GEOMETRÍA

Tomo 4

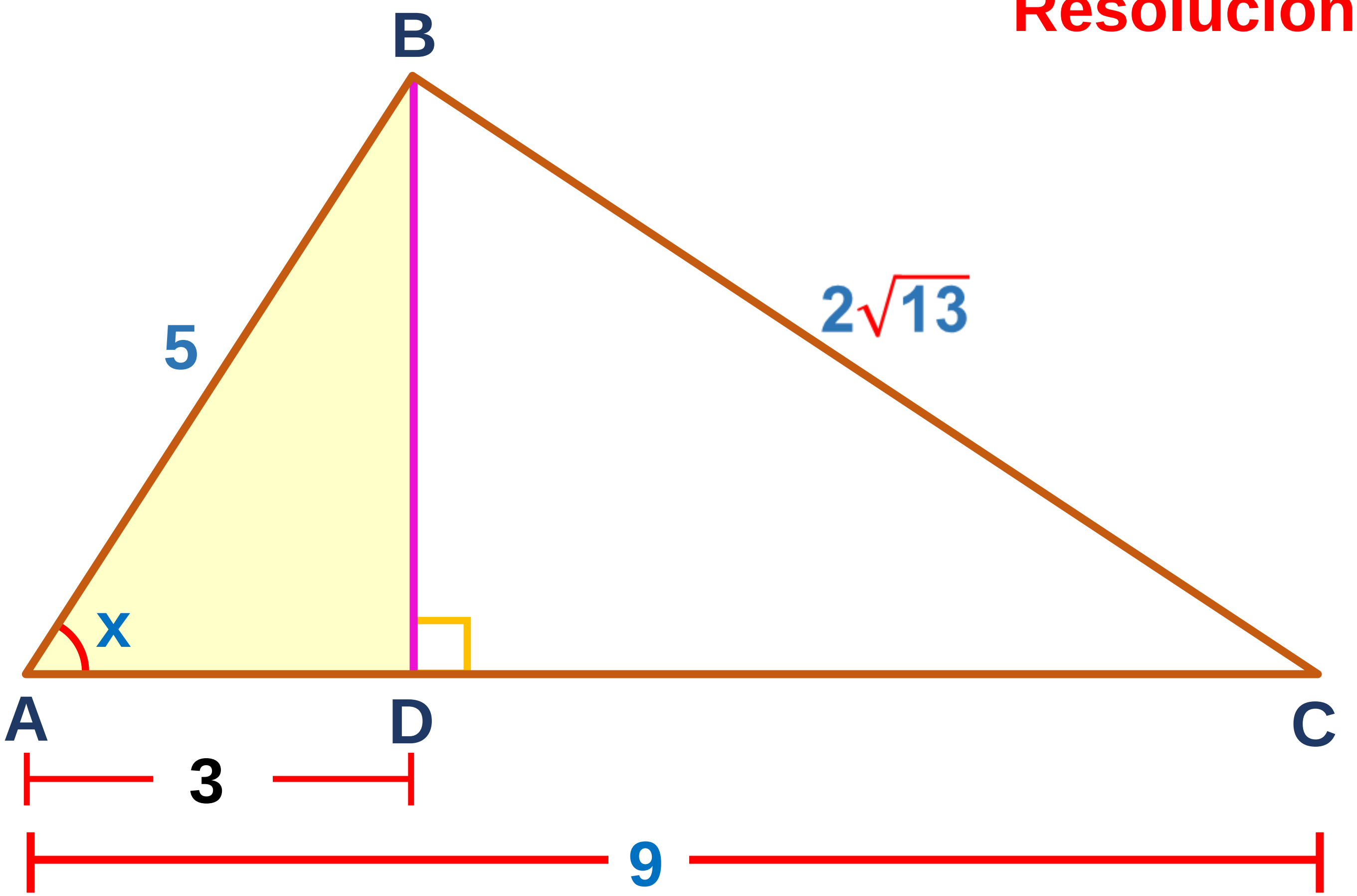
5th

SECONDARY

RETROALIMENTACIÓ
N

SACO OLIVEROS

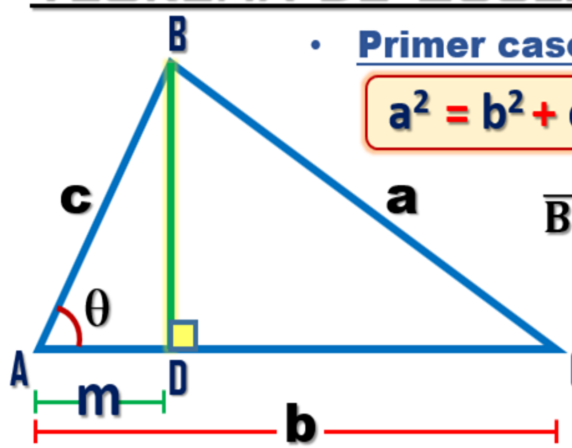
1. En la figura, calcule x.



* Se traza la altura $\overline{BD}$

Resolución:

- **TEOREMA DE EUCLIDES**



- Primer caso

$$a^2 = b^2 + c^2 - 2bm$$

$\overline{BD}$: Altura

$\theta < 90°$

$$(2\sqrt{13})^2 = 9^2 + 5^2 - 2(9)(m)$$

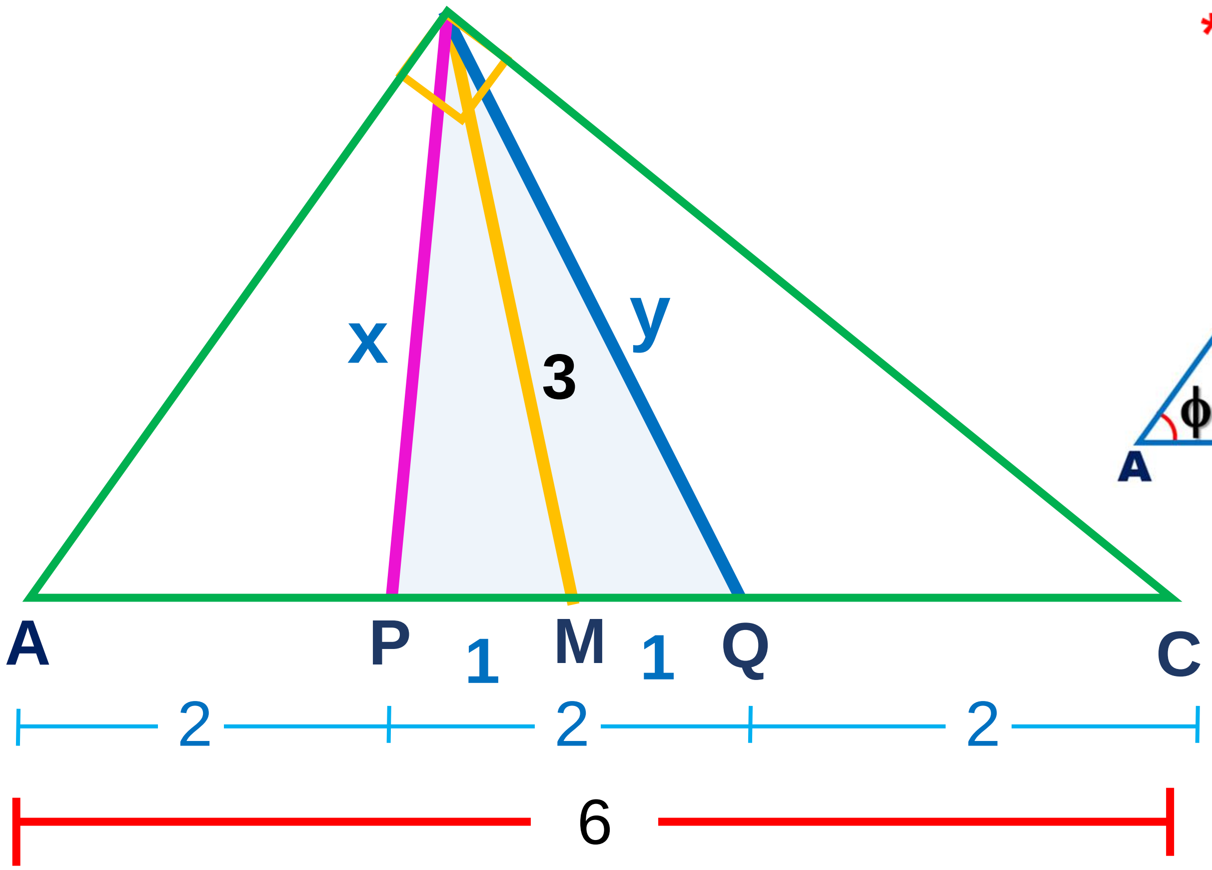
$$52 = 81 + 25 - 18m$$

$$18m = 54 \quad \Rightarrow \quad m = 3$$

* ABD aproximado de 37° y 53°

$$\therefore\ x = 53°$$

2. En la figura, calcule $x^2 + y^2$.

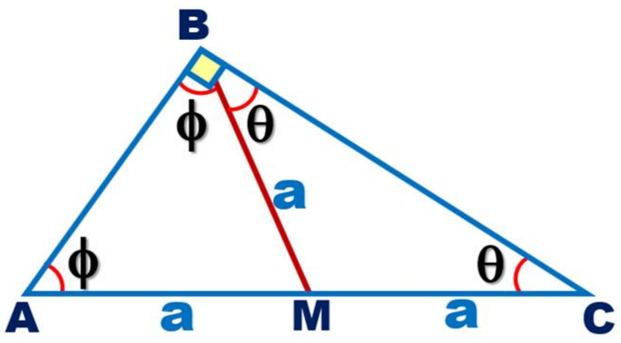


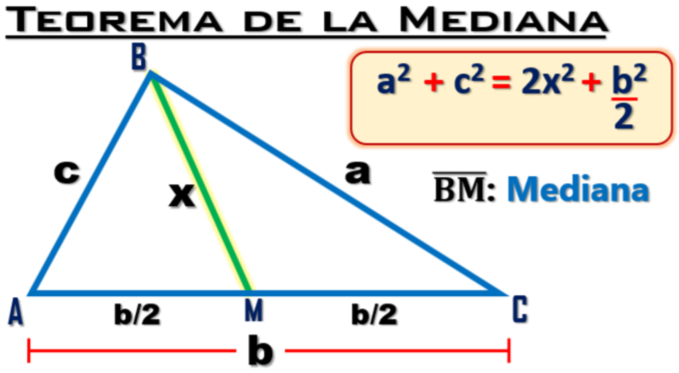
* Del gráfico:

AM = MC = BM = 3

Resolución:

* $\triangle$ ABC, se traza la mediana $\overline{BM}$





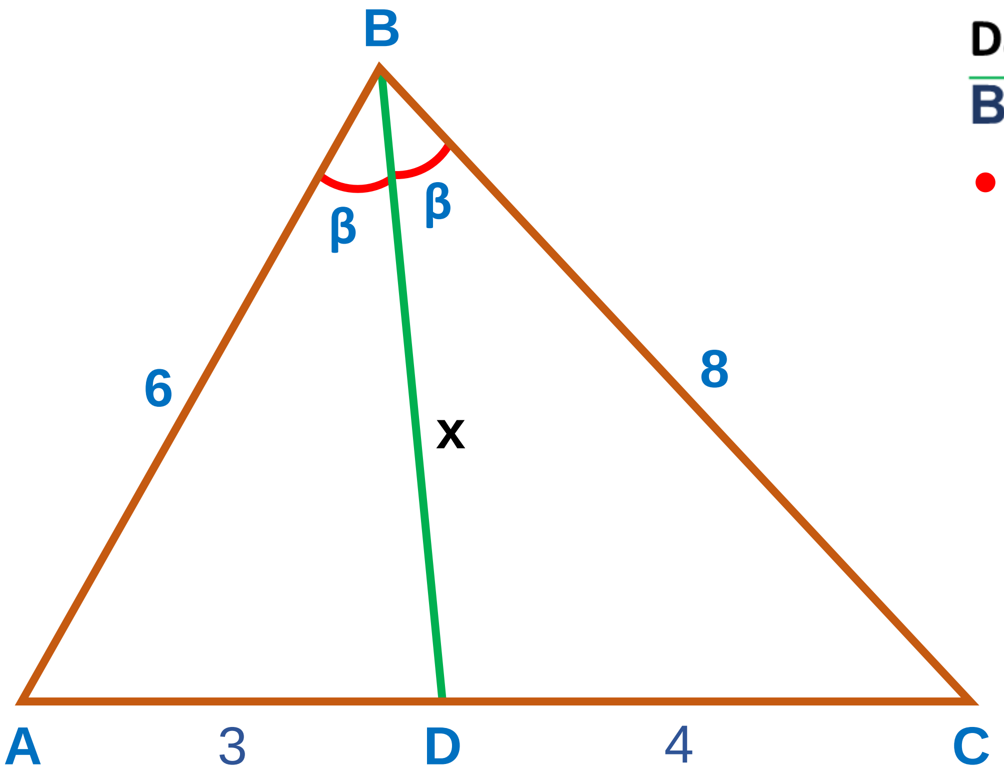
$\triangle$ PBQ:

$$x^2 + y^2 = 2(3)^2 + \frac{(2)^2}{2}$$

$$x^2 + y^2 = 18 + 2$$

$$\therefore x^2 + y^2 = 20$$

3. En un triángulo ABC, se traza la bisectriz interior $\overline{BD}$. Si AB = 6, BC = 8 y DC = 4. Halle BD.



Resolución

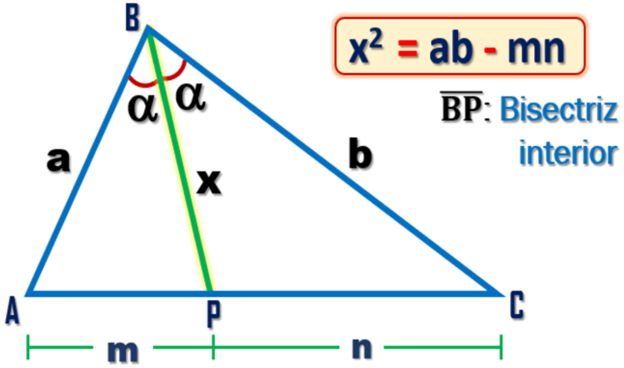
Dato:

$\overline{BD}$: **bisectriz interior**

- **Por teorema de la bisectriz interior**

$$\frac{\cancel{6}^{\,3}}{\cancel{8}_{\,4}} = \frac{AD}{4} \Rightarrow AD = 3$$

T. de la longitud de la bisectriz interior



En el △ ABC:

