menelao



$$a \,.\, b \,.\, c = m \,.\, n \,.\, p$$

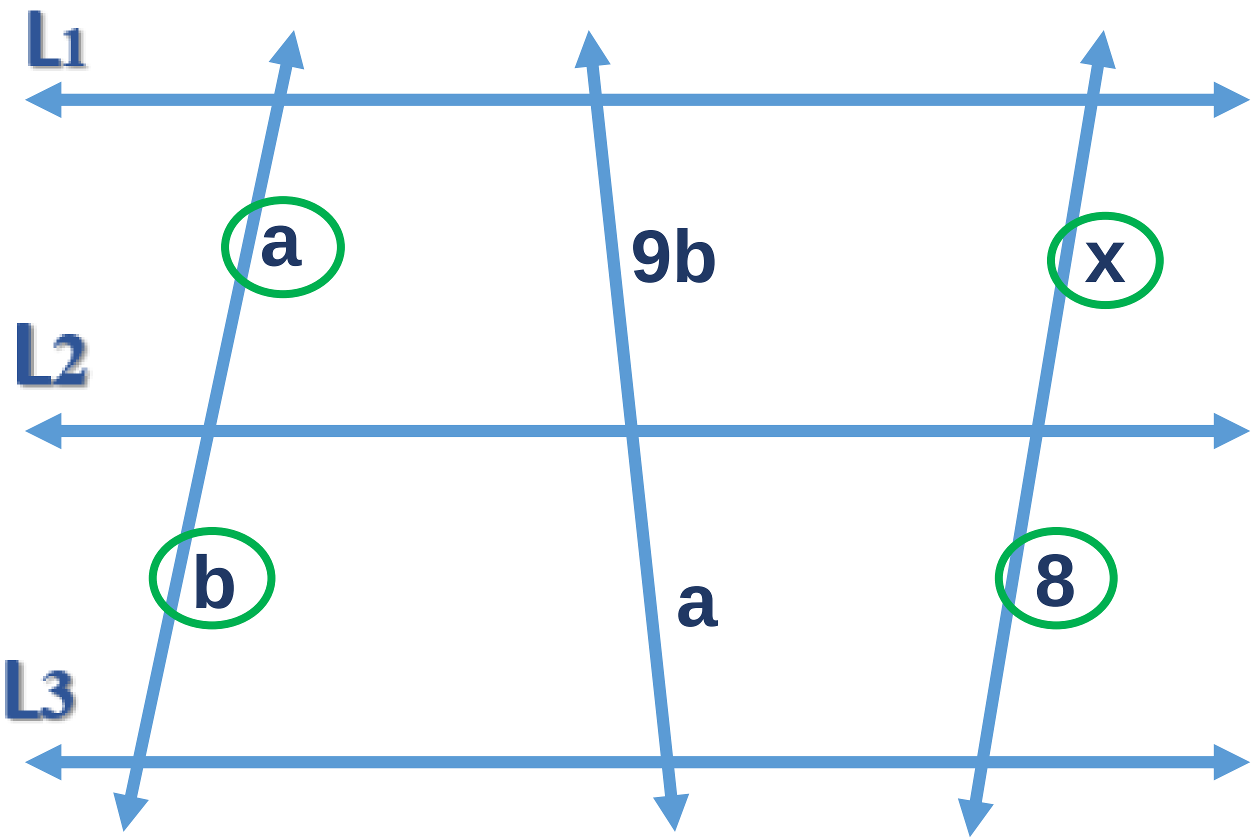
Teorema de Ceva



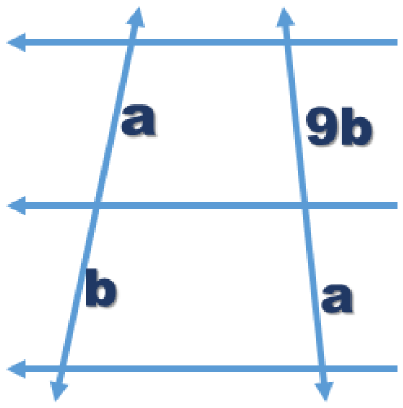
$$a \,.\, b \,.\, c = m \,.\, n \,.\, p$$

1. Halle el valor de x, si $\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2} \,//\, \overleftrightarrow{L_3}$.

Resolución:



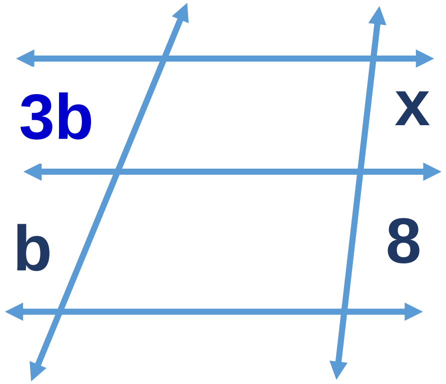
- **Piden:** x
- **Aplicamos teorema de Tales**



$$\frac{a}{b} = \frac{9b}{a}$$

$$a^2 = 9b^2$$

$$\boxed{a = 3b}$$

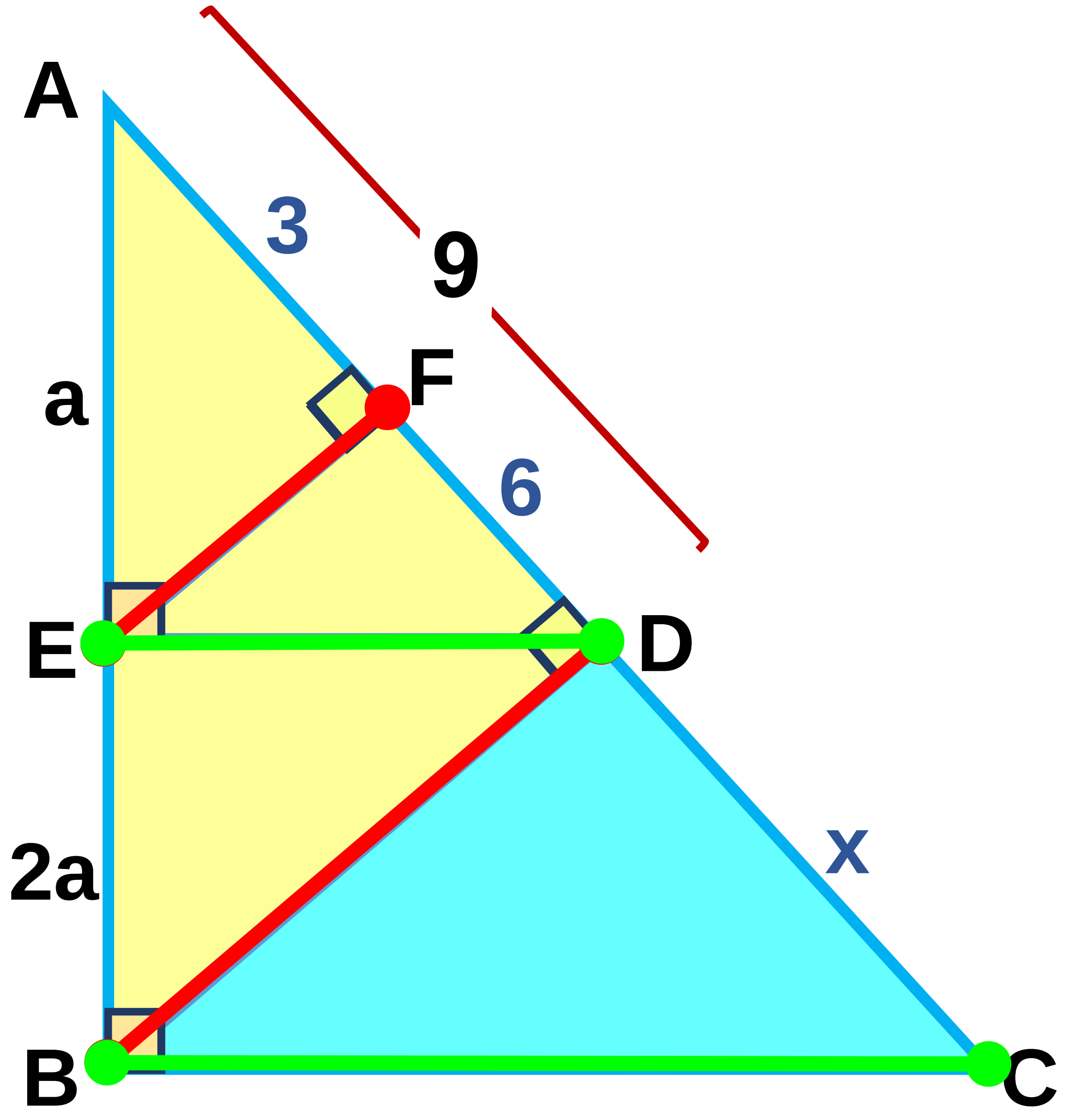


$$\frac{3\cancel{b}}{\cancel{b}} = \frac{x}{8}$$

$$3(8) = x$$

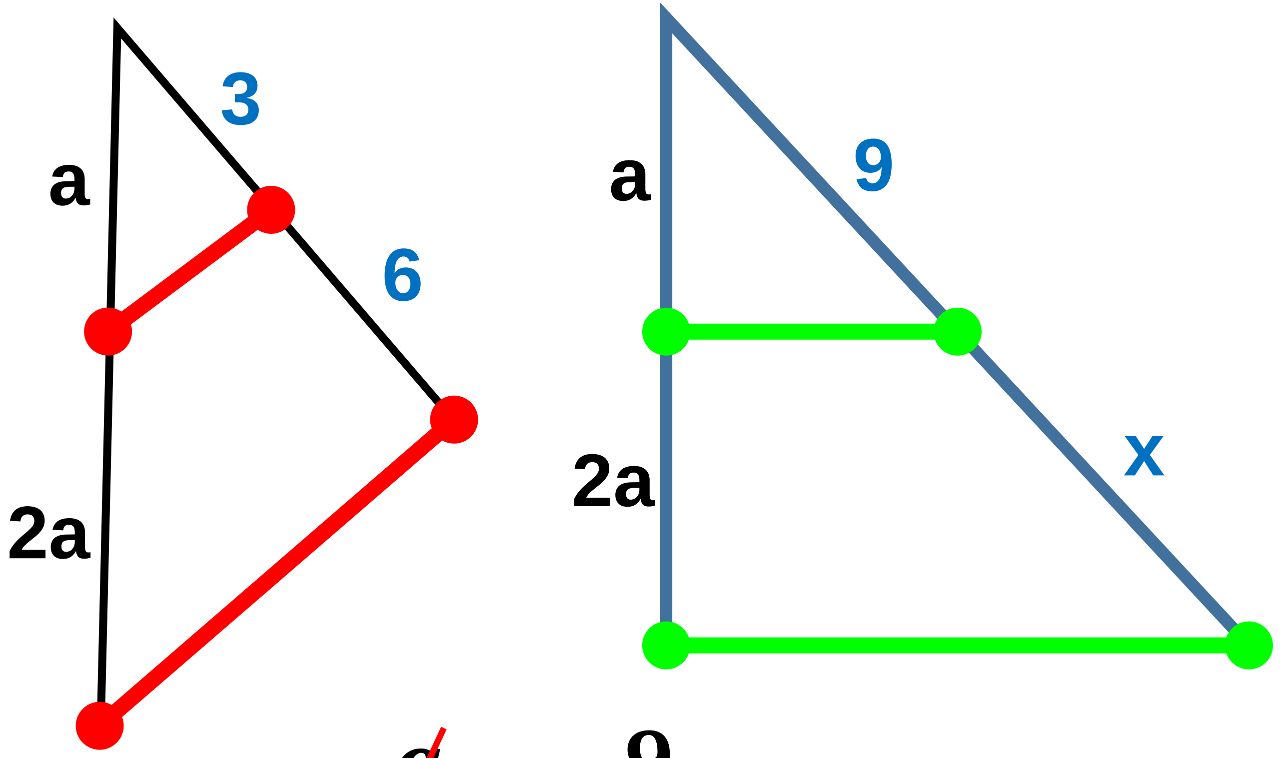
$$\boxed{x = 24}$$

2. Halle el valor de x.



