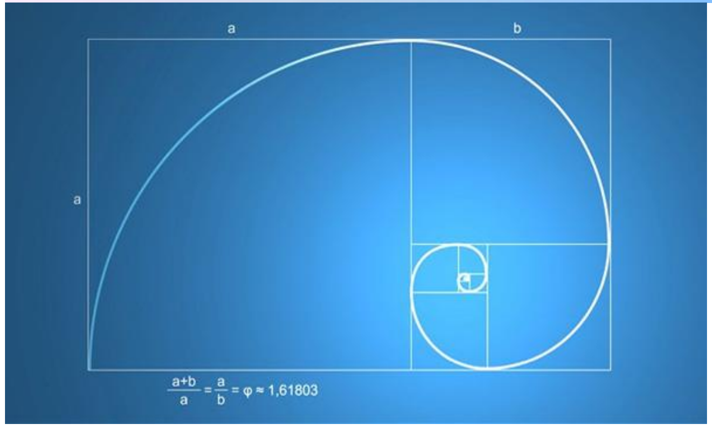
GEOMETRÍA

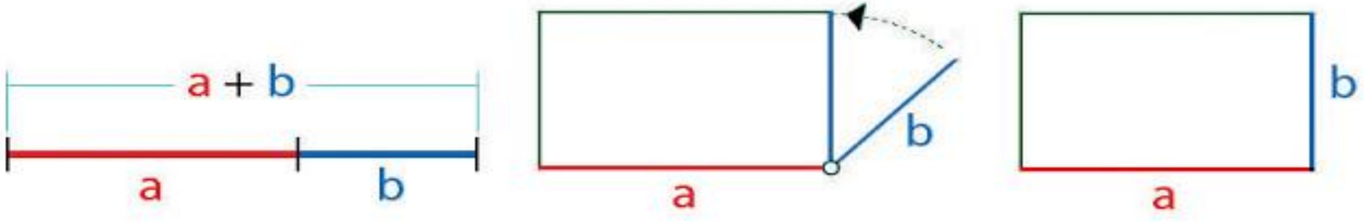
Capítulo 7

5th
SECONDARY

SEGMENTOS PROPORCIONALES

SACO OLIVEROS





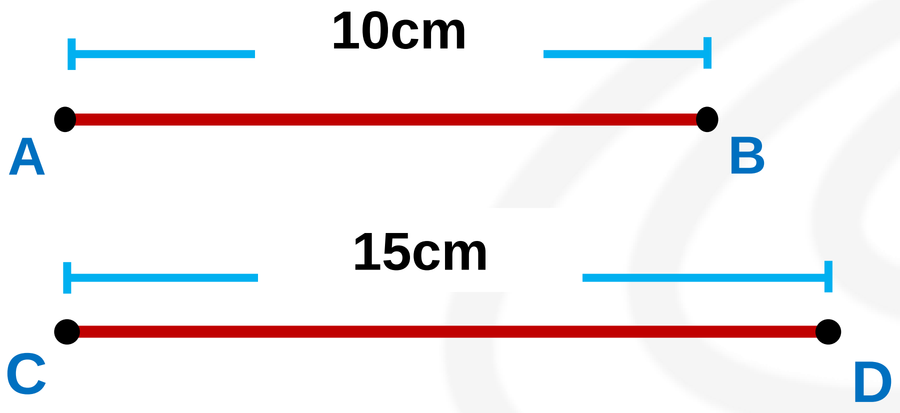
$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\ldots$$

SEGMENTOS PROPORCIONALES

Razón geométrica de dos segmentos

Es el cociente que se obtiene al dividir las longitudes de dos segmentos que tienen la misma unidad de medida.

Ejemplo:

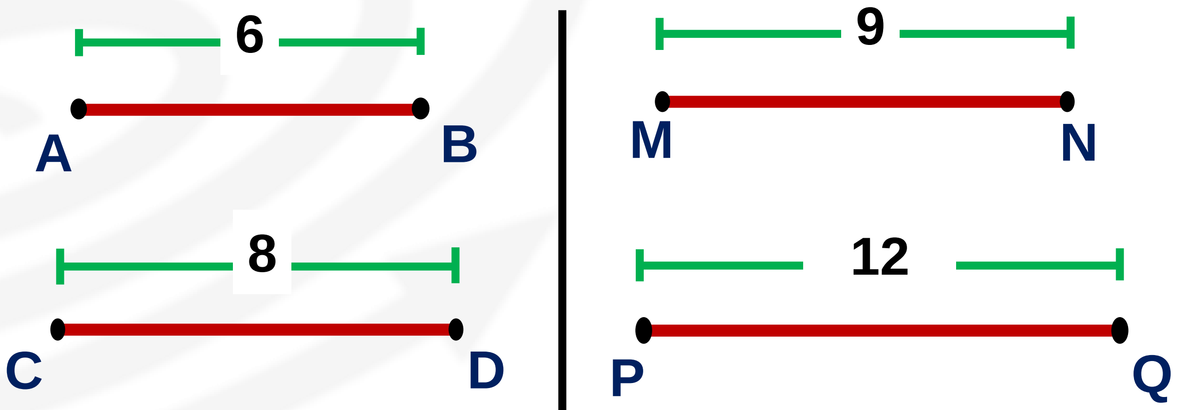


$$\frac{\overline{AB}}{\overline{CD}} = \frac{\cancel{1}0cm}{\cancel{1}5cm} \Rightarrow \frac{\overline{AB}}{\overline{CD}} = \frac{2}{3}$$

$\frac{2}{3}$: razón geométrica de $\overline{AB}$ y $\overline{CD}$

Segmentos proporcionales

Si la razón geométrica de 2 segmentos es igual a la de otros dos, dichos pares de segmentos son proporcionales.

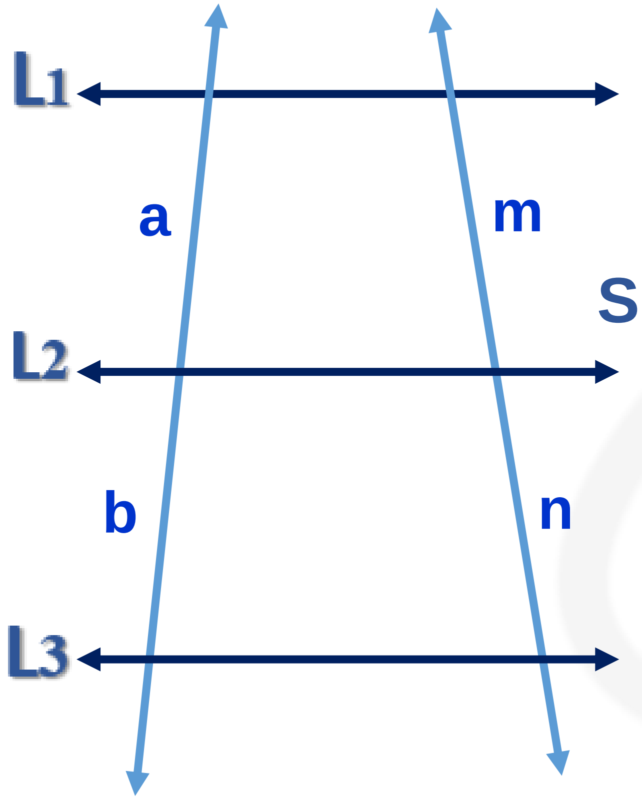


$$\frac{\overline{AB}}{\overline{CD}} = \frac{\cancel{6}}{\cancel{8}} = \boxed{\frac{3}{4}}$$

$$\frac{\overline{MN}}{\overline{PQ}} = \frac{\cancel{9}}{\cancel{12}} = \boxed{\frac{3}{4}}$$

$$\boxed{\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{MN}}{\overline{PQ}}}$$ ➡ Son proporcionales

