



ALGEBRA

Chapter 11

4th
SECONDARY

NUMEROS COMPLEJOS



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HELICO

MOTIVATING



¿Puedes multiplicar mentalmente el siguiente números complejos y dar la respuesta en menos de 10 segundos

$$z_1 = 7 + i$$

$$z_2 = 7 - i$$

Rpta. 50

HELICO THEORY

CHAPTER 01

NÚMEROS COMPLEJOS

I) UNIDAD IMAGINARIA

$$i^2 = -1$$

y

$$i = \sqrt{-1}$$

$$\diamond \sqrt{-9} = \sqrt{9} \cdot \underbrace{\sqrt{-1}}_i = 3i$$

$$\diamond \sqrt{-25} = \sqrt{25} \cdot \underbrace{\sqrt{-1}}_i = 5i$$



POTENCIAS DE LA UNIDAD IMAGINARIA

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i$$

$$i^8 = 1$$

$$i^9 = i$$

$$i^{10} = -1$$

$$i^{11} = -i$$

$$i^{12} = 1$$

Teorema:

$$i^{4k} = 1$$

$$i^{4k+2} = -1$$

$$i^{4k+1} = i$$

$$i^{4k+3} = -i$$



Ejemplo

$$\diamond i^{23} = i^{20+3} = i^3 = -i$$

$$\diamond i^{201} = i^{200+1} = i^1 = i$$

Teorema:

$$i + i^2 + i^3 + i^4 + \dots + i^{4k} = 0$$

Ejemplo

$$i + i^2 + i^3 + i^4 + \dots + i^{200} = 0$$

**Definición:**

$$a, b \in \mathbb{R}$$

$$z = (a; b) = a + bi \quad / i = \sqrt{-1}$$

Donde:

□ *Parte real:* $\text{Re}_{(z)} = a$

□ *Parte imaginaria:* $\text{Im}_{(z)} = b$

Ejemplo:

$$\diamond z = (3; 2) = 3 + 2i$$

$$\text{Re}_{(z)} = 3$$

$$\text{Im}_{(z)} = 2$$



Definiciones:

Sea: $z = a + bi$, $a, b \in \mathbb{R}$; entonces se define

1. complejo conjugado (\bar{z}): $\bar{z} = a - bi$
2. Complejo opuesto (z^*): $z^* = -a - bi = Op(z)$

Ejemplo

$$z = 3 - 4i \longrightarrow \bar{z} = 3 + 4i$$

$$z = 3 - 4i \longrightarrow z^* = -3 + 4i = Op(z)$$



Adición y sustracción

Ejemplo:

$$\begin{array}{l} \diamond z_1 = 2 + 4i \\ z_2 = 3 + 2i \end{array} \quad +$$

$$z_1 + z_2 = 5 + 6i$$

$$\begin{array}{l} \diamond z_1 = 2 + 4i \\ z_2 = 3 + 2i \end{array} \quad -$$

$$z_1 - z_2 = -1 + 2i$$

Multiplicación

Sea: $z_1 = 2 + 4i$
 $z_2 = 3 + 2i$

$$z_1 \cdot z_2 = (2 + 4i)(3 + 2i)$$

$$z_1 \cdot z_2 = 6 + 4i + 12i + 8i^2$$

$$\boxed{-8}$$

$$z_1 \cdot z_2 = -2 + 16i$$



$$(a + bi)(a - bi) = a^2 + b^2$$

División: $z = \frac{2 + 4i}{3 - 2i}$

$$z = \frac{(2 + 4i)(\mathbf{3 + 2i})}{(3 - 2i)(\mathbf{3 + 2i})}$$

$$z = \frac{-2 + 16i}{13} = \frac{-2}{13} + \frac{16}{13}i$$

$$\triangleright (2 + 4i)(3 + 2i) = -2 + 16i$$

$$\triangleright (3 - 2i)(3 + 2i) = 3^2 + 2^2 = 13$$

PROPIEDADES:

$$\frac{1+i}{1-i} = i$$

$$\frac{1-i}{1+i} = -i$$

$$(1+i)^2 = 2i$$

$$(1-i)^2 = -2i$$

$$(1 \mp i)^4 = -4$$

$a, b, m, n \in \mathbb{R}$ con $m, n \neq 0$

$$\frac{a+bi}{n+mi} \rightarrow \mathbb{C}. \text{ imaginario puro}$$

se cumple: $\frac{a}{m} = -\frac{b}{n}$

$$\frac{a+bi}{n+mi} \rightarrow \text{complejo real}$$

se cumple: $\frac{a}{n} = \frac{b}{m}$

HELICO PRACTICE

CHAPTER 01

PROBLEMA 1

Simplifique:

$$A = \frac{i^{32} + i^{54} + 3i^{65}}{i^{46} + 4i^{520} + i^{673}}$$

Resolución

$$A = \frac{i^{4k} + i^{4k+2} + 3i^{4k+1}}{i^{4k+2} + 4i^{4k} + i^{4k+1}}$$

$$A = \frac{1 + (-1) + 3(i)}{(-1) + 4(1) + i} = \frac{3i}{3 + i} \times \frac{3-i}{3-i} = \frac{9i + 3}{9 + 1} = \frac{9i + 3}{10}$$