Resolución

- **Piden: x**
- **Aplicamos el corolario de Tales**

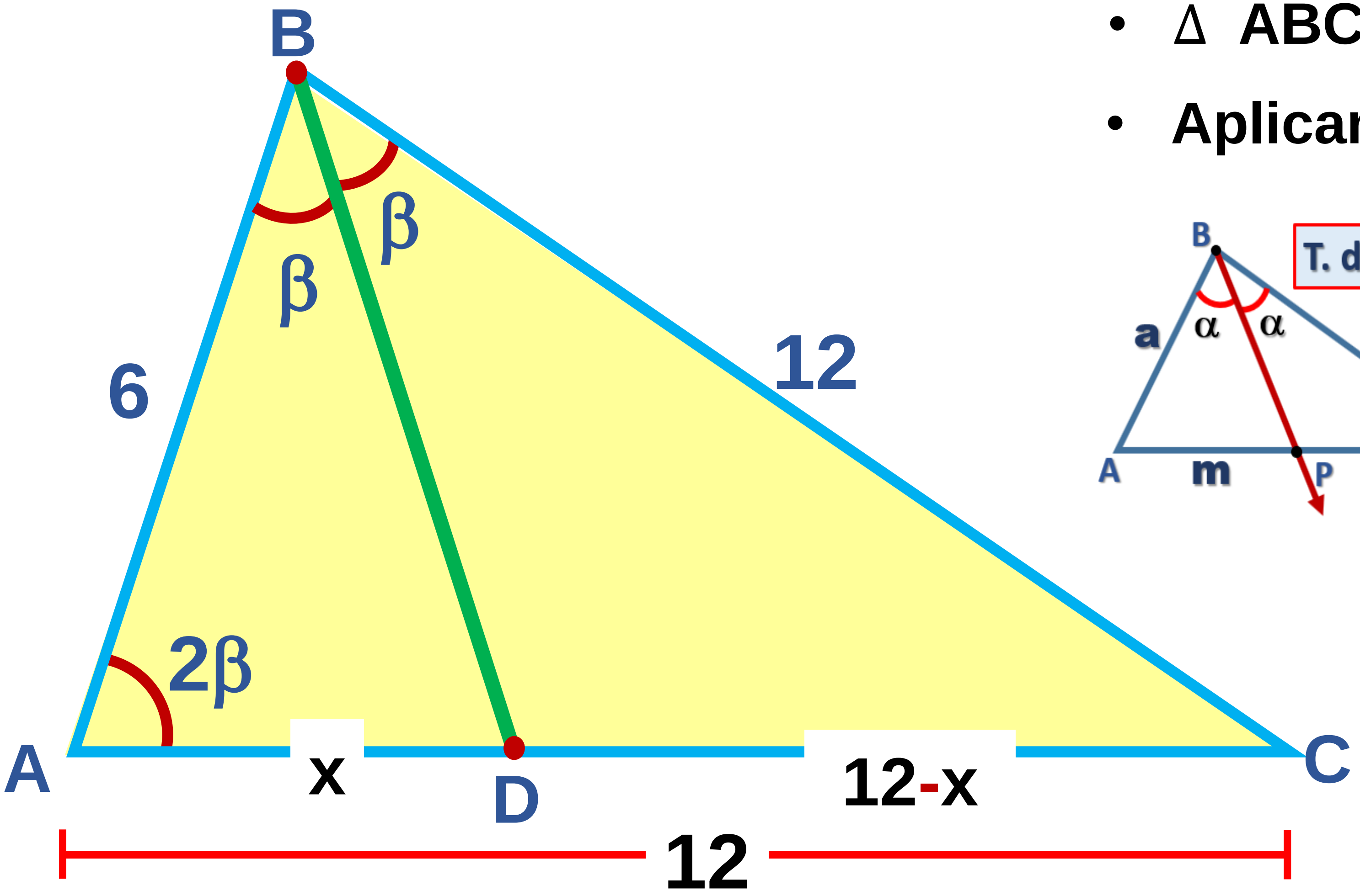


$$\frac{\cancel{a}}{2\cancel{a}} = \frac{9}{x}$$

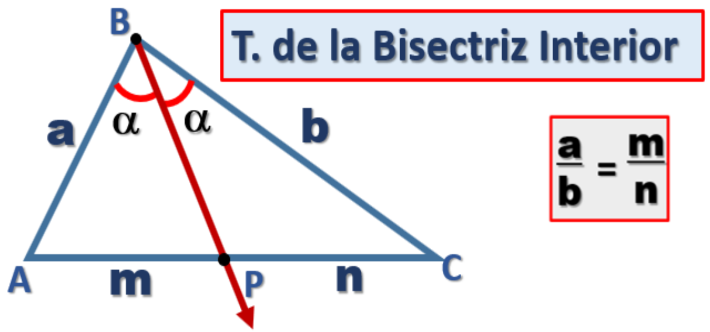
$$\boxed{x = 18u}$$

3. En un triángulo ABC, donde AB = 6 y BC = 12, se traza la bisectriz interior BD. Halle AD, si m∡BAD = m∡ABC.

Resolución