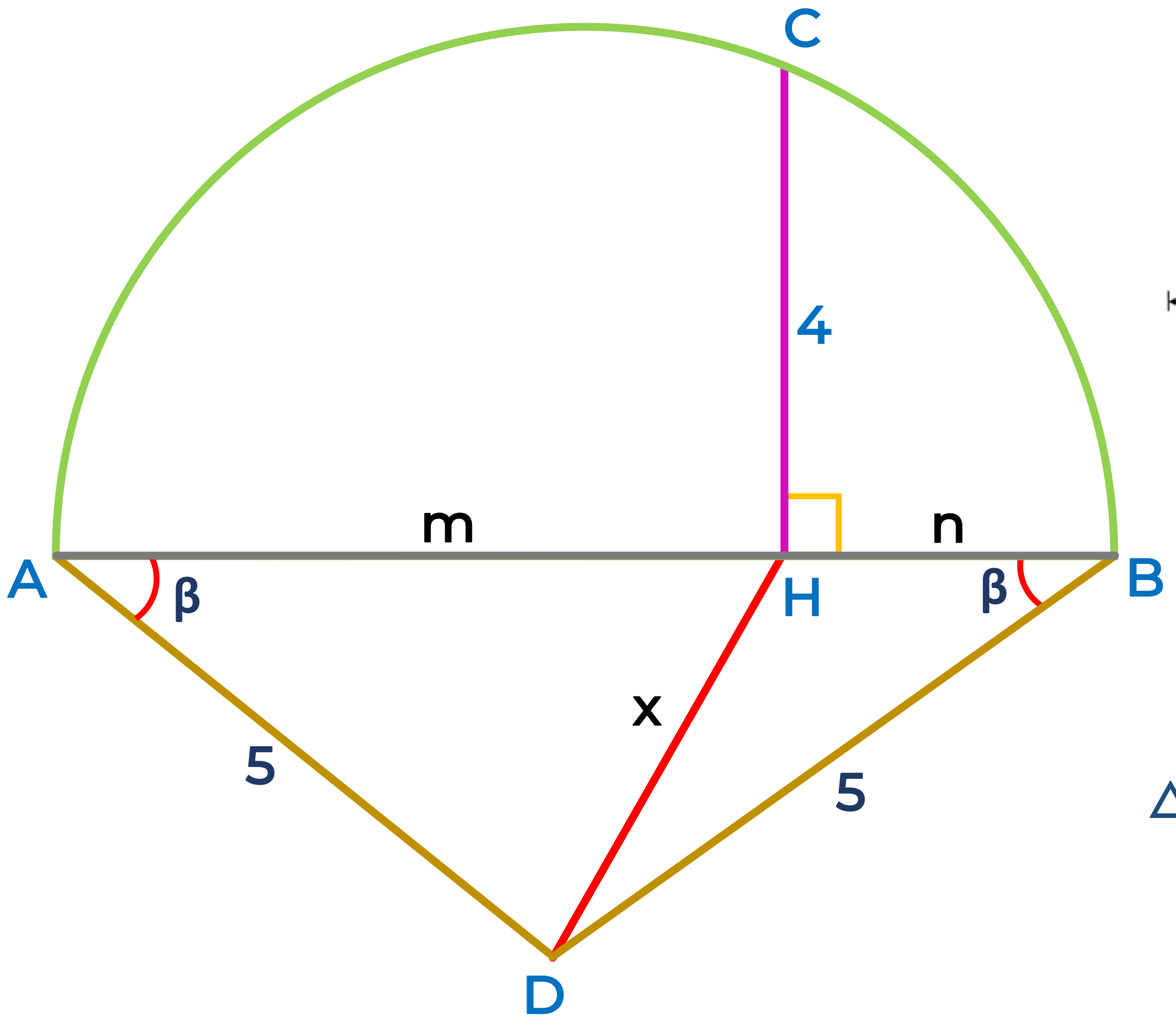
$$x^2 = 6.8 - 3.4$$

$$x^2 = 48 - 12$$

$$x^2 = 36$$

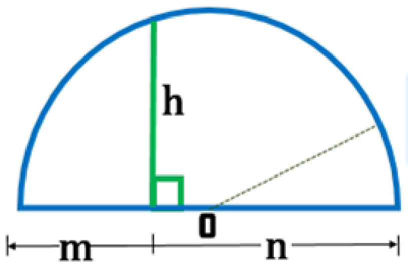
$$\boxed{.:\ x = 6}$$

4. En la figura, $\overline{AB}$ es diámetro, calcule DH.



Resolución

En la semicircunferencia

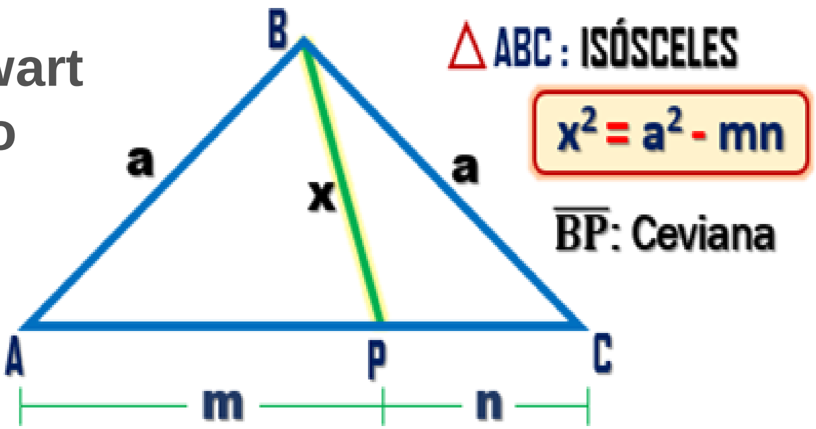


$$h^2 = mn$$

- $4^2 = m.n$

$16 = m.n$

Teorema de Stewart (para triángulo isósceles)



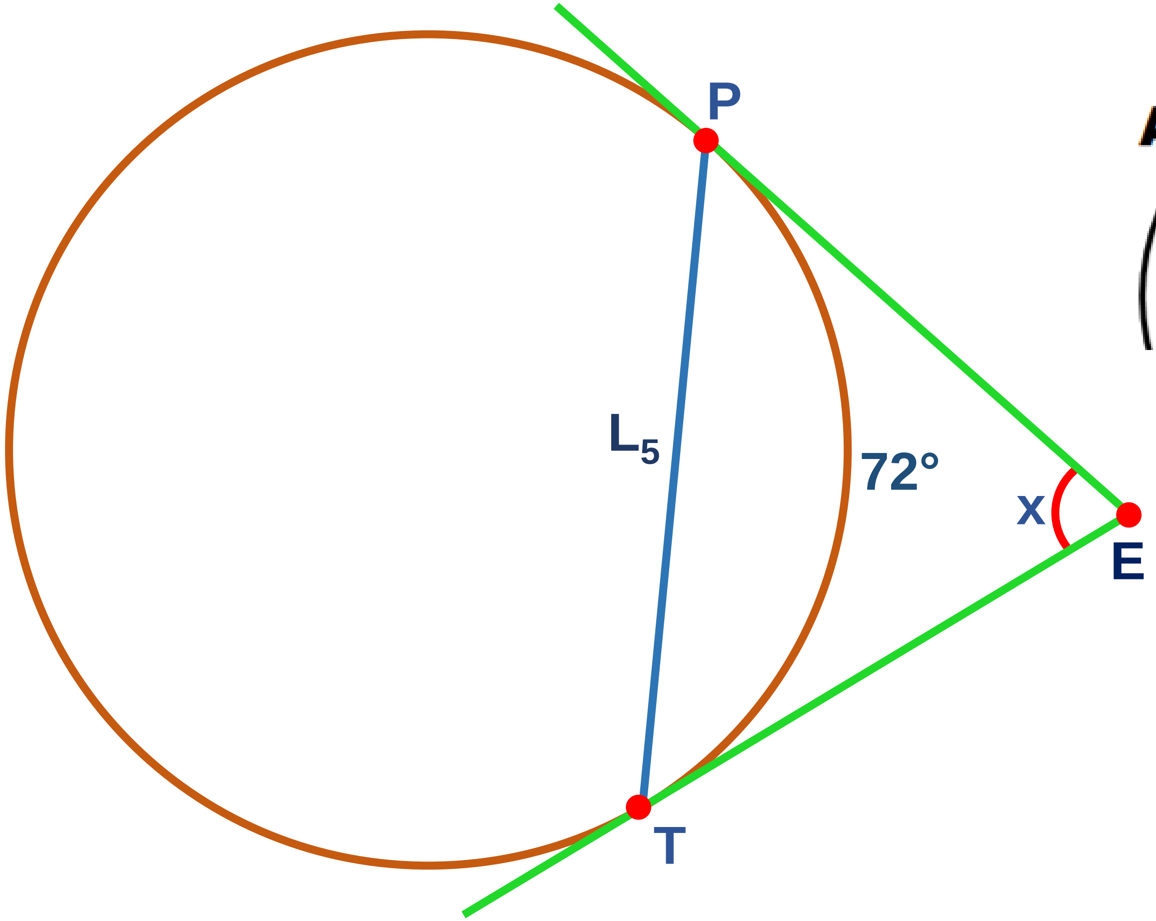
$\triangle$ ABD: $x^2 = 5^2 - \underbrace{m.n}$

$x^2 = 25 - 16$

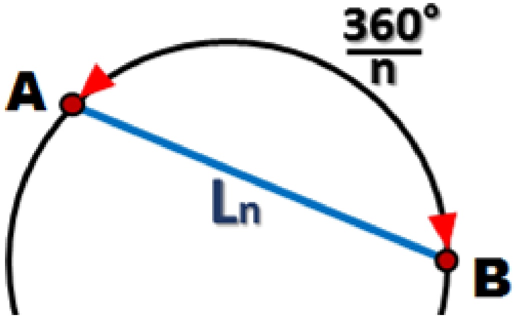
$x^2 = 9$

$\boxed{\therefore x = 3}$

5. Desde un punto E exterior a una circunferencia, se trazan los segmentos tangentes $\overline{ET}$ y $\overline{EP}$. Si PT = L_5, halle la m∡PET.



