



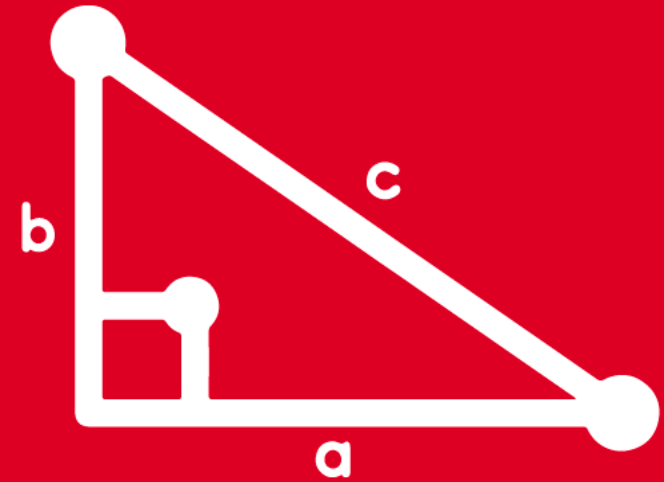
TRIGONOMETRY

Chapter 2

Verano 2021

SAN MARCOS

**Razones trigonométricas
de un ángulo agudo II**

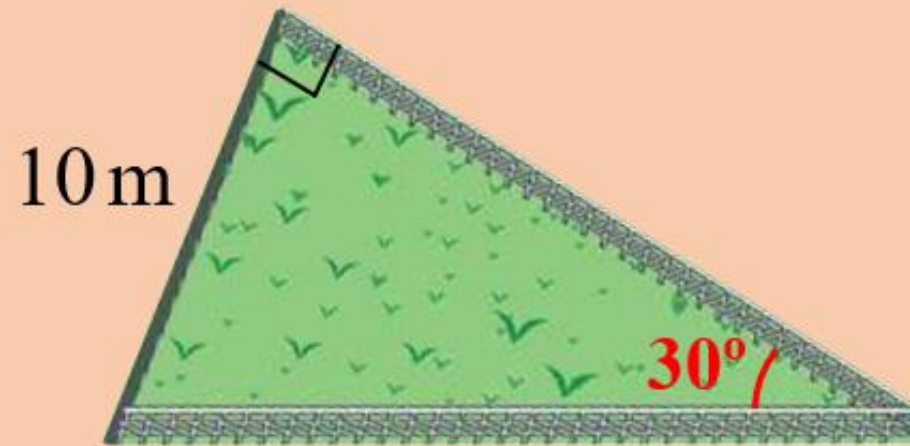


SACO OLIVEROS



La figura muestra un terreno que tiene la forma de un triángulo rectángulo.

¿puedes calcular aproximadamente el perímetro de dicho terreno?

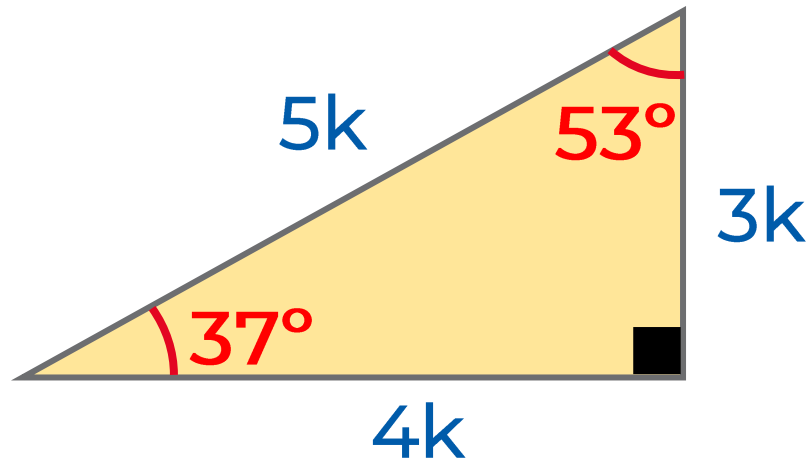


***Rpta* : 47,3 m**



RAZONES TRIGONOMÉTRICAS DE ÁNGULOS NOTABLES

RAZONES TRIGONOMÉTRICAS DE 37° y 53°



EJEMPLOS:

$$* \sin 37^\circ = \frac{3k}{5k} = \frac{3}{5}$$

$$* \sec 37^\circ = \frac{5k}{4k} = \frac{5}{4}$$

$$* \tan 53^\circ = \frac{4k}{3k} = \frac{4}{3}$$

$$* \csc 53^\circ = \frac{5k}{4k} = \frac{5}{4}$$

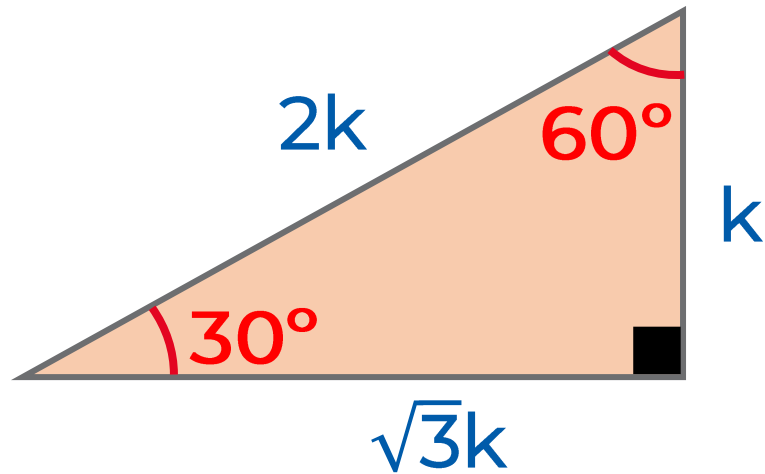
Así tenemos:

\angle	sen	cos	tan	cot	sec	csc
37°	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{5}{3}$
53°	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{5}{3}$	$\frac{5}{4}$





RAZONES TRIGONOMÉTRICAS DE 30° y 60°



EJEMPLOS:

$$* \sin 30^\circ = \frac{k}{2k} = \frac{1}{2}$$

$$* \cos 30^\circ = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$* \tan 60^\circ = \frac{\sqrt{3}k}{k} = \sqrt{3}$$

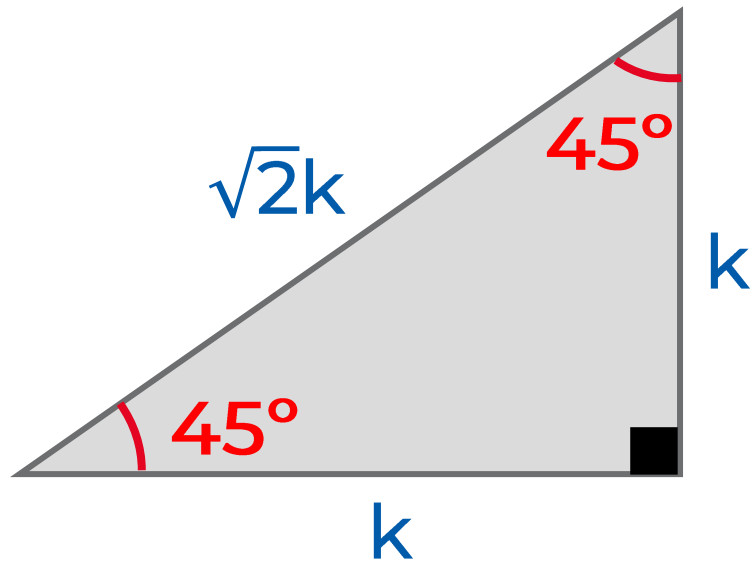
$$* \csc 60^\circ = \frac{2k}{\sqrt{3}k} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Así tenemos:

∠	sen	cos	tan	cot	sec	csc
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$



RAZONES TRIGONOMÉTRICAS DE 45°



EJEMPLOS:

$$* \sec 45^\circ = \frac{\sqrt{2}k}{k} = \sqrt{2}$$

$$* \tan 45^\circ = \frac{k}{k} = 1$$

$$* \sin 45^\circ = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$* \cos 45^\circ = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Así tenemos :

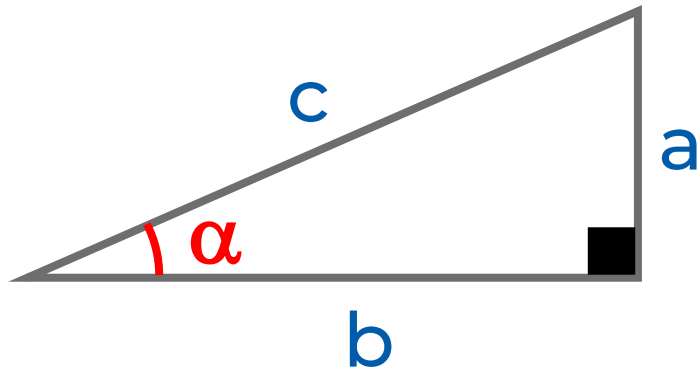
∠	sen	cos	tan	cot	sec	csc
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$





