

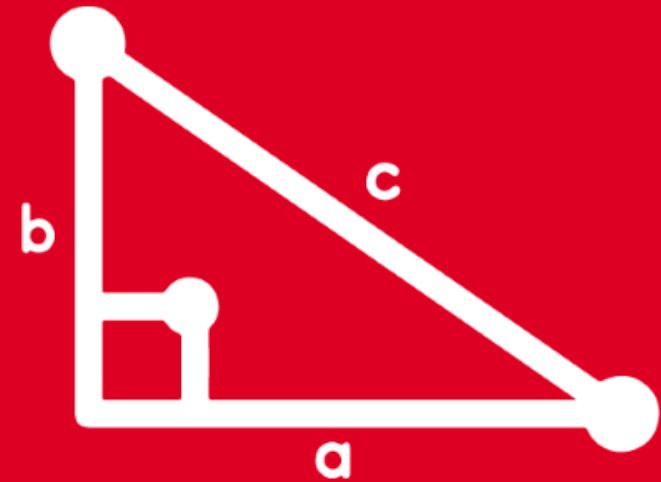


TRIGONOMETRY

Tomo 8

5th
SECONDARY

Review



 **SACO OLIVEROS**

PROBLEMA 1

Halle el valor de: $E = \arcsen(1) + \arccos\left(\frac{1}{2}\right)$

Resolución:

Piden:

$$E = \underbrace{\arcsen(1)}_{\alpha} + \underbrace{\arccos\left(\frac{1}{2}\right)}_{\theta}$$

$$\bullet \alpha = \arcsen(1) \Rightarrow \text{sen } \alpha = 1 \rightarrow \alpha = \frac{\pi}{2}$$

$$\bullet \theta = \arccos\left(\frac{1}{2}\right) \Rightarrow \cos \theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$$

Luego:

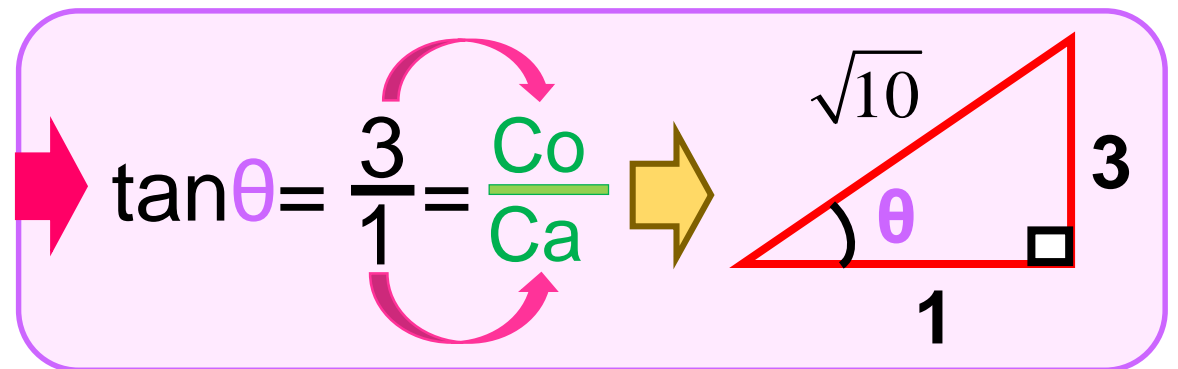
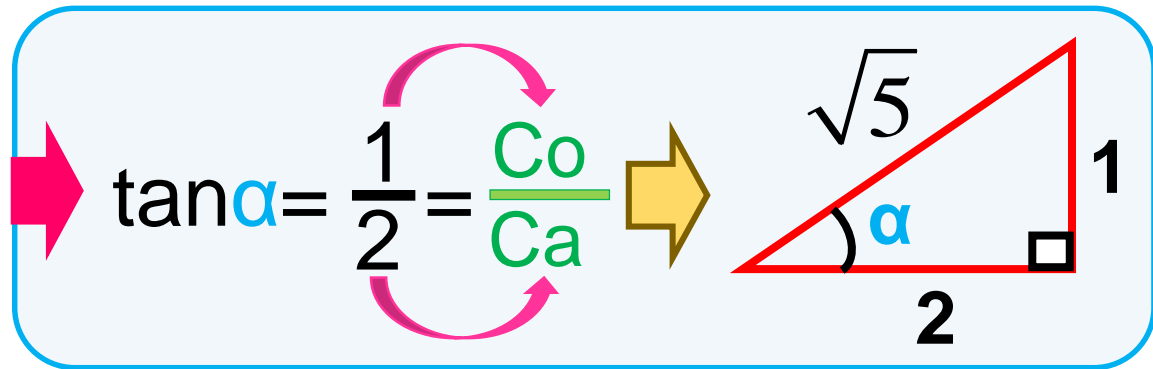
$$E = \alpha + \theta = \frac{\pi}{2} + \frac{\pi}{3}$$

