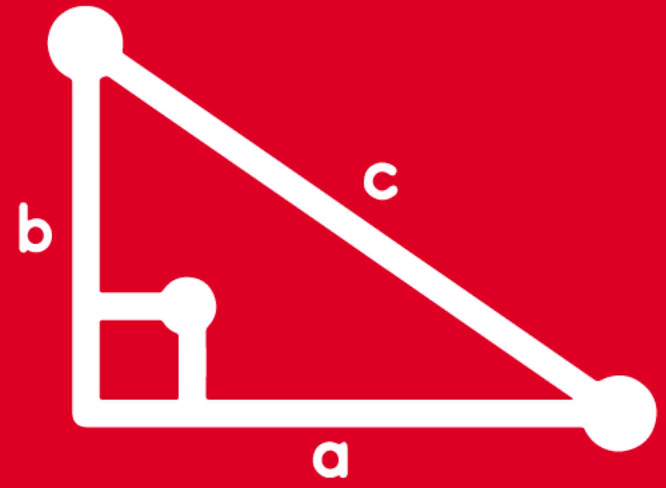
TRIGONOMETRY

Chapter 2

Verano 2022

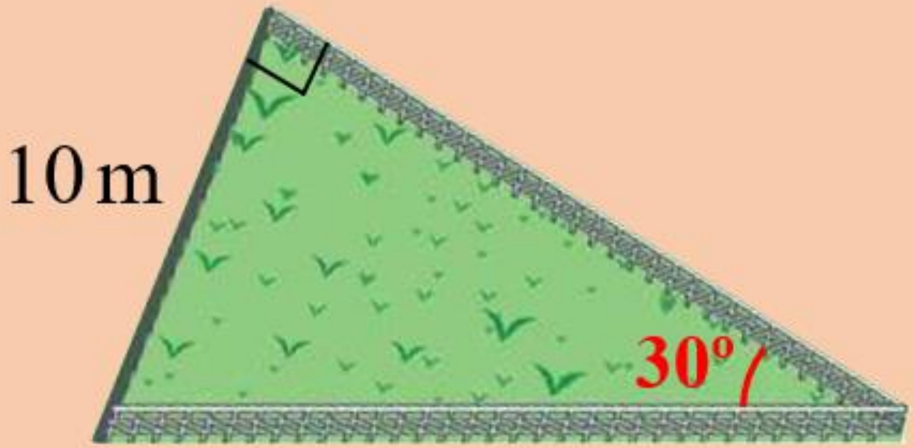
SAN MARCOS



Razones trigonométricas de un ángulo agudo II

SACO OLIVEROS

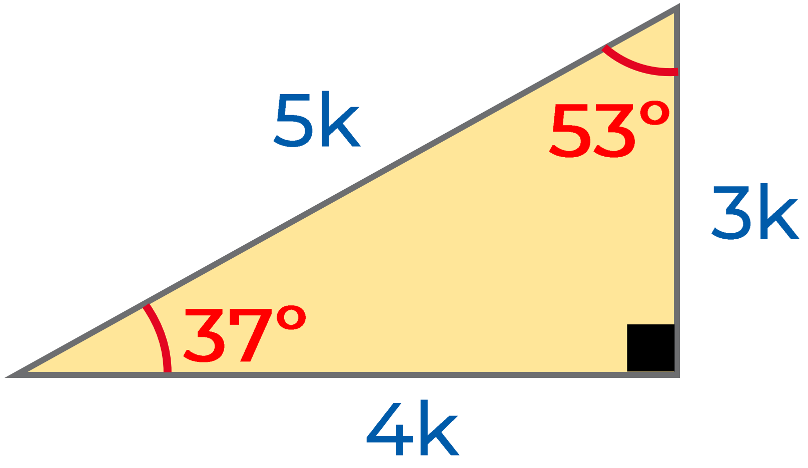
La figura muestra un terreno que tiene la forma de un triángulo rectángulo.
¿puedes calcular aproximadamente el perímetro de dicho terreno?



