ALGEBRA

5th

of Secondary

RETROALIMENTACION
(tomo 5)

SACO OLIVEROS

PROBLEMA 1

Calcular el mayor valor entero para x , que satisface

$$4 \leq \frac{3x+1}{x-2} < 7$$

Resolución

$$4 \leq \frac{3(x-2)+7}{x-2} < 7$$

$$4 \leq 3 + \frac{7}{x-2} < 7$$

$$1 \leq \frac{7}{x-2} < 4$$

$$\frac{1}{4} < \frac{x-2}{7} \leq 1$$

$$\frac{7}{4} < x-2 \leq 7$$

$$\frac{15}{4} < x \leq 9$$

$$rpta: 9$$

PROBLEMA 2

Calcular el mayor valor entero para m, que satisface

$$x^2 - 8x + 3 > m, \qquad \forall x \in R$$

Resolución

$$x^2 - 8x + 3 - m > 0$$

$$Teorema\ del\ trinomio\ Positivo$$

$$b^2 - 4ac < 0$$

$$(-8)^2 - 4(1)(3 - m) < 0$$

$$64 - 4(3 - m) < 0$$

$$64 - 12 + 4m < 0$$

$$4m < -52$$

$$m < -13$$

$$Rpta: -14$$

PROBLEMA 3

Resolver

$$(x+6)^2+(x-6)^2 > 26x$$

Resolución

$$\boxed{\begin{array}{c}\textit{Identidad de Legendre}\\ (a+b)^2+(a-b)^2 = 2(a^2+b^2)\end{array}}$$

$$2\left(x^2+36\right) > 26x$$

$$\left(x^2+36\right) > 13x$$

$$x^2-13x+36 > 0$$

$$(x-9)(x-4) > 0$$

$+ \qquad - \qquad +$

$4 \qquad 9$

$$Rpta: < -\infty ; 4 > U < 9 ; \infty >$$

PROBLEMA 4

Resolver

$$\frac{x^2 - x - 6}{x^2 - 8x + 15} \leq 0$$

Resolución

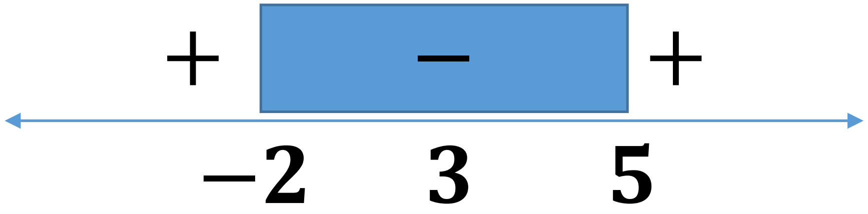
$$factorizamos\ (aspa\ simple)$$

$$\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-5)} \leq 0$$

$$Restricciones:$$

$$x \neq 3\ y x \neq 5$$

$$\frac{x+2}{x-5} \leq 0$$



$$Rpta:$$

$$CS = \ < -2; 5 > \ -\{3\}$$

Problema 5

Resolver:

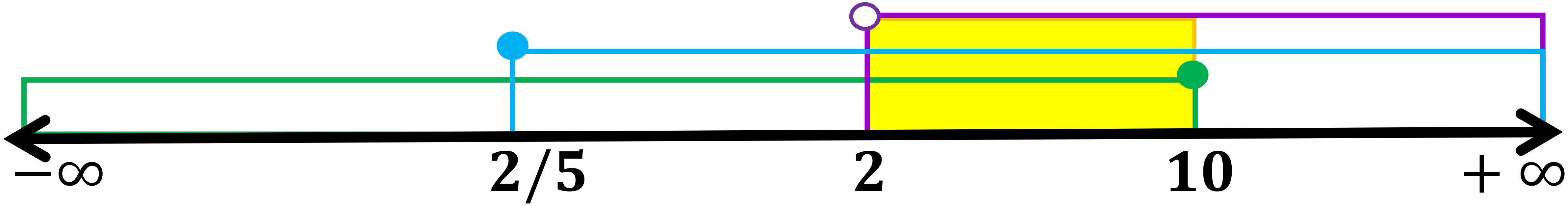
$$\sqrt{5x-2} > \sqrt{10-x}$$

$$+ > +$$

RESTRICCION Y AL CUADRADO

$$5x-2 \geq 0 \;\wedge\; 10-x \geq 0 \quad \wedge \quad 5x-2 > 10-x$$

$$x \geq 2/5 \qquad\qquad x \leq 10 \qquad\qquad x > 2$$



$$C.S. = \langle\, 2;10\,]$$

Problema 6

Resuelva:

$$\left(x^2+3\right)\left(x^3-64x\right)\leq 0$$

Resolucion:

