ALGEBRA

ASESORIA

3th
SECONDARY

3er Bimestre

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SOLVED PROBLEMS

Problema 1

Factorice e indique un factor primo de

$$P(x) = x^3 + 8x^2 + 13x - 6$$

$\boxed{a_0 = 1}$ $\boxed{a_n = 6}$

$$div(a_0) = \{1\}$$

$$div(a_n) = \{1; 2; 3; 6\}$$

$$\boxed{PC = \pm\{1; 2; 3; 6\}}$$

	1	8	13	-6
$x = -3$	$\downarrow$	-3	-15	6
$\times$	1	5	-2	0

$$\boxed{P(x) = (x + 3)(x^2 + 5x - 2)}$$

Factores primos: $\boxed{(x + 3) \quad y \quad (x^2 + 5x - 2)}$

Problema 2

Reduzca

$$L = \sqrt{7+\sqrt{40}} - \sqrt{9-\sqrt{80}} + \sqrt{11-\sqrt{72}}$$

Recordemos:

$$\sqrt{A \pm \sqrt{B}} = \sqrt{(x+y) \pm 2\sqrt{x.y}} = \sqrt{x} \pm \sqrt{y}$$

Resolución:

$$L = \sqrt{7+\sqrt{40}} - \sqrt{9-\sqrt{80}} + \sqrt{11-\sqrt{72}}$$

$$L = \sqrt{7+\sqrt{4}.\sqrt{10}} - \sqrt{9-\sqrt{4}.\sqrt{20}} + \sqrt{11-\sqrt{4}.\sqrt{18}}$$

$$L = \sqrt{\underbrace{7}_{5+2}+2\sqrt{\underbrace{10}_{5\times2}}} - \sqrt{\underbrace{9}_{5+4}-2\sqrt{\underbrace{20}_{5\times4}}} + \sqrt{\underbrace{11}_{9+2}-2\sqrt{\underbrace{18}_{9\times2}}}$$

$$L = \sqrt{5}+\sqrt{2}-(\sqrt{5}-\sqrt{4})+\sqrt{9}-\sqrt{2}$$

$$L = \cancel{\sqrt{5}}+\cancel{\sqrt{2}}-\cancel{\sqrt{5}}\underline{+\sqrt{4}}\ \underline{+\sqrt{9}}-\cancel{\sqrt{2}}$$

$$L = 2+3$$

$$\boxed{\therefore\ L = 5}$$

Problema 3

Efectúe

$$z = \frac{2(i-2)}{1+i} - (1-i)^2$$

Resolución:

$$z = \frac{2(i-2)}{1+i} - (1-i)^2$$

$$z = \frac{2(i-2)}{(1+i)} \cdot \frac{(1-i)}{(1-i)} - (\cancel{1} - 2i + i^2) \quad i^2 \to \cancel{-1}$$

$$z = \frac{2(i - i^2 - 2 + 2i)}{1 - i^2} + 2i \quad i^2 \to -1$$

$$z = \frac{\cancel{2}(3i-1)}{\cancel{2}} + 2i$$

$$z = 3i - 1 + 2i$$

$$\boxed{\therefore \quad z = 5i - 1}$$

Problema 4

Sea $z_1 = 4 + 3i$

$z_2 = 1 - 2i$

$z_3 = 5 + 5i$

Si $z = z_1 - z_3^* - \bar{z}_2$

calcule $|Z|$

Recordemos:

Sea: $z = a + bi$

Conjugado de z:

$$\boxed{\bar{z} = a - bi}$$

Opuesto de z:

$$\boxed{z^* = -a - bi}$$

Módulo de z:

$$\boxed{|z| = \sqrt{a^2 + b^2}}$$

Resolución:

$$z = z_1 - z_3^* - \bar{z}_2$$

$$z = (4 + 3i) - (-5 - 5i) - (1 + 2i)$$

$$z = 4 + 3i + 5 + 5i - 1 - 2i$$

$$\boxed{z = 8 + 6i}$$

Nos piden: $|z|$

$$|z| = \sqrt{8^2 + 6^2}$$

$$|z| = \sqrt{100}$$

$$\boxed{\therefore \quad |z| = 10}$$

Problema 5

Determine el valor de x en

$$(x+5)(2x-1)-90=(2x+1)(x-5)$$

Resolución:

$$(x+5)(2x-1)-90=(2x+1)(x-5)$$

$$\cancel{2x^2}-x+10x\cancel{-5}-90=\cancel{2x^2}-10x+x\cancel{-5}$$

$$9x-90=-9x$$

$$18x=90$$

$$\boxed{\therefore\ x=5}$$

Problema 6

Determine el valor de m en

$$\frac{m+3}{5}+\frac{3m-1}{4}=\frac{2m+1}{3}+\frac{4m+2}{15}$$

Resolución:

$$\frac{m+3}{5}+\frac{3m-1}{4}=\frac{2m+1}{3}+\frac{4m+2}{15}$$

$$\boxed{mcm(5;4;3;15)=60}$$

$$60\left(\frac{m+3}{5}\right)+60\left(\frac{3m-1}{4}\right)=60\left(\frac{2m+1}{3}\right)+60\left(\frac{4m+2}{15}\right)$$

