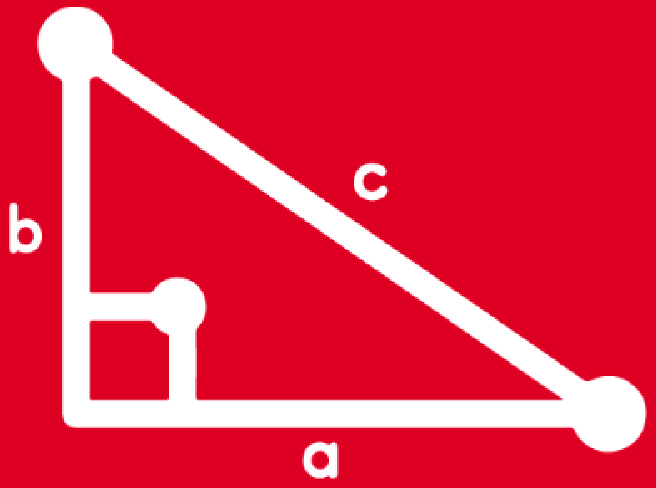
TRIGONOMETRY

TOMO VI

4TH
SECONDARY



FEEDBACK

SACO OLIVEROS


Tus metas
SON MÁS
grandes
QUE TUS
MIEDOS

$$\tan x = \frac{\operatorname{sen} x}{\cos x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cos^2 x + \operatorname{sen}^2 x = 1$$

$$\frac{1}{\operatorname{sen} x} = \csc x$$

1. Simplifique: $F = \dfrac{\cos x + \operatorname{sen} x . \tan x}{\operatorname{sen} x . \sec x}$

Resolución:

$$F = \frac{\cos x + \operatorname{sen} x . \tan x}{\operatorname{sen} x . \sec x}$$

$$F = \frac{\cos x + \operatorname{sen} x . \dfrac{\operatorname{sen} x}{\cos x}}{\operatorname{sen} x . \dfrac{1}{\cos x}}$$

$$F = \frac{\cos x + \dfrac{\operatorname{sen}^2 x}{\cos x}}{\dfrac{\operatorname{sen} x}{\cos x}}$$

$$F = \frac{\dfrac{\cos^2 x + \operatorname{sen}^2 x}{\cancel{\cos x}}}{\dfrac{\operatorname{sen} x}{\cancel{\cos x}}}$$

$$F = \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x}$$

$$F = \frac{1}{\operatorname{sen} x}$$

$$\therefore F = \csc x$$

2. Reducir: $D = \dfrac{sen^4x - cos^4x}{senx - cosx} - cosx$

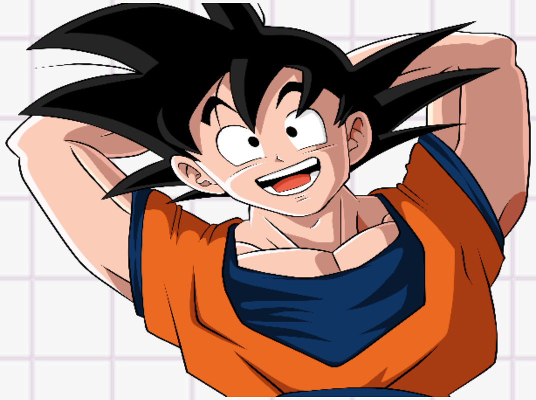
Resolución:

$$D = \frac{(sen^2x)^2 - (cos^2x)^2}{senx - cosx} - cosx$$

$$a^2 - b^2 = (a+b).(a-b)$$

$$sen^2x + cos^2x = 1$$

$$D = \frac{\overset{1}{\cancel{(sen^2x + cos^2x)}}.(sen^2x - cos^2x)}{senx - cosx} - cosx$$



$$D = \frac{(senx + cosx).\cancel{(senx - cosx)}}{\cancel{(senx - cosx)}} - cosx$$

$$D = senx + \cancel{cosx} - \cancel{cosx}$$

$$\therefore D = senx$$

3. Simplificar: $S = \dfrac{1 - senx}{cosx} + \dfrac{cosx}{1 - senx}$

Resolución:

$$S = \frac{1 - \text{senx}}{\text{cosx}} + \frac{\textbf{cosx}}{\textbf{1} - \textbf{senx}}$$

$$\frac{cosx}{1 - senx} = \frac{1 + senx}{cosx}$$

$$S = \frac{1 - \text{senx}}{\text{cosx}} + \frac{\textbf{1} + \textbf{senx}}{\textbf{cosx}}$$

$$\frac{1}{cosx} = secx$$

$$S = \frac{1 - \cancel{\text{senx}} + 1 + \cancel{\text{senx}}}{\text{cosx}}$$

$$S = \frac{2}{\text{cosx}} \Rightarrow S = 2\left(\frac{1}{\text{cosx}}\right)$$

$$\therefore S = 2secx$$

4. Si: $senx + cosx = \frac{1}{3}$

Calcule: $\mathbf{H = (1 + senx)(1 + cosx)}$

Resolución:

$$\boxed{2(1 + senx)(1 + cosx) = (1 + senx + cosx)^2}$$

Del dato: $\mathbf{senx + cosx = \frac{1}{3}}$

Calculamos:

$$\mathrm{H = (1 + senx)(1 + cosx)} \quad \ldots \times 2$$

$$2\mathrm{H} = \underbrace{2\mathrm{(1 + senx)(1 + cosx)}}_{(1 + \mathbf{senx} + \mathbf{cosx})^2}$$

$$2\mathrm{H} = (1 + \underbrace{\mathrm{senx + cosx}})^2$$

$$2\mathrm{H} = \left(1 + \frac{1}{3}\right)^2$$

$$2\mathrm{H} = \left(\frac{4}{3}\right)^2 \Rightarrow \cancel{2}\mathrm{H} = \frac{\cancel{16}^{\,8}}{9}$$

$$\boxed{\therefore H = \frac{8}{9}}$$

$tanx + cotx = secx.cscx$

$(a+b)^2 = a^2 + 2ab + b^2$

$sec^2x + csc^2x = sec^2x.csc^2x$

5. Sabiendo que: $\mathbf{tanx + cotx = 3}$
Calcule:

$$\mathbf{B = (secx + cscx)^2}$$

Resolución:

Dal dato:

$$\underbrace{\text{tanx+cotx}}_{} = 3$$

$$secx.cscx = 3 \quad \ldots(\ \ )^2$$

$$sec^2x.csc^2x = 9$$

Calculamos: $B = (secx + cscx)^2$

$$B = \mathbf{sec^2x + 2secx.cscx + csc^2x}$$

$$B = \underbrace{\mathbf{sec^2x + csc^2x}}_{} + 2secx.cscx$$

$$B = \underbrace{\mathbf{sec^2x.csc^2x}}_{(9)} + 2\underbrace{\mathbf{secx.cscx}}_{(3)}$$

$$B = 9 + 2(3) \quad = 9 + 6$$

$$\therefore \boldsymbol{B = 15}$$

6. Si: $\emptyset \in$ IIC; $\mathbf{1 - sen^4\emptyset - cos^4\emptyset = \frac{1}{2}cos^2\emptyset}$

Calcule: $E = cot\emptyset$

$$\boxed{cot\emptyset = \frac{x}{y}}$$

$$\boxed{sen^4\alpha + cos^4\alpha = 1 - 2sen^2\alpha . cos^2\alpha}$$

Resolución:

Dato: $1 - sen^4\emptyset - cos^4\emptyset = \frac{1}{2}cos^2\emptyset$

$$1 - (\underbrace{sen^4\emptyset + cos^4\emptyset}) = \frac{1}{2}cos^2\emptyset$$

$$1 - (1 - 2sen^2\emptyset . cos^2\emptyset) = \frac{1}{2}cos^2\emptyset$$

$$2sen^2\emptyset . \cancel{cos^2\emptyset} = \frac{1}{2}\cancel{cos^2\emptyset}$$

$$sen^2\emptyset = \frac{1}{4} \rightarrow sen\emptyset = \pm\frac{1}{2}$$

Como $\emptyset \in$ IIC $\Rightarrow$ $sen\emptyset = +\frac{1}{2} = \frac{y}{r}$

Recordar: $r^2 = x^2 + y^2$ $\Rightarrow 2^2 = x^2 + 1^2$

Como $\emptyset \in$ IIC $\Rightarrow$ $x = -\sqrt{3}$

Calculamos: $E = \left(\frac{-\sqrt{3}}{1}\right)$

$$\boxed{\therefore E = -\sqrt{3}}$$

7. Calcular :

$$C = \frac{\operatorname{sen}(60° + x) + \operatorname{sen}(60° - x)}{\cos x}$$

Resolución:

$$\boldsymbol{sen(x+y) = senx.cosy + cosx.seny}$$

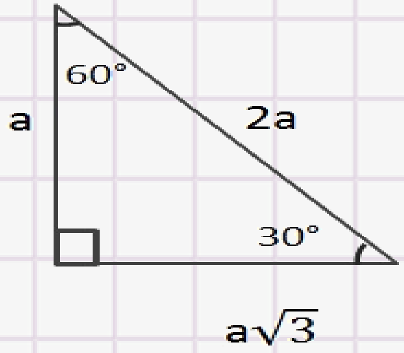
$$\boldsymbol{sen(x-y) = senx.cosy - cosx.seny}$$

$$C = \frac{sen60°.cosx + \cancel{cos60°.senx} + sen60°.cosx - \cancel{cos60°.senx}}{cosx}$$

$$C = \frac{2sen60°.\cancel{cosx}}{\cancel{cosx}}$$

$$C = 2\,sen60° \Rightarrow C = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$\boxed{\therefore C = \sqrt{3}}$$



8. Siendo : $\alpha + \beta = 45°$; $\tan\alpha = \frac{3}{5}$

Calcule: $\tan\beta$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x . \tan y}$$

$$\tan 45° = 1$$

Resolución:

Se sabe: $\tan\alpha = \frac{3}{5}$

Además: $\alpha + \beta = 45°$

$\beta = 45° - \alpha$

$\tan\beta = \tan(45° - \alpha)$

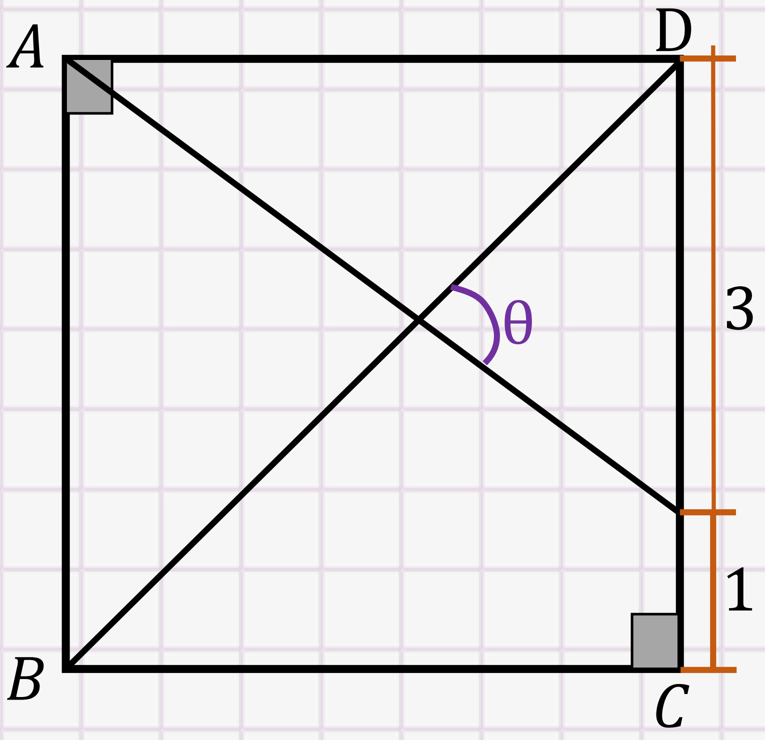
$$\tan\beta = \frac{\tan 45° - \tan\alpha}{1 + \tan 45° . \tan\alpha}$$

