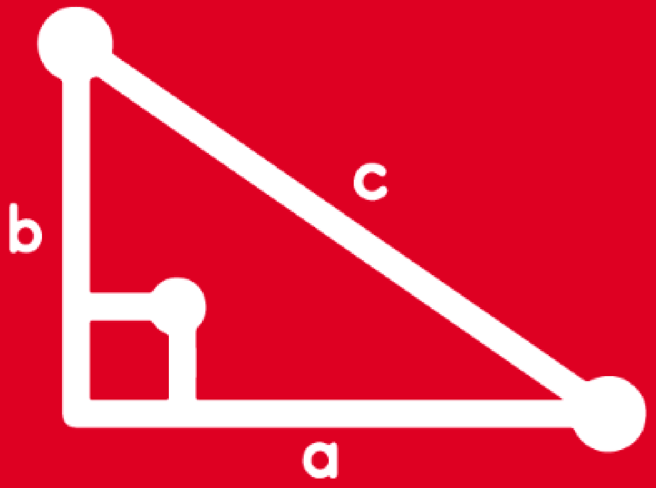
TRIGONOMETRY

VOLUME VIII

5th

SECONDARY



FEEDBACK

SACO OLIVEROS

PROBLEMA 1

Calcule el valor de $E = \text{arcsen}(1) + \text{arccos}\left(\frac{1}{2}\right)$.

Resolución:

Piden:

$$E = \underbrace{\text{arcsen}(1)}_{\alpha} + \underbrace{\text{arccos}\left(\frac{1}{2}\right)}_{\theta}$$

- $\alpha = \text{arcsen}(1) \Rightarrow \text{sen}\alpha = 1 \Rightarrow \alpha = \frac{\pi}{2}$
- $\theta = \text{arccos}\left(\frac{1}{2}\right) \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Luego:

$$E = \alpha + \theta = \frac{\pi}{2} + \frac{\pi}{3}$$

$$\therefore \boxed{E = \frac{5\pi}{6}}$$

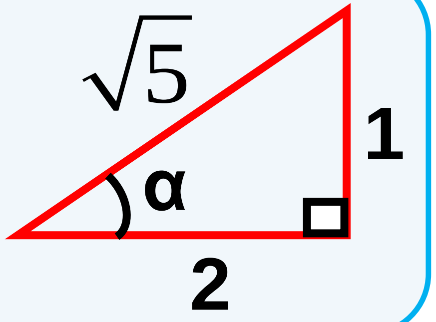
PROBLEMA 2

Calcule el valor de $E = \sqrt{5}\,\mathrm{sen}[\arctan(\frac{1}{2})] + \sqrt{10}\cos[\arctan(3)]$.

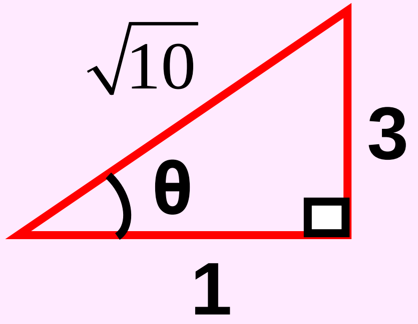
Resolución:

$$E = \sqrt{5}\,\mathrm{sen}[\underbrace{\arctan(\tfrac{1}{2})}_{\alpha}] + \sqrt{10}\cos[\underbrace{\arctan(3)}_{\theta}]$$

$\tan\alpha = \frac{1}{2} = \frac{Co}{Ca}$



$\tan\theta = \frac{3}{1} = \frac{Co}{Ca}$



Reemplazando:

$$E = \sqrt{5}\,\mathrm{sen}\alpha + \sqrt{10}\cos\theta \Rightarrow E = \cancel{\sqrt{5}}\left(\frac{1}{\cancel{\sqrt{5}}}\right) + \cancel{\sqrt{10}}\left(\frac{1}{\cancel{\sqrt{10}}}\right)$$

$\therefore \boxed{E = 2}$

PROBLEMA 3

Halle el valor de x de la igualdad: $\arccos x - \operatorname{arcsen} x = \frac{\pi}{6}$.

