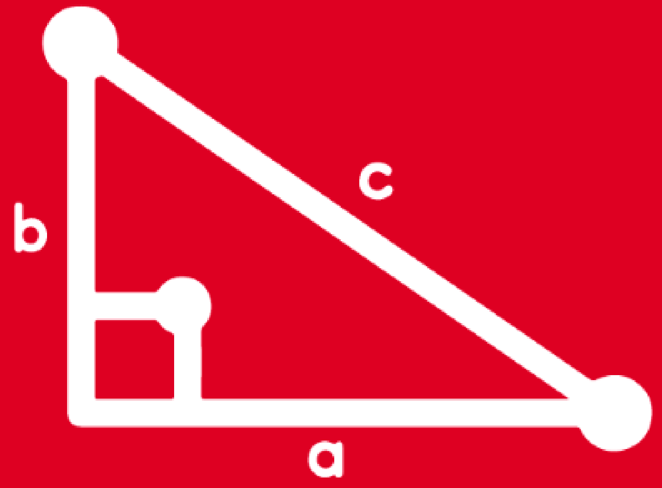
TRIGONOMETRY

Tomo 05



5th
SECONDARY

ADVISORY

SACO OLIVEROS

1. Simplifique la expresión: $E = \dfrac{\text{sen}(x+y)}{\cos x\cos y} - \tan y$

RESOLUCIÓN

$$\boxed{\text{sen}(x+y) = \text{sen}x.\cos y + \cos x.\text{sen}y}$$

$$E = \frac{\text{sen}(x+y)}{\cos x\cos y} - \tan y$$

$$E = \frac{\text{sen}x\cos y + \cos x\,\text{sen}y}{\cos x\cos y} - \tan y$$

$$E = \frac{\text{sen}x\cancel{\cos y}}{\cos x\cancel{\cos y}} + \frac{\cancel{\cos x}\,\text{sen}y}{\cancel{\cos x}\cos y} - \tan y$$