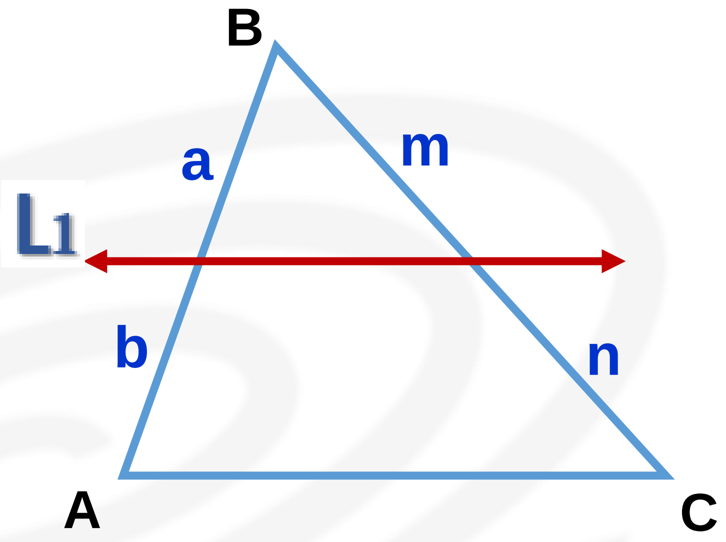
Teorema de Tales



Si: $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2} \parallel \overleftrightarrow{L_3}$

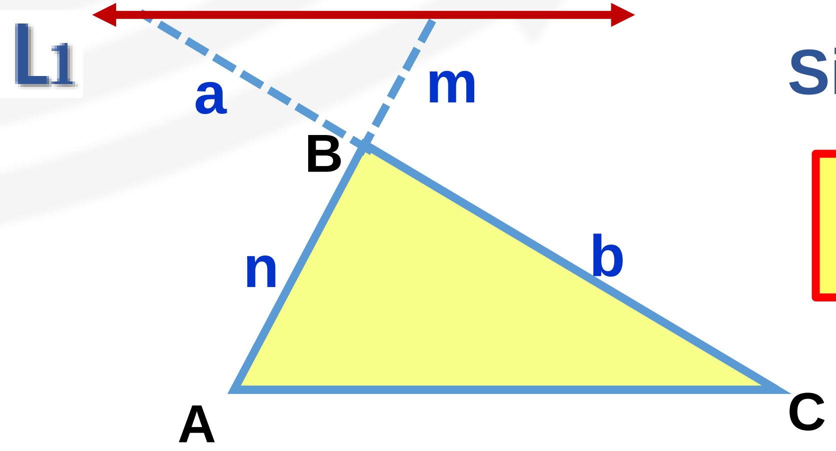
$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

Corolario de Tales



Si: $\overleftrightarrow{L_1} \parallel \overline{AC}$

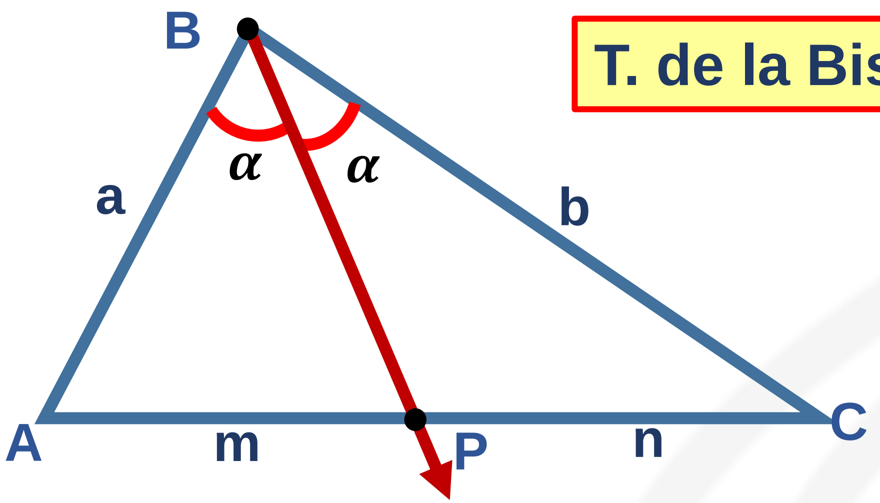
$$\boxed{\frac{a}{b} = \frac{m}{n}}$$



Si: $\overleftrightarrow{L_1} \parallel \overline{AC}$

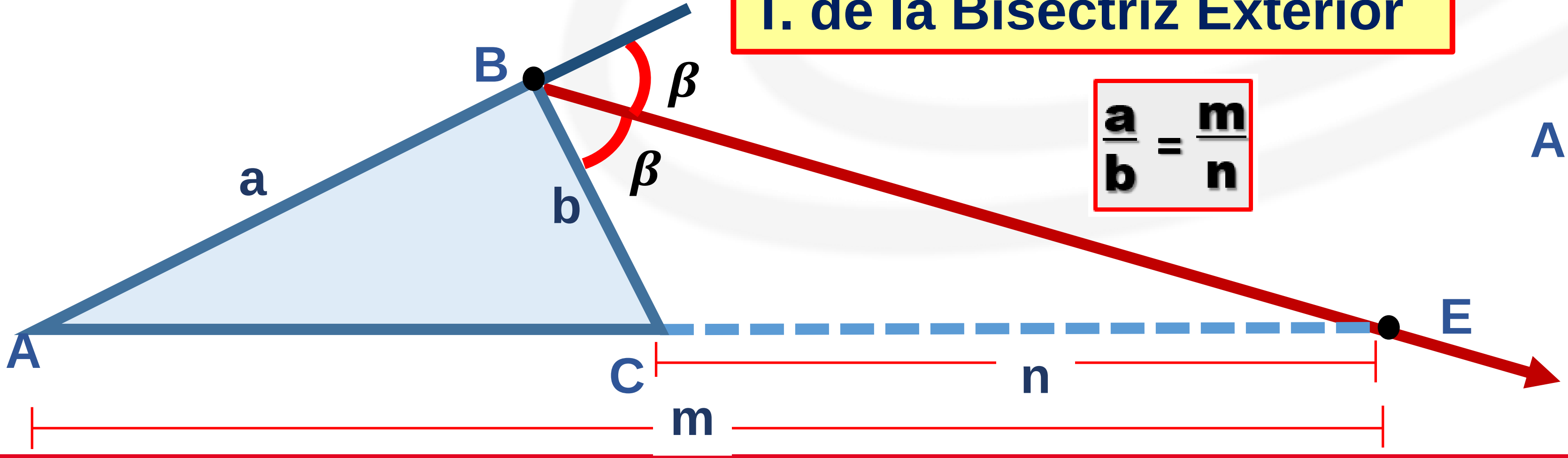
$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

