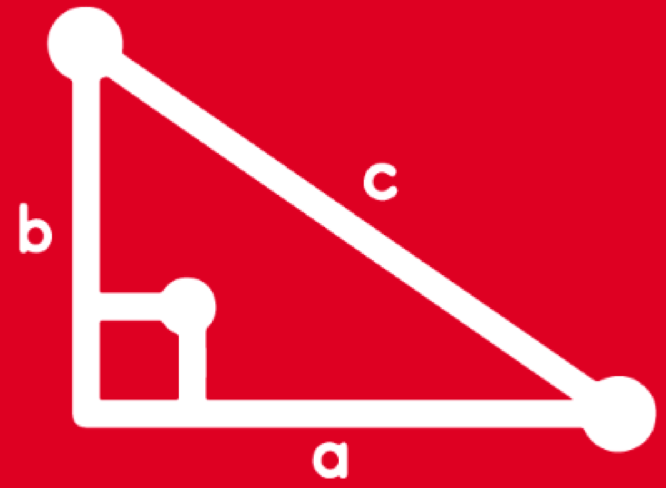
TRIGONOMETRY

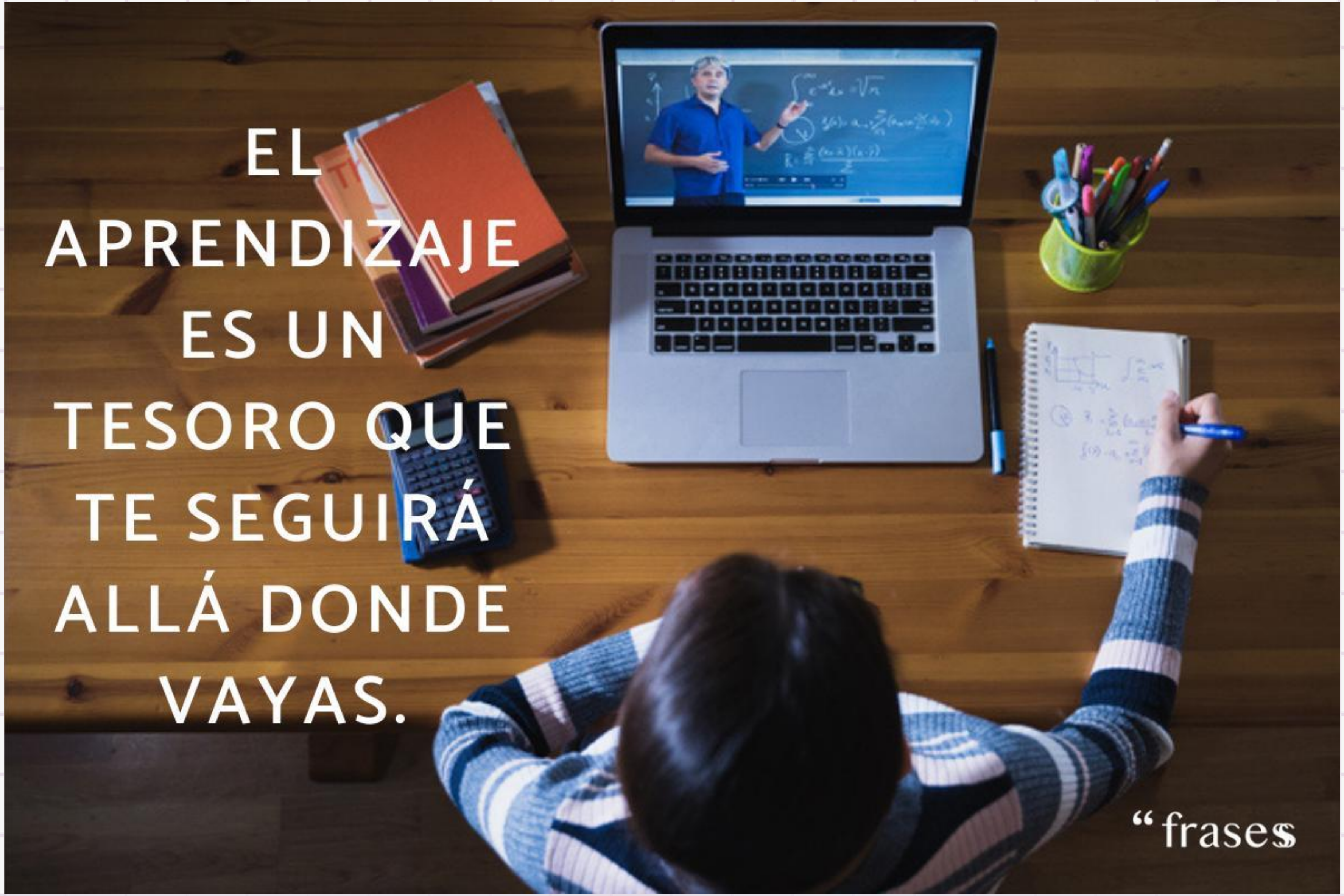
Chapter 12

5th
SECONDARY



Identidades trigonométricas II

SACO OLIVEROS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ALLÁ DONDE
VAYAS.
“frases

