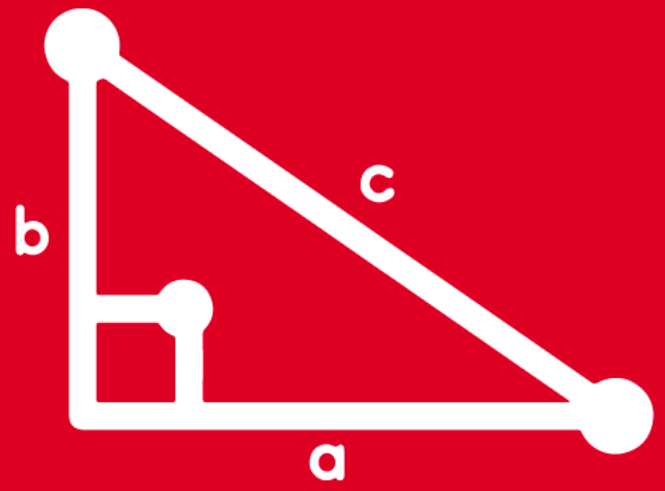
TRIGONOMETRY

Chapter 1

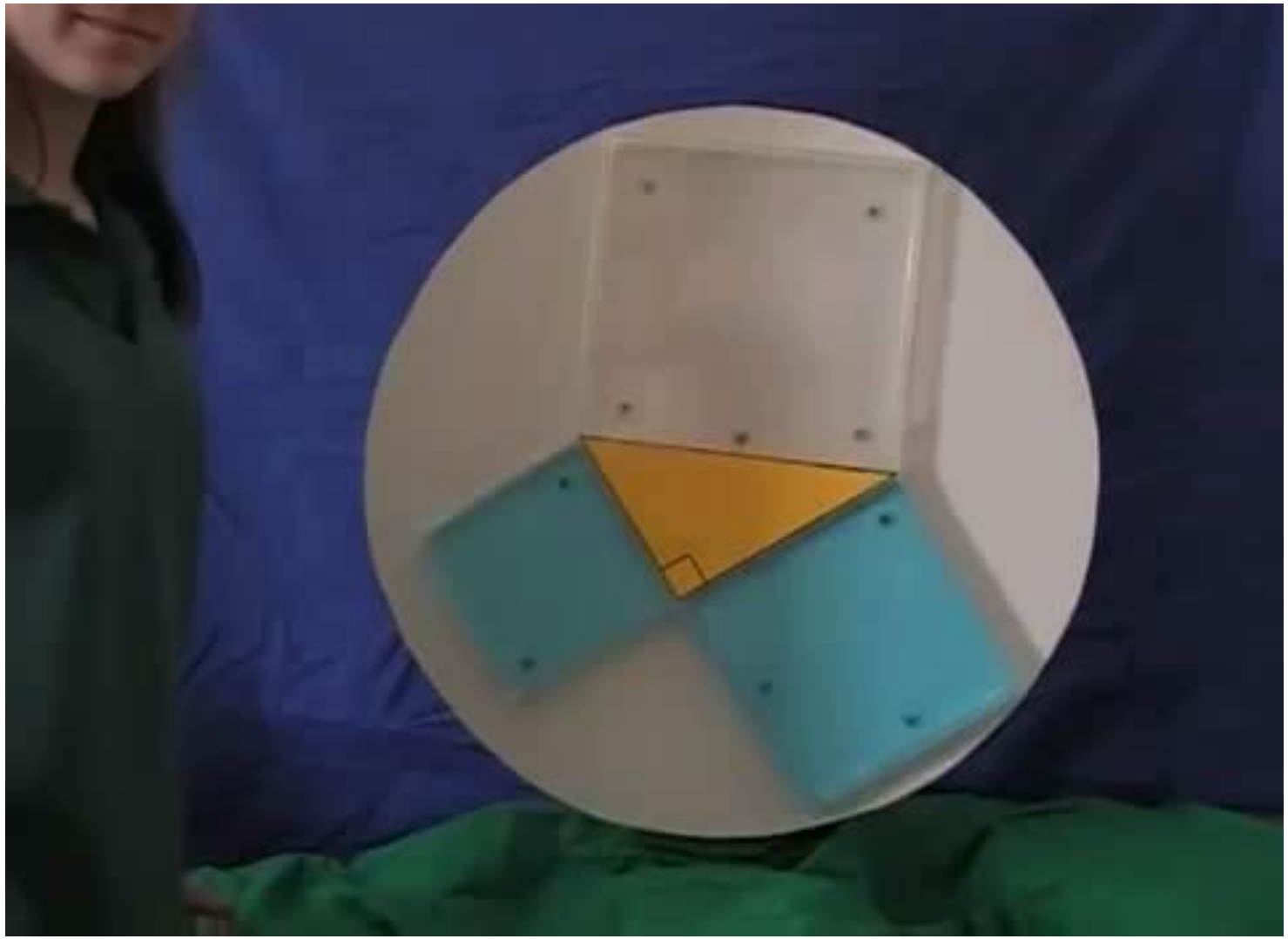
Verano 2021

SAN MARCOS

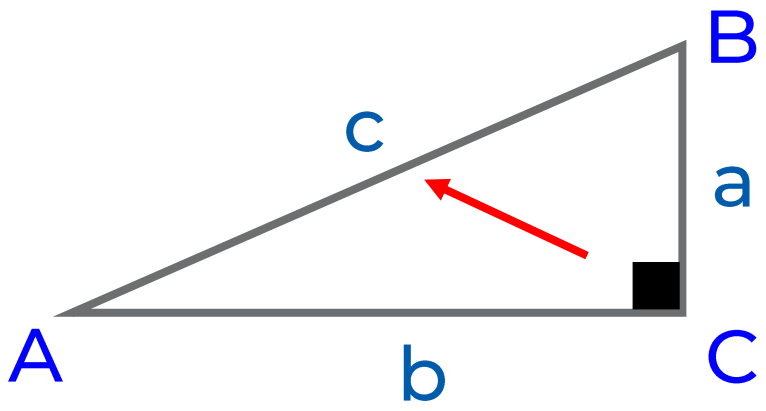


Razones trigonométricas de un ángulo agudo I

SACO OLIVEROS



TRIÁNGULO RECTÁNGULO



c es la hipotenusa

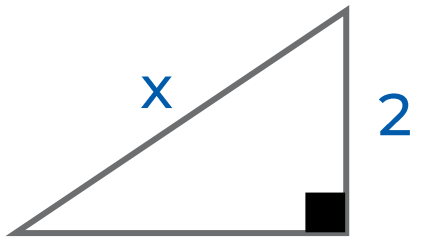
a y b son catetos

TEOREMA DE PITÁGORAS

$= \quad +$

EJEMPLOS:

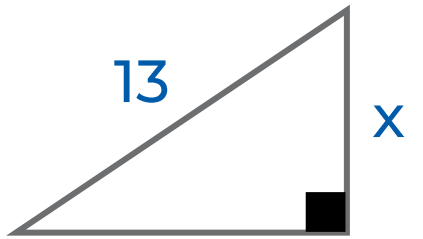
En cada figura mostrada , calcule x



$= \quad +$

$=$

$\therefore \quad = \sqrt{}$

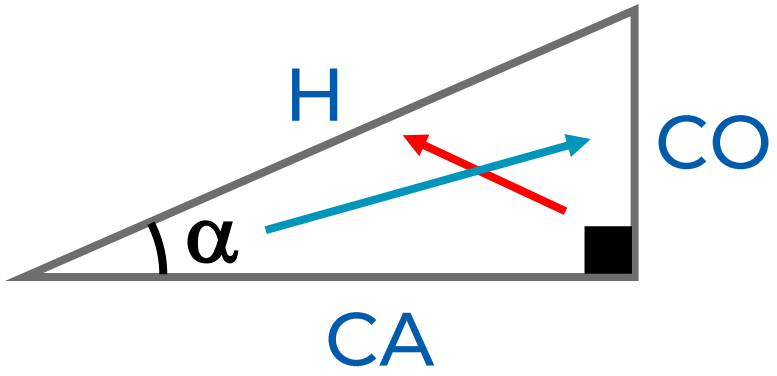


$= \quad +$

$=$

$\therefore \quad =$

RAZONES TRIGONOMÉTRICAS DE UN ÁNGULO AGUDO



H : Hipotenusa

CO : Cateto opuesto al ángulo α

CA : Cateto adyacente al ángulo α

DEFINICIONES

$\text{sen}\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\csc\alpha$
$\frac{CO}{H}$	$\frac{CA}{H}$	$\frac{CO}{CA}$	$\frac{CA}{CO}$	$\frac{H}{CA}$	$\frac{H}{CO}$

1. En un triángulo rectángulo, un cateto es el triple del otro. Determine la cosecante del mayor ángulo agudo del triángulo.

A) $\sqrt{10}$

B) $\frac{\sqrt{10}}{3}$

C) $3\sqrt{10}$

D) $\frac{\sqrt{10}}{10}$

Recordar:

$$\csc\theta = \frac{H}{CO}$$

RESOLUCIÓN

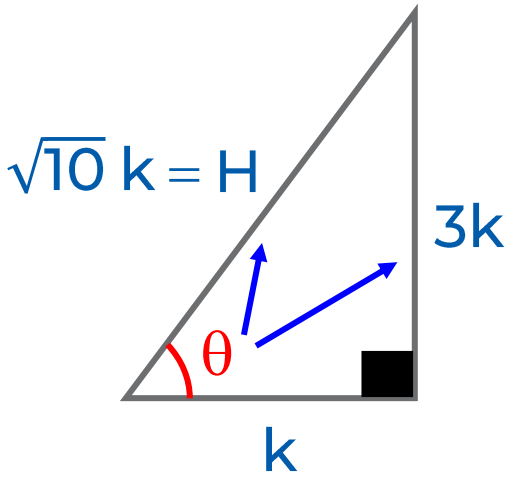
Teorema de Pitágoras:

$$H^2 = (k)^2 + (3k)^2$$

$$\Rightarrow H^2 = k^2 + 9k^2$$

$$\Rightarrow H^2 = 10k^2$$

$$\Rightarrow H = \sqrt{10}\,k$$



Piden:

$$\csc\theta = \frac{\sqrt{10}\,k}{3k}$$

$$\therefore \csc\theta = \frac{\sqrt{10}}{3}$$

2. En un triángulo rectángulo, los lados menores miden 5 cm y 12 cm. Si el menor ángulo agudo del triángulo mide α, calcule: $P = \csc\alpha + \cot\alpha$

A) 2 B) 3

C) 3 / 2 D) 5

Recordar:

$$\csc\alpha = \frac{H}{CO} \qquad \cot\alpha = \frac{CA}{CO}$$

RESOLUCIÓN

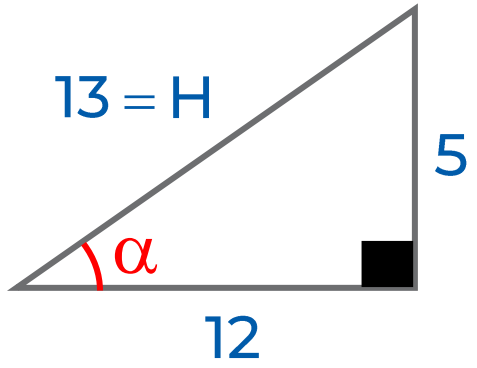
Teorema de Pitágoras:

$H^2 = (12)^2 + (5)^2$

$\Rightarrow H^2 = 144 + 25$

$\Rightarrow H^2 = 169$

$\Rightarrow H = 13$



Piden: $P = \csc\alpha + \cot\alpha$

$$\Rightarrow P = \frac{13}{5} + \frac{12}{5}$$

$$\Rightarrow P = \frac{25}{5}$$

$\therefore P = 5$

3. Dado $\cos x = 1/3$, halle:

$E = \csc^2 x + \cot^2 x$

Si x es un ángulo agudo.

