

# TRIGONOMETRY

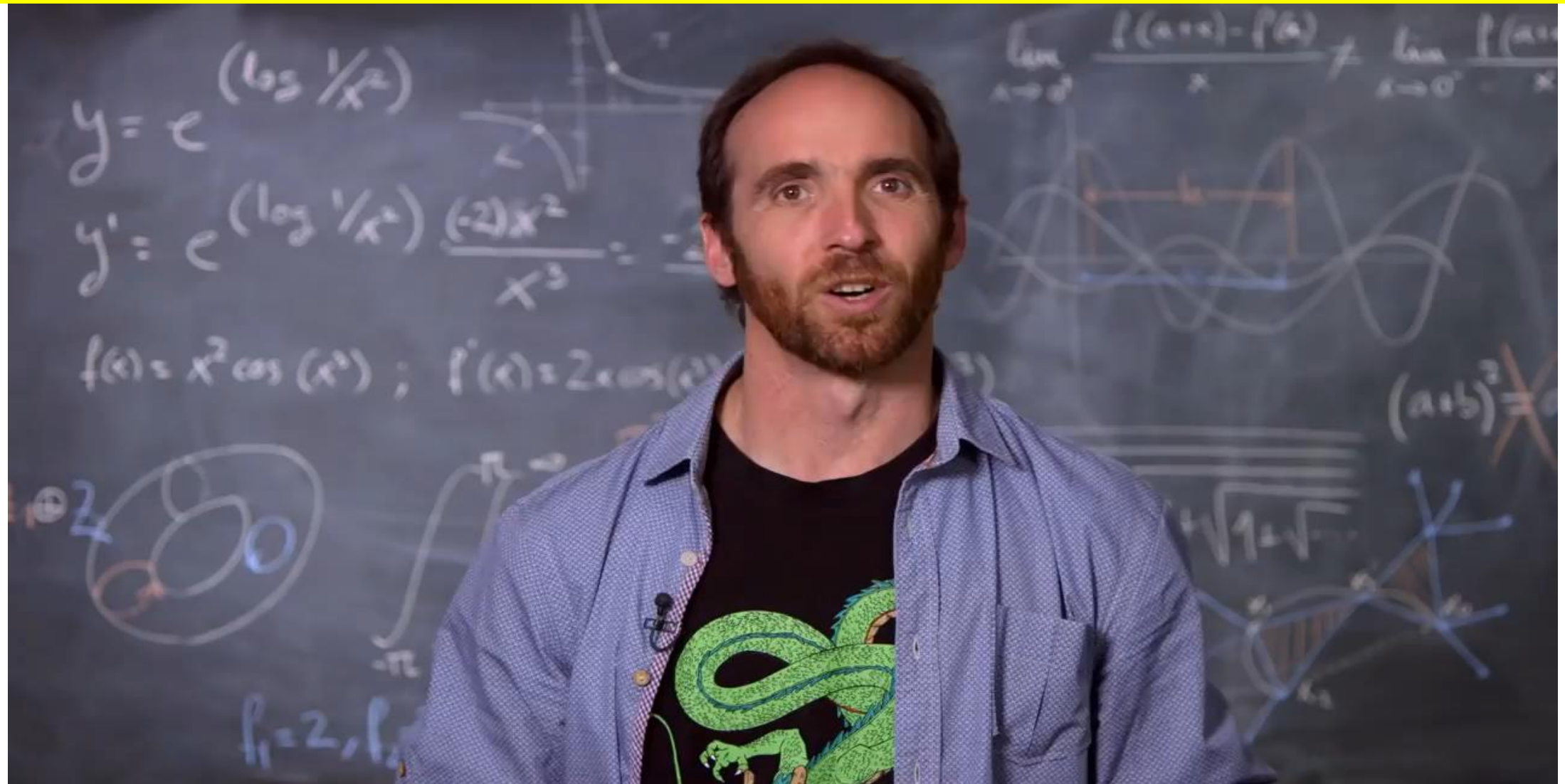
## Chapter 17

**5th**  
SECONDARY

### IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO MITAD



# ¿ QUIÉN FUE SRINIVASA RAMANUJAN ?



# IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO MITAD

## I) IDENTIDADES BÁSICAS :

$$\text{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\text{cos}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Observación :