- **Piden:** AD = x
- Δ ABC: Isósceles BC = AC = 12
- Aplicamos el teorema



$$\frac{a}{b} = \frac{m}{n}$$

$$\frac{\cancel{6}^{\,1}}{\cancel{12}_{\,2}} = \frac{x}{12-x}$$

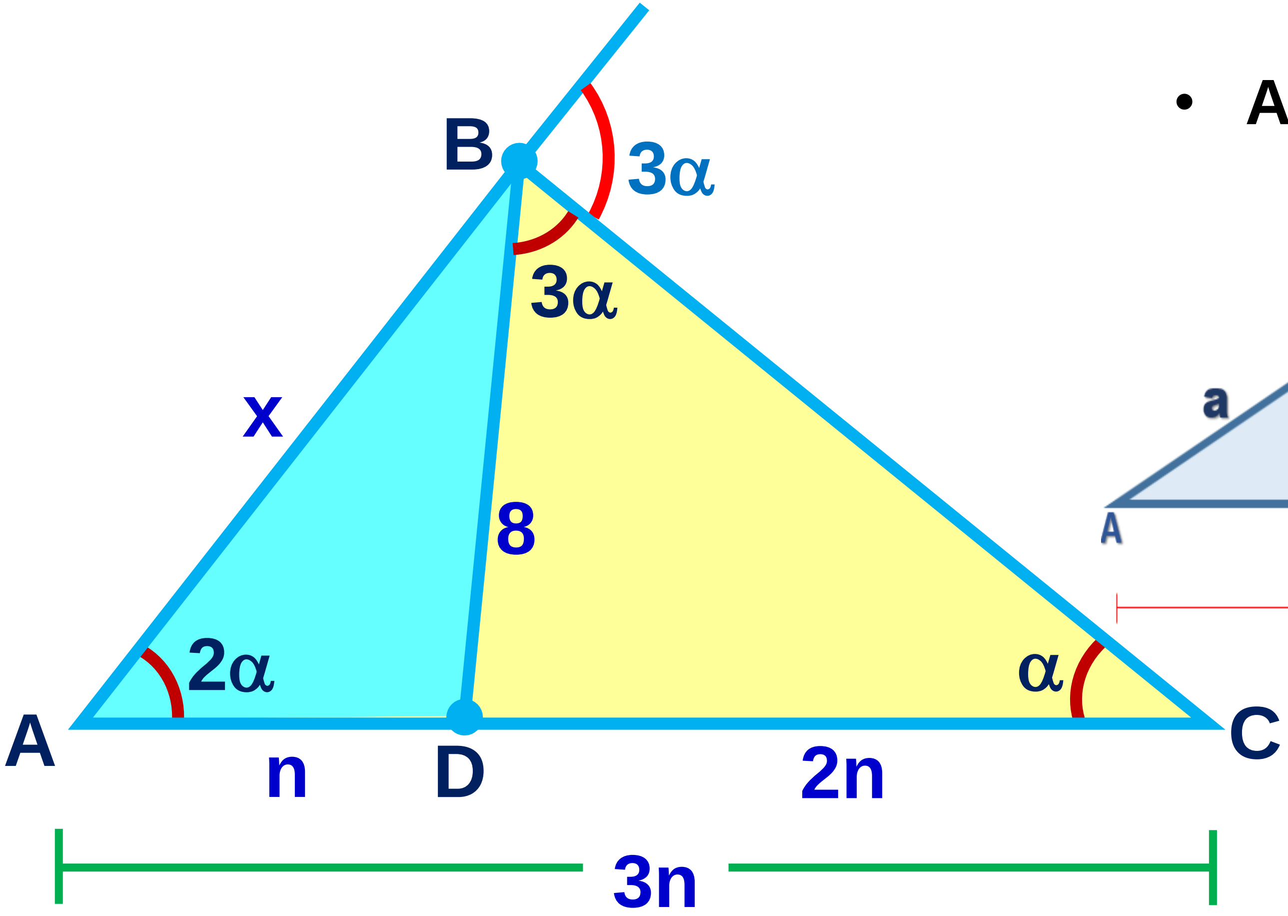
$$12 - x = 2x$$

$$12 = 3x$$

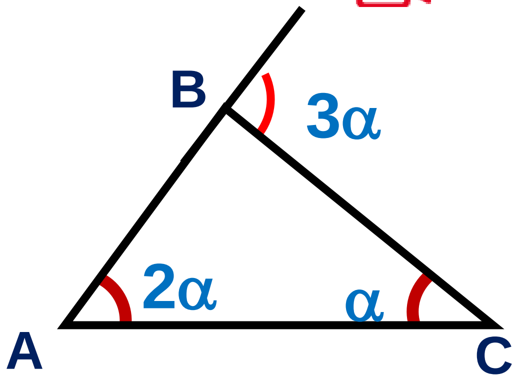
$$\boxed{x = 4}$$

4. Halle el valor de x.

Resolución

