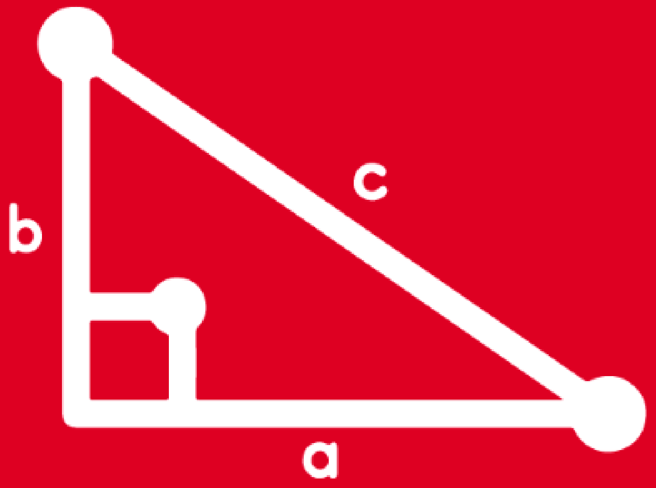
TRIGONOMETRY

Chapter 09

5th
SECONDARY



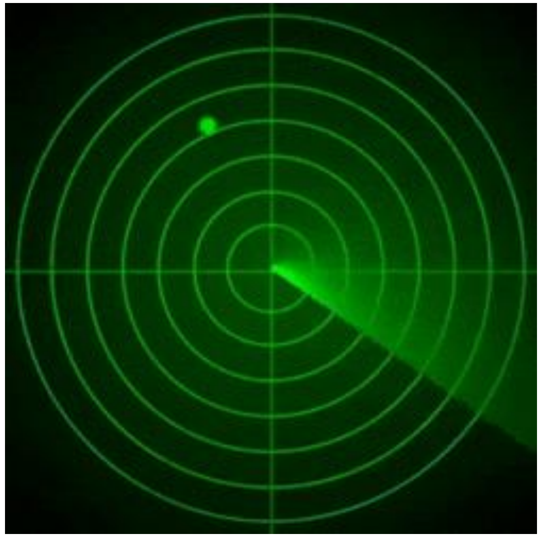
REDUCCIÓN AL PRIMER CUADRANTE II

SACO OLIVEROS

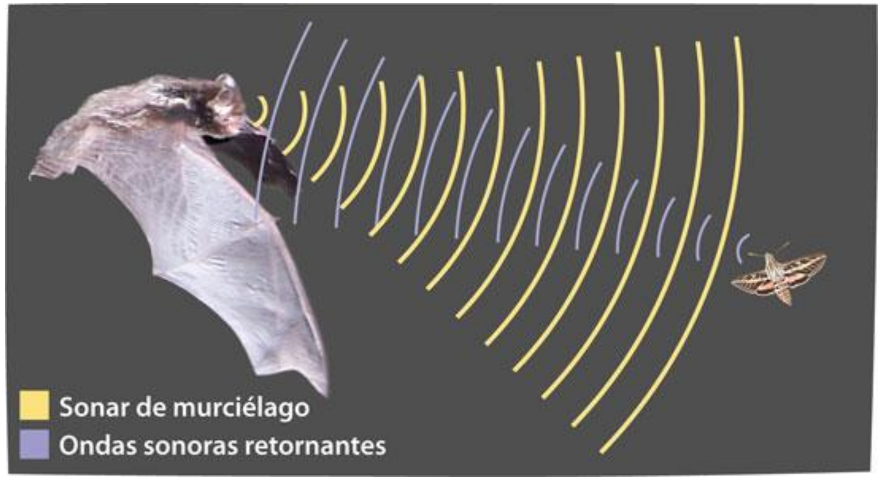
HELICO - MOTIVACIÓN

El SISTEMA DE SONAR es una técnica que principalmente usa la propagación del sonido bajo el agua para navegar, comunicarse o detectar objetos sumergidos.

El sonar funciona de forma similar al radar, con la diferencia de que en lugar de emitir ondas electromagnéticas emplea impulsos sonoros.



En la naturaleza, algunos animales como delfines y murciélagos usan el sonido para la detección de objetos



REDUCCIÓN AL PRIMER CUADRANTE

3er CASO : Para ángulos positivos mayores a una vuelta .

