ALGEBRA

3th
SECONDARY

HELICOASESORIA

Tomo I

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RESUMEN TEÓRICO

1. $x^a . x^b = x^{a+b}$

2. $\dfrac{x^a}{x^b} = x^{a-b}$

3. $(x^a)^b = x^{ab}$

4. $x^{-a} = \dfrac{1}{x^a}$

5. $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

6. $\sqrt[m]{x^a} . \sqrt[m]{x^b} = \sqrt[m]{x^{a+b}}$

7. $\sqrt[m]{x^a \sqrt[n]{x^b \sqrt[p]{x^c}}} = \sqrt[mnp]{x^{(an+b)p+c}}$

8. $Si\ x^a = x^b, entonces\ a = b$

1 Efectuar

$$P = \frac{\left(\frac{1}{5}\right)^{-2} + \left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{3}\right)^{-2}}{\left(\frac{1}{24}\right)^{-1} - \left(\frac{1}{3}\right)^{-1}}$$

Recordemos:

$$\left(\frac{A}{B}\right)^{-n} = \left(\frac{B}{A}\right)^{n}$$

Resolución:

$$P = \frac{5^2 + 2^3 + 3^2}{24^1 - 3^1} = \frac{25 + 8 + 9}{24 - 3} = \frac{42}{21}$$

$$\boxed{\therefore P = 2}$$

2 Simplifique

$$T = \frac{81^{n+1} . 27^{2n+3}}{243^{2n+2}}$$

Recordemos:

$$(a^m)^n = a^{m.n}$$

Resolución:

Descomponiendo las bases:

$$T = \frac{\left(3^4\right)^{n+1} . \left(3^3\right)^{2n+3}}{\left(3^5\right)^{2n+2}} = \frac{3^{4n+4} . 3^{6n+9}}{3^{10n+10}} = \frac{3^{\cancel{10n}+13}}{3^{\cancel{10n}+10}} = 3^3$$

$$\boxed{\therefore T = 27}$$

3

Reduzca

$$W = \frac{5^{3m+2} + 5^{3m+1} + 5^{3m+3}}{5^{3m+1}}$$

Recordemos:

$$a^{m+n} = a^m . a^n$$

Resolución:

$$W = \frac{5^{3m}.5^2 + 5^{3m}.5^1 + 5^{3m}.5^3}{5^{3m}.5^1} = \frac{\cancel{5^{3m}}(5^2 + 5^1 + 5^3)}{\cancel{5^{3m}}.5^1}$$

$$W = \frac{25 + 5 + 125}{5} = \frac{155}{5}$$

$$\boxed{\therefore W = 31}$$

4

Siendo $3^{-x} = \dfrac{1}{2}$

evalúe $Z = 27^{x} + 81^{x} - 9^{x}$

Resolución:

$$Z = 27^{x} + 81^{x} - 9^{x}$$

$$Z = \left(3^{3}\right)^{x} + \left(3^{4}\right)^{x} - \left(3^{2}\right)^{x}$$

$$Z = \left(3^{x}\right)^{3} + \left(3^{x}\right)^{4} - \left(3^{x}\right)^{2}$$

Reemplazando:

$$Z = 2^{3} + 2^{4} - 2^{2}$$

$$Z = 8 + 16 - 4$$

$$\boxed{\therefore Z = 20}$$

Recordemos:

$$\boxed{(a^{m})^{n} = (a^{n})^{m}}$$

$$\boxed{a^{-n} = \frac{1}{a^{n}}}$$

Del dato:

$$3^{-x} = \frac{1}{2}$$

$$\frac{1}{3^{x}} = \frac{1}{2} \rightarrow 3^{x} = 2$$

5

El valor reducido de A representa la edad de Alonso

$$A = \sqrt[5^{m+1}]{\sqrt[5^{m+2}]{\sqrt[5^{m+3}]{2^{5^{3m+7}}}}}$$

¿Cuántos años tiene Alonso?

