

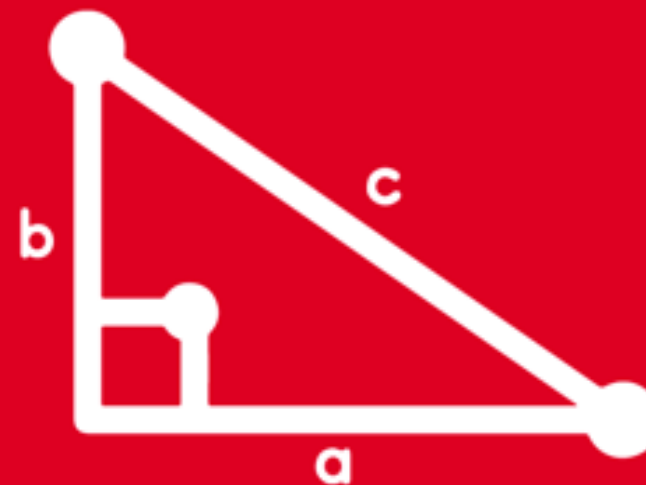


TRIGONOMETRY

ADVISORY

5th
SECONDARY

TOMOS 5 y 6



 **SACO OLIVEROS**



1. Simplifique la expresión: $E = \frac{\cos(x + y)}{\operatorname{sen}x \operatorname{cos}y} - \cot x$

RESOLUCIÓN

Recordar:

$$\cos(x + y) = \cos x \cdot \cos y - \operatorname{sen} x \cdot \operatorname{sen} y$$

$$E = \frac{\cos(x + y)}{\operatorname{sen}x \operatorname{cos}y} - \cot x$$

$$E = \frac{\cos x \cos y - \operatorname{sen} x \operatorname{sen} y}{\operatorname{sen}x \operatorname{cos}y} - \cot x$$

$$E = \frac{\cancel{\cos x \cos y}}{\cancel{\operatorname{sen}x \operatorname{cos}y}} - \frac{\cancel{\operatorname{sen} x \operatorname{sen} y}}{\cancel{\operatorname{sen}x \operatorname{cos}y}} - \cot x$$

$$E = \frac{\cos x}{\operatorname{sen}x} - \frac{\operatorname{sen} y}{\operatorname{cos}y} - \cot x$$