$$E = \frac{\text{sen}x}{\cos x} + \frac{\text{sen}y}{\cos y} - \tan y$$

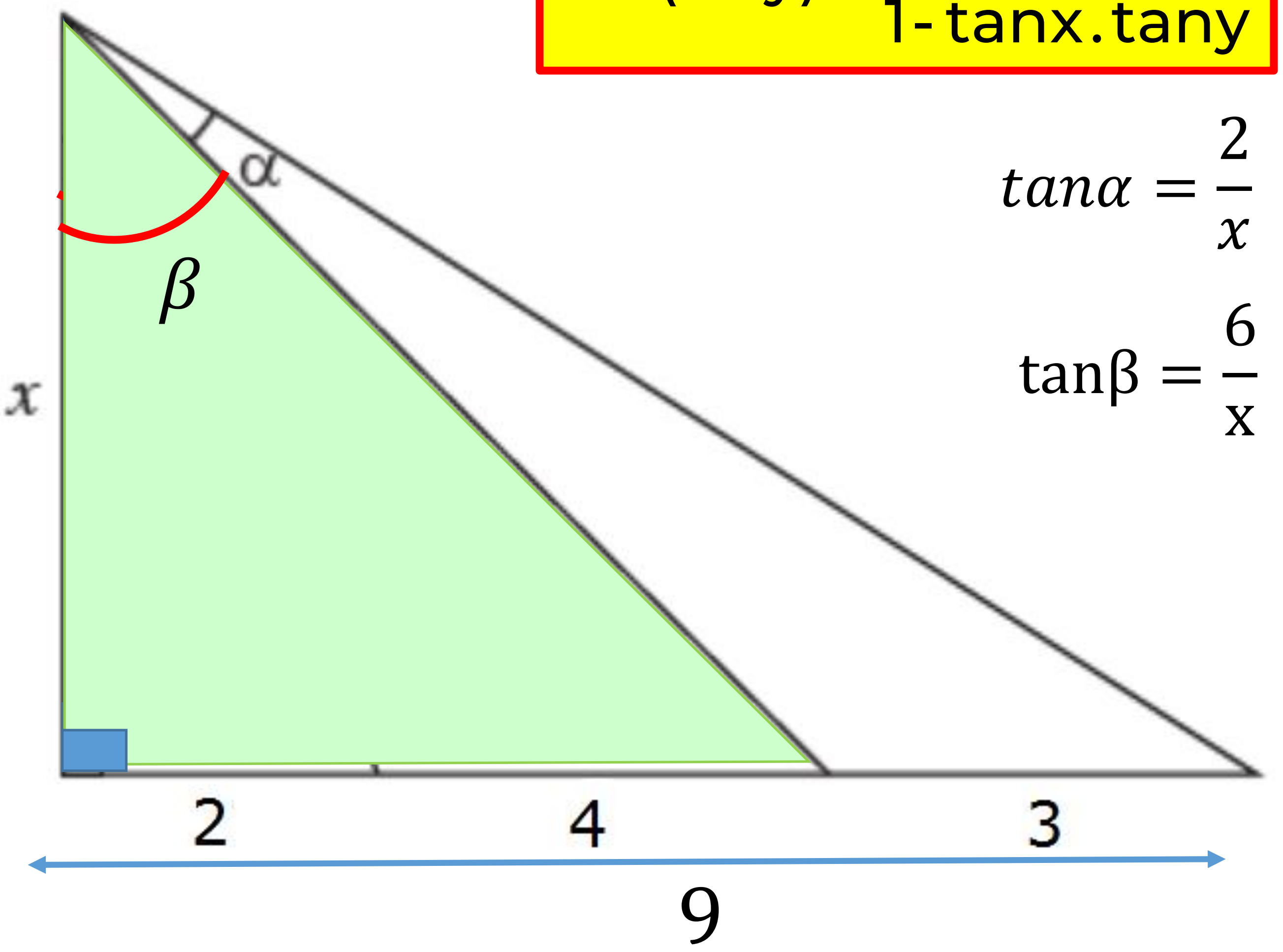
$$E = \tan x + \cancel{\tan y} - \cancel{\tan y}$$

$$\boxed{\therefore\ E = \tan x}$$

2. Del gráfico, halle el valor de *x*.

RESOLUCIÓN

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$



$$tan\alpha = \frac{2}{x}$$

$$\tan\beta = \frac{6}{x}$$

Se observa que: $\tan(\alpha+\beta) = \frac{9}{x}$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha . \tan\beta} = \frac{9}{x}$$

$$\frac{\frac{2}{x} + \frac{6}{x}}{1 - \left(\frac{2}{x}\right) . \left(\frac{6}{x}\right)} = \frac{9}{x} \Rightarrow \frac{\frac{8}{x}}{1 - \frac{12}{x^2}} = \frac{9}{x}$$

