

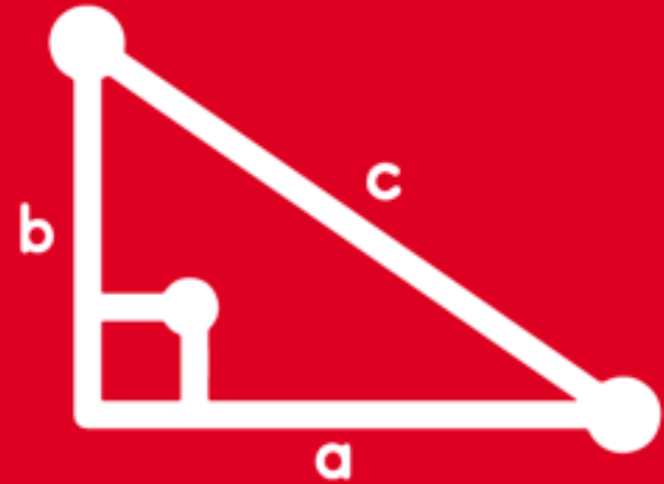


TRIGONOMETRY

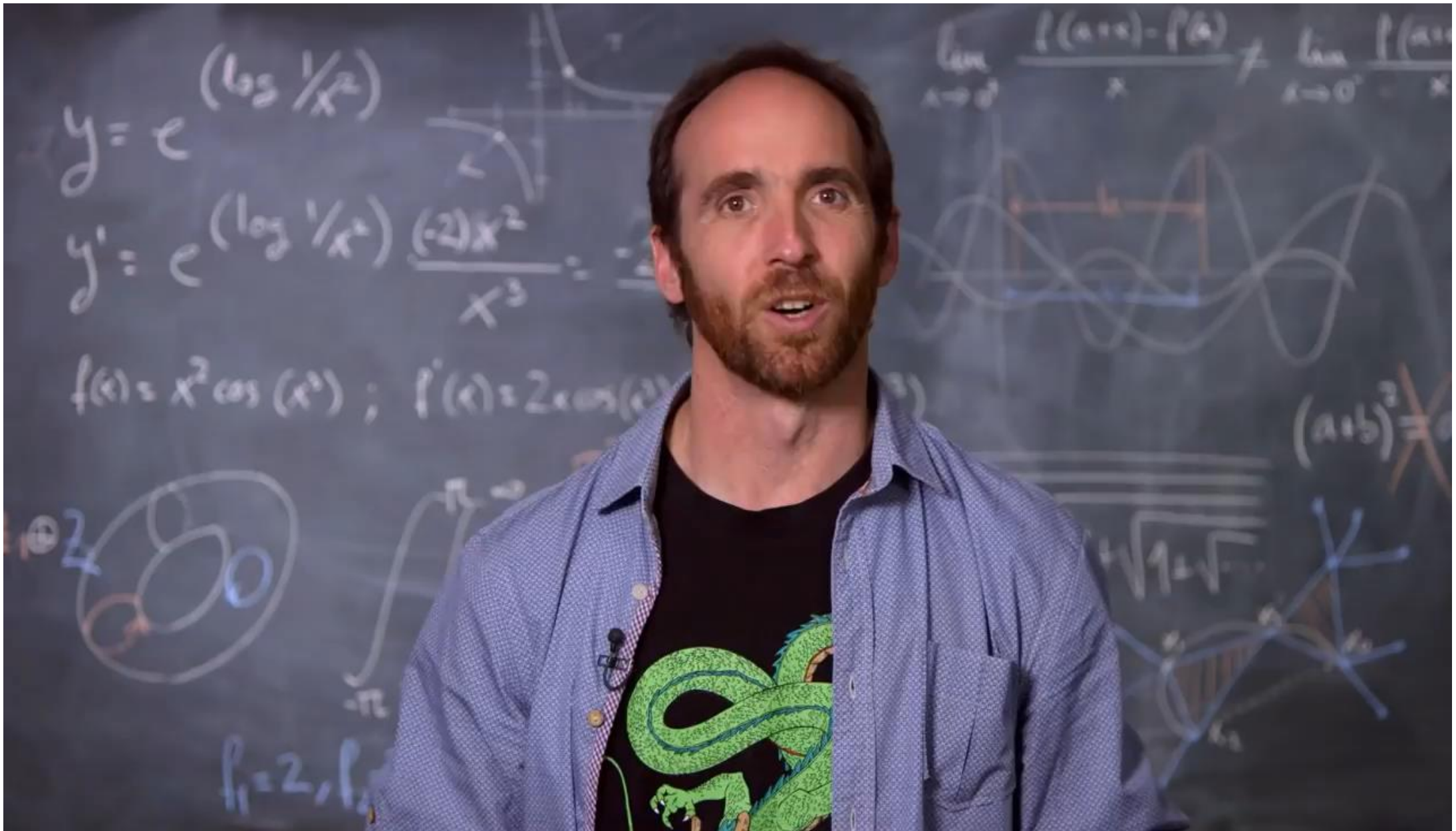
Chapter 17

5th
SECONDARY

**IDENTIDADES TRIGONOMÉTRICAS
DEL ÁNGULO MITAD**



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IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO MITAD

I. IDENTIDADES BÁSICAS

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos x}{2}}$$

$$\operatorname{cos}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1+\cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos x}{1+\cos x}}$$

Observación:

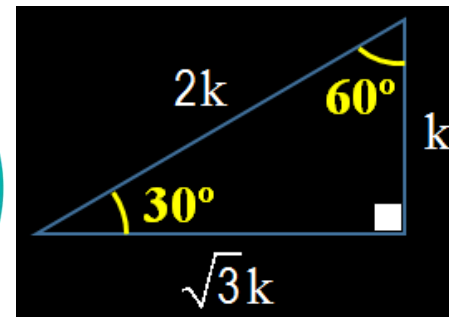
El signo \pm depende del cuadrante de $\left(\frac{x}{2}\right)$

II. IDENTIDADES AUXILIARES

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

Recordar:



Ejemplo:

$$\tan 15^\circ = \csc 30^\circ - \cot 30^\circ$$

$$\therefore \tan 15^\circ = 2 - \sqrt{3}$$



1. Reduzca: $H = \sqrt{\frac{1 + \cos 100^\circ}{2}} - \frac{\sin 80^\circ}{2 \cos 40^\circ}$

RESOLUCIÓN

Recordar:

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$H = \sqrt{\frac{1 + \cos 100^\circ}{2}} - \frac{\sin 80^\circ}{2 \cos 40^\circ}$$

Diagram illustrating the simplification of the expression for H. The first term is $\sqrt{\frac{1 + \cos 100^\circ}{2}}$, which is circled in purple. Above it, $\cos 50^\circ$ is written with a purple arrow pointing down to the square root. The second term is $\frac{\sin 80^\circ}{2 \cos 40^\circ}$. The numerator $\sin 80^\circ$ is circled in green, with a green arrow pointing down to it from $\sin 80^\circ$ written above. The denominator $2 \cos 40^\circ$ is crossed out with a red line, and $2 \sin 40^\circ \cos 40^\circ$ is written above it, also crossed out with a red line.

$$\Rightarrow H = \cos 50^\circ - \sin 40^\circ \quad \therefore H = 0$$

Diagram illustrating the simplification of the expression for H. The first term is $\cos 50^\circ$, which is circled in purple. The second term is $\sin 40^\circ$, which is circled in green. A blue arrow points from $\sin 40^\circ$ to the final result $H = 0$.



