GEOMETRÍA

Capítulo 11

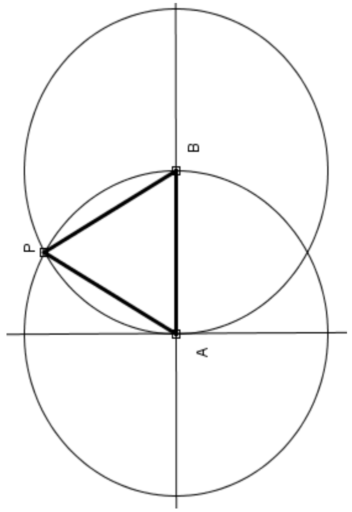
5th

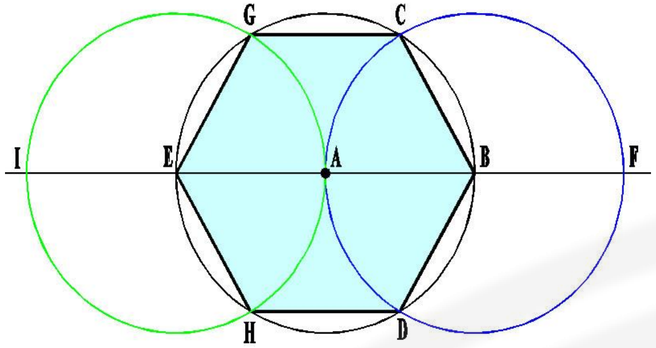
SECONDARY

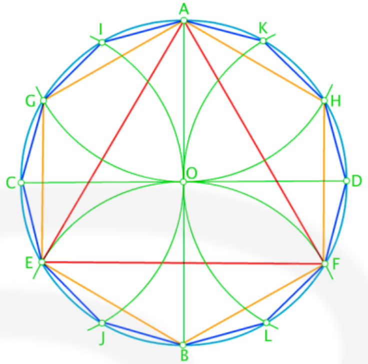
POLÍGONOS REGULARES

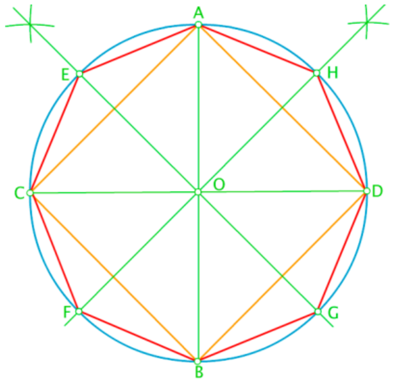
SACO OLIVEROS

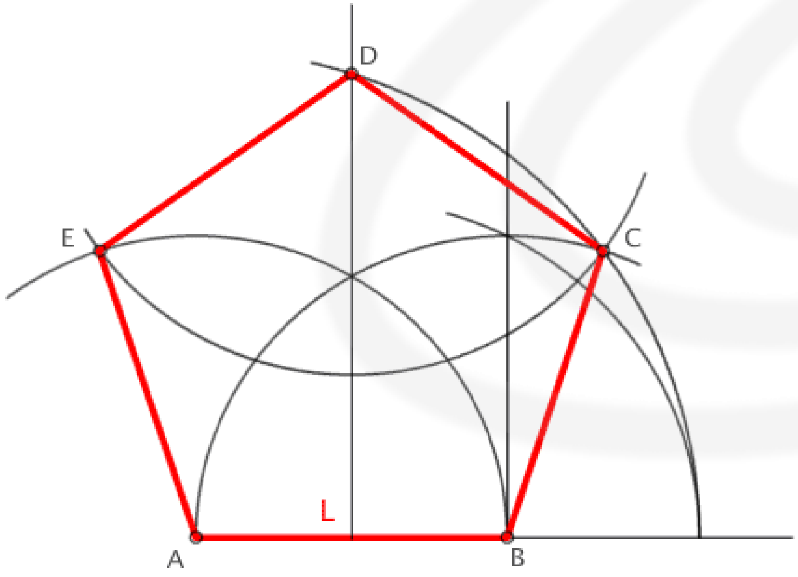
MOTIVATING | STRATEGY

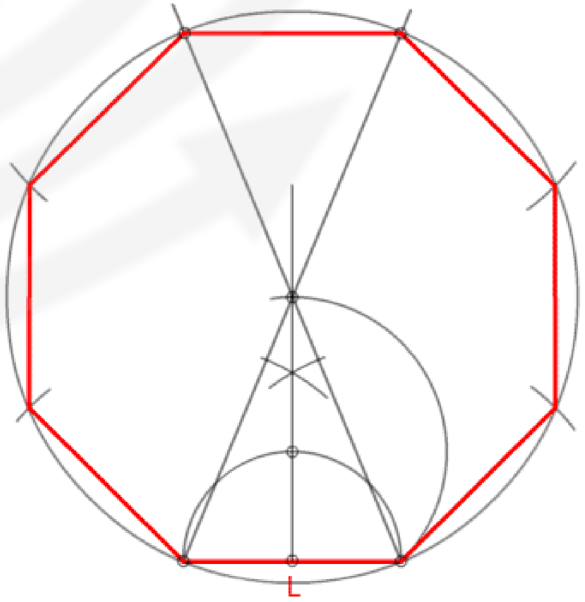






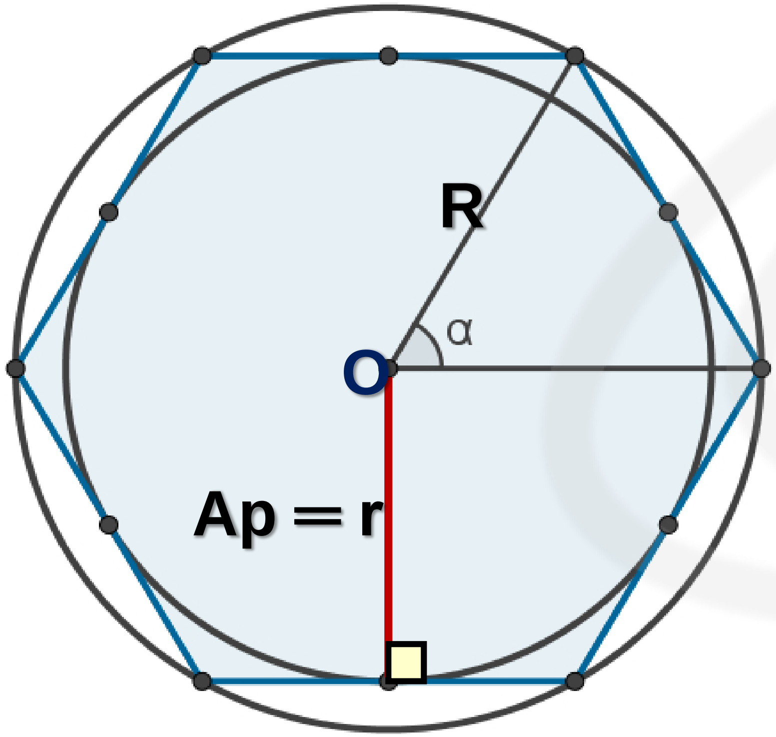






Polígonos Regulares

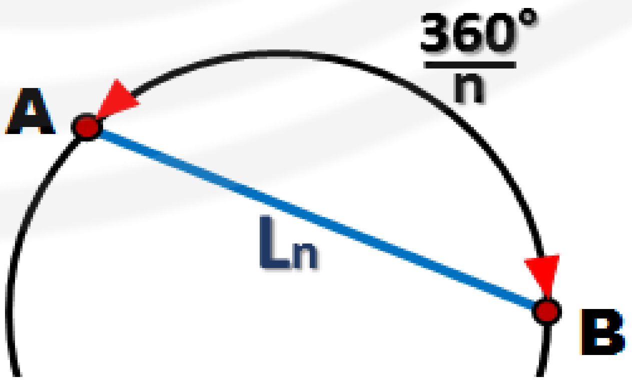
Se denomina polígono regular al polígono convexo que es equiángulo y equilátero a la vez. Todo polígono regular se puede inscribir y circunscribir a dos circunferencias concéntricas, siendo el centro de estas el centro del polígono regular.



- Para todo polígono regular, el cálculo de la medida del ángulo central es:

$$m\sphericalangle central = \frac{360°}{n}$$

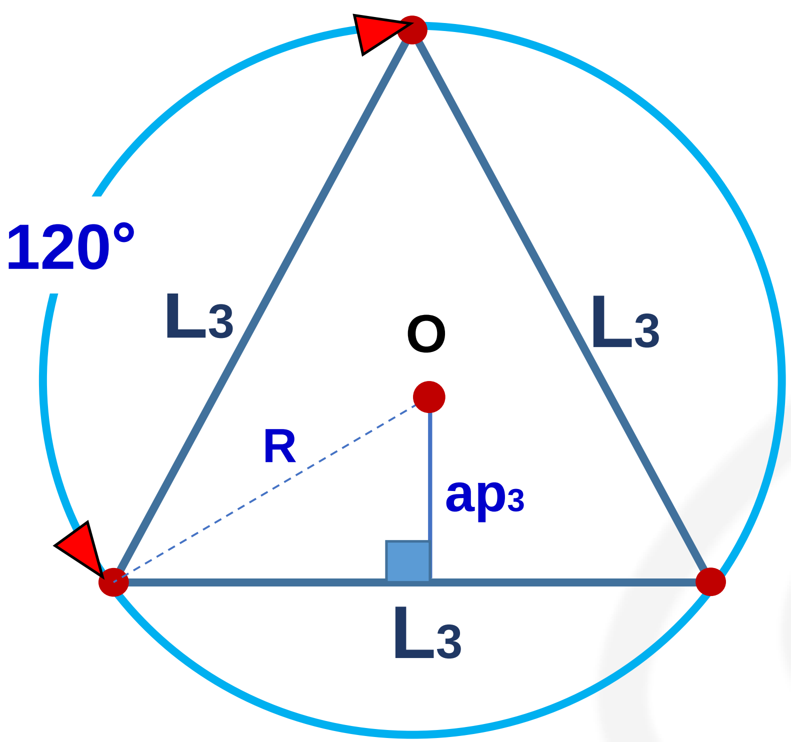
- Si: AB = Ln



$$m\overset{\frown}{AB} = \frac{360°}{n}$$

Principales Polígonos Regulares

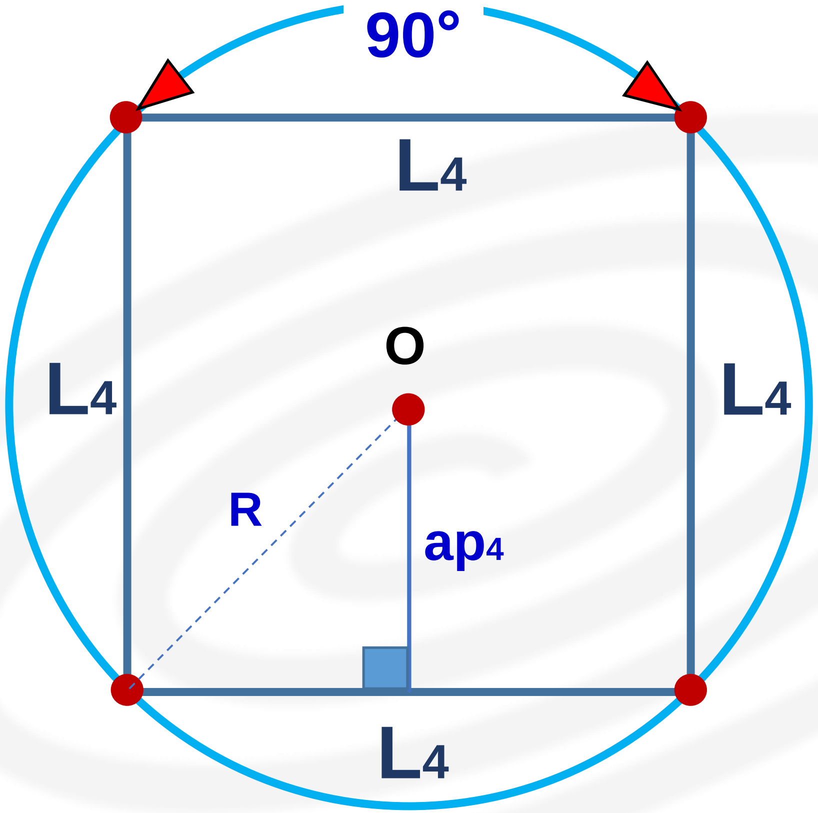
TRIÁNGULO EQUILÁTERO



$L_3 = R\sqrt{3}$

$ap_3 = R/2$

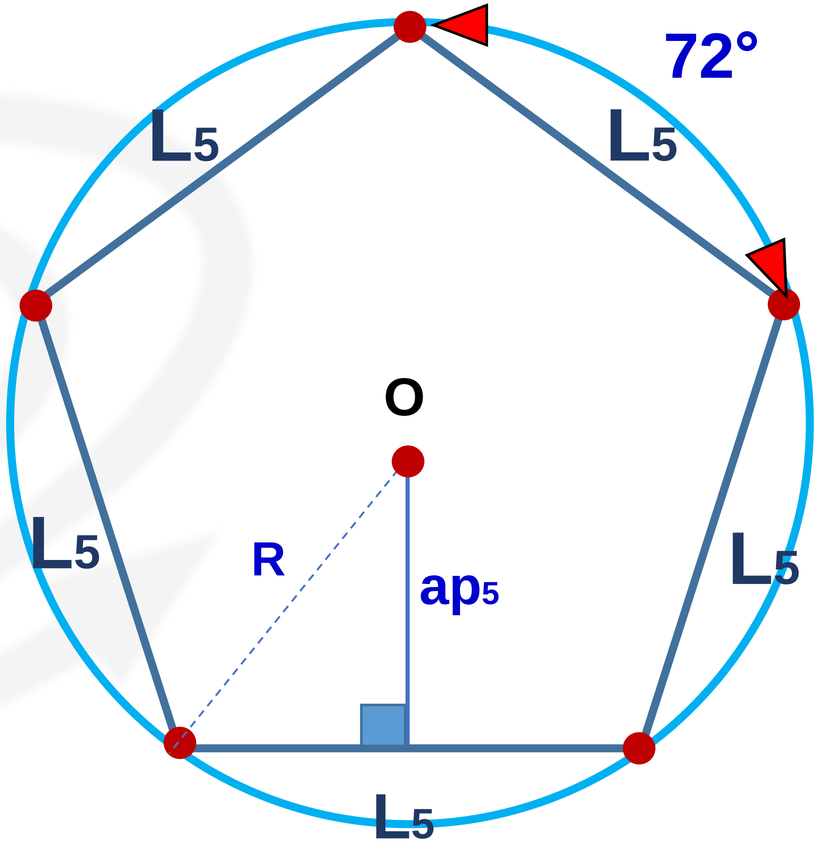
CUADRADO



$L_4 = R\sqrt{2}$

$ap_4 = R\sqrt{2}/2$

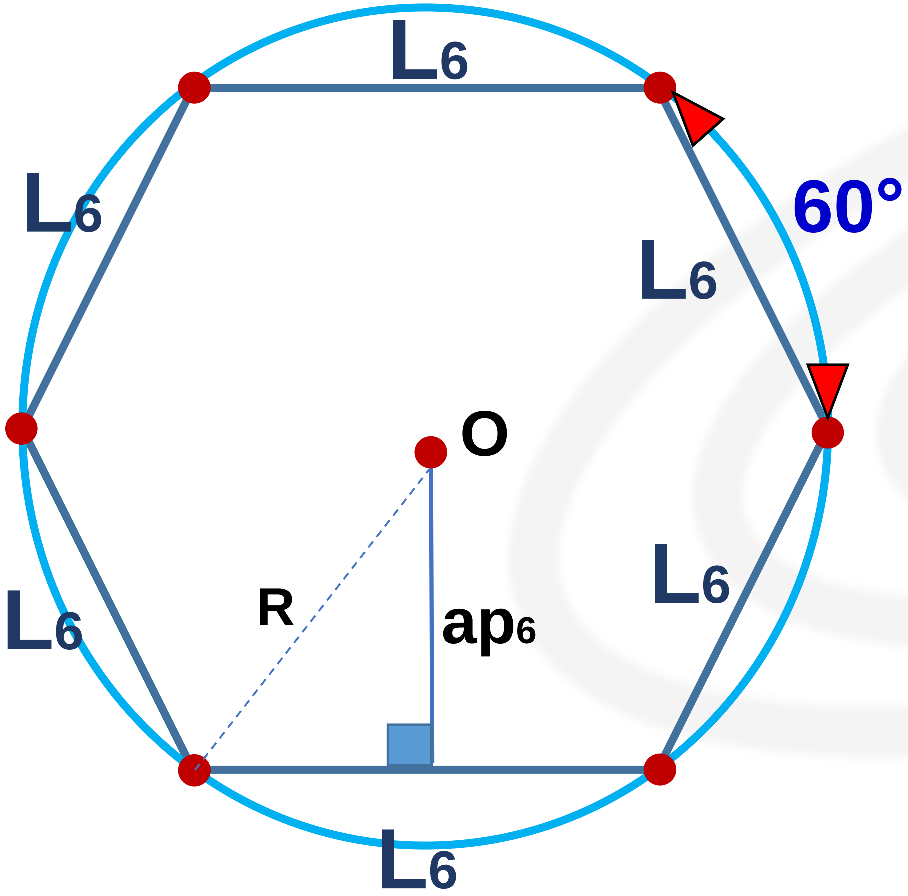
PENTÁGONO REGULAR



$L_5 = \frac{R}{2}.\sqrt{10-2\sqrt{5}}$