Recordemos:

$$\sqrt[m]{\sqrt[n]{\sqrt[p]{a}}} = \sqrt[mnp]{a}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Resolución:

$$A = \sqrt[5^{m+1}.5^{m+2}.5^{m+3}]{2^{5^{3m+7}}} = \sqrt[5^{3m+6}]{2^{5^{3m+7}}}$$

$$A = 2^{\frac{5^{3m+\cancel{7}}}{5^{3m+\cancel{6}}}} = 2^{5^1} = 2^5$$

$$\boxed{\therefore A = 32}$$

6 *Simplifique*

$$P=\sqrt[10]{\frac{\sqrt{5}.\sqrt{5}.\sqrt{5}\ldots\ldots\sqrt{5}\quad(50\ factores)}{\sqrt[5]{5}.\sqrt[5]{5}.\sqrt[5]{5}\ldots\ldots\sqrt[5]{5}\quad(25\ factores)}}$$

Resolución:

Recordemos:

$$\underbrace{a.a.a\ldots\ldots a}_{n\ factores}=a^{n}$$

$$P=\sqrt[10]{\frac{\cancel{\sqrt{5}}^{\cancel{50}}}{\cancel{\sqrt[5]{5}}^{\cancel{25}}}}=\sqrt[10]{\frac{5^{25}}{5^{5}}}=\sqrt[\cancel{10}]{5^{\cancel{20}}}=5^{2}$$

$$\boxed{\therefore P=25}$$

7

Determine el valor de "x"

$$3^{x+4} + 3^{x+3} + 3^{x+1} = 2997$$

Recordemos:

$$a^{m+n} = a^m . a^n$$

Resolución:

$$3^x . 3^4 + 3^x . 3^3 + 3^x . 3^1 = 2997$$

$$3^x(3^4 + 3^3 + 3^1) = 2997$$

$$3^x(81 + 27 + 3) = 2997$$

$$3^x(111) = 2997$$

$$3^x = \frac{2997}{111}$$

$$3^x = 27$$

$$3^x = 3^3$$

$$\boxed{\therefore x = 3}$$

8

Halle el valor de "x"

$$x^{x^{\frac{1}{3}}} = \frac{1}{3}$$

Recordemos:

$$(a^m)^n = (a^n)^m$$

Resolución:

Elevando ambos miembros a la $\frac{1}{3}$

$$\left(x^{x^{\frac{1}{3}}}\right)^{\frac{1}{3}} = \left(\frac{1}{3}\right)^{\frac{1}{3}}$$

$$\left(x^{\frac{1}{3}}\right)^{x^{\frac{1}{3}}} = \left(\frac{1}{3}\right)^{\frac{1}{3}}$$

$$x^{\frac{1}{3}} = \frac{1}{3}$$

$$x = \left(\frac{1}{3}\right)^{3}$$

$$\boxed{\therefore\ x = \frac{1}{27}}$$

9 Simplifique

$$W = 25^{8^{-27^{-3^{-1}}}}$$

Exponente de exponente

$$a^{b^{c^{d}}} = a^{b^{m}} = a^{n} = p$$

Exponente fraccionario

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \sqrt[n]{a}^{\,m}, m \in \mathbb{R} \wedge n \geq 2$$

Resolución:

$$W = 25^{\left(\frac{1}{8}\right)^{\left(\frac{1}{27}\right)^{\left(\frac{1}{3}\right)}}}$$

$$\left(\frac{1}{27}\right)^{\left(\frac{1}{3}\right)} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$$

$$W = 25^{\left(\frac{1}{8}\right)^{\left(\frac{1}{3}\right)}}$$

$$\left(\frac{1}{8}\right)^{\left(\frac{1}{3}\right)} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

$$W = 25^{\left(\frac{1}{2}\right)}$$

$$25^{\left(\frac{1}{2}\right)} = \sqrt{25} = 5$$

$$\boxed{\therefore W = 5}$$

10 Indique el exponente final de "x"

$$P = \sqrt[3]{x^3.\sqrt{x^4.\sqrt{x}}}.\sqrt[12]{x^3}$$

Resolución:

Recordemos:

RADICALES SUCESIVOS:

$$\sqrt[m]{x^a.\sqrt[n]{x^b.\sqrt[p]{x^c}}} = \sqrt[mnp]{x^{(an+b)p+c}}$$

$$P = \sqrt[3]{x^3.\sqrt[2]{x^4.\sqrt[2]{x^1}}}.\sqrt[12]{x^3}$$

Reduciendo P:

$$P = \sqrt[3.2.2]{x^{(3.2+4).2+1}}.\sqrt[12]{x^3}$$

$$P = \sqrt[12]{x^{21}}.\sqrt[12]{x^3}$$

$$P = \sqrt[\cancel{12}]{x^{2\cancel{4}}}$$

$$\boxed{P = x^2}$$

$$\boxed{\therefore Exp = 2}$$

GRACIAS POR SU ATENCIÓN!!