$$\therefore E = \frac{5\pi}{6}$$

PROBLEMA 2

Calcule el valor de: $E = \sqrt{5} \operatorname{sen}[\arctan(\frac{1}{2})] + \sqrt{10} \cos[\arctan(3)]$

Resolución: $E = \sqrt{5} \operatorname{sen}[\underbrace{\arctan(\frac{1}{2})}_{\alpha}] + \sqrt{10} \cos[\underbrace{\arctan(3)}_{\theta}]$



Reemplazando:

$$E = \sqrt{5} \operatorname{sen} \alpha + \sqrt{10} \cos \theta \Rightarrow E = \cancel{\sqrt{5}} \left(\frac{1}{\cancel{\sqrt{5}}} \right) + \cancel{\sqrt{10}} \left(\frac{1}{\cancel{\sqrt{10}}} \right) \therefore E = 2$$

PROBLEMA 3

Halle el valor de x de la igualdad: $\arccos x - \arcsen x = \frac{\pi}{6}$

Resolución:

Dato: $\arccos x - \arcsen x = \frac{\pi}{6}$

$$\arccos x - \left(\frac{\pi}{2} - \arccos x \right) = \frac{\pi}{6}$$

$$2\arccos x - \frac{\pi}{2} = \frac{\pi}{6}$$

$$2\arccos x = \frac{2\pi}{3}$$

Propiedad

$$\arcsen x + \arccos x = \frac{\pi}{2}$$

$$\arccos x = \frac{\pi}{3}$$

$$\Rightarrow x = \cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2}$$

PROBLEMA 4

Indique la menor solución positiva de:

$$2\operatorname{sen}5x - 1 = 0$$

Resolución:

Del dato: $\operatorname{sen}5x = \frac{1}{2} \dots$ ETE

Luego: $5x = \frac{\pi}{6}$



Recuerda:

$$\operatorname{sen}30^\circ = \frac{1}{2}$$

\therefore La menor solución positiva: $x = \frac{\pi}{30}$

PROBLEMA 5

Halle la solución general de: $\cot x - \tan x = 2$

Resolución:

$$\underbrace{\cot x - \tan x}_{2 \cot 2x} = 2$$

$$2 \cot 2x$$

$$\Rightarrow \cot 2x = 1$$

Luego: $\tan 2x = 1 \dots$ ETE

$$V_p = \arctan(1) = \frac{\pi}{4}$$

Solución general para la tangente:

$$X_g = k\pi + V_p ; k \in \mathbb{Z}$$

$$\Rightarrow 2x = k\pi + \frac{\pi}{4} ; k \in \mathbb{Z}$$

$$\therefore x = \frac{k\pi}{2} + \frac{\pi}{8} ; k \in \mathbb{Z}$$

PROBLEMA 6

Indique el número de soluciones: $(\operatorname{sen} x + \operatorname{cos} x)^2 = \frac{3}{2}$

