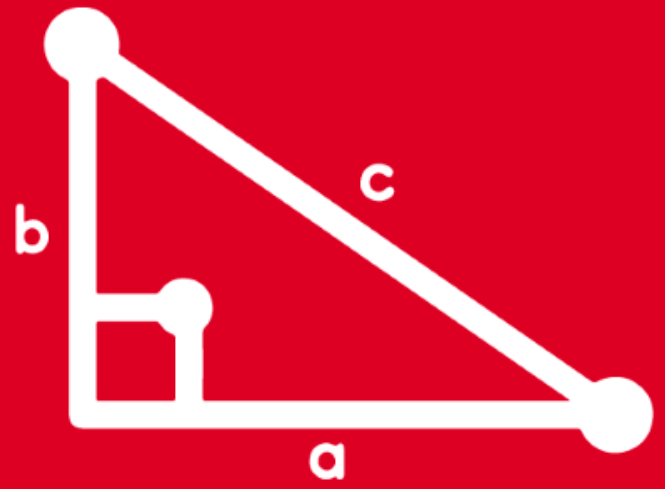
TRIGONOMETRY

VOLUME IV

4th
SECONDARY



FEEDBACK

SACO OLIVEROS

1. Indique el cuadrante al que pertenece el ángulo β, si se cumple que:

$$\sec 323° \cdot \operatorname{sen}\beta > 0 \quad \text{y} \quad \cot 162° \cdot \cos\beta > 0$$

RESOLUCIÓN

IVC

➢ $\underbrace{sec323°}_{(+)} . \underbrace{sen\beta}_{(+)} \underbrace{> 0}_{(+)} \Rightarrow sen\beta : (+)$

$\beta \in IC$

$\beta \in IIC$

IIC

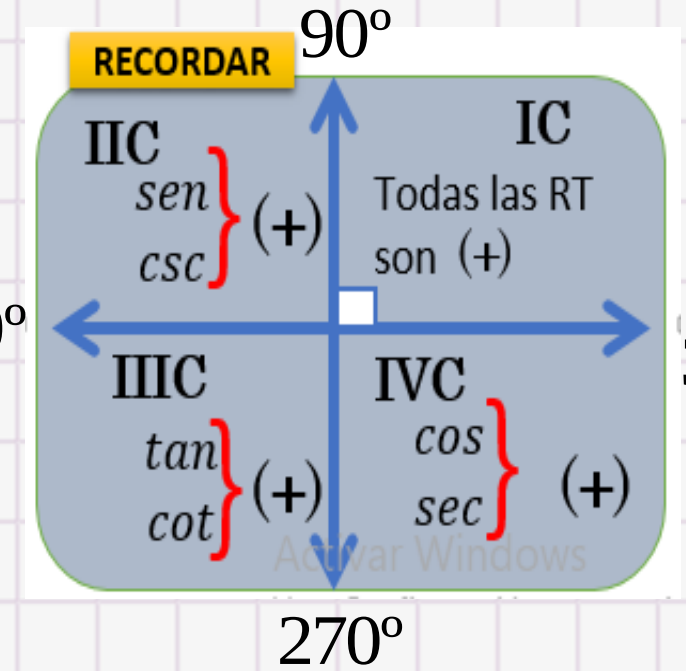
➢ $\underbrace{cot162°}_{(-)} . \underbrace{cos\beta}_{(-)} \underbrace{> 0}_{(+)} \Rightarrow cos\beta : (-)$

$\beta \in IIC$

$\beta \in IIIC$

$\therefore \boldsymbol{\beta \in IIC}$

$(+)(+) = (+)$
$(-)(-) = (+)$



2. Si $\cot\theta = -\frac{2}{3}$, donde $\theta \in$ IVC efectúe $R = \sqrt{13}(\operatorname{sen}\theta + \cos\theta)$.

RESOLUCIÓN

$$\cot\theta = -\frac{2}{3} = \frac{x}{y}$$

Como $\theta \in$ IVC se tiene que:
$x > 0 \; ; \; y < 0$

Entonces: $x = 2 \; ; \; y = -3$

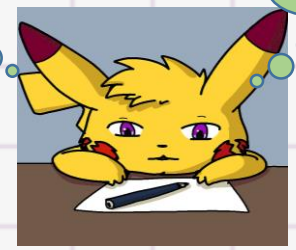
Radio vector: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(2)^2 + (-3)^2}$$

$$r = \sqrt{13}$$

Calculamos $R = \sqrt{13}(\operatorname{sen}\theta + \cos\theta)$

$\operatorname{sen}\alpha = \frac{y}{r}$ $\quad$ $\cos\alpha = \frac{x}{r}$



$$R = \cancel{\sqrt{13}}\left(\frac{-3}{\cancel{\sqrt{13}}} + \frac{2}{\cancel{\sqrt{13}}}\right)$$

$$R = -3 + 2$$

$$\therefore R = -1$$

3. Siendo α y β ángulos cuadrantales positivos y menores a una vuelta; además, $\sec\alpha + \operatorname{sen}\beta = 0$. Calcule $E = \tan\left(\frac{\alpha}{4}\right) + \sec^2\left(\frac{\beta}{3}\right)$.

