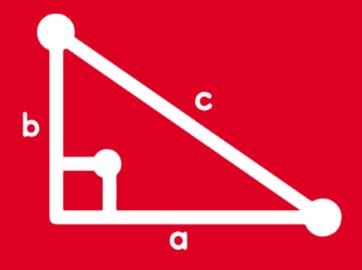
TRIGONOMETRY TOMO VI











Simplifique la expresión $E = 3(\cot x - \tan x)\tan 2x$

Resolución:





cotx - tanx =

 $2\cot(2x)$

$$E = 3 (cotx - tanx)tan2x$$

$$E = 6 \cot 2x \cdot \tan 2x$$

t

$$\therefore E = 6$$

Al copiar de la pizarra la expresión 1 + cos40°, un estudiante cometió un error y escribió sen40°. Calcule la razón entre lo que estaba escrito en la pizarra y lo que copió el estudiante.



Simplifique
$$M = \sqrt{2 + \sqrt{2 - 2\cos 80^{\circ}}}$$





 $2sen^{2}(x) = 1 - cos(2x)$

 $2\cos^2(x) = 1 + \cos(2x)$

$$M = \sqrt{2 + \sqrt{2 - 2\cos 80^{\circ}}}$$

$$M = \sqrt{2 + \sqrt{2(1 - \cos 80^\circ)}}$$

$$M = \sqrt{2 + \sqrt{2(2sen^240^\circ)}}$$

$$M = \sqrt{2 + \sqrt{4 \text{sen}^2 40^\circ}}$$

$$M = \sqrt{2 + 2 \operatorname{sen} 40^{\circ}}$$

$$M = \sqrt{2(1 + \sin 40^\circ)}$$

$$M = \sqrt{2(1 + \cos 50^\circ)}$$

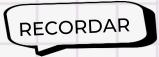
$$M = \sqrt{2(2\cos^2 25^\circ)}$$

$$M = \sqrt{4\cos^2 25^\circ}$$

 \therefore M = 2cos25°

Reduzca
$$H = \sqrt{\frac{1 + \cos 140^{\circ}}{2}} + \frac{\sin 40^{\circ}}{2\cos 20^{\circ}}$$

Resolución:





$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$H = \sqrt{\frac{1 + \cos 140^{\circ}}{2}} + \frac{\sin 40^{\circ}}{2\cos 20^{\circ}}$$

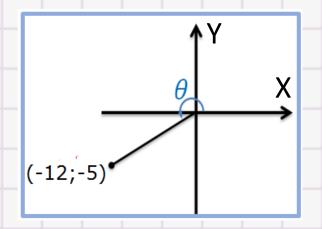
$$H = \cos 70^{\circ} + \frac{Z \operatorname{sen20^{\circ}.cos20^{\circ}}}{Z \operatorname{cos20^{\circ}}}$$

$$H = \cos 70^{\circ} + \underline{\sin 20^{\circ}}$$

$$H = \cos 70^{\circ} + \cos 70^{\circ}$$

$$\therefore H = 2\cos 70^{\circ}$$

Del gráfico, calcule: $\tan\left(\frac{\theta}{2}\right)$



Resolución:

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Del gráfico se observa:

$$180^{\circ} < \theta < 270^{\circ}$$

$$\rightarrow 90^{\circ} < \frac{\theta}{2} < 135^{\circ}$$

$$\theta \in IIC$$

Además:

$$x = -12$$
 ; $y = -5$

$$r = \sqrt{(-12)^2 + (-5)^2}$$

$$\longrightarrow$$
 r = 13

$$\cos\theta = \frac{X}{r} = -\frac{12}{13}$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

$$\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1 - \left(-\frac{12}{13}\right)}{1 + \left(-\frac{12}{13}\right)}}$$

$$\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{\frac{25}{13}}{\frac{1}{13}}} = -\sqrt{25}$$

$$\tan\left(\frac{\theta}{2}\right) = -5$$

Reduzca la expresión:
$$P = \frac{\cot\left(\frac{x}{2}\right)}{\csc x - \tan x}$$

$$= \frac{\cot\left(\frac{x}{2}\right) - \csc x}{\csc x - \tan\left(\frac{x}{2}\right)}$$