$ap_5 = \frac{R}{4}.(\sqrt{5}+1)$

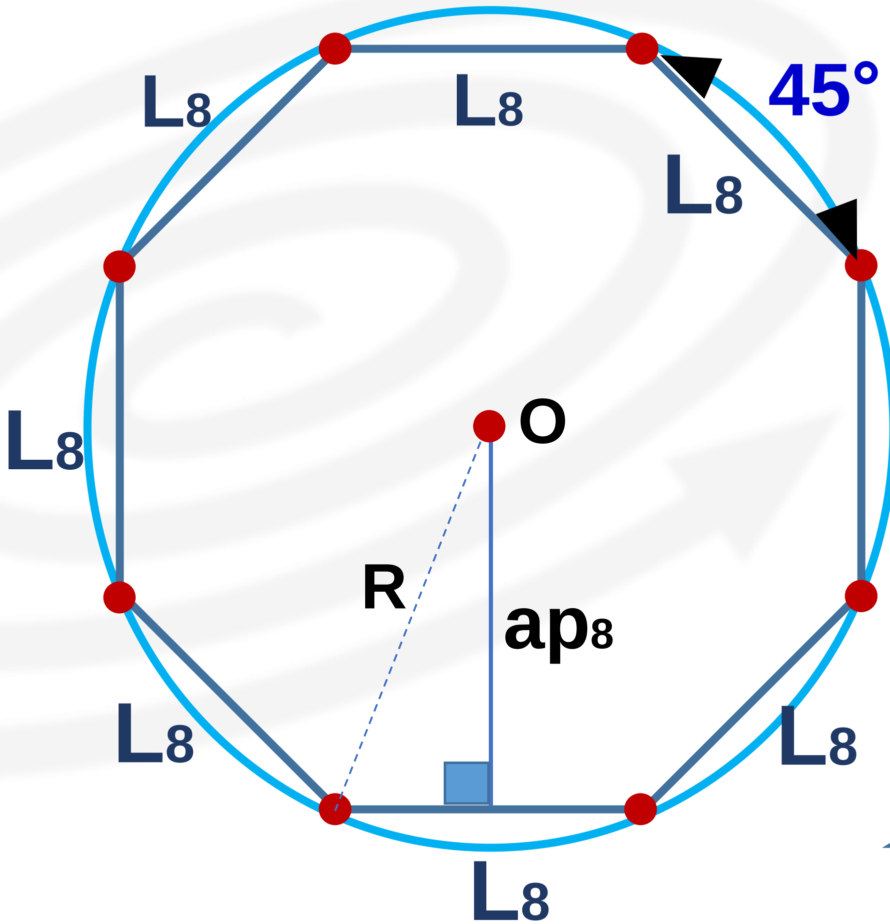
HEXÁGONO REGULAR



$$L_6 = R$$

$$ap_6 = \frac{R\sqrt{3}}{2}$$

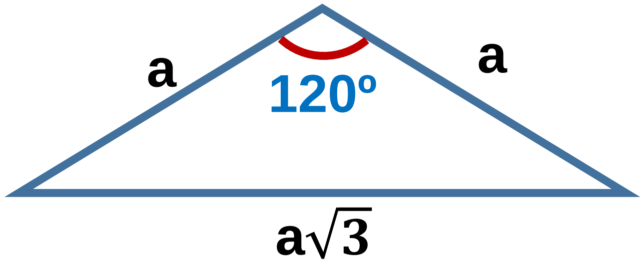
OCTÁGONO REGULAR



$$L_8 = R\sqrt{2-\sqrt{2}}$$

$$ap_8 = \frac{R}{2}\sqrt{2+\sqrt{2}}$$

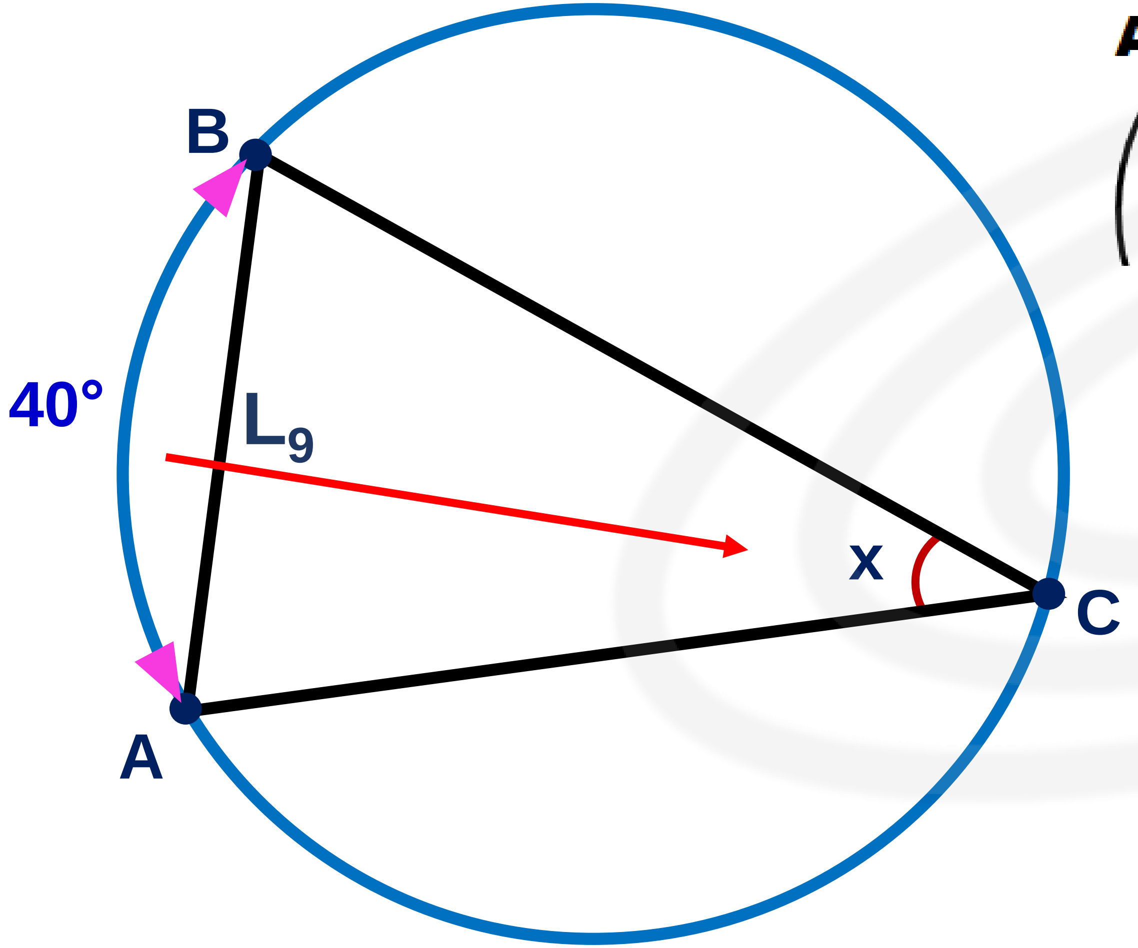
Teorema de Triángulo isósceles

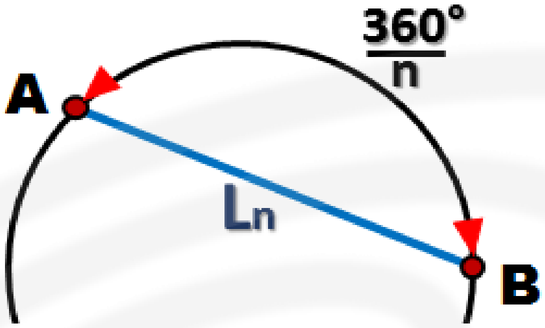


1. Se tiene un triángulo ABC inscrito en una circunferencia, tal que AB = L9. Halle la m∡ACB.