$$E = \cancel{\cot x} - \tan y - \cancel{\cot x}$$

$$\therefore E = -\tan y$$



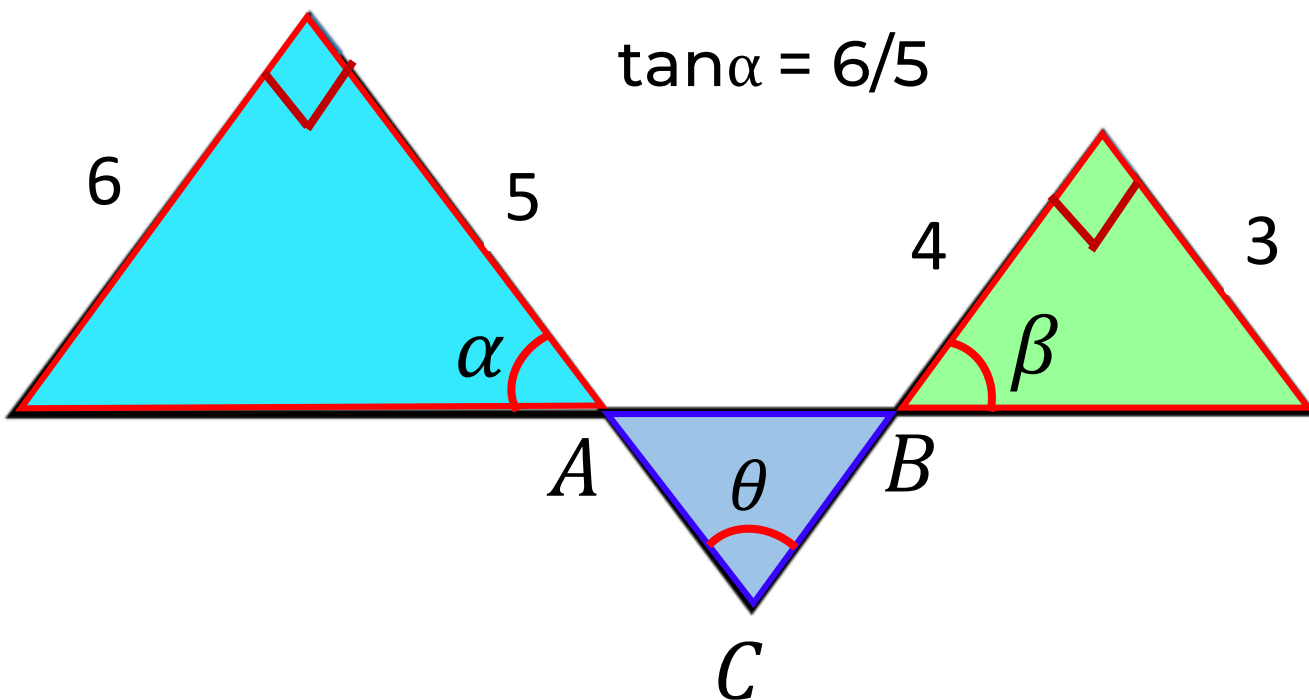


2. Del gráfico, halle el valor de $\tan\theta$.

RESOLUCIÓN

$$\tan\beta = 3/4$$

$$\tan\alpha = 6/5$$



Si $x + y + z = 180^\circ$, se cumple:

$$\tan x + \tan y + \tan z = \tan x \cdot \tan y \cdot \tan z$$

▲ ABC: $\alpha + \beta + \theta = 180^\circ$

$$\tan\alpha + \tan\beta + \tan\theta = \tan\alpha \cdot \tan\beta \cdot \tan\theta$$

$$\frac{6}{5} + \frac{3}{4} + \tan\theta = \frac{6}{5} \cdot \frac{3}{4} \cdot \tan\theta$$

$$\frac{39}{20} + \tan\theta = \frac{18}{20} \cdot \tan\theta$$

$$\frac{39}{\cancel{20}} = -\frac{2}{\cancel{20}} \cdot \tan\theta$$

$$\therefore \tan\theta = -39/2$$





3. Simplifica la expresión:

$$T = \frac{\text{sen}^3 x - \cos^3 x}{\text{sen} x - \cos x} + 5\text{sen} x \cos x$$

RESOLUCIÓN

$$T = \frac{\text{sen}^3 x - \cos^3 x}{\text{sen} x - \cos x} + 5\text{sen} x \cos x$$

$$T = \frac{(\cancel{\text{sen} x - \cos x})(\text{sen}^2 x + \text{sen} x \cos x + \cos^2 x)}{(\cancel{\text{sen} x - \cos x})} + 5\text{sen} x \cos x$$

$$T = \text{sen}^2 x + \cos^2 x + \text{sen} x \cos x + 5\text{sen} x \cos x$$

$$T = 1 + 3(\underbrace{2\text{sen} x \cos x}_{\text{sen} 2x})$$

Recordar:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{sen} 2x = 2\text{sen} x \cos x$$

$$\text{sen}^2 x + \cos^2 x = 1$$

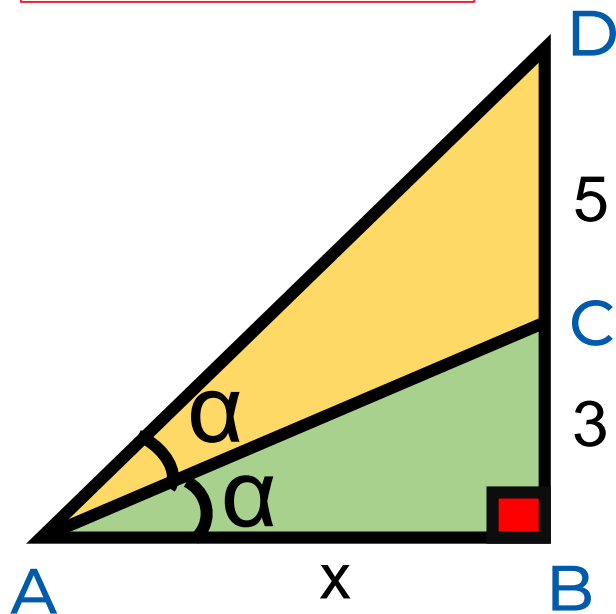
$$\therefore T = 1 + 3\text{sen} 2x$$





4. Del gráfico, calcule el valor de x.

RESOLUCIÓN



$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\triangle ABC: \tan \alpha = \frac{3}{x}$$

