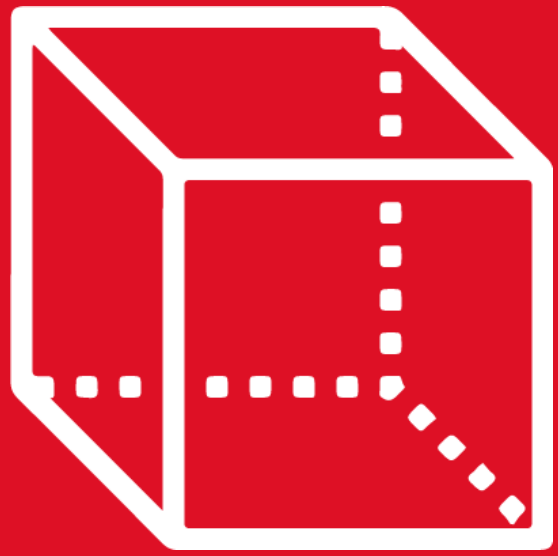
GEOMETRÍA

Capítulo 15



3rd

SECONDARY

SEGMENTOS PROPORCIONALES

SACO OLIVEROS

1. PROPORCIÓN ÁUREA

También llamada **sección áurea**, se halla presente en la naturaleza, el arte y la arquitectura.

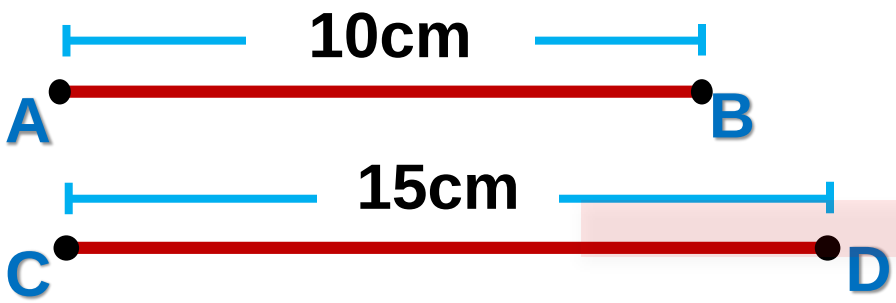
Los griegos la conocieron en **el estudio del cuerpo humano** y la utilizaron, en la escultura y la arquitectura y la definieron como una característica fundamental en su estética.

GEOMETRÍA, ESCALA Y PROPOCIÓN EN EL TIEMPO

SEGMENTOS PROPORCIONALES

RAZÓN GEOMÉTRICA DE DOS SEGMENTOS .-

Es el cociente que se obtiene al dividir las longitudes de dos segmentos que tienen la misma unidad de medida.

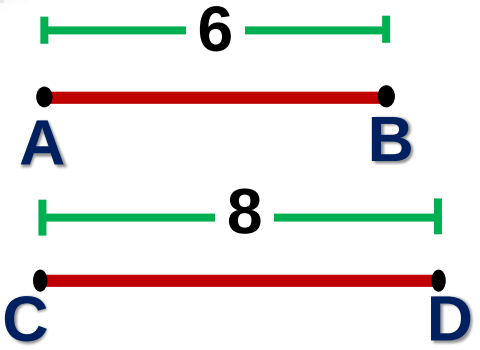


$$\frac{AB}{CD} = \frac{\cancel{10cm}}{\cancel{15cm}} \Rightarrow \frac{AB}{CD} = \frac{2}{3}$$

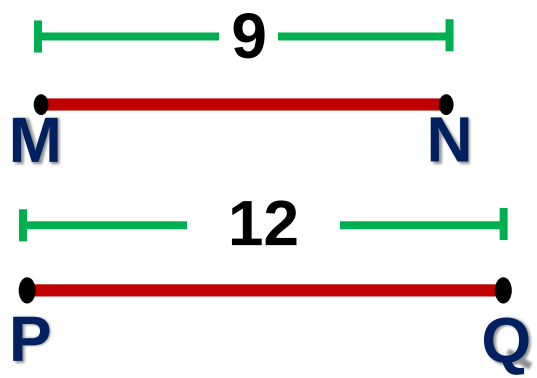
$\frac{2}{3}$: razón geométrica de $\overline{AB}$ y $\overline{CD}$

SEGMENTOS PROPORCIONALES

Es la igualdad de dos o más razones geométricas de segmentos.



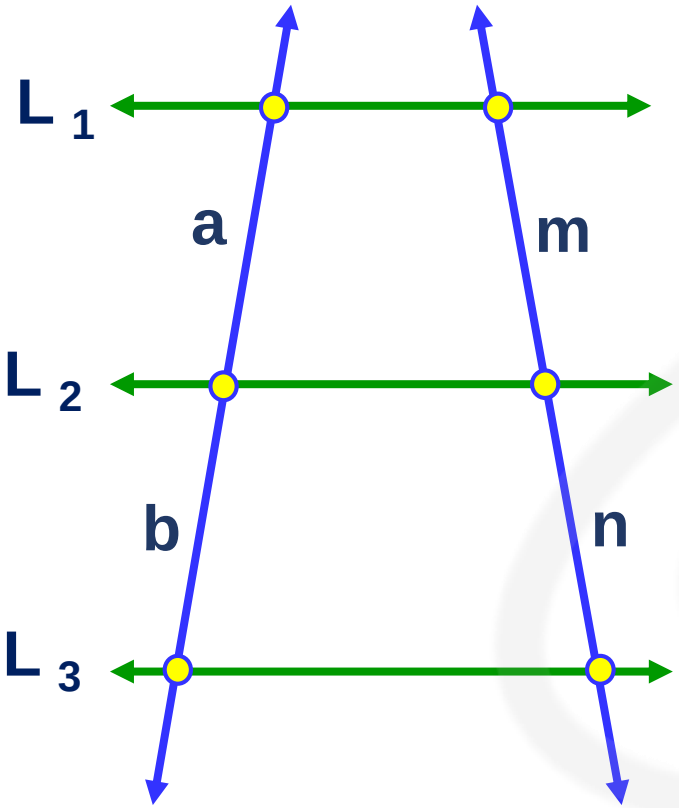
$$\frac{AB}{CD} = \frac{\cancel{6}}{\cancel{8}} = \frac{3}{4}$$



