# ALGEBRA

CHAPTER 23

$$P(x) \equiv a_0x^n + a_1x^{n-1} + \ ... + a_{n-1}x + a_n$$

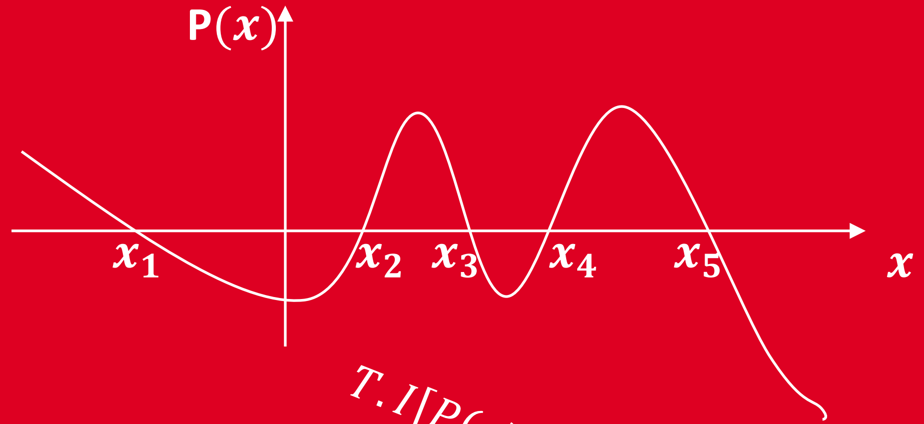
$$\sum coeficientes \ P(x) = P(1)$$

$$P(x) \equiv 0$$

$$G.A(P)$$

## 5th

of Secondary




$$T.I[P(x)] = P(0)$$

# Logaritmos I

SACO OLIVEROS

# MOTIVATING STRATEGY

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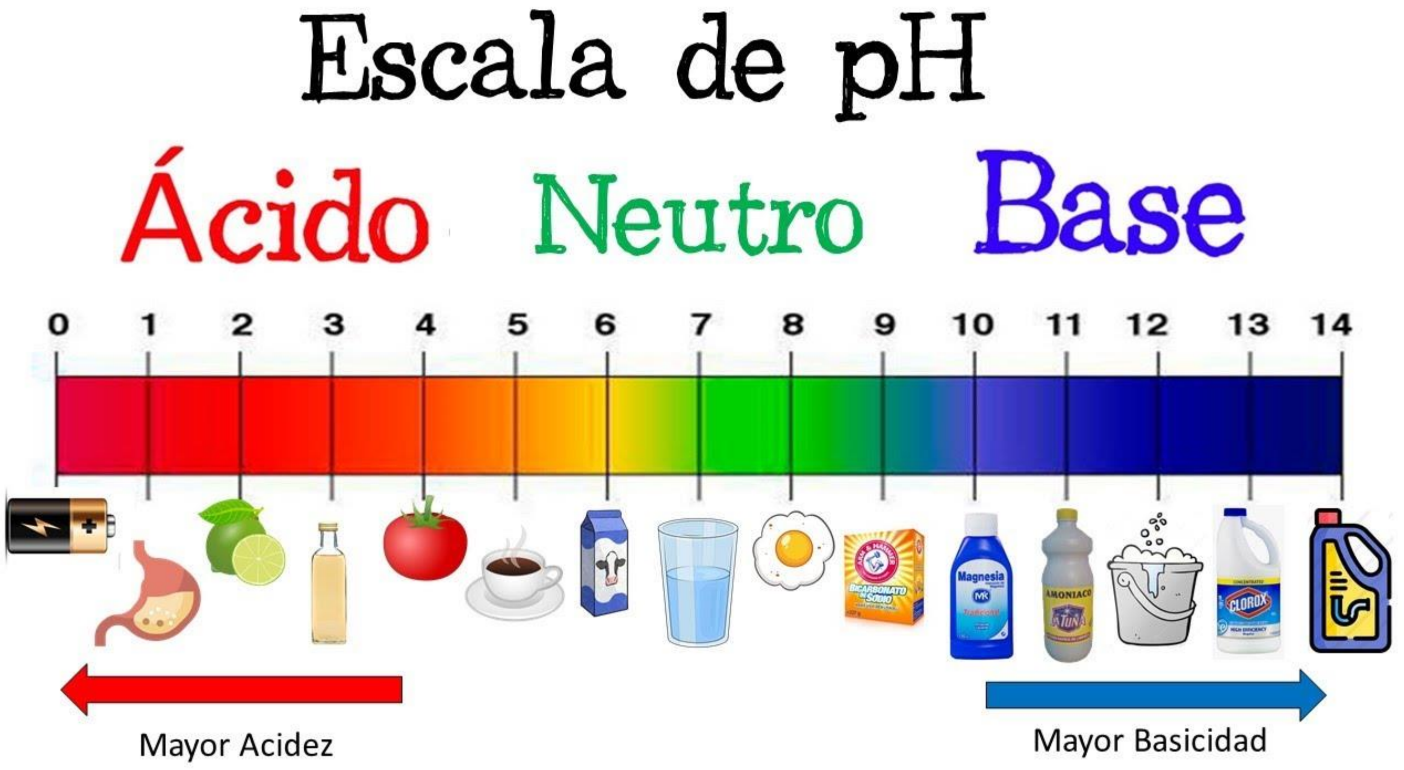
# EL pH Y LOS LOGARITMOS

