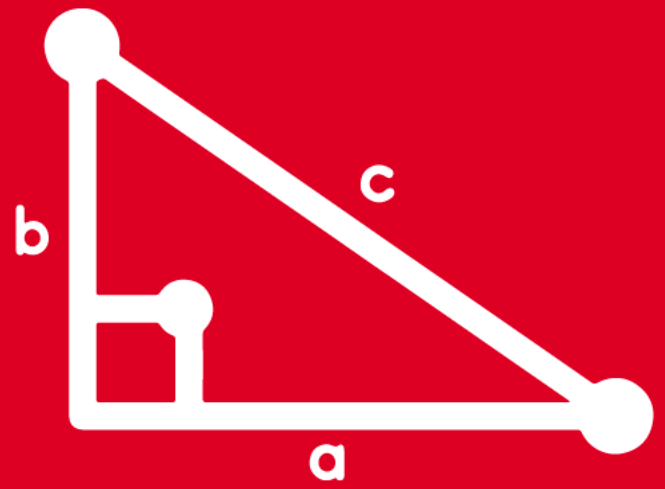
TRIGONOMETRY

Chapter 05



4th
SECONDARY

Razones trigonométricas de ángulos notables

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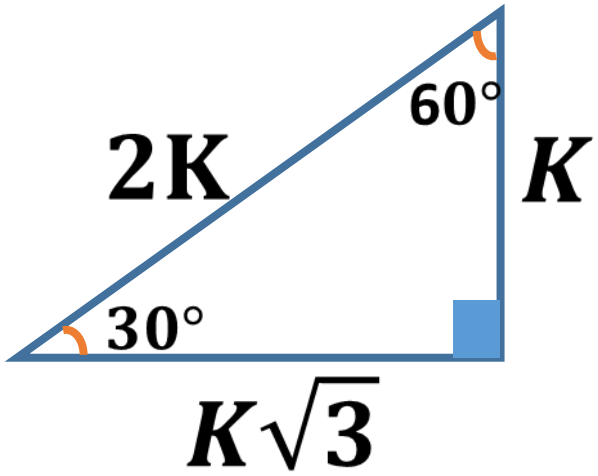
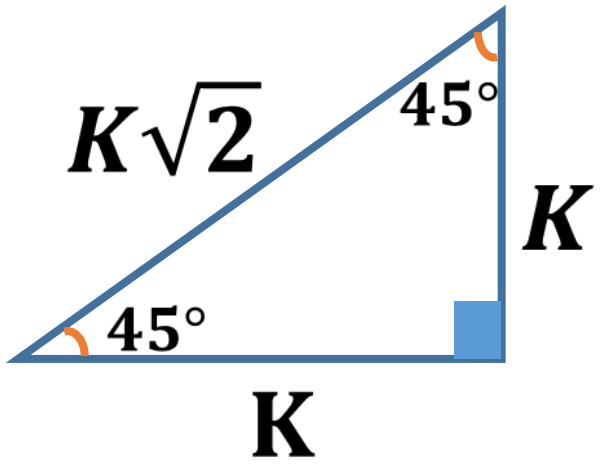
HELICO-MOTIVATION

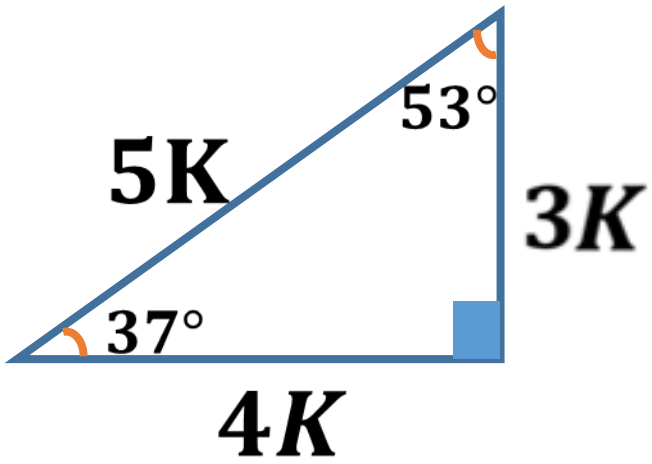
Sabías que existen varios ángulos notables, ¿Cuántas conoces?. Completemos el cuadro.

