

# ALGEBRA

## Chapter 09

### 4th

FACTORIAL Y  
NÚMERO  
COMBINATORIO



# HELICO

---

# MOTIVATING

# SABIAS QUE

El hombre-computadora Horacio Uhler<sup>®</sup>, en la década de 1950, calculó el valor de  $450!$  (factorial de 450) sin la ayuda de ordenadores. Encontró que tenía exactamente 1.001 dígitos, por lo que lo bautizó como el "Factorial de las mil y una noches".

17.333.687.331.126.326.593.447.131.461.045.793.996.778.112.652.090  
 .510.155.692.075.095.553.330.016.834.367.506.046.750.882.904.38  
 7.106.145.811.284.518.424.097.858.618.583.806.301.650.208.347.29  
 6.181.351.667.570.171.918.700.422.280.962.237.272.230.663.528.08  
 4.038.062.312.369.342.674.135.036.610.101.508.838.220.494.970.9  
 29.739.011.636.793.766.165.023.730.853.896.403.901.590.836.144.1  
 49.594.432.684.204.513.784.716.402.303.182.604.094.683.993.315.  
 061.302.563.918.385.303.341.510.606.761.462.420.205.820.006.936.  
 352.095.967.417.183.191.538.725.617.509.521.380.556.781.309.195.42  
 9.800.229.273.803.342.553.558.164.591.996.298.912.368.598.547.7  
 71.179.158.461.351.340.068.905.647.127.658.164.836.377.126.303.77  
 4.923.360.078.072.307.462.008.554.355.068.361.448.126.606.281.1  
 45.760.960.499.187.813.428.397.924.840.592.504.537.849.487.42  
 5.060.488.481.036.571.447.957.046.788.635.742.936.714.615.176.21  
 9.148.469.743.102.979.949.740.714.485.104.716.169.664.052.397.39  
 2.602.848.408.694.007.408.998.901.127.492.905.171.514.473.431.3  
 86.633.392.492.040.661.522.692.303.043.813.960.541.966.093.224.  
 243.809.225.137.268.851.717.904.303.214.058.238.447.936.111.678.5  
 68.236.973.036.238.404.626.507.890.688.000.000.000.000.000.  
 000.000.000.000.000.000.000.000.000.000.000.000.000.000.0  
 00.000.000.000.000.000.000.000.000.000.000.000.000.000.00  
 0.000.000.000

# HELICO THEORY

CHAPTHE  
R 09

---

# FACTORIAL

## DEFINICIÓN

Sea  $n \in \mathbb{N}$  (además del cero), denotado por  $n!$ ; se define como:

$$n! = \begin{cases} 0, & \text{si } n = 0 \vee n = 1 \\ 1 \times 2 \times 3 \dots n, & \text{si } n \in \mathbb{N} \wedge n \geq 2 \end{cases}$$

## Ejemplos:

$$5! = 5(4)(3)(2)(1) = 120$$

$$3! = 3(2)(1) = 6$$

$$14! = 14(13)(12)(11)(10) \underbrace{9!}_{\text{Degradación de factorial}} = 240240 \cdot 9!$$

## Propiedades

$$n! + (n+1)! = n!(n+2)$$

$$5! + 6! = 5!(5+2) = 5!(7)$$

$$n! + (n+1)! + (n+2)! = n!(n+2)^2$$

$$4! + 5! + 6! = 4!(4+2)^2 = 5!(36)$$

$$(n+1)! - n! = n!(n)$$

$$5! - 4! = 4!(4)$$

# NÚMERO COMBINATORIO

## DEFINICIÓN

El número combinatorio denotado por  $C_k^n$  representa el número total de combinaciones que se pueden realizar con  $n$  elementos tomados de  $k$  en  $k$ .

$$C_k^n = \frac{n!}{k! \cdot (n-k)!} \quad (n, k \in \mathbb{N} \wedge n \geq k)$$

### Ejemplo:

$$C_2^7 = \frac{7!}{2! \cdot (7-2)!} = \frac{7!}{2! \cdot (5)!} = \frac{7(6) \cdot 5!}{2(1) \cdot 5!} = 21$$

### Caso Práctico:

$$C_2^7 = \frac{7(6)}{2(1)} = 21$$

## Propiedades

$$C_k^n = C_{n-k}^n \quad \text{Ejemplo: } C_2^7 = C_{7-2}^7 = C_5^7$$

$$\text{Si: } C_k^n = C_p^n \Rightarrow k = p \vee n = k + p$$

$$\text{Ejemplo: Si: } C_{10}^{15} = C_p^{15} \Rightarrow p = 10 \vee 15 = p + 10$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$

$$\text{Ejemplo: } C_4^{12} + C_5^{12} = C_5^{12+1} = C_5^{13}$$

$$C_k^n = \frac{n}{k} C_{k-1}^{n-1} \quad \text{Ejemplo: } C_9^{15} = \frac{15}{9} C_{9-1}^{15-1} = \frac{5}{3} C_8^{14}$$

# HELICO PRACTICE

CHAPTHE  
R 09

---

## HELICO | PRACTICE

1. Reduzca

$$P = \left( \frac{32! + 33!}{34!} \right) \left( \frac{67!}{66! + 65!} \right)$$

