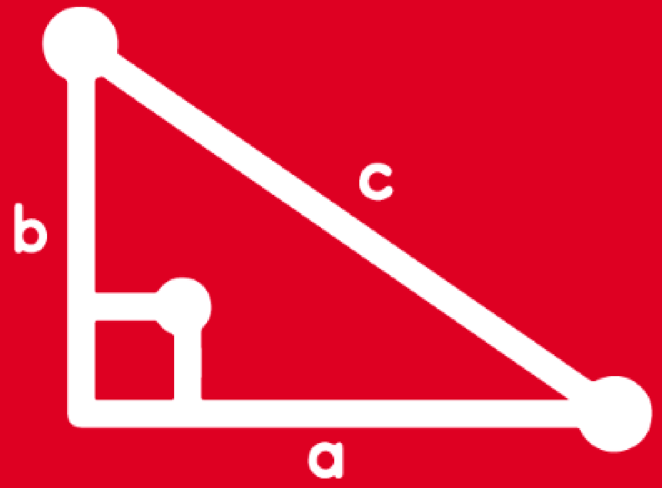
TRIGONOMETRY

Chapter 03

4th

SECONDARY

SECTOR CIRCULAR

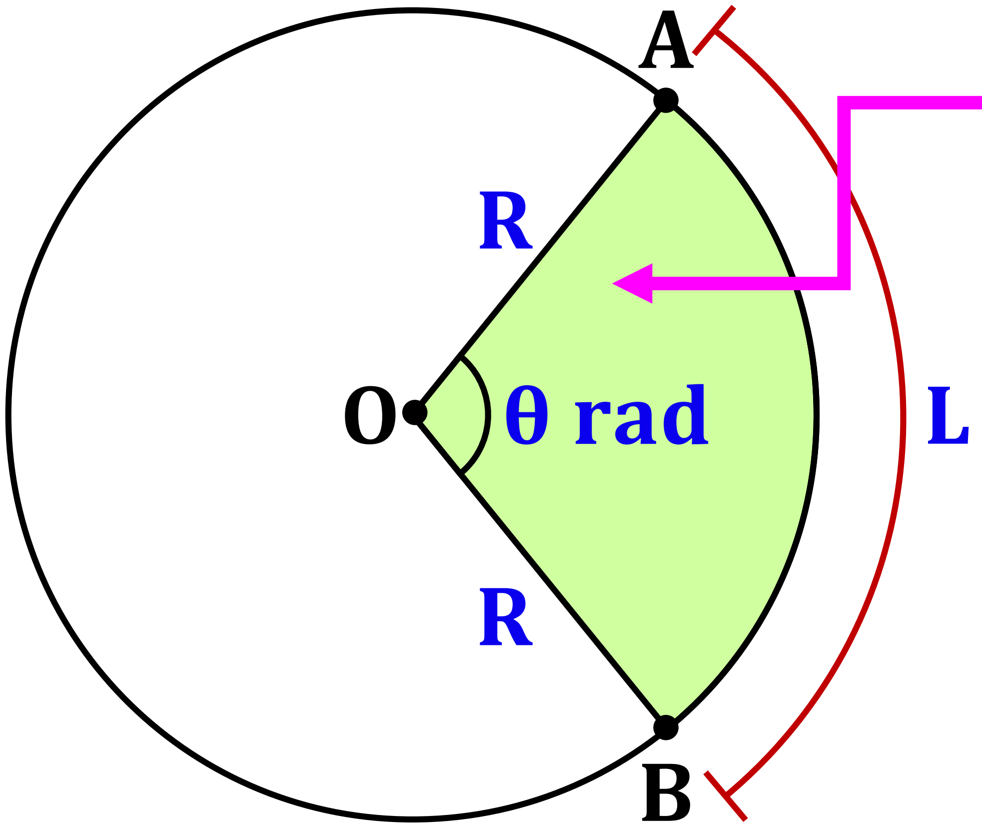


SACO OLIVEROS

SECTOR CIRCULAR Y SUS APLICACIONES

SECTOR CIRCULAR

Es la región circular limitada por dos radios y un respectivo arco correspondiente de circunferencia.



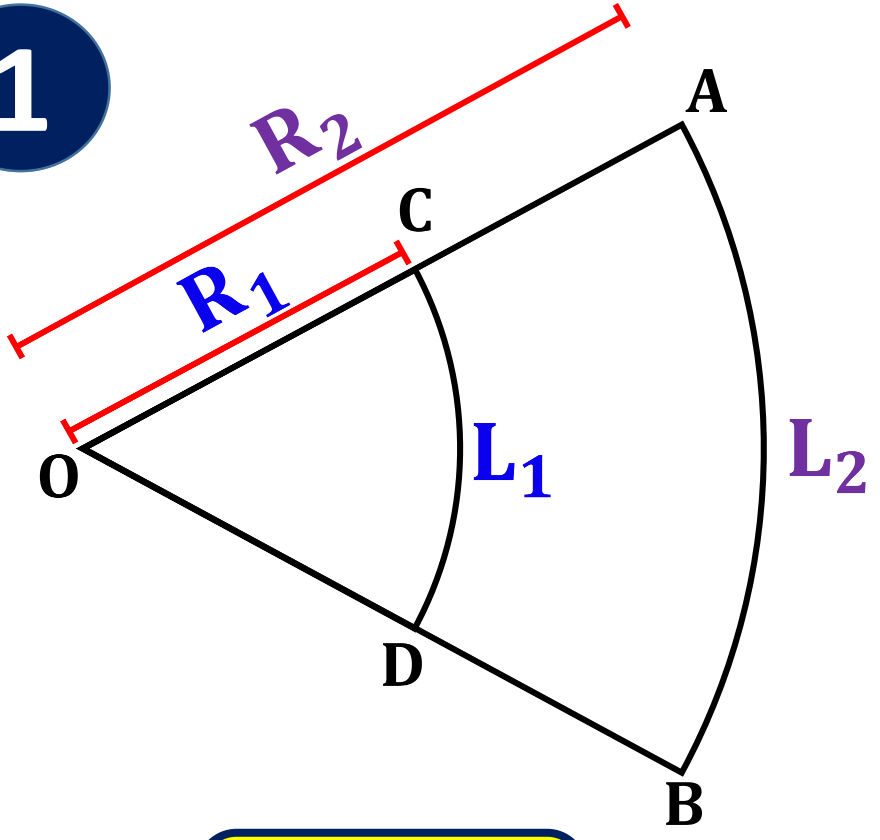
Donde :