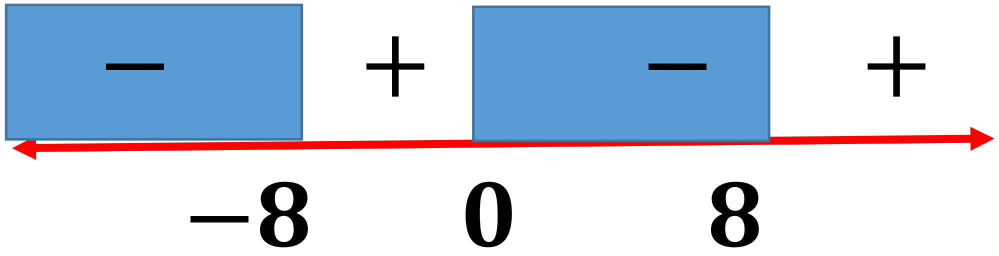
$$\underbrace{\left(x^2+3\right)}_{+}\left(x^3-64x\right)\leq 0$$

$$x\left(x^2-64\right)\leq 0$$

$$x(x-8)(x+8)\leq 0$$

puntos criticos

X=-8;   x=0;   x=8



$Rpta =< -\infty$;-8] U [0:8]

Problema 7

Resolver la desigualdad:

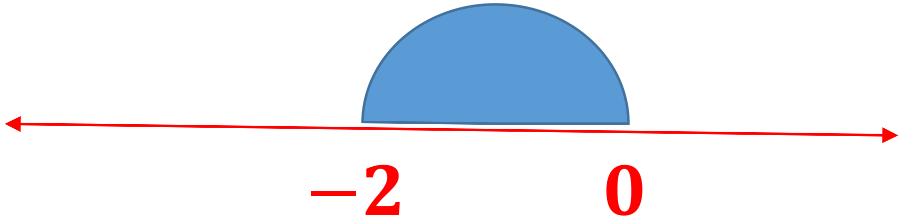
$$x + 2 \leq \sqrt[3]{x^3 + 8}$$

ELEVO AL CUBO, SIN RESTRICCION

$$(x+2)^3 \leq \left(\sqrt[3]{x^3+8}\right)^3$$

$$\cancel{x^3} + \cancel{2^3} + 3(x)(x+2) \leq \cancel{x^3} + \cancel{8}$$

$$(x)(x+2) \leq 0$$



$$C.S. = [-2; 0]$$

PROBLEMA 8

Sabiendo que $x \in \langle 1;7 \rangle$ simplifique

$$Q = \frac{|2x+3|+|5x-3|}{x}$$

$$Q = \frac{\boxed{|2x+3|} + \boxed{|5x-3|}}{x}$$

Resolución:

$Si\ x \in \langle 1;7 \rangle$

$=> 1 < x < 7$

$2 < 2x < 14$

$$5 < \underbrace{2x+3}_{(+)} < 17$$

$=> 1 < x < 7$

$5 < 5x < 35$

$$2 < \underbrace{5x-3}_{(+)} < 32$$

Luego

$$Q = \frac{(2x+3) + (5x-3)}{x}$$

$$Q = \frac{2x+3+5x-3}{x}$$

$$Q = \frac{7x}{x}$$

$$\boxed{\boldsymbol{Q = 7}}$$

PROBLEMA 9

Determine el número de raíces positivas de:

$$\left|\frac{4x-1}{x-2}\right| = 2|x|$$

$$|4x-1| = |\mathbf{2x^2 - 4x}|$$

Resolución:

$|a| = |b| \Rightarrow [a = b \lor a = -b)]$

FÓRMULA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

α: $4x - 1 = 2x^2 - 4x$

$$0 = 2x^2 - 8x + 1$$

$$x_{1,2} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(1)}}{2(2)}$$

$$x_{1,2} = \frac{8 \pm 2\sqrt{14}}{4} \quad \begin{cases} x_1 = 2 + \dfrac{\sqrt{14}}{2} \\ x_2 = 2 - \dfrac{\sqrt{14}}{2} \end{cases}$$

$\lor$ β: $4x - 1 = -2x^2 + 4x$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x_3 = +\frac{1}{\sqrt{2}} \quad \lor \quad x_4 = -\frac{1}{\sqrt{2}}$$

Las raíces positivas

$$2 + \frac{\sqrt{14}}{2}$$

$$2 - \frac{\sqrt{14}}{2}$$

$$\frac{\sqrt{2}}{2}$$

tres raices positivas

PROBLEMA 10

Resuelva la siguiente inecuación, en los enteros:

$$|8x+9|+|7x+4| \leq 10$$

$$|8x+9|+|7x+4| \leq 10$$

Aplicando la desigualdad triángular

$$|a+b| \leq |a|+|b|$$

$$|(8x+9)+(7x+4)| \leq \cancel{|8x+9|+|7x+4|} \leq 10$$

Por la propiedad transitiva

$$|(8x+9)+(7x+4)| \leq 10$$

$$|15x+13| \leq 10$$

$$|a| \leq b \quad <=> -b \leq a \leq b$$

$$-10 \leq 15x+13 \leq 10$$

$$-\frac{23}{15} \leq x \leq -\frac{1}{5}$$

$$-1,53 \qquad -1 \qquad -0.20$$

$$\boxed{\text{C.S}=\{-1\}}$$