$$8 = 9\left(1 - \frac{12}{x^2}\right) \Rightarrow 8 = 9 - \frac{9(12)}{x^2}$$

$$\frac{9(12)}{x^2} = 1 \Rightarrow x^2 = 36 * 3$$

$$\therefore\ x = 6\sqrt{3}$$

3. Efectúe: $M = (1 + \tan 19°)(1 + \tan 26°)$

RESOLUCIÓN

$$\tan x + \tan y + \tan(x+y) \cdot \tan x \cdot \tan y = \tan(x+y)$$

$$M = (1 + \tan 19°)(1 + \tan 26°)$$

$$M = 1 + \tan 26° + \tan 19° + 1 \,.\, \tan 19° . \tan 26°$$

$$M = 1 + (\tan 26° + \tan 19° + \tan 45° \,.\, \tan 19° . \tan 26°)$$

$$M = 1 + (\tan(26° + 19°))$$

$$M = 1 + \tan 45° = 1 + 1$$

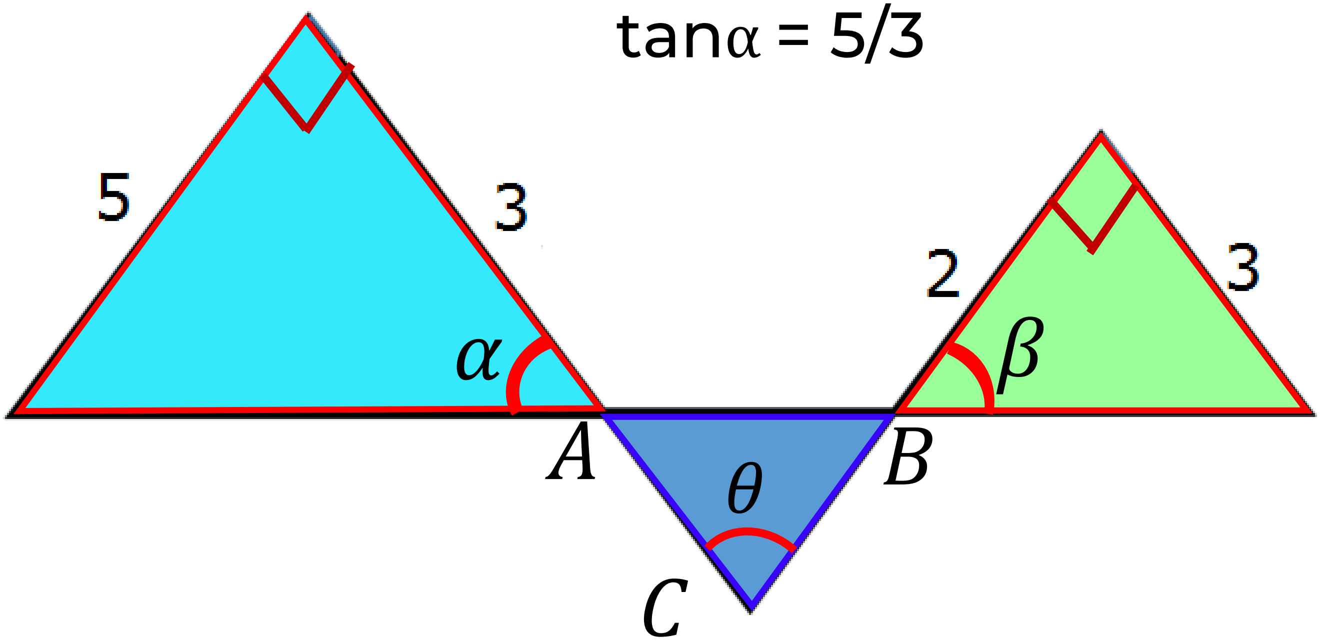
$$\therefore M = 2$$

4. Del gráfico, halle el valor de $\tan\theta$.

RESOLUCIÓN

$\tan\beta = 3/2$

$\tan\alpha = 5/3$



Si $x + y + z = 180°$, se cumple:

$\tan x + \tan y + \tan z = \tan x.\tan y.\tan z$

$\triangle ABC$: $\alpha + \beta + \theta = 180°$

$$\tan\alpha + \tan\beta + \tan\theta = \tan\alpha\tan\beta\tan\theta$$

$$\frac{5}{3} + \frac{3}{2} + \tan\theta = \frac{5}{3}.\frac{3}{2}.\tan\theta$$

$$\frac{19}{6} + \tan\theta = \frac{5}{2}.\tan\theta$$

$$\frac{19}{\cancel{6}} = \frac{3}{\cancel{2}}.\tan\theta$$

$\therefore\ \tan\theta = 19/9$

5. Sabiendo que π/24 ; determine el valor de :
$E = \operatorname{senx}\cos^3 x - \cos x \operatorname{sen}^3 x$

RESOLUCIÓN

$$E = \operatorname{senx}\cos^3 x - \cos x\operatorname{sen}^3 x$$

$$E = \operatorname{senx}\cos x\underbrace{(\cos^2 x - \operatorname{sen}^2 x)}_{\cos 2x}$$