Rpta : 47,3 m

RAZONES TRIGONOMÉTRICAS DE ÁNGULOS NOTABLES

RAZONES TRIGONOMÉTRICAS DE 37° y 53°



* $\tan 53° = \dfrac{4k}{3k} = \dfrac{4}{3}$

* $\csc 53° = \dfrac{5k}{4k} = \dfrac{5}{4}$

Así tenemos :

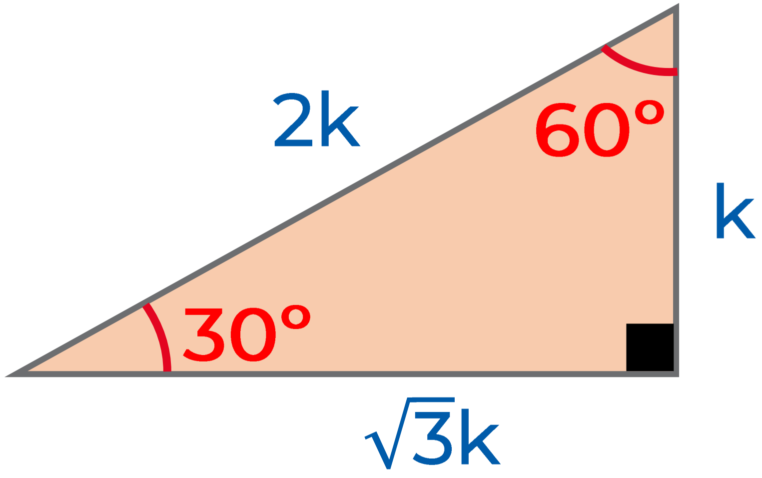
∠	sen	cos	tan	cot	sec	csc
37°	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{5}{3}$
53°	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{5}{3}$	$\frac{5}{4}$

EJEMPLOS:

* $\text{sen}37° = \dfrac{3k}{5k} = \dfrac{3}{5}$

* $\sec 37° = \dfrac{5k}{4k} = \dfrac{5}{4}$

RAZONES TRIGONOMÉTRICAS DE 30° y 60°



EJEMPLOS:

* $\text{sen}30° = \dfrac{k}{2k} = \dfrac{1}{2}$

* $\cos 30° = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

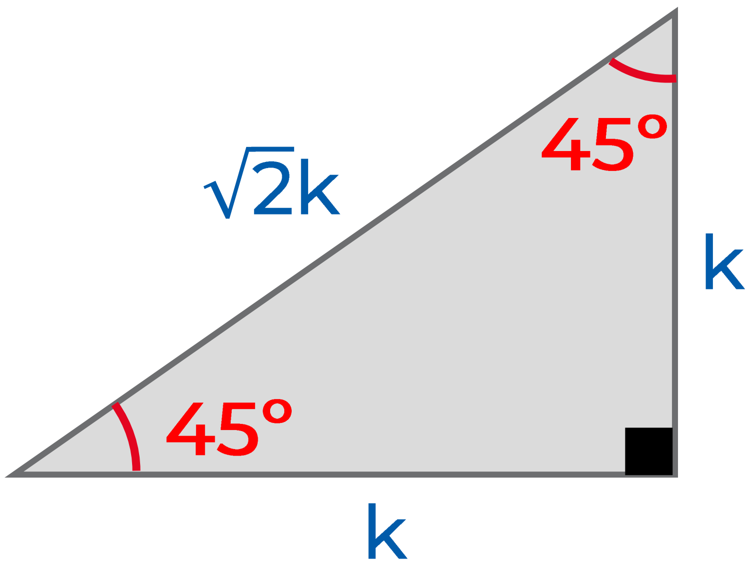
* $\tan 60° = \dfrac{\sqrt{3}k}{k} = \sqrt{3}$

* $\csc 60° = \dfrac{2k}{\sqrt{3}k} = \dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

Así tenemos :

$\angle$	sen	cos	tan	cot	sec	csc
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$

RAZONES TRIGONOMÉTRICAS DE 45°



EJEMPLOS:

* $\sec 45° = \dfrac{\sqrt{2}k}{k} = \sqrt{2}$

* $\tan 45° = \dfrac{k}{k} = 1$

* $\operatorname{sen} 45° = \dfrac{k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

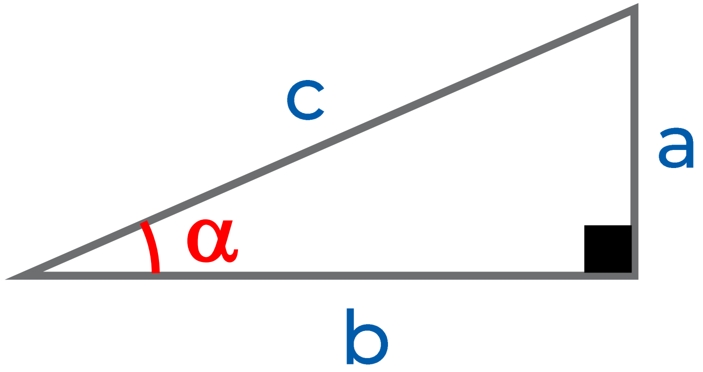
* $\cos 45° = \dfrac{k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Así tenemos :

