



TRIGONOMETRY

TOMO 6

5th
SECONDARY

REVIEW



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HELICOREVIEW 1

Simplifique la expresión $E = 3(\cot x - \tan x)\tan 2x$

Resolución:

RECORDAR



$$\cot x - \tan x = 2\cot(2x)$$

$$E = 3 \underbrace{(\cot x - \tan x)}_{2\cot 2x} \tan 2x$$

$$E = 6 \underbrace{\cot 2x \cdot \tan 2x}_1$$

$$\therefore E = 6$$





HELICOREVIEW 2

Al copiar de la pizarra la expresión $1 + \cos 40^\circ$, un estudiante cometió un error y escribió $\sin 40^\circ$. Calcule la razón entre lo que estaba escrito en la pizarra y lo que copió el estudiante.

Resolución:

RECORDAR

$$2\cos^2(x) = 1 + \cos(2x)$$

Debió escribir $\rightarrow \frac{2\cos^2 20^\circ}{1 + \cos 40^\circ}$

Escribió $\rightarrow \frac{\sin 40^\circ}{2\sin 20^\circ \cos 20^\circ}$

Luego:

$$\frac{\cancel{2}\cos^{\cancel{2}} 20^\circ}{\cancel{2}\sin 20^\circ \cancel{\cos} 20^\circ} = \frac{\cos 20^\circ}{\sin 20^\circ}$$

$$\therefore \cot 20^\circ$$



HELICOREVIEW 3

Simplifique $M = \sqrt{2 + \sqrt{2 - 2\cos 80^\circ}}$

Resolución:

$$M = \sqrt{2 + \sqrt{2 - 2\cos 80^\circ}}$$

$$M = \sqrt{2 + \sqrt{2(1 - \cos 80^\circ)}}$$

$$M = \sqrt{2 + \sqrt{2(2\sin^2 40^\circ)}}$$

$$M = \sqrt{2 + \sqrt{4\sin^2 40^\circ}}$$

RECORDAR



$$2\sin^2(x) = 1 - \cos(2x)$$

$$2\cos^2(x) = 1 + \cos(2x)$$

$$M = \sqrt{2 + 2\sin 40^\circ}$$

$$M = \sqrt{2(1 + \sin 40^\circ)}$$

$$M = \sqrt{2(1 + \cos 50^\circ)}$$

$$M = \sqrt{2(2\cos^2 25^\circ)}$$

$$M = \sqrt{4\cos^2 25^\circ}$$

$$\therefore M = 2\cos 25^\circ$$



HELICOREVIEW 4

Reduzca $H = \sqrt{\frac{1 + \cos 140^\circ}{2}} + \frac{\sin 40^\circ}{2\cos 20^\circ}$

Resolución:

RECORDAR



$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$H = \sqrt{\frac{1 + \cos 140^\circ}{2}} + \frac{\sin 40^\circ}{2\cos 20^\circ}$$

$$H = \cos 70^\circ + \frac{\cancel{2}\sin 20^\circ \cdot \cancel{\cos 20^\circ}}{\cancel{2\cos 20^\circ}}$$

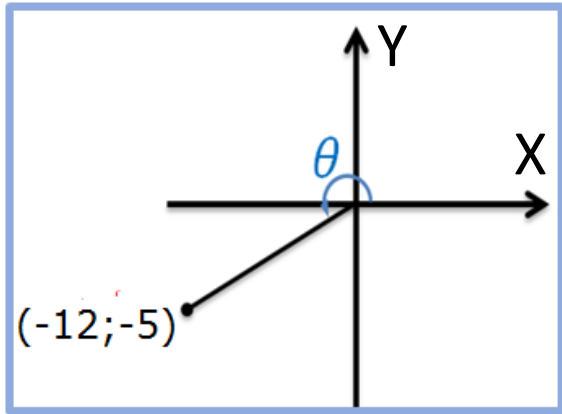
$$H = \cos 70^\circ + \underbrace{\sin 20^\circ}_{\cos 70^\circ}$$

$$H = \cos 70^\circ + \cos 70^\circ$$

$$\therefore H = 2\cos 70^\circ$$



Del gráfico, calcule: $\tan\left(\frac{\theta}{2}\right)$



Resolución:

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Del gráfico se observa:

$$180^\circ < \theta < 270^\circ$$

$$\rightarrow 90^\circ < \frac{\theta}{2} < 135^\circ$$

$$\theta \in \text{IIC}$$

Además:

$$x = -12 ; y = -5$$

$$r = \sqrt{(-12)^2 + (-5)^2}$$

$$\rightarrow r = 13$$

$$\cos \theta = \frac{x}{r} = -\frac{12}{13}$$

$$\tan\left(\frac{\theta}{2}\right) = \overset{\text{IIC}}{\uparrow} \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\tan\left(\frac{\theta}{2}\right) = - \sqrt{\frac{1 - \left(-\frac{12}{13}\right)}{1 + \left(-\frac{12}{13}\right)}}$$

$$\tan\left(\frac{\theta}{2}\right) = - \sqrt{\frac{\frac{25}{13}}{\frac{1}{13}}} = -\sqrt{25}$$

$$\therefore \tan\left(\frac{\theta}{2}\right) = -5$$



HELICOREVIEW 6

Reduzca la expresión: $P = \frac{\cot\left(\frac{x}{2}\right) - \csc x}{\csc x - \tan\left(\frac{x}{2}\right)}$