- $S_{\triangleleft AOB}$: sector circular AOB
- R : radio de la circunferencia
- θ : N° de radianes del ángulo central $(0 < \theta \leq 2\pi)$
- L : longitud del arco $\overset{\frown}{AB}$

Se cumple : $\boxed{L = \theta \cdot R}$

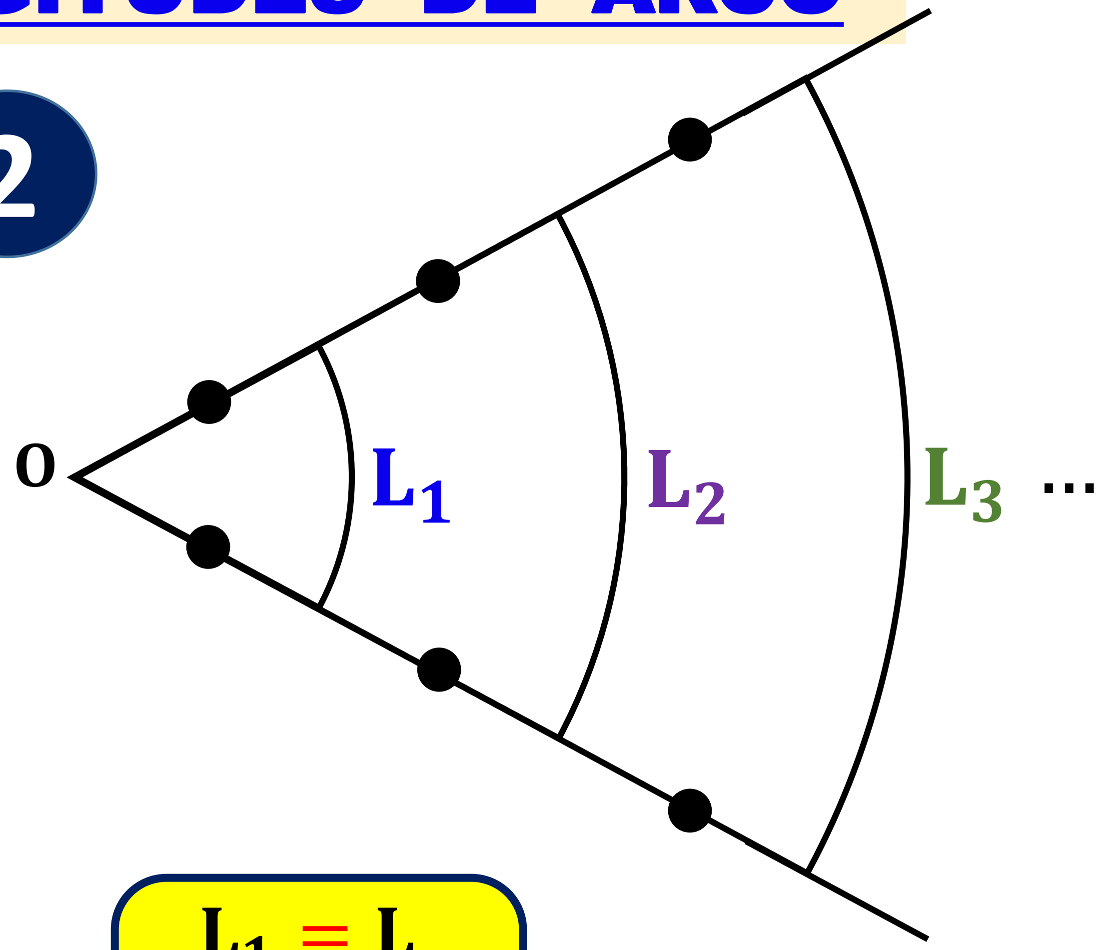
PROPIEDADES DE LONGITUDES DE ARCO

1