IDENTIDADES TRIGONOMÉTRICAS AUXILIARES

1. $\tan x + \cot x = \sec x \cdot \csc x$

2. $\sec^2 x + \csc^2 x = \sec^2 x \cdot \csc^2 x$

3. $\text{sen}^4 x + \cos^4 x = 1 - 2\text{sen}^2 x \cdot \cos^2 x$

4. $\text{sen}^6 x + \cos^6 x = 1 - 3\text{sen}^2 x \cdot \cos^2 x$

5. $(1 + \text{sen}x + \cos x)^2 = 2(1 + \text{sen}x)(1 + \cos x)$

$(1 + \text{sen}x - \cos x)^2 = 2(1 + \text{sen}x)(1 - \cos x)$

$(1 - \text{sen}x + \cos x)^2 = 2(1 - \text{sen}x)(1 + \cos x)$

$(1 - \text{sen}x - \cos x)^2 = 2(1 - \text{sen}x)(1 - \cos x)$

1. Reduzca: $G=\dfrac{\text{sen}^4x+\cos^4x+3}{\text{sen}^6x+\cos^6x+5}+\dfrac{4}{3}$

RESOLUCIÓN

Tenemos:

$$G=\frac{\text{sen}^4x+\cos^4x+3}{\text{sen}^6x+\cos^6x+5}+\frac{4}{3}$$

$$G=\frac{1-2\text{sen}^2x.\cos^2x+3}{1-3\text{sen}^2x.\cos^2x+5}+\frac{4}{3}$$

$$G=\frac{4-2\text{sen}^2x.\cos^2x}{6-3\text{sen}^2x.\cos^2x}+\frac{4}{3}$$

Identidades auxiliares

3. $\boxed{\text{sen}^4x+\cos^4x=1-2\,\text{sen}^2x.\cos^2x}$

4. $\boxed{\text{sen}^6x+\cos^6x=1-3\,\text{sen}^2x.\cos^2x}$

$$G=\frac{2\cancel{(2-\text{sen}^2x.\cos^2x)}}{3\cancel{(2-\text{sen}^2x.\cos^2x)}}+\frac{4}{3}$$

$$G=\frac{2}{3}+\frac{4}{3}\Rightarrow G=\frac{6}{3}$$

$\therefore\ \boxed{G=2}$

2. Simplifique la expresión: $T=\left(\dfrac{\sec^2x+\csc^2x}{\tan x+\cot x}\right)\cos x$

RESOLUCIÓN

Tenemos:

$$T=\left(\frac{\sec^2x+\csc^2x}{\tan x+\cot x}\right)\cos x$$

$$T=\left(\frac{\cancel{\sec^2}x.\cancel{\csc^2}x}{\cancel{\sec}x.\cancel{\csc}x}\right)\cos x$$

Identidades auxiliares

1. $\boxed{\tan x+\cot x=\sec x.\csc x}$
2. $\boxed{\sec^2x+\csc^2x=\sec^2x.\csc^2x}$

$$T=(\sec x.\csc x)\cos x$$

Ordenamos:

$$T=\underbrace{\cos x.\sec x}_{1}.\csc x$$

$\therefore\ \boxed{T=\csc x}$

3. Simplifique la expresión: $W=\dfrac{1-\cot\theta+\sec\theta.\csc\theta}{1-\tan\theta+\sec\theta.\csc\theta}$

RESOLUCIÓN

Tenemos:

$$W=\frac{1-\cot\theta+\mathbf{sec\theta.csc\theta}}{1-\tan\theta+\mathbf{sec\theta.csc\theta}}$$