Piden: m∡ACB = x

Resolución



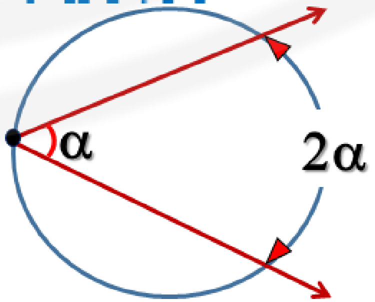


$$m\overset{\frown}{AB} = \frac{360°}{n}$$

$$n = 9$$

- $$m\overset{\frown}{AB} = \frac{360°}{9}$$

$$\boxed{m\overset{\frown}{AB} = 40°}$$



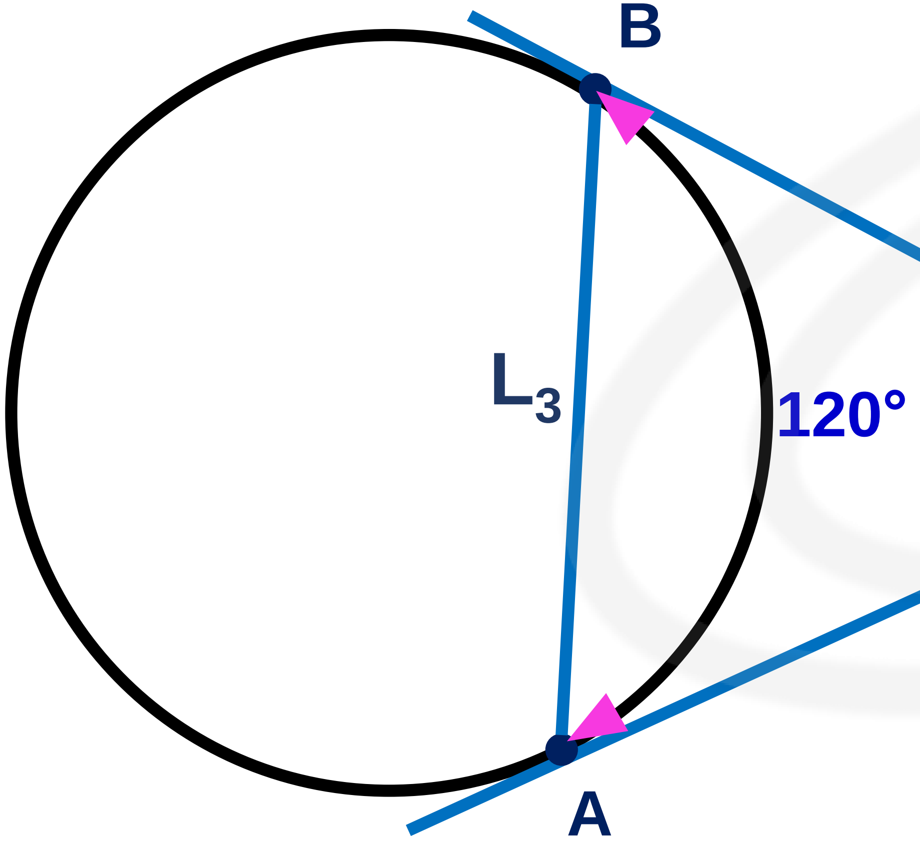
$$x = \frac{40°}{2}$$

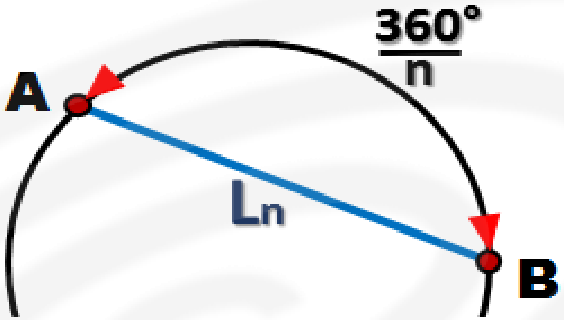
$$\boxed{x = 20°}$$

2. Desde un punto C exterior a una circunferencia, se trazan los segmentos tangentes CA y CB. Si AB = L3, halle la m∡ACB.