El signo  $\pm$  depende del cuadrante de  $\left(\frac{x}{2}\right)$

## II) IDENTIDADES AUXILIARES :



$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$



**Ejemplo :**  $\tan 15^\circ = \tan\left(\frac{30^\circ}{2}\right) = \csc 30^\circ - \cot 30^\circ$

$$\therefore \tan 15^\circ = 2 - \sqrt{3}$$

# HELICO PRACTICE 1

Reduzca  $H = \sqrt{\frac{1 + \cos 100^\circ}{2}} - \frac{\sin 80^\circ}{2 \cos 40^\circ}$

## RESOLUCIÓN

Recordar :

$$\pm \sqrt{\frac{1 + \cos x}{2}} = \cos\left(\frac{x}{2}\right)$$

$$\sin 2\theta = 2 \sin \theta \cdot \cos \theta$$

$$H = \sqrt{\frac{1 + \cos 100^\circ}{2}} - \frac{\sin 80^\circ}{2 \cos 40^\circ}$$

$$H = \cos\left(\frac{100^\circ}{2}\right) - \frac{\cancel{2} \sin 40^\circ \cdot \cancel{\cos 40^\circ}}{\cancel{2} \cos 40^\circ}$$

$$H = \cos 50^\circ - \sin 40^\circ$$

$$H = \cos 50^\circ - \cos 50^\circ$$

$$\therefore H = 0$$

# HELICO PRACTICE 2

Reduzca  $P = \sqrt{\frac{1 - \sqrt{\frac{1 + \text{sen}10^\circ}{2}}}{2}}$

## RESOLUCIÓN

Recordar :

$$\pm \sqrt{\frac{1 + \cos x}{2}} = \cos\left(\frac{x}{2}\right)$$

$$\pm \sqrt{\frac{1 - \cos x}{2}} = \sin\left(\frac{x}{2}\right)$$

$$P = \sqrt{\frac{1 - \sqrt{\frac{1 + \cos 80^\circ}{2}}}{2}}$$

$$P = \sqrt{\frac{1 - \cos\left(\frac{80^\circ}{2}\right)}{2}}$$

$$P = \sqrt{\frac{1 - \cos 40^\circ}{2}}$$

$$P = \sin\left(\frac{40^\circ}{2}\right)$$

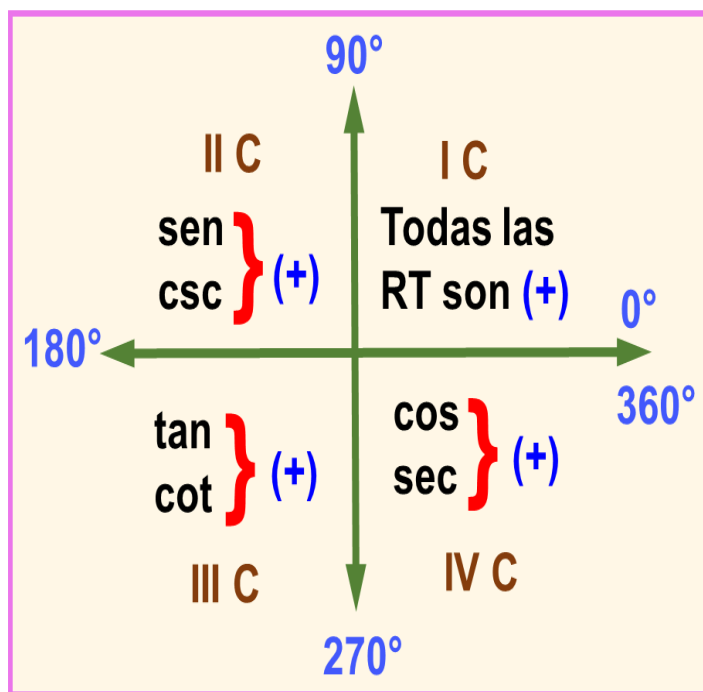
$$\therefore P = \sin 20^\circ$$

# HELICO PRACTICE 3

Para un ángulo  $\theta$  mayor a  $360^\circ$  pero menor que  $450^\circ$ , se cumple que  $\cos\theta = 0,5$  .- Según lo anterior , calcule  $\sin\left(\frac{\theta}{2}\right)$

