ALGEBRA

3th

SECONDARY

Retroalimentación

Tomo 5

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Problema 1

Calcule el número de factores primos luego de factorizar

$$P(x) = 18x^4 + x^2 - 4$$

Resolución:

$$P(x) = 18x^4 + \underline{x^2} - 4$$

$$\begin{array}{ccc} 9x^2 & & -4 \\ 2x^2 & & +1 \end{array}$$

$$P(x) = \underline{(9x^2 - 4)}(2x^2 + 1)$$

$$\boxed{P(x) = (3x + 2)(3x - 2)(2x^2 + 1)}$$

$\therefore$ *$P(x)$ tiene 3 factores primos*

Problema 2

Indique un factor primo luego de factorizar

$$R(x) = x^3 + x^2 - 13x + 3$$

Resolución:

$$R(x) = x^3 + x^2 - 13x + 3$$

$a_0 = 1$ $a_n = 3$

$div(a_0) = \{1\}$

$div(a_n) = \{1; 3\}$

$PC = \pm\{1; 3\}$

	1	1	−13	3
$x = 3$		3	12	−3
×	1	4	−1	0

$$R(x) = (x - 3)(x^2 + 4x - 1)$$

Factores primos: $(x - 3)$ y $(x^2 + 4x - 1)$

Problema 3

Calcule el número de factores primos luego de factorizar

$$Q(x) = 3x^4 + 5x^3 - 12x^2 + 8x - 8$$

Resolución:

$$Q(x) = 3x^4 \underline{+ 5x^3} \overset{TC}{- 12x^2} \underline{+ 8x} - 8$$

$$\boxed{-2x^2}$$

$$\begin{array}{ccccc} 3x^2 & -x & +2 & \rightarrow & +2x^2 \\ x^2 & +2x & -4 & \rightarrow & -12x^2 \end{array}$$

$TC = -12x^2$

$ST = +2x^2 - 12x^2 = -10x^2$

$$\begin{array}{r} -12x^2 \\ -10x^2 \\ \hline \boxed{-2x^2} \end{array} \; -$$

$$\boxed{Q(x) = (3x^2 - x + 2)(x^2 + 2x - 4)}$$

$\therefore$ $Q(x)$ *tiene 2 factores primos.*

Problema 4

Calcule:

$$A=\frac{2\sqrt{27}+\sqrt{48}+\sqrt{12}}{\sqrt{75}-\sqrt{3}}$$

Resolución:

$$A=\frac{2\sqrt{27}+\sqrt{48}+\sqrt{12}}{\sqrt{75}-\sqrt{3}}$$

$$A=\frac{2\sqrt{9}\sqrt{3}+\sqrt{16}\sqrt{3}+\sqrt{4}\sqrt{3}}{\sqrt{25}\sqrt{3}-\sqrt{3}}$$

$$A=\frac{2.3\sqrt{3}+4\sqrt{3}+2\sqrt{3}}{5\sqrt{3}-\sqrt{3}}$$

$$A=\frac{6\sqrt{3}+4\sqrt{3}+2\sqrt{3}}{5\sqrt{3}-\sqrt{3}}$$

$$A=\frac{12\cancel{\sqrt{3}}}{4\cancel{\sqrt{3}}}$$

$$\boxed{\therefore\quad A=3}$$

Problema 5

Reduzca

$$Q=\sqrt{6+2\sqrt{8}}+\sqrt{5-2\sqrt{6}}+\sqrt{12-2\sqrt{27}}$$

Recordemos:

$$\sqrt{A\pm\sqrt{B}}=\sqrt{(x+y)\pm2\sqrt{x.y}}=\sqrt{x}\pm\sqrt{y}$$

Resolución:

$$Q=\sqrt{\underset{4+2}{6}+2\sqrt{\underset{4\times2}{8}}}+\sqrt{\underset{3+2}{5}-2\sqrt{\underset{3\times2}{6}}}+\sqrt{\underset{9+3}{12}-2\sqrt{\underset{9\times3}{27}}}$$

$$Q=\sqrt{4}+\cancel{\sqrt{2}}+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}+\sqrt{9}-\cancel{\sqrt{3}}$$

$$Q=2+3$$

$$\boxed{\therefore\ Q=5}$$

Problema 6

Reduzca

$$R=\frac{6}{\sqrt{8}+\sqrt{5}}+\frac{8}{3-\sqrt{5}}-2\sqrt{8}$$

Resolución:

$$R=\frac{6}{\sqrt{8}+\sqrt{5}}+\frac{8}{3-\sqrt{5}}-2\sqrt{8}$$

$$R=\frac{6}{(\sqrt{8}+\sqrt{5})}\times\frac{(\sqrt{8}-\sqrt{5})}{(\sqrt{8}-\sqrt{5})}+\frac{8}{(3-\sqrt{5})}\times\frac{(3+\sqrt{5})}{(3+\sqrt{5})}-2\sqrt{8}$$