PROPIEDADES DE LAS R.T. DE UN ÁNGULO AGUDO

1) RAZONES TRIGONOMÉTRICAS RECÍPROCAS



$$\operatorname{sen}\alpha \cdot \operatorname{csc}\alpha = 1$$

$$\cos\alpha \cdot \sec\alpha = 1$$

$$\tan\alpha \cdot \cot\alpha = 1$$

↑ ↑
Iguales

EJEMPLOS:

$$* \operatorname{sen}53^\circ \cdot \operatorname{csc}53^\circ = 1$$

$$* \cos70^\circ \cdot \sec70^\circ = 1$$

$$* \tan3\theta \cdot \cot3\theta = 1$$

* Hallar x, si se cumple:

$$\operatorname{sen}2x \cdot \operatorname{csc}40^\circ = 1$$

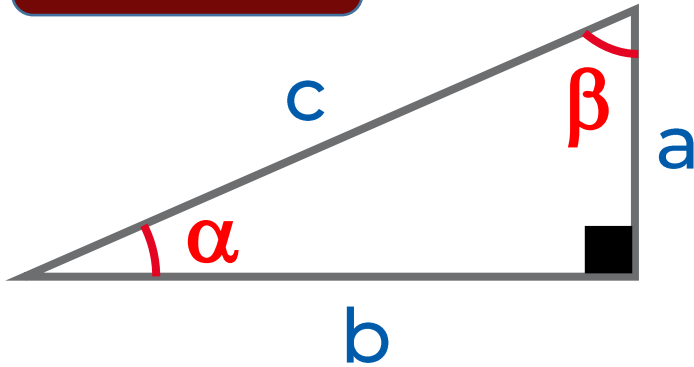
Resolución:

$$2x = 40^\circ \quad \therefore x = 20^\circ$$



2) R.T. DE ÁNGULOS COMPLEMENTARIOS

$$\alpha + \beta = 90^\circ$$



$$\operatorname{sen} \alpha = \cos \beta$$

$$\tan \alpha = \cot \beta$$

$$\sec \alpha = \csc \beta$$

↑ ↑
suman 90°

EJEMPLOS:

$$* \operatorname{sen} 60^\circ = \cos 30^\circ$$

$$* \tan 40^\circ = \cot 50^\circ$$

$$* \sec(\theta) = \csc(90^\circ - \theta)$$

* Hallar x , si se cumple:

$$\operatorname{sen} 5x = \cos 10^\circ$$

Resolución:

$$5x + 10^\circ = 90^\circ$$

$$\Rightarrow 5x = 80^\circ \quad \therefore x = 16^\circ$$



1. Calcule:

$$E = \frac{\sec 60^\circ + \tan 45^\circ + 2\cos 60^\circ}{\sec 37^\circ + \tan 37^\circ}$$

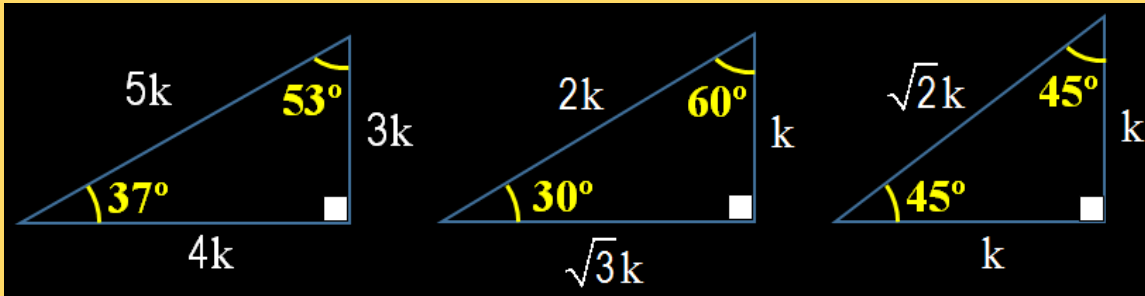
A) 0

B) 1

☒ C) 2

D) $\frac{1}{2}$

Recordar:



RESOLUCIÓN

Usando las RT de los ángulos notables, tenemos:

$$= \frac{\begin{array}{c} + \quad + \quad \text{---} \\ \text{---} \quad + \quad \text{---} \end{array}}{\text{---}}$$

$$\Rightarrow = \text{---}$$

$$\therefore E = 2$$





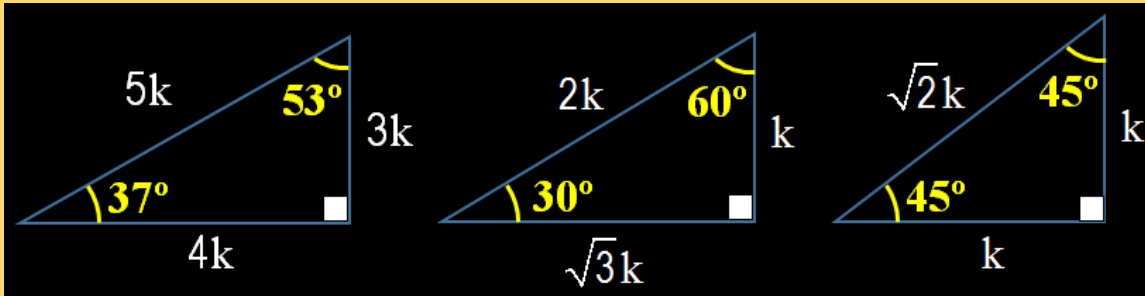
2. Resuelva:

$$\frac{+}{2} - = +$$

- ~~A) 0~~
C) 1

- B) 3
D) 2

Recordar:



RESOLUCIÓN

Usando las RT de los ángulos notables, tenemos:

$$\frac{+}{(\sqrt{-})} = x - + -$$

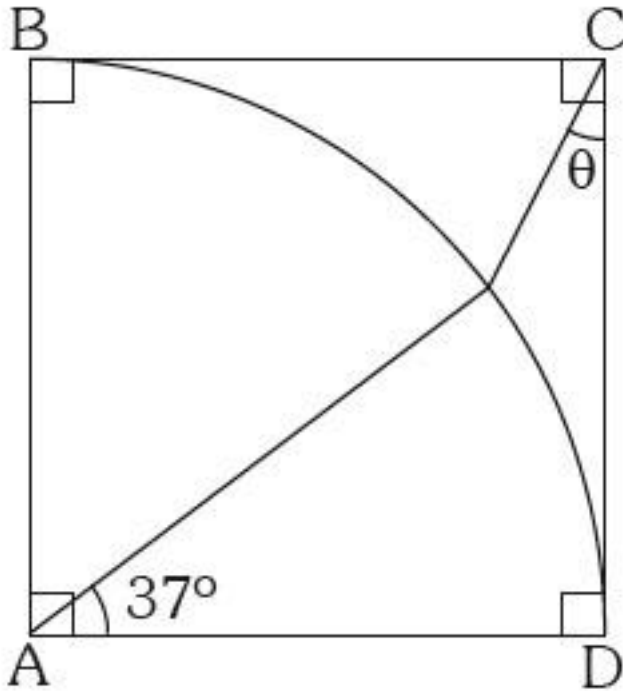
$$\Rightarrow \frac{+}{-} = - + - \Rightarrow \frac{+}{-} =$$