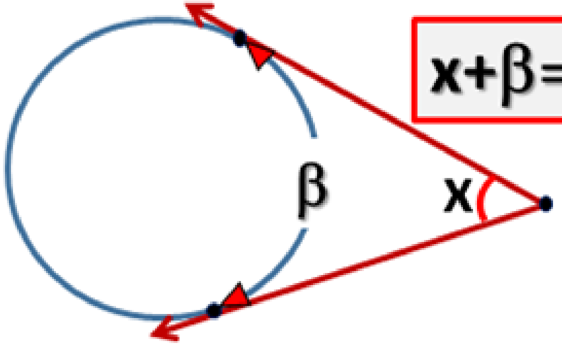
Resolución



$$m\overset{\frown}{AB} = \frac{360°}{n}$$

n = 5

$$m\overset{\frown}{PT} = 72°$$



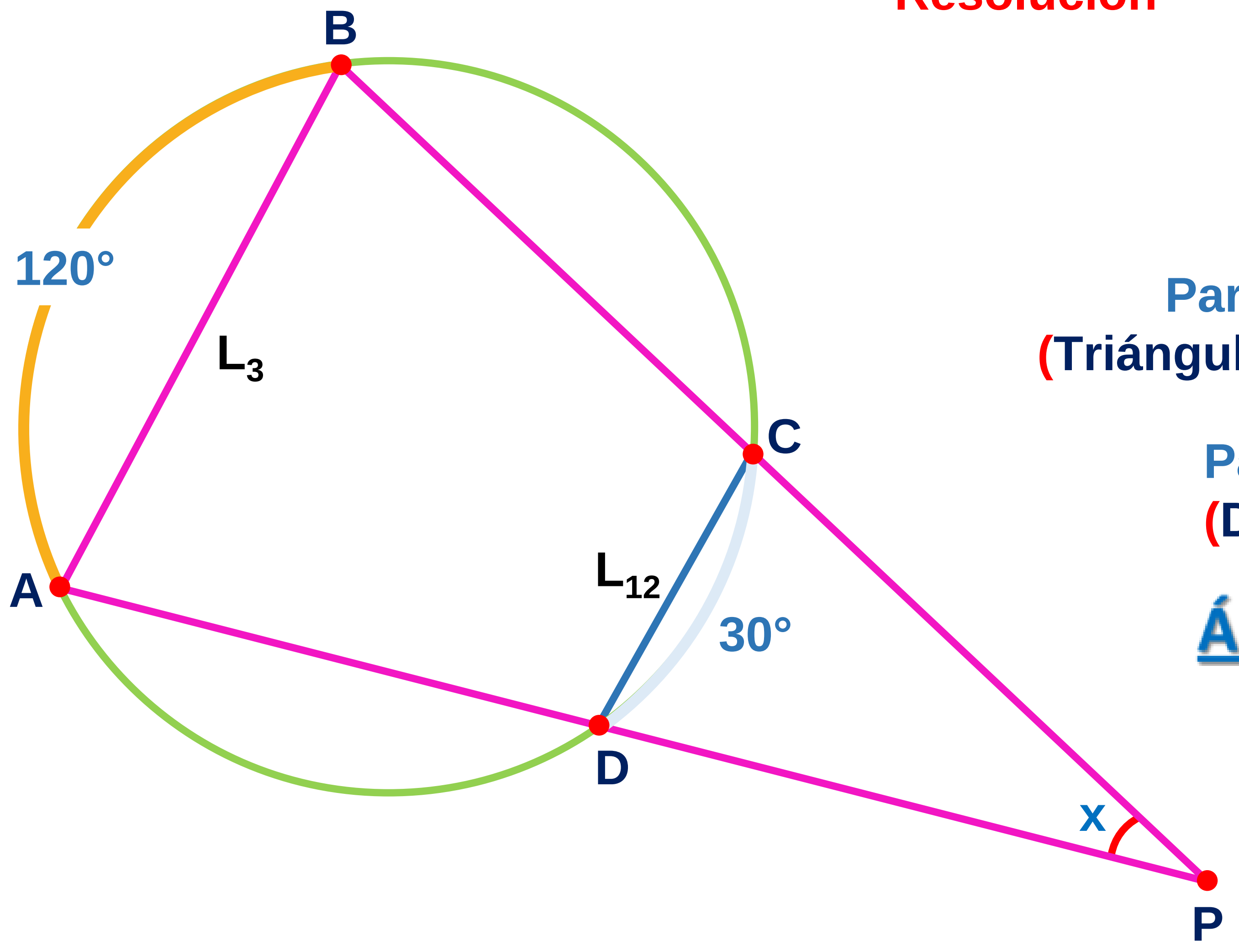
$$x+\beta=180°$$

Del gráfico:

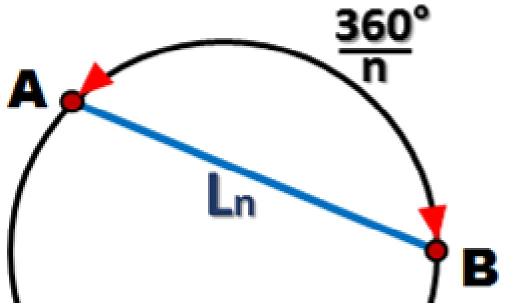
$$x + 72° = 180°$$

$$\therefore\ x = 108°$$

6. Calcule el valor de x, si $AB = L_3$ y $CD = L_{12}$.



Resolución



$$m\overset{\frown}{AB} = \frac{360°}{n}$$

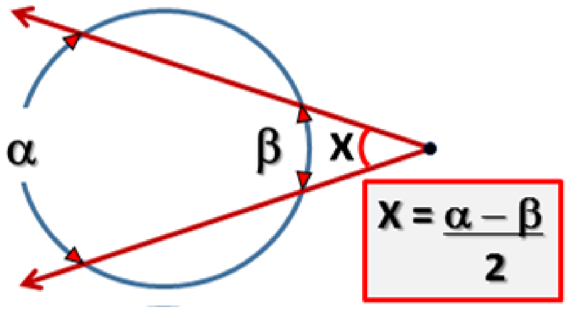
Para: $n = 3$
(Triángulo equilátero)