De forma práctica utilizaremos :

$$\forall\, k \in \mathbb{Z}^{+} :$$

$$RT[\cancel{360^\circ . k} \pm \alpha] = RT(\pm\alpha)$$

Para arcos múltiplos enteros de π :

$$RT[\underbrace{\cancel{2k}}_{\text{par}}\pi \pm \alpha] = RT(\pm\alpha)$$

$$RT[(\underbrace{2k+1}_{\text{impar}})\pi \pm \alpha] = RT(\pi \pm \alpha)$$

Ejemplo :

$$\operatorname{sen}800^\circ = \operatorname{sen}(\cancel{360^\circ . 2} + 80^\circ)$$

$$\operatorname{sen}800^\circ = \boxed{\operatorname{sen}80^\circ}$$

$$\begin{array}{r|l} 800^\circ & 360^\circ \\ \cline{2-2} 720^\circ & 2 \\ \hline 80^\circ & \end{array}$$

OBSERVACIONES

Para reducir arcos de la forma $\left(\frac{a\pi}{b}\right)$, donde $a > 2b$

Efectuamos :

$$\begin{array}{r|l} a & 2b \\ \cline{2-2} c & k \\ \cline{1-1} (r) & \end{array}$$

Luego :

$$\boxed{RT\left(\frac{a\pi}{b}\right) = RT\left(\frac{r\pi}{b}\right)}$$

$$\begin{array}{r|l} 25 & 6 \\ \cline{2-2} 24 & 4 \\ \cline{1-1} (1) & \end{array}$$

Ejemplo : $\cos\frac{25\pi}{3} = \cos\frac{1\pi}{3} = \frac{1}{2}$

$\forall\, k \in \mathbb{Z}^{+}$:

$$\boxed{RT\left[(4k+1)\frac{\pi}{2} \pm \alpha\right] = RT\left(\frac{\pi}{2} \pm \alpha\right)}$$

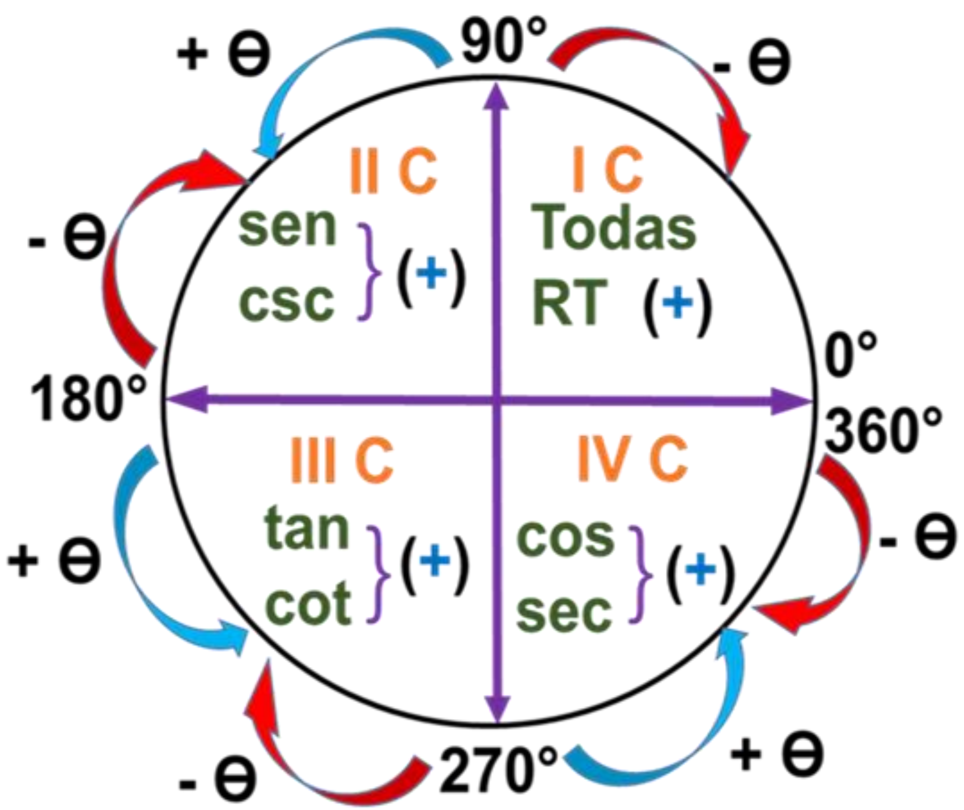
$$\boxed{RT\left[(4k+3)\frac{\pi}{2} \pm \alpha\right] = RT\left(\frac{3\pi}{2} \pm \alpha\right)}$$

