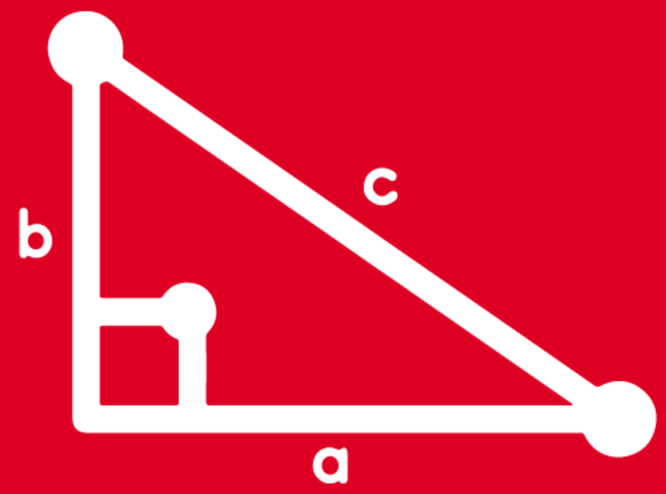
TRIGONOMETRY

Tomo 3



5th
SECONDARY

ADVISORY

SACO OLIVEROS

1. Sean α y θ las medidas de dos ángulos cuadrantales positivos y menores a una vuelta, tal que se cumple senα - tanθ = - 1
Efectué:

$$E = \frac{\tan\left(\frac{\alpha}{6}\right) + \cos\left(\frac{\theta}{3}\right)}{\text{sen}(\alpha - \theta)}$$

RESOLUCIÓN

Como $0° < \alpha$ y $\theta < 360°$ y

$$\underbrace{\text{sen}\alpha}_{-1} - \underbrace{\tan\theta}_{0} = -1$$

$\theta = 180°$

$\alpha = 270°$

Reemplazando en E:

$$E = \frac{\tan 45° + \cos 60°}{\text{sen} 90°}$$

$$E = \frac{1 + \frac{1}{2}}{1}$$

$$\boxed{\therefore\ E = \frac{3}{2}}$$

2. Si para un ángulo α en posición normal se cumple $\tan\alpha\sqrt{\operatorname{sen}\alpha}<0$, indique el signo de $M = \operatorname{sen}\alpha + \csc\alpha$ y $N = \cos\alpha \cdot \cot\alpha$.

RESOLUCIÓN

$$\underbrace{\tan\alpha}_{(-)}\underbrace{\sqrt{\operatorname{sen}\alpha}}_{(+)}<0$$

Si $\tan\alpha < 0$ ➡ $\alpha \in IIC \vee \alpha \in IVC$

Si $\operatorname{sen}\alpha > 0$ ➡ $\alpha \in IC \vee \alpha \in IIC$

por lo tanto: **$\alpha \in IIC$**

Nos piden el signo:

$$M = \underbrace{\operatorname{sen}\alpha}_{(+)} + \underbrace{\csc\alpha}_{(+)}$$ ➡ $M = (+)$

$$N = \underbrace{\cos\alpha}_{(-)} \,.\, \underbrace{\cot\alpha}_{(-)}$$ ➡ $N = (+)$

$\therefore$ $\boxed{(+)\ ;\ (+)}$

3. Si senϕ = –5/13 , además $\phi \in \langle 270°; 360° \rangle$, halle el valor de P = cscϕ + cotϕ

RESOLUCIÓN

- $\phi \in \langle 270°; 360° \rangle$

$\phi \in$ IVC ➡ x(+), y(–) , r(+)

- Además:

$$\text{sen}\phi = \frac{-5}{13} = \frac{y}{r}$$

$$\text{sen}\phi = \frac{y}{r}$$

Luego: $y = -5$ y $r = 13$

Sabemos: $r = \sqrt{x^2 + y^2}$

$$13 = \sqrt{(x)^2 + (-5)^2} \Rightarrow x = 12$$

Calculamos: P = cscϕ + cotϕ

$$P = \frac{13}{-5} + \frac{12}{-5}$$

$$\therefore P = -5$$

4. A Juan se le entregó S/x como incentivo por sus buenas calificaciones. Resolviendo la siguiente ecuación podrá averiguar con cuanto se le premió.