Para $x \in [0; \pi]$

Resolución:

$$\underbrace{(\operatorname{sen} x + \operatorname{cos} x)^2}_{1 + \operatorname{sen} 2x} = \frac{3}{2}$$

$$1 + \operatorname{sen} 2x = \frac{3}{2}$$

$$\Rightarrow \operatorname{sen} 2x = \frac{1}{2} \dots \text{ETE}$$

$$\Rightarrow V_p = \operatorname{arcsen}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

La solución general para el seno:

$$X_g = k\pi + (-1)^k \cdot V_p ; k \in \mathbb{Z}$$

$$\Rightarrow 2x = k\pi + (-1)^k \cdot \frac{\pi}{6}$$

$$x = \frac{k\pi}{2} + (-1)^k \cdot \frac{\pi}{12} ; k \in \mathbb{Z}$$

Tabular:
 $k = 0 ; 1 / x \in [0; \pi]$

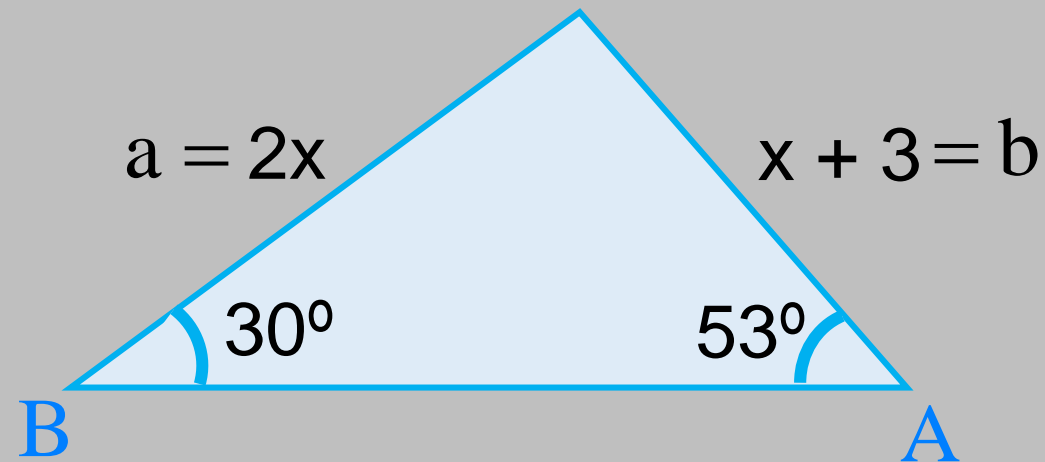
$$\Rightarrow x = \left\{ \frac{\pi}{12} ; \frac{5\pi}{12} \right\}$$



Hay 2 soluciones

PROBLEMA 7

De la figura, calcule el valor de x .



Resolución:

Ley de Senos:

$$\frac{a}{\text{sen}A} = \frac{b}{\text{sen}B}$$

$$\Rightarrow \frac{2x}{\text{sen}53^\circ} = \frac{x+3}{\text{sen}30^\circ}$$

$$\Rightarrow 2x \cdot \text{sen}30^\circ = (x+3) \cdot \text{sen}53^\circ$$

$$\Rightarrow 2x \cdot \frac{1}{2} = (x+3) \cdot \frac{4}{5}$$

$$\Rightarrow 5x = 4(x+3)$$

$$\Rightarrow 5x = 4x + 12$$

$$\Rightarrow x = 12$$



$$x = 12$$

PROBLEMA 8

Calcule la longitud de la circunferencia circunscrita al

triángulo ABC, si: $\frac{5a}{\text{sen}A} - \frac{2b}{\text{sen}B} + \frac{c}{\text{sen}C} = 24\text{m}$

Resolución:

Ley de senos:

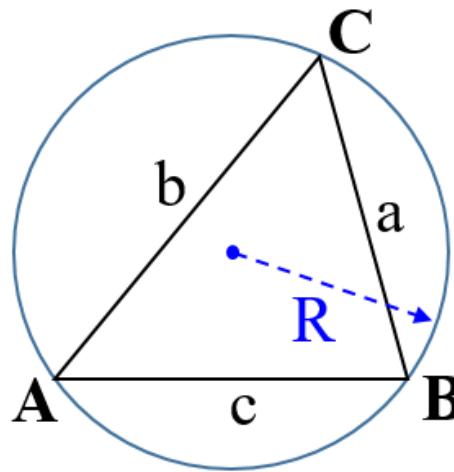
$$a = 2R\text{sen}A$$

$$b = 2R\text{sen}B$$

$$c = 2R\text{sen}C$$

En el DATO :

$$\frac{5(2R\text{sen}A)}{\cancel{\text{sen}A}} - \frac{2(2R\text{sen}B)}{\cancel{\text{sen}B}} + \frac{2R\text{sen}C}{\cancel{\text{sen}C}} = 24\text{m}$$



$$\Rightarrow 5(2R) - 2(2R) + (2R) = 24\text{m}$$

$$\Rightarrow 8R = 24\text{m}$$

$$\Rightarrow R = 3\text{m}$$

PIDEN :

Longitud de la circunferencia circunscrita

$$L_{\square} = 2\pi R \Rightarrow L_{\square} = 2\pi(3)$$

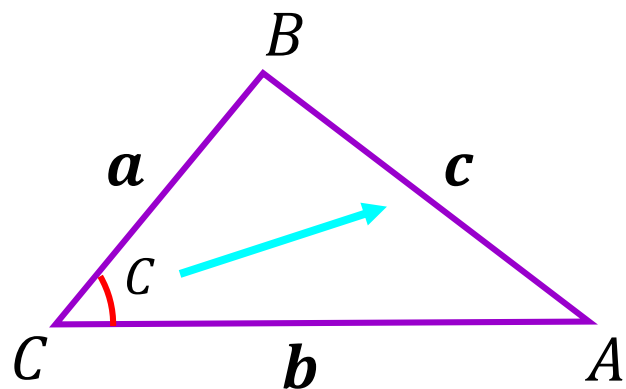
$$\therefore L_{\square} = 6\pi\text{m}$$

PROBLEMA 9

$$(a + b)^2 + (a - b)^2 = 2c^2 -$$

$2ab$

Resolución:



Dato:

$$(a + b)^2 + (a - b)^2 = 2c^2 - 2ab$$

$$\Rightarrow 2(a^2 + b^2) = 2c^2 - 2ab$$

$$\Rightarrow a^2 + b^2 = c^2 - ab$$

$$\Rightarrow c^2 = a^2 + b^2 + ab \dots (I)$$

Ley de cosenos:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C \dots (II)$$

Igualando (I) y (II):

$$\cancel{a^2} + \cancel{b^2} + ab = \cancel{a^2} + \cancel{b^2} - 2ab \cdot \cos C$$