Ejemplos :

$$\operatorname{sen}\left(\frac{\overbrace{17}^{4k+1}\pi}{2} + \alpha\right) = \operatorname{sen}\left(\frac{\pi}{2} + \alpha\right)$$

$$\cot\left(\frac{\overbrace{71}^{4k+3}\pi}{2} - \alpha\right) = \cot\left(\frac{3\pi}{2} - \alpha\right)$$

RECORDAR



$$RT\begin{bmatrix}180° \pm \theta \\ 360° - \theta\end{bmatrix} = \pm RT(\theta)$$

$$RT\begin{bmatrix}90° + \theta \\ 270° \pm \theta\end{bmatrix} = \pm CO\text{-}RT(\theta)$$

$$\cos(-x) = \cos(x)$$

$$\sec(-x) = \sec(x)$$

Co - RT

sen ↔ cos

tan ↔ cot

sec ↔ csc

HELICO PRACTICE 1

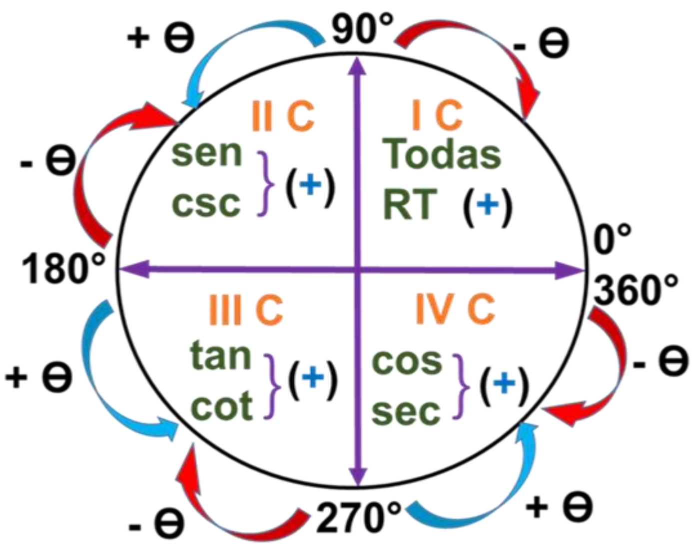
Efectúe $P = \dfrac{\text{sen } 1500° \, . \, \cos 1110°}{\tan 3645°}$

RESOLUCIÓN

$$P = \frac{\text{sen } 1500° \, . \, \cos 1110°}{\tan 3645°} = \frac{\text{sen } 60° \, . \, \cos 30°}{\tan 45°} = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{1}$$

$$\therefore \quad P = \frac{3}{4}$$

$$\begin{array}{r|l} 1500^0 & 360^o \\ \cline{2-2} 1440^0 & \ \ 4 \\ \hline (60^0) & \end{array} \qquad \begin{array}{r|l} 1110^0 & 360^o \\ \cline{2-2} 1080^0 & \ \ 3 \\ \hline (30^0) & \end{array} \qquad \begin{array}{r|l} 3645^0 & 360^o \\ \cline{2-2} 3600^0 & \ \ 10 \\ \hline (45^0) & \end{array}$$



HELICO PRACTICE 2

Simplifique la expresión

$$E = \frac{sen(8\pi + x).cos(7\pi + x)}{cos\left(\frac{15\pi}{2} + x\right)}$$

RESOLUCIÓN

$$E = \frac{sen(\underbrace{8}_{par}\pi + x).cos(\underbrace{7}_{impar}\pi + x)}{cos\left(\frac{15\pi}{2} + x\right)}$$

$15 \rightarrow$ 4k + 3

$$E = \frac{senx.\ cos(\underbrace{\pi + x}_{IIIC})}{\underbrace{cos\left(\frac{3\pi}{2} + x\right)}_{IVC}}$$

