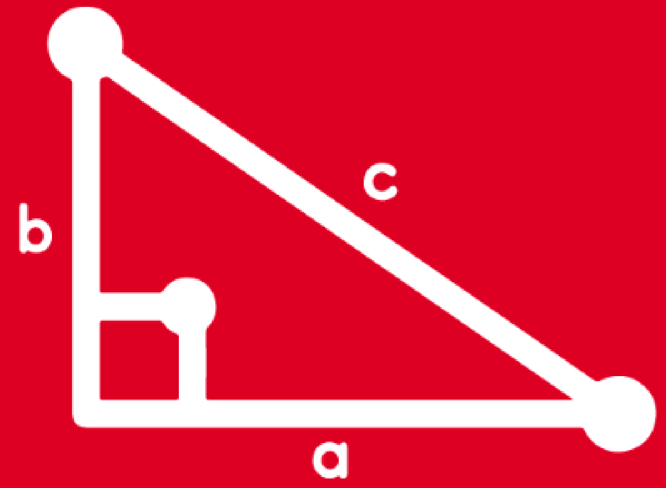
TRIGONOMETRY

VOLUME II

4th
SECONDARY



FEEDBACK

SACO OLIVEROS

1. Si $5\cos\alpha - 2 = 0$, donde α es la medida de un ángulo agudo, efectúe:

$$Q = \sqrt{21}(\cot\alpha + \csc\alpha)$$

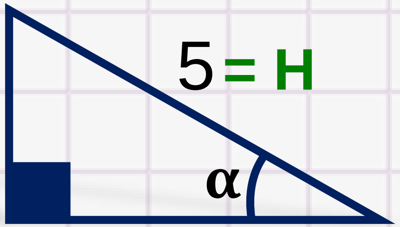
RESOLUCIÓN

Recordar

$$\cos\alpha = \frac{CA}{H} \quad \cot\alpha = \frac{CA}{CO} \quad \csc\alpha = \frac{H}{CO}$$

Dato:

$$5\cos\alpha - 2 = 0 \;\Rightarrow\; \cos\alpha = \frac{2}{5} = \frac{CA}{H}$$



Por teorema de Pitágoras

$$5^2 = 2^2 + (CO)^2 \;\Rightarrow\; \boxed{CO = \sqrt{21}}$$

Efectuamos:

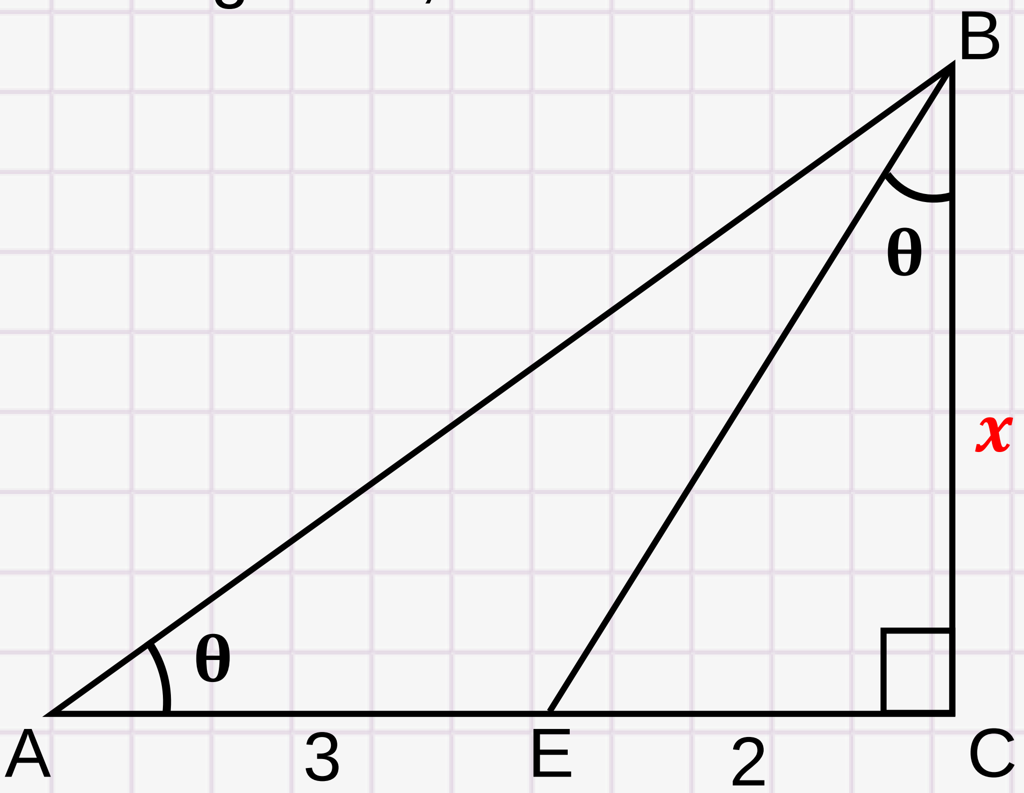
$$Q = \sqrt{21}(\cot\alpha + \csc\alpha)$$