$$\Rightarrow x + 4 = 4 - 2x \Rightarrow 3x = 0$$

$$\therefore x = 0$$

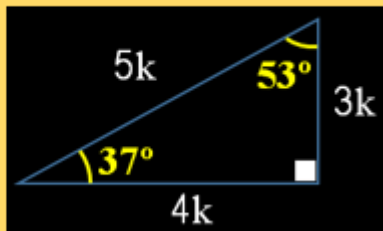


3. Si ABCD es un cuadrado, halle $\tan\theta$

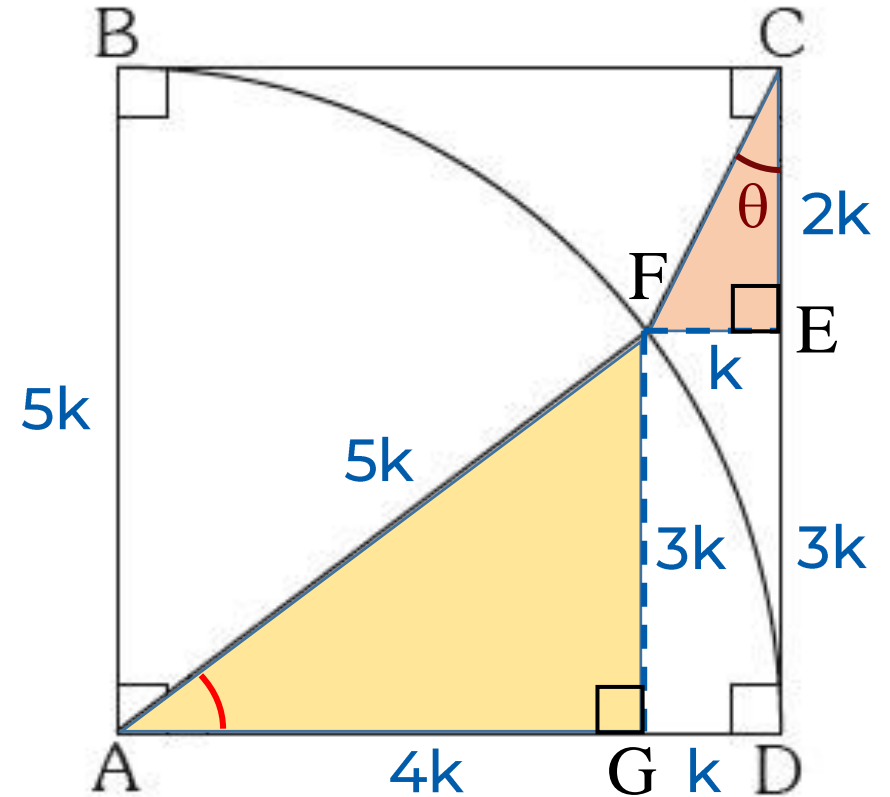


- A) $\frac{1}{2}$
- B) $\frac{1}{3}$
- C) $\frac{2}{3}$
- D) $\frac{1}{4}$

Recordar:



RESOLUCIÓN



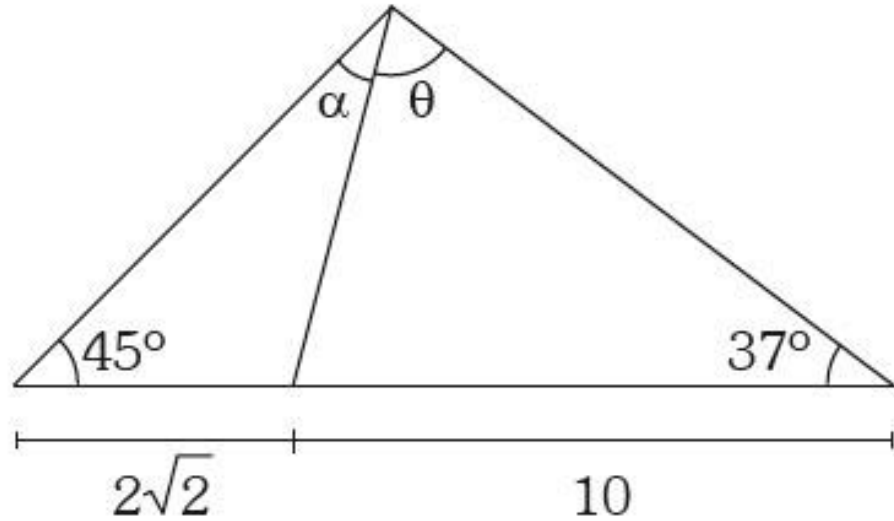
* $\triangle CEF: \tan\theta = \frac{k}{2k}$

$\therefore \tan\theta = \frac{1}{2}$



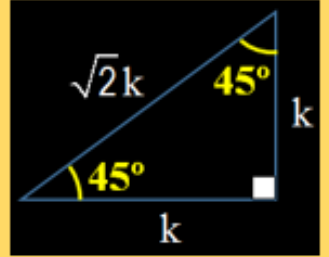
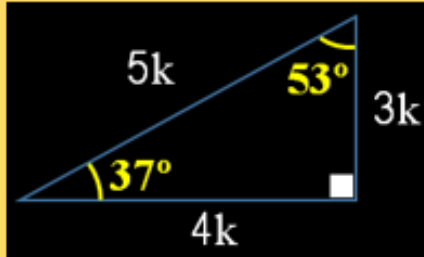