$$\frac{MN}{PQ} = \frac{\cancel{9}}{\cancel{12}} = \frac{3}{4}$$

$$\frac{AB}{CD} = \frac{MN}{PQ} \Rightarrow \text{Son proporcionales}$$

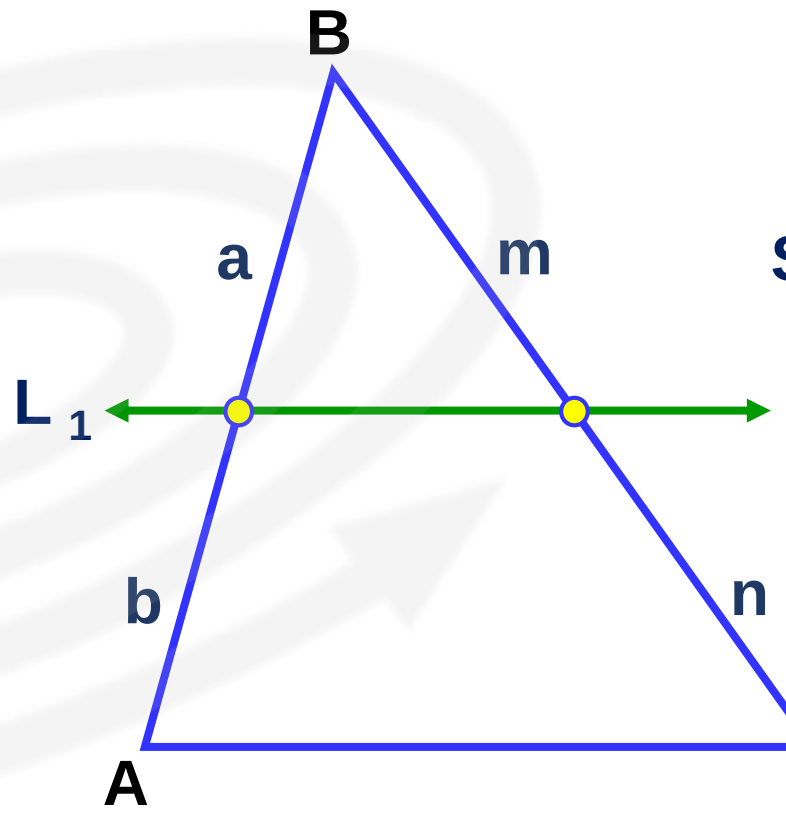
Teorema de Thales



Si: $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2} \parallel \overleftrightarrow{L_3}$