Identidad auxiliar

1. $\tan x+\cot x=\sec x.\csc x$

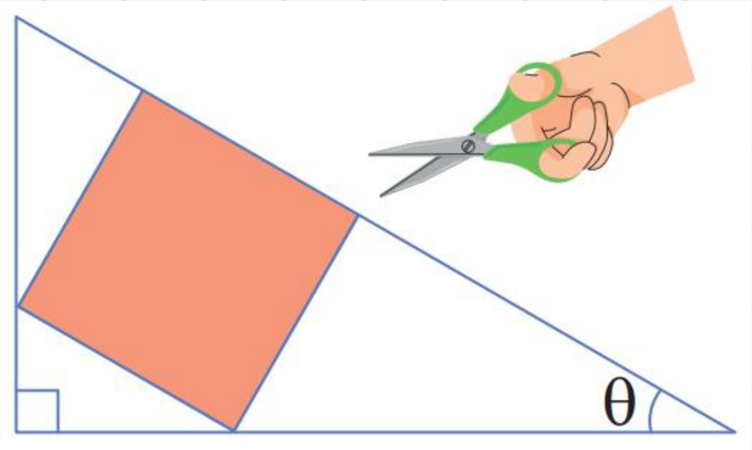
$$W=\frac{\cancel{1-\cot\theta}+\mathbf{\tan\theta}+\cancel{\mathbf{\cot\theta}}}{\cancel{1-\tan\theta}+\cancel{\mathbf{\tan\theta}}+\mathbf{\cot\theta}}$$

$$W=\frac{1+\tan\theta}{1+\cot\theta}\ \Rightarrow\ W=\frac{1+\dfrac{\text{sen}\theta}{\cos\theta}}{1+\dfrac{\cos\theta}{\text{sen}\theta}}$$

$$W=\frac{\dfrac{\cancel{\cos\theta+\text{sen}\theta}}{\cos\theta}}{\dfrac{\cancel{\text{sen}\theta+\cos\theta}}{\text{sen}\theta}}=\frac{\text{sen}\theta}{\cos\theta}$$

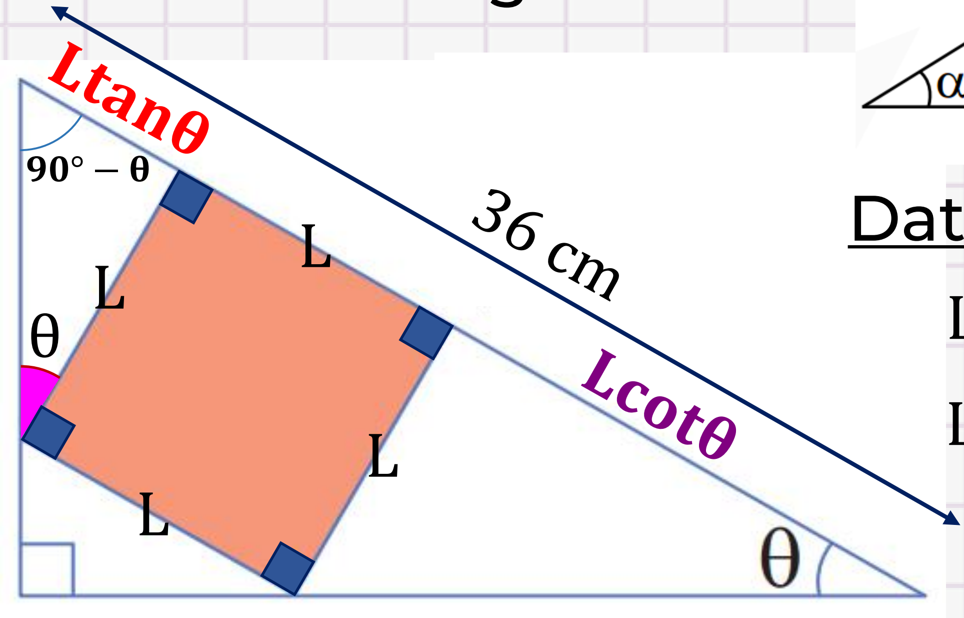
$$\therefore\ \boxed{W=\tan\theta}$$

4. De un papel que tiene la forma de un triángulo rectángulo, se cortará un cuadrado sombreado como indica la figura. Calcule el área de dicho cuadrado, si $\operatorname{sen}\theta \cdot \cos\theta = 2/7$ y la hipotenusa de dicho triángulo mide 36 cm.

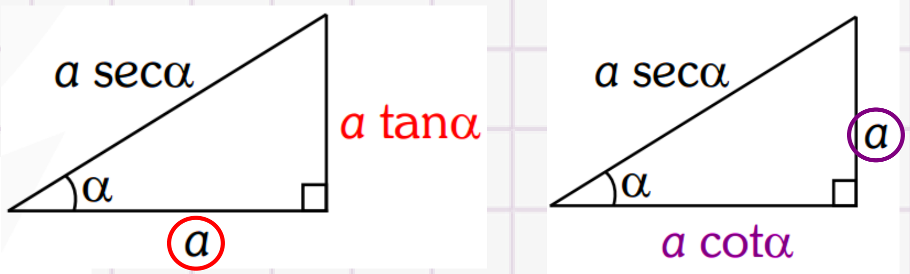


RESOLUCIÓN

Analizamos la figura:



¡Recordamos!



Dato 1: Hipotenusa = 36 cm

$$L\tan\theta + L\cot\theta + L = 36\text{ cm}$$

$$L(\underbrace{\boxed{\tan\theta + \cot\theta}}_{\sec\theta \cdot \csc\theta} + 1) = 36\text{ cm}$$

$$L(\sec\theta \cdot \csc\theta + 1) = 36\text{ cm}$$

Dato 2:

$$\operatorname{sen}\theta \cdot \cos\theta = 2/7$$

inversa

$$\csc\theta \cdot \sec\theta = 7/2$$

$$\Rightarrow L\left(\frac{7}{2} + 1\right) = 36\text{ cm}$$

$$\boxed{L = 8\text{ cm}}$$

$$\Rightarrow \text{Área}\,\square = (8\text{ cm})^2$$

$$\therefore \text{Área}\,\square = 64\text{ cm}^2$$

5. De la condición: $\operatorname{sen}x + \cos x = \sqrt{\frac{2}{3}}$

Determine: $F = \operatorname{sen}^4x + \cos^4x$

Identidad auxiliar

3. $\operatorname{sen}^4x + \cos^4x = 1 - 2\operatorname{sen}^2x.\cos^2x$

$(a+b)^2 = a^2 + 2ab + b^2$

RESOLUCIÓN

A partir de: $F = \operatorname{sen}^4x + \cos^4x$

$F = 1 - 2\operatorname{sen}^2x\cos^2x \ldots (*)$

Dato: $\operatorname{sen}x + \cos x = \sqrt{\frac{2}{3}} \ldots (\quad)^2$

$\Rightarrow (\operatorname{sen}x + \cos x)^2 = \left(\sqrt{\frac{2}{3}}\right)^2$

$\underbrace{\operatorname{sen}^2x + \cos^2x}_{1} + 2\operatorname{sen}x\cos x = \frac{2}{3}$

$\Rightarrow 1 + 2\operatorname{sen}x\cos x = \frac{2}{3}$

$\Rightarrow 2\operatorname{sen}x\cos x = \frac{2}{3} - 1$

$\Rightarrow 2\operatorname{sen}x\cos x = -\frac{1}{3}$

$\Rightarrow \operatorname{sen}x\cos x = -\frac{1}{6}$

Reemplazamos en $(*)$:

$F = 1 - 2\left(-\frac{1}{6}\right)^2$

$\therefore \boxed{F = \frac{17}{18}}$

6. Si se cumple que: $\operatorname{sen}x - \cos x = \dfrac{1}{3}$

Calcule: $E = (1+\operatorname{sen}x)(1-\cos x)$

RESOLUCIÓN

A partir de:

$$E = (1 + \operatorname{sen}x)(1 - \cos x) \ldots \times 2$$

$$\Rightarrow 2E = \underbrace{2(1 + \operatorname{sen}x)(1 - \cos x)}$$

$$2E = (1 + \underbrace{\operatorname{sen}x - \cos x}_{\text{Dato} \rightarrow \frac{1}{3}})^2$$

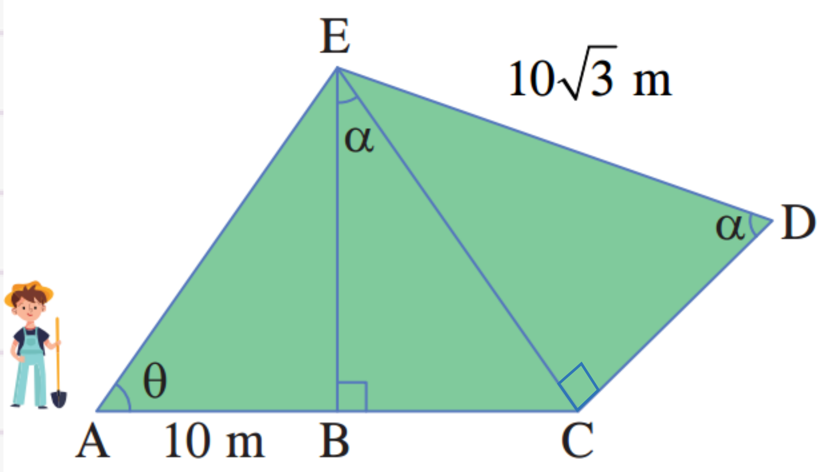
Identidad auxiliar

5. $(1+\operatorname{sen}x - \cos x)^2 = 2(1+\operatorname{sen}x)(1-\cos x)$

$$\Rightarrow 2E = \left(1 + \frac{1}{3}\right)^2 \Rightarrow 2E = \left(\frac{4}{3}\right)^2$$