Resolución

Piden: m∡ACB = x



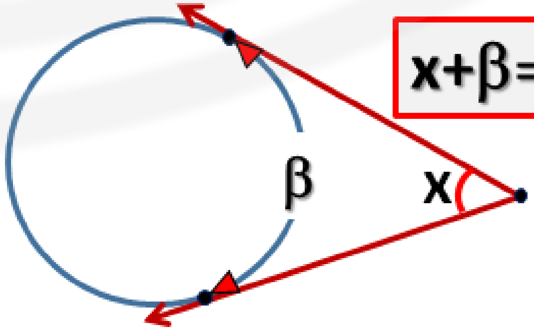


$m\overset{\frown}{AB} = \dfrac{360°}{n}$

$n = 3$

- $m\overset{\frown}{AB} = \dfrac{360°}{3}$

$\boxed{m\overset{\frown}{AB} = 120°}$



$x + \beta = 180°$

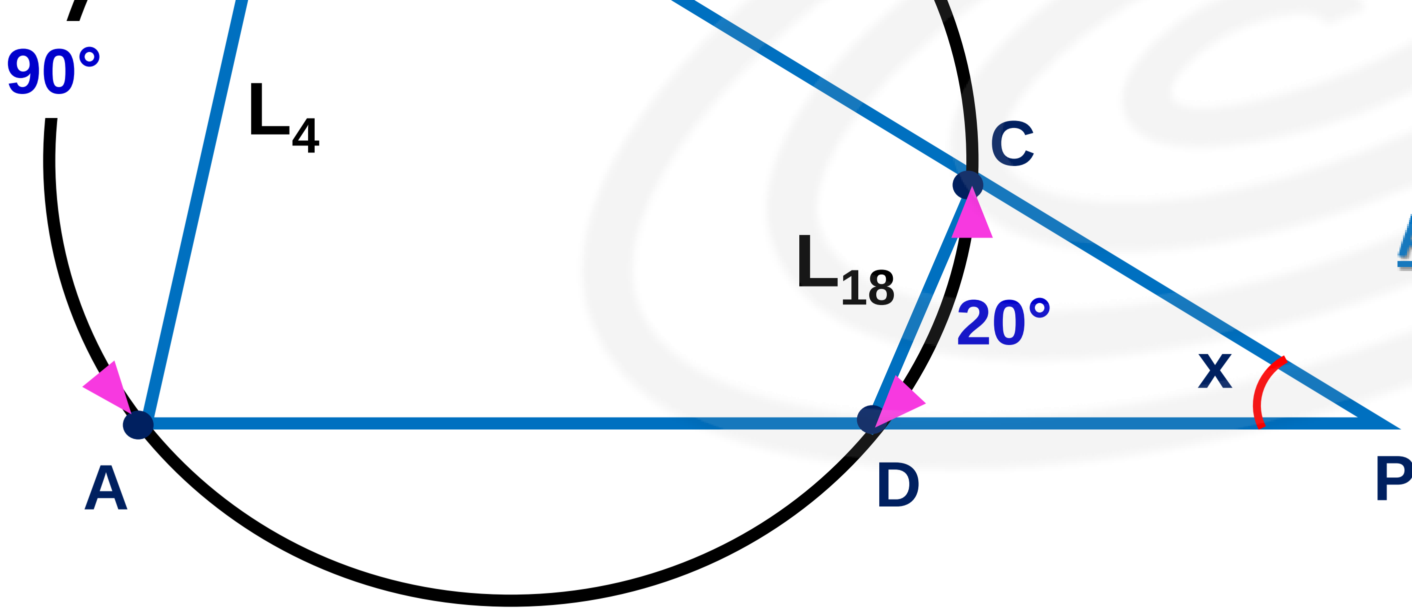
$x + 120° = 180°$

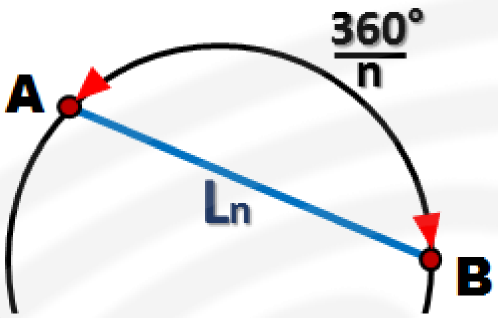
$\boxed{x = 60°}$

3. Halle el valor de x, si $AB = L_4$ y $CD = L_{18}$.

Resolución

Piden: el valor de x





$n = 4$

$m\overset{\frown}{AB} = \frac{360°}{4}$

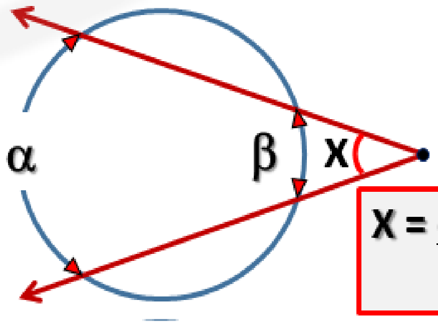
$\boxed{m\overset{\frown}{AB} = 90°}$

$n = 18$

$m\overset{\frown}{CD} = \frac{360°}{18}$

$\boxed{m\overset{\frown}{CD} = 20°}$