RESOLUCIÓN

- Del dato:

$$0° < \alpha\ ;\ \beta < 360°$$

- Además:

$$\underline{\sec\alpha} + \underline{\operatorname{sen}\beta} = 0$$

-1 $\quad$ 1

$\alpha = 180°$ $\quad$ $\beta = 90°$

Calculamos

$$E = \tan\left(\frac{\mathbf{180°}}{4}\right) + \sec^2\left(\frac{\mathbf{90°}}{3}\right)$$

R.T	0° ; 360°	90°	180°	270°
SEN	O	1	0	-1
COS	1	O	-1	0
TAN	0	N.D	0	N.D
COT	N.D	0	N.D	0
SEC	1	N.D	-1	N.D
CSC	N	1	N.D	-1

4. Simplifique $P = \sqrt{3}\sec(-30°) - 5\cot(-53°) \cdot \cos(-37°)$.

RESOLUCIÓN

$$P = \sqrt{3}\sec(-30°) - 5\cot(-53°) \cdot \cos(-37°)$$

$$P = \sqrt{3}(\mathbf{sec30°}) - 5(\mathbf{-cot53°})(\mathbf{cos37°})$$

$$P = \cancel{\sqrt{3}}\left(\frac{2}{\cancel{\sqrt{3}}}\right) - \cancel{5}\left(-\frac{3}{\cancel{4}}\right)\left(\frac{\cancel{4}}{\cancel{5}}\right)$$

$$P = 2 - (-3)$$

$$P = 2 + 3$$

$$\therefore \mathbf{P = 5}$$

$sen(-x) = -senx$	$csc(-x) = -cscx$
$\cos(-x) = cosx$	$sec(-x) = secx$
$tan(-x) = -tanx$	$cot(-x) = -cotx$

5. Reduzca $L = \dfrac{3sen\,(180^\circ - x)}{cos(270^\circ + x)} + \dfrac{2sec(90^\circ - x)}{csc(180^\circ + x)}$.

RESOLUCIÓN

IIC: $180^\circ - x$ | IC: $90^\circ - x$

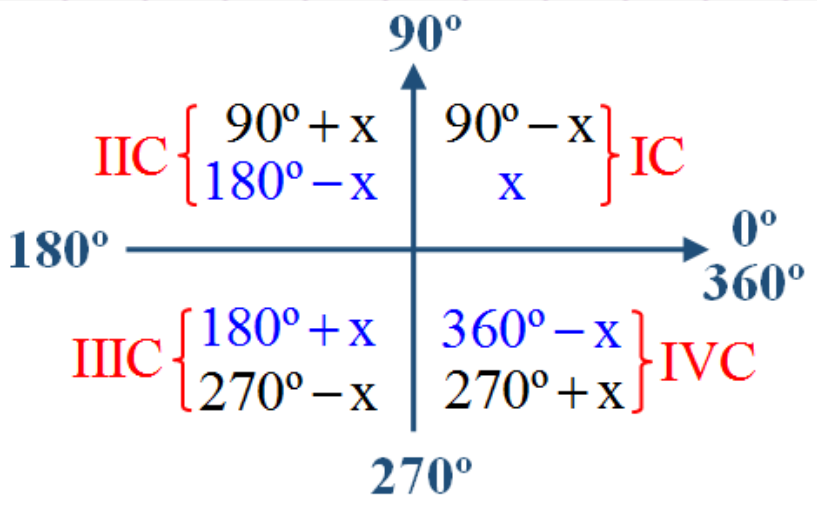
IVC: $270^\circ + x$ | IIIC: $180^\circ + x$

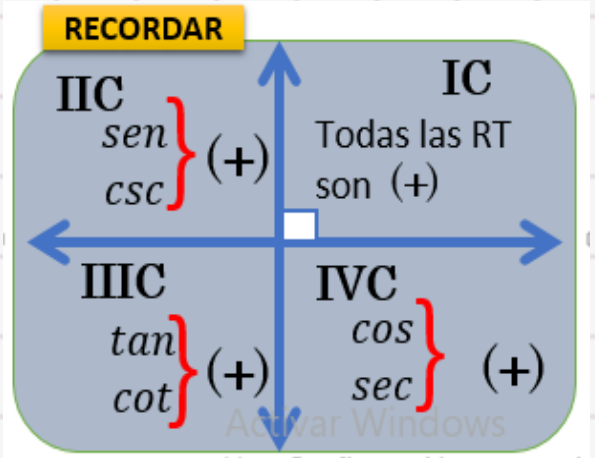
$$L = \frac{3sen\,(180^\circ - x)}{cos(270^\circ + x)} + \frac{2sec(90^\circ - x)}{csc(180^\circ + x)}$$

$$L = \frac{3(+senx)}{(+senx)} + \frac{2(+cscx)}{(-cscx)}$$

$$L = 3 + (-2)$$

$$\therefore L = 1$$





6. Si $\alpha - \beta = 90°$, reduzca $E = \frac{tan\alpha}{cot\beta} + sec\alpha \cdot sec\beta$.

RESOLUCIÓN

Dato:

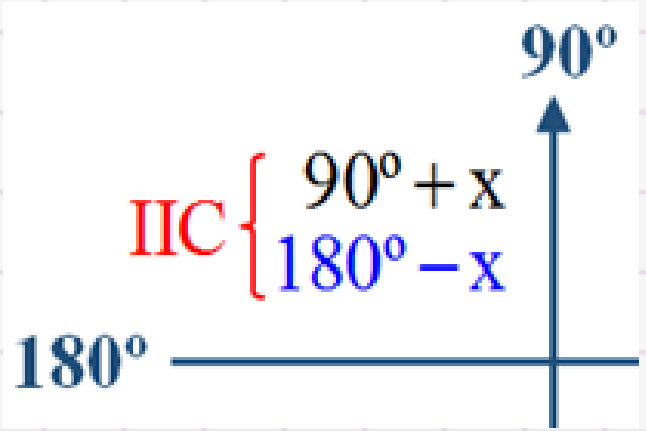
$$\alpha - \beta = 90° \Rightarrow \boxed{\alpha = 90° + \beta}$$

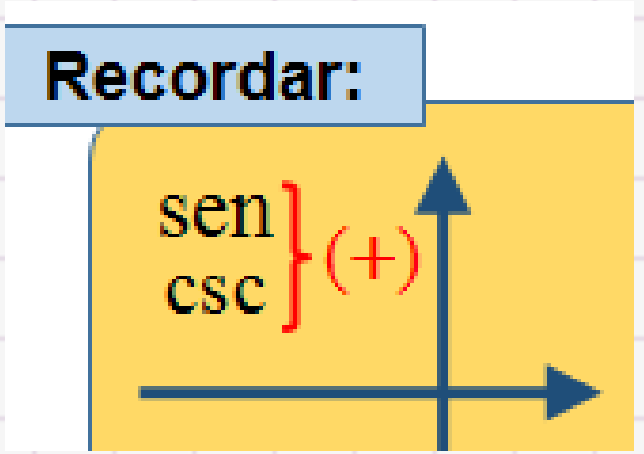
Piden:

$$E = \frac{\tan(\overbrace{90° + \beta}^{\text{IIC}})}{\cot\beta} + \operatorname{sen}(\overbrace{90° + \beta}^{\text{IIC}}) \cdot \sec\beta$$

$$E = \frac{\underbrace{-cot\beta}_{-1}}{cot\beta} + \underbrace{(cos\beta)}_{1}(sec\beta)$$

$$\therefore E = 0$$





7. Efectúe $E = tan2115° + sec1320°$.

RESOLUCIÓN

$$\begin{array}{r|l} 2115° & 360° \\ 1800° & 5 \\ \hline 315° & \end{array} \qquad \begin{array}{r|l} 1320° & 360° \\ 1080° & 3 \\ \hline 240° & \end{array}$$