$m\overset{\frown}{AB} = 120°$

Para: $n = 12$
(Dodecágono)

$m\overset{\frown}{CD} = 30°$

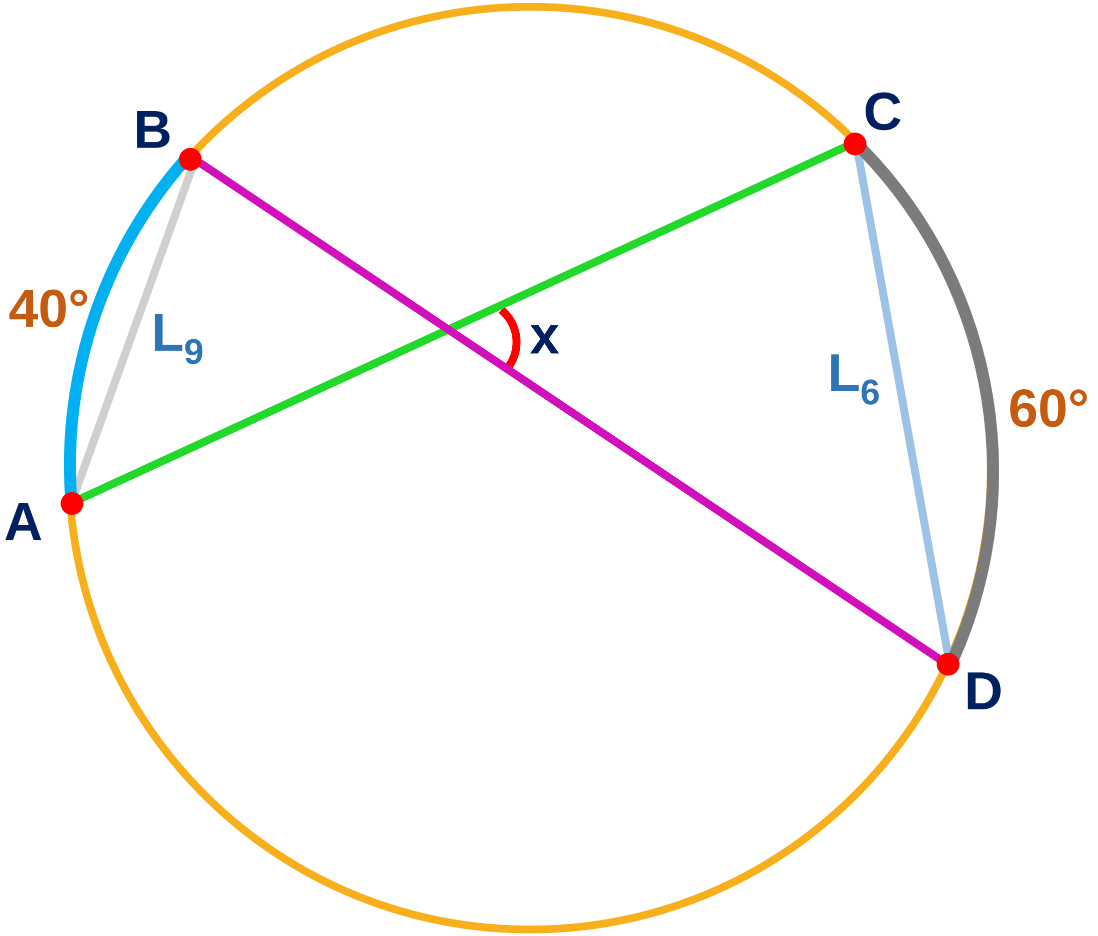
Ángulo exterior



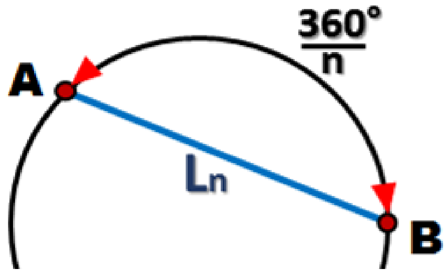
$$x = \frac{120° - 30°}{2}$$

$\therefore x = 45°$

7. Si $AB = L_9$ y $CD = L_6$, calcule la medida del ángulo que forman $\overline{BD}$ y $\overline{AC}$.



Resolución



n = 9

(nonágono)

$$m\overset{\frown}{AB} = \frac{360°}{9}$$

$$m\overset{\frown}{AB} = 40°$$

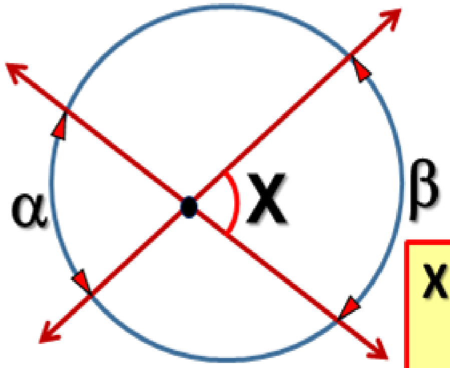
n = 6

(hexágono)