A) 1 / 8 B) 3 / 4

C) 9 / 4 D) 5 / 4

Recordar:

$\csc = \frac{H}{CO}$ $\cot = \frac{CA}{CO}$

RESOLUCIÓN

Dato: $\cos x = \frac{1}{3} = \frac{CA}{H}$

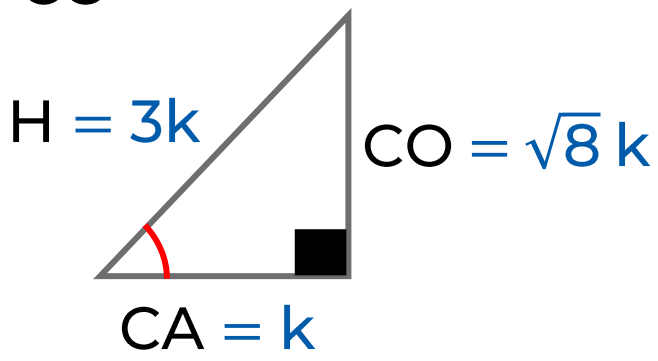
Teorema de Pitágoras $H^2 = CA^2 + CO^2$

$\Rightarrow (3k)^2 = (k)^2 + CO^2$

$\Rightarrow 9k^2 = k^2 + CO^2$

$\Rightarrow 8k^2 = CO^2$

$\Rightarrow CO = \sqrt{8}\,k$



Piden: $E = \csc^2 x + \cot^2 x$

$\Rightarrow E = \left(\frac{3k}{\sqrt{8}k}\right)^2 + \left(\frac{k}{\sqrt{8}k}\right)^2$

$\Rightarrow E = \frac{9}{8} + \frac{1}{8} = \frac{10}{8}$

$\therefore E = \frac{5}{4}$

4. Siendo $\cos x = 8/17$ y x es agudo, calcule:

$$E = \frac{7\sec x}{\tan x - 1}$$

A) 9 B) 8

C) 13 D) 17

Recordar:

$\sec = \frac{H}{CA}$ $\tan = \frac{CO}{CA}$

RESOLUCIÓN

Dato: $\cos x = \frac{8}{17} = \frac{CA}{H}$

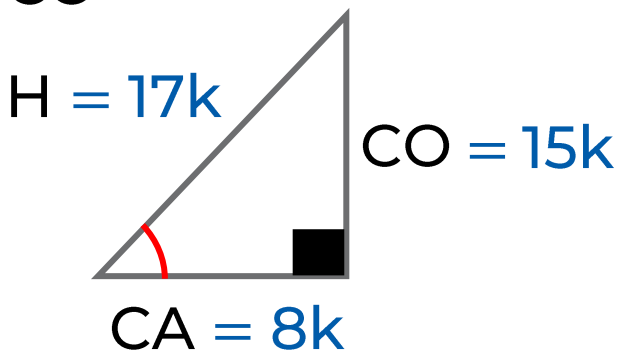
Teorema de Pitágoras: $H^2 = CA^2 + CO^2$

$\Rightarrow (17k)^2 = (8k)^2 + CO^2$

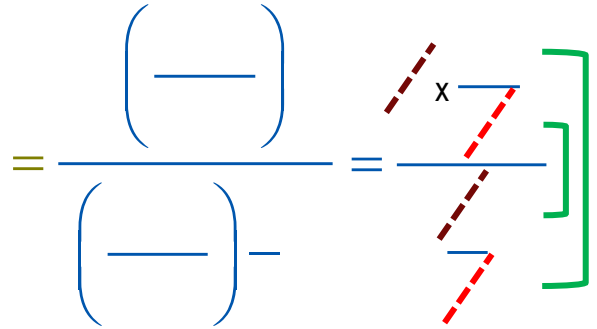
$\Rightarrow 289k^2 = 64k^2 + CO^2$

$\Rightarrow 225k^2 = CO^2$

$\Rightarrow CO = 15k$

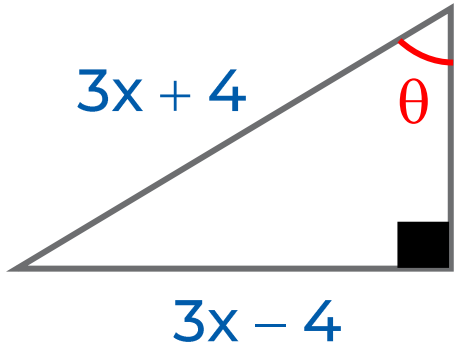


Piden:

$E = \frac{7\sec x}{\tan x - 1} \Rightarrow$ 

$\therefore E = 17$

5. Halle el valor de x, si $\text{sen}\theta = \frac{4}{5}$



A) 10

B) 12

C) 13

D) 11

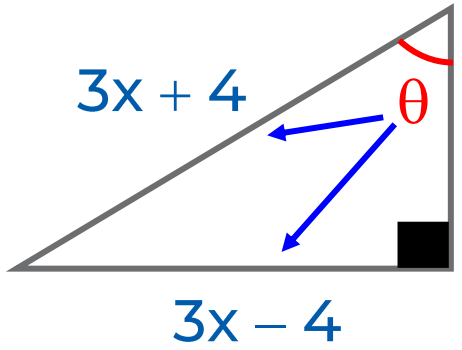
Recordar:

$$\text{sen}\theta = \frac{CO}{H}$$

RESOLUCIÓN

Del gráfico:

$$\begin{cases} CO = 3x - 4 \\ H = 3x + 4 \end{cases}$$



Luego:

$$\text{sen}\theta = \frac{3x-4}{3x+4} \Rightarrow \frac{4}{5} = \frac{3x-4}{3x+4}$$

$$\Rightarrow 4(3x+4) = 5(3x-4)$$

$$\Rightarrow 12x + 16 = 15x - 20$$

$$\Rightarrow 36 = 3x$$

$$\therefore x = 12$$

6. En un triángulo rectángulo ABC(B=90°). Reduzca: E = a.secC + b.senA + c, si su perímetro es 20 cm.

A) 20 cm

B) 10 cm

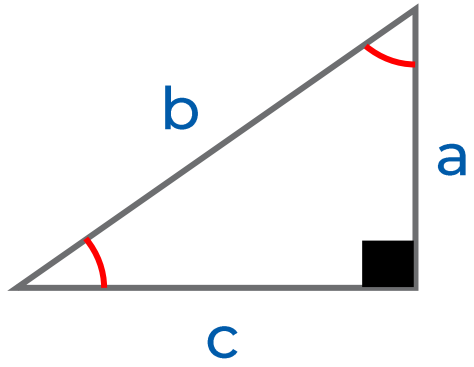
C) 5 cm

D) 40 cm

Recordar:

$$\sec\theta = \frac{H}{CA} \qquad \operatorname{sen}\theta = \frac{CO}{H}$$

RESOLUCIÓN



Piden: E = a.secC + b.senA + c

$$\Rightarrow E = \cancel{a}\cdot\frac{b}{\cancel{a}} + \cancel{b}\cdot\frac{a}{\cancel{b}} + c$$

$$\Rightarrow E = \underbrace{b + a + c}_{\text{Perímetro}}$$

$$\therefore E = 20 \text{ cm}$$

7. En un triángulo rectángulo ABC(B=90°).
Reduzca: $E = \sec^2 C - \cot^2 A$

A) 1
B) 2
C) 3
D) $a^2 - c^2$

Recordar:

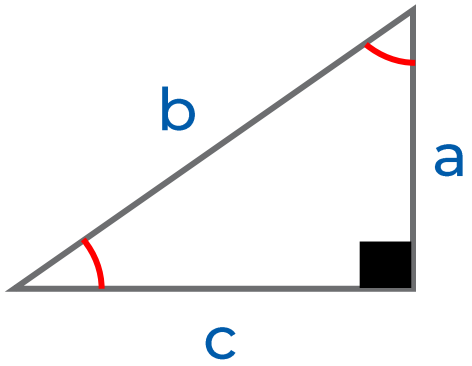
$\sec = \frac{H}{CA}$ $\quad \cot = \frac{CA}{CO}$