Resolución:

Dato: $\arccos x - \operatorname{arcsen} x = \frac{\pi}{6}$

$\arccos x - \left(\frac{\pi}{2} - \arccos x\right) = \frac{\pi}{6}$

$2\arccos x - \frac{\pi}{2} = \frac{\pi}{6}$

$2\arccos x = \frac{2\pi}{3}$

Propiedad

$\operatorname{arcsen} x + \arccos x = \frac{\pi}{2}$

$\arccos x = \frac{\pi}{3}$

$\Rightarrow x = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow x = \frac{1}{2}$

$\therefore \boxed{x = \frac{1}{2}}$

PROBLEMA 4

Indique la menor solución positiva de

$$2\text{sen}5x - 1 = 0$$

Resolución:

Del dato: $\text{sen}5x = \frac{1}{2} \ldots$ ETE

Luego: $5x = \frac{\pi}{6}$

Recuerda:

$$\text{sen}30° = \frac{1}{2}$$

$\therefore$ La menor solución positiva: $x = \frac{\pi}{30}$

PROBLEMA 5

Halle la solución general de: cotx – tanx = 2.

Resolución:

$$\underbrace{\cot x - \tan x}_{2\cot 2x} = 2$$

$\cot 2x = 1$

Luego: $\tan 2x = 1 \ldots \text{ETE}$

$$Vp = \arctan(1) = \frac{\pi}{4}$$

Solución general para la tangente:

$$\boldsymbol{X_g = k\pi + V_p \ ; k \in \mathbb{Z}}$$

$$\boldsymbol{2x = k\pi + \frac{\pi}{4} \ ; k \in \mathbb{Z}}$$

$$\therefore \boxed{x = \frac{k\pi}{2} + \frac{\pi}{8} \ ; k \in \mathbb{Z}}$$

PROBLEMA 6

Indique el número de soluciones: $(\text{senx} + \text{cosx})^2 = \dfrac{3}{2}$

para x ϵ [0;π].

Resolución:

$$\underbrace{(\text{senx} + \text{cosx})^2}\ = \frac{3}{2}$$

$$1 + sen2x = \frac{3}{2}$$

$$\Rightarrow sen2x = \frac{1}{2} \ldots \text{ETE}$$

$$\text{Vp} = \text{arcsen}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

La solución general para el seno:

$$\boldsymbol{X_g = k\pi + (-1)^k . V_p \ ; k \in \mathbb{Z}}$$

$$\boldsymbol{2x = k\pi + (-1)^k . \frac{\pi}{6}}$$

$$\boxed{\boldsymbol{x} = \frac{k\pi}{2} + (-\boldsymbol{1})^{\boldsymbol{k}} . \frac{\pi}{12} \ ; \ \boldsymbol{k} \in \mathbb{Z}}$$

Tabular:

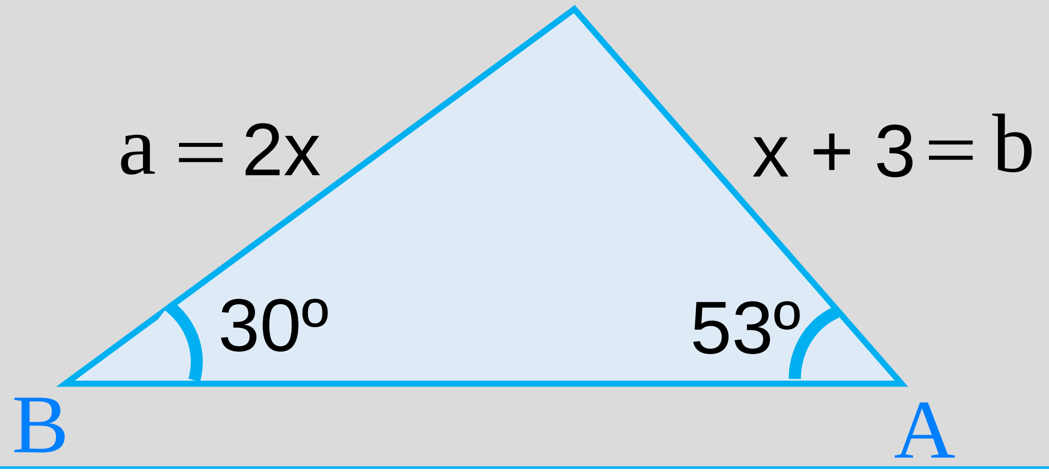
$\boldsymbol{k = 0 ; 1}$ / x ϵ [0;π]

$$x = \left\{\frac{\pi}{12}; \frac{5\pi}{12}\right\}$$

∴ ***Hay 2 soluciones***

PROBLEMA 7

De la figura, calcule el valor de *x*.