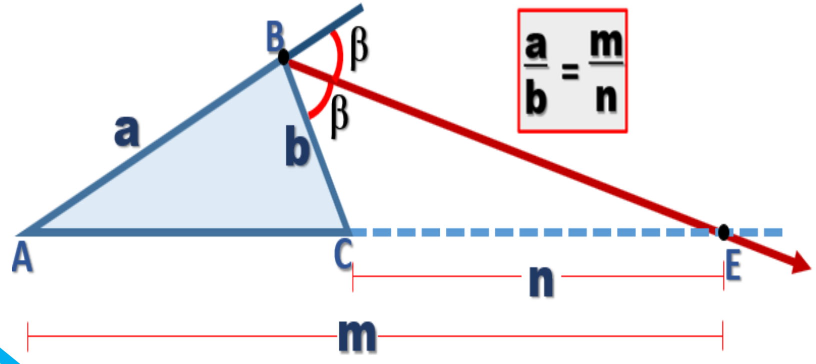


- **Piden: x**
- **Δ ABC: ángulo externo**



- **Aplicamos el teorema**

T. de la Bisectriz Exterior



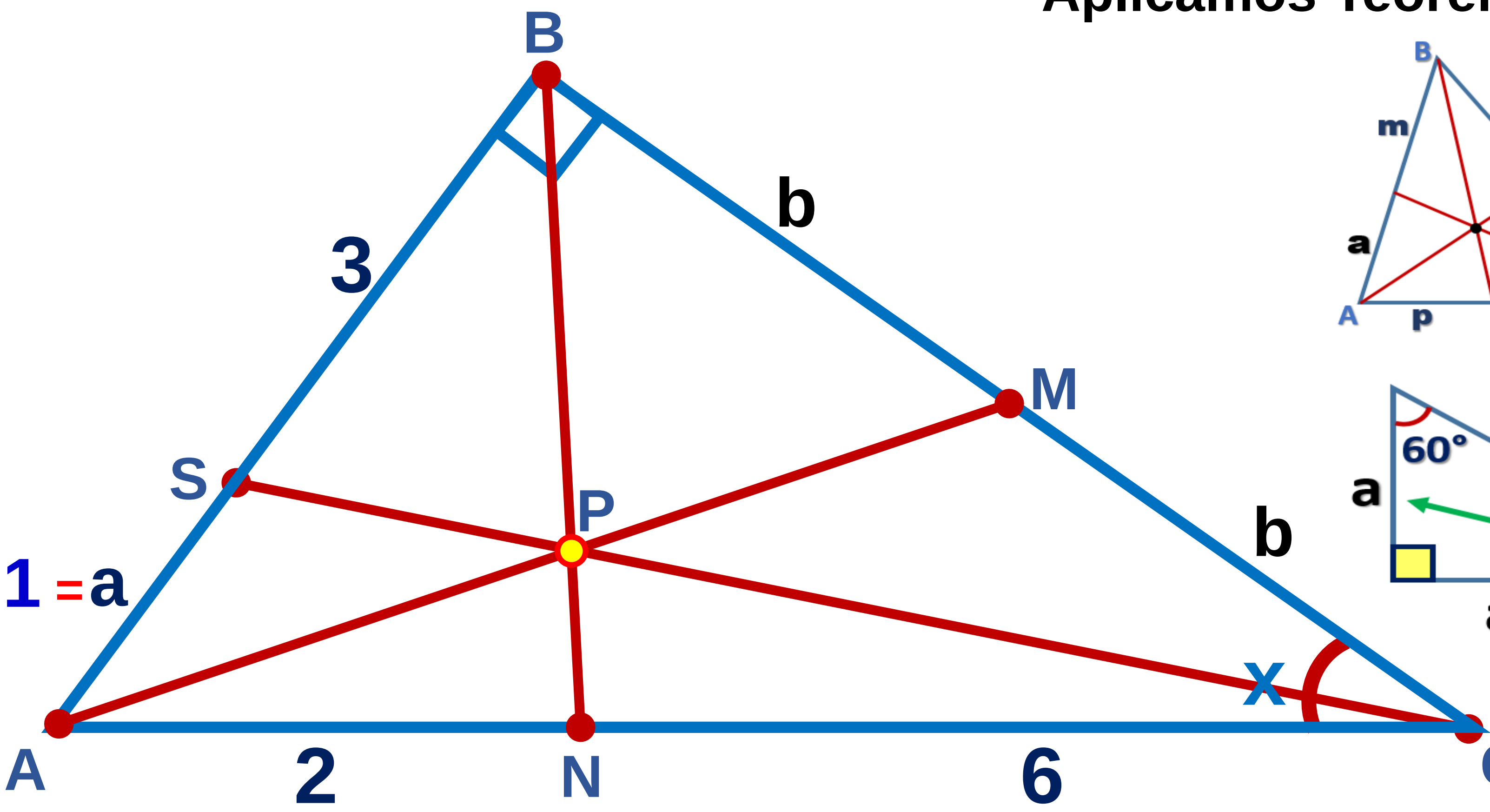
$$\frac{x}{8} = \frac{3\cancel{n}}{2\cancel{n}}$$

$$2x = 24$$

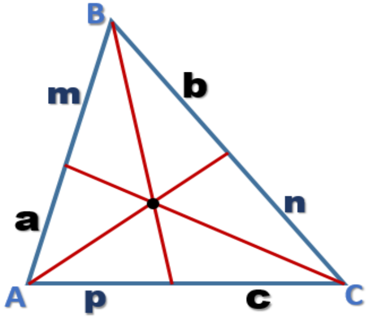
$$\boxed{x = 12u}$$

