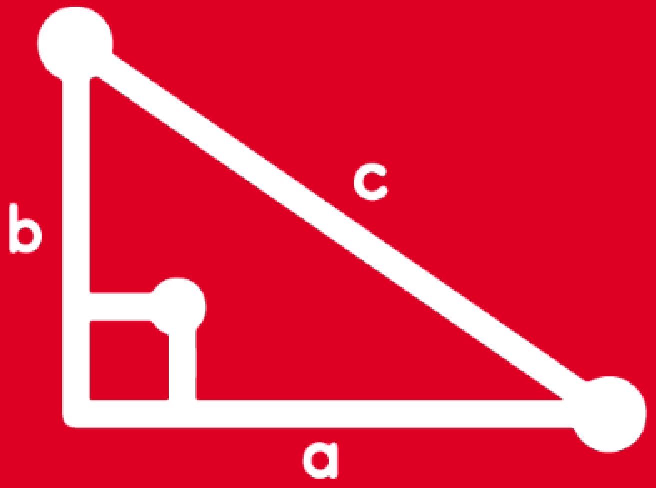
TRIGONOMETRY

VOLUME III

5th
SECONDARY



FEEDBACK

SACO OLIVEROS

1) **Siendo θ y β las medidas de dos ángulos cuadrantales diferentes, positivos y menores o iguales a 360°, se cumple que**

$$\sqrt{1-\operatorname{sen}\theta}+\sqrt{\operatorname{sen}\theta-1}=1+\cos\beta \quad \ldots\ldots (*)$$

calcule $\theta + \beta$.

Resolución:

$$1-\operatorname{sen}\theta \geq 0 \quad \wedge \quad \operatorname{sen}\theta-1 \geq 0$$

$$\underbrace{1 \geq \operatorname{sen}\theta \quad \wedge \quad \operatorname{sen}\theta \geq 1}$$

$$\operatorname{sen}\theta = 1$$

$$\text{como } 0° < \theta \leq 360°$$

$$\boxed{\theta = 90°}$$

Reemplazamos en (*)

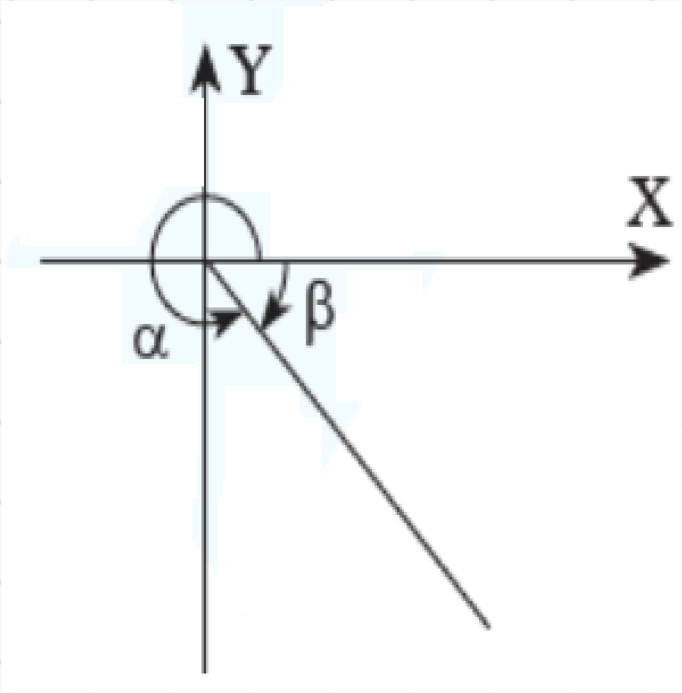
$$\underbrace{\cancel{\sqrt{1-\operatorname{sen}\theta}}}_{0}+\underbrace{\cancel{\sqrt{\operatorname{sen}\theta-1}}}_{0}=1+\cos\beta$$

$$-1 = \cos\beta \quad \boxed{\beta = 180°}$$

Recordar

RT \ ∢	0°	90°	180°	270°	360°
sen	0	1	0	−1	0
cos	1	0	−1	0	1

2) **En la figura, se cumple que $\cot\alpha \cdot \cot\beta + \cos\alpha \cdot \sec\beta = 10$. Calcule $\cot\alpha$.**

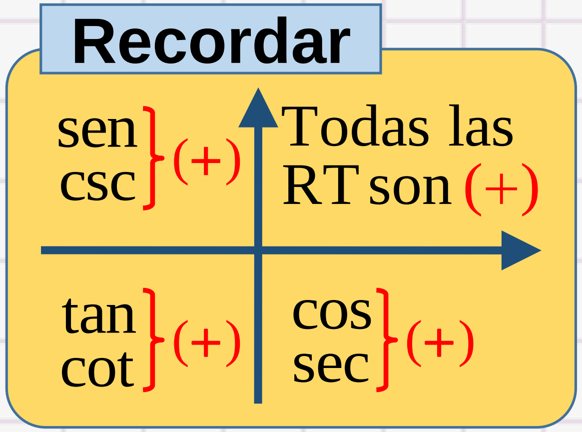


Resolución:

Del gráfico se observa que α y β son las medidas de dos ángulos coterminales:

$$RT(\alpha) = RT(\beta)$$

Recordar



Del dato:

$$\cot\alpha \cdot \cot\beta + \cos\alpha \cdot \sec\beta = 10$$

$$\cot\alpha \cdot \boldsymbol{\cot\alpha} + \underbrace{\boxed{\cos\alpha \cdot \boldsymbol{\sec\alpha}}}_{1} = 10$$

$$\cot^2\alpha + 1 = 10$$

$$\cot^2\alpha = 9 \Rightarrow \cot\alpha = \pm 3$$

Como $\alpha \in$ IVC

$$\therefore \boxed{\cot\alpha = -3}$$

3) **Si $\cos\theta > 0$, además $16^{\cot\theta} = 0,25$, efectúe**

$$\mathbf{P = \sqrt{5}(\operatorname{sen}\theta - \cos\theta)}$$

Resolución:

Del dato: $16^{\cot\theta} = \dfrac{1}{4}$

$$16^{\cot\theta} = 4^{-1}$$

$$(4^2)^{\cot\theta} = 4^{-1}$$

$$4^{2\cot\theta} = 4^{-1}$$

$$\rightarrow 2\cot\theta = -1$$

$$\left.\rightarrow \cot\theta = -\frac{1}{2}\right\} \begin{array}{l}\cot\theta: (-)\\ \cos\theta: (+)\end{array} \Rightarrow \underbrace{\theta \in \mathrm{IVC}}_{\begin{array}{l}x: (+)\\ y: (-)\end{array}}$$

$$\cot\theta = \frac{1}{-2} = \frac{x}{y}$$

Por $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{1^2 + (-2)^2}$$

$$r = \sqrt{5}$$

Efectuamos $P = \sqrt{5}\left(\dfrac{-2}{\sqrt{5}} - \dfrac{1}{\sqrt{5}}\right) = \cancel{\sqrt{5}}\left(\dfrac{-3}{\cancel{\sqrt{5}}}\right)$

$$\therefore \boxed{\mathbf{P = -3}}$$

4) Simplifique

$$E = \frac{\text{sen}330°.\cos120°}{\tan225°}$$

Resolución:

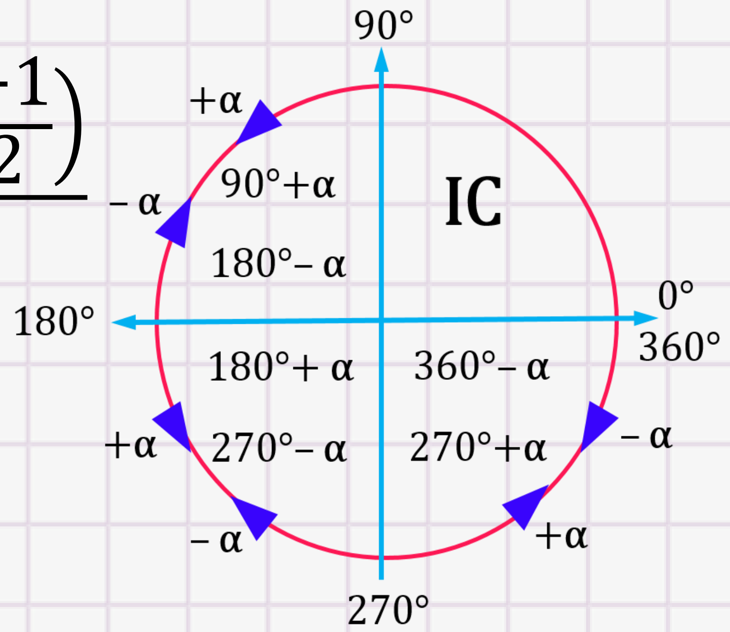
$$E = \frac{\overset{(-)}{\text{sen}}\overbrace{(360° - 30°)}^{\text{IVC}} \cdot \overset{(-)}{\cos}\overbrace{(180° - 60°)}^{\text{IIC}}}{\underset{(+)}{\tan}\underbrace{(180° + 45°)}_{\text{IIIC}}}$$