Sean los números complejos:

$$z_1 = 5 + 7i \quad z_2 = 8 - 4i$$

$$\text{Calcule: } Op(z_1) + \overline{z_2} - 2\overline{z_1}$$

Resolución

$$z_1 = 5 + 7i \Rightarrow \overline{z_1} = 5 - 7i \Rightarrow Op(z_1) = -5 - 7i$$

$$z_2 = 8 - 4i \Rightarrow \overline{z_2} = 8 + 4i$$

$$\text{Luego: } -5 - 7i + 8 + 4i - 2(5 - 7i)$$

$$\Rightarrow -7 + 11i$$



Sean:

$$z_1 = -7 + 2i \quad z_2 = 4 - 3i$$

Calcule: $z_1 \cdot z_2 + Op(z_2) + \overline{z_1}$ Resolución

$$z_1 = -7 + 2i \Rightarrow \overline{z_1} = -7 - 2i$$

$$z_2 = 4 - 3i \Rightarrow Op(z_2) = -4 + 3i$$

$$z_1 \cdot z_2 = (-7 + 2i)(4 - 3i) = -28 + 21i + 8i - 6i^2$$

$$z_1 \cdot z_2 = -22 + 29i$$

$$\Rightarrow -22 + 29i - 4 + 3i - 7 - 2i = -33 + 30i$$



Si: $\frac{5+2i}{3+4i} = a + bi$ Calcule: $\frac{b}{a}$

Resolución

$$\frac{(5 + 2i)}{(3 + 4i)} \times \frac{(3 - 4i)}{(3 - 4i)} = \frac{(15 - 20i + 6i - 8i^2)}{9 - 16i^2} = \frac{23 - 14i}{25}$$

$$\Rightarrow \frac{23}{25} - \frac{14}{25}i = a + bi$$

$$\Rightarrow \frac{b}{a} = \frac{-14}{23}$$



Simplifica:

$$M = \left[\frac{1+i}{1-i} - \frac{1-i}{1+i} \right]^4$$

Resolución

RECORDANDO:

$$\frac{1+i}{1-i} = i$$

$$\frac{1-i}{1+i} = -i$$



Reemplazando:

$$M = \left[\frac{1+i}{1-i} - \frac{1-i}{1+i} \right]^4$$

$$M = [i - (-i)]^4$$

$$M = [i + i]^4$$

$$M = [2i]^4$$

$$M = 16$$



La edad de Carlos hace 15 años coincide con la parte imaginaria de $z_1 \cdot \overline{z_2}$, donde: $z_1 = 4 - 3i$; $z_2 = -7 - \overline{z_1}$
 ¿Qué edad tiene Carlos?

Resolución

$$z_1 = 4 - 3i \quad \longrightarrow \quad \overline{z_1} = 4 + 3i$$

$$z_2 = -7 - \overline{z_1}$$

Hallando $\text{Imag}(z_1 \cdot \overline{z_2})$:

$$z_2 = -7 - (4 + 3i)$$

$$z_1 \cdot \overline{z_2} = (4 - 3i)(-11 + 3i)$$

$$z_2 = -7 - 4 - 3i$$

$$z_1 \cdot \overline{z_2} = -44 + 12i + 33i + 9$$

$$z_2 = -11 - 3i$$

$$z_1 \cdot \overline{z_2} = -35 + 45i$$

$$\longrightarrow \overline{z_2} = -11 + 3i$$

$$\text{Imag}(z_1 \cdot \overline{z_2}) = 45$$

Rpta: 60 años



Al reducir $T = \frac{(1+i)^{12} + (1-i)^4}{17}$, El valor de $T^2 + 4$ representa el precio de una entrada a un concierto. Si asisten 100 personas, ¿cuánto fue lo recaudado?

Recordar:

$$(1+i)^2 = 2i$$

$$(1-i)^2 = -2i$$

$$i^2 = -1$$

$$i^6 = -1$$

Resolución

$$T = \frac{(1+i)^{12} + (1-i)^4}{17}$$

$$T = \frac{[(1+i)^2]^6 + [(1-i)^2]^2}{17}$$

$$T = \frac{[2i]^6 + [-2i]^2}{17}$$

$$T = \frac{2^6 \cdot i^6 + (-2)^2 \cdot i^2}{17}$$

$$T = \frac{64(-1) + 4(-1)}{17}$$

$$T = \frac{-68}{17}$$

$$T = -4$$

$$\therefore T^2 + 4 = 20$$