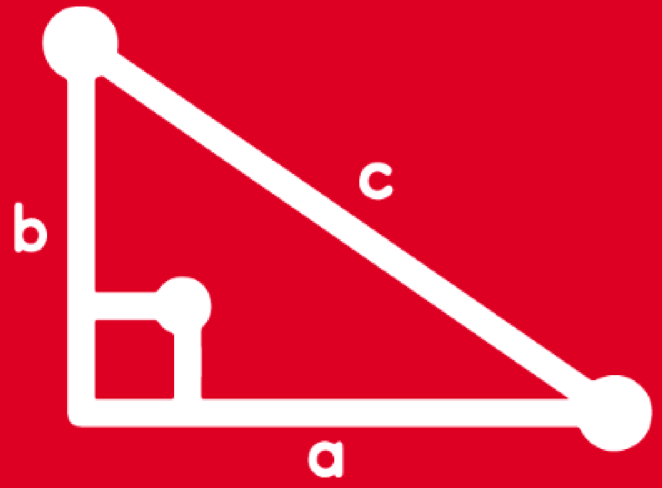
TRIGONOMETRY

Tomo 05

5th
SECONDARY



FEEDBACK

SACO OLIVEROS

HELICOPRACTICE 1

Halle el valor de sen16° y cos67°

Resolución:

Como: $16° = 53° - 37°$ ⟹ $\text{sen}(16°) = \text{sen}(53° - 37°)$

⟹ $\text{sen}16° = \underbrace{\text{sen}53°}_{\left(\frac{4}{5}\right)}\underbrace{\cos37°}_{\left(\frac{4}{5}\right)} - \underbrace{\cos53°}_{\left(\frac{3}{5}\right)}\underbrace{\text{sen}37°}_{\left(\frac{3}{5}\right)}$

$\therefore \text{sen}16° = \boxed{\frac{7}{25}}$

Como: $67° = 37° + 30°$ ⟹ $\cos(67°) = \cos(37° + 30°)$

⟹ $\cos67° = \underbrace{\cos37°}_{\left(\frac{4}{5}\right)}\underbrace{\cos30°}_{\left(\frac{\sqrt{3}}{2}\right)} - \underbrace{\text{sen}37°}_{\left(\frac{3}{5}\right)}\underbrace{\text{sen}30°}_{\left(\frac{1}{2}\right)}$

$\therefore \cos67° = \boxed{\frac{4\sqrt{3} - 3}{10}}$

HELICOPRACTICE 2

Si se cumple que $3\text{sen}(x+45°)=\sqrt{2}$, calcule $\text{sen}x\cos x$

Resolución:

$$sen(x+y)=senx.cosy+cosx.seny$$

$$3[\text{senx}\underbrace{\cos 45°}_{\frac{\sqrt{2}}{2}}+\text{cosx}\underbrace{\text{sen}45°}_{\frac{\sqrt{2}}{2}}]=\sqrt{2} \Rightarrow \frac{3}{2}\cancel{\sqrt{2}}[\text{senx}+\text{cosx}]=\cancel{\sqrt{2}}$$

$$\left\{\text{senx}+\text{cosx}=\frac{2}{3}\right\}^{2} \Rightarrow 1+2\text{senxcosx}=\frac{4}{9} \Rightarrow 2\text{senxcosx}=-\frac{5}{9}$$

$$\therefore \boxed{\text{senxcosx}=-\frac{5}{18}}$$

HELICOPRACTICE 3

Siendo $\alpha + \beta = 60°$, calcule el valor de $M = (\cos\alpha + \cos\beta)^2 + (\text{sen}\alpha - \text{sen}\beta)^2$

Resolución:

$$M = (\cos\alpha + \cos\beta)^2 + (\text{sen}\alpha - \text{sen}\beta)^2$$

$$M = \cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta + \text{sen}^2\alpha - 2\text{sen}\alpha\text{sen}\beta + \text{sen}^2\beta$$

$$M = \underbrace{\text{sen}^2\alpha + \cos^2\alpha}_{1} + 2\underbrace{(\cos\alpha\cos\beta - \text{sen}\alpha\text{sen}\beta)}_{\cos(\alpha+\beta)} + \underbrace{\cos^2\beta + \text{sen}^2\beta}_{1}$$

$$M = 2 + 2\cos 60°$$

$$M = 2 + \cancel{2}\left(\frac{1}{\cancel{2}}\right)$$

$\therefore$ $\boxed{M = 3}$

HELICOPRACTICE 4

Reduzca $T = \cos(x+30°).\cos(x-30°) - \text{sen}^2x$

Resolución:

$$\boxed{\cos(x+y)\cdot\cos(x-y) = \cos^2 x - \text{sen}^2 y}$$

$$T = \cos(x+30°).\cos(x-30°) + \text{sen}^2x$$

$$T = \cos^2 x - \text{sen}^2 30° + \text{sen}^2 x$$

$$T = \underbrace{\text{sen}^2 x + \cos^2 x}_{1} - \underbrace{\text{sen}^2 30°}_{\left(\frac{1}{2}\right)^2} \quad T = 1 - \frac{1}{4}$$

$$\therefore \boxed{T = \frac{3}{4}}$$

HELICOPRACTICE 5

Calcule el máximo valor de

$$E = 13\text{senx} + \sqrt{2}\text{sen}(45° - x) + 4\cos x$$

Resolución:

$$E = 13\text{senx} + \sqrt{2}\,(\text{sen}45°\cos x - \cos45°\text{senx}) + 4\cos x$$