$$Q = \sqrt{21}\left(\frac{2}{\sqrt{21}} + \frac{5}{\sqrt{21}}\right)$$

$$Q = \cancel{\sqrt{21}}\left(\frac{7}{\cancel{\sqrt{21}}}\right)$$

$$\boxed{\therefore Q = 7}$$

2. Del gráfico, calcule cotθ.



Recordar

$$\cot\alpha = \frac{CA}{CO}$$

RESOLUCIÓN

Sea BC = x

En el ◣ BCE:

$$\cot\theta = \frac{x}{2} \quad \dots \textbf{(1)}$$

En el ◣ BCA:

$$\cot\theta = \frac{5}{x} \quad \dots \textbf{(2)}$$

Igualamos las ecuaciones **(1)** y **(2)**:

$$\frac{x}{2} = \frac{5}{x} \Rightarrow x = \sqrt{10}$$

$$\therefore \cot\theta = \frac{\sqrt{10}}{2}$$

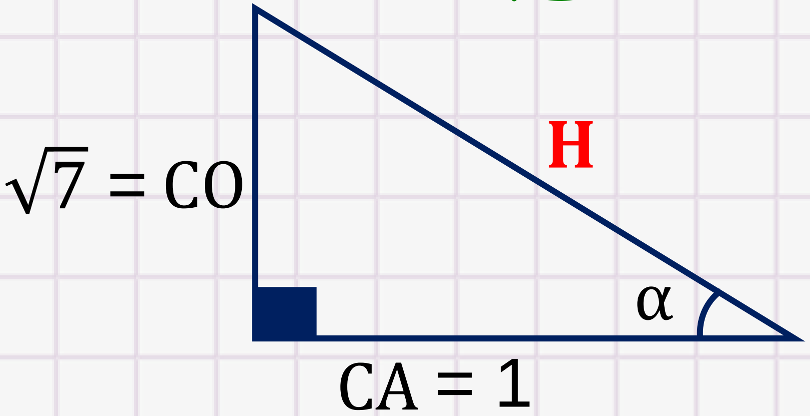
3. Si $\tan\alpha = \sqrt{7}$, donde $0° < \alpha < 90°$, efectúe $E = \tan^2\alpha + 2\sqrt{8}\cos\alpha$.

RESOLUCIÓN

Recordar

$$\cos\alpha = \frac{CA}{H} \qquad \tan\alpha = \frac{CO}{CA}$$

Dato: $\tan\alpha = \dfrac{\sqrt{7}}{1} = \dfrac{CO}{CA}$



Por teorema de Pitágoras:

$$H^2 = (\sqrt{7})^2 + (1)^2$$

$$H^2 = 7 + 1$$

$$\boxed{H = \sqrt{8}}$$

Efectuamos:

$$E = \tan^2\alpha + 2\sqrt{8}\cos\alpha$$

$$E = \cancel{(\sqrt{7})^2} + (2\cancel{\sqrt{8}})\left(\frac{1}{\cancel{\sqrt{8}}}\right)$$

$$E = 7 + 2$$

$$\therefore E = 9$$

4. Calcule el valor de x , si

$$2x \cdot \sec^2 45° \cdot \operatorname{sen}^2 30° + \sec 60° = 3x \cdot \csc^2 60° \cdot \tan 37$$