4. Del gráfico mostrado, calcule $\text{sen}\theta \cdot \text{csc}\alpha$

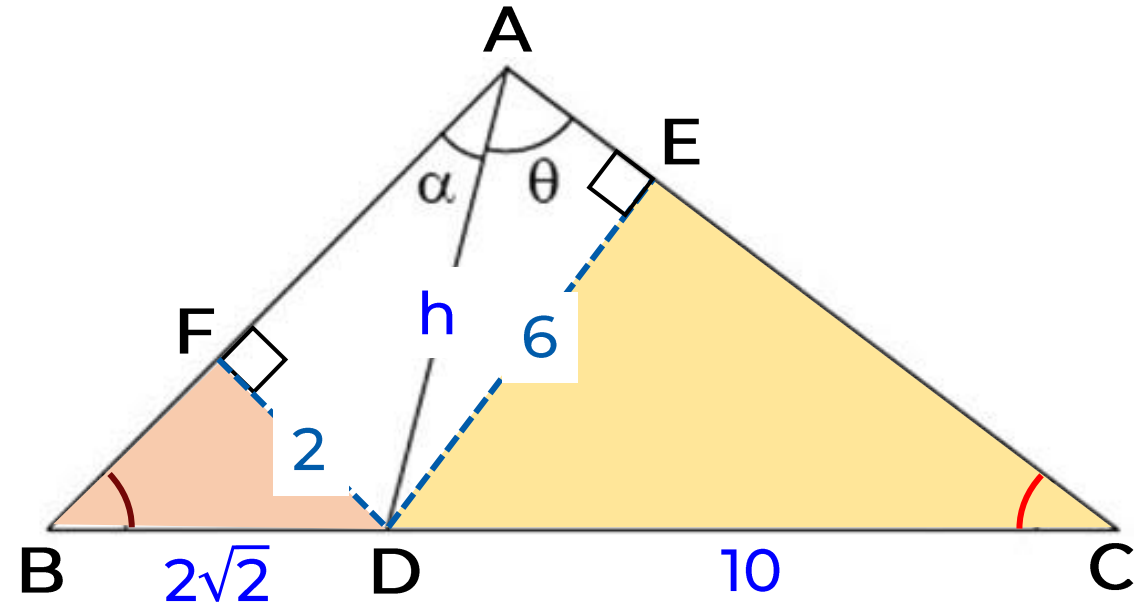


A) 1 B) 2 **C) 3** D) 4

Recordar:



RESOLUCIÓN



* $\triangle AED: \text{sen}\theta = \frac{6}{h}$

* $\triangle AFD: \text{csc}\alpha = \frac{h}{2}$

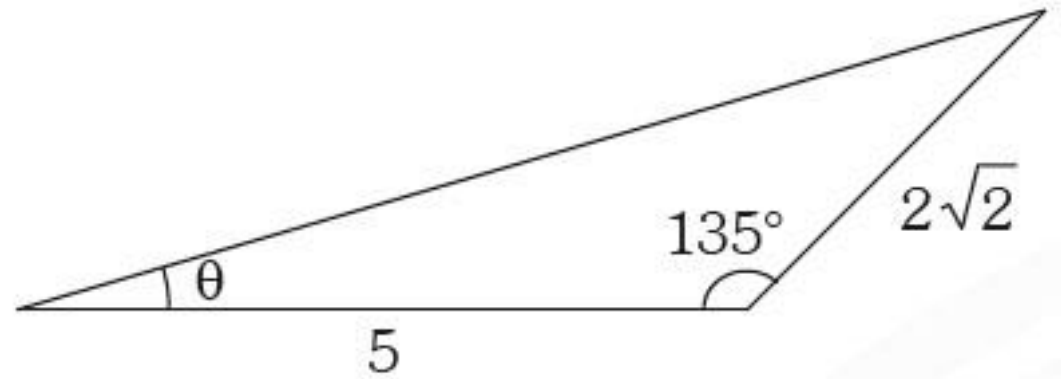
Piden:

$\text{sen}\theta \cdot \text{csc}\alpha = \frac{6}{h} \times \frac{h}{2}$

$\therefore \text{sen}\theta \cdot \text{csc}\alpha = 3$



5. Calcule $\tan\theta$, en:



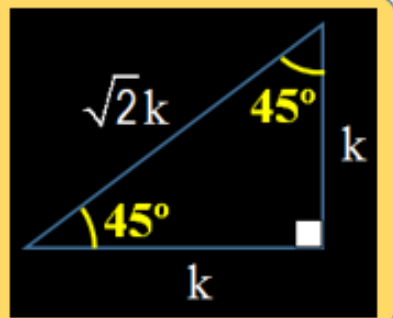
A) $\frac{1}{7}$

☒ B) $\frac{2}{7}$

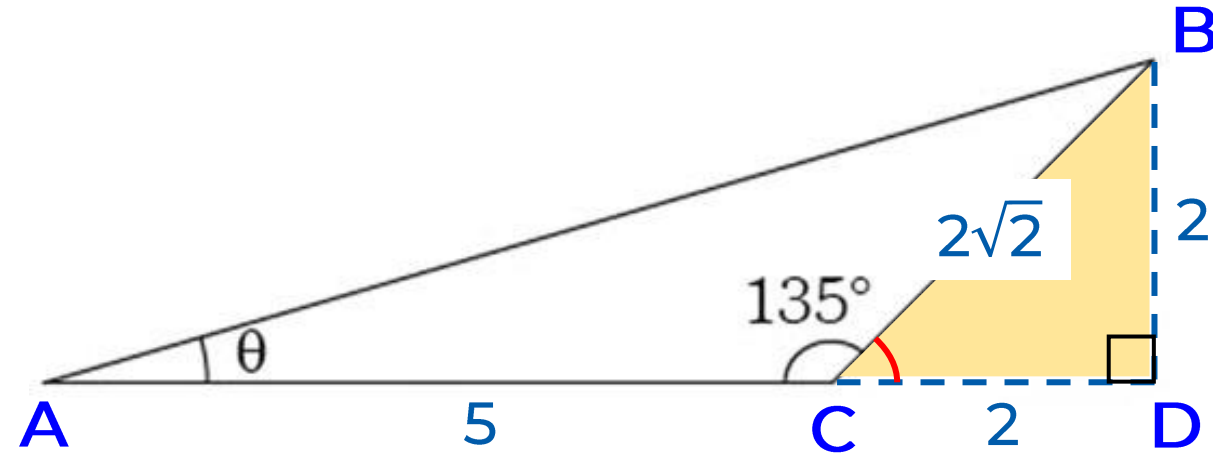
C) $\frac{3}{7}$

D) $\frac{4}{7}$

Recordar:



RESOLUCIÓN



* $\triangle ADB: \tan\theta = \frac{BD}{AD} = \frac{2}{5+2}$

$\therefore \tan\theta = \frac{2}{7}$

**6.** Calcule:

$$E = \operatorname{sen}25^\circ \cdot \operatorname{sec}65^\circ + 2 \tan40^\circ \cdot \tan50^\circ$$

A) 1

B) 2

~~C) 3~~

D) 4

1) R.T. RECÍPROCAS

$$\operatorname{sen}\alpha \cdot \operatorname{csc}\alpha = 1$$

$$\operatorname{cos}\alpha \cdot \operatorname{sec}\alpha = 1$$

$$\tan\alpha \cdot \cot\alpha = 1$$

↑ ↑
Iguales

2) R.T. DE ÁNGULOS**COMPLEMENTARIOS**

$$\operatorname{sen}\alpha = \operatorname{cos}\beta$$

$$\tan\alpha = \cot\beta$$

$$\operatorname{sec}\alpha = \operatorname{csc}\beta$$

↑ ↑
suman 90°

RESOLUCIÓN**Piden:**

$$E = \operatorname{sen}25^\circ \cdot \operatorname{sec}65^\circ + 2 \tan40^\circ \cdot \tan50^\circ$$

Usando propiedad **2)** :

$$E = \underbrace{\operatorname{sen}25^\circ \cdot \operatorname{csc}25^\circ}_1 + 2 \underbrace{\tan40^\circ \cdot \cot40^\circ}_1$$

Usando propiedad **1)** :

$$\Rightarrow E = 1 + 2 \times 1$$

$$\therefore E = 3$$



7. Halle el valor de x , si se cumple la siguiente ecuación:

$$\sec(90^\circ - x) = \csc(3x - 18^\circ)$$

A) 4° B) 5° C) 7° D) 9°

**2) R.T. DE ÁNGULOS
COMPLEMENTARIOS**

$$\operatorname{sen} \alpha = \cos \beta$$

$$\tan \alpha = \cot \beta$$

$$\sec \alpha = \csc \beta \quad \dots (*)$$

↑ ↑
suman 90°

RESOLUCIÓN

Dato: $\sec(90^\circ - x) = \csc(3x - 18^\circ)$

Usando (*):