Teorema de la Bisectriz



T. de la Bisectriz Interior

$$\frac{a}{b} = \frac{m}{n}$$

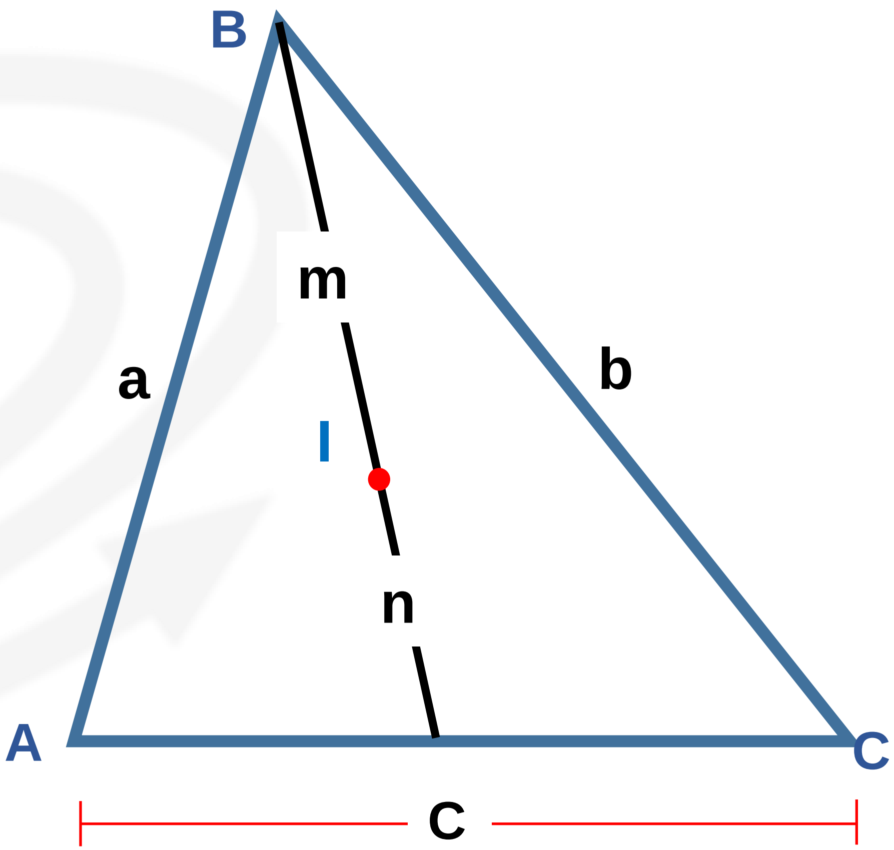


T. de la Bisectriz Exterior

$$\frac{a}{b} = \frac{m}{n}$$

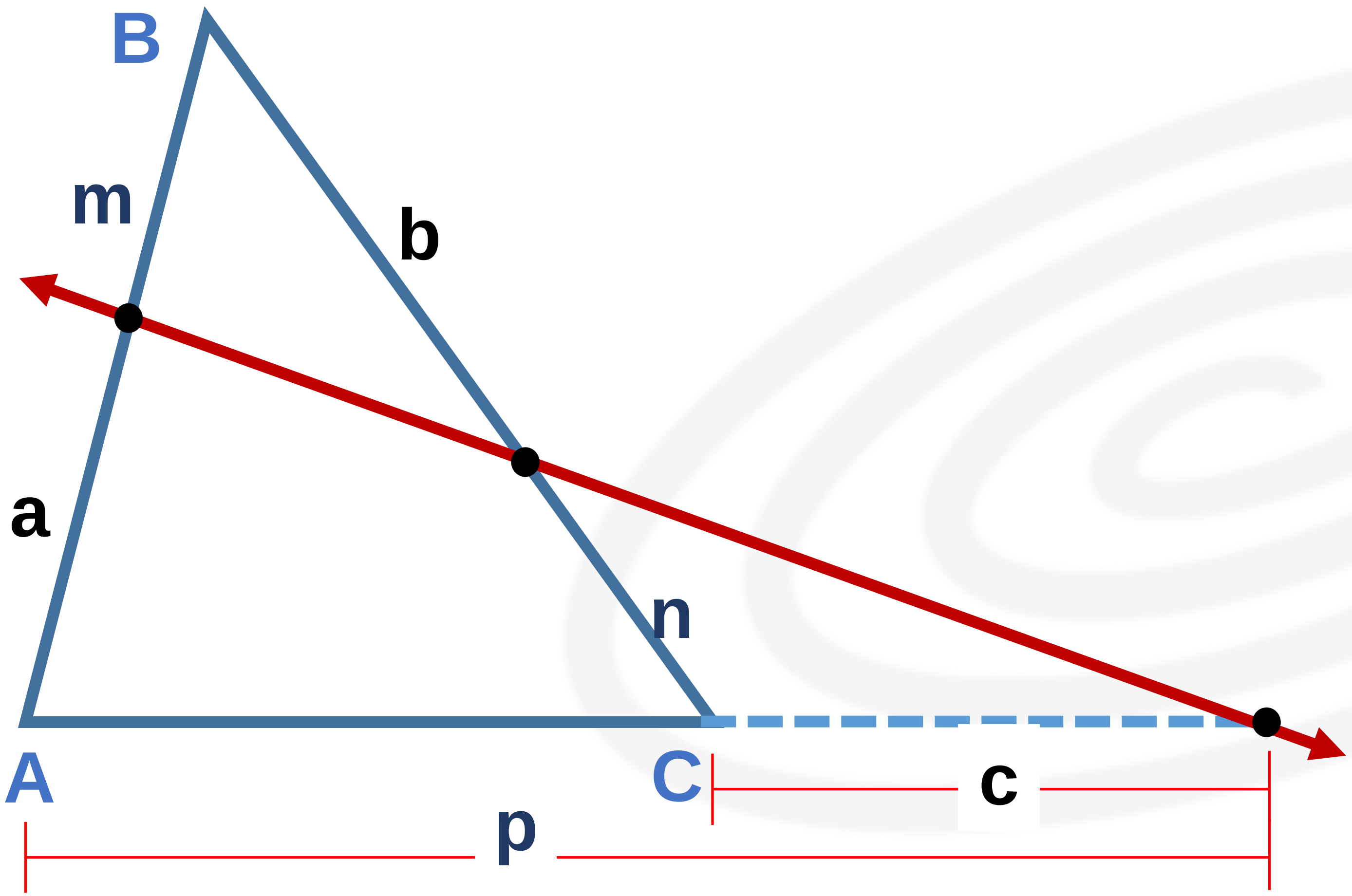
Teorema del Incentro

I: Incentro del Δ ABC



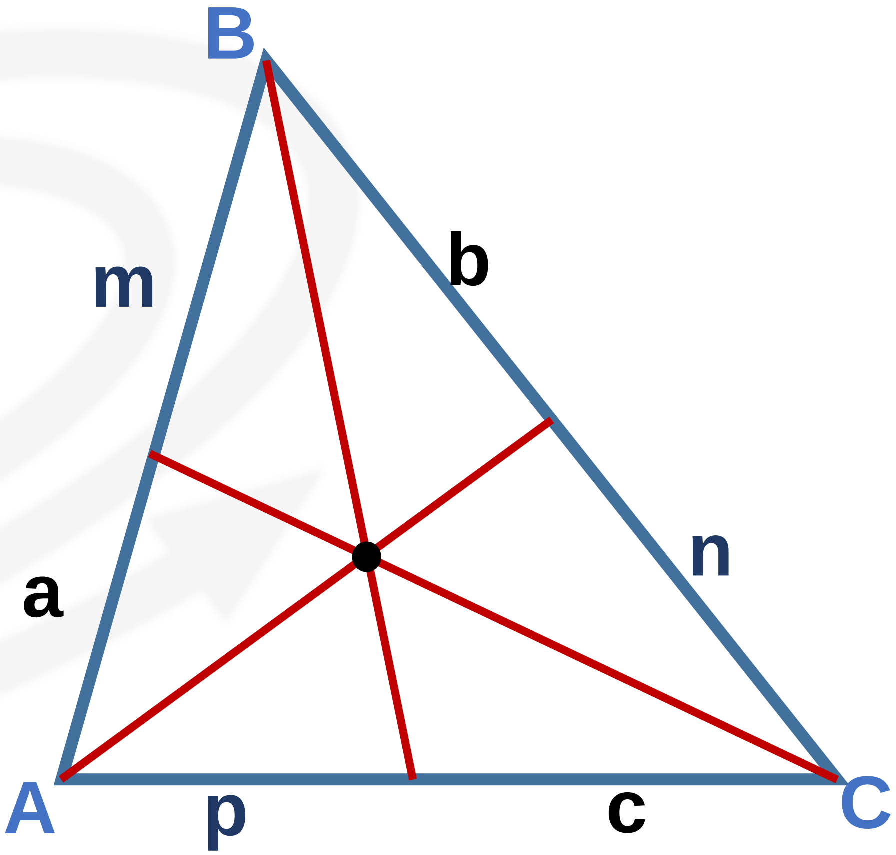
$$\frac{m}{n} = \frac{a+b}{c}$$

Teorema de Menelao



(a)(b)(c) = (m)(n)(p)