5. En un triángulo rectángulo ABC, recto en B, la mediana $\overline{AM}$ y las cevianas interiores $\overline{BN}$ y $\overline{CS}$ se intersecan en P. Si SB = 3, AN = 2 y NC = 6, halle m∡BCA.

Resolución

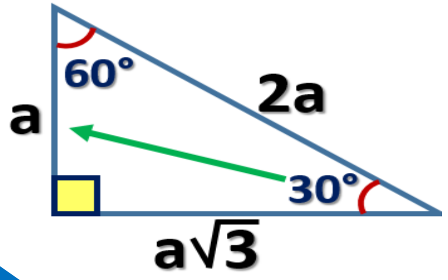


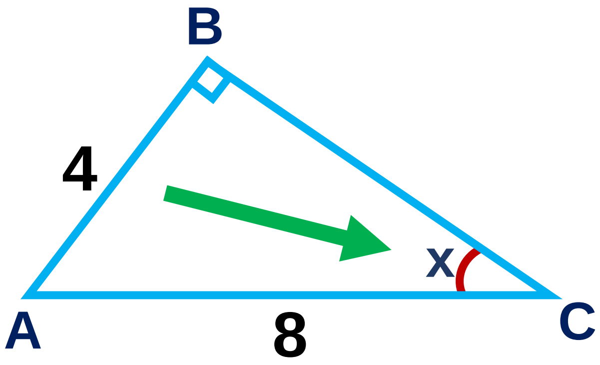
- **Piden:** m∡BCA = x
- **Aplicamos Teorema de Ceva** $\boxed{a \cdot b \cdot c = m \cdot n \cdot p}$