$$\cancel{(90^\circ - x)} + (3x - 18^\circ) = \cancel{90^\circ}$$

$$\Rightarrow 2x - 18^\circ = 0$$

$$\Rightarrow 2x = 18^\circ$$

$$\therefore x = 9^\circ$$





8. Halle el valor de “ m ”, si:

$$\tan(\sqrt{m} - 3)^\circ \cdot \cot 13^\circ = 1$$

A) 220

B) 224

C) 226

~~D) 256~~

1) R.T. RECÍPROCAS

$$\operatorname{sen} \alpha \cdot \operatorname{csc} \alpha = 1$$

$$\cos \alpha \cdot \sec \alpha = 1$$

$$\tan \alpha \cdot \cot \alpha = 1 \quad \dots (*)$$

Iguales

RESOLUCIÓN

Dato: $\tan(\sqrt{m} - 3)^\circ \cdot \cot 13^\circ = 1$

Usando (*):

$$(\sqrt{m} - 3)^\circ = 13^\circ$$

$$\Rightarrow \sqrt{m} - 3 = 13$$

$$\Rightarrow \sqrt{m} = 16$$

Elevando al cuadrado:

$$\therefore m = 256$$



9. Si se verifica que:

$$\operatorname{sen} 3x - \operatorname{cos} y = 0$$

$$\tan 2y \cdot \cot 30^\circ - 1 = 0$$

Calcule: $H = \sqrt{3} \sec(x + 5^\circ) + \tan(x + y + 5^\circ)$

- A) 1 B) $\frac{3}{4}$ C) $\frac{3}{2}$ ~~D) 3~~

1) R.T. RECÍPROCAS

$$\operatorname{sen} \alpha \cdot \operatorname{csc} \alpha = 1$$

$$\operatorname{cos} \alpha \cdot \operatorname{sec} \alpha = 1$$

$$\tan \alpha \cdot \cot \alpha = 1 \quad \dots (**)$$

Iguales

2) R.T. DE ÁNGULOS COMPLEMENTARIOS

$$\operatorname{sen} \alpha = \operatorname{cos} \beta \quad \dots (*)$$

$$\tan \alpha = \cot \beta$$

$$\operatorname{sec} \alpha = \operatorname{csc} \beta$$

suman 90°

RESOLUCIÓN

Dato 1: $\operatorname{sen} 3x = \operatorname{cos} y$

Usando (*): $3x + y = 90^\circ \dots (1)$

Dato 2: $\tan 2y \cdot \cot 30^\circ = 1$

Usando (**): $2y = 30^\circ \Rightarrow y = 15^\circ \dots (2)$

(2) en (1): $3x + 15^\circ = 90^\circ \Rightarrow x = 25^\circ$

Reemplazando en H:

$$\Rightarrow H = \sqrt{3} \sec(30^\circ) + \tan(45^\circ)$$

$$\Rightarrow H = \cancel{\sqrt{3}} \times \frac{2}{\cancel{\sqrt{3}}} + 1$$

$$\therefore H = 3$$





10. Si $\cos A = \frac{3x+2}{3x+1}$ y $\sin B = \frac{x+1}{x+2}$.
Determine el valor de $\tan A$,
sabiendo que A y B son ángulos
agudos complementarios.

A) 5

B) $2\sqrt{6}$

C) $2\sqrt{6}$

D) $\frac{\sqrt{6}}{8}$

RESOLUCIÓN

Dato: $A + B = 90^\circ$

R.T de Ángulos complementarios: $\cos A = \sin B$

Así tenemos: $\frac{3x+2}{3x+1} = \frac{x+1}{x+2}$

$$\Rightarrow (3x+2)(x+2) = (3x+1)(x+1)$$

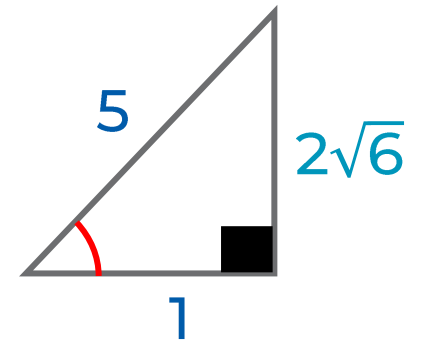
$$\Rightarrow \cancel{3x^2} + 8x + 4 = \cancel{3x^2} + 4x + 1$$

$$\Rightarrow 4x = -3 \Rightarrow x = -\frac{3}{4}$$

Luego:

$$\cos A = \frac{3x+2}{3x+1} \Rightarrow \cos A = \frac{3\left(-\frac{3}{4}\right) + 2}{3\left(-\frac{3}{4}\right) + 1}$$

$$\Rightarrow \cos A = \frac{-\frac{1}{4}}{-\frac{5}{4}} = \frac{1}{5}$$



Piden: $\tan A = \frac{2\sqrt{6}}{1}$

$$\therefore \tan A = 2\sqrt{6}$$