## RESOLUCIÓN

### Recordar :



$$\begin{aligned} & \div 2 \\ & 360^\circ < \theta < 450^\circ \\ & 180^\circ < \frac{\theta}{2} < 225^\circ \end{aligned}$$

$$\frac{\theta}{2} \in \text{III C}$$

$$\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1 - \cos\theta}{2}}$$

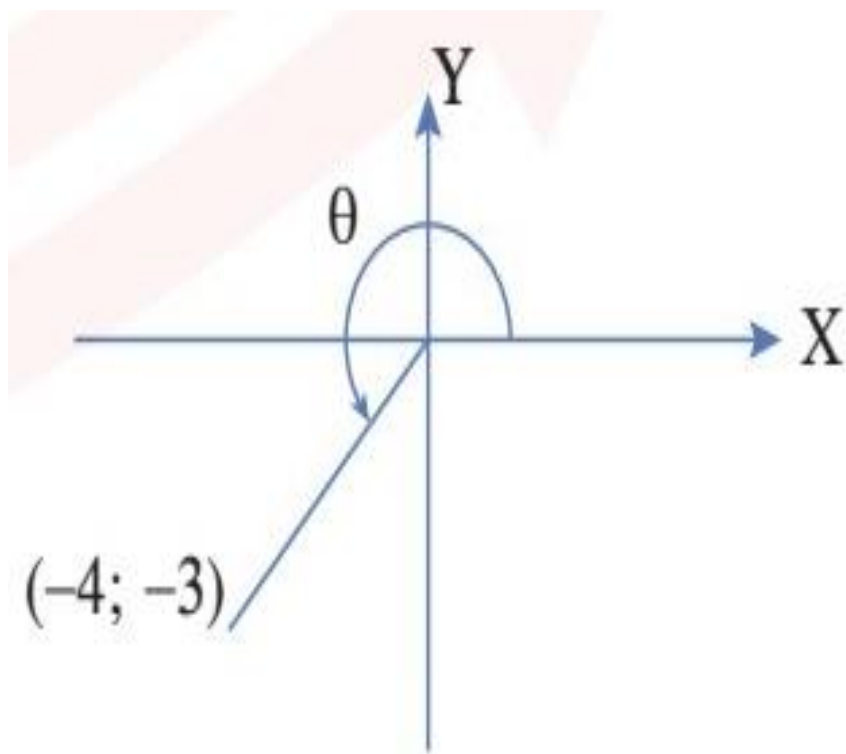
$$\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1 - \frac{1}{2}}{2}} = -\sqrt{\frac{\frac{1}{2}}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1}{4}}$$

$$\therefore \sin\left(\frac{\theta}{2}\right) = -\frac{1}{2}$$

# HELICO PRACTICE 4

Del gráfico, calcule  $\tan\left(\frac{\theta}{2}\right)$ .



## RESOLUCIÓN

Según gráfico :  $x = -4$        $y = -3$

Luego:  $r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + (-3)^2}$

$$r = \sqrt{16 + 9} \rightarrow r = 5$$

$$\tan\left(\frac{\theta}{2}\right) = \csc\theta - \cot\theta = \frac{r}{y} - \frac{x}{y}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{5}{-3} - \frac{-4}{-3} = \frac{5+4}{-3} = \frac{9}{-3}$$

$$\therefore \tan\left(\frac{\theta}{2}\right) = -3$$



# HELICO PRACTICE 5

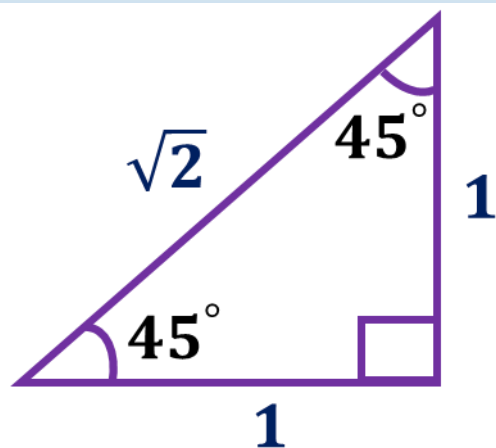
**Dar el valor de  $E = \cot\left(\frac{\pi}{8}\right) - \sec\left(\frac{\pi}{4}\right)$**

## RESOLUCIÓN

Recordar :

$$\pi \text{ rad} \leftrightarrow 180^\circ$$

$$\cot\left(\frac{\theta}{2}\right) = \csc\theta + \cot\theta$$



$$E = \cot\left(\frac{180^\circ}{8}\right) - \sec\left(\frac{180^\circ}{4}\right)$$

$$E = \cot\left(\frac{45^\circ}{2}\right) - \sec 45^\circ$$

$$E = \cancel{\csc 45^\circ} + \cot 45^\circ - \cancel{\sec 45^\circ}$$

$$\therefore E = 1$$

# HELICO PRACTICE 6

El niño Pepito recibe de propina diaria la suma de  $12(\csc\theta - \csc 2\theta - \cot 2\theta)^2$  soles.- Indique el valor del ángulo agudo  $\theta$  para que la propina diaria sea de 4 soles.

