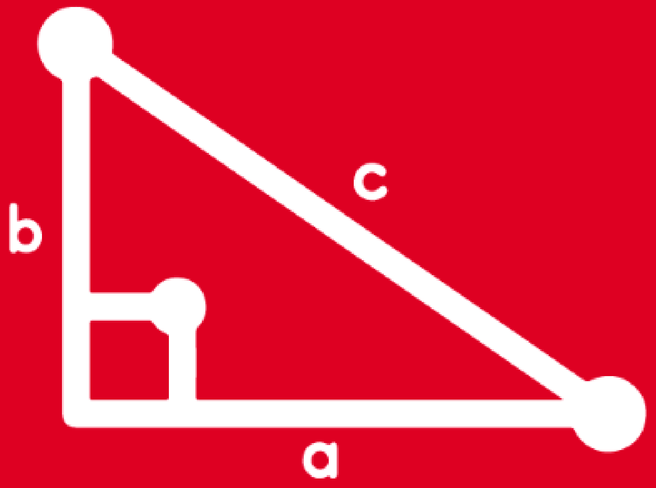
TRIGONOMETRY

TOMO VII



4th
SECONDARY

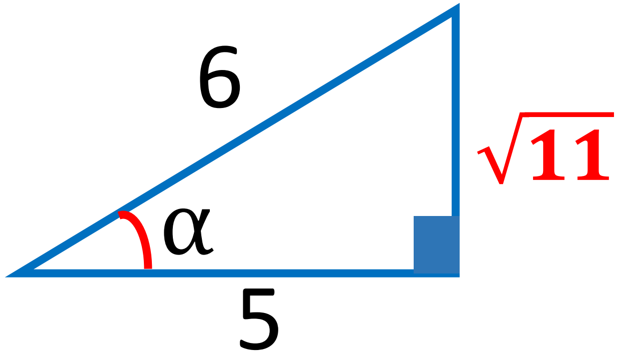
Feedback

SACO OLIVEROS

1. Si $\cos\alpha = \frac{5}{6}$, donde $0 < \alpha < 90°$, calcule $\mathrm{sen}2\alpha$.

Resolución

Del dato: $\cos\alpha = \frac{5}{6} = \frac{\mathbf{CA}}{\mathbf{H}}$



Recuerda:

$$\boxed{\mathbf{sen2x = 2senx \cdot cosx}}$$

Del gráfico: $\mathbf{sen\alpha = \frac{\sqrt{11}}{6}}$

Piden $\mathrm{sen}2\alpha = 2\mathrm{sen}\alpha \cdot \cos\alpha$

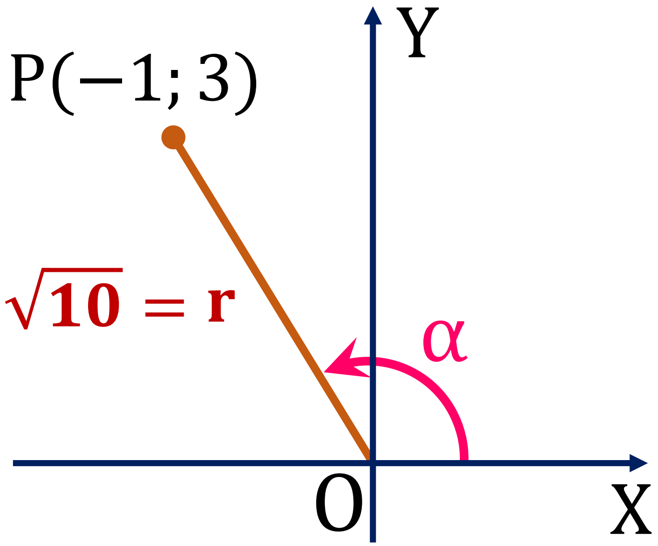
$$\mathrm{sen}2\alpha = \overset{1}{\cancel{2}}\left(\frac{\sqrt{11}}{\underset{3}{\cancel{6}}}\right)\left(\frac{5}{6}\right)$$

$$\therefore \boxed{\mathbf{sen2\alpha = \frac{5\sqrt{11}}{18}}}$$

2. Si el punto $P(-1;3)$ pertenece al lado final de un ángulo en posición normal α, calcule $5\cos 2\alpha$.

Resolución

Graficando, según la condición:



Calculamos r:

$$\mathbf{r = \sqrt{x^2 + y^2}}$$

$$\mathbf{r = \sqrt{(-1)^2 + (3)^2}}$$

$$\boxed{\mathbf{r = \sqrt{10}}}$$

Recuerda: $\mathbf{\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha}$

Así: $\mathbf{\operatorname{sen}\alpha = \dfrac{3}{\sqrt{10}}}$ $\mathbf{\cos\alpha = \dfrac{-1}{\sqrt{10}}}$