RT \ α	30^0	60^0	45^0	37^0	53^0	16^0	74^0
Sen α	1/2	$\sqrt{3}/2$	$\sqrt{2}/2$	3/5	4/5	7/25	24/25
Cos α	$\sqrt{3}/2$	1/2	$\sqrt{2}/2$	4/5	3/5	24/25	7/25
Tan α	$\sqrt{3}/3$	$\sqrt{3}$	1	3/4	4/3	7/24	24/7
Cot α	$\sqrt{3}$	$\sqrt{3}/3$	1	4/3	3/4	24/7	7/24
Sec α	$2\sqrt{3}/3$	2	$\sqrt{2}$	5/4	5/3	25/24	25/7
Csc α	2	$2\sqrt{3}/3$	$\sqrt{2}$	5/3	5/4	25/7	25/24

Razones trigonométricas de ángulos notables



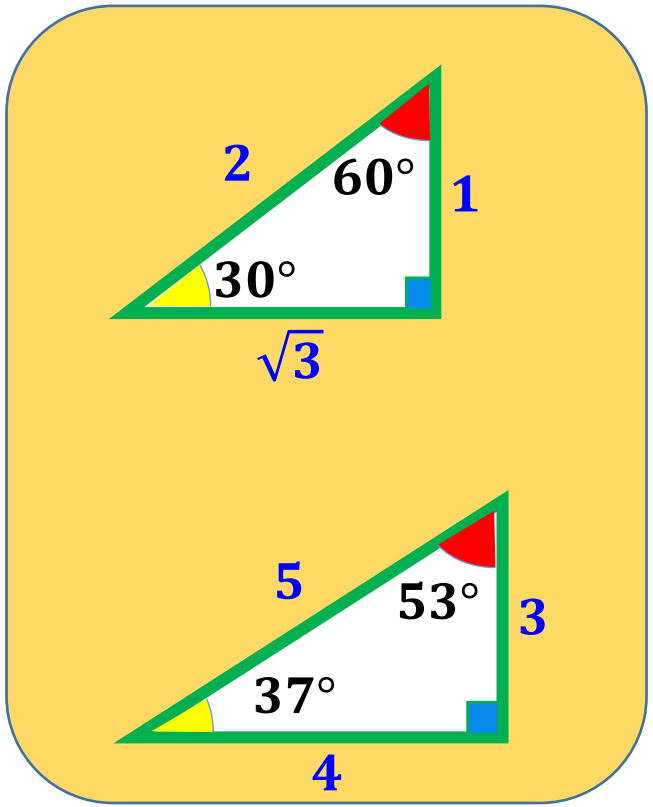




RT \ ∠	30°	60°	37°	53°	45°
sen	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{\sqrt{2}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{\sqrt{2}}{2}$
tan	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{3}{4}$	$\frac{4}{3}$	1
cot	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	$\frac{4}{3}$	$\frac{3}{4}$	1
sec	$\frac{2\sqrt{3}}{3}$	2	$\frac{5}{4}$	$\frac{5}{3}$	$\sqrt{2}$
csc	2	$\frac{2\sqrt{3}}{3}$	$\frac{5}{3}$	$\frac{5}{4}$	$\sqrt{2}$