$$\frac{a}{b} = \frac{m}{n}$$

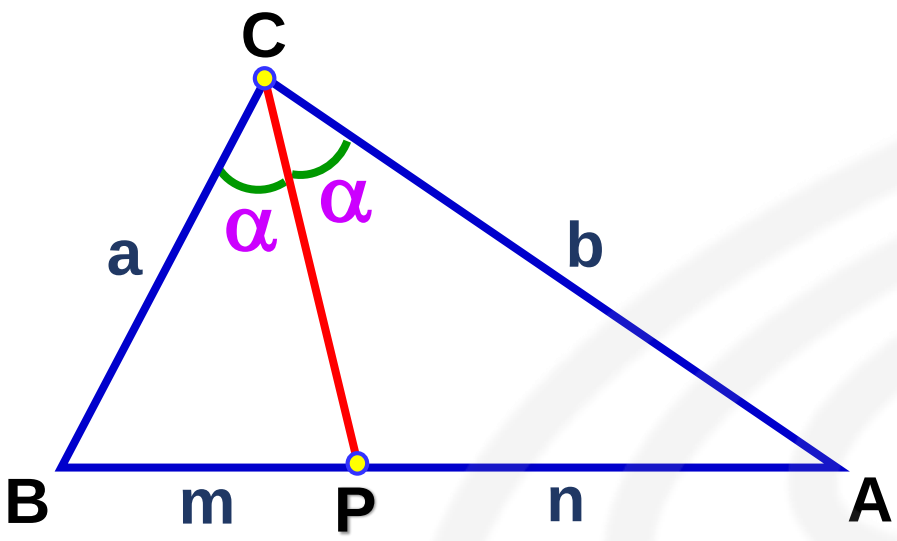
Corolario de Thales



Si: $\overleftrightarrow{L_1} \parallel \overline{AC}$

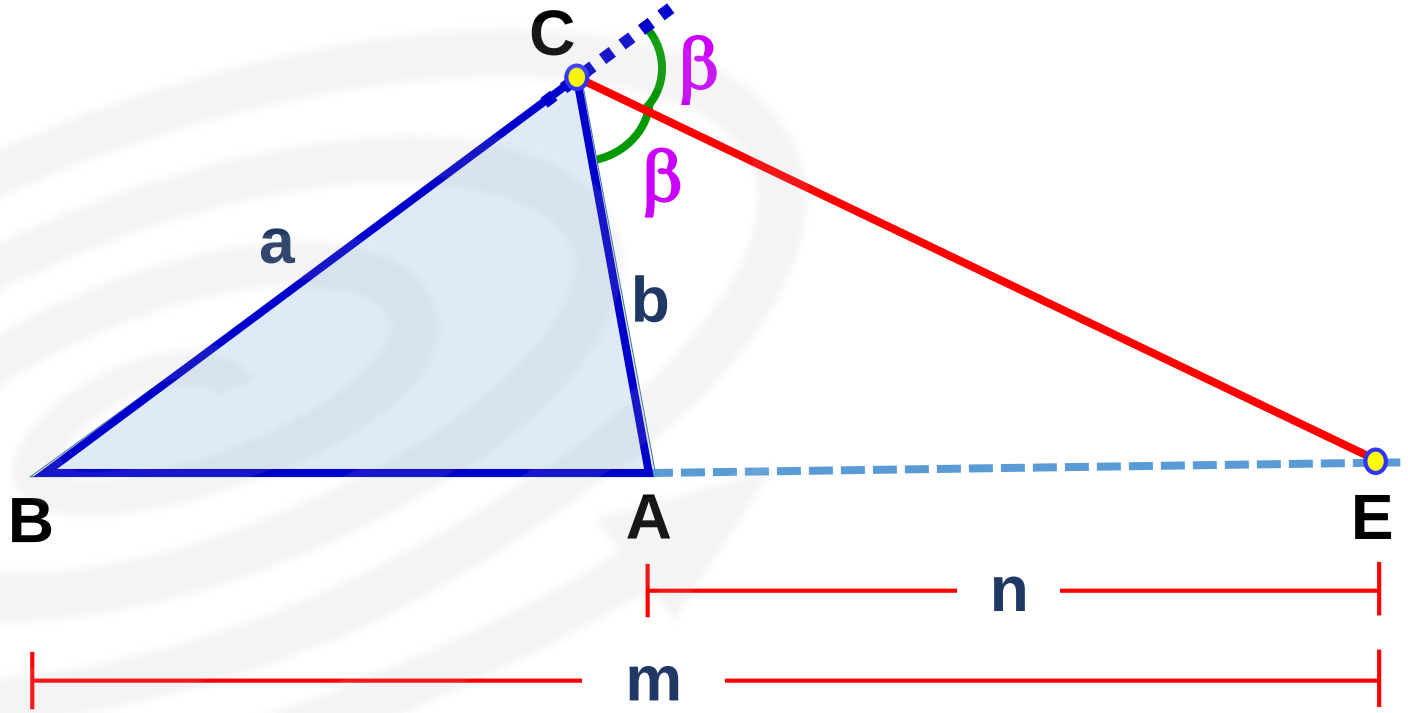
$$\frac{a}{b} = \frac{m}{n}$$

Teorema de la bisectriz Interior



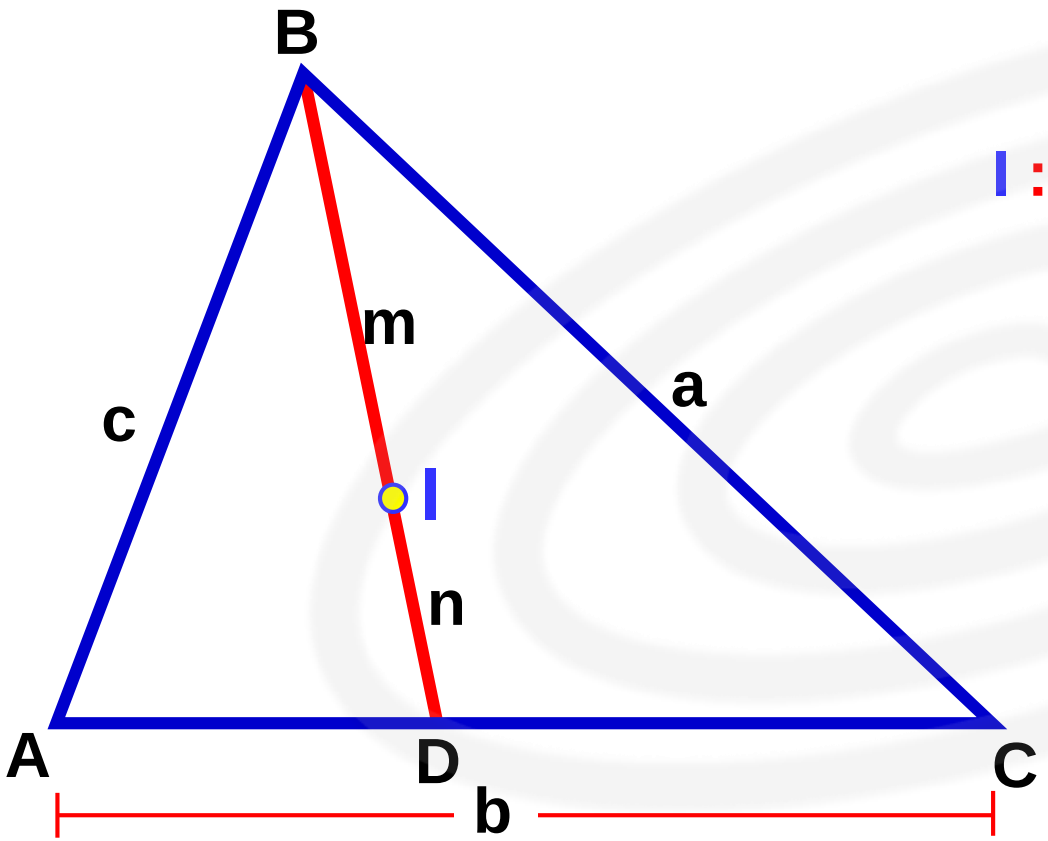
$$\frac{a}{b} = \frac{m}{n}$$

Teorema de la bisectriz Exterior



$$\frac{a}{b} = \frac{m}{n}$$

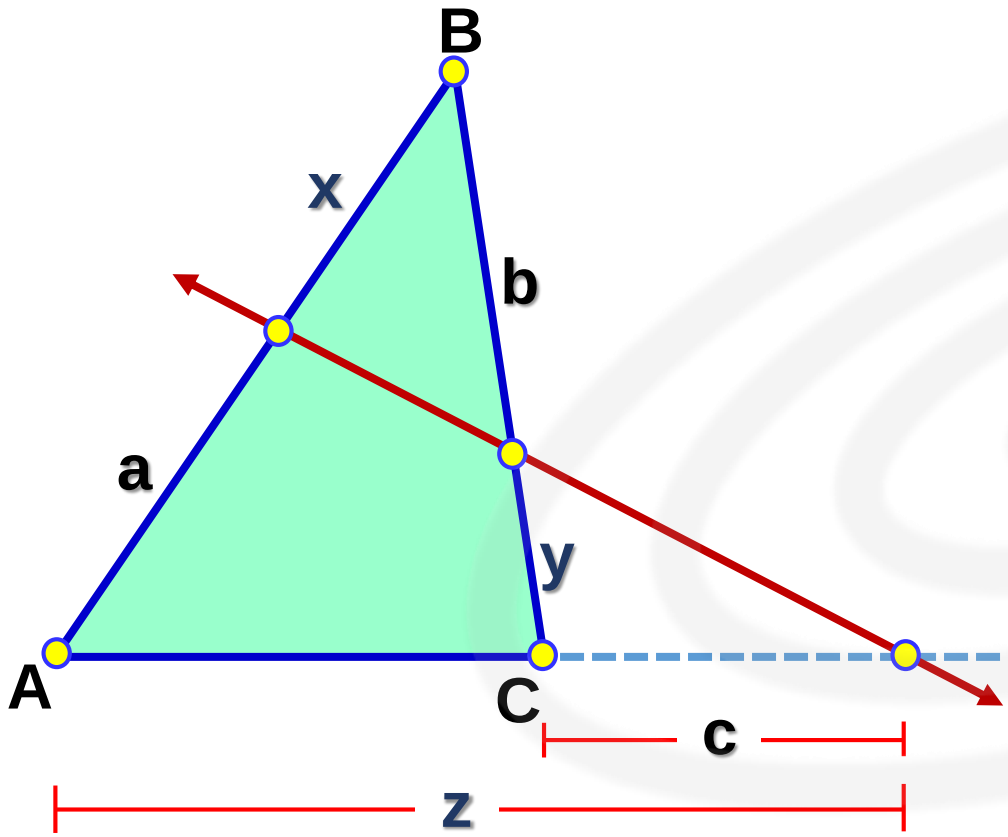
Teorema del incentro