$$8\csc(-30°) - x.\cot(-45°) = 15\cos(-37°)$$

RESOLUCIÓN

Resolviendo la ecuación:

$$8\underbrace{(-\csc 30°)}_{-2} - x\,\underbrace{(-\cot 45°)}_{-1} = 15\,\underbrace{(\cos 37°)}_{4/5}$$

$$-16 + x = 12 \Rightarrow x = 28$$

$\cos(-\alpha) = \cos\alpha$
$\cot(-\alpha) = -\cot\alpha$
$\csc(-\alpha) = -\csc\alpha$

$\therefore$ Juan recibió S/ 28 de incentivo

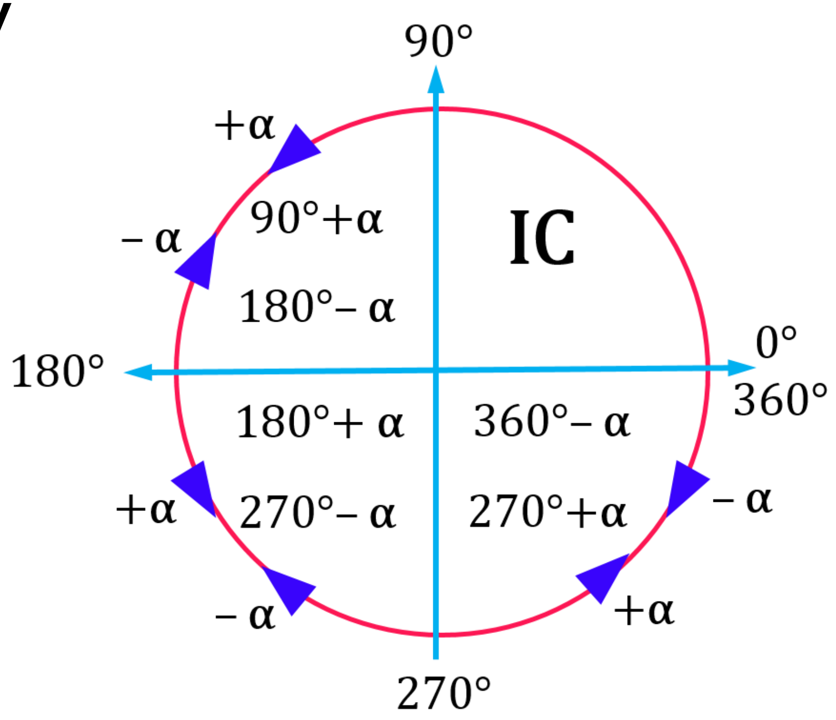
5. Si: x – y = π / 2, reduzca: $E=\dfrac{3\cos x}{\operatorname{sen} y}-\dfrac{\cot x}{\tan y}$

RESOLUCIÓN

Despejamos x, luego reemplazar en E:

$$E=\frac{3\cos\overbrace{\left(\frac{\pi}{2}+y\right)}^{IIC}}{\operatorname{sen} y}-\frac{\cot\overbrace{\left(\frac{\pi}{2}+y\right)}^{IIC}}{\tan y}$$

$$E=\frac{3(-\cancel{\operatorname{sen} y})}{\cancel{\operatorname{sen} y}}-\frac{-\cancel{\tan y}}{\cancel{\tan y}}\rightarrow E=-3+1$$



$$\boxed{E=-2}$$

6. Si $\theta \in$ IIC, además, $\cos\theta = -\frac{\sqrt{5}}{3}$, reduzca: $Q = \dfrac{\sqrt{5}\cot(270° + \theta)}{\csc(180° - \theta)}$