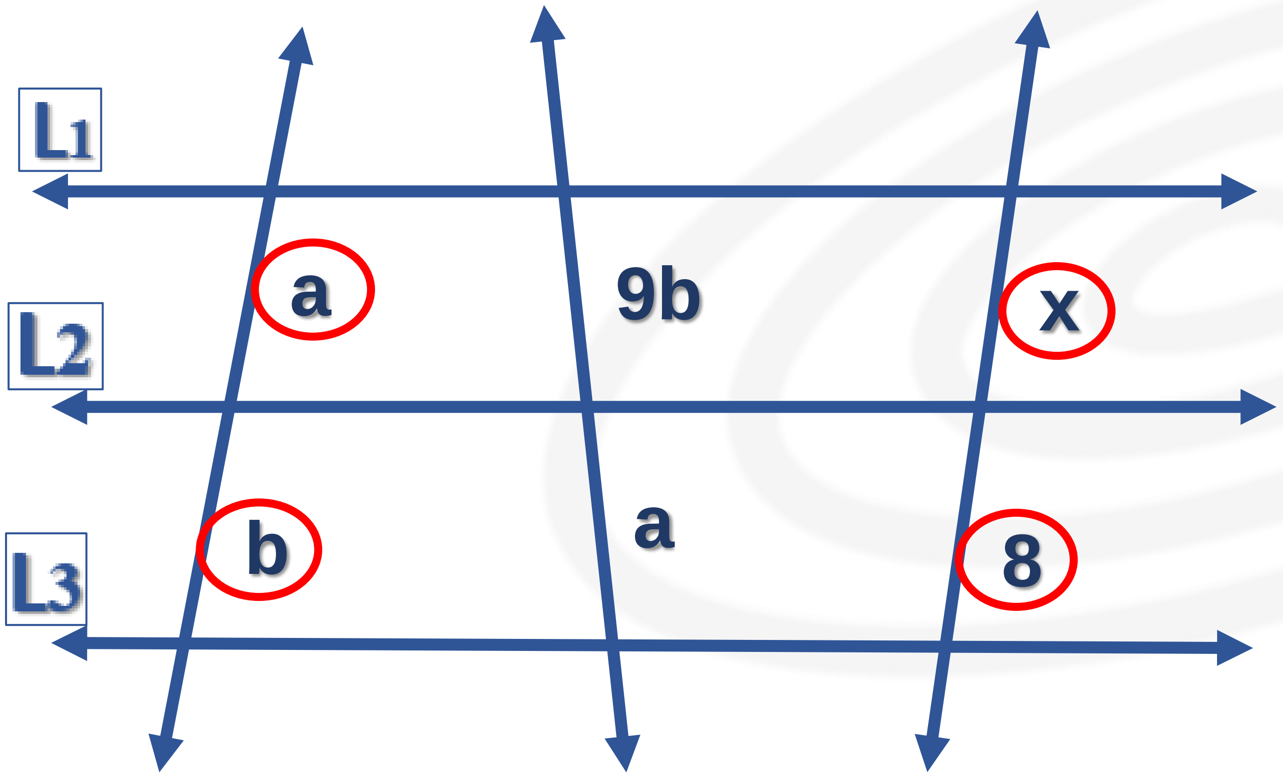
Teorema de Ceva



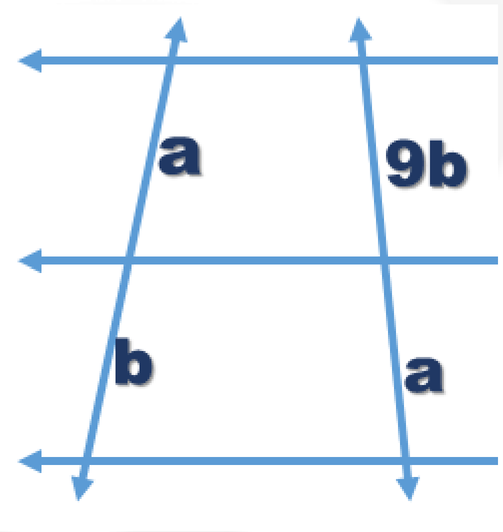
(a)(b)(c) = (m)(n)(p)

1. Halle el valor de x, si $\overleftrightarrow{L1} \parallel \overleftrightarrow{L2} \parallel \overleftrightarrow{L3}$.