RESOLUCIÓN

Teorema de Pitágoras:

$b^2 = a^2 + c^2$

$b^2 - c^2 = a^2$ ... (*)



Piden:

$E = \sec^2 C - \cot^2 A \Rightarrow E = \left(\frac{b}{a}\right)^2 - \left(\frac{c}{a}\right)^2$

$\Rightarrow E = \frac{b^2}{a^2} - \frac{c^2}{a^2} = \frac{b^2 - c^2}{a^2}$

Usando (*): $E = \frac{a^2}{a^2}$

$\therefore E = 1$

8. Si $\cos\phi = \frac{\sqrt{3}}{4}$; ϕ es agudo, calcule:

$$E = 13\csc^2\phi + 3\tan^2\phi$$

A) 23 B) 25

C) 27 D) 29

Recordar:

$$\csc\phi = \frac{H}{CO} \qquad \tan\phi = \frac{CO}{CA}$$

RESOLUCIÓN

Dato: $\cos\phi = \frac{\sqrt{3}}{4} = \frac{CA}{H}$

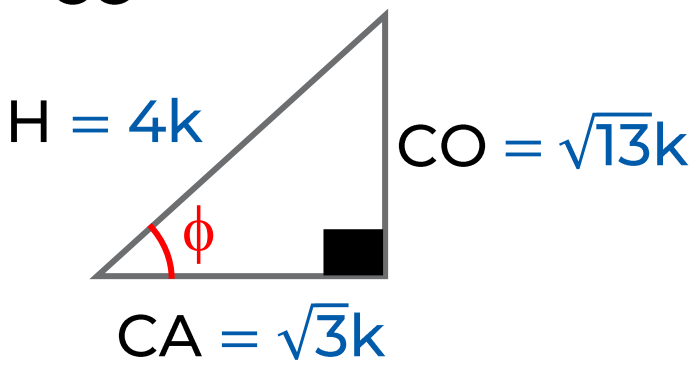
Teorema de Pitágoras $H^2 = CA^2 + CO^2$

$\Rightarrow (4k)^2 = (\sqrt{3}k)^2 + CO^2$

$\Rightarrow 16k^2 = 3k^2 + CO^2$

$\Rightarrow 13k^2 = CO^2$

$\Rightarrow CO = \sqrt{13}\,k$



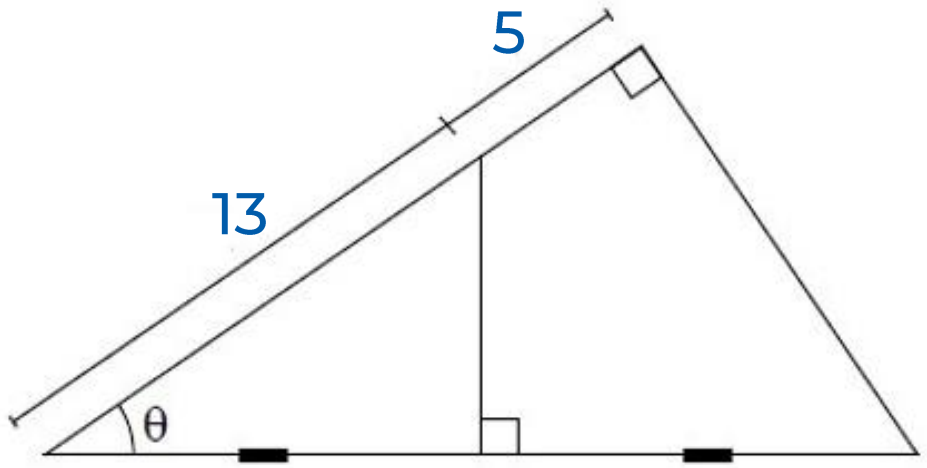
Piden: $E = 13\csc^2\phi + 3\tan^2\phi$

$$\Rightarrow E = 13\left(\frac{4k}{\sqrt{13}k}\right)^2 + 3\left(\frac{\sqrt{13}k}{\sqrt{3}k}\right)^2$$