$(a)(\cancel{b})(6) = (3)(\cancel{b})(2)$

$\boxed{a = 1}$



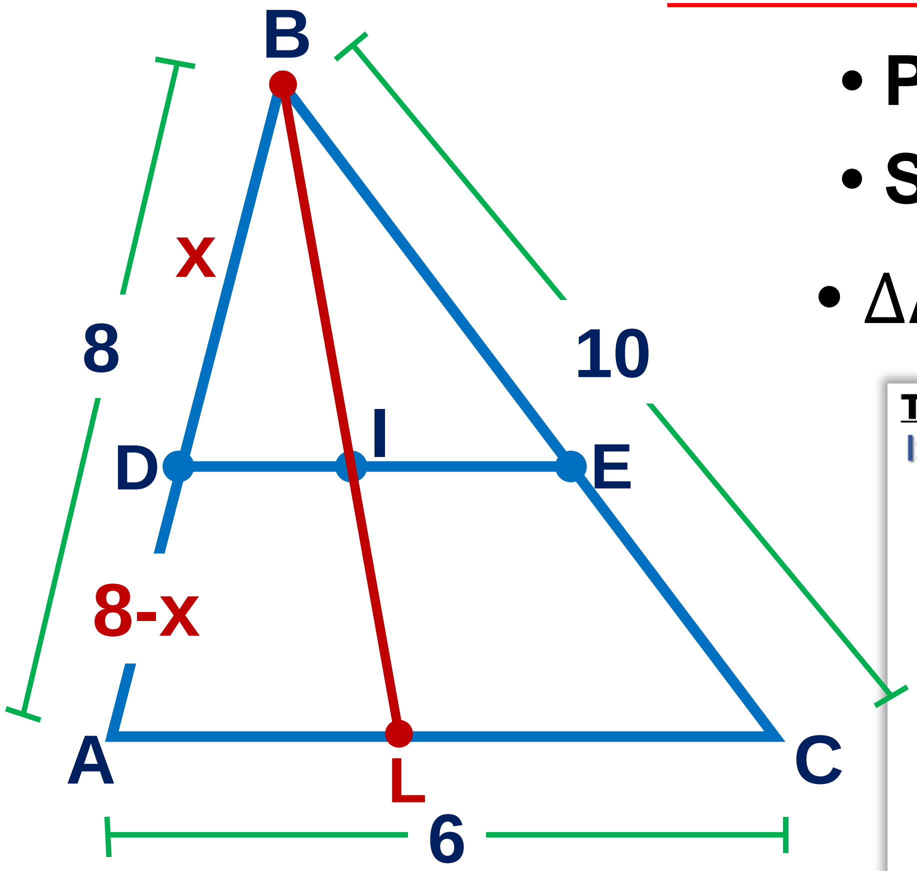


- ⊿ABC: notable de 30° y 60°

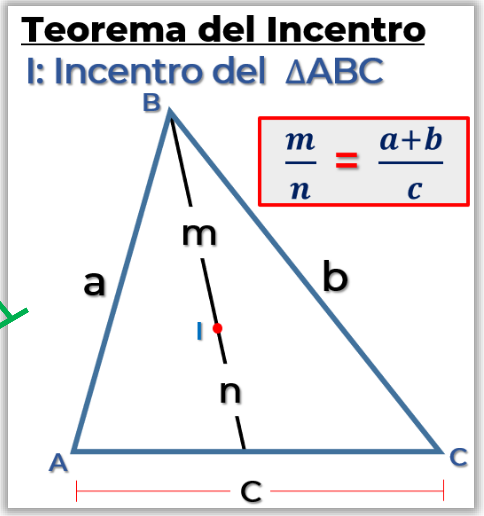
$\boxed{x = 30°}$

6. En la figura el triángulo ABC representa el contorno de un jardín donde I es su incentro. AB=8m, BC=10m y AC=6m. Luego se traza el $\overline{DE} \parallel \overline{AC}$ para dividir al jardín en dos partes para cultivar flores de diferente color. Halle la longitud del $\overline{BD}$.

Resolución



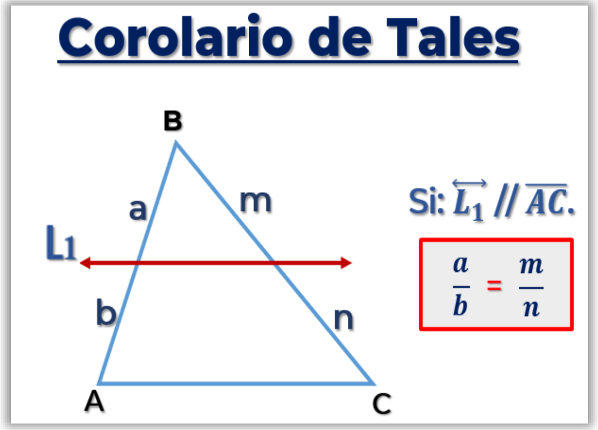
- Piden: BD
- Se traza $\overline{BL}$
- ΔABC: aplicamos