$$R=\frac{6(\sqrt{8}-\sqrt{5})}{8-5}+\frac{8(3+\sqrt{5})}{9-5}-2\sqrt{8}$$

$$R=\frac{\overset{2}{\cancel{6}}(\sqrt{8}-\sqrt{5})}{\underset{1}{\cancel{3}}}+\frac{\overset{2}{\cancel{8}}(3+\sqrt{5})}{\underset{1}{\cancel{4}}}-2\sqrt{8}$$

$$R=\cancel{2\sqrt{8}}-\cancel{2\sqrt{5}}+6+\cancel{2\sqrt{5}}-\cancel{2\sqrt{8}}$$

$$\boxed{\therefore\ R=6}$$

Problema 7

Reduzca

$$M = \frac{5i^{2325} - 7i^{7455}}{2i^{9412} - i^{5474}}; \quad (i = \sqrt{-1})$$

Recordemos:

POTENCIAS DE i:

$$i^{4k} = 1$$

$$i^{4k+1} = i$$

$$i^{4k+2} = -1$$

$$i^{4k+3} = -i$$

Resolución:

$$M = \frac{5i^{2325} - 7i^{7455}}{2i^{9412} - i^{5474}}$$

- $i^{2325} = i^{2324+1} = i^{4k+1} = i$
- $i^{7455} = i^{7452+3} = i^{4k+3} = -i$
- $i^{9412} = i^{9412} = i^{4k} = 1$
- $i^{5474} = i^{5472+2} = i^{4k+2} = -1$

$$M = \frac{5(i) - 7(-i)}{2(1) - (-1)} = \frac{5i + 7i}{2 + 1} = \frac{12i}{3}$$

$$\boxed{\therefore \quad M = 4i}$$

Problema 8

Sea $z_1 = 3 - 2i$
$z_2 = 3 - 8i$
$z_3 = 5 - 6i$

Si $z = z_1 + z_2^* + \bar{z}_3$

calcule $|Z|$

Recordemos:

Sea: $z = a + bi$

Conjugado de z:

$$\boxed{\bar{z} = a - bi}$$

Opuesto de z:

$$\boxed{z^* = -a - bi}$$

Módulo de z:

$$\boxed{|z| = \sqrt{a^2 + b^2}}$$

Resolución:

$$z = z_1 + z_2^* + \bar{z}_3$$

$$z = (3 - 2i) + (-3 + 8i) + (5 + 6i)$$

$$z = \cancel{3} - 2i - \cancel{3} + 8i + 5 + 6i$$

$$\boxed{z = 5 + 12i}$$

Nos piden: $|z|$

$$|z| = \sqrt{5^2 + 12^2}$$

$$|z| = \sqrt{169}$$

$$\boxed{\therefore \quad |z| = 13}$$

Problema 9

Luego de efectuar

$$z = \frac{2(3-i)}{2+i} + 5 - 3i$$

calcule $Re(z)$

Resolución:

$$z = \frac{2(3-i)}{2+i} + 5 - 3i$$

$$z = \frac{2(3-i)}{(2+i)} \cdot \frac{(2-i)}{(2-i)} + 5 - 3i$$

$$z = \frac{2(6 - 3i - 2i + i^2)}{4 - i^2} + 5 - 3i \qquad i^2 \to -1$$

$$z = \frac{2(6 - 3i - 2i - 1)}{4 + 1} + 5 - 3i$$

$$z = \frac{2(5-5i)}{5} + 5 - 3i$$

$$z = \frac{2.\cancel{5}(1-i)}{\cancel{5}} + 5 - 3i$$

$$z = 2 - 2i + 5 - 3i$$

$$\boxed{z = 7 - 5i}$$

$$\boxed{\therefore \quad Re(z) = 7}$$

Problema 10

Si $z_1 = 3 + 4i$
$z_2 = 5 - 2i$

al efectuar

$$M = z_1^{*}.\bar{z}_2 + 10 + 26i$$

el valor de M en soles representa el precio de 1 Kg. de azúcar. Si José compró un saco de 25 Kg, ¿cuál es el precio que pagó?

Recordemos:

Sea: $z = a + bi$

Conjugado de z:

$$\boxed{\bar{z} = a - bi}$$

Opuesto de z:

$$\boxed{z^{*} = -a - bi}$$

Resolución:

$$M = z_1^{*}.\bar{z}_2 + 10 + 26i$$

$$M = (-3 - 4i)(5 + 2i) + 10 + 26i$$

$$M = -15 - 6i - 20i - 8i^2 + 10 + 26i$$

$i^2 \to -1$

$$M = -15 - \cancel{6i} - \cancel{20i} + 8 + 10 + \cancel{26i}$$

$$\boxed{M = 3}$$ (Precio de 1 Kg de azúcar en soles)

Precio de 1 saco de 25 Kg: $25 \times 3 = 75$

$\therefore$ José pagó $S/.75$