Resolución



TEOREMA DE TALES



$$\frac{a}{b} = \frac{9b}{a}$$

$$a^2 = 9b^2$$

$$\boxed{a = 3b}$$

$$\frac{a}{b} = \frac{x}{8}$$

$$\frac{3\cancel{b}}{\cancel{b}} = \frac{x}{8}$$

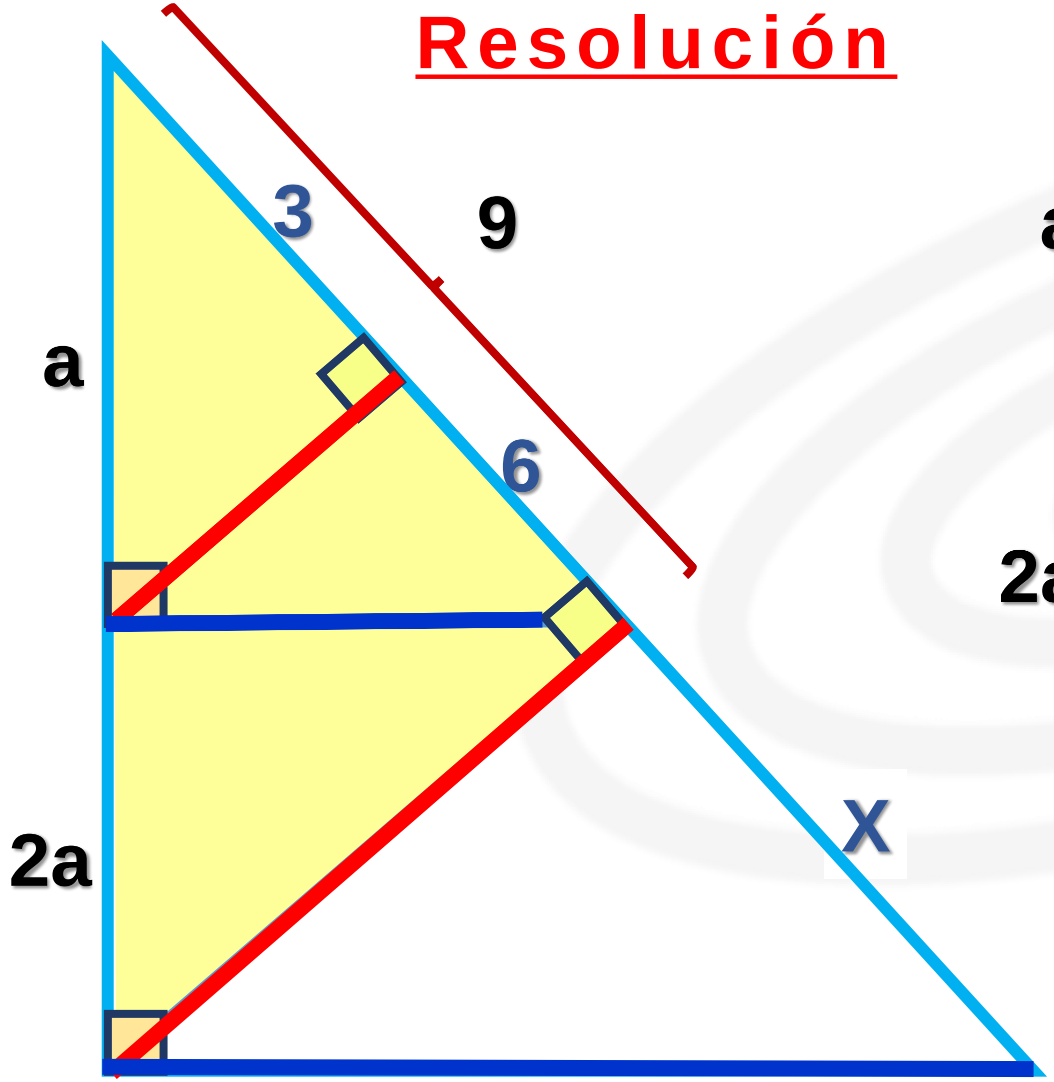
$$3(8) = x$$

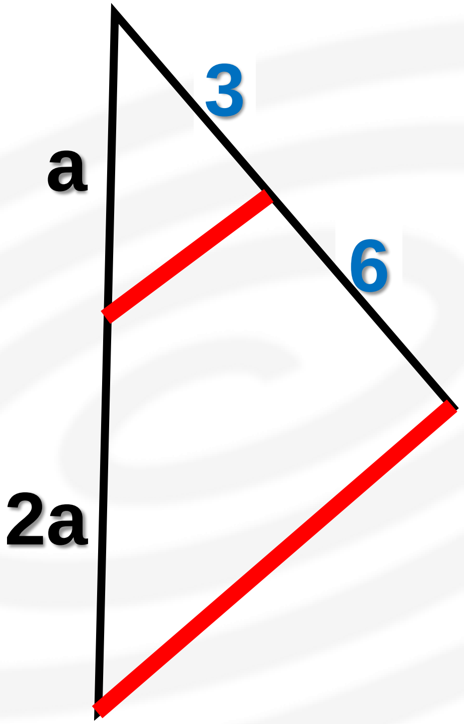
$$\boxed{x = 24}$$

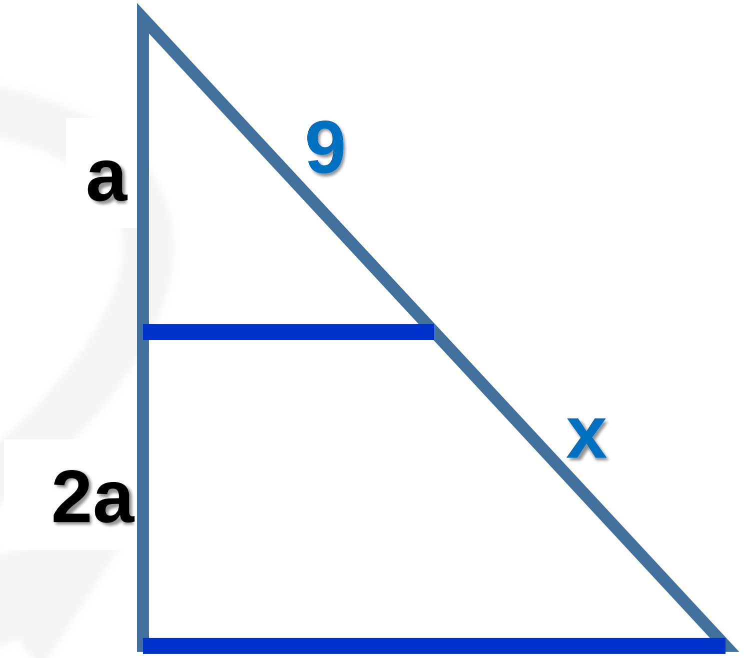
2. Halle el valor de x.