## RESOLUCIÓN

$$12(\csc\theta - \csc 2\theta - \cot 2\theta)^2 = 4$$

$$(\csc\theta - (\csc 2\theta + \cot 2\theta))^2 = \frac{4}{12}$$

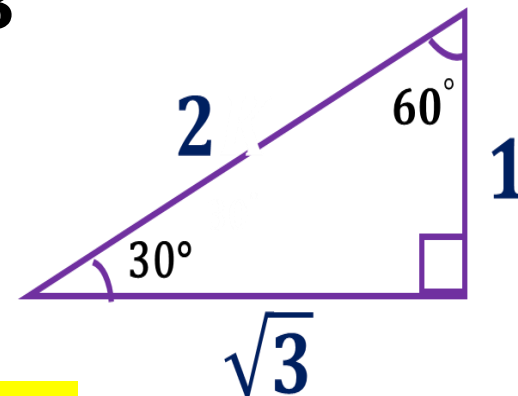
$$\left(\csc\theta - \cot\left(\frac{2\theta}{2}\right)\right)^2 = \frac{1}{3}$$

$$\csc\theta - \cot\theta = \frac{1}{\sqrt{3}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

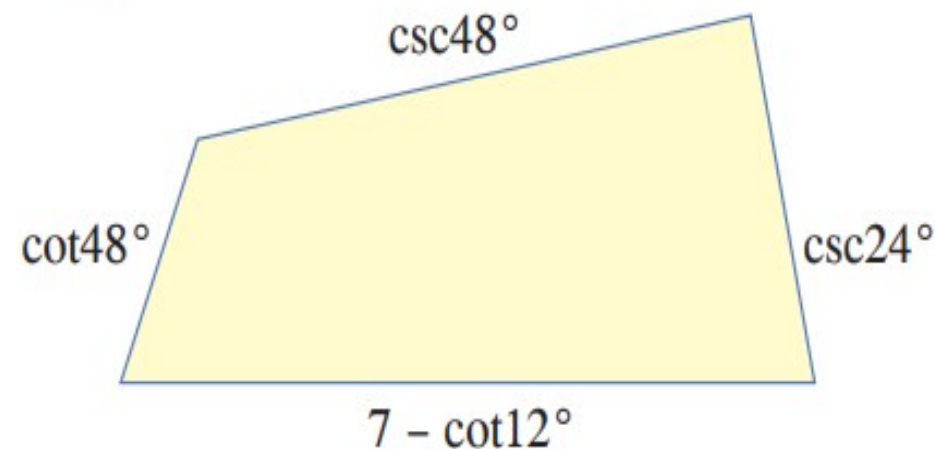
$$\frac{\theta}{2} = 30^\circ$$

$$\therefore \theta = 60^\circ$$



# HELICO PRACTICE 7

El contorno de la mesa en la sala de espera de una clínica dental , tiene las siguientes dimensiones ( en metros ) .  
¿Cuál es el perímetro de dicho contorno?



## RESOLUCIÓN

$$2p = ( 7 - \cot 12^\circ + \csc 24^\circ + \underbrace{\csc 48^\circ + \cot 48^\circ}_{\csc 24^\circ} ) \text{ m}$$

$$2p = ( 7 - \cot 12^\circ + \underbrace{\csc 24^\circ + \cot 24^\circ}_{\csc 12^\circ} ) \text{ m}$$

$$2p = ( 7 - \cancel{\cot 12^\circ} + \cancel{\cot 12^\circ} ) \text{ m}$$

$$\therefore 2p = 7 \text{ m}$$



Recordar :

$$\csc \theta + \cot \theta = \cot \left( \frac{\theta}{2} \right)$$



**SACO**  
**OLIVEROS**