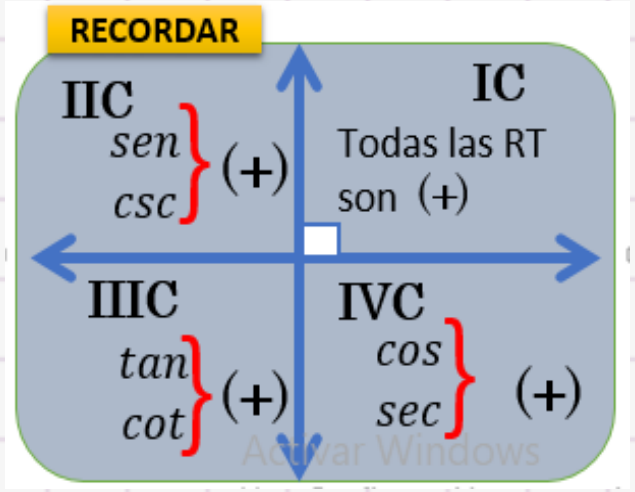
$$E = tan315° + sec240°$$

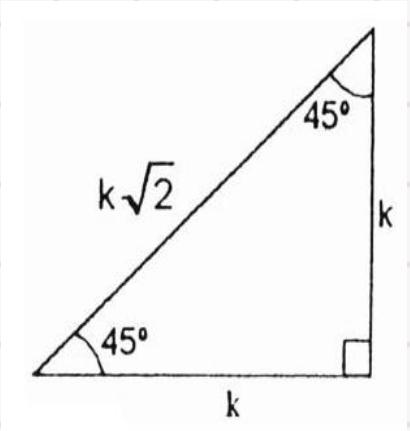
$$E = \tan(\underbrace{360° - 45°}_{\text{IVC}}) + sec(\underbrace{180° + 60°}_{\text{IIIC}})$$

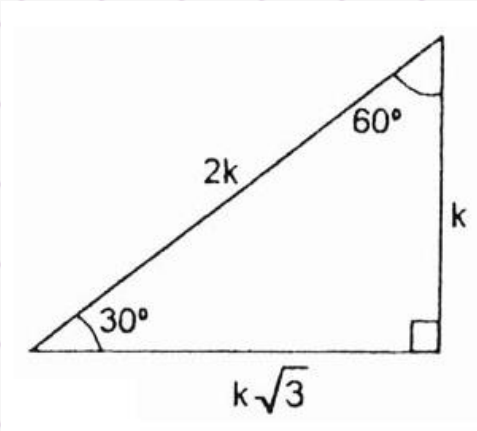
$$E = (-\tan45°) + (-sec60°)$$

$$E = (-1) + (-2)$$

$$\therefore E = -3$$







8. Si $x + y = 51\pi$, reduzca $M = \frac{senx}{seny} + \frac{cscx}{cscy}$.

RESOLUCIÓN

Dato:

$$x + y = 51\pi$$

IMPAR

$$x + y = \pi$$

$$y = \pi - x$$

Calculamos $M = \frac{senx}{seny} + \frac{cscx}{cscy}$

$$M = \frac{senx}{\text{sen}(\boldsymbol{\pi - x})} + \frac{cscx}{csc(\boldsymbol{\pi - x})}$$

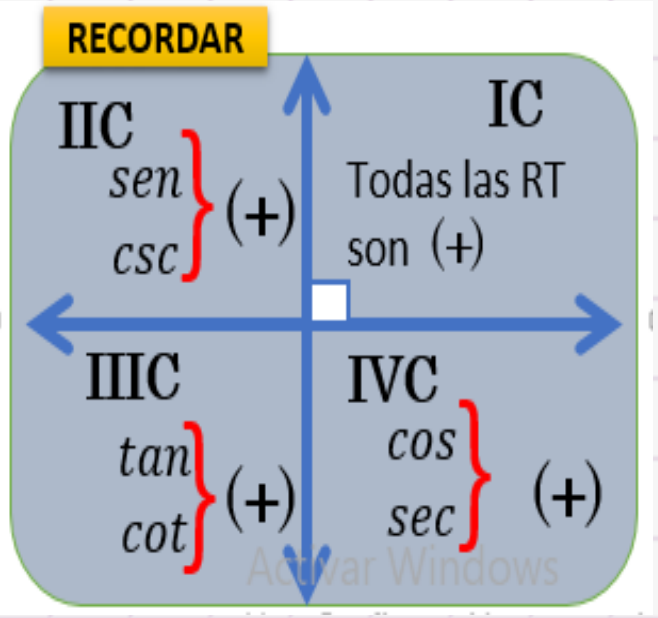
IIC IIC

$$M = \frac{\cancel{senx}}{\cancel{\boldsymbol{senx}}} + \frac{\cancel{cscx}}{\cancel{\boldsymbol{cscx}}}$$

1 1

$$\therefore M = 2$$

$\pi\, rad = 180°$



9. Simplifique

$$L = \frac{\tan\left(31\frac{\pi}{2} - x\right)}{\cot(18\pi + x)} + \sec 60°$$

RESOLUCIÓN

- $\tan\left(31\frac{\pi}{2} - x\right) = \tan\left(3\frac{\pi}{2} - x\right) = \mathbf{cotx}$