$$E = \frac{(-\text{sen}30°)(-\cos60°)}{\tan45°}$$

$$E = \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)}{1}$$

$$E = \frac{\frac{1}{4}}{1}$$

$$\therefore \boxed{E = \frac{1}{4}}$$

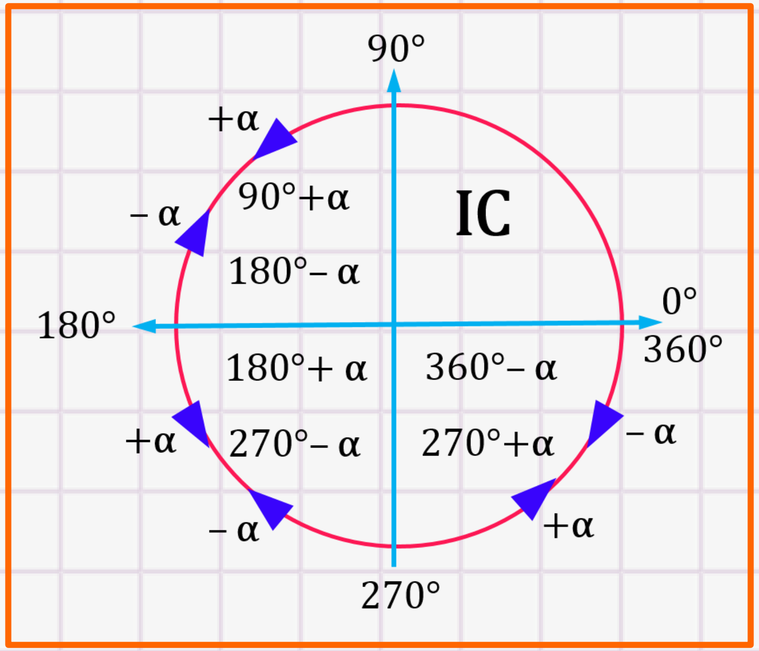


Recordar

sen, csc } (+) | Todas las RT son (+)

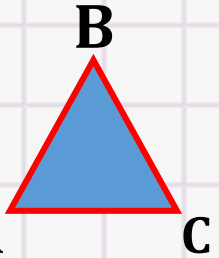
tan, cot } (+) | cos, sec } (+)

5) En un triángulo ABC, reduzca $M = \dfrac{\operatorname{sen}(B+C)}{\cos\left(\dfrac{3A+B+C}{2}\right)}$



Resolución:

Del dato:



$$A + B + C = 180°$$

$$\rightarrow \mathbf{B + C = 180° - A}$$

Reducimos:

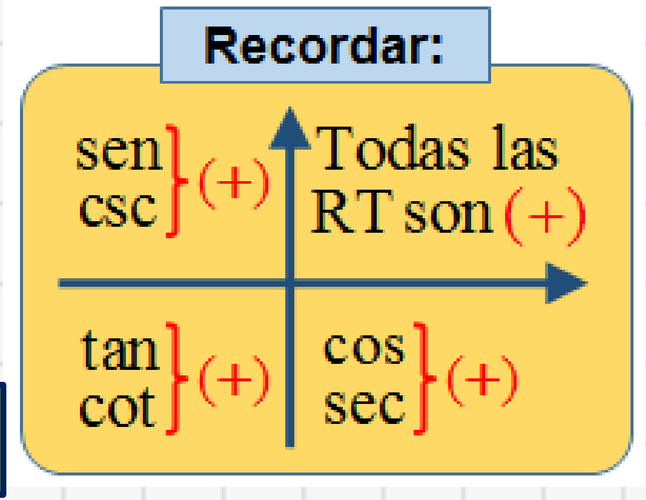
$$M = \frac{\overbrace{\operatorname{sen}(\underbrace{180° - A}_{\text{IIC}})}^{(+)}}{\cos\left(\dfrac{2A + \boxed{A + B + C}^{\,180°}}{2}\right)}$$

$$M = \frac{\operatorname{sen}A}{\cos\left(\dfrac{2A + 180°}{2}\right)}$$

$$M = \frac{\operatorname{sen}A}{\underbrace{\cos(\underbrace{90° + A}_{\text{IIC}})}_{(-)}}$$

$$M = \frac{\overset{1}{\cancel{\operatorname{sen}A}}}{\underset{-1}{\cancel{-\operatorname{sen}A}}}$$

$$\therefore \boxed{M = -1}$$



6) Si $\alpha \in$ IVC, además $\mathrm{sen}(270° + \alpha) = -0,8$, reduzca

$$T = \csc(180° - \alpha) + \tan(270° + \alpha)$$

Recordar

$$\csc\alpha = \frac{r}{y}$$

$$\cot\alpha = \frac{x}{y}$$

Resolución:

$$\rightarrow T = \underbrace{\csc(180° - \alpha)}_{\text{IIC}} + \underbrace{\tan(270° + \alpha)}_{\text{IVC}}$$