$$\frac{L_1}{L_2} = \frac{R_1}{R_2}$$

2

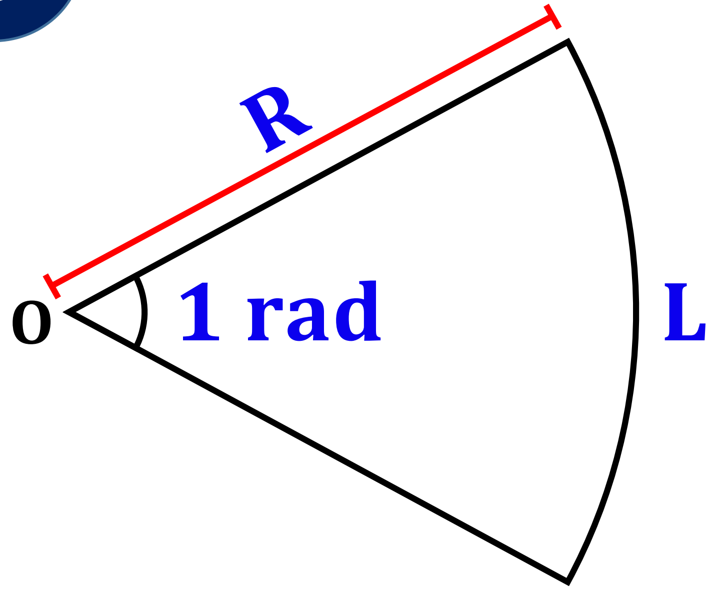


$$L_1 = L$$
$$L_2 = 2L$$
$$L_3 = 3L$$

; $L_n = nL$

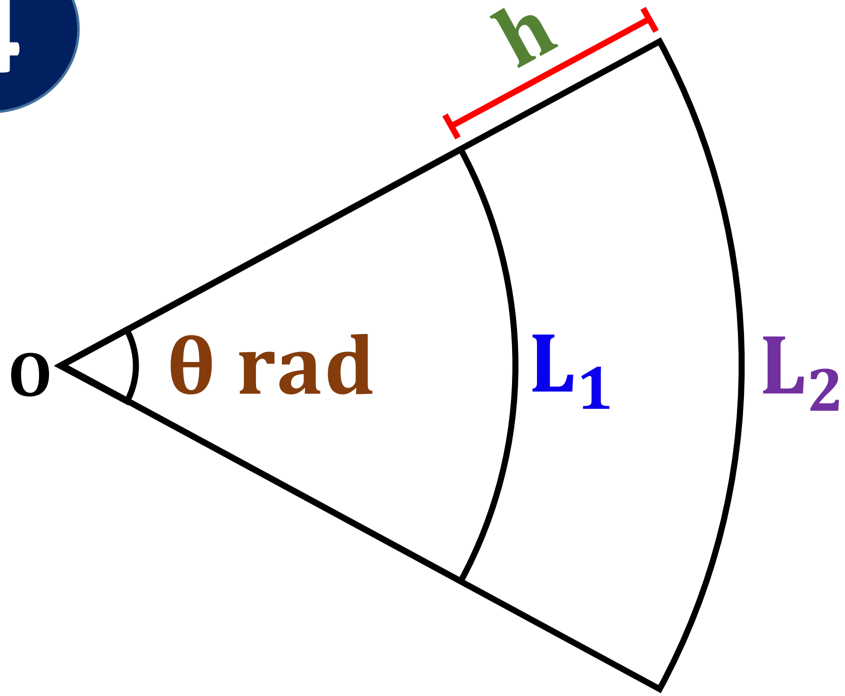
PROPIEDADES DE LONGITUDES DE ARCO

3



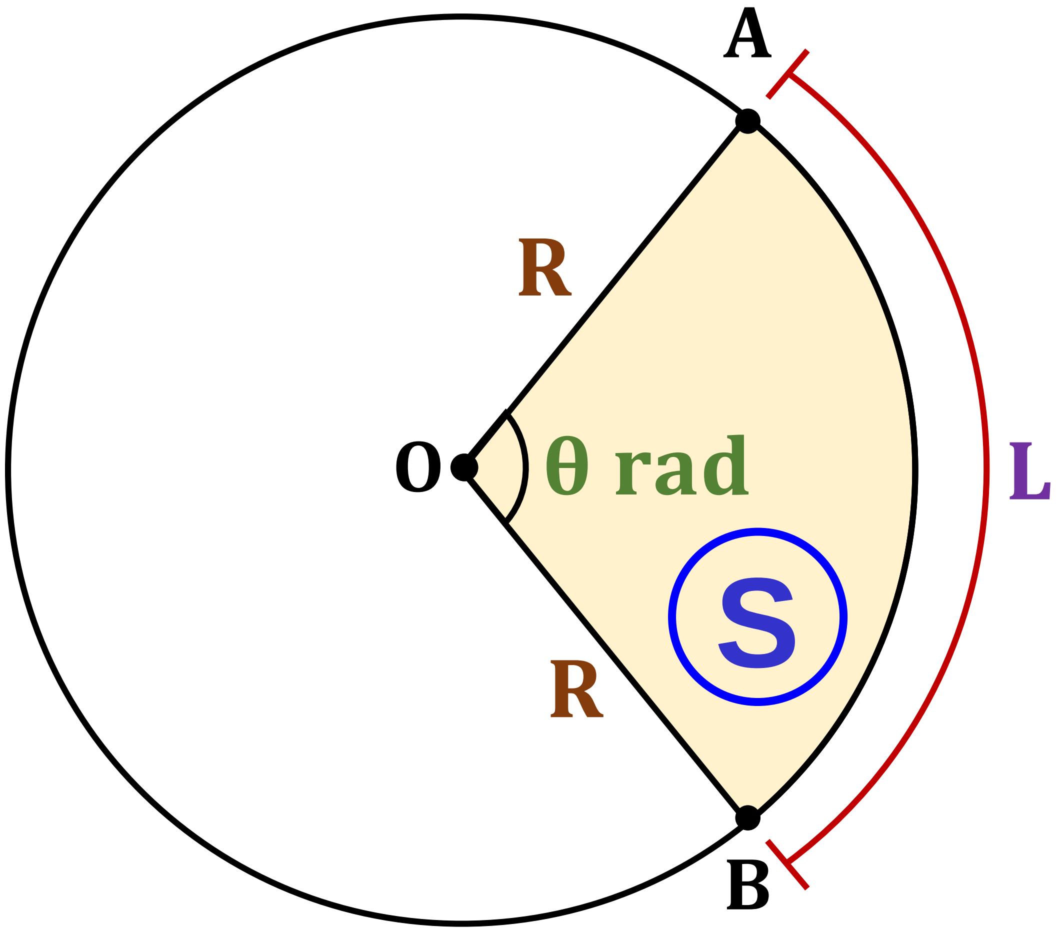
$$L = R$$

4



$$\theta = \frac{L_2 - L_1}{h}$$

ÁREA DEL SECTOR CIRCULAR