1. Efectúe: $P = \left(5\text{sen}37° + \sqrt{3}\tan 60° + \cot^2 30°\right)^{\cos 60°}$

RESOLUCIÓN



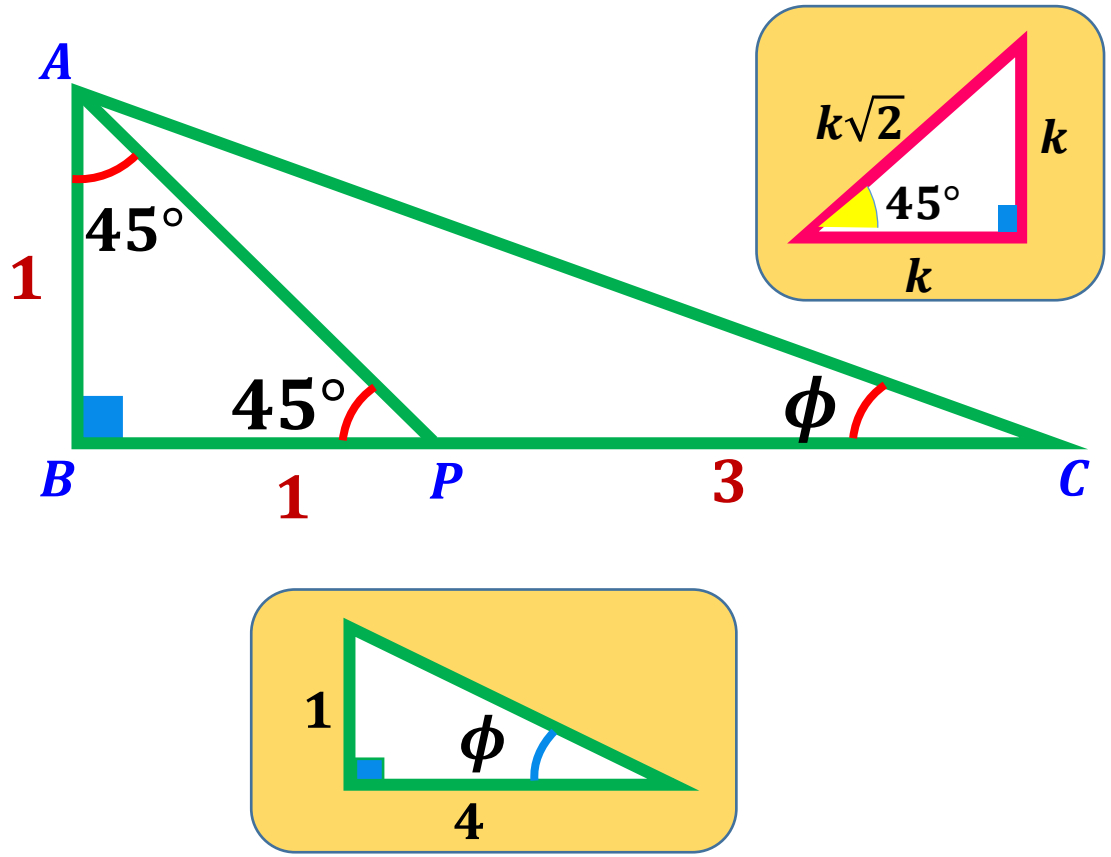
$$P = \left(\cancel{5} \times \left(\frac{3}{\cancel{5}}\right) + \sqrt{3} \times (\sqrt{3}) + (\sqrt{3})^2\right)^{1/2}$$

$$P = (3 + 3 + 3)^{1/2}$$

$$P = \sqrt{9}$$

$$\therefore P = 3$$

2. Del gráfico, calcule cot ϕ si PC = 3BP.



RESOLUCIÓN

Del dato:

$PC = 3BP \Rightarrow \frac{PC}{BP} = \frac{3}{1}$

En el ΔABP (Notable de 45°)