Resolución:

Ley de Senos:

$$\boxed{\frac{a}{\text{sen}A} = \frac{b}{\text{sen}B}}$$

$$\Rightarrow \frac{2x}{\text{sen}53°} = \frac{x+3}{\text{sen}30°}$$

$$\Rightarrow 2x.\,\text{sen}30° = (x+3).\,\text{sen}53°$$

$$\Rightarrow \cancel{2}x.\frac{1}{\cancel{2}} = (x+3).\frac{4}{5}$$

$$\Rightarrow 5x = 4(x+3)$$

$$\Rightarrow 5x = 4x + 12$$

$$\Rightarrow x = 12$$

$$\therefore \boxed{x = 12}$$

PROBLEMA 8

Calcule la longitud de la circunferencia circunscrita al triángulo ABC, si: $\dfrac{5a}{\text{senA}} - \dfrac{2b}{\text{senB}} + \dfrac{c}{\text{senC}} = 24\text{m}$

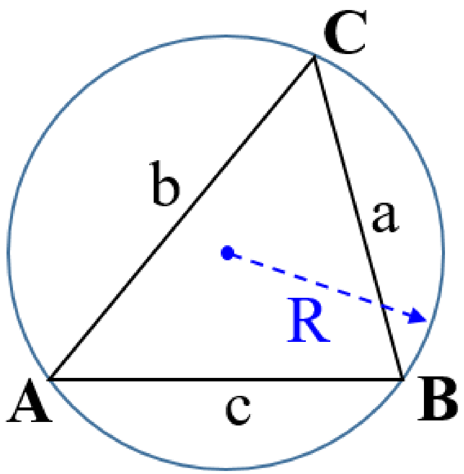
Resolución:

Ley de senos:

$a = 2RSenA$

$b = 2RSenB$

$c = 2RSenC$



En el **DATO** :

$$\frac{5(2R\cancel{\text{senA}})}{\cancel{\text{senA}}} - \frac{2(2R\cancel{\text{senB}})}{\cancel{\text{senB}}} + \frac{2R\cancel{\text{senC}}}{\cancel{\text{senC}}} = 24\text{m}$$

$\Rightarrow 5(2R) - 2(2R) + (2R) = 24\,\text{m}$

$\Rightarrow 8R = 24\,\text{m}$

$\Rightarrow R = 3\,\text{m}$

PIDEN :

Longitud de la circunferencia circunscrita

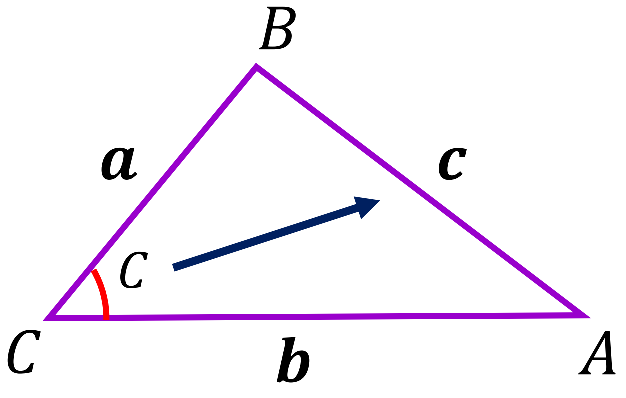
$L_{\bigcirc} = 2\pi R \Rightarrow L_{\bigcirc} = 2\pi(3)$

$$\boxed{\therefore\ L_{\bigcirc} = 6\pi\,\text{m}}$$

PROBLEMA 9

Halle la medida del ángulo C en un triángulo ABC de lados a, b y c; si se cumple $(a+b)^2+(a-b)^2=2c^2-2ab$.

Resolución:



Dato:

$$(a+b)^2+(a-b)^2=2c^2-2ab$$

$$\Rightarrow 2(a^2+b^2)=2c^2-2ab$$

$$\Rightarrow a^2+b^2=c^2-ab$$

$$\Rightarrow c^2=a^2+b^2+ab\ \ldots\text{(I)}$$

Ley de cosenos:

$$\boxed{c^2=a^2+b^2-2ab.cosC}\ \ldots\text{(II)}$$

si: $x+y=180°$

$cosx=-cosy$

Igualando (I) y (II):

