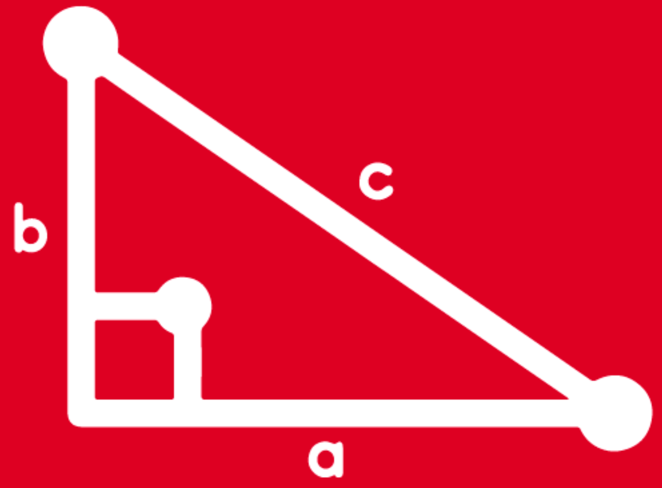
# TRIGONOMETRY

## TOMO 3 y 4




**5th**
**SECONDARY**

## ADVISORY

SACO OLIVEROS

**1.** Si sen$\theta$ = – 9/41 , además $\theta \in \langle 270°;360° \rangle$, halle el valor de M = csc$\theta$ + cot$\theta$

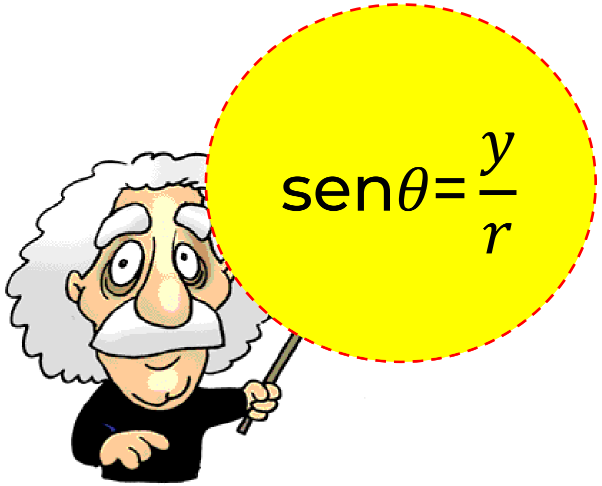
## RESOLUCIÓN

- Como $\theta \in \langle 270°;360° \rangle$

$\theta \in$ IVC $\Rightarrow$ x( + ), y( – ) , r(+)

- Además:

$$\text{sen}\theta = \frac{-9}{41} = \frac{y}{r}$$




Luego: y = – 9 y r = 41

Sabemos: $\boxed{r = \sqrt{x^2 + y^2}}$

$$41 = \sqrt{x^2 + (-9)^2} \Rightarrow x = 40$$

Piden: M = csc$\theta$ + cot$\theta$

$$M = \frac{41}{-9} + \frac{40}{-9}$$

$$\boxed{\therefore M = -9}$$


**2.** Si $\beta \in$ IIIC, además $\tan(270 - \beta) = 0{,}75$

Reduzca: $E = \csc(270° - \beta) + \tan(180° + \beta)$

**RESOLUCIÓN**

$$E = \underbrace{\csc(\overbrace{270° - \beta}^{\text{IIIC}})}_{-\sec\beta} + \underbrace{\tan(\overbrace{180° + \beta}^{\text{IIIC}})}_{\tan\beta}$$

$$T = -\sec\beta + \tan\beta \quad \ldots(*)$$

Del dato:

$$\tan(\overbrace{270° - \beta}^{\text{IIIC}}) = 0{,}75$$

$$\cot\beta = \frac{3}{4}$$

$$\cot\beta = \frac{-3}{-4} = \frac{x}{y}$$

**Por radio vector:**

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-3)^2 + (-4)^2} \qquad r = 5$$