$$12(m+3)+15(3m-1)=20(2m+1)+4(4m+2)$$

$$12m+36+45m-15=40m+20+16m+8$$

$$57m+21=56m+28$$

$$\boxed{\therefore\ m=7}$$

Problema 6

Determine el valor de m en

$$\frac{m+3}{5}+\frac{3m-1}{4}=\frac{2m+1}{3}+\frac{4m+2}{15}$$

Resolución:

$$\frac{m+3}{5}+\frac{3m-1}{4}=\frac{2m+1}{3}+\frac{4m+2}{15}$$

$$\boxed{mcm(5;4;3;15)=60}$$

$$\cancel{60}\left(\frac{m+3}{\cancel{5}}\right)+\cancel{60}\left(\frac{3m-1}{\cancel{4}}\right)=\cancel{60}\left(\frac{2m+1}{\cancel{3}}\right)+\cancel{60}\left(\frac{4m+2}{\cancel{15}}\right)$$

$$12(m+3)+15(3m-1)=20(2m+1)+4(4m+2)$$

$$12m+36+45m-15=40m+20+16m+8$$

$$57m+21=56m+28$$

$$\boxed{\therefore\quad m=7}$$

Problema 7

Halle el valor de x

$$(x-3)^2+(x+1)^2=12$$

Resolución:

$$(x-3)^2+(x+1)^2=12$$

$$x^2-6x+9+x^2+2x+1=12$$

$$\cancel{2}x^2-\cancel{4}x-\cancel{2}=0$$

$x^2-2x-1=0$, *donde:* $\begin{cases} a=1 \\ b=-2 \\ c=-1 \end{cases}$

Cálculo del discriminante:

$$\boxed{\Delta=b^2-4ac}$$

$$\Delta=(-2)^2-4(1)(-1)$$

$$\boxed{\Delta=8}$$

Fórmula general:

$$\boxed{x=\frac{-b\pm\sqrt{\Delta}}{2a}}$$

$$x=\frac{-(-2)\pm\sqrt{8}}{2(1)}=\frac{2\pm\sqrt{8}}{2}=\frac{\cancel{2}\pm\cancel{2}\sqrt{2}}{\cancel{2}}=\boxed{1\pm\sqrt{2}}$$

$$\therefore\ \boxed{x_1=1+\sqrt{2}}\ \vee\ \boxed{x_2=1-\sqrt{2}}$$

Problema 8

Calcule el valor de n ***si las raíces de la ecuación***

$$(2n+15)x^2+2nx+1=0$$

son iguales.

Recordemos:

Sea: $ax^2+bx+c=0$

La ecuación tiene raíces iguales si y solo si $\Delta=0$:

$$\boxed{b^2-4ac=0}$$

Resolución:

$$\boxed{\underbrace{(2n+15)}_{a}x^2+\underbrace{2n}_{b}x+\underbrace{1}_{c}=0}$$

La ecuación tiene raíces iguales $\Rightarrow$ $\boxed{b^2-4ac=0}$

$$(2n)^2-4(2n+15)(1)=0$$

$$\cancel{4}n^2-\cancel{8}n-\cancel{60}=0$$

$$n^2-2n-15=0$$

$$\begin{array}{ll} n & -5 \\ n & +3 \end{array}$$

$$n-5=0 \quad \vee \quad n+3=0$$

$$\therefore \boxed{n=5} \quad \vee \quad \boxed{n=-3}$$

Problema 9

Luego de resolver

$$\begin{vmatrix} x & 2 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 3 & 6 \end{vmatrix};\ \boxed{x > 0}$$

el valor de x representa la edad del hijo mayor de Miguel. ¿Cuántos años tiene Miguel si se sabe que tuvo a su primer hijo a los 25 años?

Resolución:

$$\begin{vmatrix} x & 2 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 3 & 6 \end{vmatrix}$$

$$x^2 - 10 = 24 - 9$$

$$x^2 = 25$$

$$\boxed{x = \pm 5}$$

Edad del hijo mayor de Miguel: 5 *años*

$\therefore$ *Miguel tiene 30 años.*

Problema 10

Determine el valor de x ***en***

$$\begin{vmatrix} 1 & -2 & 2 \\ 1 & 2x & 5 \\ 3 & -1 & 1 \end{vmatrix} = 5$$

Resolución:

$$\begin{vmatrix} 1 & -2 & 2 \\ 1 & 2x & 5 \\ 3 & -1 & 1 \end{vmatrix} = 5$$

$$\begin{vmatrix} 1 & -2 & 2 \\ 1 & 2x & 5 \\ 3 & -1 & 1 \end{vmatrix} \begin{matrix} 1 & -2 \\ 1 & 2x \\ 3 & -1 \end{matrix} = 5$$

$$(2x - 30 - 2) - (12x - 5 - 2) = 5$$

$$2x - 32 - 12x + 7 = 5$$

$$-10x = 30$$

$$\boxed{\therefore \quad x = -3}$$