Resolución:





$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

$$P = \frac{\cot\left(\frac{x}{2}\right) - \csc x}{\csc x - \tan\left(\frac{x}{2}\right)}$$

$$P = \frac{\csc x + \cot x - \csc x}{\csc x - (\csc x - \cot x)}$$

$$P = \frac{\csc x + \cot x - \csc x}{\csc x - \csc x + \cot x}$$

$$P = \frac{\cot x}{\cot x}$$

Reduzca:
$$T = \frac{4\cos^3 20^\circ - 3\cos 20^\circ}{3\sin 15^\circ - 4\sin^3 15^\circ}$$

Resolución:





$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$T = \frac{4\cos^{3}20^{\circ} - 3\cos20^{\circ}}{3\sin15^{\circ} - 4\sin^{3}15^{\circ}}$$

$$T = \frac{\cos 3(20^{\circ})}{\sin 3(15)}$$

$$T = \frac{\cos 60^{\circ}}{\sin 45^{\circ}}$$

$$T = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}$$

$$\therefore T = \frac{\sqrt{2}}{2}$$

De la condición: $\operatorname{senx} - \cos x = \frac{\sqrt{2}}{2}$; calcule sen6x.

Resolución:

Dato:

$$\operatorname{senx} - \cos x = \frac{\sqrt{2}}{2}$$

Elevamos al cuadrado:

$$(\mathbf{senx} - \mathbf{cosx})^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

$$1 - \sin 2x = \frac{2}{4}$$

$$sen2x = \frac{1}{2}$$

Calculamos:

$$sen6x = sen3(2x)$$

$$sen6x = 3sen2x - 4sen^32x$$

$$sen6x = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$sen6x = \frac{3}{2} - \frac{1}{2}$$

$$\therefore$$
 sen6x = 1

RECORDAR

$$(\operatorname{senx} - \operatorname{cosx})^2 = 1 - \operatorname{sen2x}$$

$$sen3\alpha = 3sen\alpha - 4sen^3\alpha$$

De la siguiente identidad:

$$\frac{3\text{sen3x}}{\text{senx}} - \frac{2\text{cos3x}}{\text{cosx}} = M + N\text{cos(Px)}$$

Calcule: M + N + P

Resolución:

Dato:

$$\frac{3\text{sen3x}}{\text{senx}} - \frac{2\cos 3x}{\cos x} = M + N\cos(Px)$$

$$\frac{3\operatorname{senx}(2\cos 2x + 1)}{\operatorname{senx}} - \frac{2\cos x(2\cos 2x - 1)}{\cos x} = M + N\cos(Px)$$

$$3(2\cos 2x + 1) - 2(2\cos 2x - 1) = M + N\cos(Px)$$

$$6\cos 2x + 3 - 4\cos 2x + 2 = M + N\cos(Px)$$





sen3x = senx(2cos2x + 1)

$$\cos 3x = \cos x(2\cos 2x - 1)$$

$$5 + 2\cos 2x = M + N\cos(Px)$$

Comparando:

$$M = 5$$
; $N = 2$; $P = 2$

$$\therefore M + N + P = 9$$

Un científico observa el movimiento de una mariposa en el aire y ve que en un instante de tiempo t, la altura en metros respecto al suelo está dado por la siguiente expresión: h(t)=16sentcos2tcos4tcos8t, si t está en segundos. ¿A qué altura se encuentra para t = $\frac{\pi}{30}$ seg?

Resolución:

sen2t

sen4t

$$h(\frac{\pi}{30}).\cos 6^{\circ} = \sec 96^{\circ}$$

$$h(\frac{\pi}{30}).\cos 6^\circ = \sin(90^\circ + 6^\circ)$$

$$h(\frac{\pi}{30}).\cos 6^\circ = \cos 6^\circ \Rightarrow h(\frac{\pi}{30}) = 1$$

∴ La altura de la mariposa para
$$t = \frac{\pi}{30}$$
seg es 1m.

Dato:

