1 Calcule los valores de sen16° y cos67°.

Resolución:

Como: $16° = 53° - 37°$ ➡️ **sen**(16°) = **sen**(53° - 37°)

➡️ $\text{sen}16° = \underbrace{\text{sen}53°}_{\left(\dfrac{4}{5}\right)}\underbrace{\cos37°}_{\left(\dfrac{4}{5}\right)} - \underbrace{\cos53°}_{\left(\dfrac{3}{5}\right)}\underbrace{\text{sen}37°}_{\left(\dfrac{3}{5}\right)}$

∴ $\text{sen}16° = \boxed{\dfrac{7}{25}}$

Como: $67° = 37° + 30°$ ➡️ **cos**(67°) = **cos**(37° + 30°)

➡️ $\cos67° = \underbrace{\cos37°}_{\left(\dfrac{4}{5}\right)}\underbrace{\cos30°}_{\left(\dfrac{\sqrt{3}}{2}\right)} - \underbrace{\text{sen}37°}_{\left(\dfrac{3}{5}\right)}\underbrace{\text{sen}30°}_{\left(\dfrac{1}{2}\right)}$

∴ $\cos67° = \boxed{\dfrac{4\sqrt{3} - 3}{10}}$

2 Si se cumple que $3\text{sen}(x + 45°) = \sqrt{2}$, calcule $\text{senx}\cos x$.

Resolución:

$$\text{sen}(x + y) = \text{senx}.\cos y + \cos x.\text{seny}$$

$$3\left[\text{senxcos}45° + \cos x\text{sen}45°\right] = \sqrt{2} \quad \Rightarrow \quad \frac{3}{2}\sqrt{2}\left[\text{senx} + \cos x\right] = \sqrt{2}$$

$$\frac{\sqrt{2}}{2} \qquad\qquad \frac{\sqrt{2}}{2}$$

$$\left\{\text{senx} + \cos x = \frac{2}{3}\right\}^2 \quad \Rightarrow \quad 1 + 2\text{senxcosx} = \frac{4}{9} \quad \Rightarrow \quad 2\text{senxcosx} = -\frac{5}{9}$$

$$\therefore \text{senxcosx} = -\frac{5}{18}$$

3 Siendo $\alpha + \beta = 60°,$ calcule el valor de $M = (\cos\alpha + \cos\beta)^2 + (\text{sen}\alpha - \text{sen}\beta)^2.$

Resolución:

$M = (\cos\alpha + \cos\beta)^2 + (\text{sen}\alpha - \text{sen}\beta)^2$

$M = \cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta + \text{sen}^2\alpha - 2\text{sen}\alpha\text{sen}\beta + \text{sen}^2\beta$

$M = \underbrace{sen^2\alpha + cos^2\alpha}_{1} + \underbrace{2(cos\alpha cos\beta - sen\alpha sen\beta)}_{cos(\alpha + \beta)} + \underbrace{cos^2\beta + sen^2\beta}_{1}$

$M = 2 + 2cos60°$

$M = 2 + \cancel{2}\left(\dfrac{1}{2}\right)$ $\therefore$ $\boxed{M = 3}$

4 Reduzca $T = \cos(x + 30°) \cdot \cos(x - 30°) - \text{sen}^2 x$.

Resolución:

$$\cos(x + y) \cdot \cos(x - y) = \cos^2 x - \text{sen}^2 y$$

$T = \cos(x+30°)\cos(x-30°) + \text{sen}^2 x$

$T = \cos^2 x - \text{sen}^2 30° + \text{sen}^2 x$

$T = \underbrace{\text{sen}^2 x + \cos^2 x}_{1} - \underbrace{\text{sen}^2 30°}_{\left(\frac{1}{2}\right)^2}$ ➡ $T = 1 - \dfrac{1}{4}$

$\therefore \boxed{T = \dfrac{3}{4}}$

5 Calcule el máximo valor de

$$E = 13\,\text{senx} + \sqrt{2}\,\text{sen}(45° - x) + 4\cos x$$

Resolución:

$$E = 13\,\text{senx} + \sqrt{2}\,\textbf{(sen45°cosx - cos45°senx)} + 4\text{cosx}$$

$$E = 13\,\text{senx} + \sqrt{2}\cdot\frac{1}{\sqrt{2}}\cdot\textbf{.cosx -} \sqrt{2}\cdot\frac{1}{\sqrt{2}}\cdot\textbf{.senx} + 4\text{cosx}$$

$$E = 13\,senx + \cos x - senx + 4\cos x$$

$$\underbrace{-\sqrt{a^2 + b^2}}_{\textbf{mínimo}} \leq \ a.\textbf{senx} + b.\textbf{cosx} \leq \underbrace{\sqrt{a^2 + b^2}}_{\textbf{máximo}}$$

$$E = 12\,\text{senx} + 5\text{cosx}$$

Calculamos: $E_{máx} = \sqrt{12^2 + 5^2}$

∴ $E_{máx} = 13$

6 Reduzca $M = \dfrac{2\tan 50° + \tan 80°}{\cot 40°.\cot 10°}$.

Resolución:

Si $x + y + z = 180°$, se cumple:
$\tan x + \tan y + \tan z = \tan x.\tan y.\tan z$

$M = \dfrac{\tan 50° + \tan 50° + \tan 80°}{\cot 40°\cot 10°}$

Se observa que:

$50° + 50° + 80° = 180°$

Entonces:

$\tan 50° + \tan 50° + \tan 80° = \tan 50°\tan 50°\tan 80°$

$M = \dfrac{\tan 50°\tan 50°\tan 80°}{\cot 40°\cot 10°}$

$M = \dfrac{\cancel{\tan 50°}\tan 50°\cancel{\tan 80°}}{\tan 50°\tan 80°}$

∴ $M = \tan 50°$

7 Si $\tan\alpha = -\dfrac{1}{2}$ y $\alpha \in$ IIC, calcule sen2α

Resolución:

$$\tan\alpha = -\frac{1}{2} = \frac{y}{x}$$

Como $\alpha \in$ IIC

$x = -2 \; ; \quad y = 1$

Por radio vector:

$r = \sqrt{(-2)^2 + (1)^2}$

$r = \sqrt{5}$

Calculamos:

$$\text{sen2α} = 2\underbrace{\text{senα}}_{\frac{y}{r}}.\underbrace{\text{cosα}}_{\frac{x}{r}}$$

$$\text{sen2α} = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right)$$

$$\text{sen2α} = -\frac{4}{5}$$

$$\mathbf{sen2\alpha = -\frac{4}{5}}$$

8 Reduzca: $N = \dfrac{(\cos 40° + \text{sen}40°)(\cos 40° - \text{sen}40°)}{\text{sen}5°.\cos 5°}$.

Resolución:

$$(a + b)(a - b) = a^2 - b^2$$

$$N = \frac{(\cos 40° + \text{sen}40°)(\cos 40° - \text{sen}40°)}{\text{sen}5°.\cos 5°}$$

$$2\text{sen}(x).\cos(x) = \text{sen}2x$$

$$\blacktriangleright \quad N = \frac{\mathbf{2}\overbrace{(\cos^2 40° - \text{sen}^2 40°)}^{\textbf{cos80°}}}{\underbrace{\mathbf{2}\;\text{sen}5°.\cos 5°}_{\textbf{sen10°}}}$$

$$\blacktriangleright \quad N = \frac{\mathbf{2}\overbrace{\cancel{\cos 80°}}^{\textbf{sen10°}}}{\cancel{\text{sen}10°}}$$

$$\therefore \boxed{N = 2}$$

9 Calcule sen2x si se cumple que $\cos\left(\dfrac{\pi}{4}+x\right)=\dfrac{1}{\sqrt{5}}$.

Resolución:

Dato: $\cos(45° + x) = \dfrac{1}{\sqrt{5}}$

Usamos la identidad:

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \text{sen}\alpha.\text{sen}\beta$$

$\Rightarrow \cos 45°\cos x - \text{sen}45°\text{sen}x = \dfrac{1}{\sqrt{5}}$

$\Rightarrow \dfrac{1}{\sqrt{2}}\cos x - \dfrac{1}{\sqrt{2}}\text{sen}x = \dfrac{1}{\sqrt{5}}$

$\Rightarrow \dfrac{1}{\sqrt{2}}(\cos x - \text{sen}x) = \dfrac{1}{\sqrt{5}}$

$\Rightarrow \cos x - \text{sen}x = \dfrac{\sqrt{2}}{\sqrt{5}}$

Elevamos al cuadrado:

$$\underbrace{\cos^2 x + \text{sen}^2 x}_{1} - \underbrace{2\cos x \text{sen}x}_{\text{sen}2x} = \dfrac{2}{5}$$

$$\therefore \text{sen2x} = \dfrac{3}{5}$$

10 Simplificar la expresión: $\cos^4 8° - 6\text{sen}^2 8° \cos 8° + \text{sen}^4 8°$.

Resolución:

$E = \cos^4 8° - 6sen^2 8°\cos^2 8° + sen^4 8°$

$E = \underbrace{sen^4 8° + cos^4 8°}_{} - 6sen^2 8°\cos^2 8°$

$E = 1 - 2sen^2 8°\cos^2 8° - 6sen^2 8°\cos^2 8°$

$E = 1 - 8sen^2 8°\cos^2 8°$

$E = 1 - 2(\underbrace{4sen^2 8°\cos^2 8°}_{})$

$E = 1 - 2(sen16°)^2$

$E = 1 - 2sen^2 16°$

$\boxed{\cos(2\theta) = 1 - 2sen^2(\theta)}$

$E = cos32°$

$\therefore \boxed{E = cos32°}$