COROLARIO DE TALES

Resolución





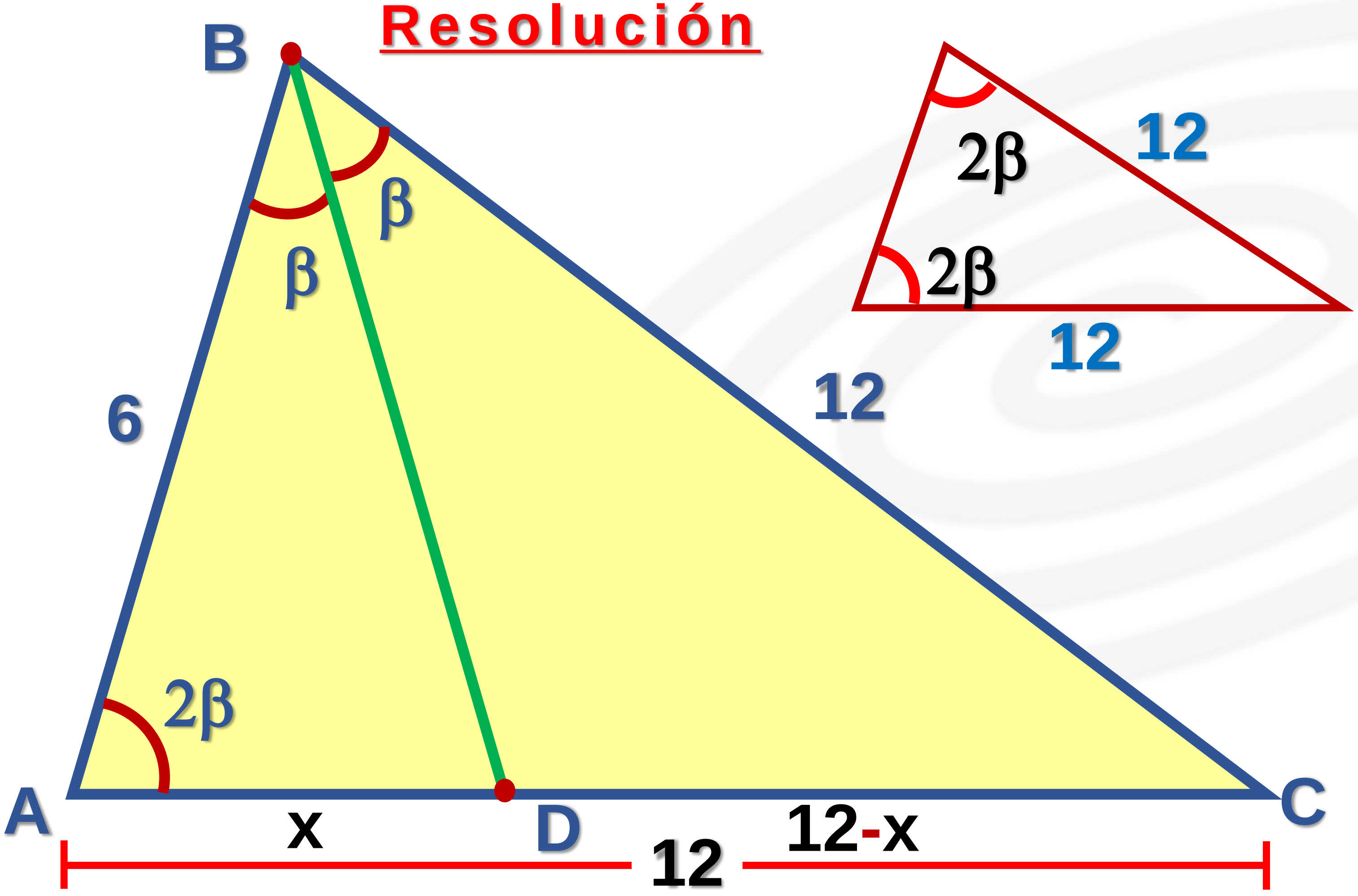


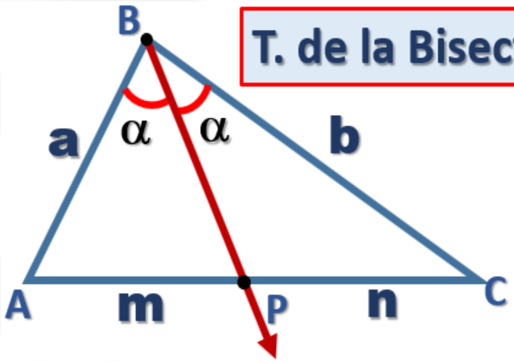
$$\frac{\cancel{a}}{2\cancel{a}} = \frac{9}{x}$$

$$\boxed{x = 18}$$

3. En un triángulo ABC, donde AB = 6 y BC = 12, se traza la bisectriz interior BD. Halle AD, si m∡BAD = m ∡ ABC.

Resolución





T. de la Bisectriz Interior

$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

$$\frac{\cancel{6}^{\,1}}{\cancel{12}_{\,2}} = \frac{x}{12 - x}$$

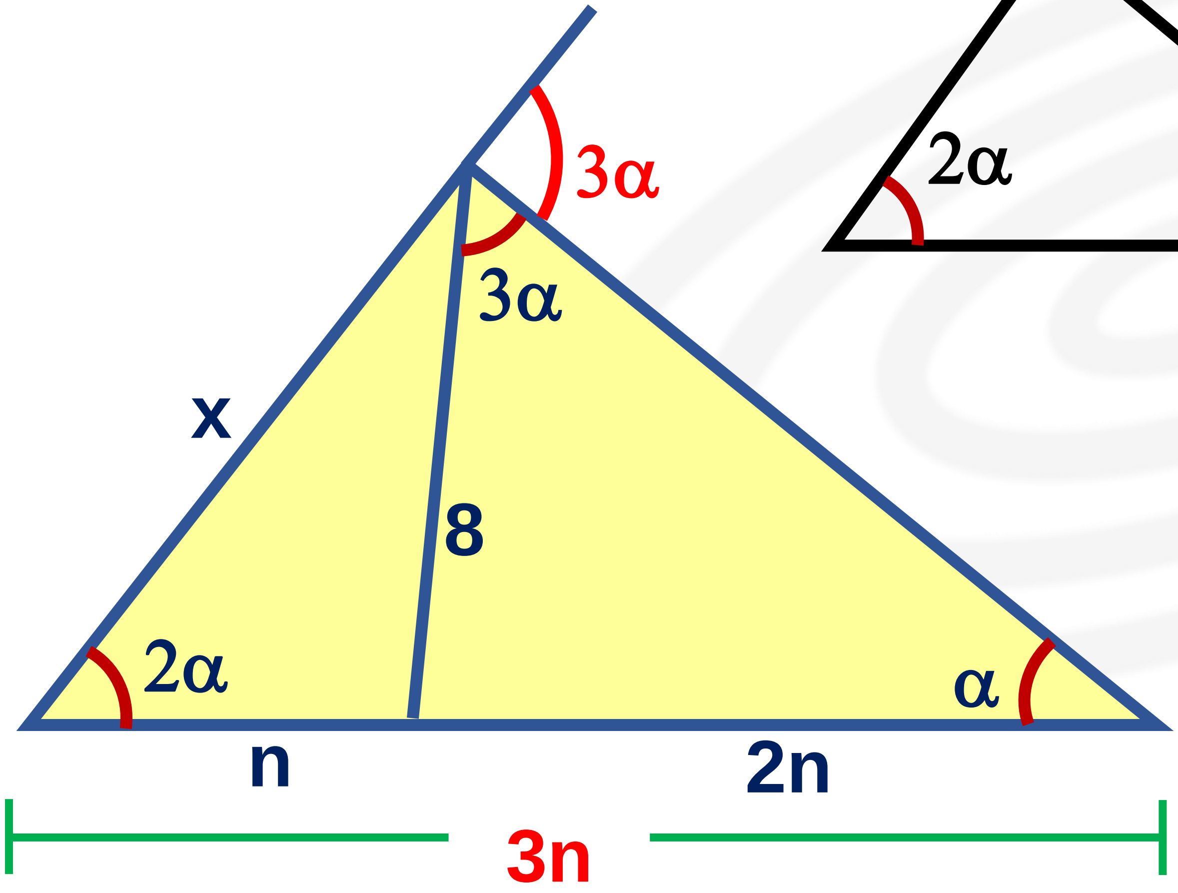
$$12 - x = 2x$$

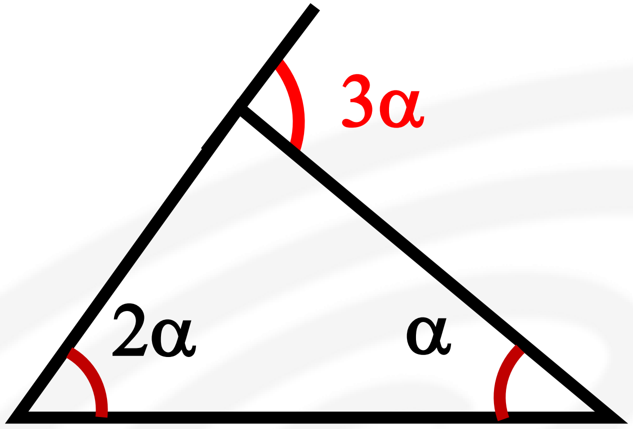
$$12 = 3x$$

$$\boxed{x = 4}$$

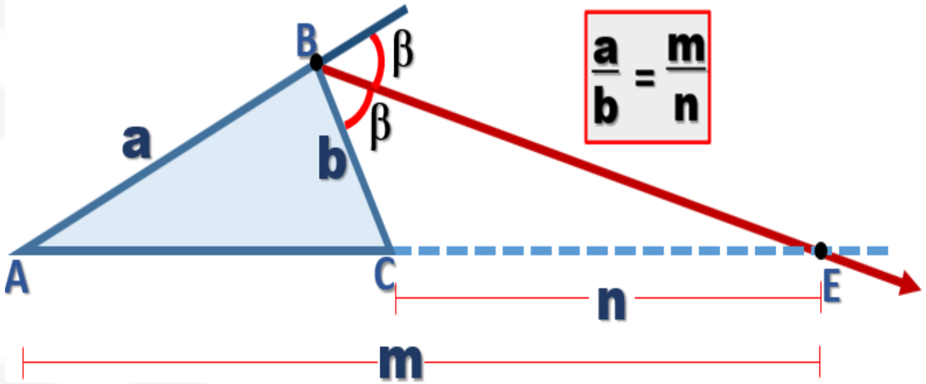
4. Halle el valor de X.