∠	sen	cos	tan	cot	sec	csc
45°	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$

PROPIEDADES DE LAS R.T. DE UN ÁNGULO AGUDO

1) RAZONES TRIGONOMÉTRICAS RECÍPROCAS



$$\text{sen}\alpha.\text{csc}\alpha = 1$$

$$\cos\alpha.\sec\alpha = 1$$

$$\tan\alpha.\cot\alpha = 1$$

Iguales

EJEMPLOS:

* $\text{sen}53°.\text{csc}53° = 1$

* $\cos70°.\sec70° = 1$

* $\tan3\theta.\cot3\theta = 1$

* Hallar x, si se cumple:

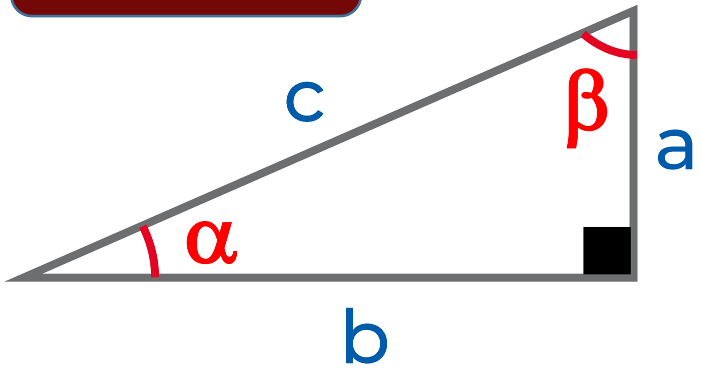
$\text{sen}2x.\text{csc}40° = 1$

Resolución:

$2x = 40°$ $\therefore$ $\boxed{x = 20°}$

2) R.T. DE ÁNGULOS COMPLEMENTARIOS

$\alpha + \beta = 90°$



$\text{sen}\alpha = \cos\beta$

$\tan\alpha = \cot\beta$

$\sec\alpha = \csc\beta$

suman 90°

EJEMPLOS:

* $\text{sen}60° = \cos30°$
* $\tan40° = \cot50°$
* $\sec(\theta) = \csc(90° - \theta)$

* Hallar x, si se cumple:

$\text{sen}5x = \cos10°$

Resolución:

$5x + 10° = 90°$

$\Rightarrow 5x = 80° \quad \therefore \boxed{x = 16°}$

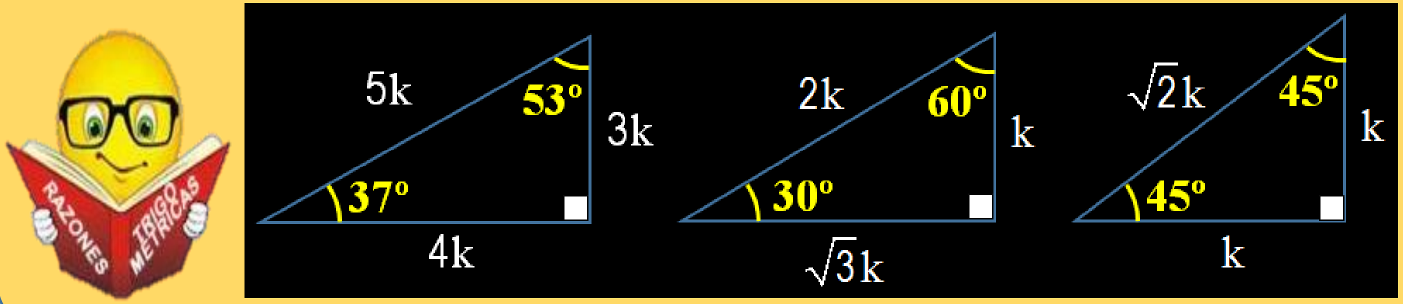
1. Calcule:

$$E = \frac{\sec 60^\circ + \tan 45^\circ + 2\cos 60^\circ}{\sec 37^\circ + \tan 37^\circ}$$

A) 0 B) 1

C) 2 D) $\frac{1}{2}$

Recordar:



RESOLUCIÓN

Usando las RT de los ángulos notables, tenemos:

$$= \frac{\quad + \quad + \quad \times\ —}{—\ +\ —}$$

$\Rightarrow \quad = —$

$\boxed{\therefore\ E = 2}$

2. Resuelva:

$$\frac{+}{2 \qquad -} = \qquad +$$

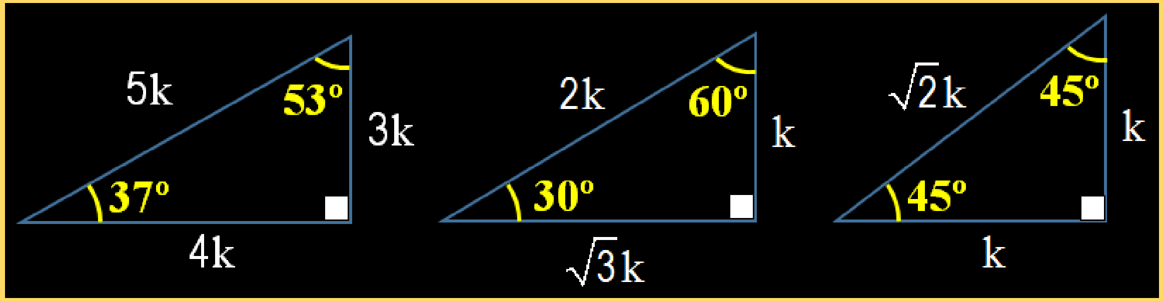
A) 0

B) 3

C) 1

D) 2

Recordar:



RESOLUCIÓN

Usando las RT de los ángulos notables, tenemos:

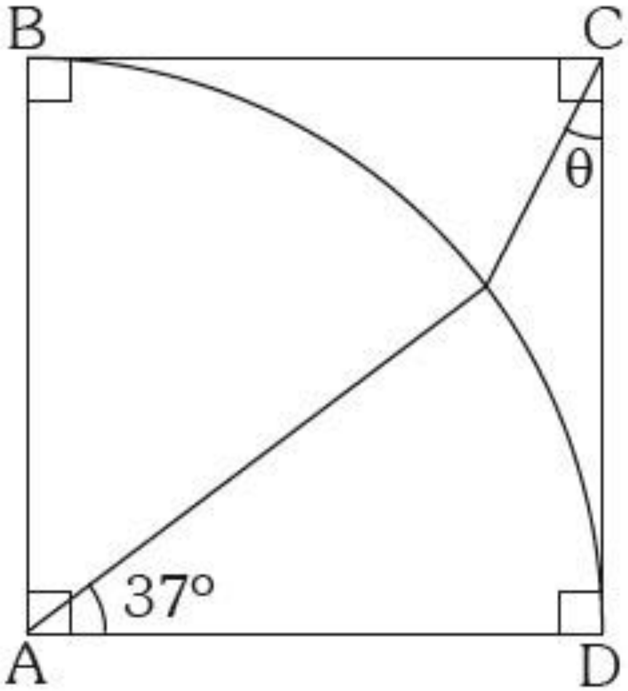
$$\frac{+ \quad x\,-}{(\sqrt{\ })\quad -} = \quad x - + -$$