Siendo S el área sombreada del sector circular AOB :

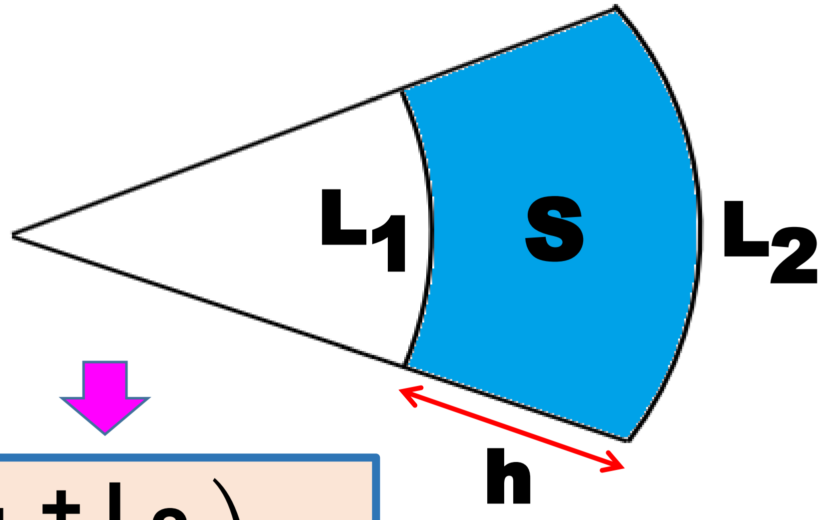
$$S = \frac{\theta \cdot R^2}{2}$$

$$S = \frac{L \cdot R}{2}$$

$$S = \frac{L^2}{2\theta}$$

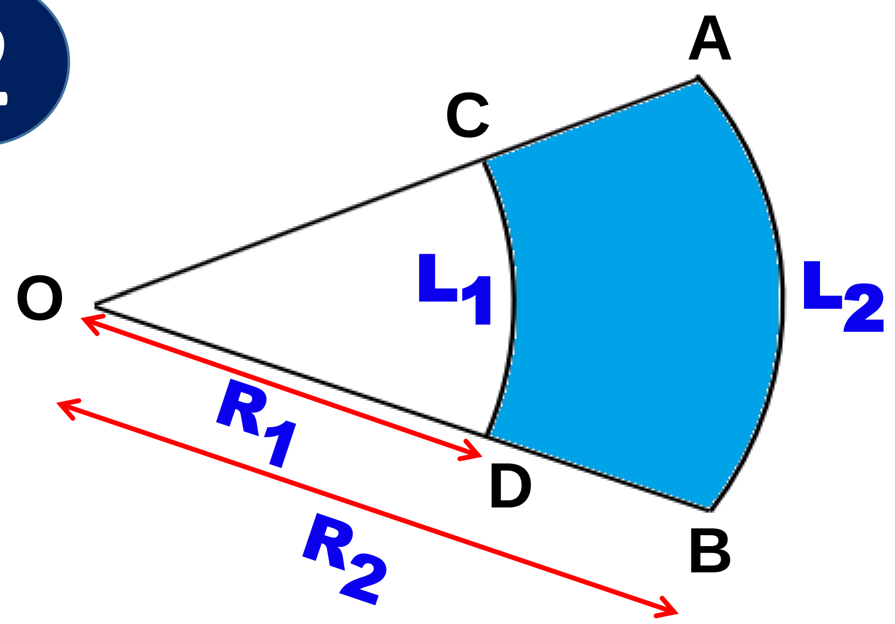
PROPIEDADES DE ÁREAS

1



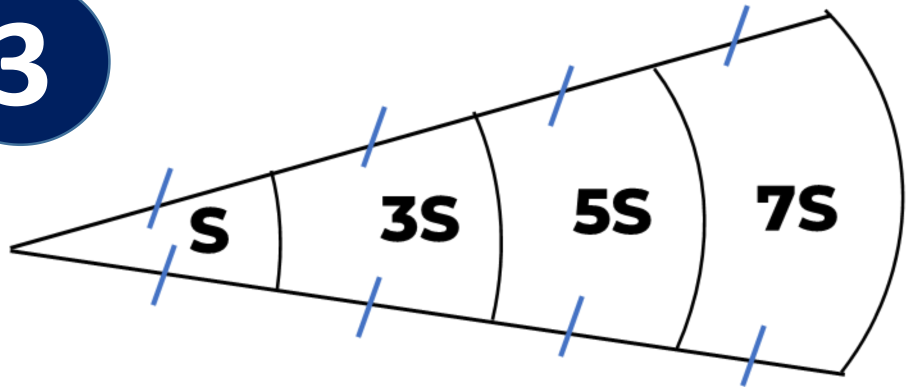
$$S = \left(\frac{L_1 + L_2}{2}\right) h$$

