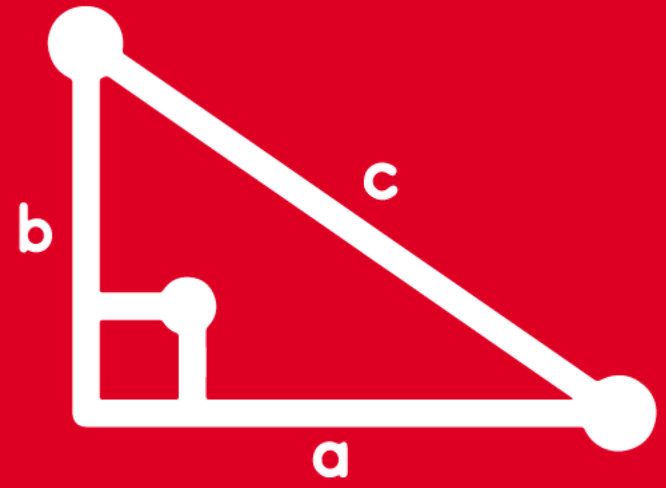
TRIGONOMETRY

Chapter 7, 8 and 9

5th
SECONDARY



REVIEW

SACO OLIVEROS

HELICOREVIEW 1

Siendo θ y β las medidas de dos ángulos cuadrantales diferentes, positivos y menores o iguales a 360°, se cumple que

$$\sqrt{1-\operatorname{sen}\theta}+\sqrt{\operatorname{sen}\theta-1}=1+\cos\beta \;\ldots\ldots\; (*)$$

calcule θ + β.

Resolución:

$$1-\operatorname{sen}\theta \geq 0 \quad \wedge \quad \operatorname{sen}\theta-1 \geq 0$$

$$\underbrace{1 \geq \operatorname{sen}\theta \quad \wedge \quad \operatorname{sen}\theta \geq 1}$$

$$\operatorname{sen}\theta = 1$$

$$\text{como } 0°<\theta \leq 360°$$

$$\Rightarrow \theta = 90°$$

Reemplazando en (*)

$$\underbrace{\cancel{\sqrt{1-\operatorname{sen}\theta}}}_{0}+\underbrace{\cancel{\sqrt{\operatorname{sen}\theta-1}}}_{0}=1+\cos\beta$$

Recordar

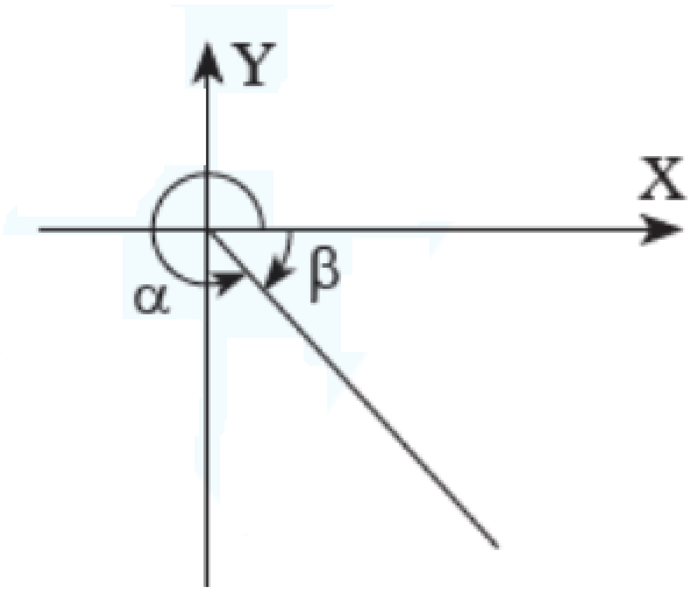
RT \ ∢	0°	90°	180°	270°	360°
sen	0	1	0	–1	0
cos	1	0	–1	0	1

HELICOREVIEW 2

En la figura, se cumple que

$\cot\alpha.\cot\beta + \cos\alpha.\sec\beta = 10$. Calcule $\cot\alpha$

Resolución:



α y $\beta \in$ IVC

Del gráfico se observa que α y β son las medidas de dos ángulos coterminales, se cumple:

$$Rt(\alpha) = Rt(\beta)$$

Recordar

sen, csc (+)	Todas las RT son (+)
tan, cot (+)	cos, sec (+)

Del dato:

$$\cot\alpha.\cot\beta + \cos\alpha.\sec\beta = 10$$

$$\cot\alpha.\cot\alpha + \underbrace{\cos\alpha.\sec\alpha}_{1} = 10$$

$$\cot^2\alpha + 1 = 10$$

$$\cot^2\alpha = 9 \Rightarrow \cot\alpha = \pm 3$$

Como $\alpha \in$ IVC

$$\therefore \boxed{\cot\alpha = -3}$$

HELICOREVIEW 3

Si $\cos\theta > 0$, además $16^{\cot\theta} = 0,25$, efectúe

$$M = \sqrt{5}(\operatorname{sen}\theta - \cos\theta)$$

Recordar

sen, csc } (+) | Todas las RT son (+)
tan, cot } (+) | cos, sec } (+)

Resolución:

Del dato:

$$16^{\cot\theta} = \frac{1}{4}$$

$$4^{2\cot\theta} = 4^{-1}$$

$$2\cot\theta = -1$$

$$\cot\theta = -\frac{1}{2}$$

Como $\cos\theta$ es $(+)$ y $\cot\theta$ es $(-)$

$\theta \in \text{IVC}$ → $x(+), y(-), r(+)$

- $\cot\theta = \frac{1}{-2} = \frac{x}{y}$ → $x = 1, y = -2$

$r = \sqrt{1^2 + (-2)^2}$ → $r = \sqrt{5}$

$$\boxed{r = \sqrt{x^2 + y^2}}$$

Calculamos: $M = \sqrt{5}(\operatorname{sen}\theta - \cos\theta)$

$$M = \cancel{\sqrt{5}}\left(\frac{-2}{\cancel{\sqrt{5}}} - \frac{1}{\cancel{\sqrt{5}}}\right) = -2 - 1$$

$\therefore$ $\boxed{M = -3}$

HELICOREVIEW 4

Halle el valor de:

$$E = \frac{\operatorname{sen}330°.\cos120°}{\tan225°}$$

Resolución:

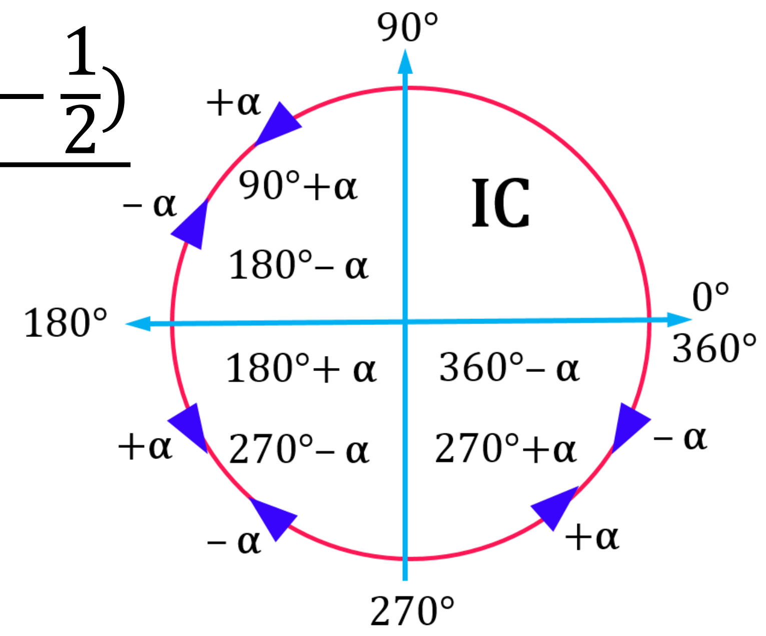
$$E = \frac{Sen\,\overbrace{(360° - 30°)}^{IVC}\,Cos\,\overbrace{(180° - 60°)}^{IIC}}{tan\underbrace{(180° + 45°)}_{IIIC}}$$