Ángulo exterior



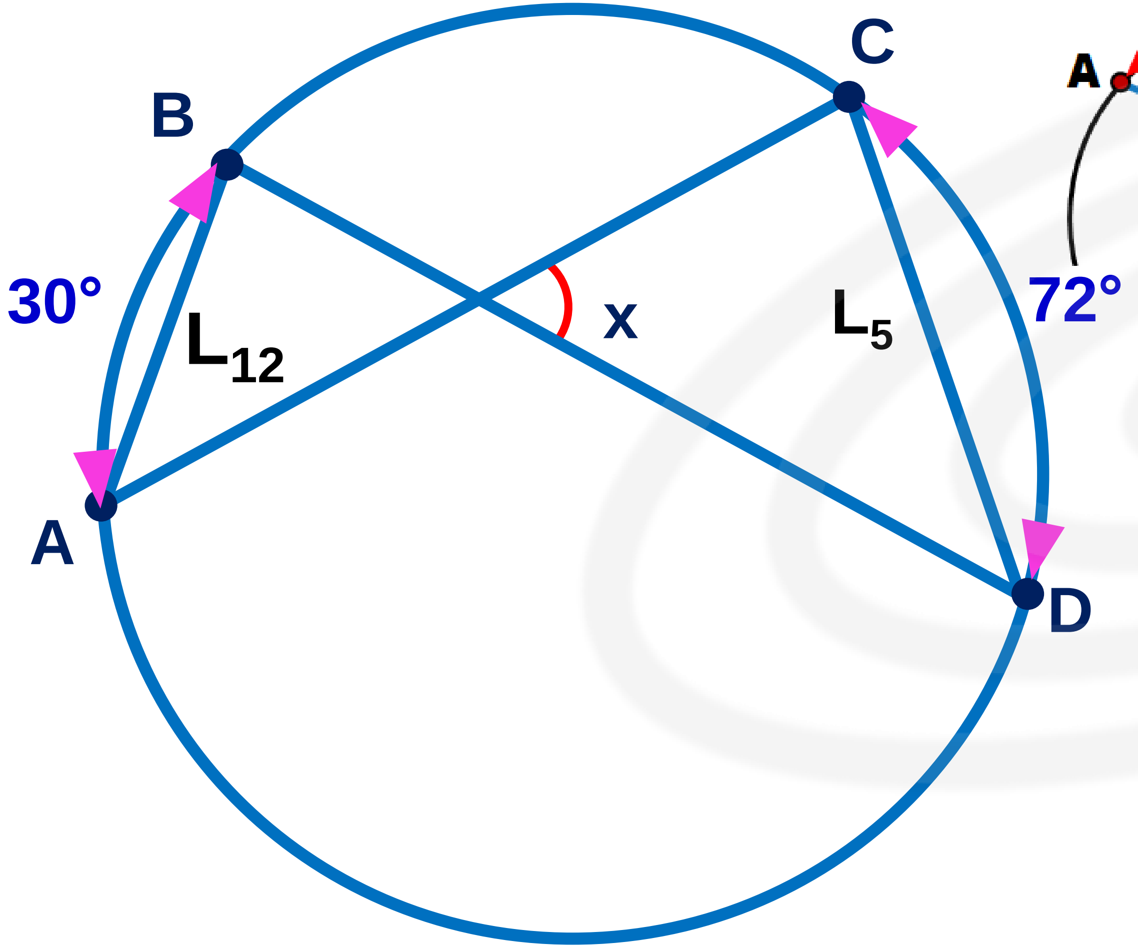
$\boxed{x = \frac{\alpha - \beta}{2}}$

$x = \frac{90° - 20°}{2}$

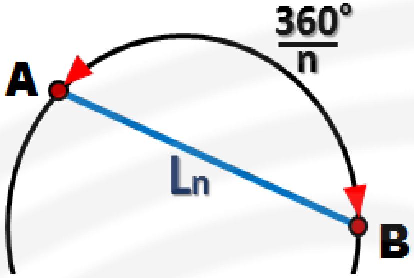
$\boxed{x = 35°}$

4. Halle el valor de x, si AB = L12 y CD = L5.

Resolución



Piden: el valor de x



$\boxed{n = 12}$

$$m\overset{\frown}{AB} = \frac{360°}{12}$$

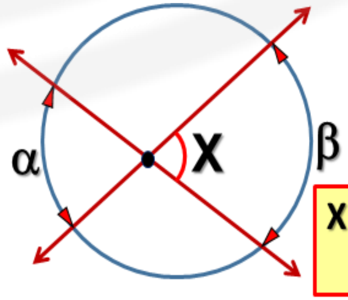
$$\boxed{m\overset{\frown}{AB} = 30°}$$

$\boxed{n = 5}$

$$m\overset{\frown}{CD} = \frac{360°}{5}$$

$$\boxed{m\overset{\frown}{CD} = 72°}$$

Ángulo interior



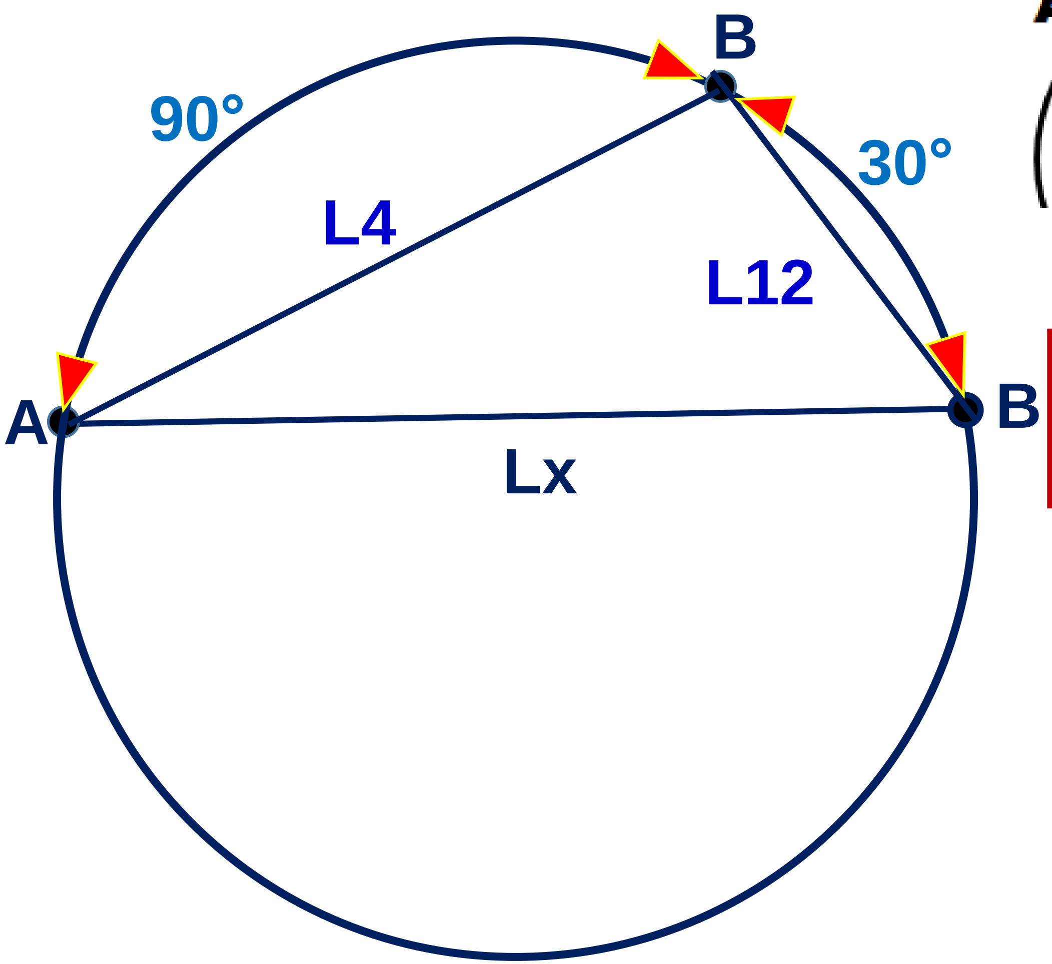
$$\boxed{X = \frac{\alpha+\beta}{2}}$$

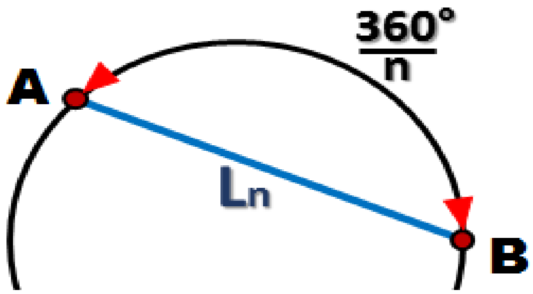