2



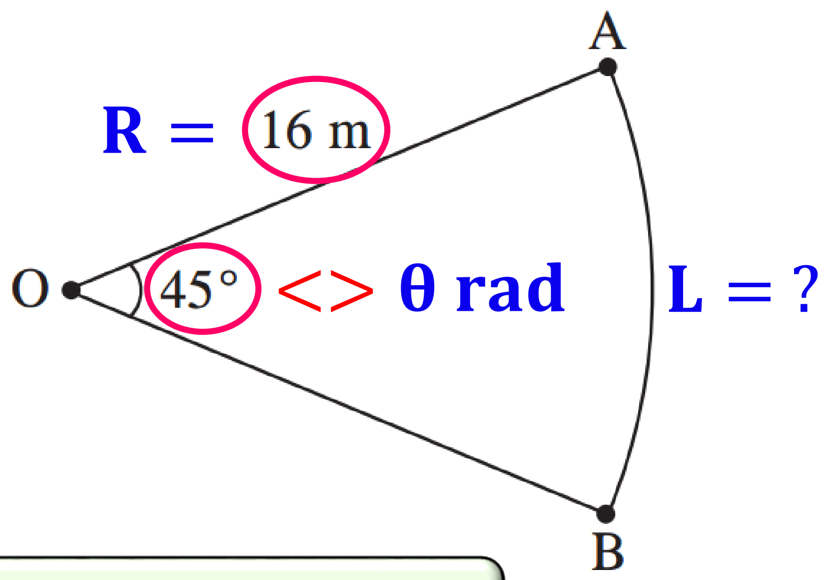
$$\frac{S_{\triangleleft COD}}{S_{\triangleleft AOB}} = \frac{R_1^{\,2}}{R_2^{\,2}} = \frac{L_1^{\,2}}{L_2^{\,2}}$$

3



HELICO PRACTICE 1

Determine la longitud del arco AB en el gráfico mostrado .



RESOLUCIÓN

Convertimos 45° a radianes :

$$45^\circ \quad <> \quad \cancel{45}^{1}{}^\circ\left(\frac{\pi \text{ rad}}{\cancel{180}_{4}{}^\circ}\right) \quad <> \quad \overbrace{\frac{\pi}{4}}^{\theta} \text{ rad}$$

Aplicamos fórmula :

$$\boxed{L = \theta \cdot R} \quad \rightarrow \quad L = \frac{\pi}{\cancel{4}_{1}} \cdot \cancel{16}^{4} \text{ m}$$

$$\boxed{\therefore \quad L = 4\pi \text{ m}}$$

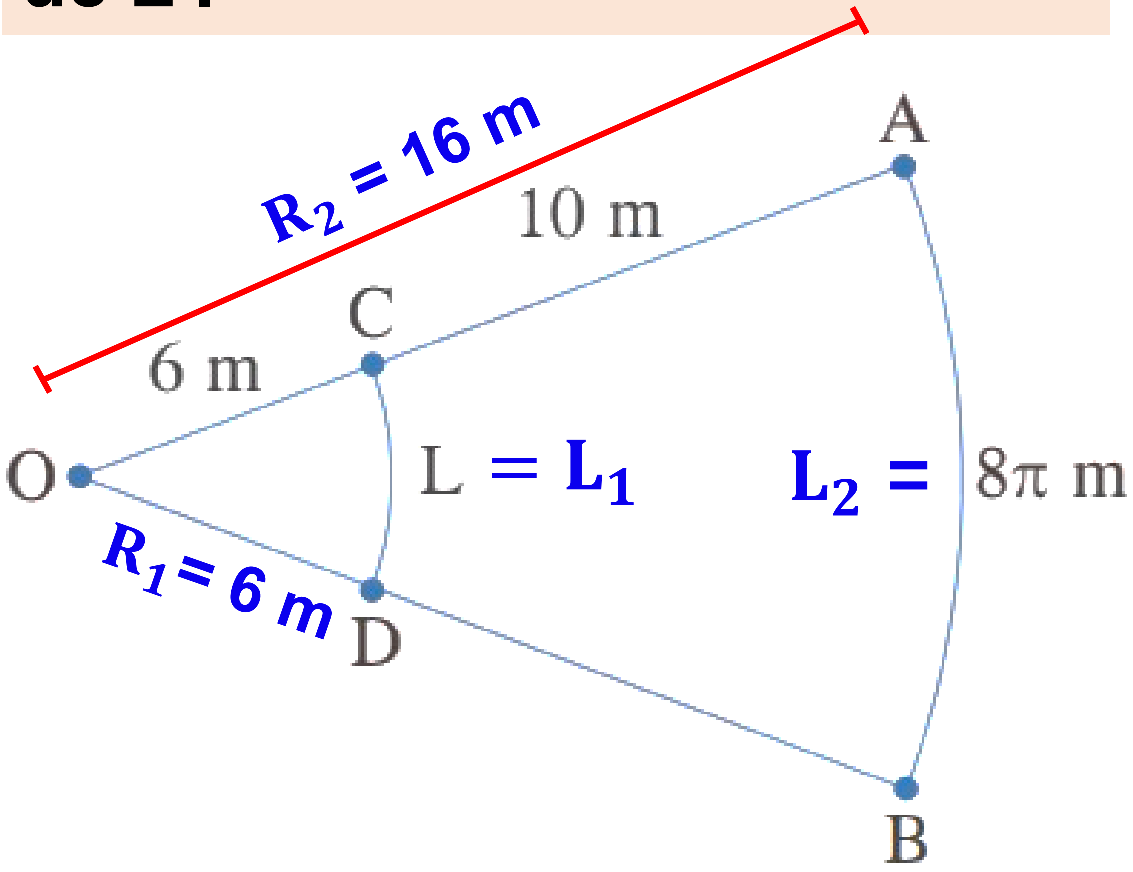
Recordamos :