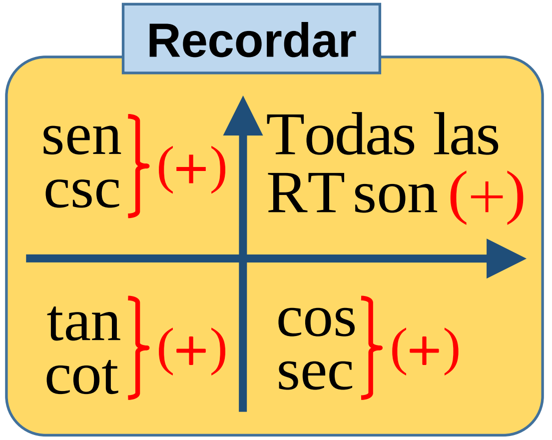
$$E = \frac{(-Sen\,30°)(-Cos\,60°)}{tan45°}$$

$$E = \frac{(-\frac{1}{2})(-\frac{1}{2})}{1}$$

$$E = \frac{\frac{1}{4}}{1}$$

$$\therefore \boxed{E = \frac{1}{4}}$$



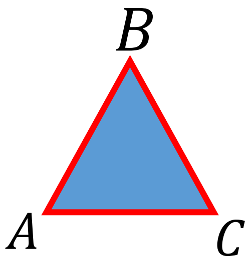


HELICOREVIEW 5

En un triángulo ABC, reduzca $M=\dfrac{\operatorname{sen}(B+C)}{\cos\left(\dfrac{3A+B+C}{2}\right)}$

Resolución:

Del dato:



$$A+B+C=180°$$

Calculamos:

$$M=\frac{sen(B+C)}{\cos\left(\frac{3A+B+C}{2}\right)}$$

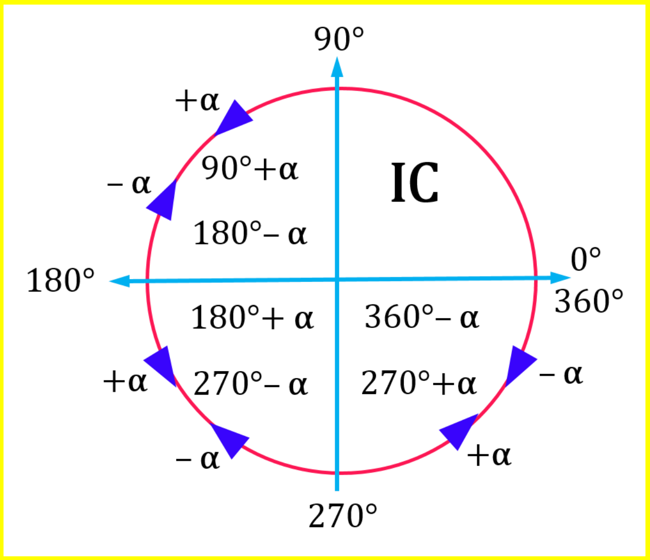
$$M=\frac{sen(B+C)}{\cos\left(\frac{3A+B+C}{2}\right)}$$

$$M=\frac{sen(180°-A)}{\cos\left(\frac{3A+180°-A}{2}\right)}$$

$$M=\frac{sen\overbrace{(180°-A)}^{IIC}}{\cos\left(\underbrace{90°+A}_{IIC}\right)}$$

$$M=\frac{\cancel{senA}}{-\cancel{senA}}$$

$$\therefore\ \boxed{M=-1}$$



Recordar:

sen, csc (+) | Todas las RT son (+)

tan, cot (+) | cos, sec (+)

HELICOREVIEW 6

Si $\alpha \in$ IVC, además $\operatorname{sen}(270° + \alpha) = -0{,}8$, reduzca