$AB = BP$

Pero

$BP = 1 \Rightarrow AB = 1$

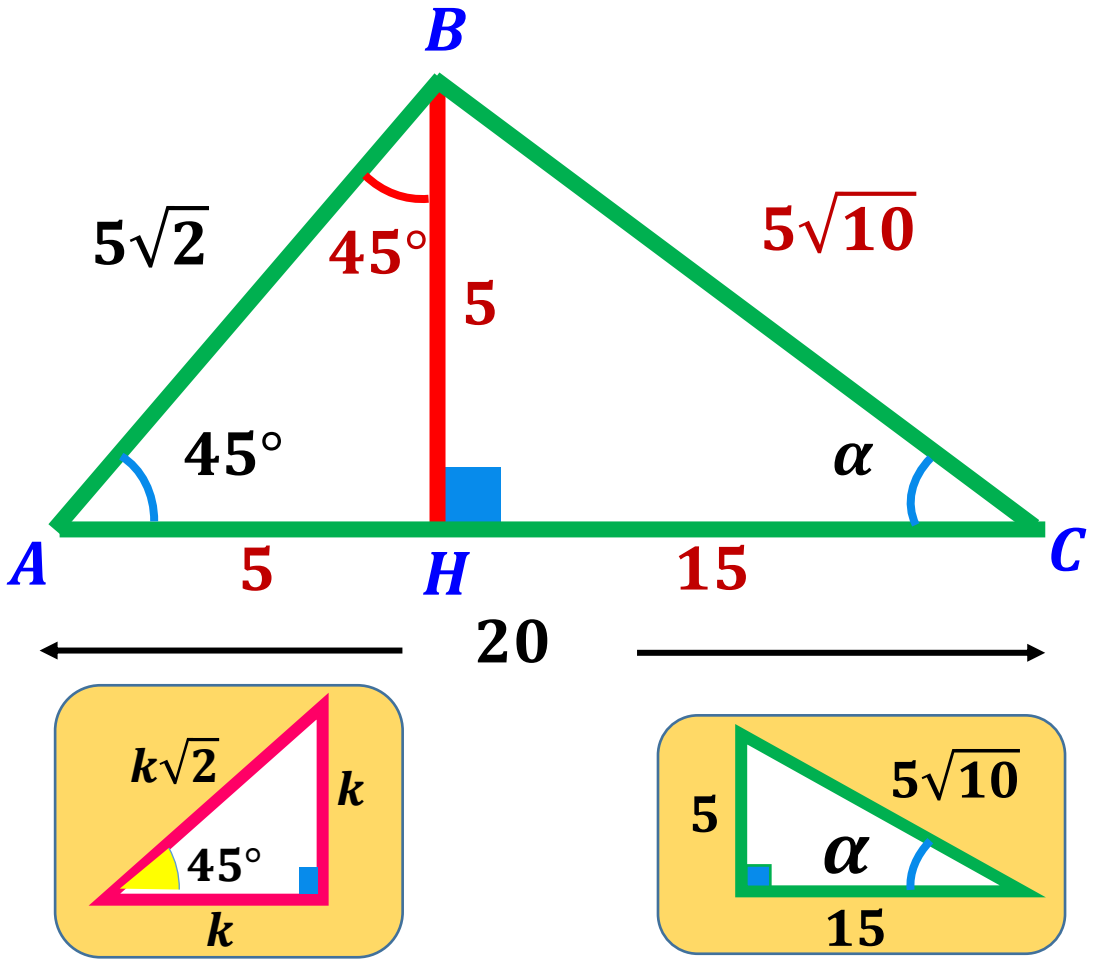
Calculamos:

$\cot\phi = \frac{4}{1}$

$\therefore \cot\phi = 4$

3. Del gráfico, efectúe:

$E = \sqrt{10}\,\text{sen}\alpha + \cot\alpha$



RESOLUCIÓN

$\triangle AHB$ (Notable de 45°)

$$\boldsymbol{AB = k\sqrt{2}} \Rightarrow 5\sqrt{2} = k\sqrt{2}$$

$$\Rightarrow k = 5$$

Pero: $AH = HB = k \Rightarrow \boldsymbol{AH = HB = 5}$

$\triangle BHC$ (Teorema de Pitágoras)