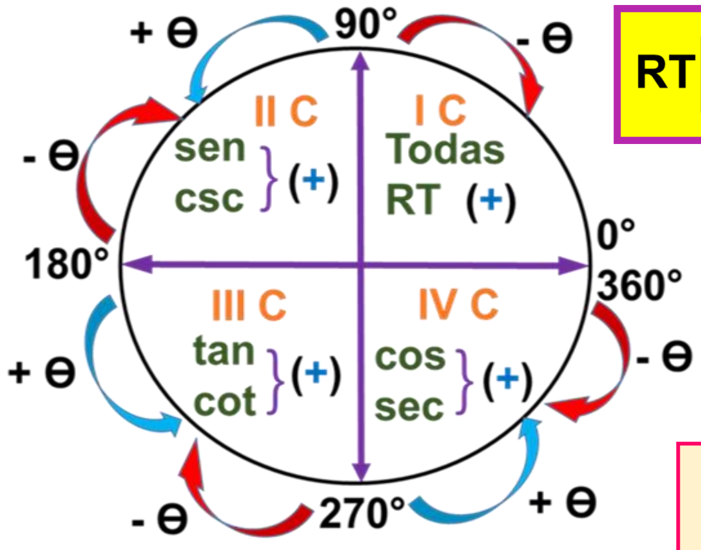
$$E = \frac{\cancel{senx}(-cosx)}{\cancel{senx}}$$

$$\therefore\ E = -cosx$$

$$RT[\underbrace{\cancel{2k}}_{par}\pi \pm \alpha] = RT(\pm\alpha)$$

$$RT[(\underbrace{2k+1}_{impar})\pi \pm \alpha] = RT(\pi \pm \alpha)$$

$$RT\left[(4k+3)\frac{\pi}{2} \pm \alpha\right] = RT\left(\frac{3\pi}{2} \pm \alpha\right)$$



$$RT\begin{bmatrix}180° \pm \theta \\ 360° - \theta\end{bmatrix} = \pm RT(\theta)$$

$$RT\begin{bmatrix}90° + \theta \\ 270° \pm \theta\end{bmatrix} = \pm CO\text{-}RT(\theta)$$

HELICO PRACTICE 3

A Manuel se le entregó S/. x como incentivo por sus buenas calificaciones. Resolviendo la siguiente ecuación podrá averiguar con cuánto se le premió.

$$\sec 420° + x \tan 2565° = 20 \operatorname{sen} 2213°$$

RESOLUCIÓN

$$\sec 420° + x \tan 2565° = 20 \operatorname{sen} 2213°$$

$$\sec 60° + x \tan 45° = 20 \operatorname{sen} 53°$$

$$2 + x(1) = 20\left(\frac{4}{5}\right)$$

$$2 + x = 16 \Rightarrow x = 14$$

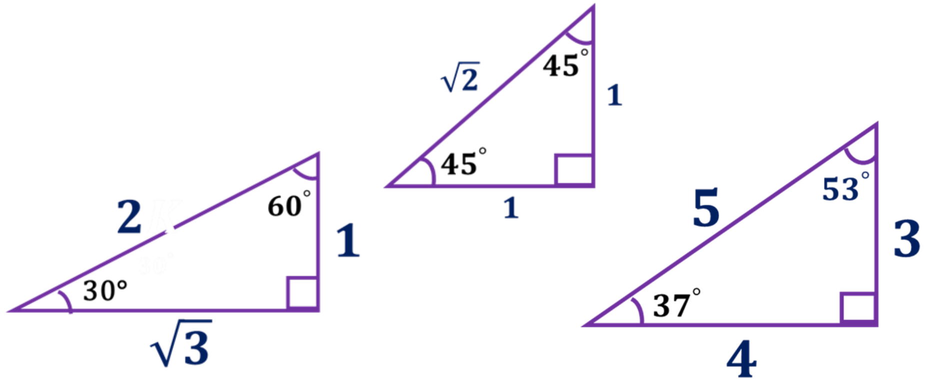
∴ A Manuel se le premió con S/. 14

$$RT[\cancel{360°.k} \pm \alpha] = RT(\pm\alpha)$$

$$\begin{array}{r|l} 420^0 & 360^0 \\ \hline 360^0 & 1 \\ (60^0) & \end{array}$$

$$\begin{array}{r|l} 2565^0 & 360^0 \\ \hline 2520^0 & 7 \\ (45^0) & \end{array}$$

$$\begin{array}{r|l} 2213^0 & 360^0 \\ \hline 2160^0 & 6 \\ (53^0) & \end{array}$$