$$m\overset{\frown}{CD} = \frac{360°}{6}$$

$$m\overset{\frown}{CD} = 60°$$

Ángulo interior

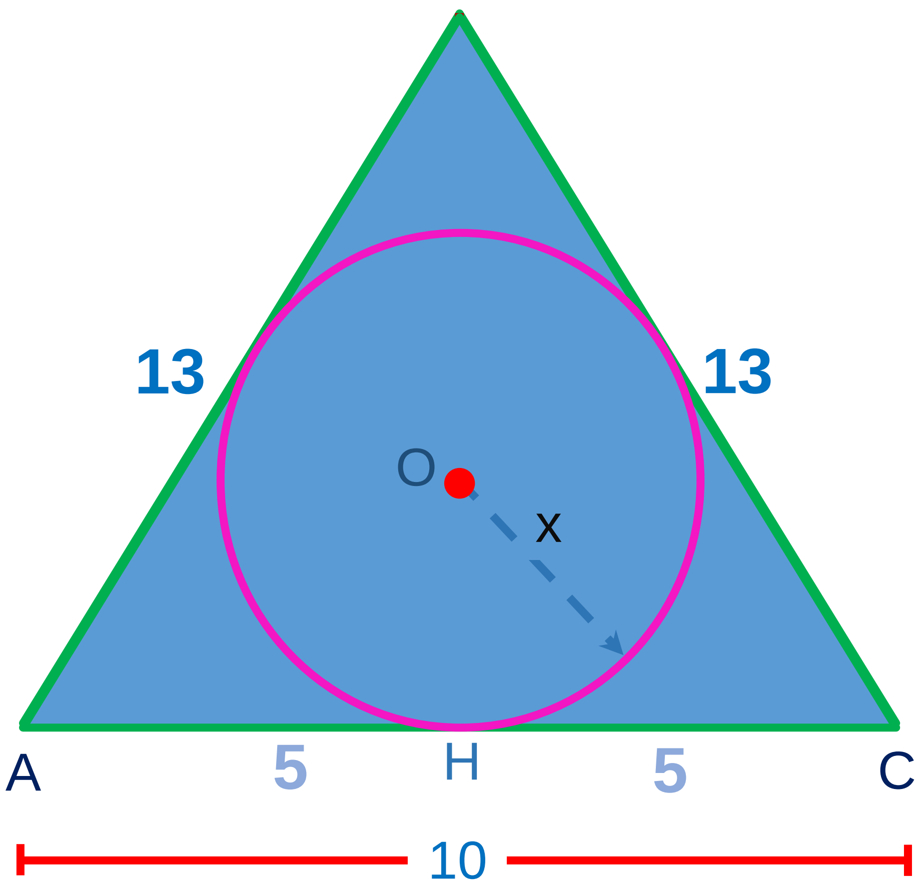


$$\boxed{X = \frac{\alpha+\beta}{2}}$$

$$x = \frac{40° + 60°}{2}$$

$$\boxed{.:\ x = 50°}$$

8. Las longitudes de los lados de un triángulo son 13, 13 y 10 cm. Calcule la longitud de su inradio.



Resolución

△ ABC es isósceles

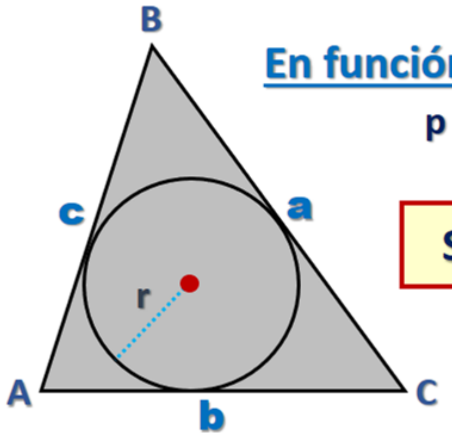
△BHC por T. Pitágoras:

$$13^2 = (BH)^2 + 5^2$$

$$144 = (BH)^2 \Rightarrow 12 = BH$$

$$S_{(\triangle ABC)} = \frac{10.\ 12}{2}$$

$$S_{(\triangle ABC)} = 60u^2$$



En función al inradio

$$p = \frac{a+b+c}{2}$$

$$\boxed{S_{ABC} = p.r}$$

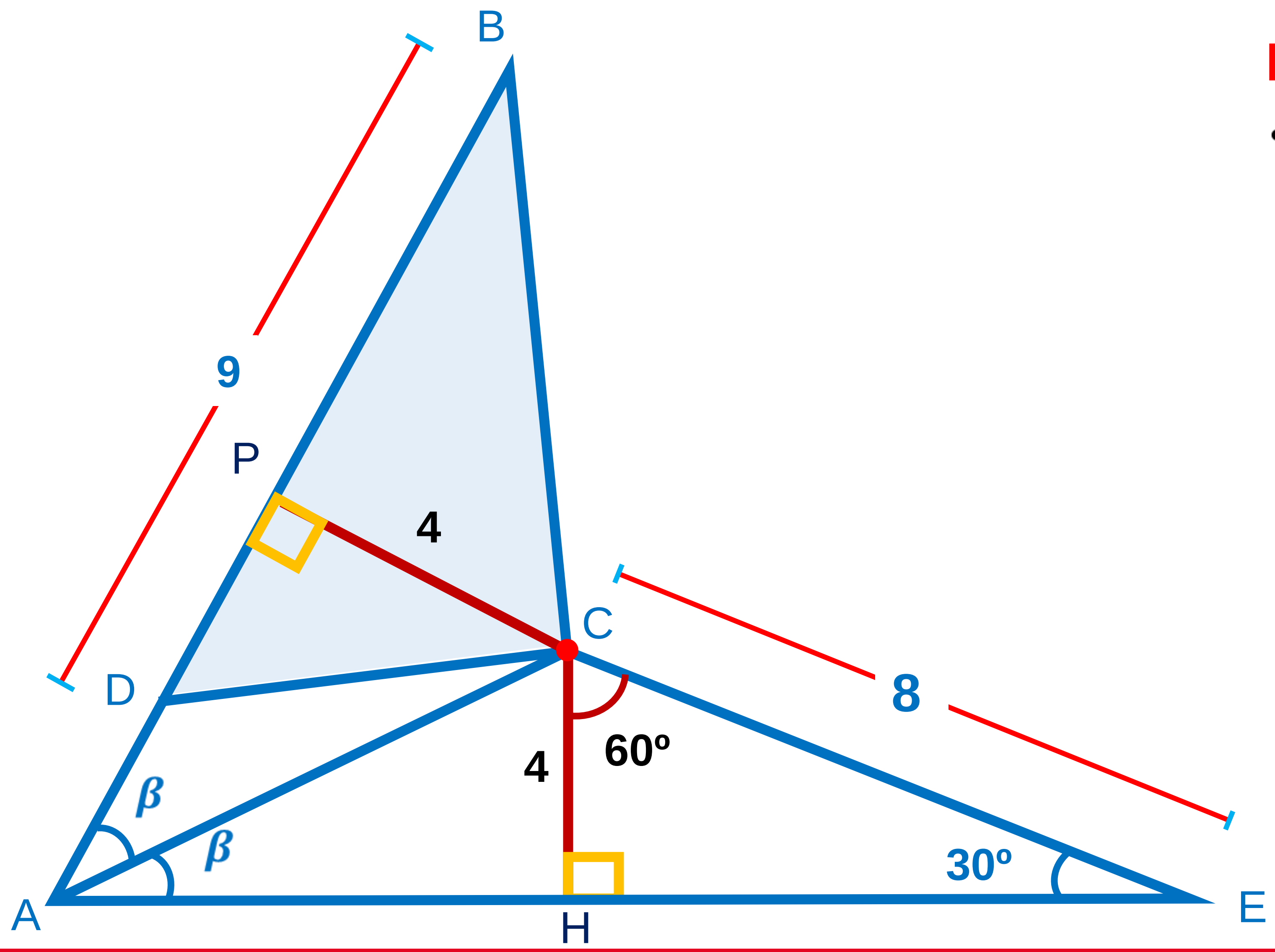
$$S_{\triangle ABC} = \frac{(13+13+10).x}{2}$$