$$\frac{BI}{IL} = \frac{8+10}{6}$$

$$\frac{BI}{IL} = 3$$

- ΔABL: aplicamos

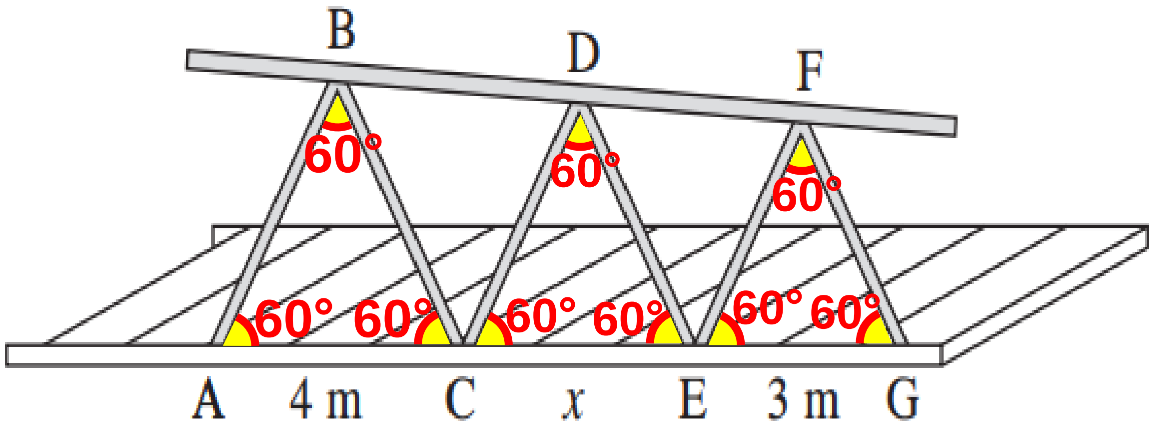


$$\frac{BI}{IL} = \frac{x}{8-x}$$

$$3 = \frac{x}{8-x}$$

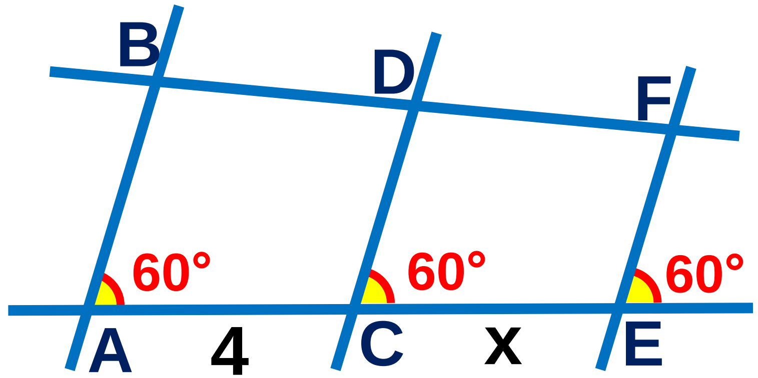
$$\boxed{x = 6m}$$

7. La estructura de un puente tiene el diseño que se muestra en la figura, tal que los triángulos ABC, CDE y EFG son equiláteros. Halle el valor de x.



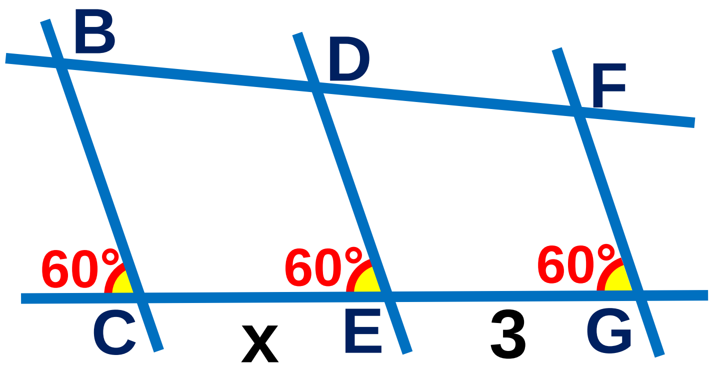
Resolución

$\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ (Áng. correspondientes)

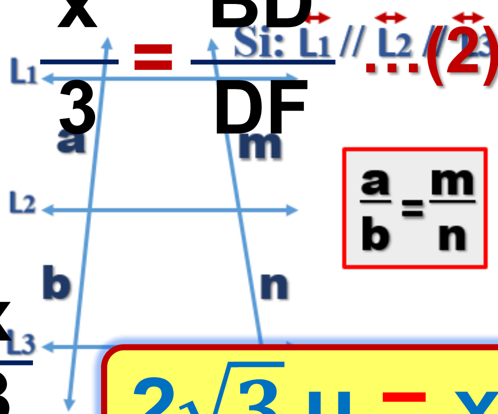


$$\frac{4}{x} = \frac{BD}{DF} \quad \ldots(1)$$

$\overline{BC} \parallel \overline{DE} \parallel \overline{FG}$ (Áng. correspondientes)



Teorema de Tales



$$\frac{x}{3} = \frac{BD}{DF} \quad \ldots(2)$$

Igualando (1) y (2) $\frac{4}{x} = \frac{x}{3}$

$$\boxed{2\sqrt{3}\ u = x}$$