Longitud de arco (L):

$$L = \theta \cdot R$$

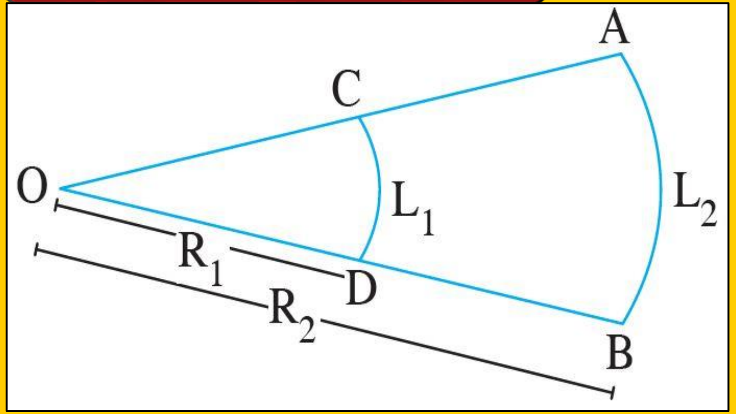
HELICO PRACTICE 2

Del gráfico, determine el valor de L .



RESOLUCIÓN

Recordamos :



$$\frac{L_1}{L_2} = \frac{R_1}{R_2}$$

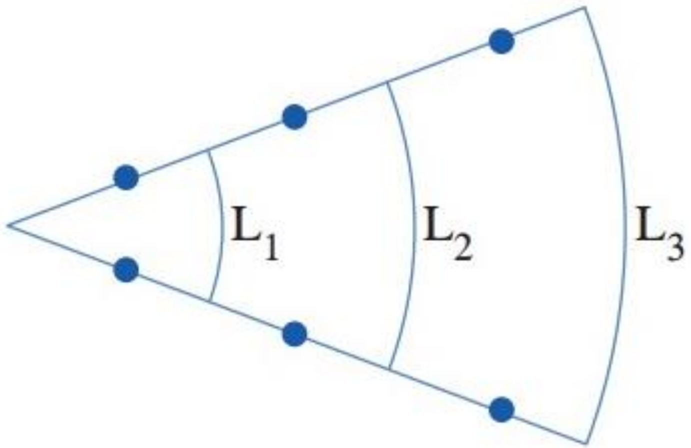
Luego : $$\frac{L}{\cancel{8}\pi\ m} = \frac{6\ \cancel{m}}{\cancel{16}_{2}\ \cancel{m}}$$

$$\therefore\ L = 3\pi\ m$$

HELICO PRACTICE 3

Del gráfico, reduzca :

$$M = \frac{5L_1 + 2L_2 + L_3}{L_3 - L_1}$$



Recordamos :

Del gráfico, por propiedad :

$\boxed{L_1 = L}$ $\boxed{L_2 = 2L}$ $\boxed{L_3 = 3L}$

RESOLUCIÓN

Reemplazamos en M :