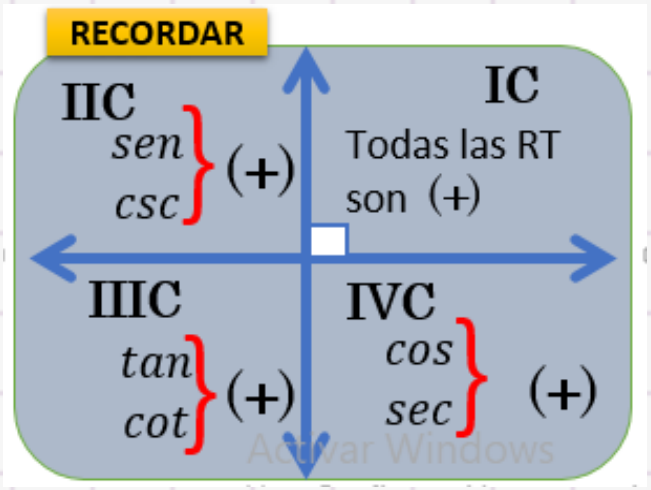
IIIC

$31 \div 4 \rightarrow$ cociente 7, resto 3

$\frac{3\pi}{2}\,\mathbf{rad} = \mathbf{270°}$

- $\cot(18\pi + x) = \mathbf{cot}\,x$

PAR



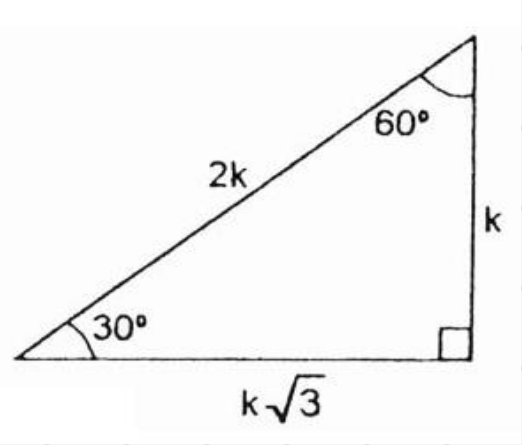
Calculamos:

$$L = \frac{\tan\left(31\frac{\pi}{2} - x\right)}{\cot(18\pi + x)} + \sec 60°$$

$$L = \frac{\cancel{\mathbf{cotx}}}{\cancel{\mathbf{cotx}}} + 2$$

$$L = 1 + 2$$

$$\therefore L = 3$$



10. La empresa "MIL OFICIOS" desea invertir en un proyecto donde pueda producir la mayor utilidad posible. El ingeniero a cargo deberá elegir entre 2 proyectos. Se sabe que la empresa esta dispuesta a desembolsar "A" soles, y cada proyecto ofrece una utilidad de B% y C% de la cantidad invertida. ¿Qué proyecto generará mayor utilidad? ¿Cuánta utilidad generará?

$A = 5000 \cdot \csc 1230°$

$B = 12\operatorname{sen}90° + \sec 180°$

$C = 7\cos 360° - 5\csc 270°$

RESOLUCIÓN

$A = 5000 \cdot \csc 1230° = 5000 \cdot \csc \mathbf{150°}$

$1230° \mid 360°$

$1080 \quad 3$

$150°$

$A = 5000 \cdot \csc(\mathbf{180°} - \mathbf{30°})$

$A = 5000 \cdot \csc \mathbf{30°} = 5000(\mathbf{2})$

$A = 10000$

- $B = 12\operatorname{sen}90° + \sec 180°$

  $B = 12(\mathbf{1}) + (\mathbf{-1})$ → $\mathbf{B = 11\%}$

- $C = 7\cos 360° - 5\csc 270°$

  $C = 7(\mathbf{1}) - 5(\mathbf{-1})$ → $\mathbf{C = 12\%}$ **Mayor utilidad**

R.T	0° ; 360°	90°	180°	270°
SEN	0	1	0	-1
COS	1	0	-1	0
TAN	0	N.D	0	N.D
COT	N.D	[illegible]	[illegible]	[illegible]
SEC	1	N[illegible]	[illegible]	[illegible]
CSC	N	1	N.D	-1

Calculamos la utilidad del proyecto:

$U_{total} = \mathbf{A} * \mathbf{C} = 10000 * 12\%$

$U_{total} = \overset{100}{10000} * \dfrac{12}{100}$ **= 1200 soles**


SACO
OLIVEROS