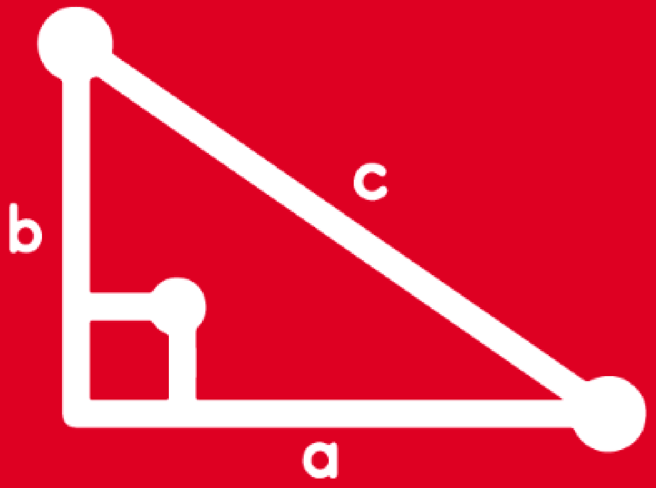
TRIGONOMETRY

Chapter 18

5th
SECONDARY



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO TRIPLE

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MOTIVATING STRATEGY

Para deducir las identidades del cos(2x), cos(3x), cos(4x), cos(5x) etc; se puede usar la siguiente expresión:

$$\cos(nx) = 2\cos(x)\cos(nx - x) - \cos(nx - 2x)$$

* Para n = 2

$$\Rightarrow \cos(2x) = 2\cos(x)\cos(2x - x) - \cos(2x - 2x)$$

$$\Rightarrow \cos(2x) = 2\cos(x)\cos(x) - \cos(0x)$$

$$\therefore\ \cos(2x) = 2\cos^2(x) - 1$$

* Para n = 3

$$\Rightarrow \cos(3x) = 2\cos(x)\cos(3x - x) - \cos(3x - 2x)$$

$$\Rightarrow \cos(3x) = 2\cos(x)\cos(2x) - \cos(x)$$

$$\Rightarrow \cos(3x) = 2\cos(x)\left[2\cos^2(x) - 1\right] - \cos(x)$$

$$\therefore\ \cos(3x) = 4\cos^3(x) - 3\cos(x)$$

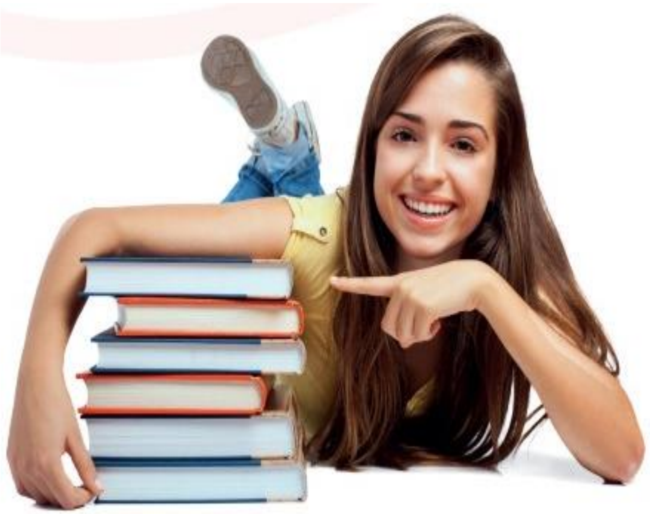
IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO TRIPLE

I) IDENTIDADES BÁSICAS :

$$\text{sen}3x = 3\,\text{sen}x - 4\,\text{sen}^3x$$

$$\cos 3x = 4\cos^3x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3x}{1 - 3\tan^2x}$$



II) IDENTIDADES AUXILIARES :

$$\text{sen}3x = \text{sen}x\,(\,2\cos 2x + 1\,)$$

$$\cos 3x = \cos x\,(\,2\cos 2x - 1\,)$$

$$4\,\text{sen}x \,.\, \text{sen}(\,60° - x\,) \,.\, \text{sen}(\,60° + x\,) = \text{sen}3x$$

$$4\cos x \,.\, \cos(\,60° - x\,) \,.\, \cos(\,60° + x\,) = \cos 3x$$

$$\tan x \,.\, \tan(\,60° - x\,) \,.\, \tan(\,60° + x\,) = \tan 3x$$

HELICO PRACTICE 1

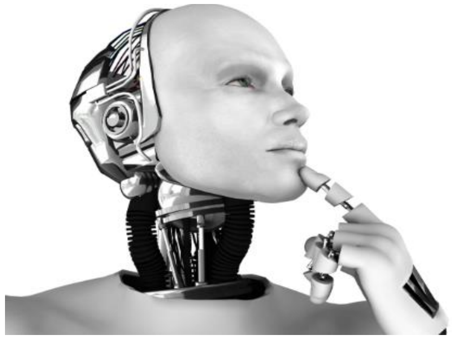
Reduzca $E = \dfrac{4\cos^3 15° - 3\cos 15°}{3\,\mathrm{sen}10° - 4\,\mathrm{sen}^3 10°}$