$$M = \frac{5(L) + 2(2L) + (3L)}{(3L) - (L)}$$

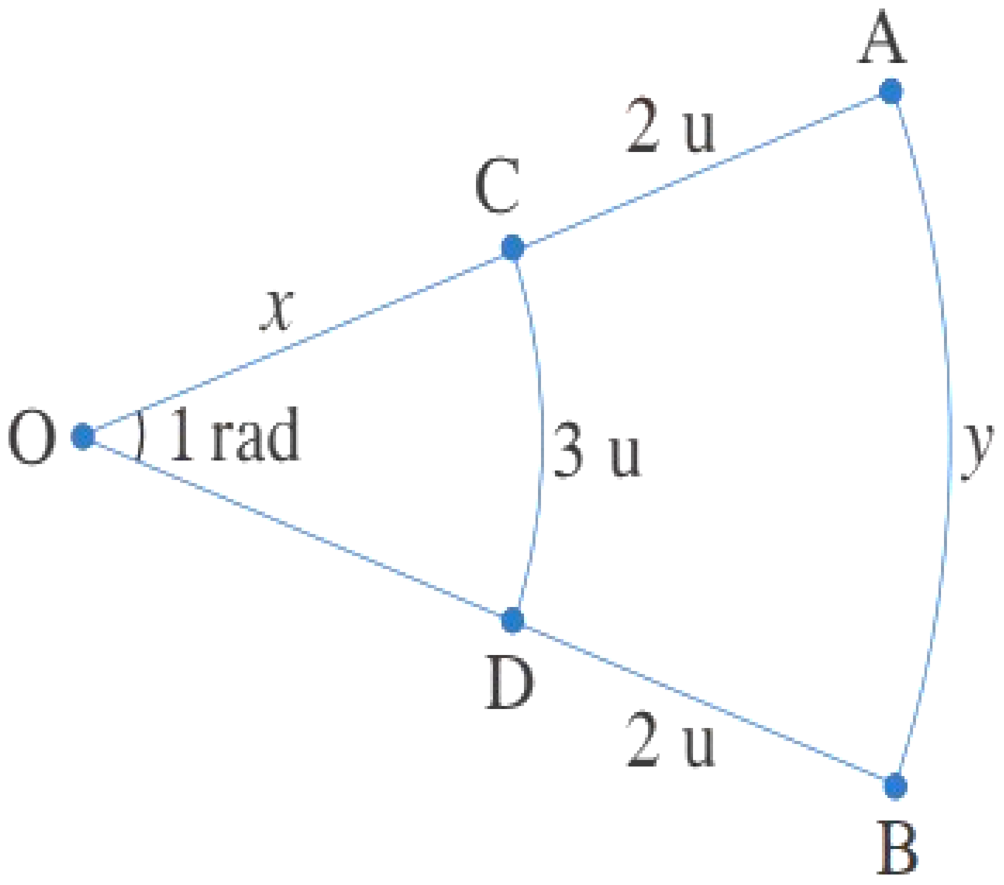
$$M = \frac{5L + 4L + 3L}{2L}$$

$$M = \frac{12\cancel{L}}{2\cancel{L}}$$

$\boxed{\therefore\ M = 6}$

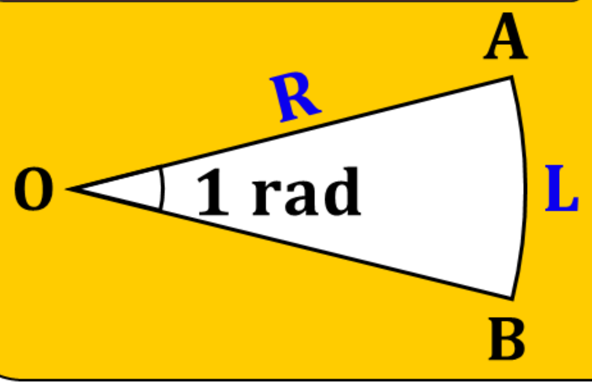
HELICO PRACTICE 4

Del gráfico, calcule $x + y$.



RESOLUCIÓN

Recordamos :



$\boxed{R = L}$

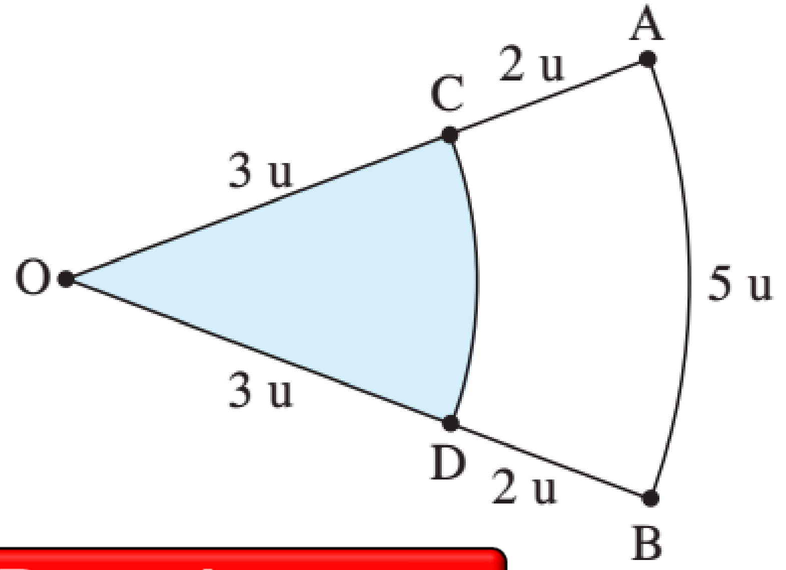
$\sphericalangle$ COD : $\boxed{x = 3\ u}$

$\sphericalangle$ AOB : $\underbrace{x}_{3\ u} + 2\ u = y \rightarrow \boxed{y = 5\ u}$

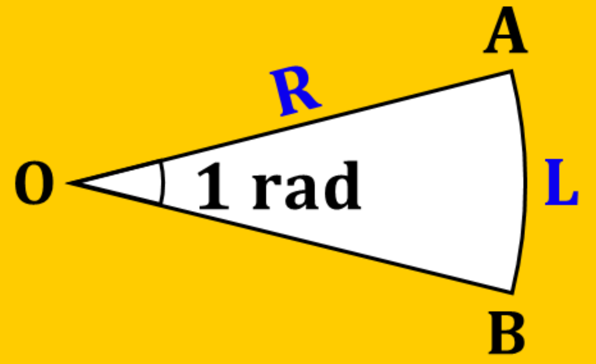
$\boxed{\therefore\ x + y = 8\ u}$

HELICO PRACTICE 5

Del gráfico, calcule el área de la región sombreada .