Reemplazando en $(*)$ :

$$T = -\frac{r}{x} + \frac{y}{x}$$

$$T = -\frac{5}{-3} + \frac{-4}{-3} = \frac{-5 - 4}{-3}$$

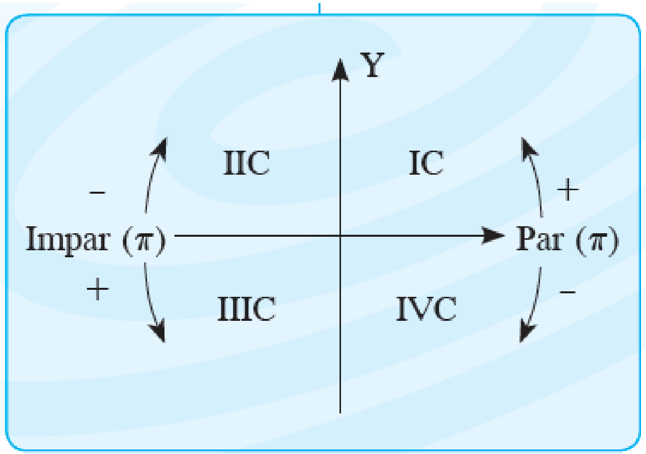
$$T = \frac{\cancel{-}9}{\cancel{-}3}$$

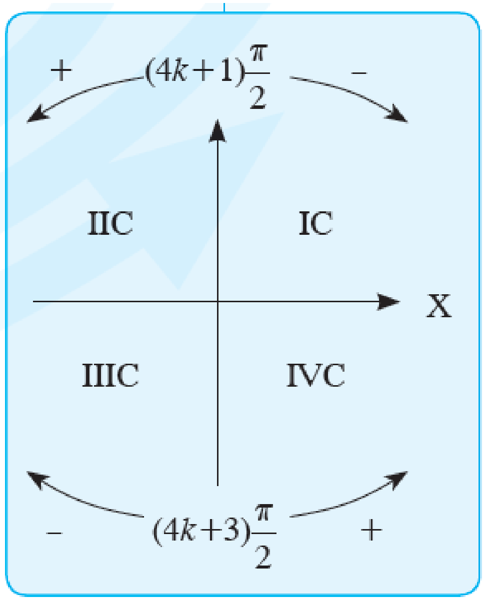
$$\therefore T = 3$$

**3.** Se cumple que $\tan(11\pi + x) = 3$

Efectúe: $K = \cos\left(\frac{15\pi}{2} - x\right).\sec(26\pi - x)$

si x es un ángulo agudo.




## RESOLUCIÓN

$$K = \cos\left(\overset{4k+3}{15}\frac{\pi}{2} - x\right).\sec(\overset{\text{PAR}}{26}\pi - x)$$

$$K = \underbrace{\cos\left(\overbrace{3\frac{\pi}{2} - x}^{\text{IIIC}}\right)}_{-\operatorname{sen}x}.\underbrace{\sec\overbrace{(2\pi - x)}^{\text{IVC}}}_{\sec x}$$

$K = -\operatorname{senx}.\sec x$ ...(*)

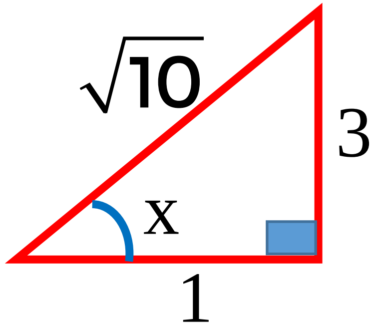
Del dato:

$$\tan(\overset{\text{IMPAR}}{11}\pi + x) = 3$$

$$\underbrace{\tan\overbrace{(\pi + x)}^{\text{IIIC}}}_{\tan x} = 3$$

$\tan x = 3$

$\tan x = \frac{3}{1}$




Reemplazamos en (*):