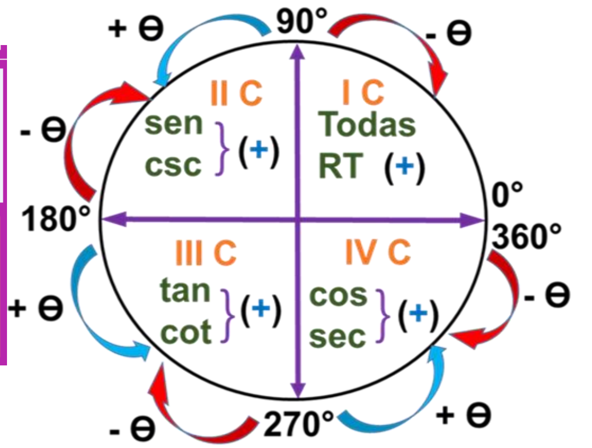
HELICO PRACTICE 4

Halle el valor de "n" si se cumple que : $\text{sen}(21\pi - \alpha) = \dfrac{n-1}{3}$;

$$\cos\left(\frac{41\pi}{2} + \alpha\right) = \frac{n}{2} - 3$$

$$RT[\underbrace{(2k+1)}_{\text{impar}}\pi \pm \alpha] = RT(\pi \pm \alpha)$$

$$RT\left[(4k+1)\frac{\pi}{2} \pm \alpha\right] = RT\left(\frac{\pi}{2} \pm \alpha\right)$$



RESOLUCIÓN

$$\frac{n-1}{3} = \text{sen}(\overbrace{21}^{\text{impar}}\pi - \alpha) = \text{sen}(\overbrace{\pi - \alpha}^{\text{IIC}}) = \text{sen}\alpha$$

4k + 1

$$\frac{n}{2} - 3 = \cos\left(\frac{41\pi}{2} + \alpha\right) = \cos\left(\overbrace{\frac{\pi}{2} + \alpha}^{\text{IIC}}\right) = -\text{sen}\alpha$$

$$3 - \frac{n}{2} = \text{sen}\alpha \Rightarrow \frac{6-n}{2} = \text{sen}\alpha$$

Luego : $\text{sen}\alpha = \text{sen}\alpha$

$$\frac{n-1}{3} = \frac{6-n}{2}$$

$$2n - 2 = 18 - 3n$$

$$5n = 20$$

$$\therefore n = 4$$

HELICO PRACTICE 5

Halle el valor de $E = \cos\left(\frac{37\pi}{3}\right) + \tan\left(\frac{59\pi}{4}\right)$

RESOLUCIÓN

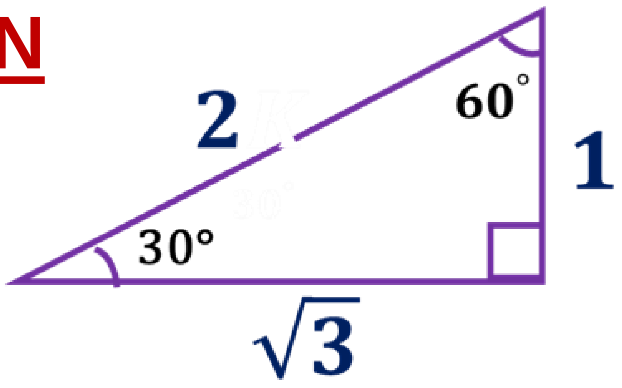
$$E = \cos\left(\frac{37\pi}{3}\right) + \tan\left(\frac{59\pi}{4}\right)$$