Resolución





T. de la Bisectriz Exterior



$\frac{a}{b} = \frac{m}{n}$

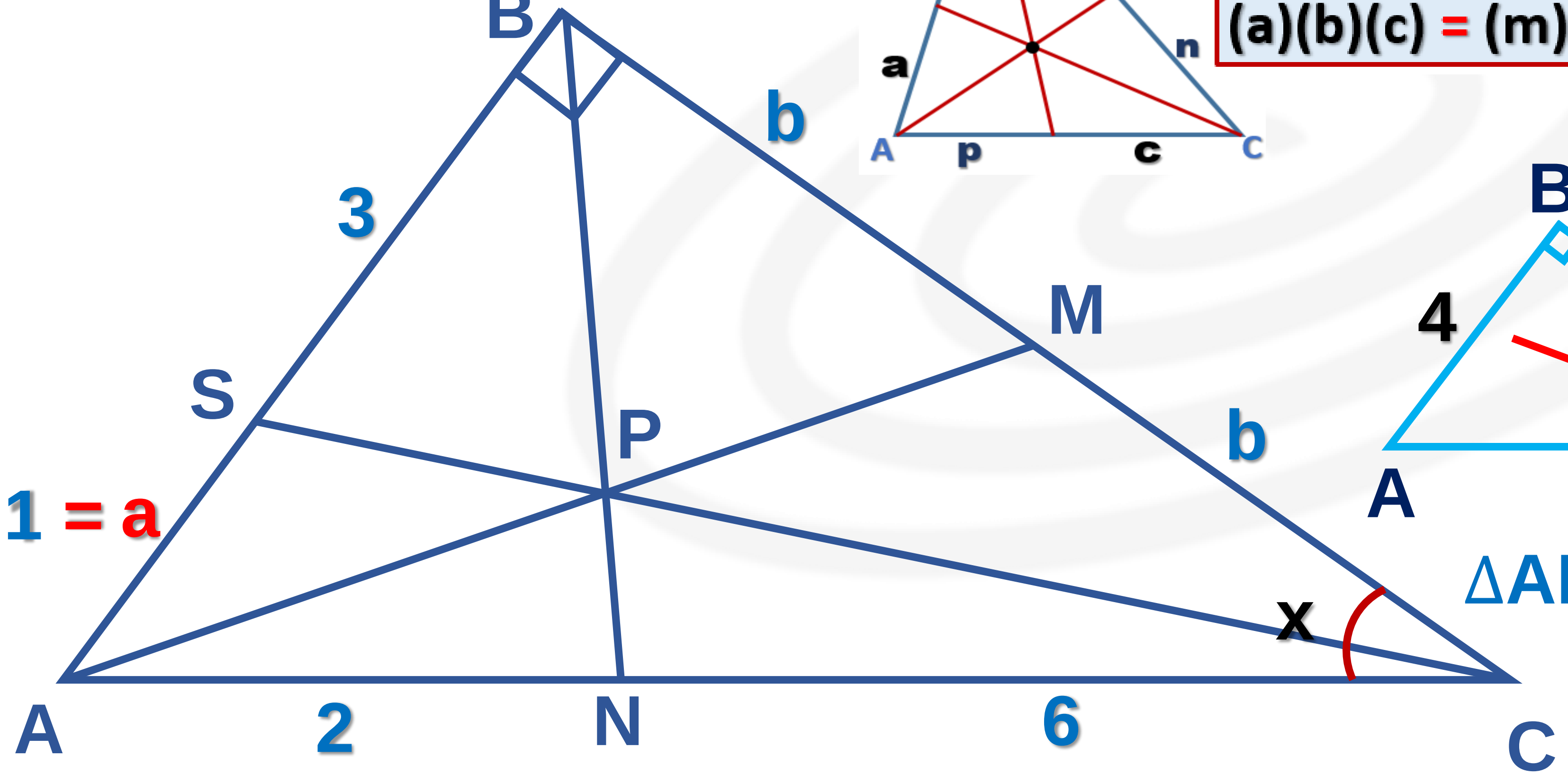
$$\frac{x}{8} = \frac{3\cancel{n}}{2\cancel{n}}$$

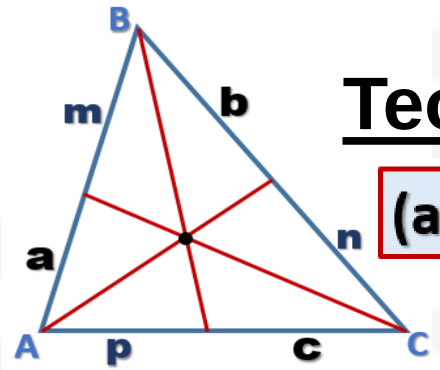
$$2x = 24$$

$$\boxed{x = 12}$$

5. En un triángulo rectángulo ABC, recto en B, la mediana $\overline{AM}$ y las cevianas interiores $\overline{BN}$ y $\overline{CS}$ se intersecan en P. Si SB = 3, AN = 2 y NC = 6, halle m∡BCA.