$$2E = \underbrace{2\operatorname{senx}\cos x}_{\operatorname{sen}2x}\cos 2x$$

$$2E = \operatorname{sen}2x\cos 2x$$

$$4E = \underbrace{2\operatorname{sen}2x\cos 2x}_{\operatorname{sen}4x}$$

$$E = \frac{1}{4}\operatorname{sen}(30°)$$

$$E = \frac{1}{4}\left(\frac{1}{2}\right)$$

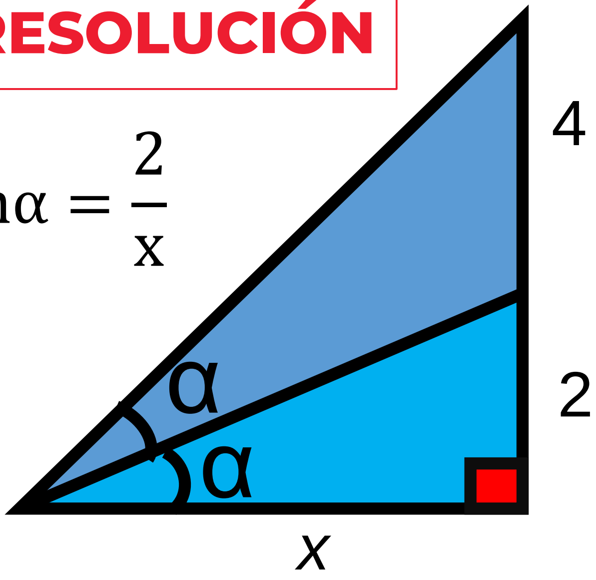
$$\boxed{4x = \frac{\pi}{6} rad}$$

$$\boxed{4x = 30°}$$

$$\therefore E = 1/8$$

6. Del gráfico, calcule el valor de x.

RESOLUCIÓN



$$\tan\alpha = \frac{2}{x}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan 2\alpha = \frac{6}{x}$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{6}{x}$$

$$\frac{2\left(\frac{2}{x}\right)}{1-\left(\frac{2}{x}\right)^2} = \frac{6}{x}$$

$$\frac{\frac{4}{x}}{1-\frac{4}{x^2}} = \frac{6}{x}$$

$$2 = 3\left(1-\frac{4}{x^2}\right)$$

$$2 = 3 - \frac{12}{x^2}$$

$$\frac{12}{x^2} = 1 \Rightarrow x^2 = 12$$

$$\therefore x = 2\sqrt{3}$$

7. Si : $\alpha + \beta = 45°$; halle: $E = (1 - \cot\alpha)(1 - \cot\beta)$

RESOLUCIÓN

$$E = (1 - \cot\alpha)(1 - \cot\beta)$$

$$E = \left(1 - \frac{1}{\tan\alpha}\right)\left(1 - \frac{1}{\tan\beta}\right)$$

$$E = \left(\frac{\tan\alpha - 1}{\tan\alpha}\right)\left(\frac{\tan\beta - 1}{\tan\beta}\right)$$

$$E = \frac{\tan\alpha.\tan\beta - \tan\alpha - \tan\beta + 1}{\tan\alpha.\tan\beta}$$

$$E = \frac{\tan\alpha.\tan\beta + 1 - \tan\alpha - \tan\beta}{\tan\alpha.\tan\beta}$$

Del dato: $\tan(\alpha + \beta) = \tan 45°$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = 1$$

$$\tan\alpha + \tan\beta = 1 - \tan\alpha.\tan\beta$$

$$\tan\alpha.\tan\beta = 1 - \tan\alpha - \tan\beta$$

Reemplazando en E:

$$E = \frac{\tan\alpha.\tan\beta + \tan\alpha.\tan\beta}{\tan\alpha.\tan\beta}$$

$$E = \frac{2\cancel{\tan\alpha.\tan\beta}}{\cancel{\tan\alpha.\tan\beta}}$$

$$\boxed{\therefore E = 2}$$

8. Simplifica la expresión:

$$T = \frac{\text{sen}^3x + \cos^3x}{\text{senx} + \text{cosx}} + 3\text{senxcosx}$$