$T = \csc(180° - \alpha) + \tan(270° + \alpha)$

Resolución:

$$T = \underbrace{\overbrace{\csc(180° - \alpha)}^{IIC}}_{\csc\alpha} + \underbrace{\overbrace{\tan(270° + \alpha)}^{IVC}}_{-\cot\alpha}$$

$$T = \csc\alpha - \cot\alpha \quad \ldots(*)$$

Del dato:

$$sen(\overbrace{270° + \alpha}^{IVC}) = -0{,}8$$

$$\cancel{-}\cos\alpha = \cancel{-}\frac{4}{5}$$

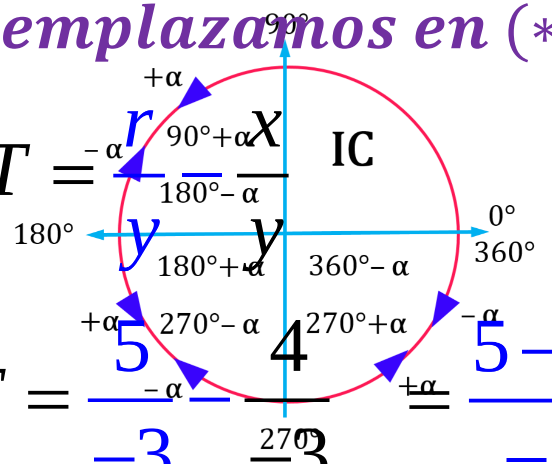
$$\cos\alpha = \frac{4}{5} = \frac{x}{r}$$

Por radio vector:

$$r = \sqrt{x^2 + y^2}$$

$$5 = \sqrt{4^2 + y^2} \rightarrow y = -3$$

Reemplazamos en $(*)$:

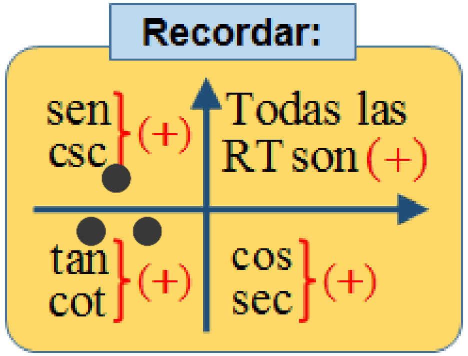


$$T = \frac{r}{y} - \frac{x}{y}$$

$$T = \frac{5}{-3} - \frac{4}{-3} = \frac{5-4}{-3}$$

$$T = \frac{1}{-3}$$

Recordar:



HELICOREVIEW 7

Efectúe

$$P = \frac{\cos 1470° . \operatorname{sen} 1140°}{\cot 3285°}$$

Recordar:

$$RT(\cancel{360°k} + x) = RT(x) \ ; k \in Z$$

Resolución:

$$\begin{array}{r|l} 1470 & 360 \\ \hline 1440 & 4 \\ \hline 30 & \end{array} \qquad \begin{array}{r|l} 1140 & 360 \\ \hline 1080 & 3 \\ \hline 60 & \end{array}$$

$$\begin{array}{r|l} 3285 & 360 \\ \hline 3240 & 9 \\ \hline 45 & \end{array}$$

$$P = \frac{\cos 30° . \operatorname{sen} 60°}{\cot 45°}$$

$$P = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{1}$$

$$\therefore \boxed{P = \frac{3}{4}}$$

HELICOREVIEW 8

Halle el valor de $E = sen\left(\frac{37\pi}{6}\right) + \cos\left(\frac{59\pi}{3}\right)$

Resolución:

Dando forma a los ángulos

$$E = \text{sen}\left(\frac{36\pi+\pi}{6}\right) + \cos\left(\frac{60\pi-\pi}{3}\right)$$

$$E = \text{sen}\left(\frac{36\pi}{6}+\frac{\pi}{6}\right) + \cos\left(\frac{60\pi}{3}-\frac{\pi}{3}\right)$$

$$E = \text{sen}\left(\underset{PAR}{6\pi}+\frac{\pi}{6}\right) + \cos\left(\underset{PAR}{20\pi}-\frac{\pi}{3}\right)$$