$$\triangle ABD: \tan 2\alpha = \frac{8}{x}$$

$$\Rightarrow \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{8}{x}$$

$$\Rightarrow \frac{2 \left(\frac{3}{x} \right)}{1 - \left(\frac{3}{x} \right)^2} = \frac{8}{x}$$

$$\Rightarrow \frac{\frac{6}{x}}{1 - \frac{9}{x^2}} = \frac{8}{x}$$

$$\Rightarrow x \left(\frac{6}{x} \right) = 8 \left(1 - \frac{9}{x^2} \right)$$

$$\Rightarrow 6 = 8 - \frac{72}{x^2} \quad \Rightarrow \quad \frac{72}{x^2} = 2$$

$$\Rightarrow x^2 = 36$$

$$\therefore x = 6$$



5. De la condición: $\tan x + \cot x = 12$
Calcule: $\sin 2x$

RESOLUCIÓN

Recordar: $\cot(x) + \tan(x) = 2\csc(2x)$

$$\csc(x) = \frac{1}{\sin(x)}$$



$$\cot x + \tan x = 12$$

$$2\csc 2x = 12$$

$$\csc 2x = 6$$

$$\therefore \sin 2x = \frac{1}{6}$$





6. Reduzca la expresión: $P = \frac{4(1 + \cos 4\theta)}{1 - \cos 8\theta} - \frac{1 + \cos 4\theta}{1 - \cos 4\theta}$

RESOLUCIÓN

$$P = \frac{4(1 + \cos 4\theta)}{1 - \cos 8\theta} - \frac{1 + \cos 4\theta}{1 - \cos 4\theta}$$

$$P = \frac{4(\cancel{2}\cos^2 2\theta)}{\cancel{2}\sin^2 4\theta} - \frac{\cancel{2}\cos^2 2\theta}{\cancel{2}\sin^2 2\theta}$$

$$P = \frac{4\cos^2 2\theta}{(\sin 4\theta)^2} - \frac{\cos^2 2\theta}{\sin^2 2\theta}$$

$$P = \frac{4\cos^2 2\theta}{(2\sin 2\theta \cos 2\theta)^2} - \cot^2 2\theta$$

$$P = \frac{\cancel{4}\cos^2 2\theta}{\cancel{4}\sin^2 2\theta \cancel{\cos^2 2\theta}} - \cot^2 2\theta$$

$$P = \frac{1}{\sin^2 2\theta} - \cot^2 2\theta$$

$$P = \underbrace{\csc^2 2\theta - \cot^2 2\theta}_1$$

$$\therefore P = 1$$

Recordar:

$$2\cos^2(x) = 1 + \cos(2x)$$

$$2\sin^2(x) = 1 - \cos(2x)$$

$$\sin 2x = 2\sin x \cos x$$

$$\csc^2 x - \cot^2 x = 1$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$





7. Reduzca: $P = \sqrt{\frac{1 + \sqrt{\frac{1 - \sin 50^\circ}{2}}}{2}}$

RESOLUCIÓN

Recordar:

$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$P = \sqrt{\frac{1 + \sqrt{\frac{1 - \cos 40^\circ}{2}}}{2}} \xrightarrow{\text{sen } 20^\circ} P = \sqrt{\frac{1 + \cos 70^\circ}{2}}$$

$$\therefore P = \cos 35^\circ$$



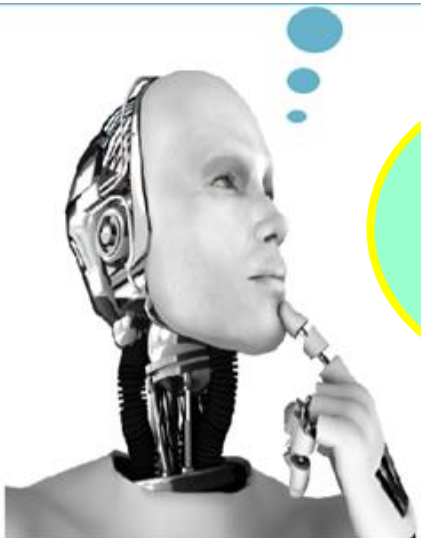


8. Dar el valor de: $E = \cot\left(\frac{37^\circ}{2}\right) - \sec\left(\frac{\pi}{3}\right)$

RESOLUCIÓN

Recordar:

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$



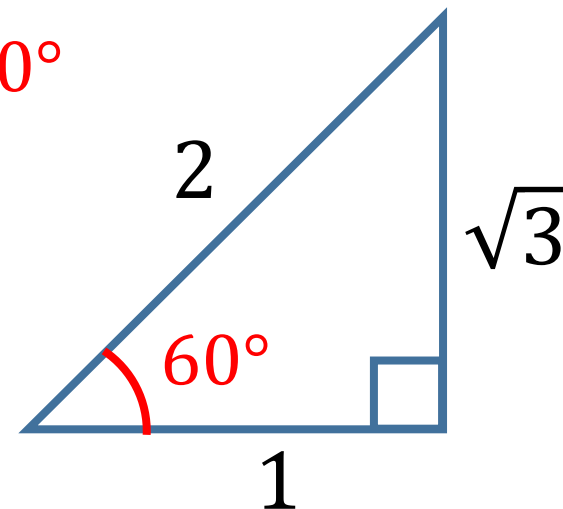
$$\frac{\pi}{3} = 60^\circ$$

$$E = \cot\left(\frac{37^\circ}{2}\right) - \sec\left(\frac{\pi}{3}\right)$$

$$E = \csc(37^\circ) + \cot(37^\circ) - \sec 60^\circ$$

$$E = \frac{5}{3} + \frac{4}{3} - 2$$

$$\therefore E = 1$$

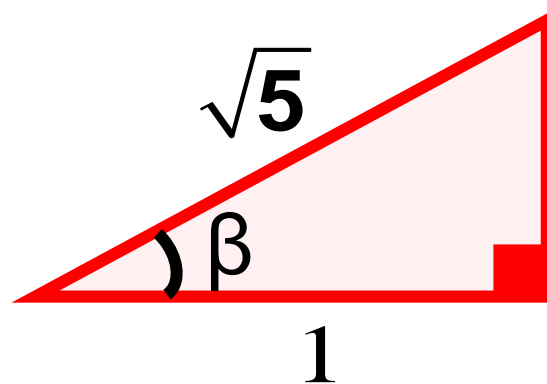




9. Si un ángulo β , cumple que $\sec\beta = \sqrt{5}$. Calcule $\tan 3\beta$

RESOLUCIÓN

Dato: $\sec\beta = \frac{\sqrt{5}}{1} = \frac{\text{Hip}}{\text{CA}}$



$$\Rightarrow \tan\beta = 2$$

Recordar:

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$



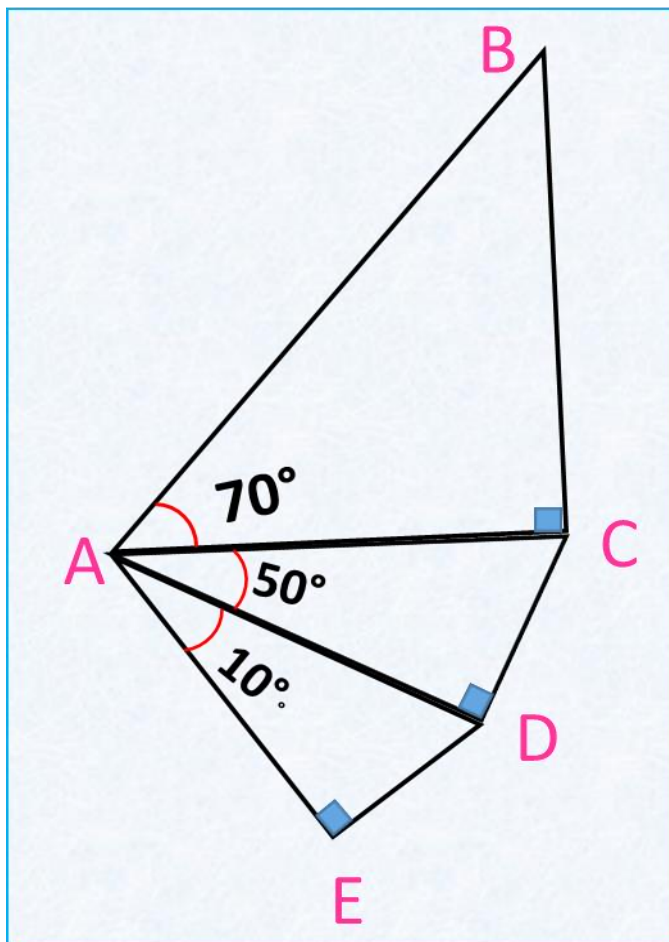
Calculamos:

$$\tan 3\beta = \frac{3(2) - (2)^3}{1 - 3(2)^2} = \frac{-2}{-11}$$

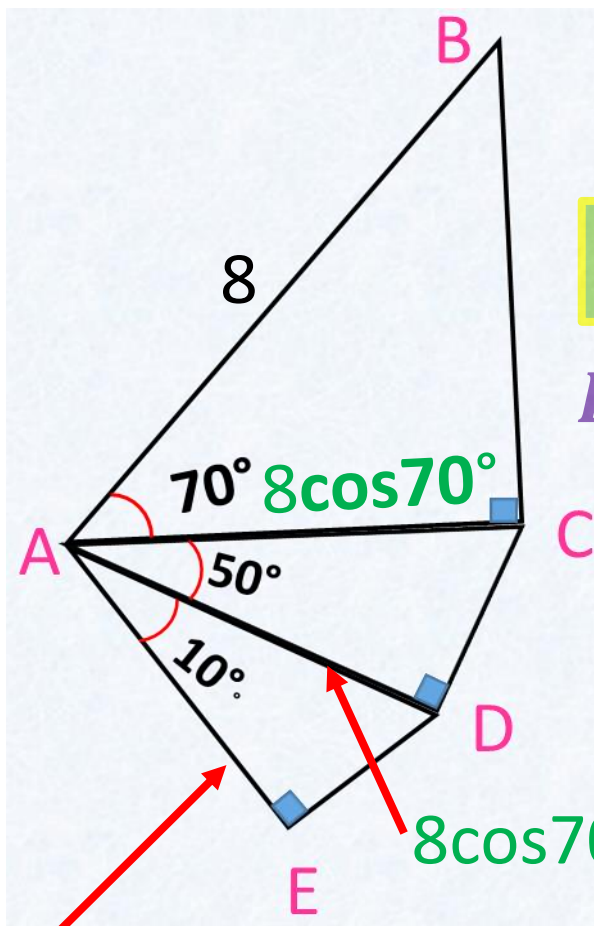
$$\therefore \tan 3\beta = \frac{2}{11}$$



10. En la figura, $AB = 8$.
Calcule AE

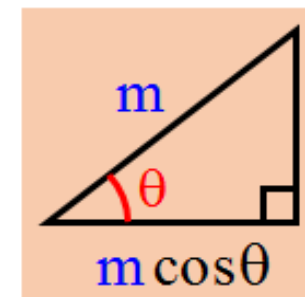


RESOLUCIÓN



$$8\cos 70^{\circ}\cos 50^{\circ}\cos 10^{\circ}$$

Recordar:



$$\cos 3x = 4\cos x \cdot \cos(60^{\circ} - x) \cdot \cos(60^{\circ} + x)$$

Luego: $AE = 8 \cdot \cos 70^{\circ} \cos 50^{\circ} \cos 10^{\circ}$

$$AE = 2 \cdot \underbrace{4\cos 10^{\circ} \cos 50^{\circ} \cos 70^{\circ}}$$

$$AE = 2\cos(3(10^{\circ}))$$

$$AE = 2\cos 30^{\circ} = 2 \cdot \frac{\sqrt{3}}{2}$$

$$\therefore AE = \sqrt{3}$$