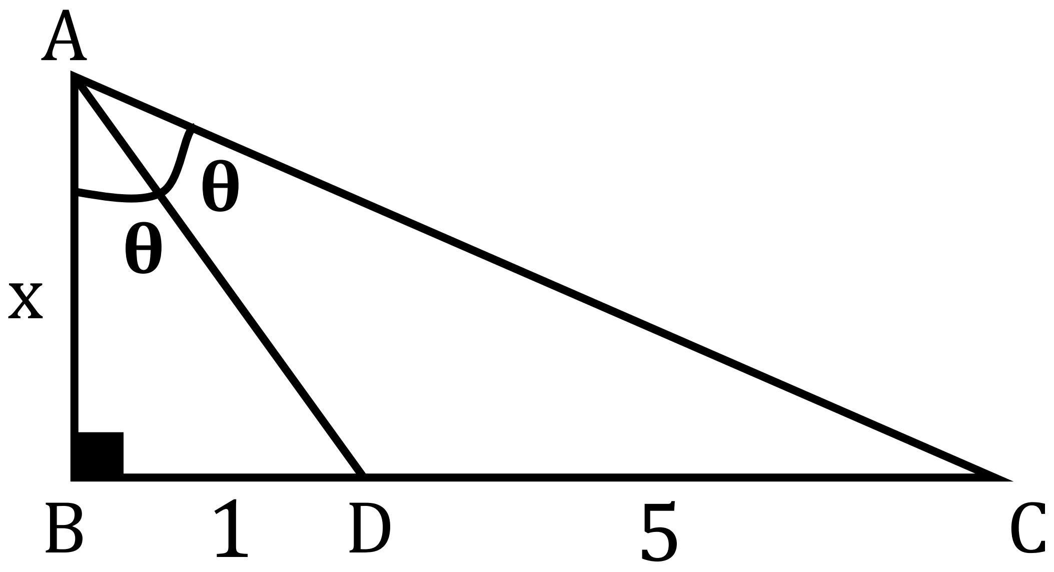
$$\rightarrow \cos 2\alpha = \left(\frac{-1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2$$

$$\cos 2\alpha = \frac{1}{10} - \frac{9}{10} = \frac{-8}{10}$$

$$\cos 2\alpha = \frac{-4}{5}$$

$$\therefore \boxed{\mathbf{5\cos 2\alpha = -4}}$$

3. A partir del gráfico, determine el valor de $2x$.



Resolución

ΔABD: $\tan\theta = \dfrac{1}{x}$ ΔABC: $\tan 2\theta = \dfrac{6}{x}$

Aplicamos: $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\frac{6}{x} = \frac{2\left(\frac{1}{x}\right)}{1-\left(\frac{1}{x}\right)^2}$$

$$\frac{6}{x} = \frac{\frac{2}{x}}{\frac{x^2-1}{x^2}}$$

$$\frac{\overset{3}{\cancel{6}}}{\cancel{x}} = \frac{\overset{1}{\cancel{2}}x^2}{\cancel{x}(x^2-1)}$$

Tenemos:

$$3x^2 - 3 = x^2$$

$$2x^2 = 3$$

$$x^2 = \frac{3}{2}$$

$$x = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

2

$$\therefore \boxed{2x = \sqrt{6}}$$

4. Si $\boldsymbol{cot\theta + tan\theta = 7}$, calcule

$$\boldsymbol{K = 14sen2\theta.}$$

Resolución

Recordar:

$$\boxed{\boldsymbol{cot\theta + tan\theta = 2csc2\theta}}$$

$$\underbrace{cot\theta + tan\theta}_{} = 7$$

$$2csc2\theta = 7$$

$$csc2\theta = \frac{7}{2} \Rightarrow \boxed{sen2\theta = \frac{2}{7}}$$

Luego: $K = 14sen2\theta$

$$K = 14\left(\frac{2}{7}\right)$$

$$\therefore \boxed{\boldsymbol{K = 4}}$$

5. Si $m = 4cos^3 20° - 3cos20°$

$n = 3sen40° - 4sen^3 40°$

Calcule $E = m^2 + n^2$

Resolución

$$m = \underbrace{4cos^3 20° - 3cos20°}_{\cos(3.20°)}$$

$$m = cos60° \Rightarrow m = \frac{1}{2}$$

$$n = \underbrace{3sen40° - 4sen^3 40°}_{sen(3.40°)}$$

$$n = sen120° \Rightarrow n = \frac{\sqrt{3}}{2}$$

Calculamos:

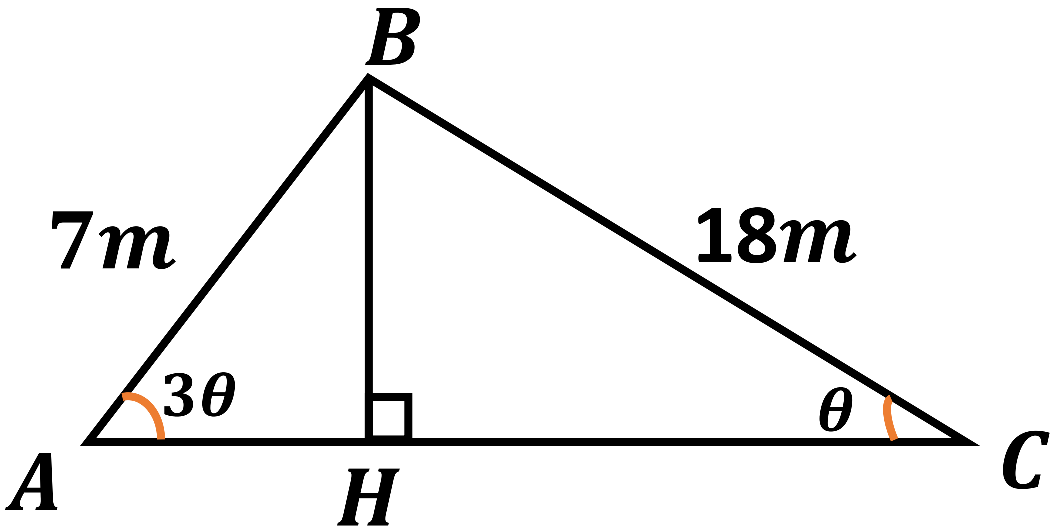
$$E = m^2 + n^2$$

$$E = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

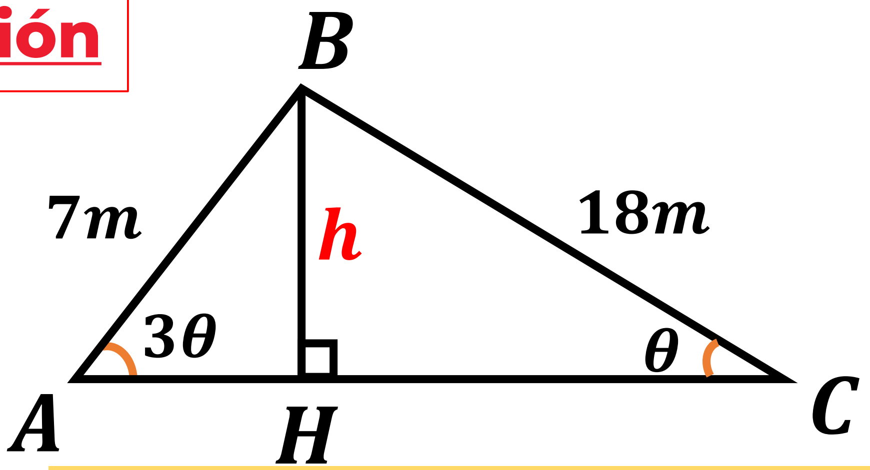
$$E = \frac{1}{4} + \frac{3}{4}$$

$$\therefore \boxed{E = 1}$$

6. *Se construye un minimarket sobre un terreno que tiene la forma de un triángulo ABC, tal como se muestra en la figura. Determine el valor de $\boldsymbol{cos2\theta}$.*



Resolución



Recordar: $\boldsymbol{sen3\theta = sen\theta(2cos2\theta + 1)}$

$$\frac{h}{7} = \frac{h}{18}(2cos2\theta + 1)$$

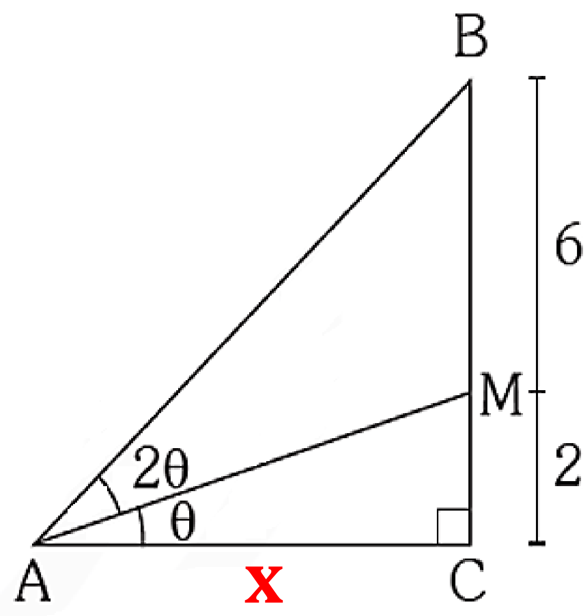
$$18.\cancel{h} = 7.\cancel{h}(2cos2\theta + 1)$$