Resolución



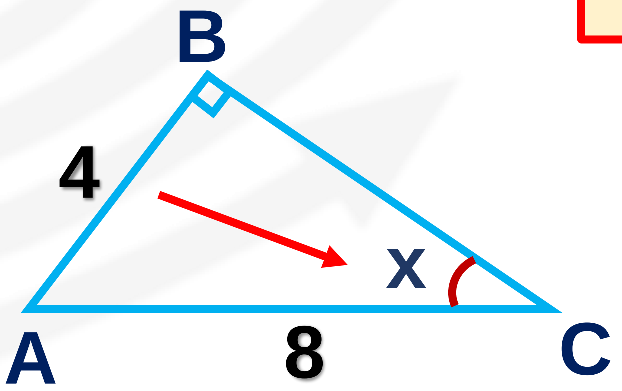


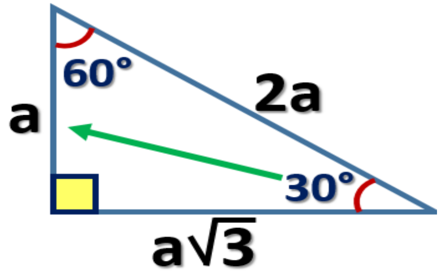
Teorema de Ceva

$\boxed{(a)(b)(c) = (m)(n)(p)}$

$(a)(\cancel{b})(6) = (3)(\cancel{b})(2)$

$\boxed{a = 1}$

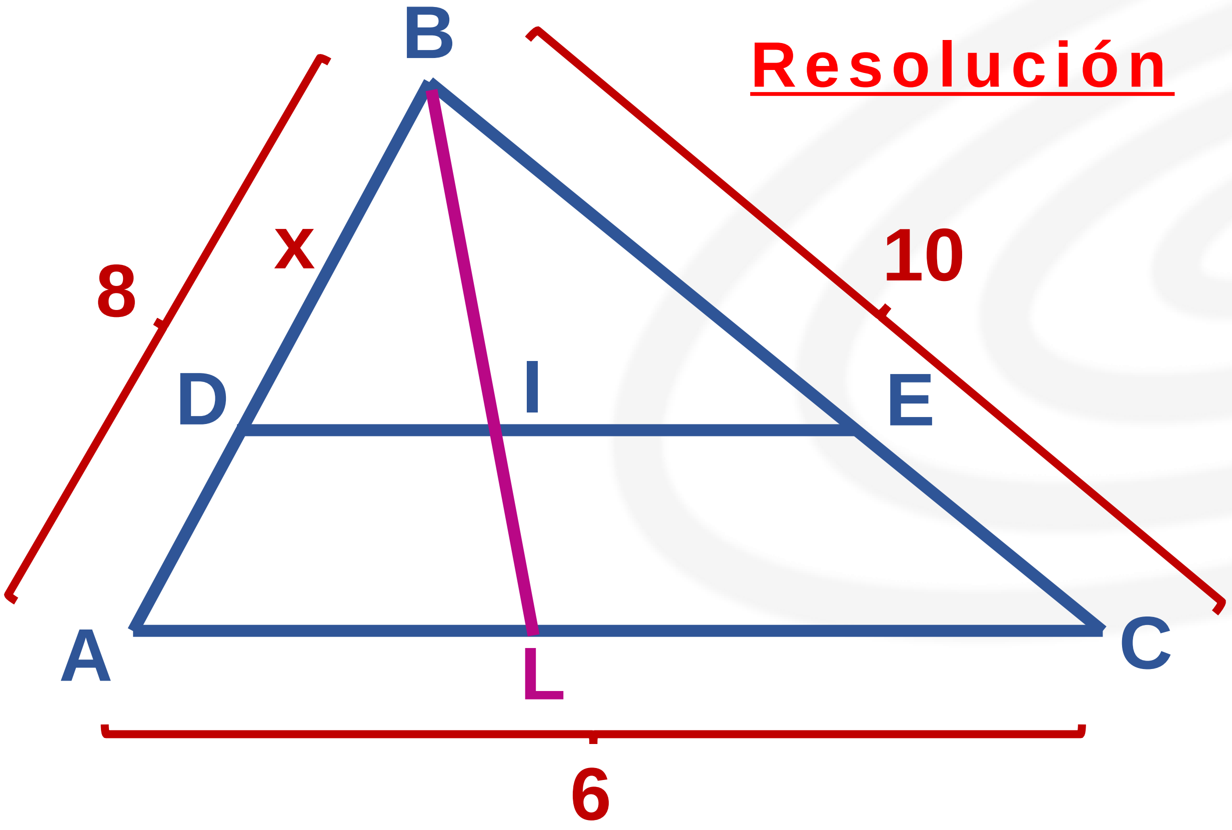




ΔABC: Notable de 30° y 60°

$\boxed{x = 30°}$

6. En la figura el triángulo ABC representa el contorno de un jardín donde I es su incentro. AB=8 m, BC=10 m y AC=6 m. Luego se traza el $\overline{DE}$ // $\overline{AC}$ para dividir al jardín en dos partes para cultivar flores de diferente color. Halle la longitud del $\overline{BD}$.



Resolución

- Piden: BD = x
- Se traza $\overline{BIL}$
- Δ ABC: teorema del incentro

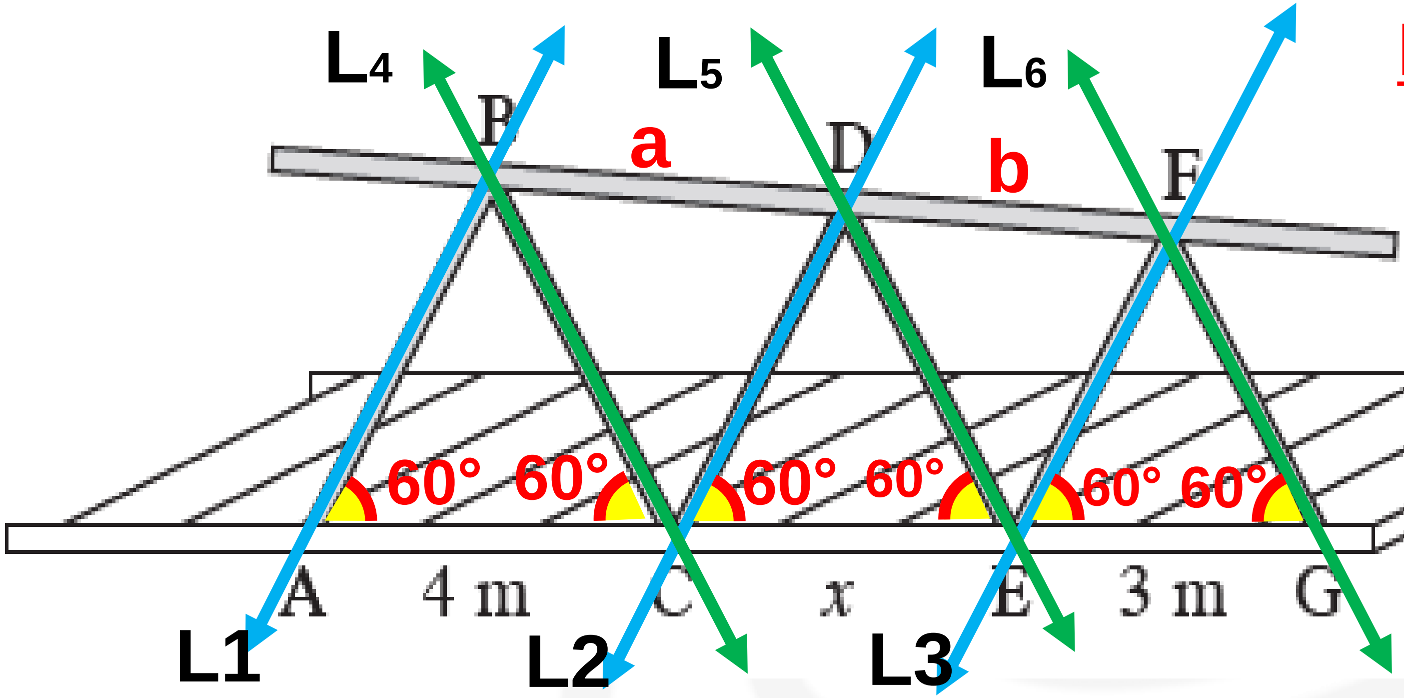
$$\frac{BI}{IL} = \frac{8+10}{6}$$

- Δ ABL: corolario de tales

$$\frac{BI}{IL} = \frac{x}{8-x}$$

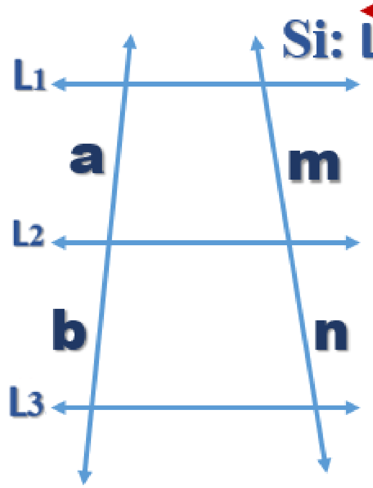
$$\boxed{x = 6}$$

7. Los triángulos ABC, CDE y EFG son equiláteros. Halle el valor de x.



Resolución

Teorema de Tales



$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

$$\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2} \,//\, \overleftrightarrow{L_3}$$

$$\overleftrightarrow{L_4} \,//\, \overleftrightarrow{L_5} \,//\, \overleftrightarrow{L_6}$$

$$\frac{a}{b} = \frac{4}{x} \quad \cdots (1)$$

$$\frac{a}{b} = \frac{x}{3} \quad \cdots (2)$$

Igualando 1 y 2

$$\frac{4}{x} = \frac{x}{3}$$

$$12 = x^2$$

$$\boxed{x = 2\sqrt{3}}$$