El pH es la medida de la acidez o alcalinidad de una solución.

$$pH = -\log[H^{+}]$$



HELICO
THEORY

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# LOGARITMOS

## DEFINICIÓN

$\forall a, n \in \mathbb{R}^{+} \wedge \ a \neq 1$

$\boxed{\log_a n = L}$ <-> $a^L = n$

$a$: base

$n$: argumento

$L$: logaritmo

## EJEMPLOS

$\log_3 81 = 4$

$\log_2 32 = 5$

$\log_{16} 4 = \dfrac{1}{2}$

## OBSERVACIÓN

$\boxed{\log_{10} n = \log n}$

## IDENTIDAD FUNDAMENTAL DEL LOGARITMO

$$a^{\log_a n} = n$$

## EJEMPLOS

$$7^{\log_7 4} = 4$$

## TEOREMAS

$\forall x, y \in \mathbb{R}^+$

1) $\log_a(xy) = \log_a x + \log_a y$

$\log_7 5 + \log_7 6 = \log_7 30$

2) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_3 20 - \log_3 4 = \log_3 5$

3) $\log_{a^n}(x^m) = \frac{m}{n}\log_a x$

$\log_{16} 125 = \log_{2^4}(5^3) = \frac{3}{4}\log_2 5$

4) $x^{\log_a y} = y^{\log_a x}$ $3^{\log_4 5} = 5^{\log_4 3}$

5) $\log_a x = \dfrac{1}{\log_x a}$ $\dfrac{1}{\log_2 5} = \log_5 2$

## OBSERVACIÓN

$\boxed{\log_a 1 = 0}$ $\log_8 1 = 0$

$\boxed{\log_a a = 1}$ $\log_5 5 = 1$

# HELICO PRACTICE

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1) **Calcule** $\log_B A$ **si** $A = \log_2 512 \;\; y \;\; B = \log_5 125$