$\Rightarrow 18 = 14cos2\theta + 7$

$\Rightarrow 11 = 14cos2\theta$

$$\therefore \boxed{cos2\theta = \frac{11}{14}}$$

7. A partir del gráfico, determine AC.



Recordamos

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Resolución

$\Delta ABC:\ \tan 3\theta = \frac{8}{x}$ $\qquad \Delta ACM:\ \tan\theta = \frac{2}{x}$

Reemplazando:

$$\frac{8}{x} = \frac{3\left(\frac{2}{x}\right) - \left(\frac{2}{x}\right)^3}{1 - 3\left(\frac{2}{x}\right)^2}$$

Cambio de variable: $\frac{2}{x} = a$

$$\rightarrow \quad 4a = \frac{3a - a^3}{1 - 3a^2}$$

SACO OLIVEROS

7. Continuación

De la anterior:

$$4\mathbf{a} = \frac{3\mathbf{a} - \mathbf{a}^3}{1 - 3\mathbf{a}^2}$$

$$4a - 12a^3 = 3a - a^3$$

$$a = 11a^3$$

$$\frac{1}{11} = a^2$$

Pero: $\mathbf{a} = \frac{\mathbf{2}}{\mathbf{x}}$

$$\rightarrow \quad \frac{1}{11} = \frac{4}{x^2}$$

$$x^2 = 44$$

$$\therefore \quad \boxed{\mathbf{x = 2\sqrt{11}}}$$

8. Reduzca

$$T = \frac{\cos 11x + \cos 9x + \cos 7x + \cos 5x}{\cos 3x + \cos x}$$

Resolución

Recordamos:

$$\mathbf{\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)}$$

Aplicando la IT en el numerador:

$$T = \frac{\mathbf{2\cos 8x \cdot \cos 3x} + \mathbf{2\cos 8x \cdot \cos x}}{\cos 3x + \cos x}$$

Factorizando "2cos8x":

$$T = \frac{\mathbf{2\cos 8x}\cancel{(\cos 3x + \cos x)}}{\cancel{\cos 3x + \cos x}}$$

$$\therefore \boxed{\mathbf{T = 2\cos 8x}}$$

9. Efectúe

$$A = \frac{2\text{sen}20° + \text{sen}40°}{\text{sen}50°}$$

Resolución

Descomponiendo "2sen20°"

$$A = \frac{\text{sen}40° + \text{sen}20° + \text{sen}20°}{\text{sen}50°}$$

Recordamos:

$$\textbf{senA} + \textbf{senB} = \textbf{2sen}\left(\frac{\textbf{A}+\textbf{B}}{\textbf{2}}\right) \cdot \textbf{cos}\left(\frac{\textbf{A}-\textbf{B}}{\textbf{2}}\right)$$

Aplicando la IT en el numerador:

$$A = \frac{\overset{1}{\cancel{2\text{sen}30°}} \cdot \overset{\text{sen}80°}{\text{cos}10°} + \text{sen}20°}{\text{sen}50°}$$

$$A = \frac{\cancel{2\text{sen}50°} \cdot \text{cos}30°}{\cancel{\text{sen}50°}}$$

$$A = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$\therefore \boxed{A = \sqrt{3}}$$

SACO OLIVEROS

10. Gerald va al mercado y compra (3A) kg de fresa, (2B) kg de naranjas y (C) kg de manzanas. Si $\text{sen}11x \cdot \cos 3x - \text{sen}9x \cdot \cos 5x = A\text{sen}(Bx) \cdot \cos(Cx)$. Determine la cantidad total de frutas que compró Gerald.

Resolución

Recordamos

$$2\text{sen}A \cdot \cos B = \text{sen}(A+B) + \text{sen}(A-B)$$

Dando forma al 1er miembro:

$$\frac{2\text{sen}11x \cdot \cos 3x - 2\text{sen}9x \cdot \cos 5x}{2} = \frac{\text{sen}14x + \text{sen}8x - (\text{sen}14x + \text{sen}4x)}{2}$$

$$= \frac{\cancel{\text{sen}14x} + \underbrace{\text{sen}8x - \cancel{\text{sen}14x} - \text{sen}4x}_{2\text{sen}2x \cdot \cos 6x}}{2}$$

$$A\text{sen}(Bx) \cdot \cos(Cx) = \text{sen}2x \cdot \cos 6x \quad \boxed{A = 1} \quad \boxed{B = 2} \quad \boxed{C = 6}$$

→ Cantidad total $= 3(1) + 2(2) + 6$ $\therefore$ **Cantidad total = 13 kg**