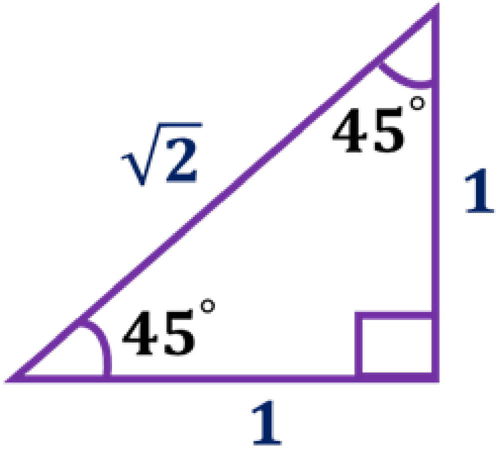
$$E = \cos\left(\frac{1\pi}{3}\right) + \tan\left(\frac{3\pi}{4}\right)$$

$$E = \cos 60° + \tan 135°$$

$$E = \frac{1}{2} + \tan(\overbrace{180° - 45°}^{\text{IIC}})$$

$$E = \frac{1}{2} + (-\tan 45°) = \frac{1}{2} - 1$$





$$\therefore E = -\frac{1}{2}$$

Para reducir arcos de la forma $\left(\frac{a\pi}{b}\right)$, donde $a > 2b$

Efectuamos :

$$\begin{array}{r|l} a & 2b \\ \hline c & k \end{array}$$

(r)

Luego :

$$\boxed{RT\left(\frac{a\pi}{b}\right) = RT\left(\frac{r\pi}{b}\right)}$$

$$\begin{array}{r|l} 37 & 6 \\ \hline 36 & 6 \end{array}$$

(1)

$$\begin{array}{r|l} 59 & 8 \\ \hline 56 & 7 \end{array}$$

(3)

HELICO PRACTICE 6

Siendo $x + y = 1170°$, reduzca :

$$G = \frac{\tan y}{\cot x} + \operatorname{sen}x \cdot \sec y$$

RESOLUCIÓN

Dato : $x + y = 1170°$

$$y = 1170° - x$$

$$RT(y) = RT[\,\cancel{3(360°)} + 90° - x\,]$$

$$RT(y) = RT[\,\underbrace{90° - x}_{IC}\,]$$

$$RT(y) = CO - RT(x)$$

Recordar :

$$RT[\,\cancel{360° \cdot k} \pm \alpha\,] = RT(\pm\alpha)$$

$$RT\begin{Bmatrix} 90° \pm \theta \\ 270° \pm \theta \end{Bmatrix} = \pm\, Co\_RT(\theta)$$

Luego : $G = \dfrac{\tan y}{\cot x} + \operatorname{sen}x \cdot \sec y$

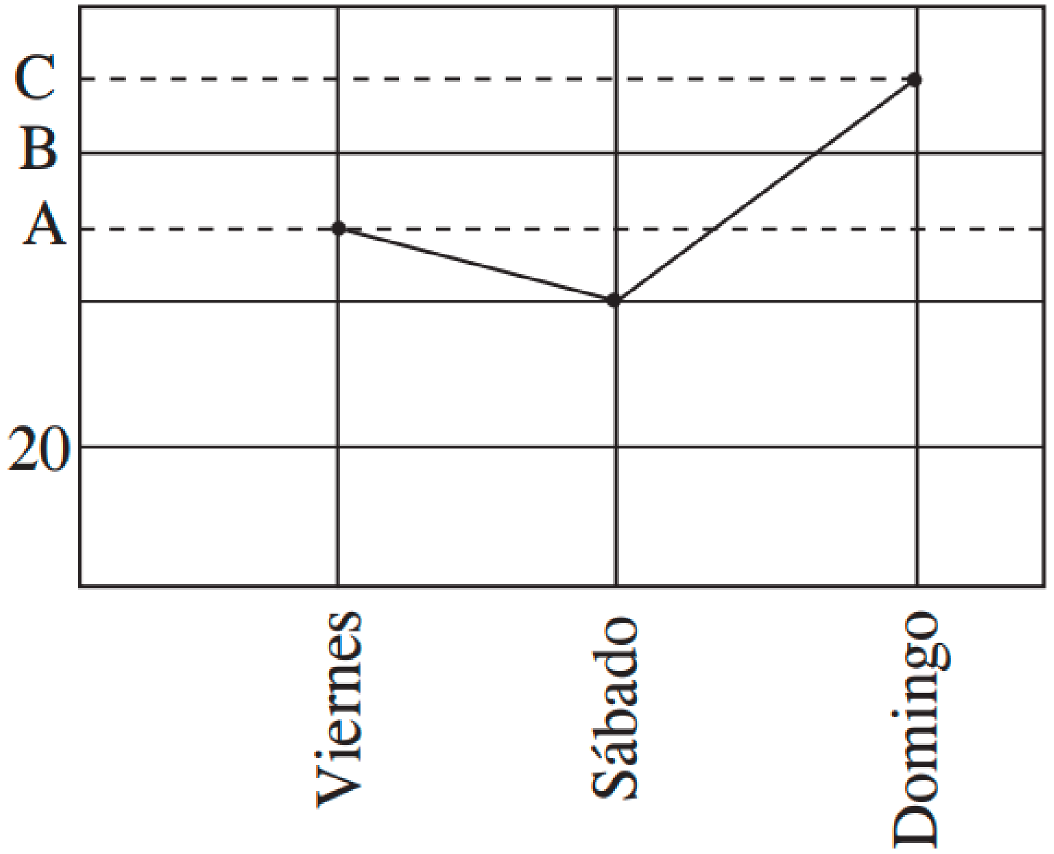
$$G = \frac{\cot x}{\cot x} + \operatorname{sen}x \cdot \csc x$$