$$\underbrace{S_{\triangle ABC}} = (18).x$$

$$\cancel{6}0 = 1\cancel{8}x$$

$$\boxed{\therefore \frac{10}{3} = x}$$

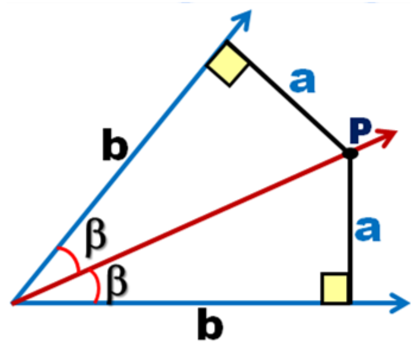
9. En el gráfico, BD = 9 y CE = 8, calcule el área de la región sombreada.



Resolución

- Se traza la altura $\overline{CH}$.

⊿ CHE es notable de 30° y 60°



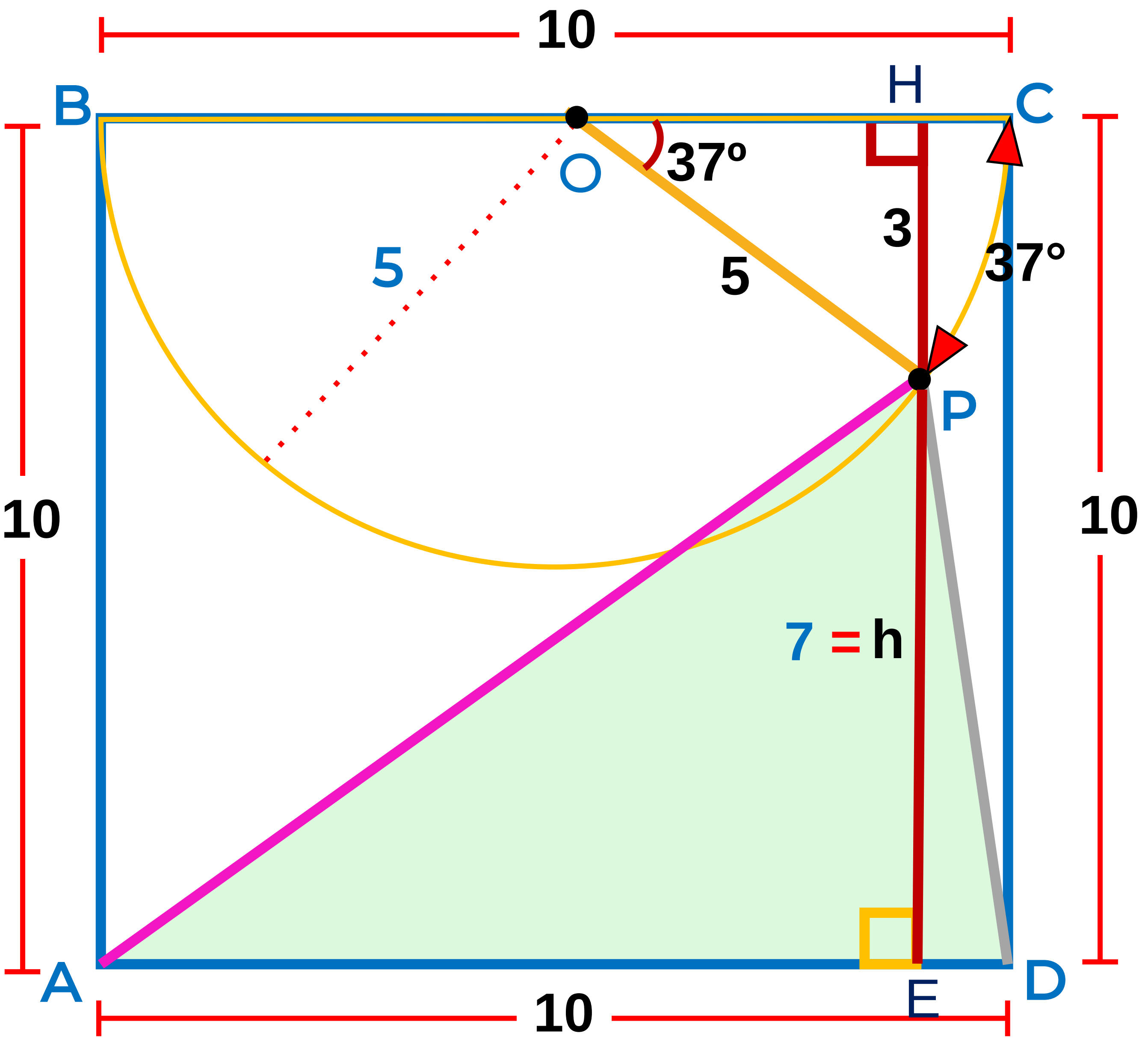
CH = CP = 4

- △ BCD, teorema:

$$S_{\triangle}BCD = \frac{9 \cdot 4}{2}$$

.: $S_{\triangle}BCD = 18\ u^2$

10. En la figura, ABCD es un cuadrado, si m $\overset{\frown}{CP}$ = 37º, calcule el área de la región sombreada.



Resolución

- Se traza $\overline{OP}$.
- Se traza $\overline{PH}$ perpendicular a $\overline{BC}$.
- ⊿OHP de 37° y 53°
- Se prolonga $\overline{HP}$ hasta E.

▌CDEH es rectángulo

HE = CD = 10

h + 3 = 10 → h = 7

- Teorema: $S_{\blacktriangle APD} = \dfrac{10 \cdot 7}{2}$

.: $S_{\blacktriangle APD} = 35\ u^2$