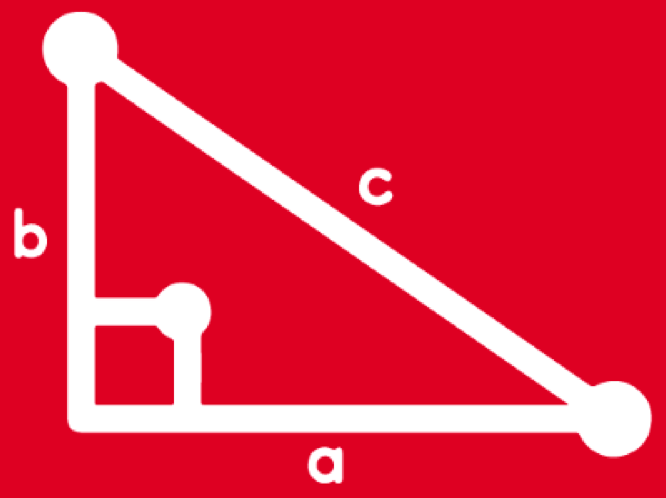
TRIGONOMETRY

VOLUME III

5th
SECONDARY



FEEDBACK

SACO OLIVEROS

1) **Siendo θ y β las medidas de dos ángulos cuadrantales diferentes, positivos y menores o iguales a 360°, se cumple que**

$$\sqrt{1-\operatorname{sen}\theta}+\sqrt{\operatorname{sen}\theta-1}=1+\cos\beta \;\ldots\ldots (*)$$

calcule θ + β.

Resolución:

$$\underbrace{\begin{array}{ccc} 1-\operatorname{sen}\theta\geq 0 & \wedge & \operatorname{sen}\theta-1\geq 0 \\ 1\geq \operatorname{sen}\theta & \wedge & \operatorname{sen}\theta\geq 1 \end{array}}$$

$$\operatorname{sen}\theta=1$$

$$\text{como } 0°<\theta\leq 360°$$

$$\boxed{\theta=90°}$$

Reemplazamos en (*)

$$\underbrace{\cancel{\sqrt{1-\operatorname{sen}\theta}}}_{0}+\underbrace{\cancel{\sqrt{\operatorname{sen}\theta-1}}}_{0}=1+\cos\beta$$

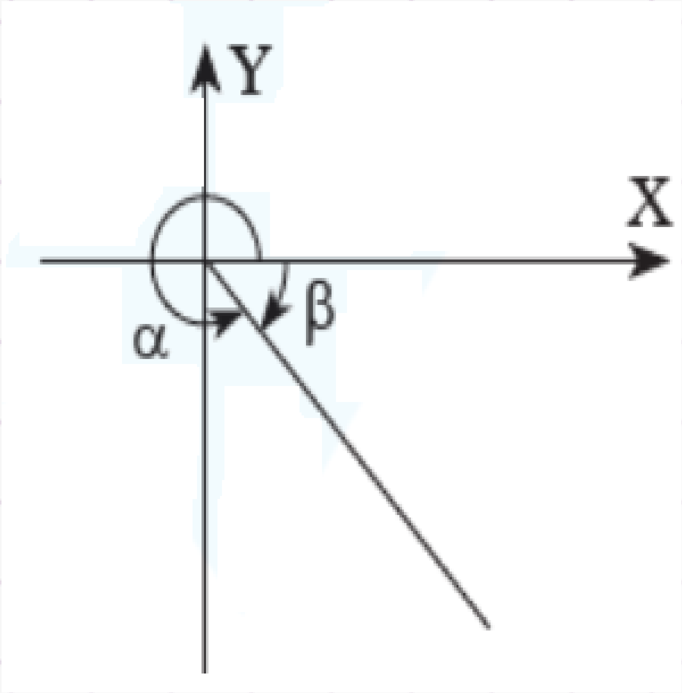
$$-1=\cos\beta$$

$$\boxed{\beta=180°}$$

Recordar

RT ∡	0°	90°	180°	270°	360°
sen	0	1	0	−1	0
cos	1	0	−1	0	1

2) **En la figura, se cumple que $\cot\alpha \cdot \cot\beta + \cos\alpha \cdot \sec\beta = 10$. Calcule $\cot\alpha$.**



Resolución:

Del gráfico se observa que α y β son las medidas de dos ángulos coterminales:

$$RT(\alpha) = RT(\beta)$$

Recordar

sen, csc (+)	Todas las RT son (+)
tan, cot (+)	cos, sec (+)

