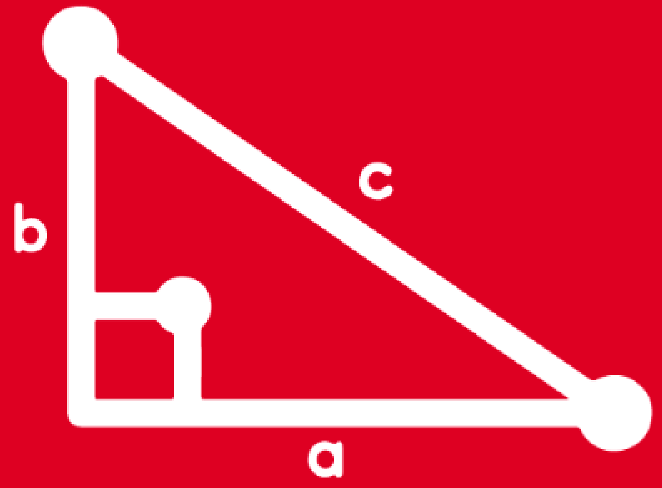
TRIGONOMETRY

Chapter 18

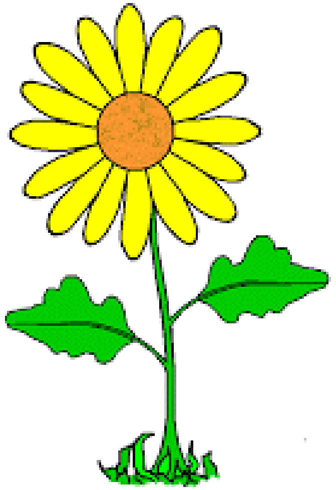


4th
SECONDARY

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO

SACO OLIVEROS

MOTIVATING | STRATEGY



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO (FUNDAMENTALES)

Para la suma de dos ángulos:

$$\operatorname{sen}(x+y)=\operatorname{sen}x\cdot\cos y+\cos x\cdot\operatorname{sen}y$$

$$\cos(x+y)=\cos x\cdot\cos y-\operatorname{sen}x\cdot\operatorname{sen}y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\cdot\tan y}$$

Para la resta de dos ángulos:

$$\operatorname{sen}(x-y)=\operatorname{sen}x\cdot\cos y-\cos x\cdot\operatorname{sen}y$$

$$\cos(x-y)=\cos x\cdot\cos y+\operatorname{sen}x\cdot\operatorname{sen}y$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\cdot\tan y}$$

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO (AUXILIARES)

$$\boxed{\mathrm{sen}(x+y)\cdot \mathrm{sen}(x-y)=\mathrm{sen}^2x-\mathrm{sen}^2y}$$

$$\boxed{\cos(x+y)\cdot \cos(x-y)=\cos^2x-\mathrm{sen}^2y}$$

$$\boxed{\tan x\pm \tan y\pm \tan(x\pm y)\cdot \tan x\cdot \tan y=\tan(x\pm y)}$$

Para tres ángulos:

$$\mathrm{Si}\ \alpha+\beta+\theta=180°$$

$$\boxed{\tan\alpha+\tan\beta+\tan\theta=\tan\alpha\cdot\tan\beta\cdot\tan\theta}$$

$$\boxed{\cot\alpha\cdot\cot\beta+\cot\alpha\cdot\cot\theta+\cot\beta\cdot\cot\theta=1}$$

Para tres ángulos:

Si $\alpha + \beta + \theta = 90°$

$$\boxed{\cot\alpha + \cot\beta + \cot\theta = \cot\alpha \cdot \cot\beta \cdot \cot\theta}$$

$$\boxed{\tan\alpha \cdot \tan\beta + \tan\alpha \cdot \tan\theta + \tan\beta \cdot \tan\theta = 1}$$