RESOLUCIÓN

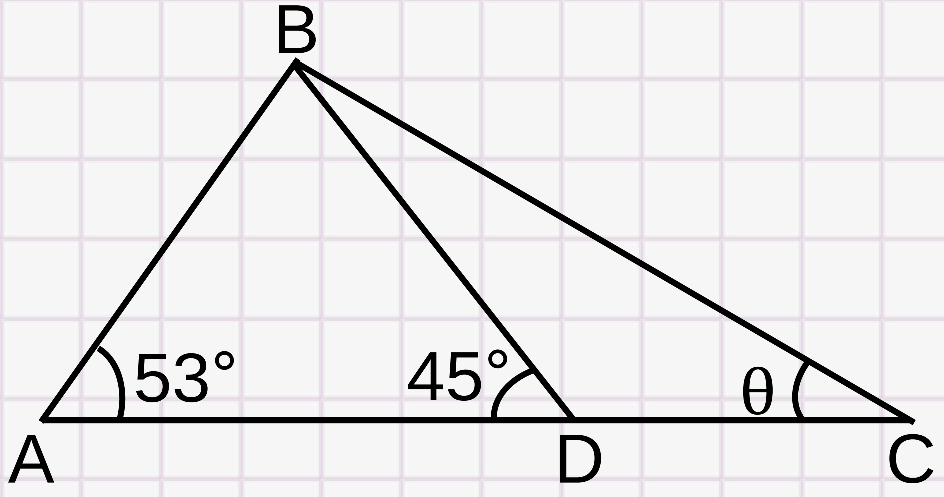
$$2x \cdot (\sqrt{2})^2 \cdot \left(\frac{1}{2}\right)^2 + 2 = 3x \cdot \left(\frac{2}{\sqrt{3}}\right)^2 \cdot \left(\frac{3}{4}\right)$$

$$\cancel{2}x \cdot (\cancel{2}) \cdot \left(\frac{1}{\cancel{4}}\right) + 2 = 3x \left(\frac{\cancel{4}}{\cancel{3}}\right) \cdot \left(\frac{\cancel{3}}{\cancel{4}}\right)$$

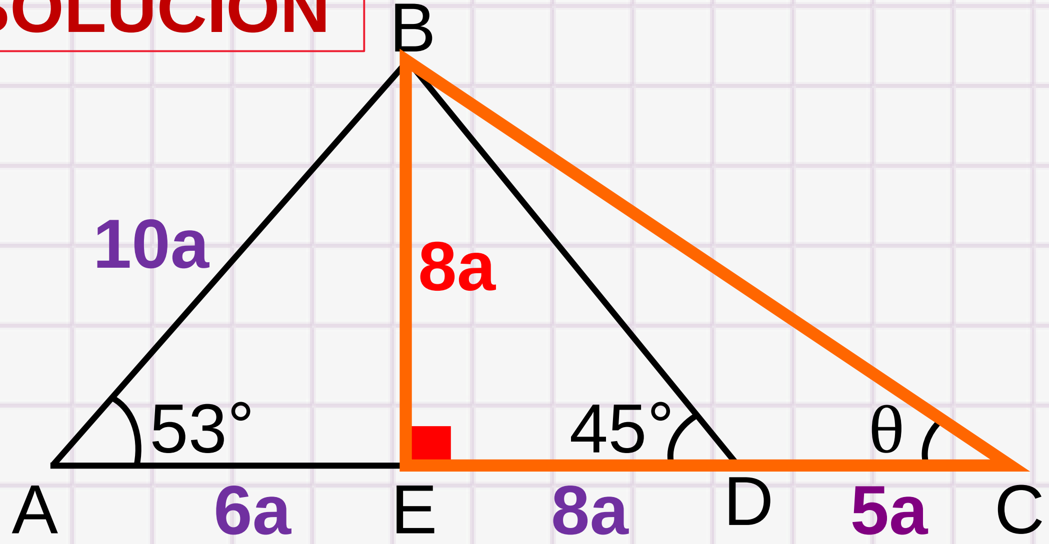
$$x + 2 = 3x$$

$$\boxed{\therefore\ x = 1}$$

5. Del gráfico, calcule cotθ, si $AB = 2DC$.



RESOLUCIÓN



Trazamos la altura $\overline{BE}$

- En el ◣ ABE (37°- 53°):

$$AB = 10a\ ;\ BE = 8a\ ;\ AE = 6a$$

- En el ◣ BED (45°- 45°):

$$ED = 8a$$

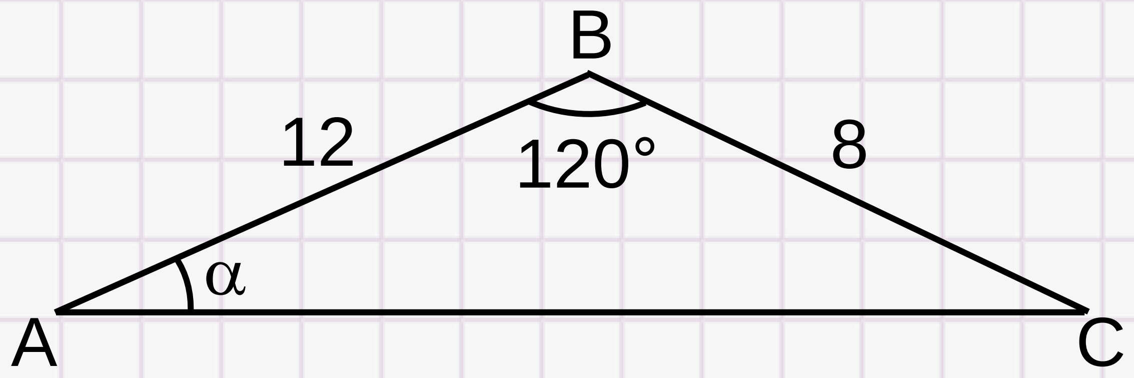
Dato: $AB = 2DC$ ➡ $DC = 5a$

Finalmente:

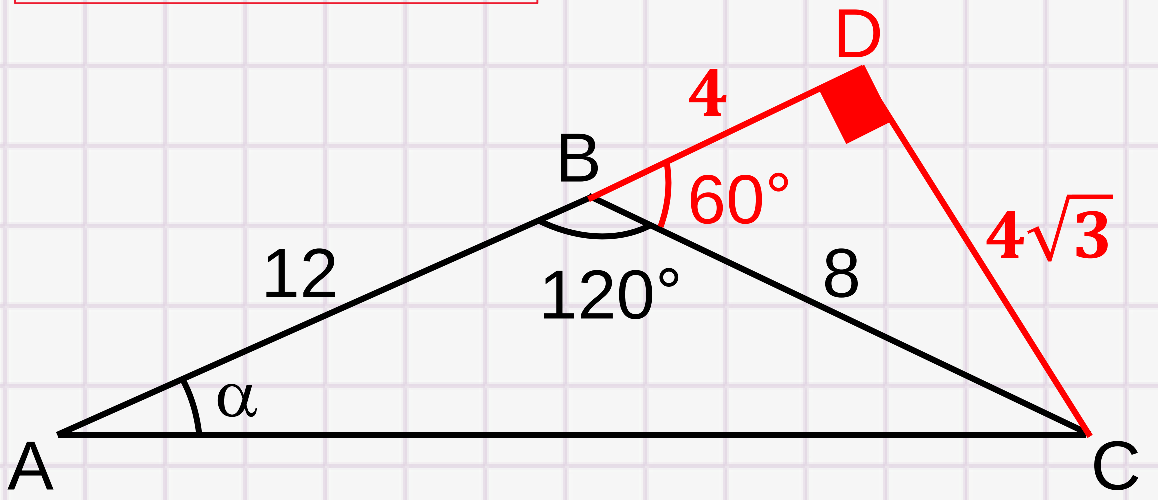
- En el ◣BEC: $\cot\theta = \dfrac{13\cancel{a}}{8\cancel{a}}$

$$\boxed{\therefore\ \cot\theta = \frac{13}{8}}$$

6. Del gráfico, calcule tanα.



RESOLUCIÓN



Trazamos las líneas auxiliares $\overline{BD}$ y $\overline{DC}$ formando un ángulo de $90°$.

Completamos el ◺BDC $(60° - 30°)$:

Del ◺ADC: $\tan\alpha = \dfrac{\cancel{4}\sqrt{3}}{1\cancel{6}}$

$$\therefore \boldsymbol{\tan\alpha = \dfrac{\sqrt{3}}{4}}$$

7. Si $\tan 9x = \cot 6x$, efectúe $\mathrm{Q} = \tan^2 10x + \csc 5x$.

RESOLUCIÓN

Por RT de ángulos complementarios en:

$$9x + 6x = 90°$$

$$15x = 90°$$

$$\boldsymbol{x = 6°}$$

Efectuamos

$$\mathrm{Q} = \tan^2 10(6°) + \csc 5(6°)$$

$$\mathrm{Q} = \tan^2 60° + \csc 30°$$

$$\mathrm{Q} = (\sqrt{3})^2 + 2$$

$$\boldsymbol{\therefore \mathrm{Q} = 5}$$

8. Efectúe $Q = \operatorname{sen}(x + y)$ si

$$\operatorname{sen}(x + 15°) \cdot \csc(35° - x) = 1 \quad \text{y} \quad \tan(3y - 20°) = \cot(30° + y)$$

RESOLUCIÓN

Dato:

$$\operatorname{sen}(x + 15°) \cdot \csc(35° - x) = 1$$

Por RT recíprocas:

$$x + 15° = 35° - x$$

$$2x = 20°$$

$$\boldsymbol{x = 10°}$$

Dato:

$$\tan(3y - 20°) = \cot(30° + y)$$

Por RT de ángulos complementarios:

$$3y - 20 + 30° + y = 90°$$

$$4y = 80°$$

$$\boldsymbol{y = 20°}$$

Efectuamos:

$$Q = \operatorname{sen}(x + y)$$

$$Q = \operatorname{sen} 30°$$

$$\boxed{\therefore \mathbf{Q = \frac{1}{2}}}$$

9. Si $\operatorname{sen}5\theta \cdot \csc(2\theta + 45°) = \frac{\operatorname{sen}20° \cdot \sec 70°}{\tan 55° \cdot \tan 35°}$. Efectúe: $M = \sec 4\theta + \tan 3\theta$.