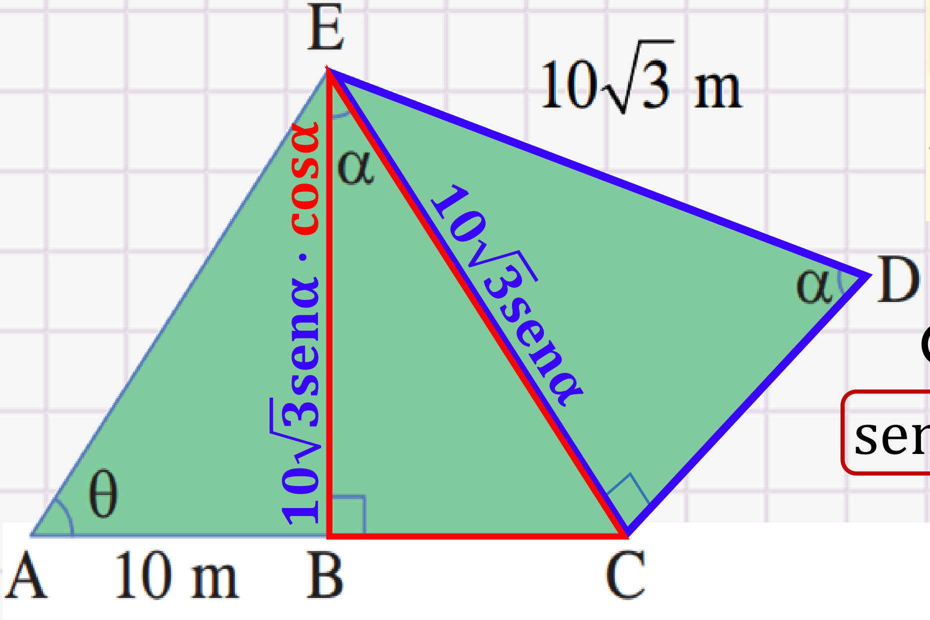
$$\Rightarrow 2E = \frac{16}{9} \qquad \therefore \boxed{E = \frac{8}{9}}$$

7. El joven Philip recibió como herencia una pequeña huerta; la cual es dividida en tres partes para sembrar distintas plantas, tal como muestra la figura. Con los datos obtenidos de la figura, obtenga el valor de $\operatorname{sen}^6\alpha + \cos^6\alpha + \tan^2\theta$.

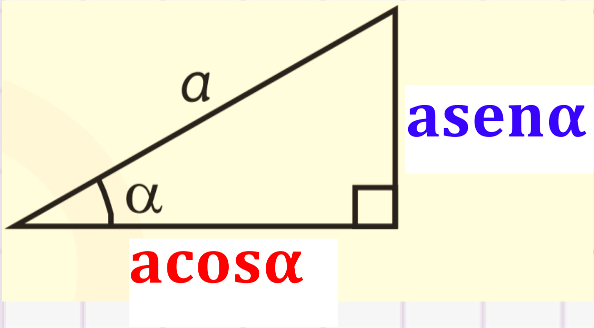


RESOLUCIÓN

Analizamos la figura:



¡Recordamos!



En el ⊿ ABC:

$$\tan\theta = \frac{\cancel{10}\sqrt{3}\operatorname{sen}\alpha \cdot \cos\alpha}{\cancel{10}}$$

$$\tan\theta = \sqrt{3}\operatorname{sen}\alpha \cdot \cos\alpha$$

$(\quad)^2$

$$\tan^2\theta = 3\operatorname{sen}^2\alpha \cdot \cos^2\alpha$$

Calculamos:

$$\boxed{\operatorname{sen}^6\alpha + \cos^6\alpha} + \boxed{\tan^2\theta} = 1 \cancel{- 3\operatorname{sen}^2\alpha \cdot \cos^2\alpha} \cancel{+ 3\operatorname{sen}^2\alpha \cdot \cos^2\alpha}$$

$$\therefore \boxed{\operatorname{sen}^6\alpha + \cos^6\alpha + \tan^2\theta = 1}$$

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