HELICOPRACTICE 1

Calcule $\cos 16°$

$$\cos(x-y) = \cos x.\cos y + \operatorname{sen} x.\operatorname{sen} y$$

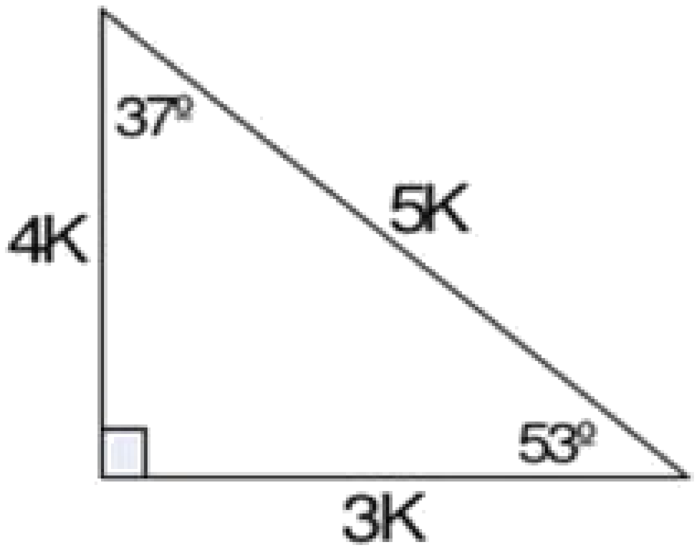
Resolución:

$$\cos 16° = \cos(53° - 37°)$$

$$\cos 16° = \underbrace{\cos 53°}.\underbrace{\cos 37°} + \underbrace{\operatorname{sen} 53°}.\underbrace{\operatorname{sen} 37°}$$

$$\cos 16° = \underbrace{\frac{3}{5} \cdot \frac{4}{5}} + \underbrace{\frac{4}{5} \cdot \frac{3}{5}}$$

$$\cos 16° = \frac{12}{25} + \frac{12}{25}$$



$$\therefore \cos 16° = \frac{24}{25}$$

HELICOPRACTICE 2

Reducir $R = \sqrt{2}\cos(x - 45°) - \operatorname{sen}x$

Resolución:

$$\boxed{\cos(x - y) = \cos x . \cos y + \operatorname{sen}x . \operatorname{sen}y}$$

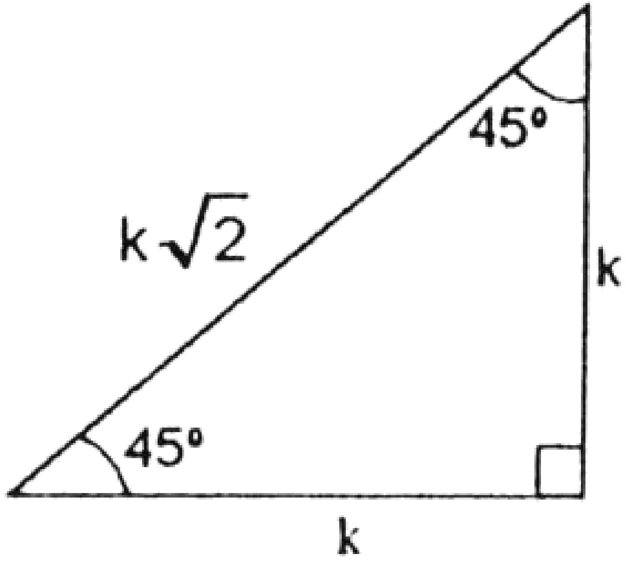
$$R = \sqrt{2}\cos(x - 45°) - \operatorname{sen}x$$

$$R = \sqrt{2}\ [\cos x . \underbrace{\cos 45°} + \operatorname{sen}x . \underbrace{\operatorname{sen}45°}] - \operatorname{sen}x$$

$$R = \cancel{\sqrt{2}}\,[\cos x \ . \ \frac{1}{\cancel{\sqrt{2}}} + \operatorname{sen}x \ . \ \frac{1}{\cancel{\sqrt{2}}}\ ] - \operatorname{sen}x$$