RESOLUCIÓN

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$T = \frac{\text{sen}^3x + \cos^3x}{\text{senx} + \text{cosx}} + 3\text{senxcosx}$$

$$T = \frac{\cancel{(\text{senx} + \text{cosx})}(\text{sen}^2x - \text{senxcosx} + \cos^2x)}{\cancel{\text{senx} + \text{cosx}}} + 3\text{senxcosx}$$

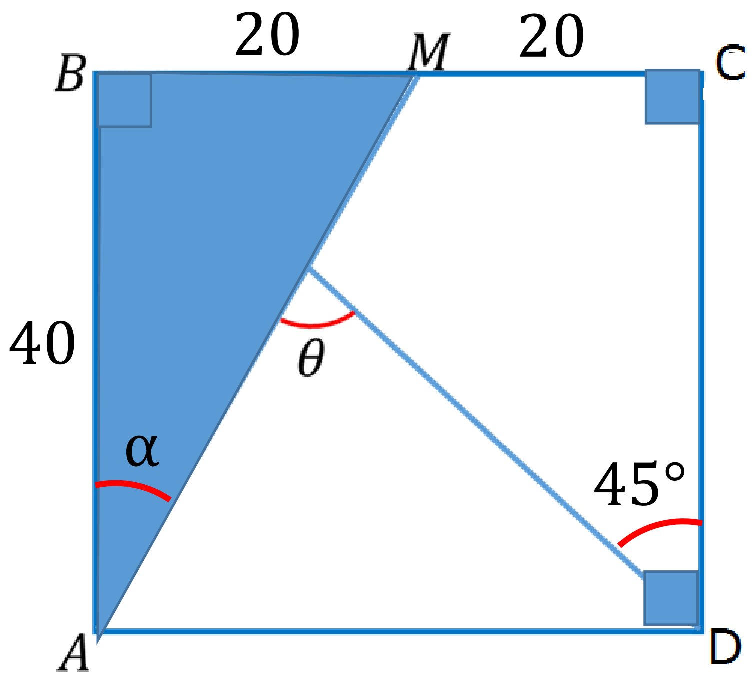
$$T = \text{sen}^2x + \cos^2x - \text{senxcosx} + 3\text{senxcosx}$$

$$T = 1 + \underbrace{\mathbf{2senxcosx}}_{\text{sen2x}}$$

$\therefore$ T = 1 + sen2x

9. Una baldosa de forma cuadrada ABCD es dividida para que sus partes sean pintadas de diferentes colores; de acuerdo con un cierto diseño. Para dividirla se consideran los trazos BD y AM, siendo M el punto medio de BC. Si AB=40 cm, hallas tanθ.

RESOLUCIÓN



$$\theta = \alpha + 45°$$

$$\tan\theta = \tan(\alpha + 45°)$$

$$\tan\theta = \frac{\tan\alpha + \tan 45°}{1 - \tan\alpha.\tan 45°} \qquad \tan\alpha = \frac{20}{40} \qquad \tan\alpha = \frac{1}{2}$$

$$\tan\theta = \frac{\frac{1}{2} + 1}{1 - \frac{1}{2}.1} \Rightarrow \tan\theta = \frac{\frac{3}{2}}{\frac{1}{2}}$$

$$\therefore \tan\theta = 3$$

10. Si A, B, y C son los ángulos internos de un triangulo y sen(A+B)cos(A+B)=1/2 ; calcule tanC

RESOLUCIÓN

Dato:

$$\operatorname{sen}(A+B)\cos(A+B) = \frac{1}{2}$$

$$\underbrace{2\operatorname{sen}(A+B)\cos(A+B)}_{\operatorname{sen}(2A+2B)} = 1$$

$$\Rightarrow \operatorname{sen}(2A+2B) = 1$$

$$2A + 2B = 90°$$

$$A + B = 45°$$

Además:

$$\underbrace{A + B}_{45°} + C = 180° \longrightarrow C = 135°$$

Calculamos: $\tan C = \tan 135°$

$$\tan C = \tan\underbrace{(180° - 45°)}_{IIC}$$

$$\tan C = -\tan 45°$$

$$\boxed{\therefore\ \tan C = -1}$$