I : Incentro del △ABC

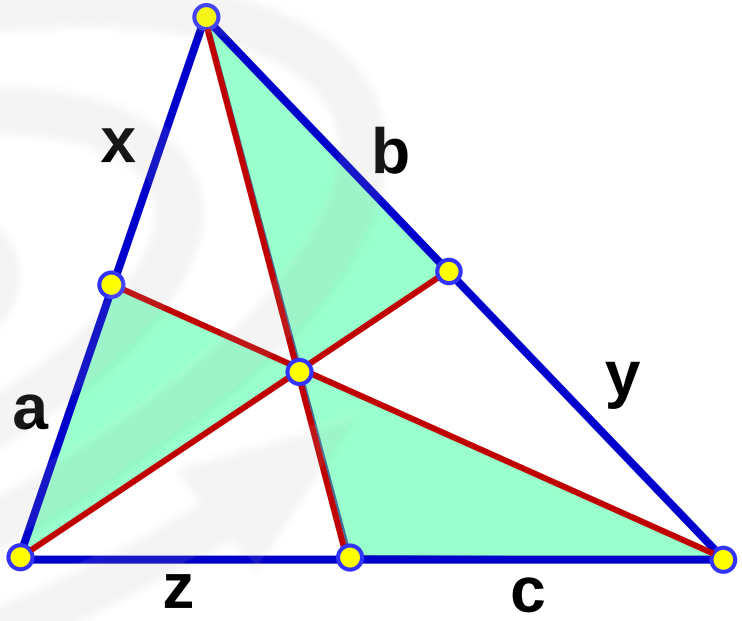
$$\frac{m}{n} = \frac{a+c}{b}$$

Teorema de Menelao



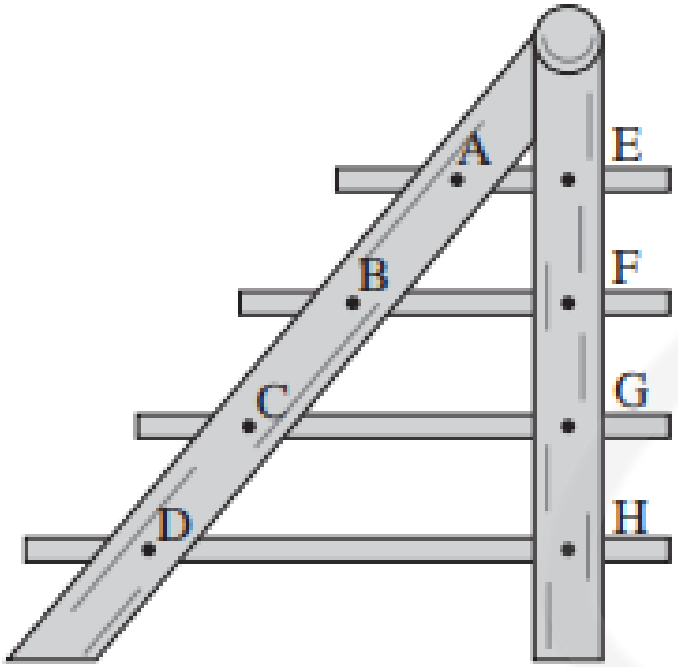
$$a \cdot b \cdot c = x \cdot y \cdot z$$

Teorema de Ceva

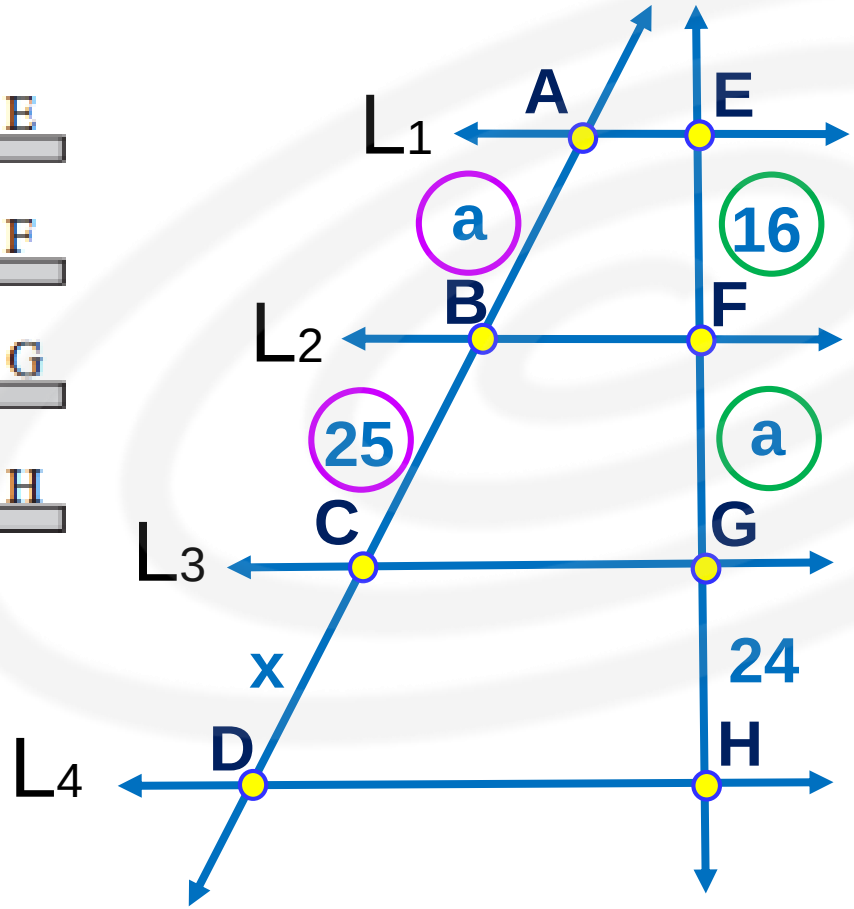


$$a \cdot b \cdot c = x \cdot y \cdot z$$

1. Se tiene una escalera con peldaños paralelos tal que AB = FG, BC = 25 cm, EF = 16 cm y GH = 24 cm. Determine la longitud del $\overline{CD}$.