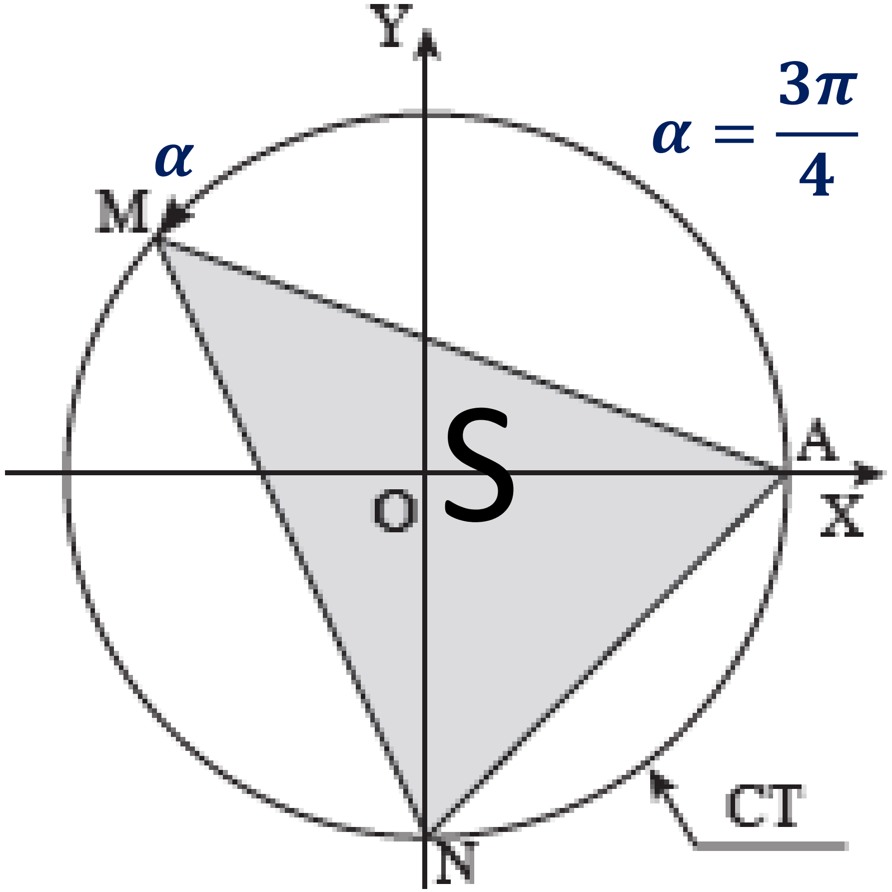
$K = -\operatorname{senx}.\sec x$

$$K = -\frac{3}{\cancel{\sqrt{10}}}\cdot\frac{\cancel{\sqrt{10}}}{1}$$

$$\boxed{\therefore K = -3}$$

**4.** En la circunferencia trigonométrica mostrada. Halle el área de la región triangular AMN .

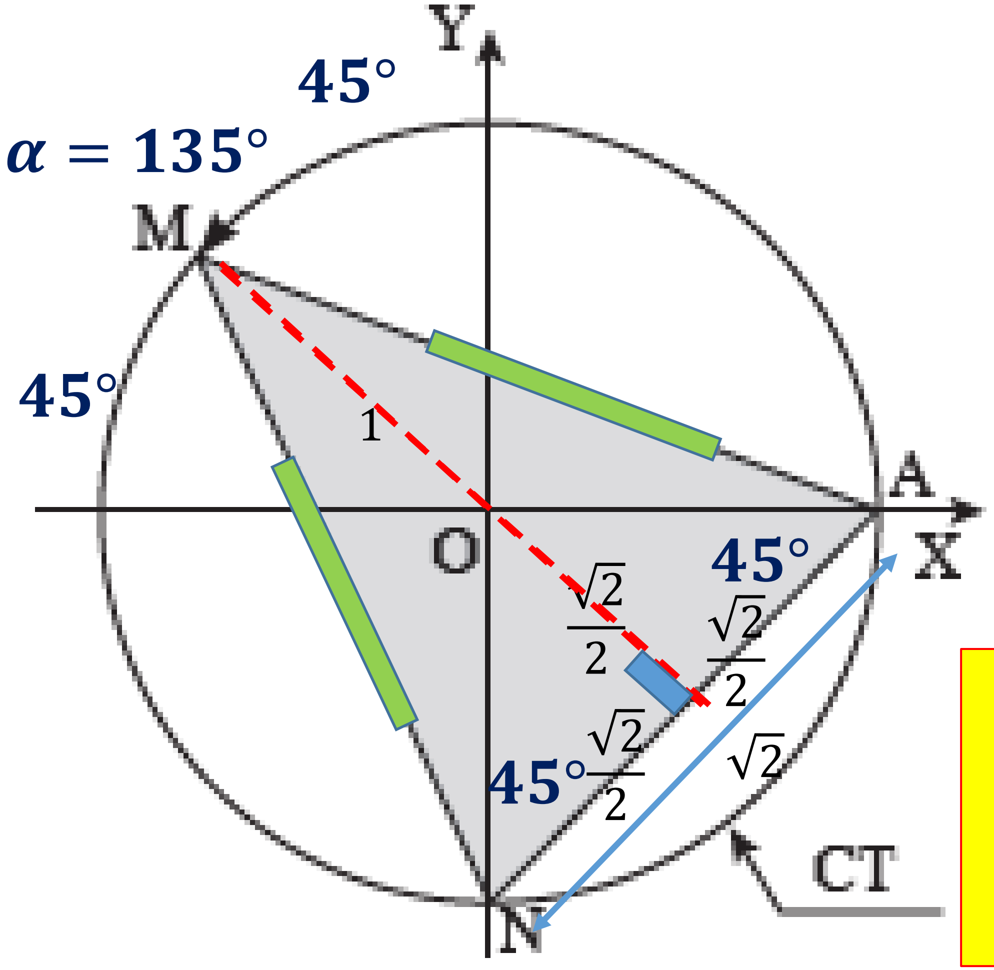
**RESOLUCIÓN**




$\alpha = \dfrac{3\pi}{4}$

*Calculamos el área S*:

$\alpha = 135°$




***Entonces***:

$$S = \frac{\sqrt{2}\left(1 + \frac{\sqrt{2}}{2}\right)}{2}$$

**5.** Si se cumple que: $\operatorname{sen}x - \cos x = a$ y $\operatorname{sen}x.\cos x = b$ determine una relación entre m y n independiente de x.

**RESOLUCIÓN**

$$\operatorname{sen}x - \cos x = a$$

$$(\operatorname{sen}x - \cos x)^2 = a^2$$

$$\operatorname{sen}^2x - 2\operatorname{sen}x\cos x + \cos^2x = a^2$$

$$\operatorname{sen}^2x + \cos^2x - 2\operatorname{sen}x\cos x = a^2$$

$$1 - \underbrace{2\operatorname{sen}x\cos x}_{b} = a^2$$

$$\therefore\ \boxed{1 - 2b = a^2}$$

**6.** Si se cumple que $\text{senx} + \text{cosx} = \frac{2}{3}$ , calcule

$$E = (1 - \text{senx})(1 - \text{cosx})$$

## RESOLUCIÓN

$$E = (1 - \text{senx})(1 - \text{cosx})$$

$$2E = \underbrace{2(1 - \text{senx})(1 - \text{cosx})}$$

$$2E = (1 - senx - \text{cosx})^2$$

$$2E = (1 - \underbrace{(\text{senx} + \text{cosx})}_{\frac{2}{3}})^2$$

$$2E = \left(1 - \frac{2}{3}\right)^2$$

$$2E = \left(\frac{1}{3}\right)^2$$

$$\boxed{\therefore E = \frac{1}{18}}$$

**7.** Calcule el valor de:

$$E = \frac{\csc 2730°}{\text{sen}4005°}$$

**RESOLUCIÓN**

$$\begin{array}{r|l} 2730° & 360° \\ \cline{2-2} 2520° & 7 \\ 210° & \end{array}$$

$$\begin{array}{r|l} 4005° & 360° \\ \cline{2-2} 3960° & 11 \\ \hline 45° & \end{array}$$