Del dato:

$$\cot\alpha \cdot \cot\beta + \cos\alpha \cdot \sec\beta = 10$$

$$\cot\alpha \cdot \mathbf{\cot\alpha} + \underbrace{\boxed{\cos\alpha \cdot \mathbf{\sec\alpha}}}_{1} = 10$$

$$\cot^2\alpha + 1 = 10$$

$$\cot^2\alpha = 9 \Rightarrow \cot\alpha = \pm 3$$

Como $\alpha \in$ IVC

$$\therefore \boxed{\cot\alpha = -3}$$

3) **Si $\cos\theta > 0$, además $16^{\cot\theta} = 0,25$, efectúe**

$$\mathbf{P = \sqrt{5}(sen\theta - \cos\theta)}$$

$x: (+)$

$y: (-)$

Resolución:

Del dato: $16^{\cot\theta} = \dfrac{1}{4}$

$$16^{\cot\theta} = 4^{-1}$$

$$(4^2)^{\cot\theta} = 4^{-1}$$

$$4^{2\cot\theta} = 4^{-1}$$

$$\rightarrow 2\cot\theta = -1$$

$$\rightarrow \cot\theta = -\frac{1}{2}$$

$\cot\theta: (-)$

$\cos\theta: (+)$

$\Rightarrow \theta \in \mathbf{IVC}$

$$\cot\theta = \frac{1}{-2} = \frac{x}{y}$$

Por $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{1^2 + (-2)^2}$$

$$r = \sqrt{5}$$

Efectuamos $P = \sqrt{5}\left(\dfrac{-2}{\sqrt{5}} - \dfrac{1}{\sqrt{5}}\right) = \cancel{\sqrt{5}}\left(\dfrac{-3}{\cancel{\sqrt{5}}}\right)$

$\therefore$ $\boxed{\mathbf{P = -3}}$

4) Simplifique

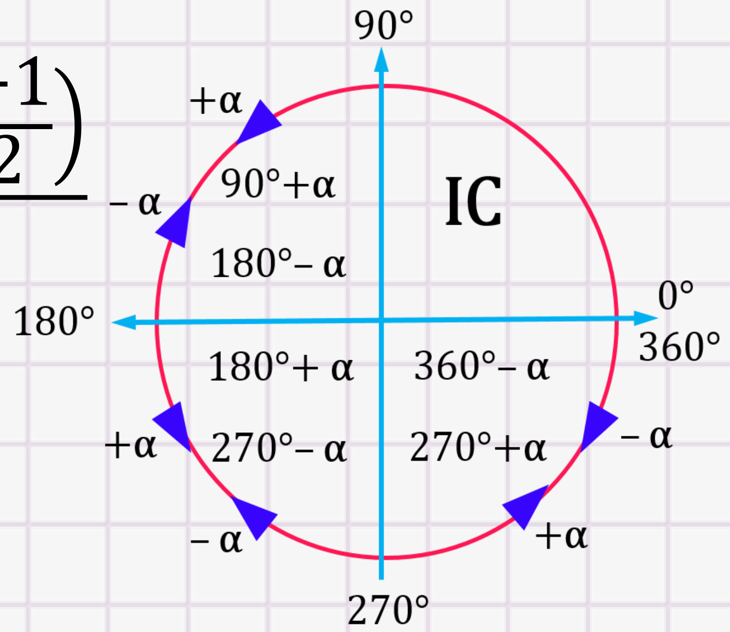
$$E = \frac{\mathbf{sen330°.\,cos120°}}{\mathbf{tan225°}}$$

Resolución:

$$E = \frac{\text{sen}\overbrace{(360° - 30°)}^{\text{IVC}\ (-)} \cdot \cos\overbrace{(180° - 60°)}^{\text{IIC}\ (-)}}{\tan\underbrace{(180° + 45°)}_{\text{IIIC}\ (+)}}$$

$$E = \frac{(-\text{sen}30°)(-\cos60°)}{\tan45°}$$

$$E = \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)}{1}$$



$$E = \frac{\frac{1}{4}}{1}$$

$$\therefore \boxed{E = \frac{1}{4}}$$

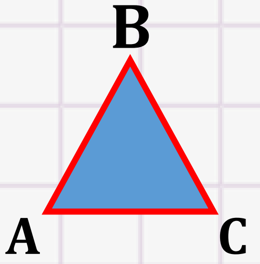
Recordar

sen, csc (+)	Todas las RT son (+)
tan, cot (+)	cos, sec (+)

5) En un triángulo ABC, reduzca $M = \dfrac{\operatorname{sen}(B+C)}{\cos\left(\dfrac{3A+B+C}{2}\right)}$

Resolución:

Del dato:



$$A + B + C = 180°$$

$$\rightarrow \mathbf{B + C = 180° - A}$$

Reducimos:

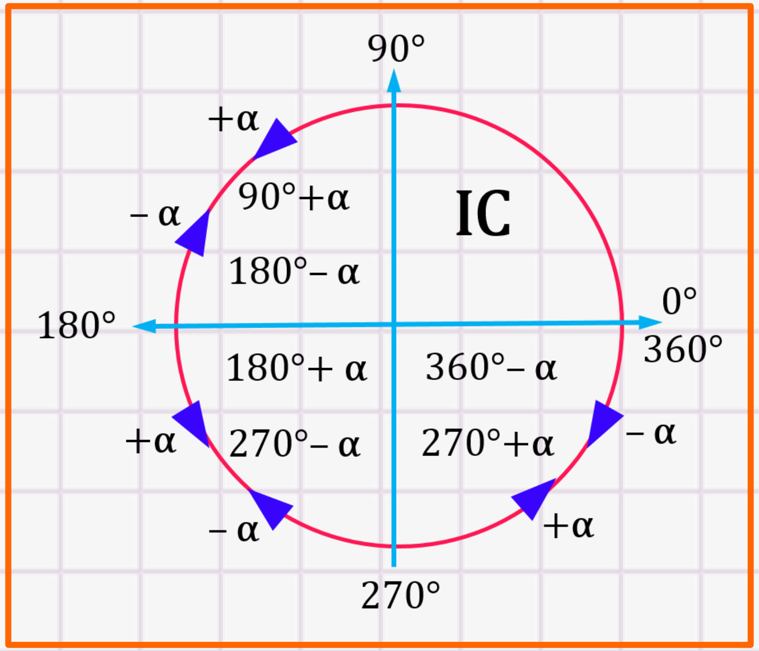
$$M = \frac{\overbrace{\operatorname{sen}(\underbrace{180° - A}_{IIC})}^{(+)}}{\cos\left(\dfrac{2A + \boxed{A + B + C}^{\,180°}}{2}\right)}$$

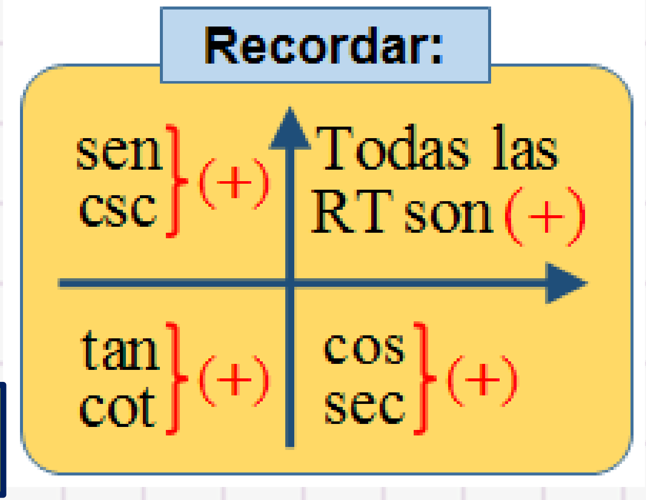
$$M = \frac{\operatorname{sen}A}{\cos\left(\dfrac{2A + 180°}{2}\right)}$$

$$M = \frac{\operatorname{sen}A}{\underbrace{\cos(\underbrace{90° + A}_{IIC})}_{(-)}}$$

$$M = \frac{\overset{1}{\cancel{\operatorname{sen}A}}}{\underset{-1}{\cancel{-\operatorname{sen}A}}}$$

$$\therefore \boxed{M = -1}$$





6) Si $\alpha \in$ IVC, además $\mathrm{sen}(270° + \alpha) = -0,8$, reduzca

$$T = \csc(180° - \alpha) + \tan(270° + \alpha)$$

Recordar

$$\csc\alpha = \frac{r}{y}$$

$$\cot\alpha = \frac{x}{y}$$

Resolución:

$$\rightarrow T = \underbrace{\csc(180° - \alpha)}_{(+)\ \mathrm{IIC}} + \underbrace{\tan(270° + \alpha)}_{(-)\ \mathrm{IVC}}$$

$$T = \csc\alpha + (-\cot\alpha)$$

$$T = \csc\alpha - \cot\alpha \ \ldots (*)$$

Del dato: $\underbrace{\mathrm{sen}(270° + \alpha)}_{(-)\ \mathrm{IVC}} = -0,8$

$$\rightarrow \cancel{-}\cos\alpha = \cancel{-}\frac{4}{5} = \frac{x}{r}$$

Por $r^2 = x^2 + y^2$ $\quad y \in$ IVC

$$5^2 = 4^2 + y^2 \rightarrow \boxed{y = -3}$$

En $(*)$: $T = \csc\alpha - \cot\alpha$

$$T = \frac{5}{-3} - \left(\frac{4}{-3}\right) = \boxed{-\frac{1}{3}}$$

7) **Simplifique**

$$P = \frac{\cos 1470° \cdot \operatorname{sen} 1140°}{\cot 3285°}$$

Resolución:

Eliminamos el número de vueltas:

$$\begin{array}{r|l} 1470 & 360 \\ \hline 1440 & 4 \\ \hline 30 & \end{array} \qquad \begin{array}{r|l} 1140 & 360 \\ \hline 1080 & 3 \\ \hline 60 & \end{array}$$

$$\begin{array}{r|l} 3285 & 360 \\ \hline 3240 & 9 \\ \hline 45 & \end{array}$$

$$\rightarrow P = \frac{\cos 30° \cdot \operatorname{sen} 60°}{\cot 45°}$$

$$P = \frac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}}{1}$$

$$\therefore \boxed{P = \frac{3}{4}}$$

8) Calcule

$$\mathbf{E = sen\left(\frac{37\pi}{6}\right) + cos\left(\frac{59\pi}{3}\right)}$$

Resolución:

Eliminamos el número de vueltas:

$$\begin{array}{r|l} 37 & 12 \\ \hline 36 & 3 \\ \hline 1 & \end{array} \qquad \begin{array}{r|l} 59 & 6 \\ \hline 54 & 9 \\ \hline 5 & \end{array}$$

$$\rightarrow \mathrm{E} = \mathrm{sen}\left(\frac{1\pi}{6}\right) + \cos\left(\frac{5\pi}{3}\right)$$

Expresamos en grados:

$$\mathrm{E} = \mathrm{sen}30° + \cos300° \ldots (*)$$

- $\cos300° = \cos\underbrace{(360° - 60°)}_{\text{IVC}}$

$(+)$

$$= \cos60° = \frac{\mathbf{1}}{\mathbf{2}}$$

$$\rightarrow \mathrm{E} = \frac{1}{2} + \frac{1}{2} \qquad \therefore \boxed{\mathbf{E = 1}}$$

9) **Si $\theta \in$ IVC, además $\cos\theta = \frac{1}{2}$, reduzca**

$$\mathbf{M = \sec\left(\frac{13\pi}{2} + \theta\right) \cdot \tan(22\pi + \theta)}$$

Resolución:

- $\sec\left(\frac{13\pi}{2} + \theta\right) = \sec\underbrace{\left(\frac{1\pi}{2} + \theta\right)}_{\text{IIC}}$ $(-)$

$$\begin{array}{r|l} 13 & 4 \\ \hline 12 & 3 \\ \hline 1 & \end{array}$$

$= -\mathbf{csc\theta}$

PAR

- $\tan(\cancel{22}\pi + \theta) = \mathbf{tan\theta}$

$\rightarrow \mathrm{M} = -\csc\theta \cdot \tan\theta$

$$\mathrm{M} = \frac{-r}{\cancel{y}} \cdot \frac{\cancel{y}}{x} = -\frac{r}{x} \;\ldots (*)$$

Del dato: $\cos\theta = \frac{1}{2} = \frac{x}{r} \rightarrow \frac{\boldsymbol{r}}{\boldsymbol{x}} = \mathbf{2}$

Reemplazamos en **(*)**:

$\therefore$ $\boxed{\mathbf{M = -2}}$

10) Si $\cot\theta = -0,75$ y $\cos\theta < 0$, efectúe

$$M = 5\operatorname{sen}\theta + 3\sec\theta + 1$$

Recordar

$$\operatorname{sen}\alpha = \frac{y}{r}$$

$$\sec\alpha = \frac{r}{x}$$

Resolución:

Del dato: $\cot\theta = -\dfrac{3}{4}$ $\left.\begin{matrix} \cot\theta: (-) \\ \cos\theta: (-) \end{matrix}\right\} \Rightarrow \theta \in IIC \left\{\begin{matrix} x: (-) \\ y: (+) \end{matrix}\right.$

$$\cot\theta = \frac{-3}{4} = \frac{x}{y}$$

Por $r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2}$

$$r = 5$$

Efectuamos

$$M = 5\operatorname{sen}\theta + 3\sec\theta + 1$$

$$M = \cancel{5}\left(\frac{4}{\cancel{5}}\right) + \overset{1}{\cancel{3}}\left(\frac{5}{\underset{-1}{-\cancel{3}}}\right) + 1$$

$$M = 4 + (-5) + 1$$

$\therefore$ $\boxed{M = 0}$

SACO OLIVEROS