$$G = 1 + 1$$

$$\boxed{\therefore G = 2}$$

HELICO PRACTICE 7

La gráfica muestra las temperaturas (en C°) registradas al mediodía en la ciudad de Piura, los días viernes, sábado y domingo de la primera semana de febrero.



Donde :

$$A = 62\ \text{sen}\left(\frac{13\pi}{6}\right)$$

$$B = 16\ \csc^2\left(\frac{81\pi}{4}\right)$$

$$C = 18\ \sec\left(\frac{19\pi}{3}\right)$$

¿Cuál es el promedio de las temperaturas?

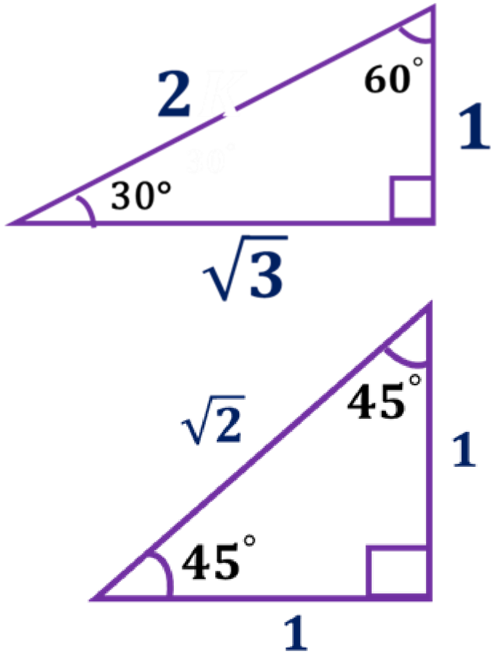
HELICO PRACTICE 7

RESOLUCIÓN

$$A = 62\,\text{sen}\left(\frac{1\pi}{6}\right) = 62\left(\frac{1}{2}\right) = 31$$

$$B = 16\,\csc^2\left(\frac{1\pi}{4}\right) = 16\,(\sqrt{2}\,)^2 = 32$$

$$C = 18\,\sec\left(\frac{1\pi}{3}\right) = 18(\,2) = 36$$



Luego : Promedio $= \left(\dfrac{31 + 32 + 36}{3}\right)^{0}$ C

$\therefore$ **Promedio = 33°C**

Para reducir arcos de la forma $\left(\frac{a\pi}{b}\right)$, donde $a > 2b$

Efectuamos :

$$\begin{array}{r|l} a & 2b \\ \hline c & k \\ (r) & \end{array}$$

Luego :

$$\boxed{RT\left(\frac{a\pi}{b}\right) = RT\left(\frac{r\pi}{b}\right)}$$

$$\begin{array}{r|l} 13 & 12 \\ \hline 12 & 1 \\ (1) & \end{array} \qquad \begin{array}{r|l} 81 & 8 \\ \hline 80 & 10 \\ (1) & \end{array} \qquad \begin{array}{r|l} 19 & 6 \\ \hline 18 & 3 \\ (1) & \end{array}$$

SACO OLIVEROS