$$(BC)^2 = (5)^2 + (15)^2$$

$$(BC)^2 = 250 \Rightarrow \boldsymbol{BC = 5\sqrt{10}}$$

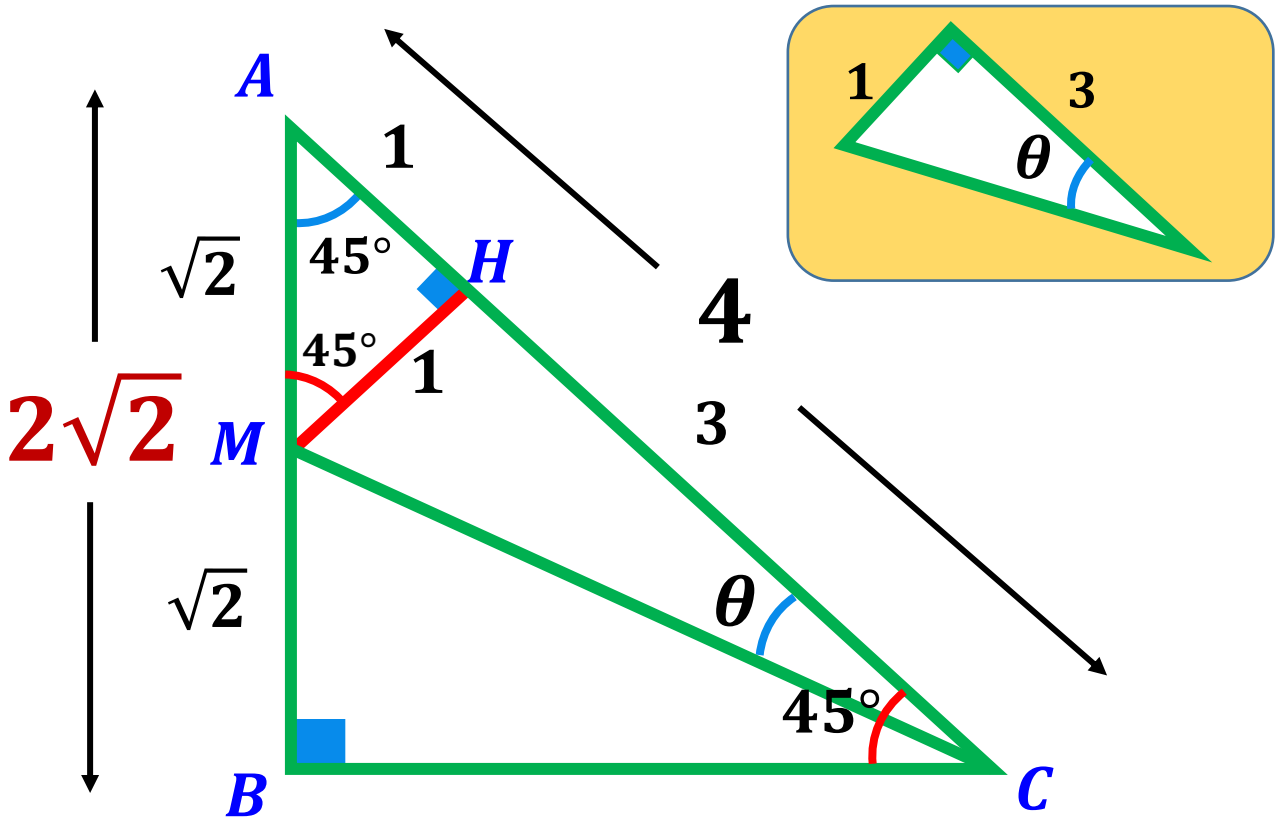
Calculamos: $\boldsymbol{E = \sqrt{10}\,sen\alpha + cot\alpha}$

$$E = \cancel{\sqrt{10}} \times \left(\frac{5}{5\cancel{\sqrt{10}}}\right) + \frac{15}{5}$$

$$E = 1 + 3$$

$$\therefore \boldsymbol{E = 4}$$

4. Del gráfico, calcule tan θ si AM=MB.



RESOLUCIÓN

Sea: $AM = MB = \sqrt{2}$

$\blacktriangle ABC$ (Notable de 45°)

$AB = a \; ; \; AC = a\sqrt{2}$

Pero: $AB = 2\sqrt{2} \Rightarrow a = 2\sqrt{2}$

Luego:

$AC = a\sqrt{2} = (2\sqrt{2})\sqrt{2} \Rightarrow AC = 4$

$\blacktriangle AHM$ (Notable de 45°)