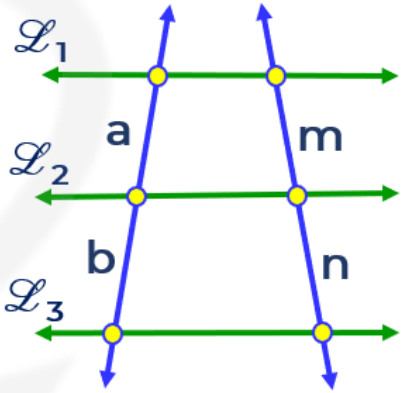


Resolución



Teorema de Tales

$\mathcal{L}_1 // \mathcal{L}_2 // \mathcal{L}_3$



$$\frac{a}{b} = \frac{m}{n}$$

$$\frac{a}{25} = \frac{16}{a}$$

$$a^2 = 400$$

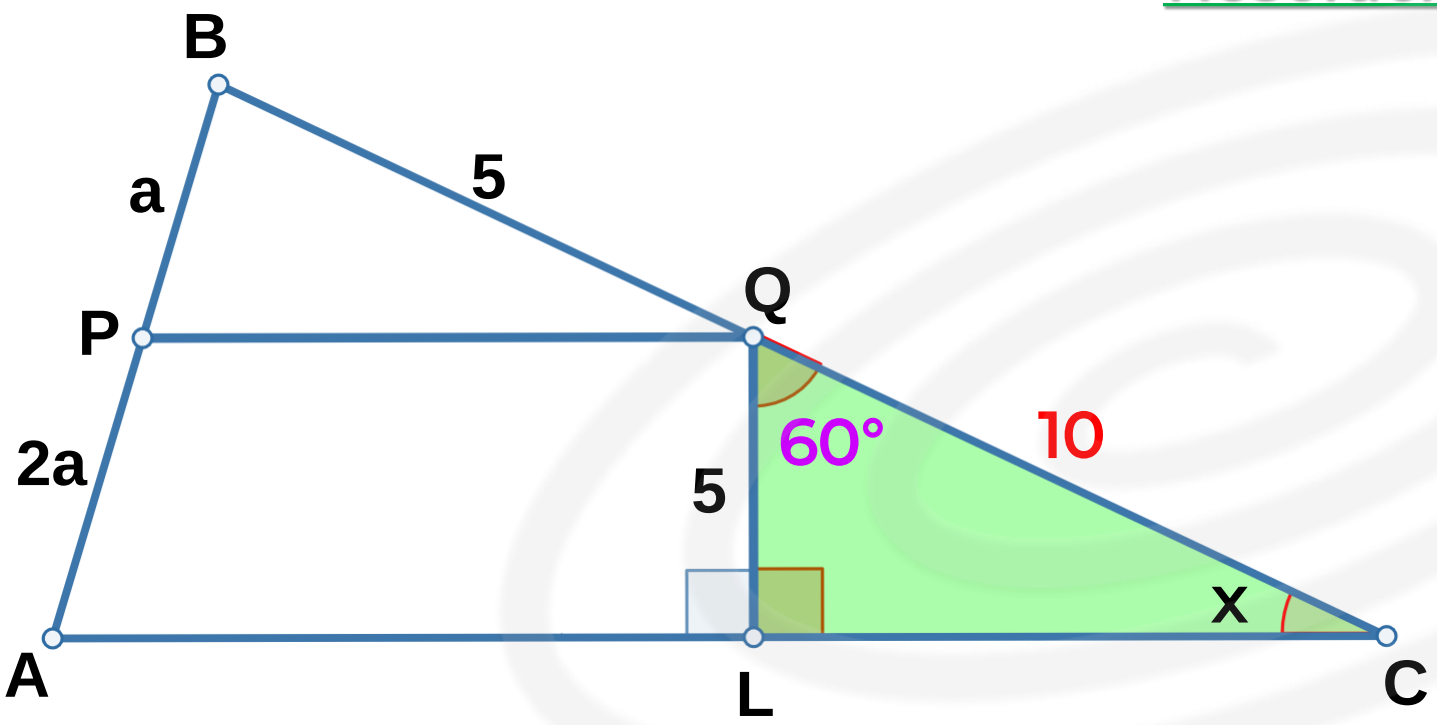
$$a = 20$$

$$\frac{25}{x} = \frac{20}{24}$$

$$20x = 600$$

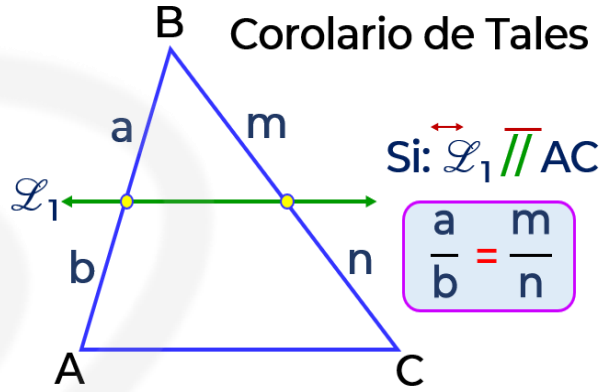
$$x = 30 \text{ cm}$$

2. Del gráfico, halle el valor de x, si $\overline{PQ} \parallel \overline{AC}$.



Resolución

Corolario de Tales



$$\frac{\cancel{a}}{\cancel{2a}} = \frac{5}{QC}$$

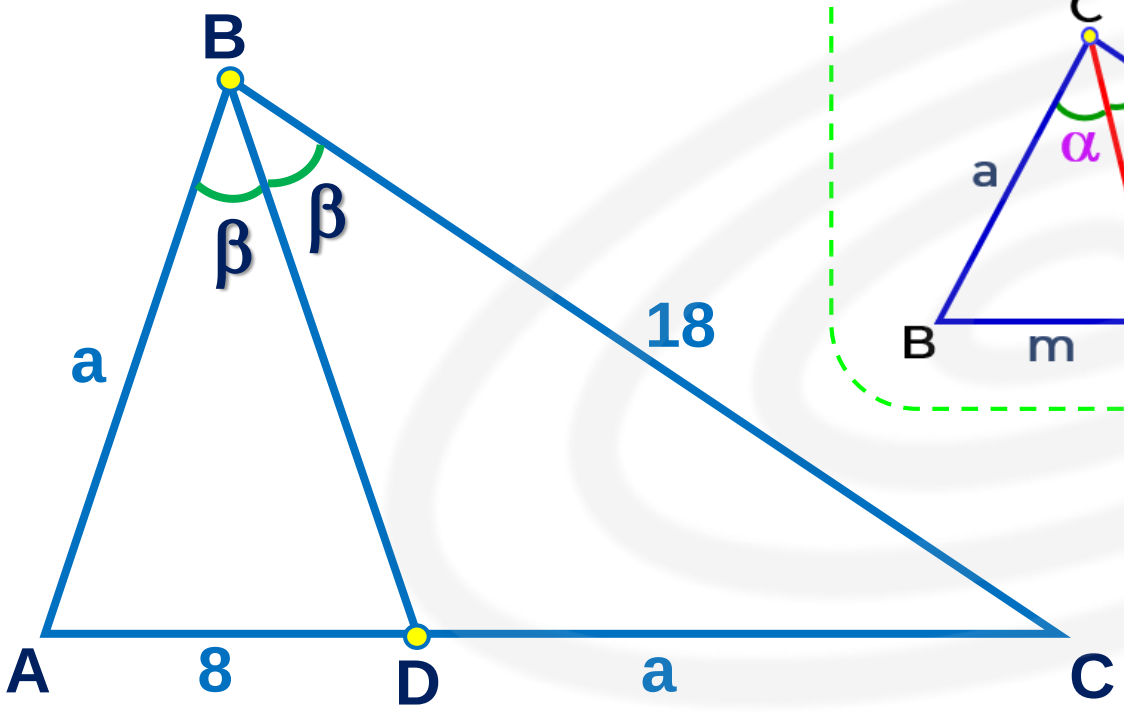
$$QC = 10$$

$\triangle QLC$: Notable de 30° y 60°

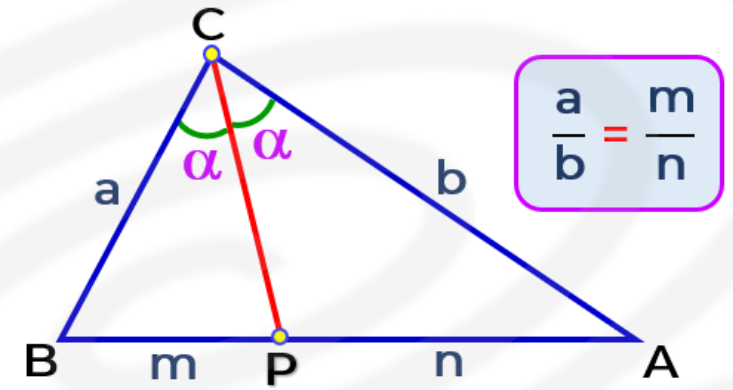
$$x = 30°$$

3. En un triángulo ABC, se traza la bisectriz interior $\overline{BD}$. Calcule el perímetro del triángulo ABC, si AB = DC, AD = 8 m y BC = 18 m.