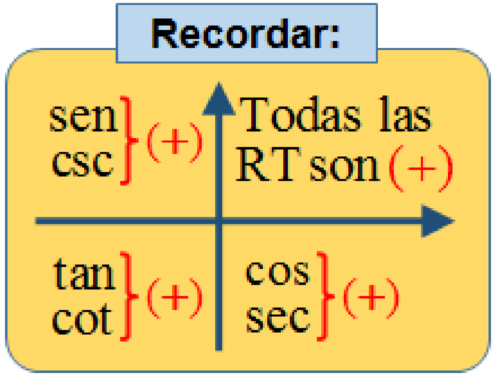
$$E = \underbrace{\text{sen}\overbrace{\left(6\pi+\frac{\pi}{6}\right)}^{IC}}_{sen\frac{\pi}{6}} + \underbrace{\cos\overbrace{\left(20\pi-\frac{\pi}{3}\right)}^{IVC}}_{\cos\frac{\pi}{3}}$$

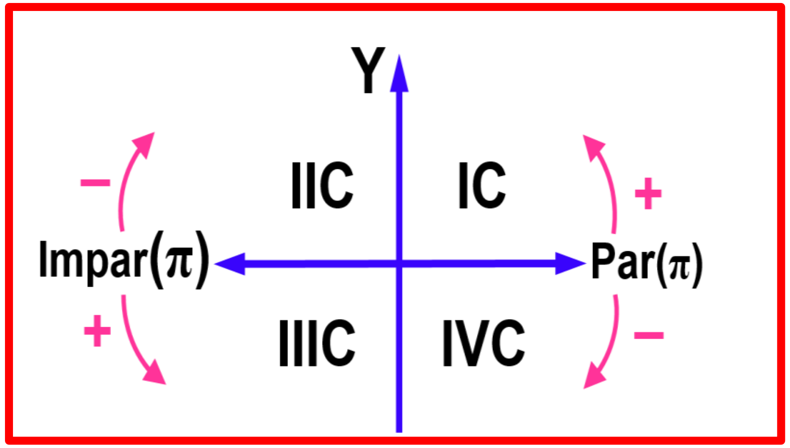
$$E = \text{sen}\frac{\pi}{6} + \cos\frac{\pi}{3}$$

$$E = \text{sen}30^{o} + \cos60^{o}$$

$$E = \frac{1}{2} + \frac{1}{2}$$

$$\therefore \boxed{E = 1}$$





HELICOREVIEW 9

Efectúe:

$$M=\sec\left(\frac{13\pi}{2}+\theta\right).\tan(22\pi+\theta),\quad \text{si } \cos\theta=\frac{1}{2},\ \text{donde } \theta\in IVC.$$

Resolución:

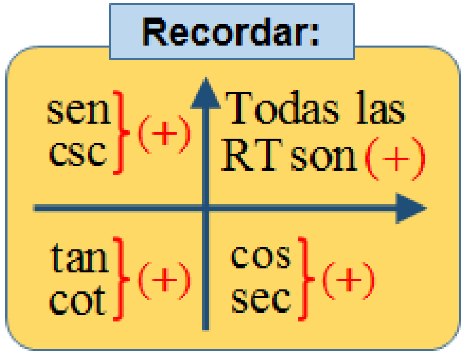
$$M=\sec\left(\overset{\mathring{4}+1}{13}\frac{\pi}{2}+\theta\right).\tan(\overset{PAR}{22}\pi+\theta)$$