2. Reduzca la expresión: $P = \sqrt{\frac{1 - \sqrt{\frac{1 + \operatorname{sen} 10^\circ}{2}}}{2}}$

RESOLUCIÓN

Recordar:

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$P = \sqrt{\frac{1 - \sqrt{\frac{1 + \cos 80^\circ}{2}}}{2}}$$

$\cos 40^\circ$

$\Rightarrow P = \sqrt{\frac{1 - \cos 40^\circ}{2}}$

$$\therefore P = \operatorname{sen} 20^\circ$$



3. Si para un ángulo θ mayor a 360° pero menor que 450° , se cumple que $\cos \theta = 0.5$; calcule $\text{sen}\left(\frac{\theta}{2}\right)$

RESOLUCIÓN

Recordar:

$$\text{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$\in \text{III C}$

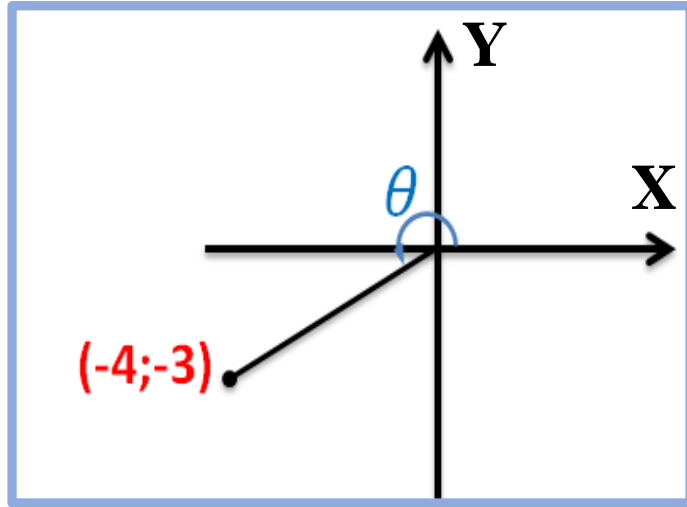
Dato: $360^\circ < \theta < 450^\circ \rightarrow$ Si $180^\circ < \frac{\theta}{2} < 225^\circ$

$$\text{sen}\left(\frac{\theta}{2}\right) = \overset{\text{III C}}{-} \sqrt{\frac{1 - \cos \theta}{2}} = -\sqrt{\frac{1 - \frac{1}{2}}{2}} = -\sqrt{\frac{\frac{1}{2}}{2}} = -\sqrt{\frac{1}{4}}$$

$\therefore \text{sen}\left(\frac{\theta}{2}\right) = -\frac{1}{2}$



4. Del gráfico, calcule $\tan\left(\frac{\theta}{2}\right)$



RESOLUCIÓN

Del gráfico: $180^\circ < \theta < 270^\circ$

$\in \text{IIC}$

$\Rightarrow 90^\circ < \frac{\theta}{2} < 135^\circ$

Además: $x = -4$; $y = -3$

$$r = \sqrt{(-4)^2 + (-3)^2} = 5 \Rightarrow \cos\theta = \frac{x}{r} = -\frac{4}{5}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

$$\tan\left(\frac{\theta}{2}\right) = - \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = - \sqrt{\frac{1 - (-\frac{4}{5})}{1 + (-\frac{4}{5})}}$$

IIC

$$\tan\left(\frac{\theta}{2}\right) = - \sqrt{\frac{9}{1}} = -\sqrt{9} \therefore \tan\left(\frac{\theta}{2}\right) = -3$$



5. Dar el valor de: $E = \cot\left(\frac{\pi}{8}\right) - \sec\left(\frac{\pi}{4}\right)$

Recordar:

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

RESOLUCIÓN

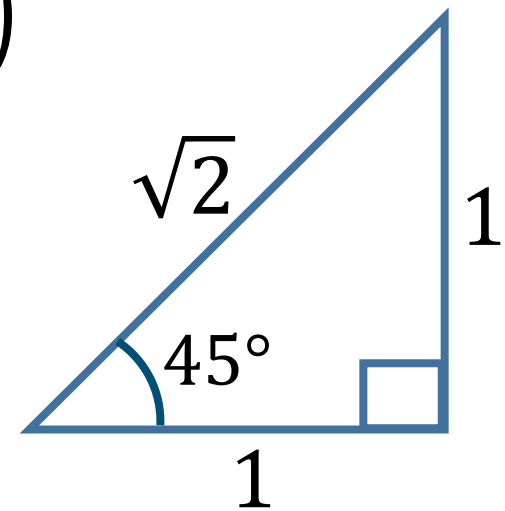
$$E = \cot\left(\frac{\pi}{8}\right) - \sec\left(\frac{\pi}{4}\right)$$

$$E = \csc\left(\frac{\pi}{4}\right) + \cot\left(\frac{\pi}{4}\right) - \sec\left(\frac{\pi}{4}\right)$$

$$E = \cancel{\sqrt{2}} + 1 - \cancel{\sqrt{2}}$$

$$\therefore E = 1$$

$$\frac{\pi}{4} = 45^\circ$$





6. El niño Pepito recibe de propina diaria la suma de $12(\csc\theta - \csc 2\theta - \cot 2\theta)^2$ soles. Indique el valor del ángulo agudo θ , para que la propina diaria sea de 4 soles.

RESOLUCIÓN

Del dato: $12(\csc\theta - \csc 2\theta - \cot 2\theta)^2$

$$12(\csc\theta - (\csc 2\theta + \cot 2\theta))^2$$

$$\text{Ahora hacemos: } 12(\csc\theta - \cot\theta)^2 = 4$$

$$(\csc\theta - \cot\theta)^2 = \frac{1}{3}$$

$$\rightarrow \tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}}$$

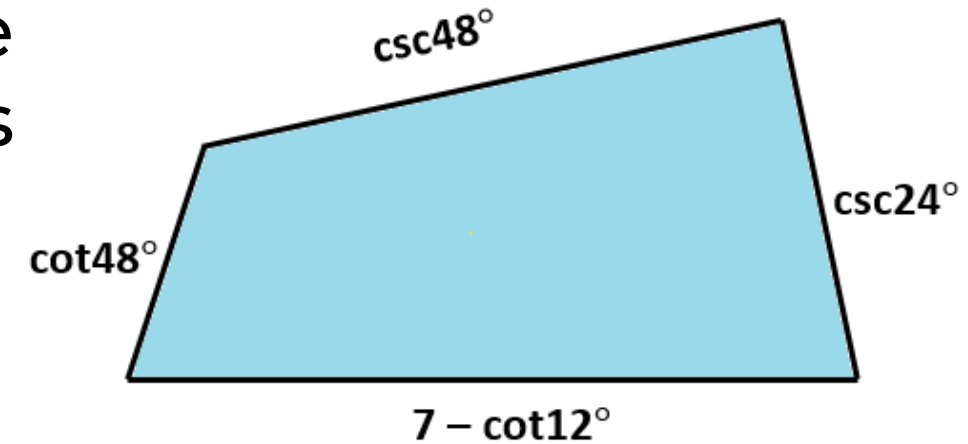
$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\therefore \theta = 60^\circ$$



7. El contorno de la mesa en la sala de espera de una clínica dental tiene las siguientes dimensiones. (en metros)
¿Cuál es el perímetro de dicho contorno?



Recordar:

$$\csc x + \cot x = \cot\left(\frac{x}{2}\right)$$

RESOLUCIÓN

$$(2p) \text{ (trapezoid)} = \underbrace{\cot 48^\circ + \csc 48^\circ + \csc 24^\circ + 7 - \cot 12^\circ}_{\text{sum of top and right sides}}$$

$$(2p) \text{ (trapezoid)} = \underbrace{\cot 24^\circ + \csc 24^\circ + 7 - \cot 12^\circ}_{\text{sum of left and bottom sides}}$$

$$(2p) \text{ (trapezoid)} = \cancel{\cot 12^\circ} + 7 - \cancel{\cot 12^\circ}$$

$$\therefore (2p) \text{ (trapezoid)} = 7m$$