$AM = k\sqrt{2} \Rightarrow \sqrt{2} = k\sqrt{2}$

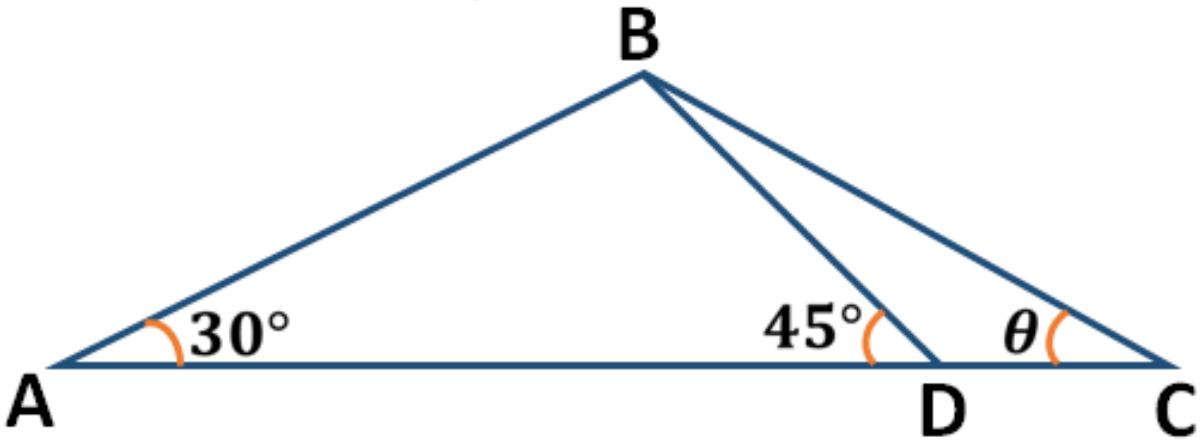
$\Rightarrow k = 1$

Luego:

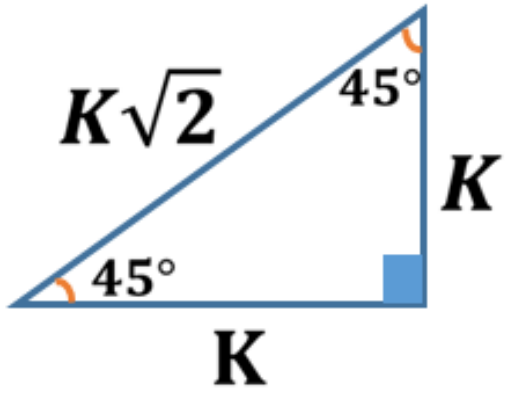
$AH = MH = k = 1$

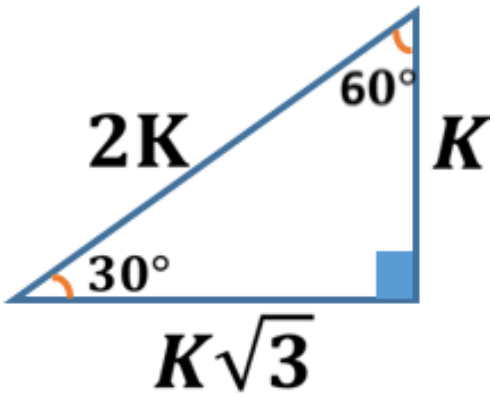
$$\therefore \tan\theta = \frac{1}{3}$$

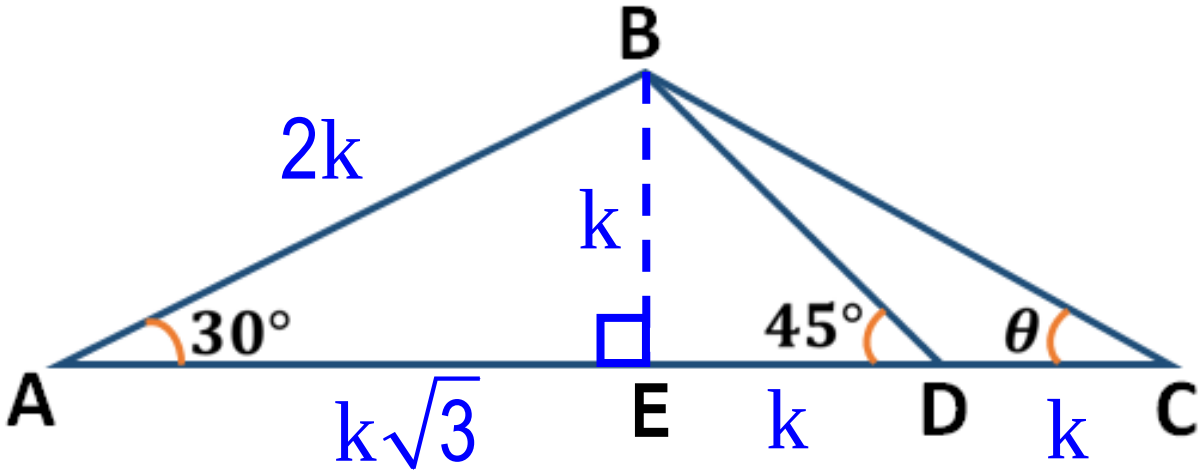
5. Del gráfico, calcule $cot\theta$, si AB = 2DC.



