# ALGEBRA


**3rd**
**SECONDARY**

## Academic advising

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## Problema 1

***Calcule la suma de los factores primos de***

$$P(x) = 36x^4 - 25x^2 + 4$$

## Resolución:

$$P(x) = 36x^4 \underline{-25x^2} + 4$$

| | |
|---|---|
| $9x^2$ | $-4$ |
| $4x^2$ | $-1$ |

$$P(x) = \underline{(9x^2 - 4)}\,\underline{(4x^2 - 1)}$$

$$\boxed{P(x) = (3x+2)(3x-2)(2x+1)(2x-1)}$$

$$\sum FP = 3x + \cancel{2} + 3x - \cancel{2} + 2x + \cancel{1} + 2x - \cancel{1}$$

$$\boxed{\therefore \quad \sum FP = 10x}$$

## Problema 2

***Indique un factor primo luego de factorizar***

$$Q(x) = 2x^3 + 5x^2 - 2x + 3$$

## Resolución:

$$Q(x) = 2x^3 + 5x^2 - 2x + 3$$

$\boxed{a_0 = 2}$ $\boxed{a_n = 3}$

$div(a_0) = \{1; 2\}$

$div(a_n) = \{1; 3\}$

$\boxed{PC = \pm\left\{1; 3; \frac{1}{2}; \frac{3}{2}\right\}}$

| | | | | |
|---|---|---|---|---|
| | 2 | 5 | −2 | 3 |
| $x = -3$ | ↓ | −6 | 3 | −3 |
| × | 2 | −1 | 1 | 0 |

$$\boxed{Q(x) = (x+3)(2x^2 - x + 1)}$$

*Factores primos:* $\boxed{(x+3)\ y\ (2x^2 - x + 1)}$

## Problema 3

***Factorice e indique el número de factores primos***

$$R(x) = x^4 - 2x^2 + 5x - 6$$

## Resolución:

$$R(x) = x^4 - 2x^2 + 5x - 6$$

*Completando el polinomio:*

$$R(x) = x^4 + 0x^3 \underbrace{- 2x^2}_{TC} + 5x - 6$$

$$\boxed{-x^2}$$

$$\begin{array}{cccccc} x^2 & +x & -3 & \rightarrow & -3x^2 & \\ x^2 & -x & +2 & \rightarrow & +2x^2 & + \\ & & & & \overline{-x^2} & \end{array}$$

$TC = -2x^2$

$ST = -x^2$

$$\begin{array}{r} -2x^2 \\ -x^2 \\ \hline \boxed{-x^2} \end{array} \quad -$$

$$\boxed{R(x) = (x^2 + x - 3)(x^2 - x + 2)}$$

$\therefore$ *$R(x)$ tiene 2 factores primos.*

## Problema 4

**Simplifique:**

$$M=\sqrt[n]{\sqrt{3}-2}\,.\,\sqrt[2n]{7-4\sqrt{3}}$$

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## Recordemos:

***TRINOMIO CUADRADO PERFECTO (Binomio al cuadrado):***

$$\boxed{(a\pm b)^2=a^2+b^2\pm 2ab}$$

***DIFERENCIA DE CUADRADOS:***

$$\boxed{(a+b)(a-b)=a^2-b^2}$$

## Resolución:

$$M=\sqrt[2n]{\left(\sqrt{3}-2\right)^2}\,.\,\sqrt[2n]{7+4\sqrt{3}}$$

$$M=\sqrt[2n]{7-4\sqrt{3}}\,.\,\sqrt[2n]{7+4\sqrt{3}}$$

$$M=\sqrt[2n]{\left(7-4\sqrt{3}\right)\left(7+4\sqrt{3}\right)}$$

$$M=\sqrt[2n]{7^2-\left(4\sqrt{3}\right)^2}$$

$$M=\sqrt[2n]{49-16\,.\,3}$$

$$M=\sqrt[2n]{1}$$

$$\boxed{\therefore\ M=1}$$

## Problema 5

***Racionalice***

$$E = \frac{10}{\sqrt[7]{16}}$$


## Resolución:

$$E = \frac{10}{\sqrt[7]{16}}$$

$$E = \frac{10}{\sqrt[7]{2^4}} \times \frac{\sqrt[7]{2^{7-4}}}{\sqrt[7]{2^{7-4}}}$$

$$E = \frac{10\ \sqrt[7]{2^3}}{\sqrt[7]{2^4}\,.\,\sqrt[7]{2^3}}$$

$$E = \frac{10\ \sqrt[7]{8}}{\sqrt[7]{2^7}}$$

$$E = \frac{10\ \sqrt[7]{8}}{2}$$

$$\boxed{\therefore\ \ E = 5\sqrt[7]{8}}$$

## Problema 6

***Reduzca***

$$R = \sqrt{8+\sqrt{28}} - \sqrt{10-\sqrt{84}} + \sqrt{7-\sqrt{48}}$$

## Recordemos:

$$\sqrt{A \pm \sqrt{B}} = \sqrt{(x+y) \pm 2\sqrt{x.y}} = \sqrt{x} \pm \sqrt{y}$$

## Resolución:

$$R = \sqrt{8+\sqrt{28}} - \sqrt{10-\sqrt{84}} + \sqrt{7-\sqrt{48}}$$

$$R = \sqrt{8 + \sqrt{4}\,.\sqrt{7}} - \sqrt{10 - \sqrt{4}\,.\sqrt{21}} + \sqrt{7 - \sqrt{4}\,.\sqrt{12}}$$