$$\Rightarrow \frac{+}{-} = - + - \qquad \Rightarrow \frac{+}{-} =$$

$$\Rightarrow x + 4 = 4 - 2x \qquad \Rightarrow 3x = 0$$

$$\boxed{\therefore\ x = 0}$$

3. Si ABCD es un cuadrado, halle $\tan\theta$



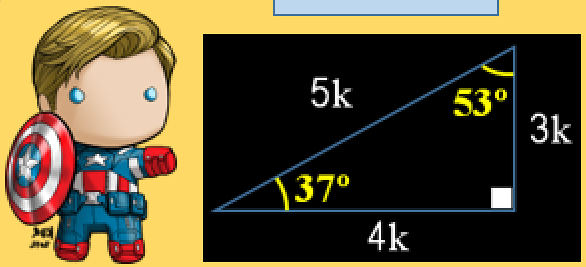
A) $\frac{1}{2}$

B) $\frac{1}{3}$

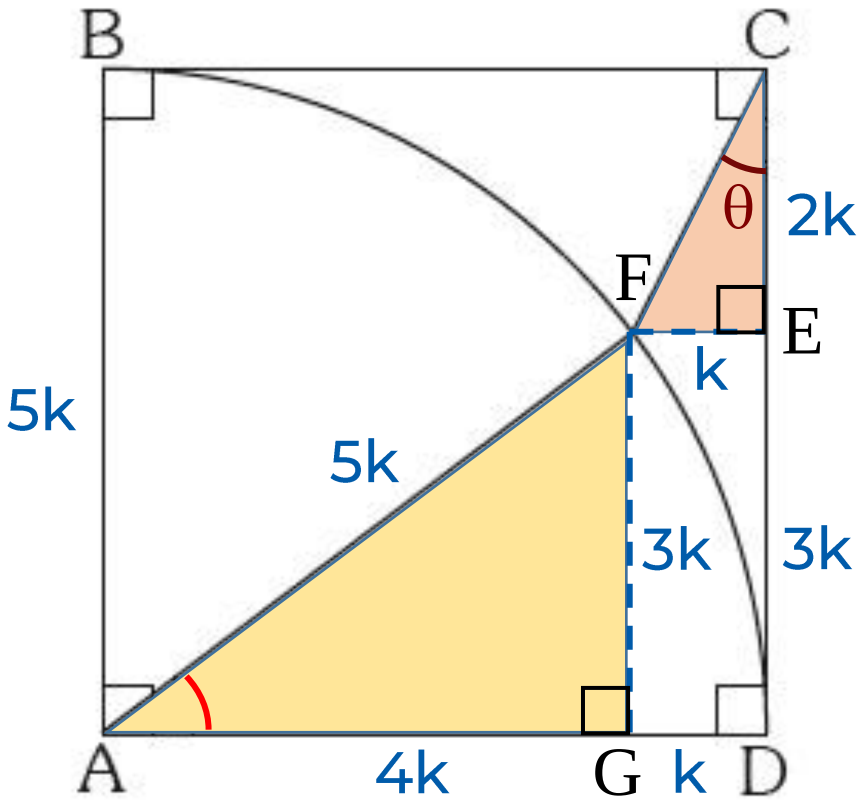
C) $\frac{2}{3}$

D) $\frac{1}{4}$

Recordar:



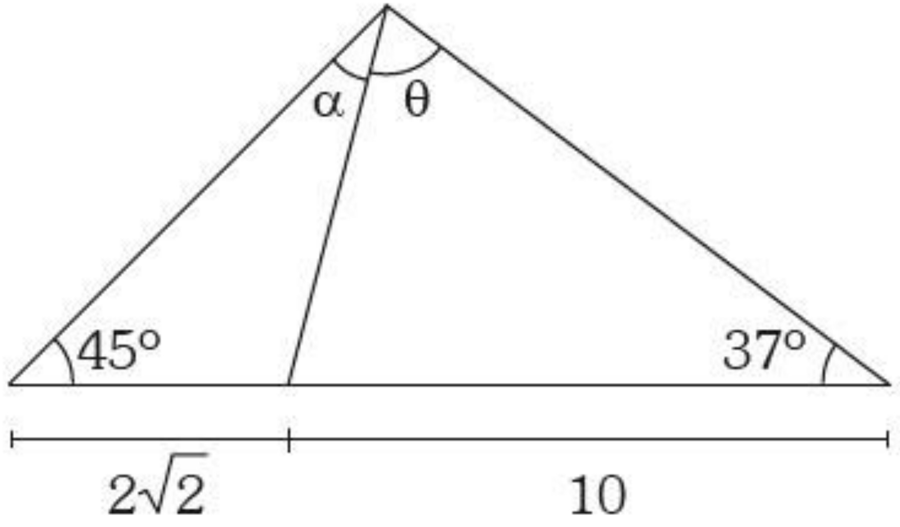
RESOLUCIÓN



* ◣ CEF: $\tan\theta = \frac{k}{2k}$

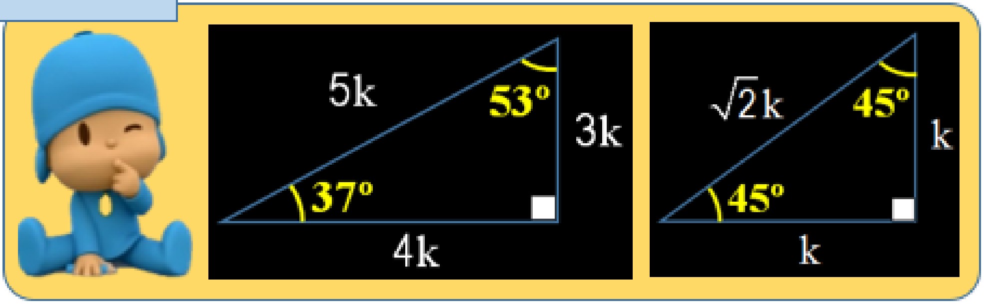
$\boxed{\therefore\ \tan\theta = \frac{1}{2}}$