RESOLUCIÓN:







DATO: $DC = k$; $AB = 2k$

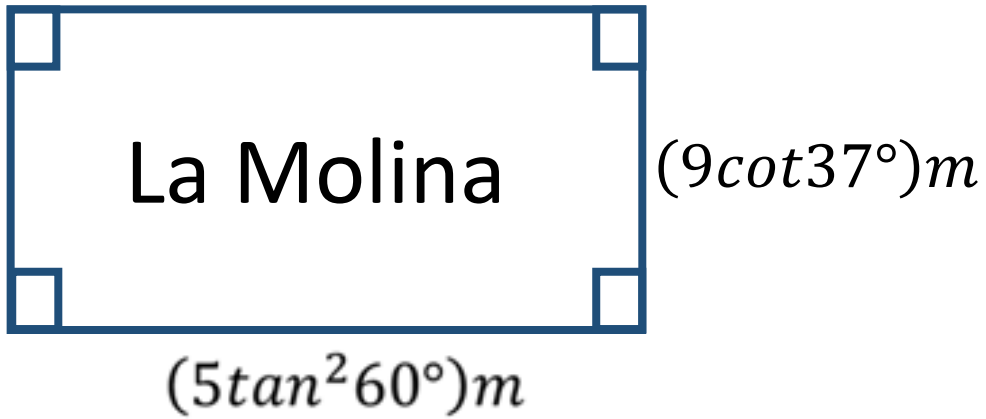
Completar en $\triangle$ 30° y $\triangle$ 45°

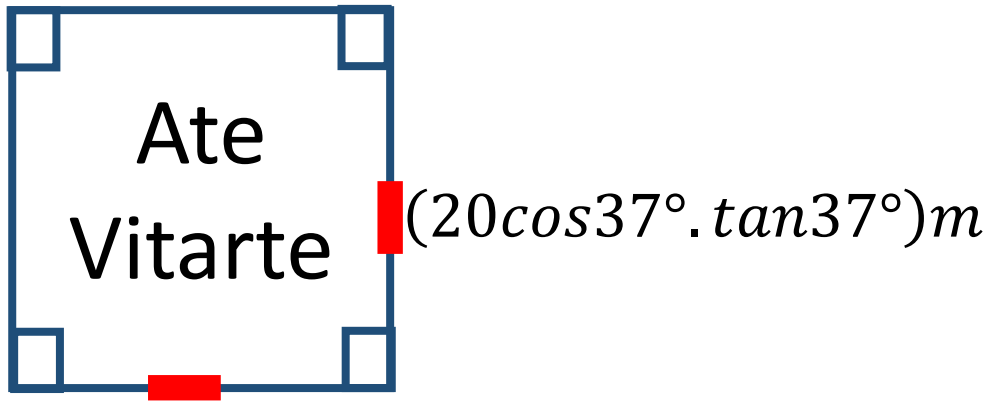
En $\triangle$ BEC:

$$\cot\theta = \frac{EC}{EB} = \frac{2k}{k}$$

$\therefore \cot\theta = 2$

6. Gigi una corredora de bienes y raíces ante el incremento del precio del dólar decide vender uno de los terrenos que tiene. Si el m^2 se valora en \$1000. Calcule el precio de venta del terreno de mayor área.