$$x = \frac{30° + 72°}{2}$$

$$\boxed{x = 51°}$$

5. Halle el valor de x, si AB = L4 y BC = L12.

Resolución





$$m\overset{\frown}{AB} = \frac{360°}{n}$$

n = 4

$$m\overset{\frown}{AB} = \frac{360°}{4}$$

$$\boxed{m\overset{\frown}{AB} = 90°}$$

n = 12

$$m\overset{\frown}{BC} = \frac{360°}{12}$$

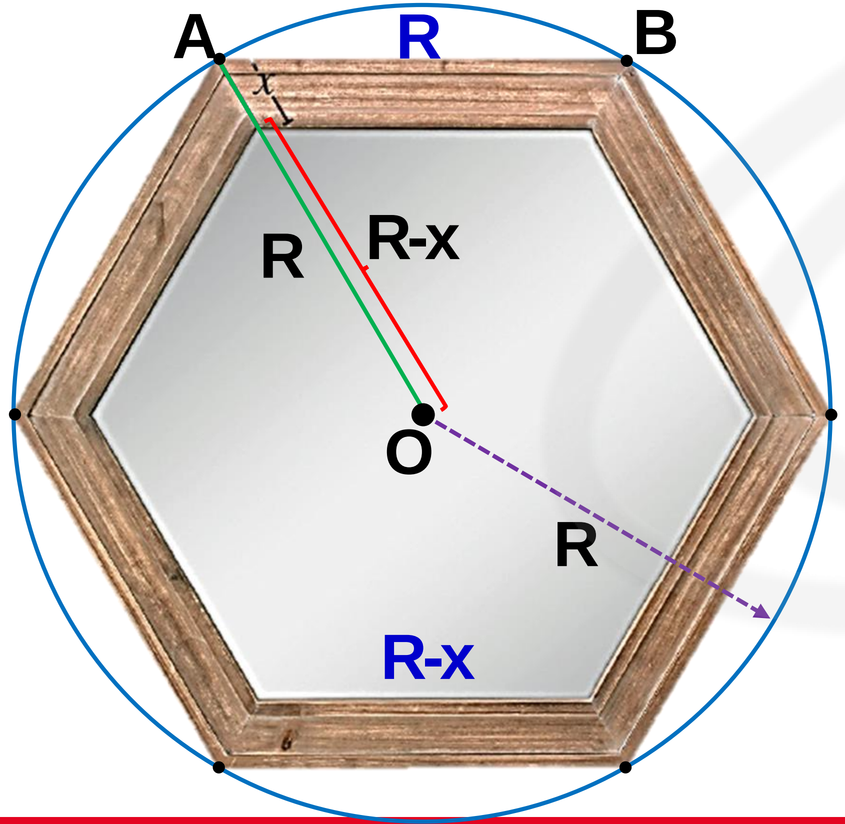
$$\boxed{m\overset{\frown}{BC} = 30°}$$

- **Para L x :** $120° = \dfrac{360°}{x}$

$$\boxed{x = 3}$$

6. En la figura se muestra el marco de un espejo, el cual tiene en sus bordes interior y exterior a dos hexágonos regulares de lados paralelos. Si la diferencia entre los perímetros de dichos hexágonos es 24 cm; calcule el valor de *x*.