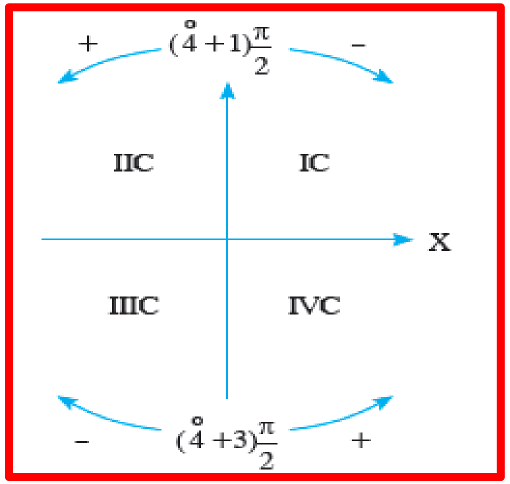
$$M=\underbrace{\sec\overbrace{\left(13\frac{\pi}{2}+\theta\right)}^{IIC}}_{-\csc\theta}.\underbrace{\tan\overbrace{(22\pi+\theta)}^{IC}}_{\tan\theta}$$

$$M=-\csc\theta.\tan\theta$$

$$M=-\frac{r}{\cancel{y}}.\frac{\cancel{y}}{x}$$

$$M=-\frac{r}{x}\ \ldots(*)$$



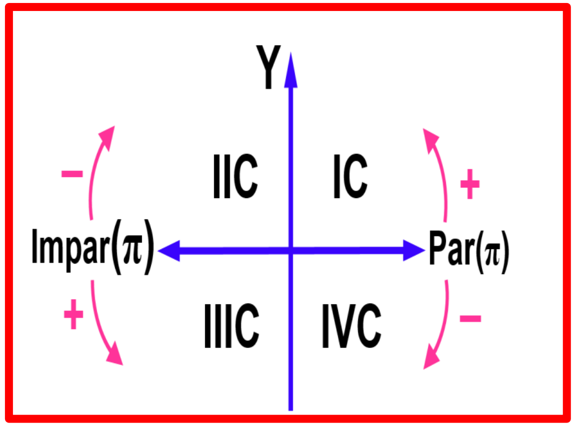


Del dato:

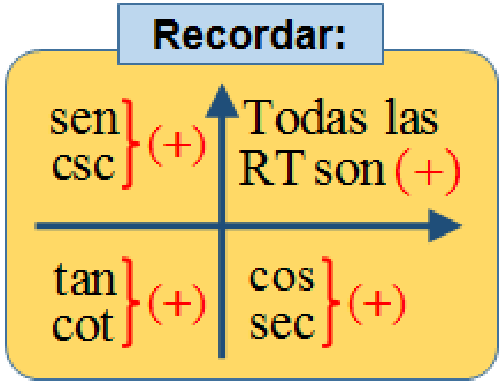
$$\cos\theta=\frac{1}{2}=\frac{x}{r}\rightarrow\frac{r}{x}=2$$

Remplazamos en $(*)$

$$\therefore\ \boxed{M=-2}$$



HELICOREVIEW 10



Se sabe que: $\cot\theta = -0{,}75 \ \ y \ \ \cos\theta < 0$

Determine: $\mathrm{M} = 5\mathrm{sen}\theta + 3\sec\theta + 1$

Resolución:

Del dato: $\cot\theta = -\dfrac{75}{100}$

$\Rightarrow \cot\theta = -\dfrac{3}{4} \ \ \mathrm{y} \ \ \cos\theta < 0$

$\rightarrow \theta \in IIC$

Sabemos:

$\mathrm{x}(-), \mathrm{y}(+), \mathrm{r}(+)$

Se tiene que:

$\cot\theta = \dfrac{-3}{4} = \dfrac{x}{y}$ $\quad \rightarrow x = -3$ $\quad \rightarrow y = 4$

Por radio vector:

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3)^2 + 4^2} \quad \rightarrow r = 5$

Calculamos:

$M = 5\mathrm{sen}\theta + 3\sec\theta + 1$

$M = 5\left(\dfrac{y}{r}\right) + 3\left(\dfrac{r}{x}\right) + 1$

$M = 5\left(\dfrac{4}{5}\right) + 3\left(\dfrac{5}{-3}\right) + 1$

$M = 4 - 5 + 1$

$\therefore \boxed{\mathrm{M} = 0}$