$$R = \sqrt{\underbrace{8}_{7+1} + 2\underbrace{\sqrt{7}}_{7\times 1}} - \sqrt{\underbrace{10}_{7+3} - 2\underbrace{\sqrt{21}}_{7\times 3}} + \sqrt{\underbrace{7}_{4+3} - 2\underbrace{\sqrt{12}}_{4\times 3}}$$

$$R = \sqrt{7} + \sqrt{1} - (\sqrt{7} - \sqrt{3}) + \sqrt{4} - \sqrt{3}$$

$$R = \cancel{\sqrt{7}} + \sqrt{1} - \cancel{\sqrt{7}} + \cancel{\sqrt{3}} + \sqrt{4} - \cancel{\sqrt{3}}$$

$$R = 1 + 2$$

$$\boxed{\therefore \quad R = 3}$$

## Problema 7

*Si* $z_1 = 7 + 3i$
$z_2 = 1 - 5i$

*al efectuar*

$$z = \bar{z}_1 . z_2^*$$

*la diferencia entre la parte real y la parte imaginaria de $z$ representa la edad de la profesora Verónica hace dos años. ¿Cuántos años tiene la profesora actualmente?*

## Recordemos:

**Sea:** $z = a + bi$

***Conjugado de $z$:*** $\boxed{\bar{z} = a - bi}$

***Opuesto de $z$:*** $\boxed{z^* = -a - bi}$

## Resolución:

$$z = \bar{z}_1 . z_2^*$$

$$z = (7 - 3i)(-1 + 5i)$$

$$z = -7 + 35i + 3i - 15i^2$$

$$z = -7 + 35i + 3i + 15$$

$$\boxed{z = 38i + 8}$$

*Edad de la profesora Verónica hace 2 años:* $38 - 8 = 30$ *años*

$\therefore$ *La profesora Verónica tiene 32 años.*

## Problema 8

***Luego de efectuar:***

$$z = \frac{15i^{72}(3-i)}{2+i^{83}}; \quad \left(i = \sqrt{-1}\right)$$

**Calcule** $Im(z)$

## Recordemos:

***POTENCIAS DE i:***

$i^{4k} = 1$ $\quad$ $i^{4k+1} = i$

$i^{4k+2} = -1$ $\quad$ $i^{4k+3} = -i$

## Resolución:

$$z = \frac{15i^{72}(3-i)}{2+i^{83}}$$

$$z = \frac{15(1)(3-i)}{(2-i)} \times \frac{(2+i)}{(2+i)}$$

$$z = \frac{15\left(6+3i-2i-i^2\right)}{4-i^2}$$

$$z = \frac{15(6+3i-2i+1)}{4+1}$$

$$z = \frac{\overset{3}{\cancel{15}}(7+i)}{\underset{1}{\cancel{5}}}$$

$$\boxed{z = 21+3i}$$

$$\boxed{\therefore \ Im(z) = 3}$$

## Problema 9

**Sea** $z_1 = -3 - 5i$
$z_2 = 6 + 4i$
$z_3 = -1 + 7i$

**Si** $z = \bar{z}_1 - z_2 + z_3^*$

**calcule** $|Z|$

### Recordemos:

Sea: $z = a + bi$

*Conjugado de z:* $\boxed{\bar{z} = a - bi}$

*Opuesto de z:* $\boxed{z^* = -a - bi}$

*Módulo de z:* $\boxed{|z| = \sqrt{a^2 + b^2}}$

### Resolución:

$$z = \bar{z}_1 - z_2 + z_3^*$$

$$z = (-3 + 5i) - (6 + 4i) + (1 - 7i)$$

$$z = -3 + 5i - 6 - 4i + 1 - 7i$$

$$\boxed{z = -8 - 6i}$$

*Nos piden:* $|z|$

$$|z| = \sqrt{(-8)^2 + (-6)^2}$$

$$|z| = \sqrt{100}$$

$$\boxed{\therefore \quad |z| = 10}$$

## Problema 10

### Efectúe

$$P = 20\left[\frac{3+i}{3-i} - \frac{3-i}{3+i}\right] \quad ;\left(i = \sqrt{-1}\right)$$

## Recordemos:

***IDENTIDAD DE LEGENDRE:***

$$\boxed{(a+b)^2 - (a-b)^2 = 4ab}$$

***DIFERENCIA DE CUADRADOS:***

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

## Resolución:

$$P = 20\left[\frac{3+i}{3-i} - \frac{3-i}{3+i}\right]$$

$$P = 20\left[\frac{(3+i)^2 - (3-i)^2}{(3-i)\ (3+i)}\right]$$

$$P = 20\left[\frac{4\ (3)\ (i)}{9 - \underbrace{i^2}_{-1}}\right]$$

$$P = \overset{2}{\cancel{20}}\left[\frac{12i}{\underset{1}{\cancel{10}}}\right]$$

$$\boxed{\therefore \quad P = 24i}$$