$$R = \cos x + \cancel{\operatorname{sen}x} - \cancel{\operatorname{sen}x}$$

$$\boxed{\therefore R = \cos x}$$



HELICOPRACTICE 3

Si $\tan\theta = \dfrac{5}{12}$; calcule $\tan(37° + \theta)$

Resolución:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x.\tan y}$$

$$\tan(37° + \theta) = \frac{\tan 37° + \tan\theta}{1 - \tan 37°.\tan\theta}$$

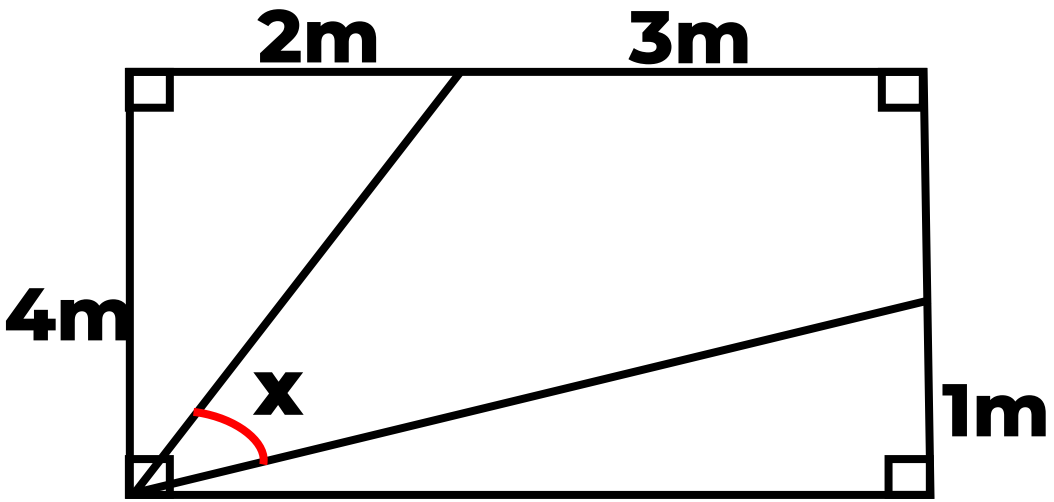
$$\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{\cancel{3}}{4}.\frac{5}{\cancel{12}_{4}}} = \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{\frac{7}{6}}{\frac{11}{16}}$$

$$\Rightarrow \tan(37° + \theta) = \frac{7 \times \overset{8}{\cancel{16}}}{\underset{3}{\cancel{6}} \times 11}$$

$$\boxed{\therefore \tan(37° + \theta) = \frac{56}{33}}$$

HELICOPRACTICE 4

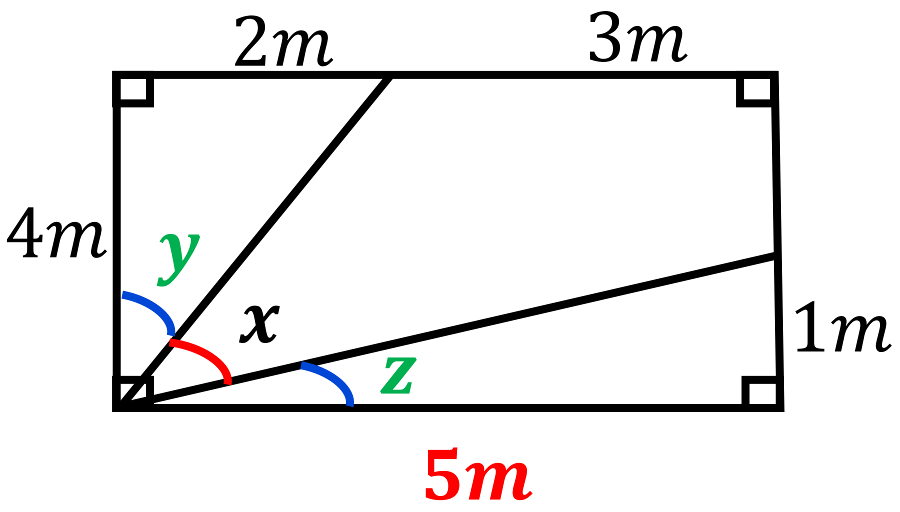
A partir del gráfico, determine el valor de tanx



Resolución:

Recordar: $\text{Si}: x + y + z = 90°$

➢ $\text{cotx} + \text{coty} + \text{cotz} = \text{cotx}.\text{coty}.\text{cotz}$



$$\mathbf{cotx + coty + cotz = cotx.coty.cotz}$$

$$\text{cotx} + \frac{4}{2} + \frac{5}{1} = \text{cotx}.\frac{4}{2}.\frac{5}{1}$$