**Resolución**

$$\log_2 \mathbf{512} = \boldsymbol{A}$$

$$\mathbf{2}^{\boldsymbol{A}} = \mathbf{512}$$

$$\cancel{\mathbf{2}}^{\boldsymbol{A}} = \cancel{\mathbf{2}}^{\mathbf{9}}$$

$$\boxed{\boldsymbol{A} = \mathbf{9}}$$

$$\log_5 \mathbf{125} = \boldsymbol{B}$$

$$\mathbf{5}^{\boldsymbol{B}} = \mathbf{125}$$

$$\cancel{\mathbf{5}}^{\boldsymbol{B}} = \cancel{\mathbf{5}}^{\mathbf{3}}$$

$$\boxed{\boldsymbol{B} = \mathbf{3}}$$

***Calculemos***: $\log_{\boldsymbol{B}} \boldsymbol{A}$

$$\log_3 \mathbf{9}$$

$$\therefore x = 2$$

**2)** **Si:** $x = \log_9\left(\log_{64}(\log_3 81)\right)$

**Hallar el valor de:**

**Resolución**

$$\boxed{\log_3 81 = \log_3(3^4) = 4}$$

$$x = \log_9(log_{64} 4)$$

$$\boxed{\log_{64} 4 = \log_{(4^3)}(4^1) = 1/3}$$

$$x = \log_9\left(\frac{1}{3}\right) = \log_{(3^2)}(3^{-1})$$

$$x = \frac{-1}{2}$$

$$\boxed{2x = -1}$$

$$\therefore M = 26$$

3) **Simplifique**

$$T = \log\left(\frac{133}{65}\right) + 2\log\left(\frac{13}{7}\right) - \log\left(\frac{143}{90}\right) + \log\left(\frac{77}{171}\right)$$

## Resolución

$$T = \log\left(\frac{133}{65}\right) + \log\left(\frac{13}{7}\right)^{2} + \log\left(\frac{143}{90}\right)^{-1} + \log\left(\frac{77}{171}\right)$$

$$T = \log\left(\frac{133}{65}\right) + \log\left(\frac{169}{49}\right) + \log\left(\frac{90}{143}\right) + \log\left(\frac{77}{171}\right)$$

$$T = \log\left(\frac{133}{65}\cdot\frac{169}{49}\cdot\frac{90}{143}\cdot\frac{77}{171}\right) \quad \rightarrow T = \log\left(\frac{19\cdot 7}{13\cdot 5}\cdot\frac{13\cdot 13}{7\cdot 7}\cdot\frac{9\cdot 5\cdot 2}{13\cdot 11}\cdot\frac{7\cdot 11}{19\cdot 9}\right)$$

$$\therefore T = \log 2$$

**4) A qué es igual?**

$$P=\frac{1}{1+\log_3 10e}+\frac{1}{1+\log_e 30}+\frac{1}{1+\log_{10} 3e}$$

## Resolución

$$P=\frac{1}{\log_3 3+\log_3 10e}+\frac{1}{\log_e e+\log_e 30}+\frac{1}{\log_{10} 10+\log_{10} 3e}$$

$$P=\frac{1}{\log_3 30e}+\frac{1}{\log_e 30e}+\frac{1}{\log_{10} 30e}$$

$$P=\log_{30e} 3+\log_{30e} e+\log_{30e} 10 \quad \rightarrow P=\log_{30e} 30e$$

$$\boxed{\therefore P=1}$$

**5) Determine la mayor raíz de x:**

$$\log_3 x^{\log_3 x} - \log_3 x^3 - 10 = 0$$

**Resolución**

$$\log_3 x \cdot \log_3 x - 3\log_3 x - 10 = 0$$

$$\log_3^2 x - 3\log_3 x - 10 = 0$$

$$\log_3 x \qquad -5$$
$$\log_3 x \qquad 2$$

$$(\log_3 x - 5)(\log_3 x + 2) = 0$$

$$\log_3 x - 5 = 0$$

$$\log_3 x = 5 \quad \rightarrow x = 3^5$$

$$\log_3 x + 2 = 0$$

$$\log_3 x = -2 \quad \rightarrow x = 3^{-2}$$

$$\therefore \text{Mayor raíz} = 3^5$$

**6)** **La edad de Rubí es 10T años, donde T se calcula como la suma de raíces de la ecuación:** $5^{\log_3(2x^2-5x+9)} = 7^{\log_3 5}$

**¿Cuál será la edad de Rubí dentro de 3 años?**

## Resolución

$$\cancel{5}^{\cancel{\log_3}(2x^2-5x+9)} = \cancel{5}^{\cancel{\log_3}7}$$

$$2x^2 - 5x + 9 = 7$$

$$2x^2 - 5x + 2 = 0$$

$$\rightarrow T = \frac{5}{2} \qquad \rightarrow 10T = 25$$

**TEOREMA DE CARDANO:**

$$ax^2 + bx + c = 0$$

**Suma de raíces** $= \dfrac{-b}{a}$

$\therefore$ *Edad dentro de 3 años: 28 años*