$(+)$ $(-)$

$$T = \csc\alpha + (-\cot\alpha)$$

$$T = \csc\alpha - \cot\alpha \;\ldots (*)$$

Del dato: $\underbrace{\mathrm{sen}(270° + \alpha)}_{\text{IVC}} = -0{,}8$

$(-)$

$$\rightarrow \cancel{-}\cos\alpha = \cancel{-}\frac{4}{5} = \frac{x}{r}$$

Por $r^2 = x^2 + y^2$ $\quad y \in$ IVC

$$5^2 = 4^2 + y^2 \rightarrow \boxed{y = -3}$$

En $(*)$: $T = \csc\alpha - \cot\alpha$

$$T = \frac{5}{-3} - \left(\frac{4}{-3}\right) = \boxed{-\frac{1}{3}}$$

7) Simplifique

$$P = \frac{\cos 1470° \cdot \operatorname{sen} 1140°}{\cot 3285°}$$

Resolución:

Eliminamos el número de vueltas:

$$\begin{array}{r|l} 1470 & \underline{360} \\ \underline{1440} & 4 \\ 30 & \end{array} \qquad \begin{array}{r|l} 1140 & \underline{360} \\ \underline{1080} & 3 \\ 60 & \end{array}$$

$$\begin{array}{r|l} 3285 & \underline{360} \\ \underline{3240} & 9 \\ 45 & \end{array}$$

$$\rightarrow P = \frac{\cos 30° \cdot \operatorname{sen} 60°}{\cot 45°}$$

$$P = \frac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}}{1}$$

$$\therefore \boxed{P = \frac{3}{4}}$$

8) Calcule

$$\mathbf{E = sen\left(\frac{37\pi}{6}\right) + cos\left(\frac{59\pi}{3}\right)}$$

Resolución:

Eliminamos el número de vueltas:

$$\begin{array}{r|l} 37 & 12 \\ \hline 36 & 3 \\ \hline 1 & \end{array} \qquad \begin{array}{r|l} 59 & 6 \\ \hline 54 & 9 \\ \hline 5 & \end{array}$$

$$\rightarrow \mathrm{E = sen}\left(\frac{1\pi}{6}\right) + \cos\left(\frac{5\pi}{3}\right)$$

Expresamos en grados:

$$\mathrm{E = sen30° + cos300°} \ldots (*)$$

- $\cos 300° = \cos\underbrace{(360° - 60°)}_{\text{IVC}}$

$(+)$

$$= \cos 60° = \frac{1}{2}$$

$$\rightarrow \mathrm{E} = \frac{1}{2} + \frac{1}{2} \qquad \therefore \boxed{\mathbf{E = 1}}$$

9) **Si $\theta \in$ IVC, además $\cos\theta = \frac{1}{2}$, reduzca**

$$\mathbf{M = \sec\left(\frac{13\pi}{2} + \theta\right) \cdot \tan(22\pi + \theta)}$$

Resolución:

- $\sec\left(\frac{13\pi}{2} + \theta\right) = \sec\left(\underbrace{\frac{1\pi}{2} + \theta}_{IIC}\right)$ $(-)$

$$\begin{array}{r|l} 13 & 4 \\ \cline{2-2} 12 & 3 \\ \hline 1 & \end{array}$$

$= -\csc\theta$

- $\tan(22\pi + \theta) = \tan\theta$

PAR

$\rightarrow M = -\csc\theta \cdot \tan\theta$

$$M = \frac{-r}{\cancel{y}} \cdot \frac{\cancel{y}}{x} = -\frac{r}{x} \; \ldots (*)$$

Del dato: $\cos\theta = \frac{1}{2} = \frac{x}{r} \rightarrow \frac{\boldsymbol{r}}{\boldsymbol{x}} = \mathbf{2}$

Reemplazamos en (*):

$\therefore$ $\boxed{\mathbf{M = -2}}$

10) **Si $\cot\theta = -0,75$ y $\cos\theta < 0$, efectúe**

$$\mathbf{M = 5sen\theta + 3sec\theta + 1}$$

Recordar

$$\mathbf{sen\alpha = \frac{y}{r}}$$

$$\mathbf{sec\alpha = \frac{r}{x}}$$

Resolución:

Del dato: $\cot\theta = -\frac{3}{4}$ $\left.\begin{array}{l}\cot\theta: (-)\\ \cos\theta: (-)\end{array}\right\}$ ⇒ $\theta \in IIC$ $\left\{\begin{array}{l}x: (-)\\ y: (+)\end{array}\right.$

$$\cot\theta = \frac{-3}{4} = \frac{x}{y}$$

Por $r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2}$

$$r = 5$$

Efectuamos

$$M = 5sen\theta + 3sec\theta + 1$$

$$M = \cancel{5}\left(\frac{4}{\cancel{5}}\right) + \overset{1}{\cancel{3}}\left(\frac{5}{\underset{-1}{-\cancel{3}}}\right) + 1$$

$$M = 4 + (-5) + 1 \quad \therefore \boxed{M = 0}$$

SACO OLIVEROS