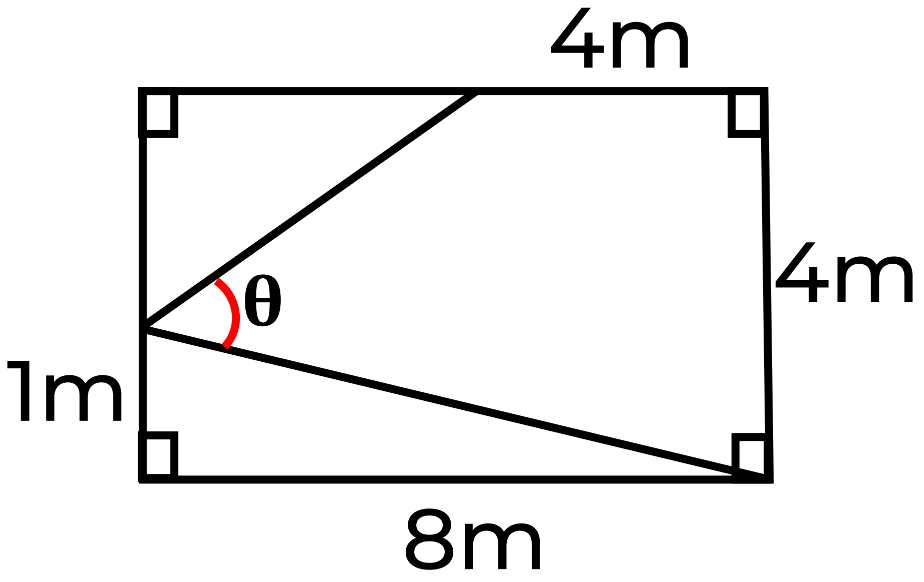
$$\text{cotx} + 7 = 10\text{cotx}$$

$$\text{cotx} = \frac{7}{9}$$

$$\therefore \text{tanx} = \frac{9}{7}$$

HELICOPRACTICE 5

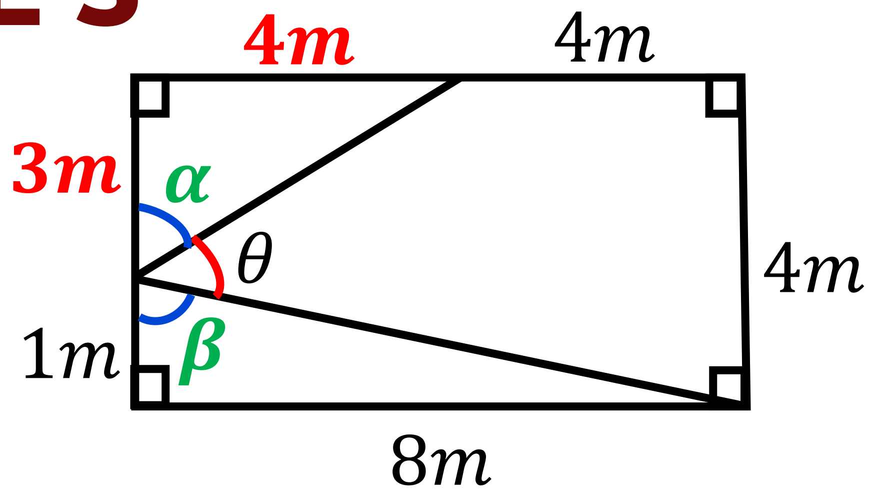
Del gráfico, determine cotθ



Resolución

Recordar: Si : $x + y + z = 180°$

$$\tan x + \tan y + \tan z = \tan x . \tan y . \tan z$$



$$\tan\alpha + \tan\theta + \tan\beta = \tan\alpha . \tan\theta . \tan\beta$$

$$\frac{4}{3} + \tan\theta + \frac{8}{1} = \frac{4}{3} . \tan\theta . \frac{8}{1}$$

$$\frac{28}{3} + \tan\theta = \frac{32}{3} . \tan\theta$$

x 3: $28 + 3\tan\theta = 32\tan\theta$

$$29\tan\theta = 28 \Rightarrow \tan\theta = \frac{28}{29}$$

$$\therefore \cot\theta = \frac{29}{28}$$

HELICOPRACTICE 6

Al copiar de la pizarra la expresión

tan30°+tan70°+tan80°; un estudiante cometió un error y escribió **tan70°.tan80°**. Calcule la razón entre lo que estaba escrito en la pizarra y lo que copió el alumno.