$$\Rightarrow \cancel{ab} = -2\cancel{ab} \cdot \cos C$$

$$\Rightarrow 1 = -2\cos C \quad \Rightarrow \cos C = -\frac{1}{2}$$

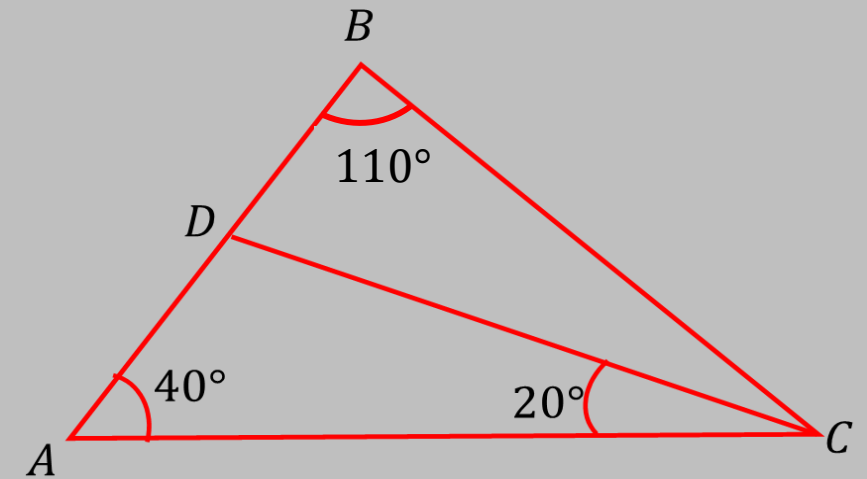
si: $x + y = 180^\circ$

$$\cos x = -\cos y$$

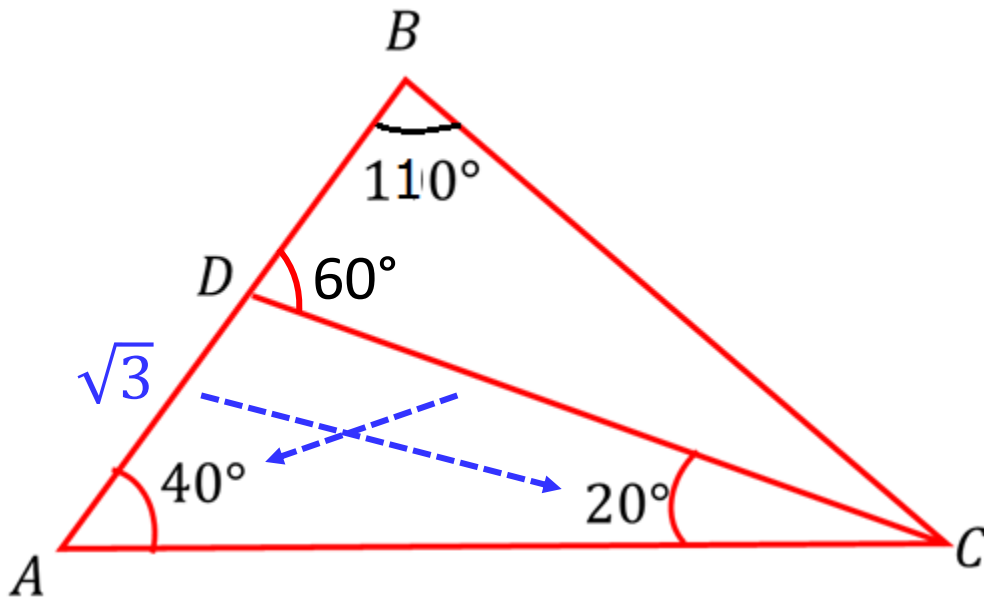
$$\therefore C = 120^\circ$$

PROBLEMA 10

$\sqrt{3}\text{cm}$. Halle BC .



Resolución:



$$\Delta \quad \frac{\sqrt{3}}{\text{sen}20^{\circ}} = \frac{DC}{\text{sen}40^{\circ}}$$

$$\frac{\sqrt{3}}{\text{sen}20^{\circ}} = \frac{DC}{2\text{sen}20^{\circ}\cos20^{\circ}}$$

$$\Rightarrow DC = \sqrt{3}\cos20^{\circ}$$

$$\text{sen}110^{\circ} = \text{sen}(90^{\circ} + 20^{\circ})$$

$$\text{sen}110^{\circ} = \cos20^{\circ}$$

$$\Delta \quad \frac{BC}{\text{sen}60^{\circ}} = \frac{DC}{\text{sen}110^{\circ}}$$

$$\frac{BC}{\text{sen}60^{\circ}} = \frac{\sqrt{3}\cos20^{\circ}}{\cos20^{\circ}}$$

$$\Rightarrow BC = 8\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore \quad \boxed{BC = 12\text{cm}}$$