RESOLUCIÓN

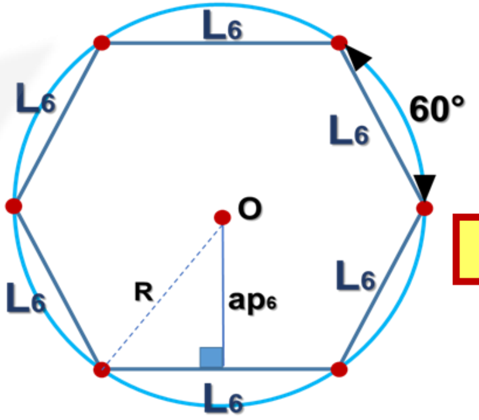


- Sea O el centro de los hexágonos regulares.

  R: longitud del circunradio del hexágono mayor

  R-x: longitud del circunradio del hexágono menor

- Por teoría del hexágono regular.



$L_6 = R$

- Por dato:

$$2P_{Hex.\ Mayor} - 2P_{Hex.\ Menor} = 24$$

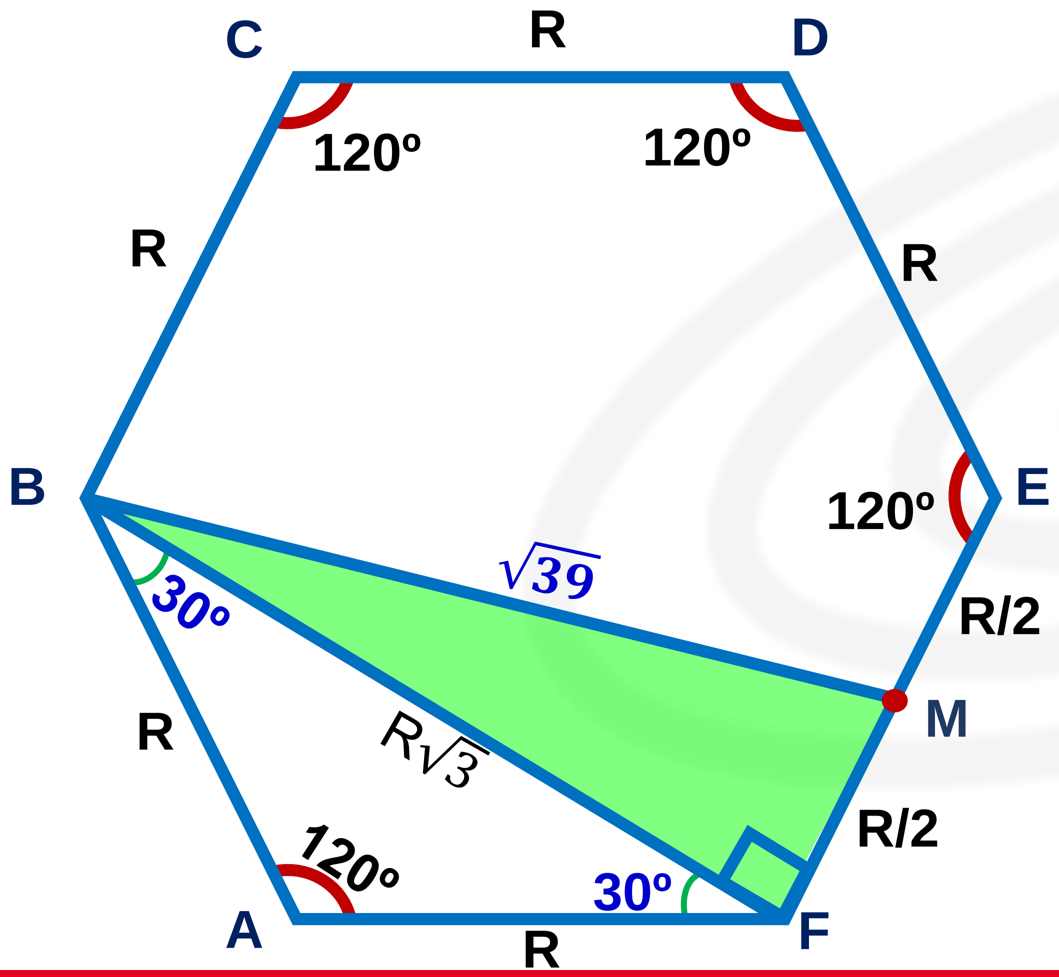
$$6R - 6(R-x) = 24$$

$$\cancel{6R} - \cancel{6R} + 6x = 24$$

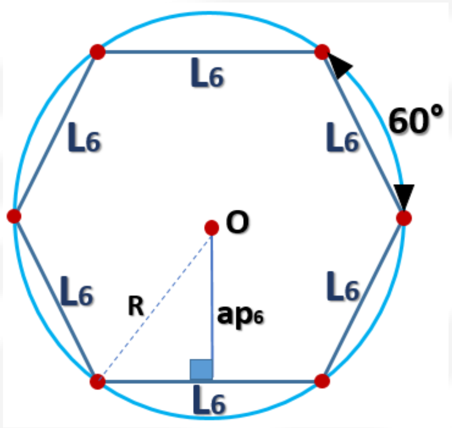
$$6x = 24$$

$$\boxed{x = 4\text{ cm}}$$

7. Una de las plazas de la ciudad de Chiclayo tiene en su contorno a un hexágono regular ABCDEF. Si M es punto medio de $\overline{FE}$ y $BM = \sqrt{39}$. Calcular la longitud de la apotema de dicho hexágono.

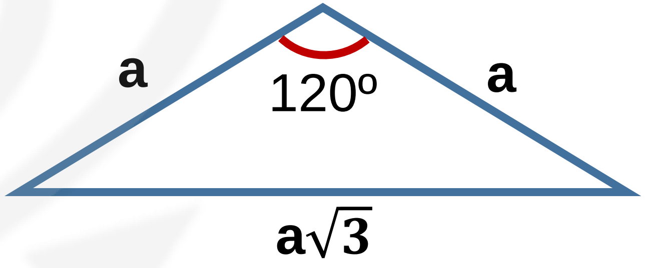


HEXÁGONO REGULAR



$$L6 = R$$

$$Ap6 = \frac{R\sqrt{3}}{2}$$



◺ BFM : T. Pitágoras

$$(\sqrt{39})^2 = (R\sqrt{3})^2 + (R/2)^2$$

$$39 = 3R^2 + R^2/4$$

$$39 = 13R^2/4$$

$$12 = R^2$$

$$2\sqrt{3} = R$$

Nos piden

$$Ap6 = \frac{R\sqrt{3}}{2}$$

$$Ap6 = \frac{(\cancel{2}\cancel{\sqrt{3}})\sqrt{3}}{\cancel{2}}$$

$$\boxed{Ap_6 = 3}$$