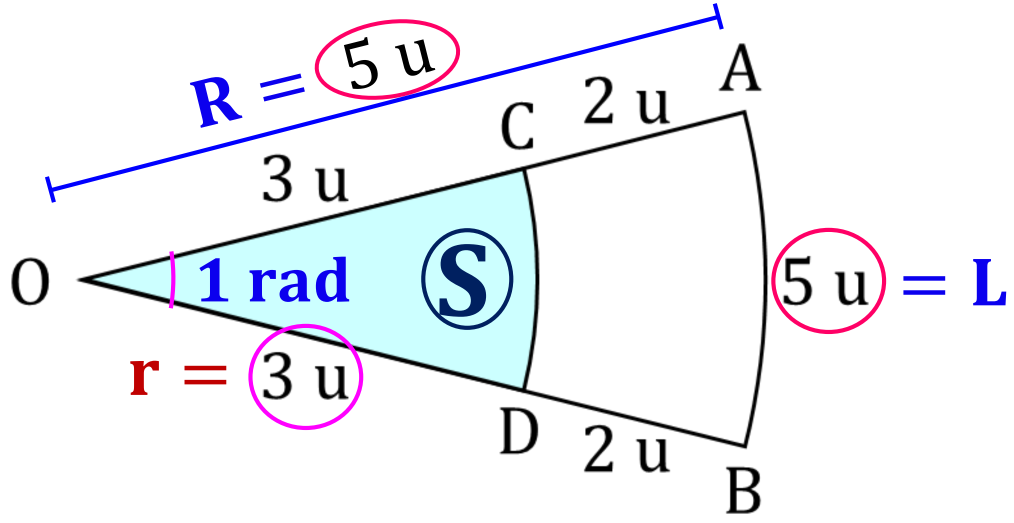
Recordamos :



$\boxed{R = L}$

RESOLUCIÓN

Analizamos el sector AOB :



Calculamos el área sombreada (S) :

Tenemos : $\boxed{\theta = 1}$ $\boxed{r = 3\ u}$

$$S = \frac{\theta \cdot r^2}{2} = \frac{1(3\ u)^2}{2} \rightarrow \boxed{S = 4,5\ u^2}$$

HELICO PRACTICE 6

Determine el área de la región que determina el borde inferior de una puerta de vaivén al girar un ángulo de 160^g, sabiendo que dicho borde mide 100 cm .

RESOLUCIÓN

Se observa que la región determinada es un sector circular :

$\boxed{m\measuredangle \text{ central} = 160^g}$ $\boxed{R = 100 \text{ cm}}$

• $160^g <> \overset{4}{\cancel{160^g}}\left(\dfrac{\pi \text{ rad}}{\underset{5}{\cancel{200^g}}}\right) = \overset{\theta}{\dfrac{4\pi}{5}} \text{rad}$

Calculamos el área S :

$$S = \frac{1}{2}\theta R^2 = \frac{1}{2}\left(\frac{4\pi}{5}\right)(100 \text{ cm})^2$$

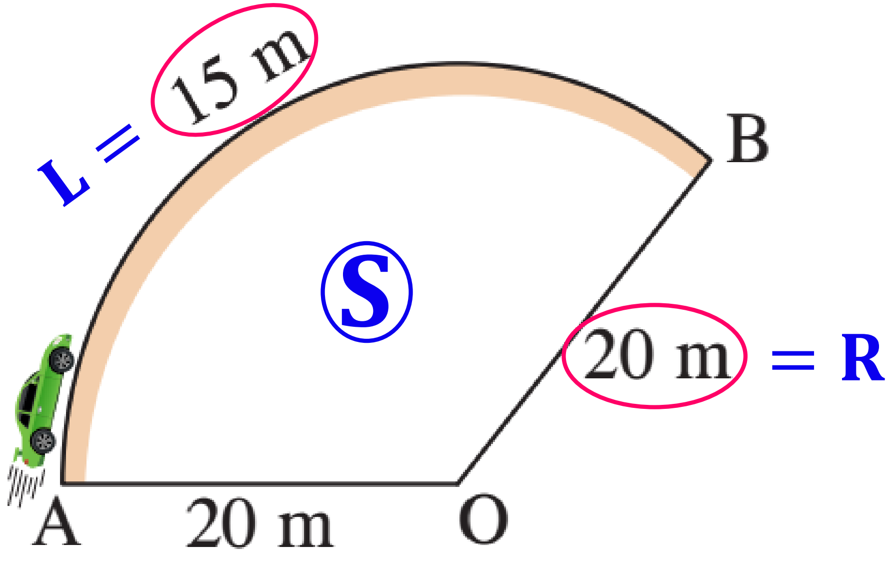
$$S = \frac{2\pi}{5}(10000 \text{ cm}^2)$$

$$\boxed{\therefore \quad S = 4000\pi \text{ cm}^2}$$

HELICO PRACTICE 7

Choper, un experimentado piloto de carrera, desea saber el costo del asfaltado de una pista circular, tal como se muestra en la figura. Sabiendo que por m^2 pagará $ 500 ,

¿ Cuánto será el costo total ?



RESOLUCIÓN

Del sector circular AOB, se tiene :

$\boxed{L = 15\text{ m}}$ $\boxed{R = 20\text{ m}}$

Calculamos el área de la pista circular (S) :

$$S = \frac{L \cdot R}{2} = \frac{15\text{ m}\,(\cancel{20}^{\,10}\text{ m})}{\cancel{2}_{\,1}} = 150\text{ m}^2$$

Calculamos el costo total (CT) del asfaltado :

$$CT = (150)(\$\,500)$$

$$\boxed{\therefore\ \text{Costo Total} = \$\,75\,000}$$

SACO
OLIVEROS