RESOLUCIÓN:

La Molina = $(5tan^260°)m \times (9cot37°)m$

$$= 5\left(\sqrt{3}\right)^2 \times 9\left(\frac{4}{3}\right) = 180m^2$$

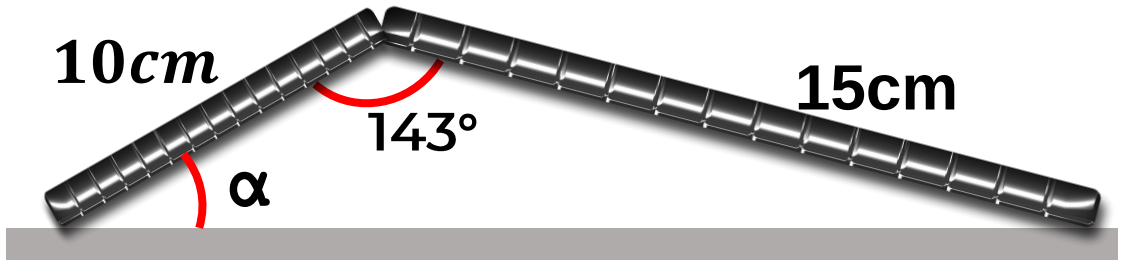
Ate Vitarte = $[(20cos37°.tan37°)m]^2$

$$= \left[20\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\right]^2 = 144m^2$$

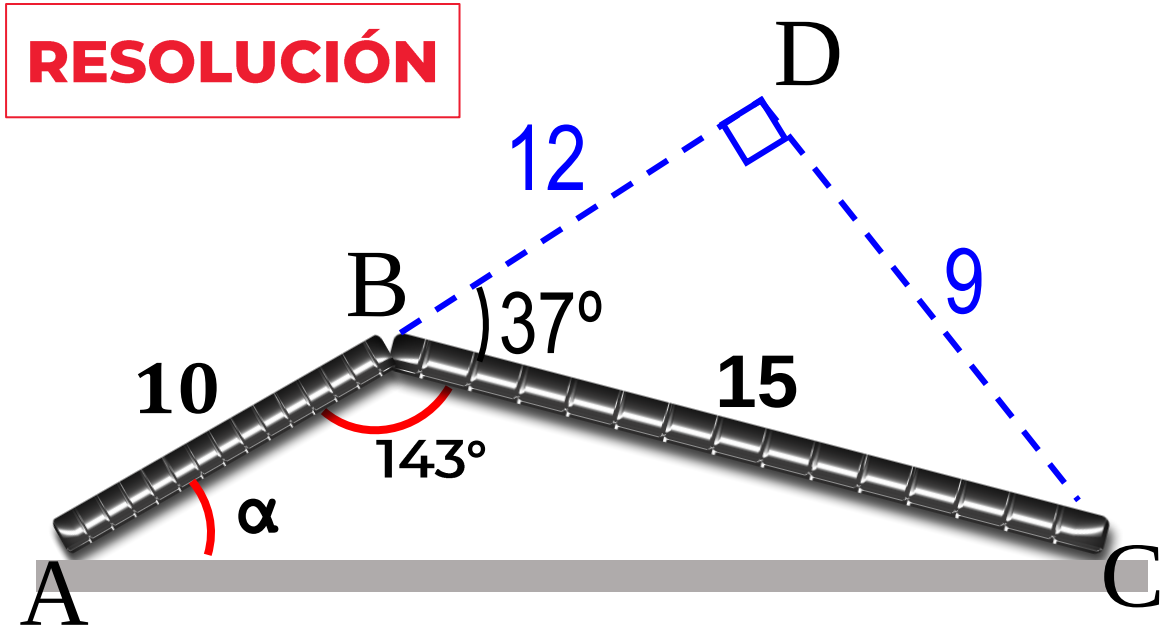
∴ El precio del terreno de La Molina es de \$180000

7. Dos barra metálicas se encuentran apoyadas en su parte superior, tal como se muestra en la figura. Si el ángulo que forman las barras en su punto de apoyo es de 143°, calcule

$$E = 11tan\alpha + \frac{1}{2} \quad \text{...(*)}$$



RESOLUCIÓN



En $\triangle$ 37° : $5K = 15 \Rightarrow K = 3$

Usando el $\triangle$ ADC en (*) :

$$E = 11x\frac{9}{22} + \frac{1}{2} = \frac{9}{2} + \frac{1}{2}$$

$$\therefore E = 5$$