$$\cancel{a^2+b^2}+ab=\cancel{a^2+b^2}-2ab.\cos C$$

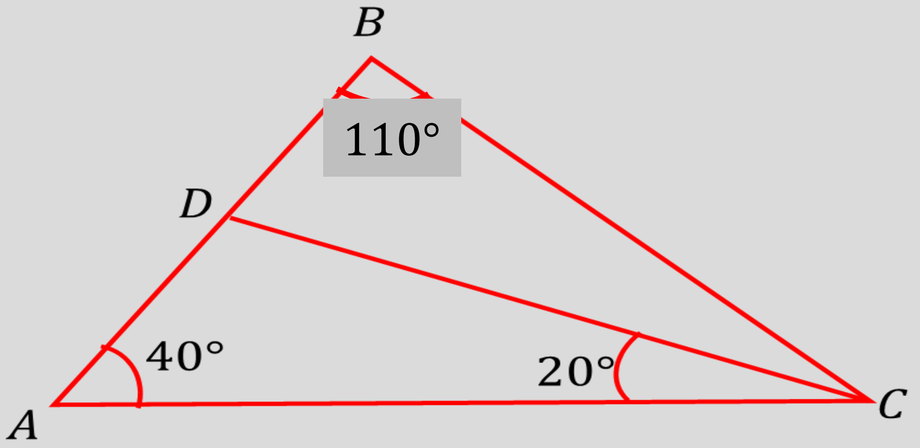
$$\Rightarrow \cancel{ab}=-2\cancel{ab}.cosC$$

$$\Rightarrow 1=-2cosC\quad \Rightarrow cosC=-\frac{1}{2}$$

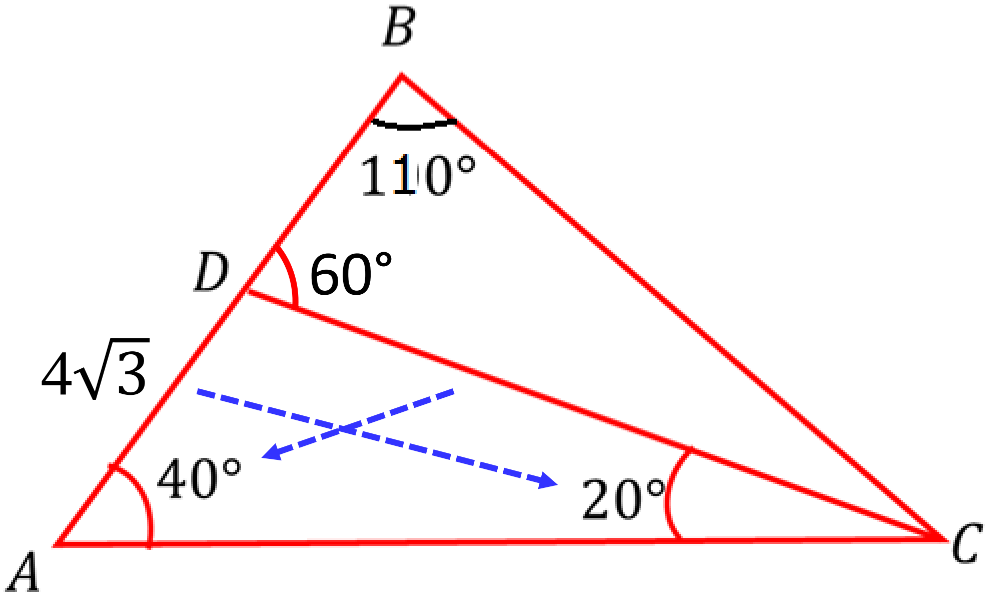
$$\therefore\ \boxed{C=120°}$$

PROBLEMA 10

En el triángulo ABC, de la figura, $AD = 4\sqrt{3}$cm. Halle BC.



Resolución:



Δ ADC: Ley de Senos

$$\frac{4\sqrt{3}}{\text{sen}20^0} = \frac{DC}{\text{sen}40^0}$$

$$\frac{4\sqrt{3}}{\cancel{\text{sen}20^0}} = \frac{DC}{2\cancel{\text{sen}20^0}cos20°}$$

$$\Rightarrow DC = 8\sqrt{3}cos20°$$

Recordar:

$\mathbf{sen110^0} = \text{sen}(90° + 20°)$

$\mathbf{sen110^0} = \cos 20°$

Δ DBC: Ley de Senos

$$\frac{BC}{\text{sen}60^0} = \frac{DC}{\mathbf{sen110^0}}$$

$$\frac{BC}{\text{sen}60^0} = \frac{8\sqrt{3}\cancel{cos}20°}{\cancel{\mathbf{cos}}\mathbf{20}}$$

$$\Rightarrow BC = 8\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore$ $\boxed{\boldsymbol{BC = 12cm}}$

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