Resolución

Recordar: $\mathrm{Si}: x + y + z = 180°$

$$\boxed{\tan x + \tan y + \tan z = \tan x . \tan y . \tan z}$$

Calculamos la razón entre ellos:

$$R = \frac{\tan 30° + \tan 70° + \tan 80°}{\tan 70° . \tan 80°}$$

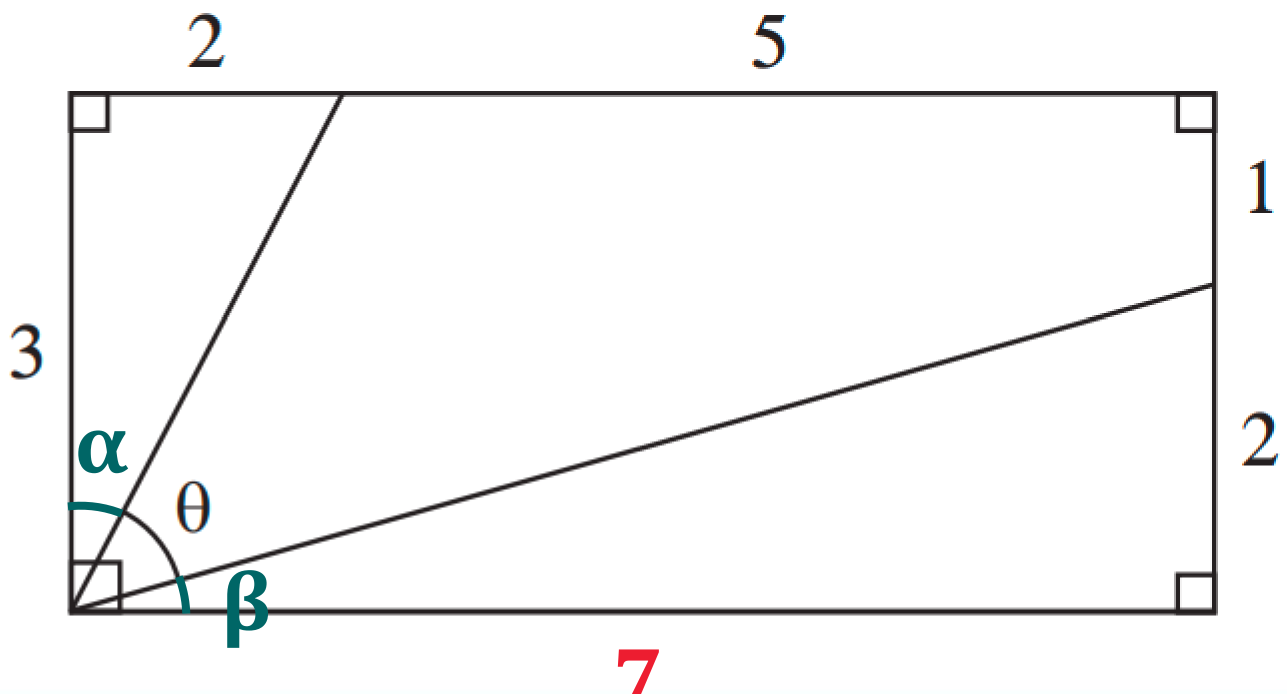
$$R = \frac{\tan 30° . \cancel{\tan 70°} . \cancel{\tan 80°}}{\cancel{\tan 70°} . \cancel{\tan 80°}}$$

$$R = \tan 30°$$

$$\boxed{\therefore R = \frac{\sqrt{3}}{3}}$$

HELICOPRACTICE 7

De la figura mostrada, calcule el valor de 2tanθ. Si se tiene una plancha metálica rectangular, la cual se desea realizar dos cortes que parten de un vértice tal que formen un ángulo θ como se aprecia en la figura adjunta



Recordar: Si : $x + y + z = 90°$

$$\mathbf{cotx + coty + cotz = cotx.\,coty.\,cotz}$$

Resolución

$$\mathbf{cot\theta + cot\alpha + cot\beta = cot\alpha \cdot cot\beta \cdot cot\theta}$$

$$\cot\theta + \frac{3}{2} + \frac{7}{2} = \cot\theta \cdot \frac{3}{2} \cdot \frac{7}{2}$$

$$\cot\theta + 5 = \frac{21}{4} \cdot \cot\theta$$

$$5 = \frac{17}{4} \cdot \cot\theta \Rightarrow \cot\theta = \frac{20}{17}$$

$$\boxed{\therefore\ 2\tan\theta = \frac{17}{10}}$$