Resolución:

RECORDAR



$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

$$P = \frac{\cot\left(\frac{x}{2}\right) - \csc x}{\csc x - \tan\left(\frac{x}{2}\right)}$$

$$P = \frac{\csc x + \cot x - \csc x}{\csc x - (\csc x - \cot x)}$$

$$P = \frac{\cancel{\csc x} + \cot x - \cancel{\csc x}}{\cancel{\csc x} - \cancel{\csc x} + \cot x}$$

$$P = \frac{\cancel{\cot x}}{\cot x}$$

$$\therefore P = 1$$



HELICOREVIEW 7

Reduzca: $T = \frac{4\cos^3 20^\circ - 3\cos 20^\circ}{3\sin 15^\circ - 4\sin^3 15^\circ}$

Resolución:

RECORDAR



$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$T = \frac{4\cos^3 20^\circ - 3\cos 20^\circ}{3\sin 15^\circ - 4\sin^3 15^\circ}$$

$$T = \frac{\cos 3(20^\circ)}{\sin 3(15^\circ)}$$

$$T = \frac{\cos 60^\circ}{\sin 45^\circ}$$

$$T = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}$$

$$\therefore T = \frac{\sqrt{2}}{2}$$



HELICOREVIEW 8

De la condición: $\text{sen}x - \text{cos}x = \frac{\sqrt{2}}{2}$; calcule $\text{sen}6x$.

Resolución:

Dato:

$$\text{sen}x - \text{cos}x = \frac{\sqrt{2}}{2}$$

Elevamos al cuadrado:

$$(\text{sen}x - \text{cos}x)^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

$$1 - \text{sen}2x = \frac{2}{4}$$

$$\text{sen}2x = \frac{1}{2}$$

Calculamos:

$$\text{sen}6x = \text{sen}3(2x)$$

$$\text{sen}6x = 3\text{sen}2x - 4\text{sen}^3 2x$$

$$\text{sen}6x = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$\text{sen}6x = \frac{3}{2} - \frac{1}{2}$$

$$\therefore \text{sen}6x = 1$$

RECORDAR

$$(\text{sen}x - \text{cos}x)^2 = 1 - \text{sen}2x$$

$$\text{sen}3\alpha = 3\text{sen}\alpha - 4\text{sen}^3\alpha$$



HELICOREVIEW 9

De la siguiente identidad: $\frac{3\text{sen}3x}{\text{sen}x} - \frac{2\text{cos}3x}{\text{cos}x} = M + N\text{cos}(Px)$

Calcule: $M + N + P$

Resolución:

Dato:

$$\frac{3\text{sen}3x}{\text{sen}x} - \frac{2\text{cos}3x}{\text{cos}x} = M + N\text{cos}(Px)$$

$$\frac{3\cancel{\text{sen}x}(2\text{cos}2x + 1)}{\cancel{\text{sen}x}} - \frac{2\cancel{\text{cos}x}(2\text{cos}2x - 1)}{\cancel{\text{cos}x}} = M + N\text{cos}(Px)$$

$$3(2\text{cos}2x + 1) - 2(2\text{cos}2x - 1) = M + N\text{cos}(Px)$$

$$6\text{cos}2x + 3 - 4\text{cos}2x + 2 = M + N\text{cos}(Px)$$

RECORDAR



$$\text{sen}3x = \text{sen}x(2\text{cos}2x + 1)$$

$$\text{cos}3x = \text{cos}x(2\text{cos}2x - 1)$$

$$5 + 2\text{cos}2x = M + N\text{cos}(Px)$$

Comparando:

$$M = 5 ; N = 2 ; P = 2$$

$$\therefore M + N + P = 9$$



Un científico observa el movimiento de una mariposa en el aire y ve que en un instante de tiempo t , la altura en metros respecto al suelo está dado por la siguiente expresión: $h(t)=16\text{sen}t\cos 2t\cos 4t\cos 8t$, si t está en segundos. ¿A qué altura se encuentra para $t = \frac{\pi}{30}$ seg?

Resolución:

$$h(t)=16\text{sen}t.\cos 2t.\cos 4t.\cos 8t$$

$$h(t).\text{cost}=8.\underbrace{2\text{sen}t.\text{cost}}_{\text{sen}2t}.\cos 2t.\cos 4t.\cos 8t$$

$$h(t).\text{cost}=4.2\underbrace{\text{sen}2t.\cos 2t}_{\text{sen}4t}.\cos 4t.\cos 8t$$

$$h(t).\text{cost}=2.2\underbrace{\text{sen}4t.\cos 4t}_{\text{sen}8t}\cos 8t$$

$$h(t).\text{cost}=2.\text{sen}8t\cos 8t$$

$$h(t).\text{cost}=\text{sen}16t$$

$$h\left(\frac{\pi}{30}\right).\text{cost}6^\circ = \text{sen}96^\circ$$

IIC

$$h\left(\frac{\pi}{30}\right).\text{cost}6^\circ = \text{sen}(90^\circ + 6^\circ)$$

$$h\left(\frac{\pi}{30}\right).\text{cost}6^\circ = \text{cos}6^\circ \Rightarrow h\left(\frac{\pi}{30}\right) = 1$$

Dato:

$$t = \frac{\pi}{30} = 6^\circ$$

∴ La altura de la mariposa para $t = \frac{\pi}{30}$ seg es 1m.