4. Del gráfico mostrado, calcule $\operatorname{sen}\theta.\csc\alpha$

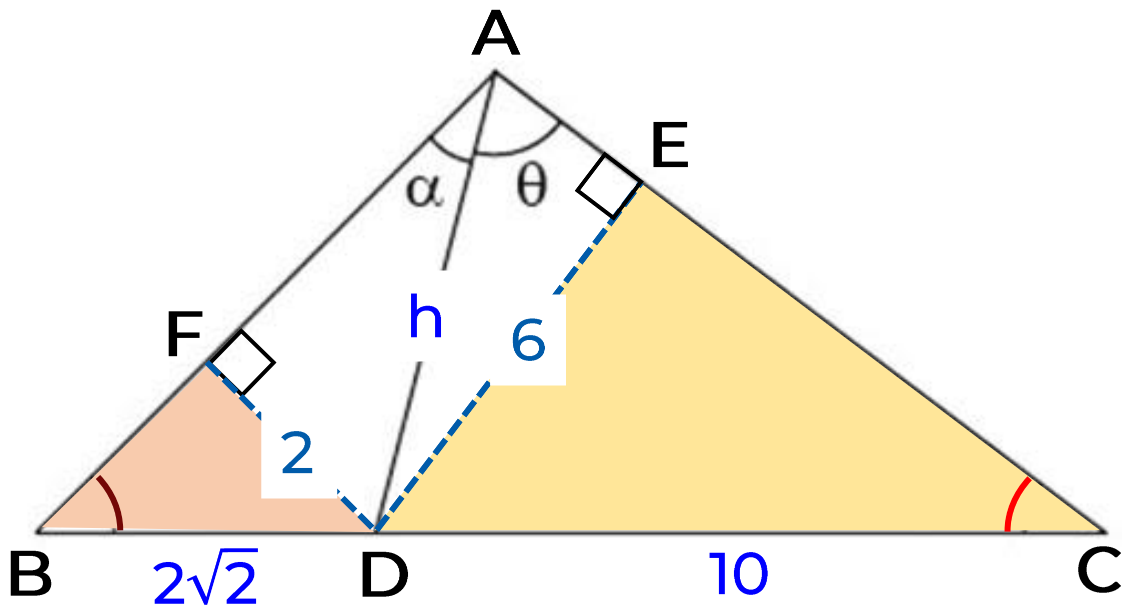


A) 1   B) 2   C) 3   D) 4

Recordar:



RESOLUCIÓN



* ◣ AED: $\operatorname{sen}\theta = \dfrac{6}{h}$

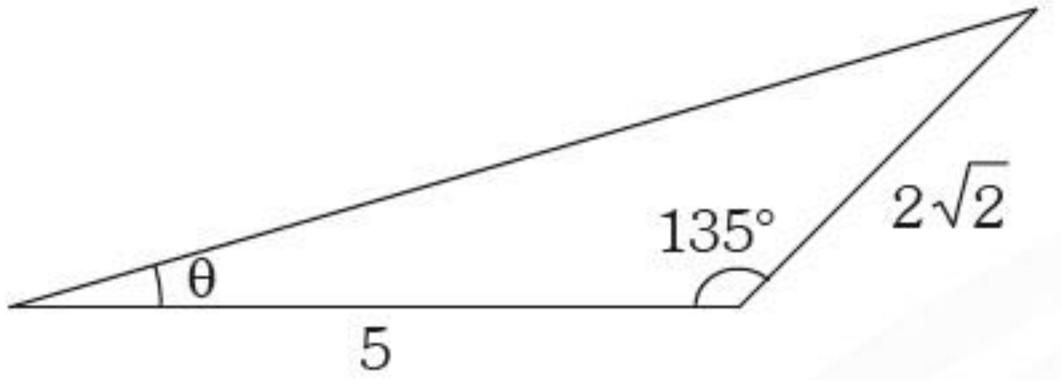
* ◣ AFD: $\csc\alpha = \dfrac{h}{2}$

Piden:

$$\operatorname{sen}\theta.\csc\alpha = \frac{6}{\cancel{h}} \times \frac{\cancel{h}}{2}$$