Resolución



Teorema de la bisectriz Interior



$$\frac{a}{b} = \frac{m}{n}$$

$$\frac{a}{18} = \frac{8}{a}$$

$$a^2 = 144$$

$$a = 12$$

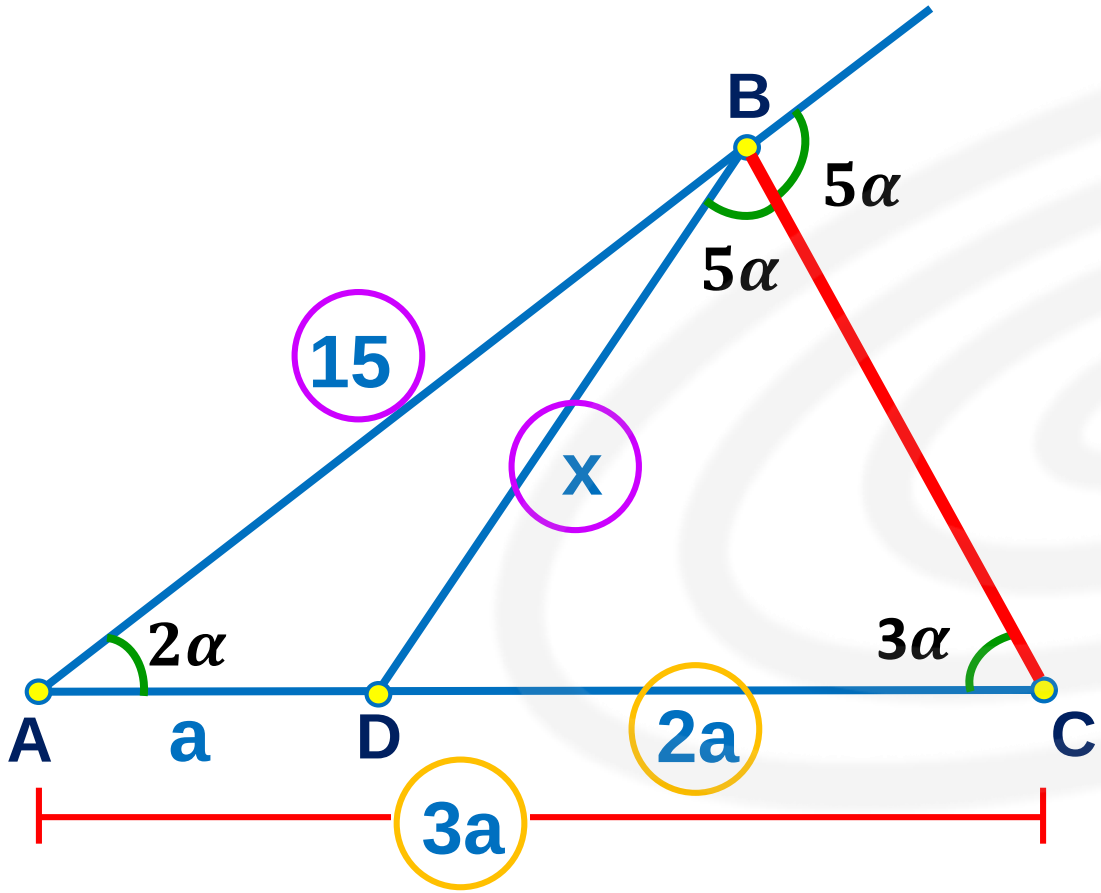
- **Piden: perímetro**

$$2p_{\triangle} = 8 + 18 + 2a$$

$$2p_{\triangle} = 8 + 18 + 2(12)$$

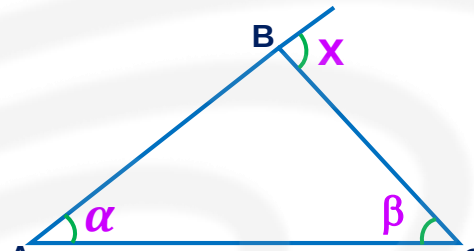
$$2p_{\triangle} = 50 \text{ m}$$

4. En la figura CD = 2(AD) y AB = 15 cm . Halle BD.



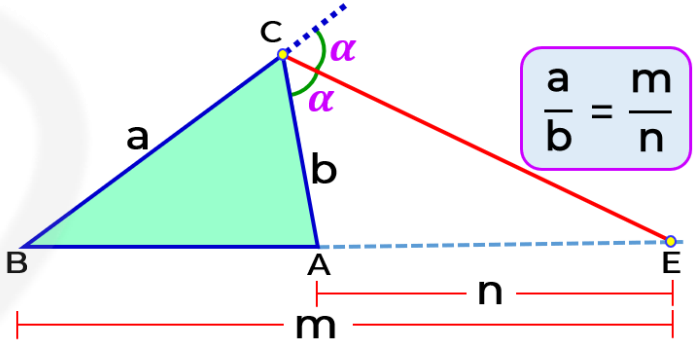
Resolución

Angulo Exterior



$x = \alpha + \beta$

Teorema de la Bisectriz Exterior



$\frac{a}{b} = \frac{m}{n}$

