ALGEBRA

5th

OF SECONDARY

ASESORÍA 3

SACO OLIVEROS

1. Factorice e indique el factor primo de mayor suma de coeficientes

$$P(x) = x^4 + 7x^3 + 17x^2 + 17x + 6$$

RESOLUCIÓN

Por Aspa Doble Especial

$$x^4 + 7x^3 + 17x^2 + 17x + 6$$

$$\begin{array}{ccccc} x^2 & & 4x & & 3 = 3x^2 \\ x^2 & & 3x & & 2 = 2x^2 \\ & & & & \boxed{5x^2} \end{array}$$

Entonces falta:

$$17x^2 - 5x^2 = \boxed{12x^2}$$

Por Aspa Simple

$$(x^2 + 4x + 3)\,(x^2 + 3x + 2)$$

$$\begin{array}{cc} x & 3 \\ x & 1 \end{array} \qquad \begin{array}{cc} x & 2 \\ x & 1 \end{array}$$

$$(x+3)\,\underline{(x+1)}\,(x+2)\,\underline{(x+1)}$$

$$(x+1)^2\,(x+3)\,(x+2)$$

Nos piden

$$\boxed{(x+3)}$$

2. Indique la suma de factores primos, luego de factorizar:

$$P(x) = x^6 - 7x^4 + 6x^3$$

RESOLUCIÓN:

$$P(x) = x^3\ \boxed{(x^3 - 7x + 6)}$$

Por divisores binómicos :

$\mathrm{P.C} = \pm\{1; 2; 3; 6\}$

Si $x = 1 \Rightarrow 1^3 - 7(1) + 6 = 0$ **(CUMPLE)**

	1	0	−7	6
$x = 1$		1	1	−6
	1	1	−6	0

$$P(x) = x^3\ (x - 1)\ (x^2 + x - 6)$$

$$\begin{array}{cc} x & 3 \\ x & -2 \end{array}$$

$$P(x) = x^3\ (x - 1)\ (x + 3)\ (x - 2)$$

FACTORES PRIMOS

- ✓ x
- ✓ $x - 1$
- ✓ $x + 3$
- ✓ $x - 2$

∴ Suma de factores primos: $4x$

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3. La suma de los factores primos resulta $ax + by + cz$

$$x^2 + 2xy + y^2 + 3xz + 3yz - 4z^2$$

Halle $a.b.c$

RESOLUCIÓN:

Por aspa doble

$\boxed{-xz + 4xz = 3xz}$

$$x^2 + 2xy + y^2 + 3xz + 3yz - 4z^2$$

$$\begin{matrix} x & & y & & 4z \\ x & & y & & -z \end{matrix}$$

$$\begin{array}{r} xy \\ + \; xy \\ \hline 2xy \end{array} \qquad \begin{array}{r} -yz \\ + \; 4yz \\ \hline 3yz \end{array}$$

$(x + y + 4z)(x + y - z)$

$\sum F.P = x + y + 4z + x + y - z$

$\sum F.P = 2x + 2y + 3z$

$a \quad b \quad c$

$\boxed{\therefore \; abc = 12}$

4. **Efectúe:**

$$K = \frac{15}{\sqrt{7}+\sqrt{2}} + \frac{14}{\sqrt{7}} - \frac{10}{\sqrt{2}}$$

$$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right) = \sqrt{7}^2 - \sqrt{2}^2 = 5$$

RESOLUCIÓN:

Multiplicamos a cada término por su factor racionalizante

$$K = \underbrace{\frac{\cancel{15}}{\sqrt{7}+\sqrt{2}} \times \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}}}_{\boxed{\cancel{5}}} + \underbrace{\frac{\cancel{14}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}}_{\boxed{\cancel{7}}} - \underbrace{\frac{\cancel{10}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}}_{\boxed{\cancel{2}}}$$

$$K = 3\left(\sqrt{7}-\sqrt{2}\right) + 2\left(\sqrt{7}\right) - 5\left(\sqrt{2}\right)$$

$$K = 3\sqrt{7} - 3\sqrt{2} + 2\sqrt{7} - 5\sqrt{2}$$

$$\boxed{\therefore\ K = 5\sqrt{7} - 8\sqrt{2}}$$

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5. Reduce la siguiente expresión:

$$R=\sqrt{15+\sqrt{216}}-\sqrt{14-6\sqrt{5}}-\sqrt{6}$$

RESOLUCIÓN:

- $\sqrt{15+\sqrt{216}}=\sqrt{\underbrace{15}_{9+6}+2\underbrace{\sqrt{54}}_{9\times 6}}$

$$=\sqrt{9}+\sqrt{6}=3+\sqrt{6}$$

- $\sqrt{14-6\sqrt{5}}=\sqrt{\underbrace{14}_{9+5}-2\underbrace{\sqrt{45}}_{9\times 5}}$

$$=\sqrt{9}-\sqrt{5}=3-\sqrt{5}$$

RECORDAR:

Si $a>b$, entonces:

$$\sqrt{(a+b)\pm 2\sqrt{a.b}}=\sqrt{a}\pm\sqrt{b}$$

$$\boxed{\sqrt{216}=\sqrt{4}\sqrt{54}=2\sqrt{54}}$$

$$\boxed{6\sqrt{5}=2.3\sqrt{5}=2\sqrt{9}\sqrt{5}=2\sqrt{45}}$$

Reemplazando en R :

$$R=3+\sqrt{6}-(3-\sqrt{5})-\sqrt{6}$$

$$R=\cancel{3}+\cancel{\sqrt{6}}-\cancel{3}+\sqrt{5}-\cancel{\sqrt{6}}$$

$$\boxed{\therefore\ R=\sqrt{5}}$$

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6. Efectúe:

$$R = \frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{5}} - \sqrt{2} - \sqrt{3}$$

$$(\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})$$
$$(\sqrt{2}+\sqrt{3})^2 - \sqrt{5}^2$$
$$\sqrt{2}^2 + 2.\sqrt{2}.\sqrt{3} + \sqrt{3}^2 - \sqrt{5}^2$$
$$5 + 2\sqrt{6} - 5 = 2\sqrt{6}$$

RESOLUCIÓN:

$$R = \underbrace{\frac{\cancel{2\sqrt{6}}}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}}_{\boxed{\cancel{2\sqrt{6}}}} - \sqrt{2} - \sqrt{3}$$

$$R = \cancel{\sqrt{2}} + \cancel{\sqrt{3}} - \sqrt{5} \cancel{-\sqrt{2}} \cancel{-\sqrt{3}}$$

$$\boxed{\therefore R = -\sqrt{5}}$$

PROBLEMA 7

Calcule :

$$M = \sqrt{14+\sqrt{180}} + \sqrt{10-\sqrt{96}} - (\sqrt{5}+\sqrt{6})$$

, dé como respuesta 2M.

Resolución

$$M = \sqrt{14+\sqrt{180}} + \sqrt{10-\sqrt{96}} - (\sqrt{5}+\sqrt{6})$$

$$M = \sqrt{\underbrace{14}_{9+5} + 2\sqrt{\underbrace{45}_{9\,.\,5}}} + \sqrt{\underbrace{10}_{6+4} - 2\sqrt{\underbrace{24}_{6\,.\,4}}} - \sqrt{5} - \sqrt{6}$$

$$M = \sqrt{9} + \cancel{\sqrt{5}} + \cancel{\sqrt{6}} - \sqrt{4} - \cancel{\sqrt{5}} - \cancel{\sqrt{6}}$$

$$M = 3 - 2$$

$$M = 1$$

RECUERDA

FORMA PRÁCTICA

$$\sqrt{A+2\sqrt{B}} = \sqrt{x}+\sqrt{y}$$

$$\sqrt{A-2\sqrt{B}} = \sqrt{x}-\sqrt{y} \qquad x > y$$

$$x+y \qquad x.y$$

$$\sqrt{180} = \sqrt{4.45} = 2\sqrt{45}$$

$$\sqrt{96} = \sqrt{4.24} = 2\sqrt{24}$$

Recuerda

Piden $\boxed{\therefore 2M = 2}$

PROBLEMA 8 **Simplifique:**

$$A = \frac{i^{428} + i^{817} + 3i^{721} + i^{342} + 2i^{755}}{i^{221} + 4i^{436} + i^{473} - 2i^{469}}$$

Resolución

$$k \text{ es } ENTERO$$

$$A = \frac{i^{4k} + i^{4k+1} + 3i^{4k+1} + i^{4k+2} + 2i^{4k+3}}{i^{4k+1} + 4i^{4k} + i^{4k+1} - 2i^{4k+1}}$$

$$A = \frac{1 + i + 3i + (-1) + 2(-i)}{i + 4(1) + i - 2i} = \frac{2i}{4} = \frac{i}{2}$$

$$\therefore A = \frac{i}{2}$$

PROBLEMA 9

Sean los números complejos:

$$z_1 = 5 + 7i \qquad z_2 = 8 - 4i$$

$$Calcule: z_1{}^{*} + \overline{z_2} - 2\overline{z_1}$$

Resolución

$$\overline{z_1} = 5 - 7i$$

$$z_1 = 5 + 7i$$

$$z_1{}^{*} = -5 - 7i$$

$$z_2 = 8 - 4i \Rightarrow \overline{z_2} = 8 + 4i$$

$$Piden: -5 - 7i + 8 + 4i - 2(5 - 7i)$$

$$\Rightarrow -7 + 11i$$

$$\therefore -7 + 11i$$

PROBLEMA 10 Si: $\frac{5+2i}{3+4i}=a+bi$ $\quad$ $Calcule: \frac{b}{a}$

Resolución

$$\frac{(5+2i)}{(3+4i)}\frac{(3-4i)}{(3-4i)}=\frac{(15-20i+6i-8\overset{-1}{i^2})}{9-16i^2}=\frac{23-14i}{25}$$

$$\Rightarrow \frac{23}{25}-\frac{14}{25}i = a+bi$$

$$\Rightarrow a=\frac{23}{25} \quad ; \quad b=\frac{-14}{25}$$

$$\Rightarrow \frac{b}{a}=\frac{-14}{23}$$

$$\therefore \frac{b}{a}=\frac{-14}{23}$$

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