### RESOLUCIÓN

$$n! + (n+1)! = n!(n+2)$$

$$P = \left( \frac{32! + 33!}{34!} \right) \left( \frac{67!}{66! + 65!} \right)$$

$$n! + (n+1)! = n!(n+2)$$

$$\Rightarrow P = \left( \frac{\cancel{32!} (\cancel{34})}{\cancel{34} (\cancel{33}) \cancel{32!}} \right) \left( \frac{\cancel{67} (\cancel{66}) \cancel{65!}}{\cancel{65!} (\cancel{67})} \right)$$

$$\Rightarrow P = \left( \frac{\cancel{66}}{\cancel{33}} \right)$$

$$P = 2$$



## HELICO | PRACTICE

2. Halle el valor de "x" en:

$$\frac{(x+4)!(x+2)!}{(x+3)! + (x+2)!} = 720$$

### RESOLUCIÓN

Degradación de factorial

$$\frac{(x+4)!(x+2)!}{(x+3)! + (x+2)!} = 720$$

$$n! + (n+1)! = n!(n+2)$$

$$\Rightarrow \frac{\cancel{(x+4)}(x+3)!\cancel{(x+2)!}}{\cancel{(x+2)!}(x+4)} = 720$$

$$\Rightarrow (x+3)! = \underbrace{720}_{6!}$$

$$\Rightarrow (x+3)! = 6!$$

$$x = 3$$

## HELICO | PRACTICE

3. Halle el valor de  $x$ , si se cumple:

$$\frac{(x+2)! + (x+3)! + (x+4)!}{(x+3)! - (x+2)!} = \frac{25}{x+2}$$

### RESOLUCIÓN

$$n! + (n+1)! + (n+2)! = n!(n+2)^2$$

$$\frac{(x+2)! + (x+3)! + (x+4)!}{(x+3)! - (x+2)!} = \frac{25}{x+2}$$

$$(n+1)! - n! = n!(n)$$

$$\Rightarrow \frac{\cancel{(x+2)!} (x+4)^2}{\cancel{(x+2)!} \cancel{(x+2)}} = \frac{25}{\cancel{x+2}}$$

$$\Rightarrow (x+4)^2 = 25$$

$$x = 1$$

## HELICO | PRACTICE

4. Halle el valor de “n” en:

$$3C_3^{2n} = 44C_2^n$$

### RESOLUCIÓN

#### Caso Práctico:

$$3 \left\{ \frac{2n(2n-1)(2n-2)}{(3)(2)(1)} \right\} = 44 \left\{ \frac{n(n-1)}{(2)(1)} \right\}$$

$$\Rightarrow n(2n-1)2(n-1) = 22n(n-1)$$

$$\Rightarrow (2n-1) = 11$$

$$n = 6$$

## 5. Calcule

$$P = \frac{C_4^{12} + C_5^{12} + C_7^{13}}{C_6^{14} + C_7^{14}}$$

## RESOLUCIÓN

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$

$$P = \frac{C_4^{12} + C_5^{12} + C_7^{13}}{C_6^{14} + C_7^{14}}$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$



$$P = \frac{C_5^{13} + C_7^{13}}{C_7^{15}}$$



$$C_7^{13} = C_6^{13}$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$



$$P = \frac{C_5^{13} + C_6^{13}}{C_7^{15}}$$



$$P = \frac{C_6^{14}}{C_7^{15}} = \frac{\frac{14!}{6!(8)!}}{\frac{15!}{7!(8)!}} = \frac{(14!)(7!)}{(6!)(15!)} = \frac{7}{15}$$

$$C_k^n = \frac{n!}{k!(n-k)!}$$



$$P = \frac{7}{15}$$

$$P = \frac{7}{15}$$

## HELICO | PRACTICE

6. Pedro le regala a su esposa una licuadora marca OSTER, cuyo precio fue el valor de  $2T$  soles, donde  $T$  está dado por:

$$T = C_5^8 + C_6^8 + C_7^9 + C_8^{10} + C_2^{11}$$

¿Cuánto le costó la licuadora a Pedro?

### RESOLUCIÓN

$$T = C_5^8 + C_6^8 + C_7^9 + C_8^{10} + C_2^{11}$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$

$$T = C_6^9 + C_7^9 + C_8^{10} + C_2^{11}$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$

$$\Rightarrow T = C_7^{10} + C_8^{10} + C_2^{11}$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$

$$T = C_8^{11} + C_2^{11}$$

$$\Rightarrow T = C_8^{11} + C_9^{11}$$

$$C_k^n = C_{n-k}^n$$

$$C_k^n + C_{k+1}^n = C_{k+1}^{n+1}$$

$$\Rightarrow T = C_9^{12}$$

$$C_k^n = \frac{n!}{k!(n-k)!}$$

$$\Rightarrow T = C_9^{12} = \frac{12!}{9!(3)!} = \frac{12(11)(10)9!}{9!(3)(2)(1)} = 220$$

*El costo de la licuadora = S/. 440*

## HELICO | PRACTICE

**7.** Halle el valor de M en:

$$M = \frac{3C_2^{11} - 5C_9^{11} + 7C_2^{11}}{C_9^{11}}$$

Si 3M representa la edad de Arturito.  
¿Cuál será su edad dentro de 5 años?

### RESOLUCIÓN

$$C_k^n = C_{n-k}^n$$

$$M = \frac{3C_2^{11} - 5C_9^{11} + 7C_2^{11}}{C_9^{11}}$$

$$C_k^n = C_{n-k}^n$$

$$\Rightarrow M = \frac{3C_2^{11} - 5C_2^{11} + 7C_2^{11}}{C_2^{11}}$$

$$\Rightarrow M = \frac{5C_2^{11}}{C_2^{11}} \Rightarrow M = 5$$

*Su edad dentro de 5 años sera: 20 años*