RESOLUCIÓN

$$Q = \frac{\sqrt{5}\cot(\overbrace{270° + \theta}^{IVC})}{\csc(\underbrace{180° - \theta}_{IIC})}$$

$$Q = \frac{\sqrt{5}(-\tan\theta)}{\csc\theta}$$

$$Q = -\frac{\sqrt{5}\tan\theta}{\csc\theta} \ldots (*)$$

Dato:

$$\cos\theta = \frac{-\sqrt{5}}{3} = \frac{x}{r}$$

$$r = \sqrt{x^2 + y^2}$$

$$3 = \sqrt{(-\sqrt{5})^2 + y^2} \rightarrow y = 2$$

Reemplazando en (*):

$$Q = -\frac{\sqrt{5}\tan\theta}{\csc\theta} = -\sqrt{5}\,\frac{\frac{y}{x}}{\frac{r}{y}} = -\sqrt{5}\,\frac{\frac{2}{-\sqrt{5}}}{\frac{3}{2}}$$

$$Q = \cancel{\sqrt{5}}\left(\frac{4}{3\cancel{\sqrt{5}}}\right)$$

$$\boxed{\therefore Q = \frac{4}{3}}$$

7. Simplifique la expresión: $E=\dfrac{\cos(3\pi+x)\,.\,\mathrm{sen}(4\pi-x)}{\mathrm{sen}\left(\dfrac{9\pi}{2}+x\right)}$

RESOLUCIÓN

$$E=\frac{\cos(\overbrace{3}^{\text{impar}}\pi+x)\,.\,\mathrm{sen}(\overbrace{4}^{\text{par}}\pi-x)}{\mathrm{sen}\left(\underbrace{\dfrac{9\pi}{2}}_{4+1}+x\right)}$$

$$E=\frac{\cos(\overbrace{3\pi+x}^{\text{IIIC}})\,.\,\mathrm{sen}(\overbrace{4\pi-x}^{\text{IVC}})}{\mathrm{sen}\underbrace{\left(\dfrac{9\pi}{2}+x\right)}_{\text{IIC}}}$$

$$E=\frac{(-\cancel{\cos}x)\,.\,(-\mathrm{sen}x)}{\cancel{\cos}x}$$

$$\boxed{\therefore E=\mathrm{sen}x}$$

8. Siendo $y - x = 1350°$; reduzca: $M = \dfrac{\text{seny}}{\text{cosx}} + \text{tanx.tany}$

RESOLUCIÓN

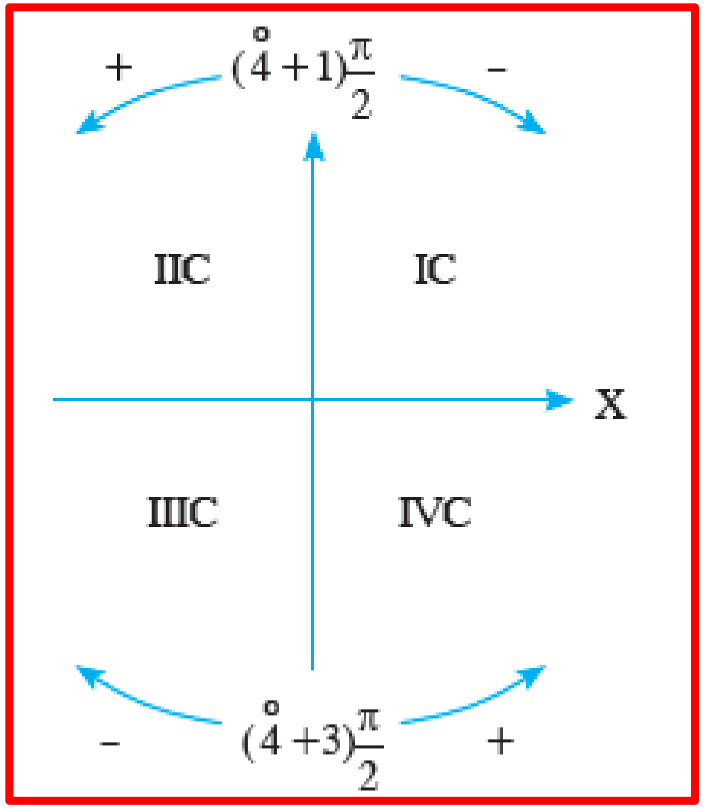
Reemplazando "y" en términos de "x": $y = 1350° + x$