Prolongamos $\overline{AB}$

$\measuredangle$ext. B = $2\alpha + 3\alpha$

$\measuredangle$ext. B = 5α

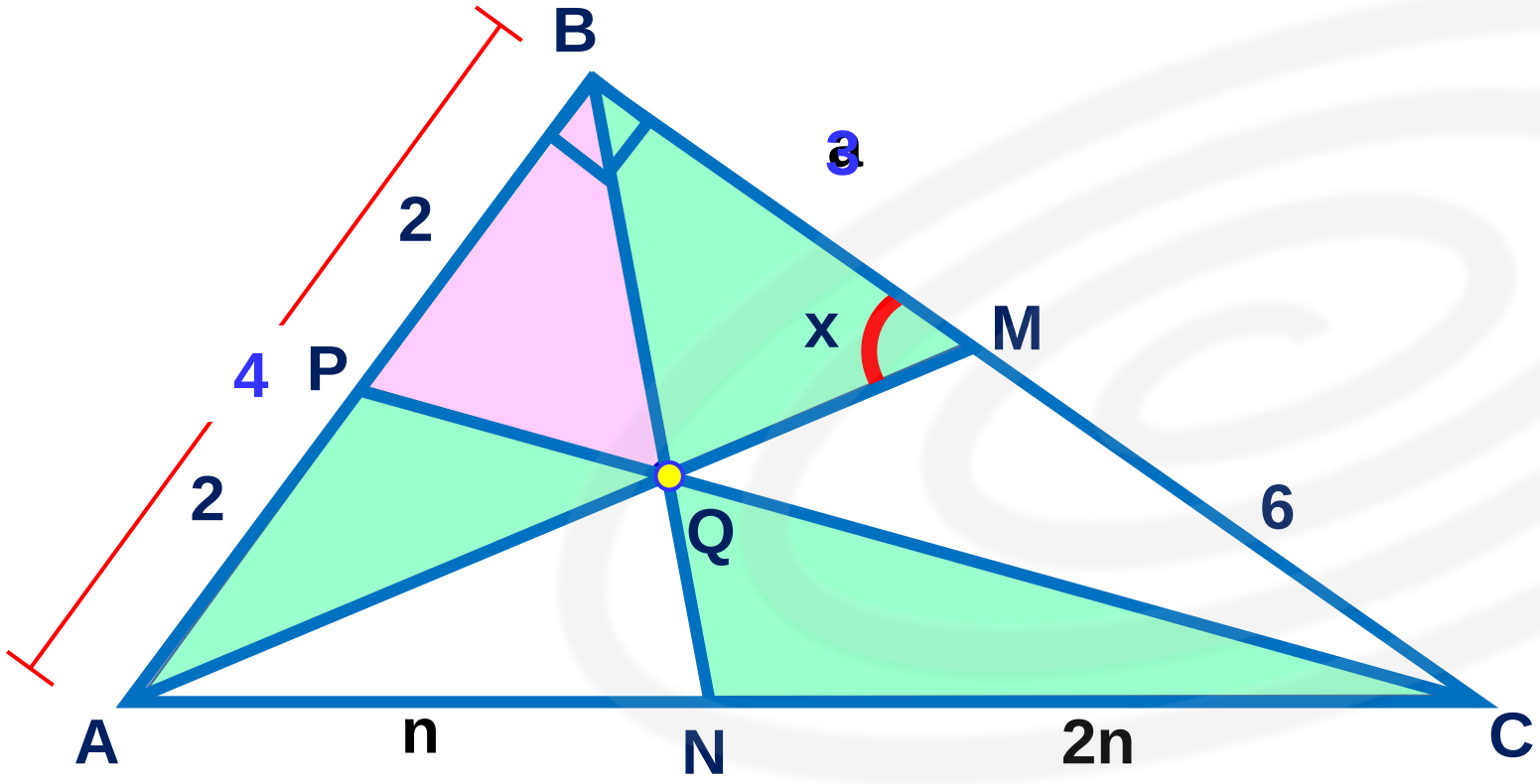
$$\frac{15}{x} = \frac{3a}{2a}$$

$3x = 30$

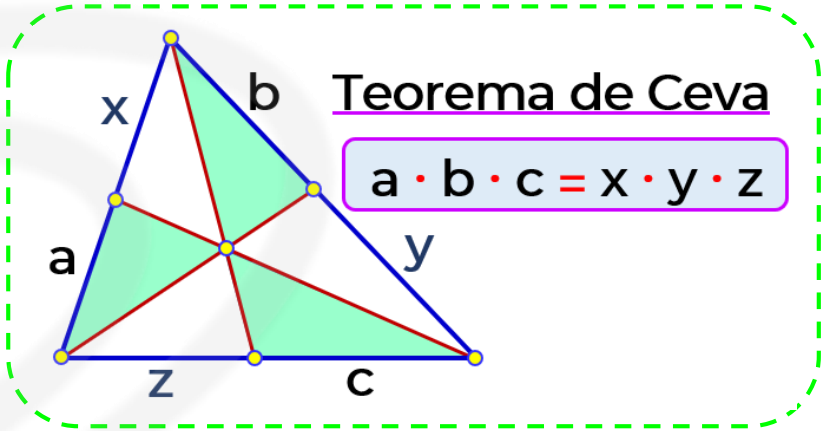
$x = 10$

BD = 10 cm

5. En la figura, halle el valor de x.



Resolución



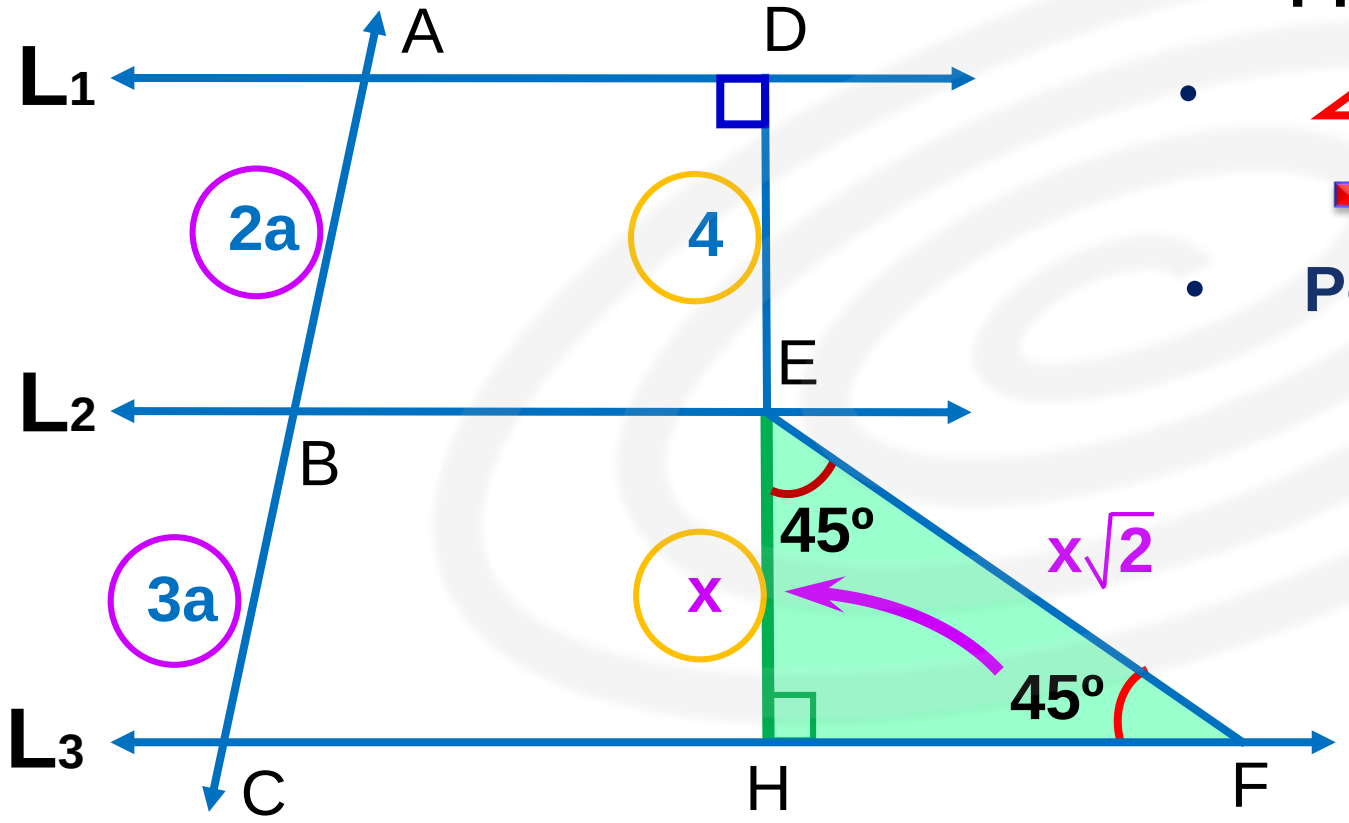
$(2)(a)(2n) = (2)(6)(n)$

$a = 3$

$\triangle ABM$: Notable de 37° y 53°

$x = 53°$

6. Se muestra las rectas paralelas L_1, L_2 y L_3. Si 3(AB) = 2(BC), DE = 4 m y EF = x $\sqrt{2}$; determine valor de x.