$$E = 13\text{senx} + \cancel{\sqrt{2}}.\frac{1}{\cancel{\sqrt{2}}}.\cos x - \cancel{\sqrt{2}}.\frac{1}{\cancel{\sqrt{2}}}.\text{senx} + 4\cos x$$

$$E = 13senx + \cos x - senx + 4\cos x$$

$$\underbrace{-\sqrt{a^2+b^2}}_{\text{mínimo}} \leq a.\text{senx} + b.\cos x \leq \underbrace{\sqrt{a^2+b^2}}_{\text{máximo}}$$

$$E = 12\text{senx} + 5\cos x$$

Nos piden: $E_{máx} = \sqrt{12^2 + 5^2}$

$$\boxed{E_{máx} = 13}$$

HELICOPRACTICE 6

Reduzca: $M = \dfrac{2\tan 50° + \tan 80°}{\cot 40° . \cot 10°}$

Resolución:

Si $x + y + z = 180°$, se cumple:

$\tan x + \tan y + \tan z = \tan x . \tan y . \tan z$

$$M = \frac{\tan 50° + \tan 50° + \tan 80°}{\cot 40° \cot 10°}$$

Se observa que:

$$50° + 50° + 80° = 180°$$

Entonces:

$$\tan 50° + \tan 50° + \tan 80° = \tan 50° \tan 50° \tan 80°$$

$$M = \frac{\tan 50° \tan 50° \tan 80°}{\cot 40° \cot 10°}$$

$$M = \frac{\cancel{\tan 50°} \tan 50° \cancel{\tan 80°}}{\cancel{\tan 50°} \cancel{\tan 80°}}$$

$$M = \tan 50°$$

$$\therefore \boxed{M = \tan 50°}$$

HELICOPRACTICE 7

Si $\tan\alpha = -\dfrac{1}{2}$ y $\alpha \in$ IIC, calcule $\text{sen}2\alpha$

Resolución:

$$\tan\alpha = -\frac{1}{2} = \frac{y}{x}$$

Como $\alpha \in IIC$

$$x = -2 \; ; \quad y = 1$$

Por Radio Vector:

$$r = \sqrt{(-2)^2 + (1)^2}$$

$$r = \sqrt{5}$$

Piden:

$$\text{sen}2\alpha = 2\underbrace{\text{sen}\alpha}_{\frac{y}{r}}.\underbrace{\cos\alpha}_{\frac{x}{r}}$$

$$\text{sen}2\alpha = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right)$$

$$\text{sen}2\alpha = -\frac{4}{5}$$

$\therefore$ $\boxed{\text{sen}2\alpha = -\dfrac{4}{5}}$

HELICOPRACTICE 8

Reduzca: $N = \dfrac{(\cos 40° + \text{sen}40°)(\cos 40° - \text{sen}40°)}{\text{sen}5°.\cos 5°}$

Resolución:

$$(a + b)(a - b) = a^2 - b^2$$

$$N = \frac{(\cos40° + \text{sen}40°)(\cos40° - \text{sen}40°)}{\text{sen}5°.\ \cos5°}$$

$$2\text{sen}(x).\cos(x) = \text{sen}2x$$

$$N = \frac{2\overbrace{(\cos^2 40° - \text{sen}^2 40°)}^{\cos80°}}{\underbrace{2\ \text{sen}5°.\ \cos5°}_{\text{sen}10°}}$$

$$N = \frac{\overbrace{2\cos80°}^{\cancel{\text{sen}10°}}}{\cancel{\text{sen}10°}}$$

$$\therefore\ \boxed{N = 2}$$

HELICOPRACTICE 9

Calcule sen2x si se cumple que $\cos\left(\frac{\pi}{4}+x\right)=\frac{1}{\sqrt{5}}$

Resolución:

Dato: $\cos(45°+x)=\frac{1}{\sqrt{5}}$

Usar identidad:

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\operatorname{sen}\alpha.\operatorname{sen}\beta$$

$$\Rightarrow \cos45°\cos x-\operatorname{sen}45°\operatorname{sen}x=\frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\operatorname{sen}x=\frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}(\cos x-\operatorname{sen}x)=\frac{1}{\sqrt{5}}$$

$$\Rightarrow \cos x-\operatorname{sen}x=\frac{\sqrt{2}}{\sqrt{5}}$$

Elevando al cuadrado:

$$\underbrace{\cos^2x+\operatorname{sen}^2x}_{1}-\underbrace{2\cos x\operatorname{sen}x}_{\operatorname{sen}2x}=\frac{2}{5}$$

$$\therefore \boxed{\operatorname{sen}2x=\frac{3}{5}}$$

HELICOPRACTICE 10

Simplificar la expresión: $\cos^4 8° - 6sen^2 8° \cos 8° + sen^4 8°$

Resolución:

$$E = cos^4 8° - 6sen^2 8° cos^2 8° + sen^4 8°$$

$$E = \underbrace{sen^4 8° + cos^4 8°} - 6sen^2 8° cos^2 8°$$

$$E = 1 - 2sen^2 8° cos^2 8° - 6sen^2 8° cos^2 8°$$

$$E = 1 - 8sen^2 8° cos^2 8°$$

$$E = 1 - 2(\underbrace{4sen^2 8° cos^2 8°})$$

$$E = 1 - 2(sen16°)^2$$

$\cos(2\theta) = 1 - 2sen^2(\theta)$

$$E = 1 - 2sen^2 16°$$

$$E = cos32°$$

$\therefore$ $\boxed{E = cos32°}$