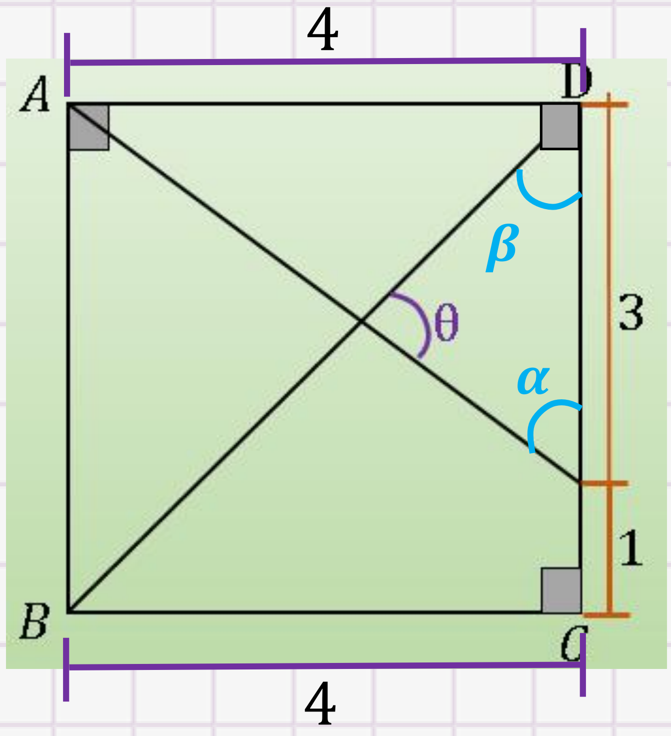
$$\tan\beta = \frac{1 - \frac{3}{5}}{1 + (1)\left(\frac{3}{5}\right)}$$

$$\Rightarrow \tan\beta = \frac{\frac{2}{\cancel{5}}}{\frac{8}{\cancel{5}}} = \frac{2}{8}$$

$$\therefore \tan\beta = \frac{1}{4}$$

9. Si ABCD es un cuadrado; calcular: $\boldsymbol{tan\theta}$





Resolución:

Observamos:

$$\theta + \alpha + \beta = 180°$$

Además:

$$\boldsymbol{tan\alpha = \frac{4}{3}} \qquad \boldsymbol{tan\beta = 1}$$

$$tan\theta + \frac{4}{3} + 1 = tan\theta.\frac{4}{3}.1 \Rightarrow \frac{7}{\cancel{3}} = \frac{1}{\cancel{3}}.tan\theta$$

$$\boldsymbol{\therefore tan\theta = 7}$$

Si $\boldsymbol{\alpha + \beta + \emptyset = 180°}$***, entonces*:**

$$\boldsymbol{tan\alpha + tan\beta + tan\emptyset = tan\alpha.tan\beta.tan\emptyset}$$

10. Carlos tiene un terreno rectangular de dimensiones "A" y "B". Calcule el área de dicho terreno en metros , si:

$$Asen^{B}x = 2\left(\frac{1-cosx}{1-senx}\right)(1-senx+cosx)^2$$

$$\boxed{(1-\mathrm{senx}+\mathrm{cosx})^2 = 2(1-\mathrm{senx})(1+\mathrm{cosx})}$$

$$\boxed{(a-b)(a+b) = a^2-b^2} \quad \boxed{1-\cos^2 x = \mathrm{sen}^2 x}$$

Resolución:

$$\mathrm{Asen^B x} = 2\left(\frac{1-\mathrm{cosx}}{1-\mathrm{senx}}\right)\underbrace{(1-\mathrm{senx}+\mathrm{cosx})^2}$$

$$\mathrm{Asen^B x} = 2\left(\frac{1-\mathrm{cosx}}{\cancel{1-\mathrm{senx}}}\right)2\cancel{(1-\mathrm{senx})}(1+\mathrm{cosx})$$

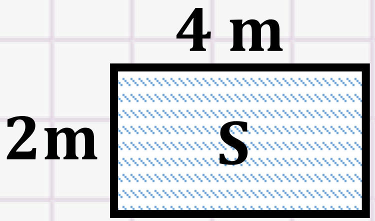
$$\mathrm{Asen^B x} = 4(1-\mathrm{cosx})(1+\mathrm{cosx})$$

$$\mathrm{Asen^B x} = 4\underbrace{(1-\cos^2 x)}$$

$$\Rightarrow \mathrm{Asen^B x} = 4\,\mathrm{sen}^2 x$$

$$\boxed{A = 4} \quad \boxed{B = 2}$$

Piden:



$$\mathrm{S = 4m.\,2m}$$

$$\boxed{\therefore S = 8\ m^2}$$

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