RESOLUCIÓN

Dato:

$$\operatorname{sen}5\theta \cdot \csc(2\theta + 45°) = \frac{\operatorname{sen}20° \cdot \boxed{\sec 70°}}{\tan 55° \cdot \boxed{\tan 35°}}$$

Por RT de ángulos complementarios en el 2° miembro:

$$\operatorname{sen}5\theta \cdot \csc(2\theta + 45°) = \frac{\overbrace{\boxed{\operatorname{sen}20° \cdot \mathbf{\csc 20°}}}^{1}}{\underbrace{\boxed{\tan 55° \cdot \mathbf{\cot 55°}}}_{1}}$$

$$\operatorname{sen}5\theta \cdot \csc(2\theta + 45°) = \mathbf{1}$$

Por RT recíprocas:

$$5\theta = 2\theta + 45°$$

$$3\theta = 45° \Rightarrow \boxed{\boldsymbol{\theta = 15°}}$$

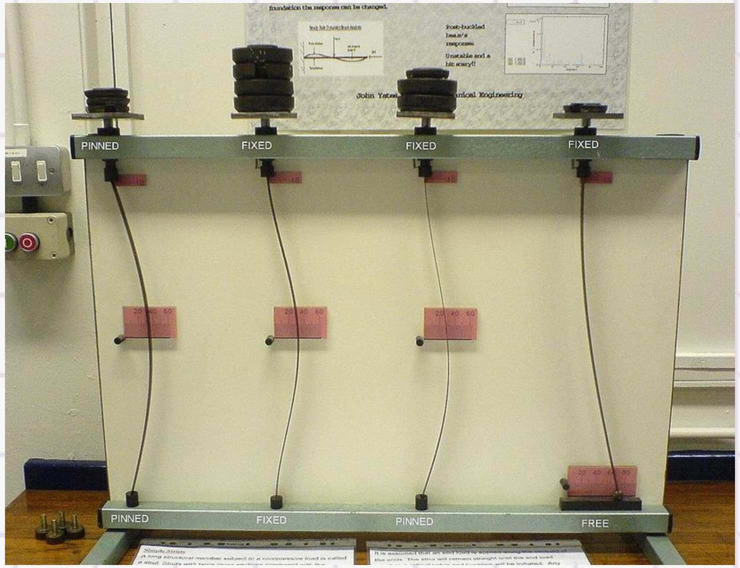
Calculamos:

$$M = \sec 4(15°) + \tan 3(15°)$$

$$M = \sec 60° + \tan 45°$$

$$M = (2) + (1) \qquad \boxed{\therefore \mathbf{M = 3}}$$

10. Se define como pandeo a la flexión producida por una carga axial pudiendo ser esta variable o crítica, sabiendo que una pieza metálica es sometida a 3 cargas axiales a, b y c definidas en Newton (N). Dar como respuesta el promedio de las cargas.



$$a = 8\text{sen}30° - 3\tan45°$$
$$b = 4\sec^2 45° - \sec60°$$
$$c = 4\csc53° + 3\cot45°$$

RESOLUCIÓN

- $\mathbf{a = \underbrace{8\text{sen}30°} - \underbrace{3\tan45°}}$

$$a = \overset{4}{\cancel{8}}\left(\frac{1}{\cancel{2}}\right) - 3(1) \Rightarrow \boxed{\mathbf{a = 1\ N}}$$

- $\mathbf{b = \underbrace{4\sec^2 45°} - \underbrace{\sec60°}}$

$$b = 4\left(\cancel{\sqrt{2}}\right)^{\cancel{2}} - (2) \Rightarrow \boxed{\mathbf{b = 6\ N}}$$

- $\mathbf{c = \underbrace{4\csc53°} + \underbrace{3\cot45°}}$

$$c = \cancel{4}\left(\frac{5}{\cancel{4}}\right) + 3(1) \Rightarrow \boxed{\mathbf{c = 8\ N}}$$

$$P = \frac{a+b+c}{3} = \frac{1+6+8}{3} \qquad \boxed{\therefore \mathbf{P = 5N}}$$

SACO
OLIVEROS