RESOLUCIÓN

Recordar :

$4\cos^3 x - 3\cos x = \cos 3x$

$3\,\mathrm{sen}x - 4\,\mathrm{sen}^3 x = \mathrm{sen}3x$



$$E = \frac{4\cos^3 15° - 3\cos 15°}{3\,\mathrm{sen}10° - 4\,\mathrm{sen}^3 10°}$$

$$E = \frac{\cos 3(15°)}{\mathrm{sen}3(10°)} = \frac{\cos 45°}{\mathrm{sen}30°}$$

$$E = \frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{1}{\cancel{2}}}$$

$$\therefore \; E = \sqrt{2}$$

HELICO PRACTICE 2

Si se cumple que $\text{sen}\theta = \frac{1}{3}$; calcular sen3θ

RESOLUCIÓN

Recordar :

$$\text{sen}3\theta = 3\,\text{sen}\theta - 4\,\text{sen}^3\theta$$

$$\text{sen}3\theta = \cancel{3}\left(\frac{1}{\cancel{3}}\right) - 4\left(\frac{1}{3}\right)^3$$

$$\text{sen}3\theta = 1 - 4\left(\frac{1}{27}\right) = \frac{27}{27} - \frac{4}{27}$$

$$\therefore\ \text{sen}3\theta = \frac{23}{27}$$

HELICO PRACTICE 3

Simplifique la expresión $E = \dfrac{\cos 3x - \cos x}{4\ \text{sen}^2 x}$

RESOLUCIÓN

Recordar :

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\text{sen}^2 x + \cos^2 x = 1$$

$$E = \frac{4\cos^3 x - 3\cos x - \cos x}{4\ \text{sen}^2 x}$$

$$E = \frac{4\cos^3 x - 4\cos x}{4\ \text{sen}^2 x} = \frac{\cancel{4}\cos x\,(\cos^2 x - 1)}{\cancel{4}\ \text{sen}^2 x}$$

$$E = \frac{\cos x\,(-\cancel{\text{sen}^2 x})}{\cancel{\text{sen}^2 x}}$$

$$\therefore\ E = -\cos x$$

HELICO PRACTICE 4

De la condición senx + cosx = $\frac{\sqrt{3}}{2}$, calcule sen6x .

RESOLUCIÓN

Recordar :

$$(\text{ senx} + \text{cosx })^2 = 1 + \text{sen2x}$$

$$\text{sen}3\theta = 3\ \text{sen}\theta - 4\ \text{sen}^3\theta$$

$\{ \text{ senx + cosx} = \frac{\sqrt{3}}{2} \}^2$

$1 + \text{sen2x} = \frac{3}{4}$ ➡ $\text{sen2x} = -\frac{1}{4}$

sen6x = sen3(2x)

$$\text{sen6x} = 3\ \text{sen2x} - 4\ \text{sen}^3\text{2x}$$

$$\text{sen6x} = 3\left(-\frac{1}{4}\right) - 4\left(-\frac{1}{4}\right)^3$$

$$\text{sen6x} = -\frac{3}{4} - 4\left(-\frac{1}{64}\right) = -\frac{12}{16} + \frac{1}{16}$$

$$\therefore\ \text{sen6x} = -\frac{11}{16}$$

HELICO PRACTICE 5

De la siguiente identidad : $\dfrac{3\,\text{sen}3x}{\text{sen}x} + \dfrac{2\cos 3x}{\cos x} \equiv A + B\cos(Cx)$; calcule $A + B + C$.

RESOLUCIÓN

Recordar : $\text{sen}3x = \text{sen}x\,(2\cos 2x + 1)$ $\qquad \cos 3x = \cos x\,(2\cos 2x - 1)$

$$A + B\cos(Cx) \equiv \frac{3\,\cancel{\text{sen}x}\,(2\cos 2x + 1)}{\cancel{\text{sen}x}} + \frac{2\,\cancel{\cos x}\,(2\cos 2x - 1)}{\cancel{\cos x}}$$

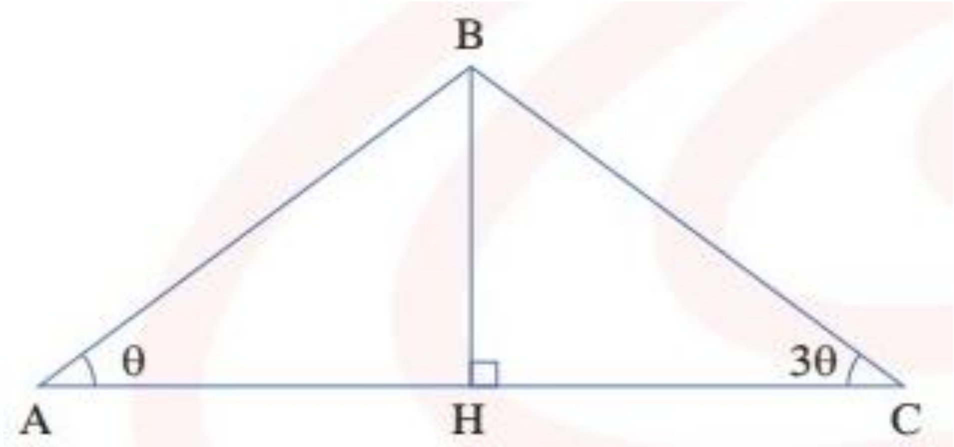
$$A + B\cos(Cx) \equiv 6\cos 2x + 3 + 4\cos 2x - 2$$