Resolución

- Prolongamos $\overline{DE}$ hasta H
- EFH : Notable de 45° y 45°

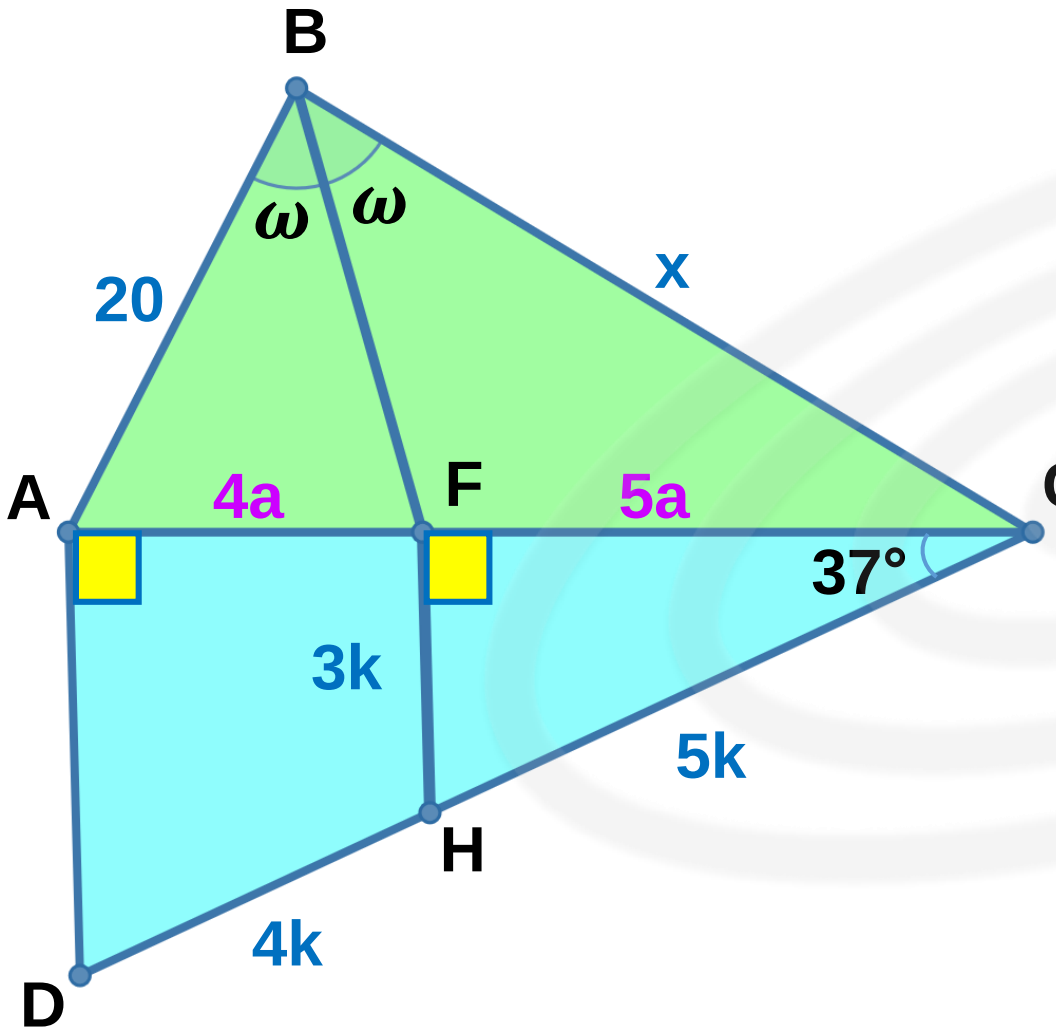
  ⇒ EH = x
- Por el teorema de Tales:

$$\frac{2a}{3a} = \frac{4}{x}$$

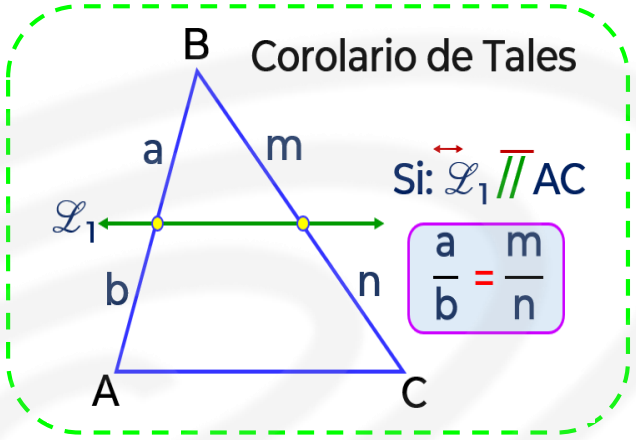
$$2x = 12$$

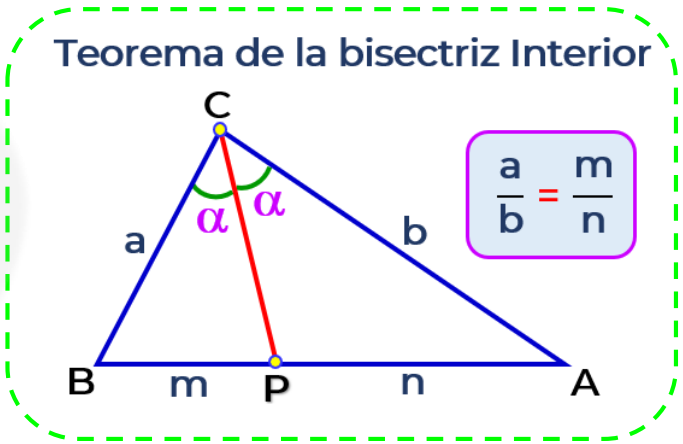
x = 6 m

7. En la figura, halle el valor de x.



Resolución





. $\triangle$ **FCH :** Notable de 37° y 53°

➡ **HC = 5K**

. $\triangle$ **ACD :** Corolario de Tales

➡ **AF = 4a** **FC = 5a**

$$\frac{20}{x} = \frac{4a}{5a}$$

$$4x = 100$$

x = 25