$$E = \frac{\csc 210°}{\text{sen}45°}$$

$$E = \frac{\csc\overbrace{(180° + 30°)}^{\text{IIIC}}}{\text{sen}45°}$$

$$E = \frac{-\csc 30°}{sen45°}$$

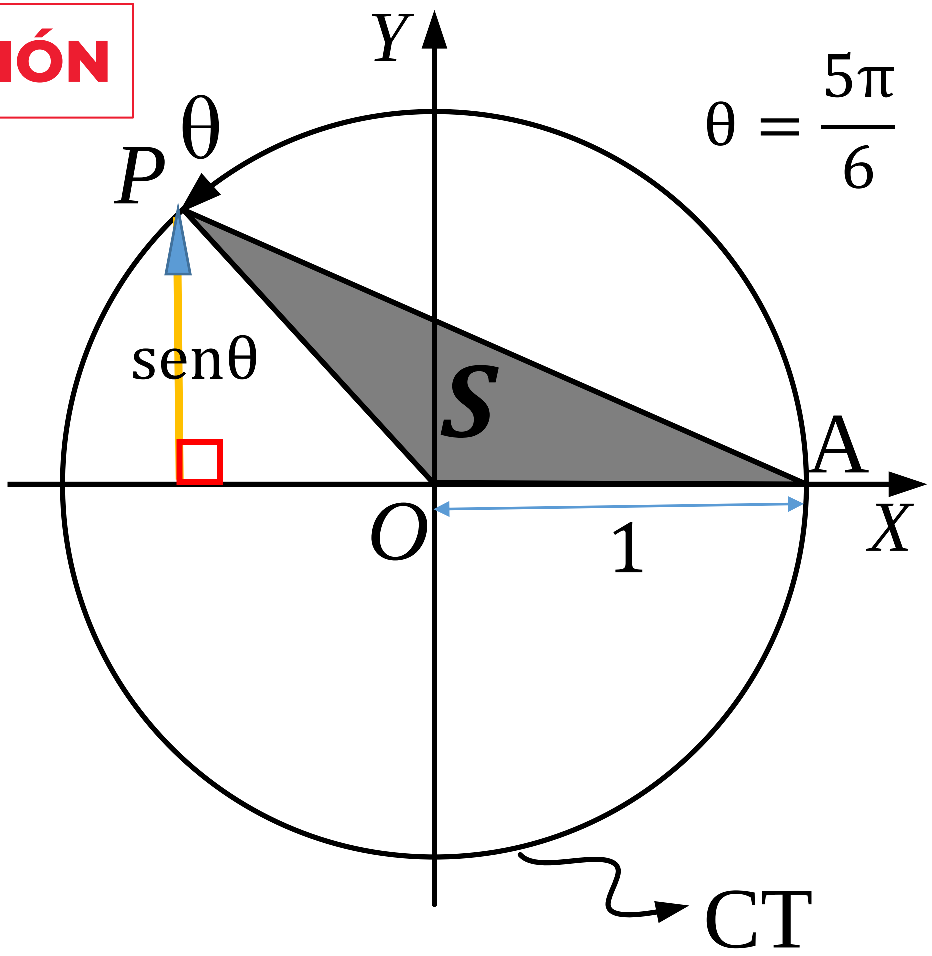
$$E = \frac{-2}{\frac{1}{\sqrt{2}}}$$

$$E = \frac{-2}{\frac{1}{\frac{1}{\sqrt{2}}}}$$

$$\boxed{\therefore E = -2\sqrt{2}}$$

**8.** De la circunferencia trigonométrica mostrada, determine el área de la región triangular sombreada AOP.

**RESOLUCIÓN**




Calculamos el área S:

$$S = \frac{1 * \text{sen}\theta}{2} \qquad \theta = \frac{5(180°)}{6} = 150°$$

$$S = \frac{1 * \text{sen}150°}{2}$$

$$S = \frac{1 * \text{sen}\overbrace{(180° - 30°)}^{\text{IIC}}}{2}$$

$$S = \frac{1 * \text{sen}30°}{2}$$

$$S = \frac{1\left(\frac{1}{2}\right)}{2}$$

$$\boxed{\therefore S = \frac{1}{4}}$$

**9.** Si: $\tan^2\alpha = 2\tan^2 x + 1$, halle el valor de

$y = \cos^2\alpha + \operatorname{sen}^2 x$ , en términos de $\alpha$

**RESOLUCIÓN**

$$\tan^2\alpha = 2\tan^2 x + 1$$

$$\underbrace{1 + \tan^2\alpha}_{} = 2\tan^2 x + 1 + 1$$

$$\sec^2\alpha = 2\tan^2 x + 2$$

$$\sec^2\alpha = 2\underbrace{(\tan^2 x + 1)}_{\sec^2 x}$$

$$\sec^2\alpha = 2\sec^2 x$$

$$\frac{1}{\cos^2\alpha} = \frac{2}{\cos^2 x}$$

$$\cos^2 x = 2\cos^2\alpha$$

$$1 - \operatorname{sen}^2 x = \cos^2\alpha + \cos^2\alpha$$

$$\underbrace{1 - \cos^2\alpha}_{\operatorname{sen}^2\alpha} = \cos^2\alpha + \operatorname{sen}^2 x$$

$$\boxed{\therefore \cos^2\alpha + \operatorname{sen}^2 x = \operatorname{sen}^2\alpha}$$

**10.** Si se cumple que $\sec x \csc x = 3$, calcule: $F = \tan^3 x + \cot^3 x$

## RESOLUCIÓN

$$\sec x \csc x = 3$$

$$\tan x + \cot x = 3$$

$$(\tan x + \cot x)^2 = 3^2$$

$$\underbrace{\tan^2 x + \cot^2 x}_{7} + \underbrace{2\tan x \cot x}_{1} = 9$$

$$\boxed{a^3 + b^3 = (a+b)(a^2 + b^2 - ab)}$$

$$F = \tan^3 x + \cot^3 x$$

$$F = \underbrace{(\tan x + \cot x)}_{3}(\underbrace{\tan^2 x + \cot^2 x}_{7} - \underbrace{\tan x \cot x}_{1})$$

$$\boxed{\therefore F = 18}$$