$$A + B\cos(Cx) \equiv 1 + 10\cos(2x)$$

$$A + B + C = 1 + 10 + 2 \qquad \boxed{\therefore\ A + B + C = 13}$$

HELICO PRACTICE 6

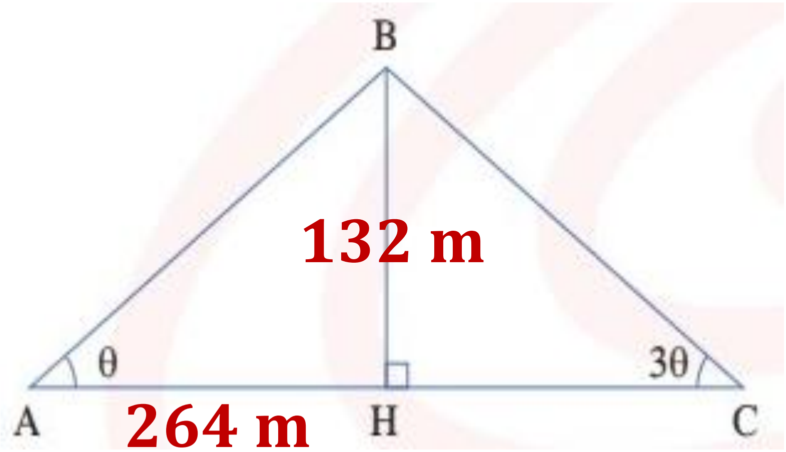
Se construye un centro comercial sobre un terreno con forma de triángulo ABC como se muestra en la figura :



Si $BH = 132$ m y AH = 264 m …
¿ Cuál es la longitud de $\overline{HC}$? .

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

RESOLUCIÓN



$$\tan\theta = \frac{132\text{ m}}{264\text{ m}}$$

$$\tan\theta = \frac{1}{2}$$

$$\tan 3\theta = \frac{132\text{ m}}{HC}$$

$$\tan 3\theta = \frac{3\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3}{1 - 3\left(\frac{1}{2}\right)^2} = \frac{\frac{3}{2} - \frac{1}{8}}{1 - 3\left(\frac{1}{4}\right)} = \frac{\frac{12}{8} - \frac{1}{8}}{\frac{4}{4} - \frac{3}{4}}$$

$$\tan 3\theta = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{2}$$

$$\frac{\overset{12}{\cancel{132}}\text{ m}}{HC} = \frac{\cancel{11}}{2}$$

$\therefore$ HC = 24 m

HELICO PRACTICE 7

Una mariposa vuela a una cierta altura h (en metros) en el instante de tiempo t (en segundos) , la cual se determina según :

$$h = 8\,\text{sen}\left(\frac{t\pi}{18}\right).\,\text{sen}\left(\frac{\pi}{3}-\frac{t\pi}{18}\right).\,\text{sen}\left(\frac{\pi}{3}+\frac{t\pi}{18}\right);\ 0 \leq t \leq 6\,.$$

Halle en qué tiempo la mariposa se encuentra una altura de 1,73 m por primera vez .

RESOLUCIÓN

Recordar :

$$4\,\text{senx}\,.\,\text{sen}\left(\frac{\pi}{3}-x\right).\,\text{sen}\left(\frac{\pi}{3}+x\right) = \text{sen}3x$$

$$h = 2\,.\,\underbrace{4\,\text{sen}\left(\frac{t\pi}{18}\right).\,\text{sen}\left(\frac{\pi}{3}-\frac{t\pi}{18}\right).\,\text{sen}\left(\frac{\pi}{3}+\frac{t\pi}{18}\right)}\,.\,m$$

$$h = 2\,.\,\text{sen}\left(\frac{3t\pi}{18}\right).\,m$$

$$h = 2\,.\,\text{sen}\left(\frac{t\pi}{6}\right).\,m$$

Cuando h = 1,73 m

$$2\,.\,\text{sen}\left(\frac{t\pi}{6}\right).\,\cancel{m} = \sqrt{3}\,\cancel{m}$$

$$\text{sen}\left(\frac{t\pi}{6}\right) = \frac{\sqrt{3}}{2} = \text{sen}\left(\frac{\pi}{3}\right)$$

$$\frac{t\cancel{\pi}}{6} = \frac{\cancel{\pi}}{3}$$

$$\therefore\ t = 2\ \text{segundos}$$

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