$$M = \frac{\text{sen}(1350° + x)}{\text{cosx}} + \tan x.\tan(1350° + x)$$

$$M = \frac{\text{sen}(\overset{\overset{\circ}{4}+3}{15(90°)} + x)}{\text{cosx}} + \tan x.\tan(\overset{\overset{\circ}{4}+3}{15(90°)} + x)$$

$$M = \frac{\text{sen}\overbrace{(15(90°) + x)}^{IVC}}{\text{cosx}} + \tan x.\tan\overbrace{(15(90°) + x)}^{IVC}$$

$$M = \frac{-\cancel{\cos x}}{\cancel{\text{cosx}}} + \underbrace{\text{tanx.(-cotx)}}_{-1}$$



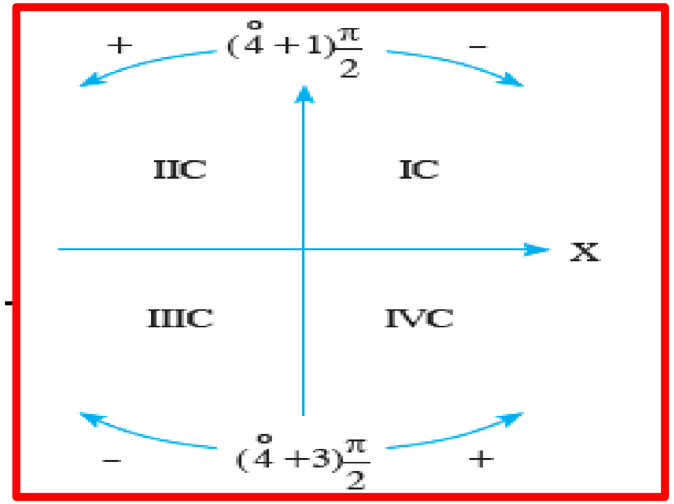
$$M = -1 - 1$$

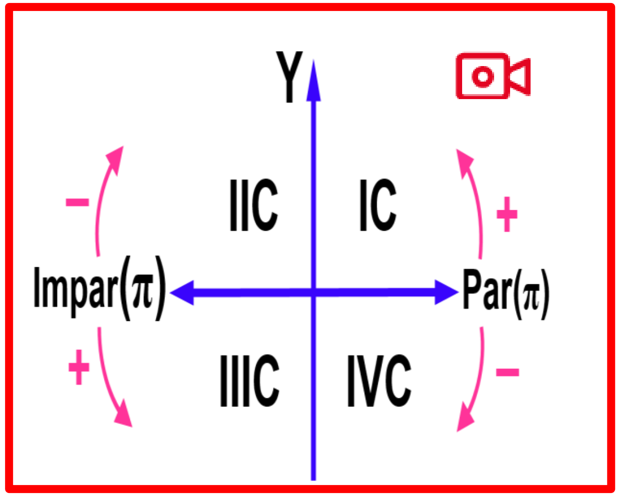
$$\boxed{\therefore M = -2}$$

9. Se cumple que $\cot(8\pi + x) = 3$.

Efectúe: $K = \text{sen}\left(\frac{15\pi}{2} + x\right).\csc(21\pi$

si x es un ángulo agudo.