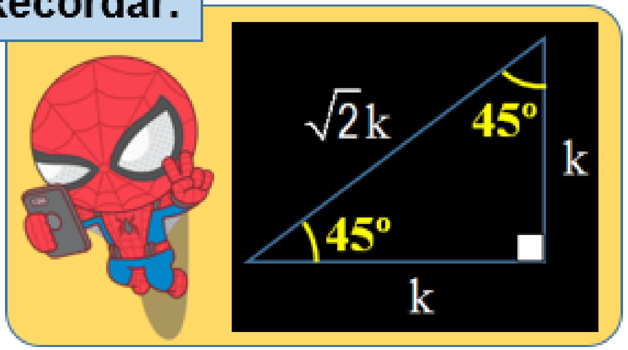
$$\boxed{\therefore\ \operatorname{sen}\theta.\csc\alpha = 3}$$

5. Calcule $\tan\theta$, en:

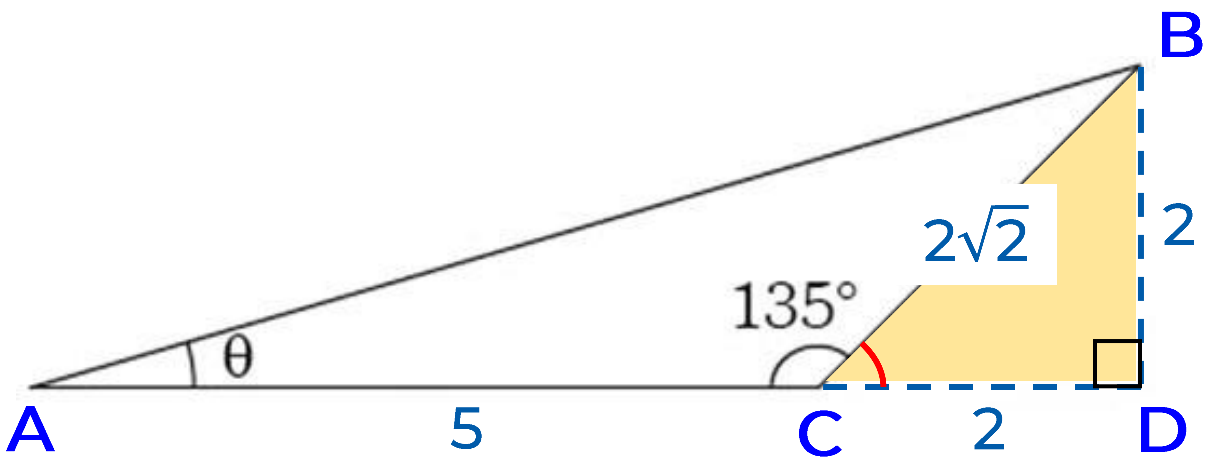


A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{3}{7}$ D) $\frac{4}{7}$

Recordar:



RESOLUCIÓN



* ◣ ADB: $\tan\theta = \dfrac{BD}{AD} = \dfrac{2}{5+2}$

$$\therefore \tan\theta = \frac{2}{7}$$

6. Calcule:

$$E = \text{sen}25°.\sec65° + 2\tan40°.\tan50°$$

A) 1

B) 2

~~C)~~ 3

D) 4

1) R.T. RECÍPROCAS

$\text{sen}\alpha.\csc\alpha = 1$
$\cos\alpha.\sec\alpha = 1$
$\tan\alpha.\cot\alpha = 1$

Iguales

2) R.T. DE ÁNGULOS COMPLEMENTARIOS

$\text{sen}\alpha = \cos\beta$
$\tan\alpha = \cot\beta$
$\sec\alpha = \csc\beta$

suman 90°

RESOLUCIÓN

Piden:

$$E = \text{sen}25°.\sec65° + 2\tan40°.\tan50°$$

Usando propiedad **2)** :

$$E = \underbrace{\text{sen}25°.\csc25°}_{1} + 2\underbrace{\tan40°.\cot40°}_{1}$$

Usando propiedad **1)** :

$$\Rightarrow E = 1 + 2 \times 1$$

$$\boxed{\therefore\ E = 3}$$

7. Halle el valor de x, si se cumple la siguiente ecuación:

$$\sec(90° - x) = \csc(3x - 18°)$$

A) 4°　　B) 5°　　C) 7°　　D) 9°

2) R.T. DE ÁNGULOS COMPLEMENTARIOS

$$\operatorname{sen}\alpha = \cos\beta$$

$$\tan\alpha = \cot\beta$$

$$\underbrace{\sec\alpha = \csc\beta}_{\text{suman } 90°} \quad \ldots (*)$$

RESOLUCIÓN

Dato: $\sec(90° - x) = \csc(3x - 18°)$

Usando **(*)**:

$$(\cancel{90°} - x) + (3x - 18°) = \cancel{90°}$$

$$\Rightarrow 2x - 18° = 0$$

$$\Rightarrow 2x = 18°$$

$$\boxed{\therefore\ x = 9°}$$

8. Halle el valor de " m ", si:

$$\tan(\sqrt{m} - 3)°.\cot 13° = 1$$

A) 220 B) 224

C) 226 D) 256

1) R.T. RECÍPROCAS

$\operatorname{sen}\alpha.\csc\alpha = 1$

$\cos\alpha.\sec\alpha = 1$

$\tan\alpha.\cot\alpha = 1$ ... (*)