$$\Rightarrow E = 13\cdot\frac{16}{13} + 3\cdot\frac{13}{3} = 16 + 13$$

$\therefore E = 29$

9. Del gráfico, calcule $\tan\theta$.



A) 1 / 3

B) 2 / 3

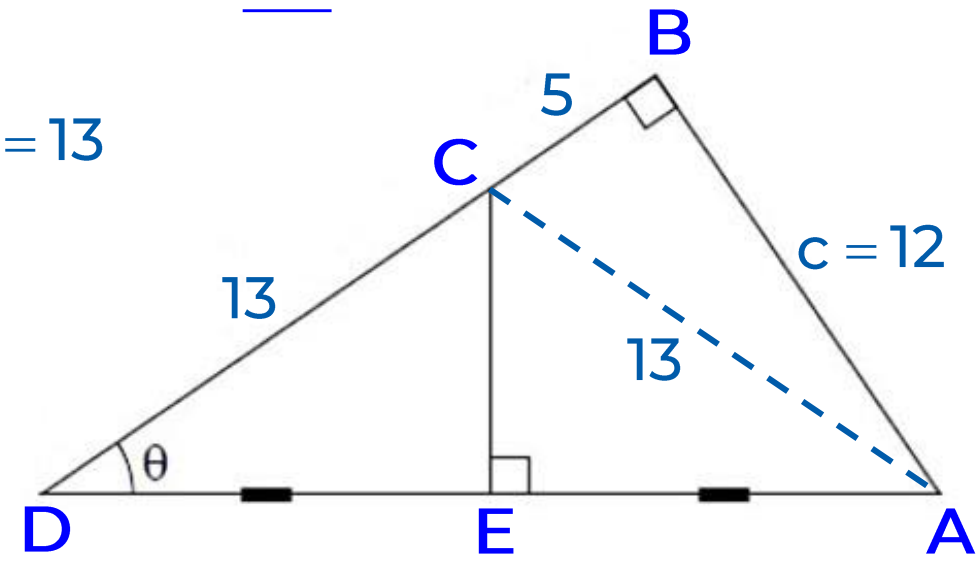
C) 1

D) 4 / 3

Recordar:

$$\tan\theta = \frac{CO}{CA}$$

RESOLUCIÓN



* $\Rightarrow CA = CD = 13$

* ◣ ABC: Teorema de Pitágoras:

$$13^2 = 5^2 + c^2 \Rightarrow 169 = 25 + c^2$$

$$\Rightarrow 144 = c^2 \Rightarrow c = 12$$

* ◣ ABD: $\tan\theta = \frac{12}{18}$

$\therefore \tan\theta = 2/3$

10. Un terreno en forma de un triángulo rectángulo el coseno de uno de sus ángulos agudos es 12/13, si el menor de sus lados es 20 m. Determine el área de dicho terreno.

A) 360 m^2 B) 450 m^2

C) 480 m^2 D) 390 m^2

RESOLUCIÓN

Dato 1: $\cos\theta = \frac{12}{13} = \frac{CA}{H}$

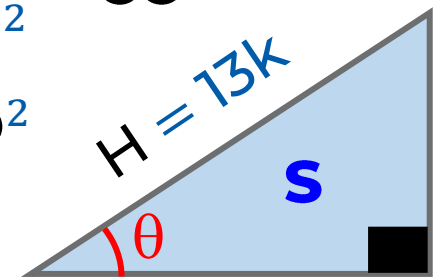
Teorema de Pitágoras $H^2 = CA^2 + CO^2$

$\Rightarrow (13k)^2 = (12k)^2 + CO^2$

$\Rightarrow 169k^2 = 144k^2 + CO^2$

$\Rightarrow 25k^2 = CO^2$

$\Rightarrow CO = 5k$



Dato 2: Lado menor $5k = 20$ ➡ $k = 4$

Piden: Área del terreno = **S**

$\Rightarrow S = \frac{48.20}{2} = 480$

$\therefore$ Área terreno = 480 m^2