RESOLUCIÓN

$$K = sen\left(\overset{\overset{\circ}{4}+3}{15}\frac{\pi}{2} + x\right).\csc(\overset{\text{impar}}{21}\pi - x)$$

$$K = \underbrace{sen\overbrace{\left(15\frac{\pi}{2} + x\right)}^{IVC}}_{-\cos x}.\underbrace{\csc\overbrace{(21\pi - x)}^{IIC}}_{\csc x}$$

$$K = -\cos x.\csc x \quad \ldots(*)$$

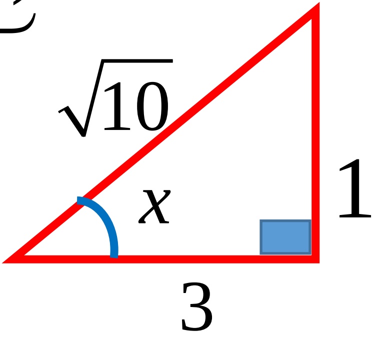
Del dato:

$$\cot(\overset{\text{par}}{8}\pi + x) = 3$$

$$\underbrace{\cot\overbrace{(8\pi + x)}^{IC}}_{\cot x} = 3$$

$$\cot x = 3$$

$$\cot x = \frac{3}{1}$$



Reemplazando

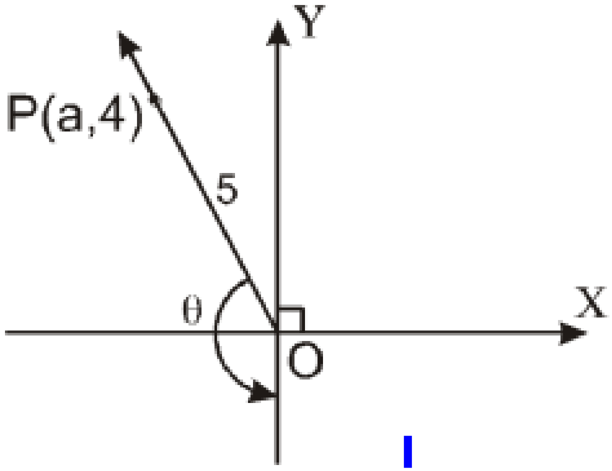
$$K = -\cos x.\csc x$$

$$K = -\frac{3}{\cancel{\sqrt{10}}}.\frac{\cancel{\sqrt{10}}}{1}$$

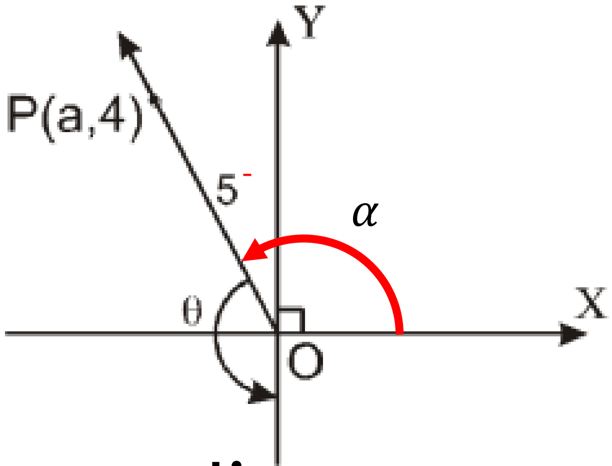
$$\boxed{\therefore K = -3}$$

10. Con la información dada en la figura, calcule el valor de

M = secθ + $tan\theta - 2$



RESOLUCIÓN



Por radio vector:

$$r=\sqrt{x^2+y^2}$$

Del dato:

$$5=\sqrt{a^2+4^2}$$

Como:$P \in IIC$

$$\rightarrow a=-3$$

Del grafico:

$$\alpha+\theta=270°$$

$$\theta=270°-\alpha$$

$$sec\theta=\sec(\underbrace{270°-\alpha}_{IIIC})$$

$$sec\theta=-csc\alpha$$

$$sec\theta=-\frac{5}{4}$$

$$tan\theta=\tan(\underbrace{270°-\alpha}_{IIIC})$$

$$tan\theta=cot\alpha$$

$$tan\theta=-\frac{3}{4}$$

Reemplazamos:

M = secθ + tanθ − 2

M =$\left(-\frac{5}{4}\right)+\left(-\frac{3}{4}\right)-2$

$$\boxed{\therefore M=-4}$$