Iguales

RESOLUCIÓN

Dato: $\tan(\sqrt{m} - 3)°.\cot 13° = 1$

Usando **(*)**:

$$(\sqrt{m} - 3)° = 13°$$

$$\Rightarrow \sqrt{m} - 3 = 13$$

$$\Rightarrow \sqrt{m} = 16$$

Elevando al cuadrado:

$$\boxed{\therefore\ m = 256}$$

9. Si se verifica que:

$$\text{sen}3x - \cos y = 0$$

$$\tan 2y.\cot 30° - 1 = 0$$

Calcule: $H = \sqrt{3}\sec(x + 5°) + \tan(x + y + 5°)$

A) 1 B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) 3

1) R.T. RECÍPROCAS

$\text{sen}\alpha.\csc\alpha = 1$

$\cos\alpha.\sec\alpha = 1$

$\underbrace{\tan\alpha.\cot\alpha}_{\text{Iguales}} = 1$ ... (**)

2) R.T. DE ÁNGULOS COMPLEMENTARIOS

$\text{sen}\alpha = \cos\beta$ ... (*)

$\tan\alpha = \cot\beta$

$\underbrace{\sec\alpha = \csc\beta}_{\text{suman } 90°}$

RESOLUCIÓN

Dato 1: $\text{sen}3x = \cos y$

Usando (*): $3x + y = 90°$ ... (1)

Dato 2: $\tan 2y.\cot 30° = 1$

Usando (**): $2y = 30° \Rightarrow \boxed{y = 15°}$ ... (2)

(2) en (1): $3x + 15° = 90° \Rightarrow \boxed{x = 25°}$

Reemplazando en H:

$\Rightarrow H = \sqrt{3}\sec(30°) + \tan(45°)$

$\Rightarrow H = \cancel{\sqrt{3}} \times \frac{2}{\cancel{\sqrt{3}}} + 1$

$\boxed{\therefore H = 3}$

10. Si $\cos A = \dfrac{3x+2}{3x+1}$ y $\operatorname{sen} B = \dfrac{x+1}{x+2}$. Determine el valor de tanA, sabiendo que A y B son ángulos agudos complementarios.

A) 5

B) $2\sqrt{6}$

C) $2\sqrt{6}$

D) $\dfrac{\sqrt{6}}{8}$

RESOLUCIÓN

Dato: $A + B = 90°$

R.T de Ángulos complem $\cos A = \operatorname{sen} B$

Así tenemos: $\dfrac{3x+2}{3x+1} = \dfrac{x+1}{x+2}$

$\Rightarrow (3x+2)(x+2) = (3x+1)(x+1)$

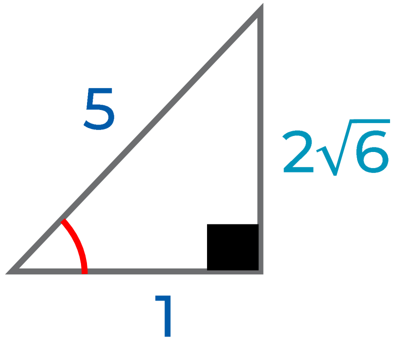
$\Rightarrow \cancel{3x^2} + 8x + 4 = \cancel{3x^2} + 4x + 1$

$\Rightarrow 4x = -3 \Rightarrow \boxed{x = -\dfrac{3}{4}}$

Luego:

$\cos A = \dfrac{3x+2}{3x+1} \Rightarrow \cos A = \dfrac{3\left(-\frac{3}{4}\right)+2}{3\left(-\frac{3}{4}\right)+1}$

$\Rightarrow \cos A = \dfrac{-\frac{1}{4}}{-\frac{5}{4}} = \dfrac{1}{5}$



Piden: $\tan A = \dfrac{2\sqrt{6}}{1}$

$\therefore \tan A = 2\sqrt{6}$