



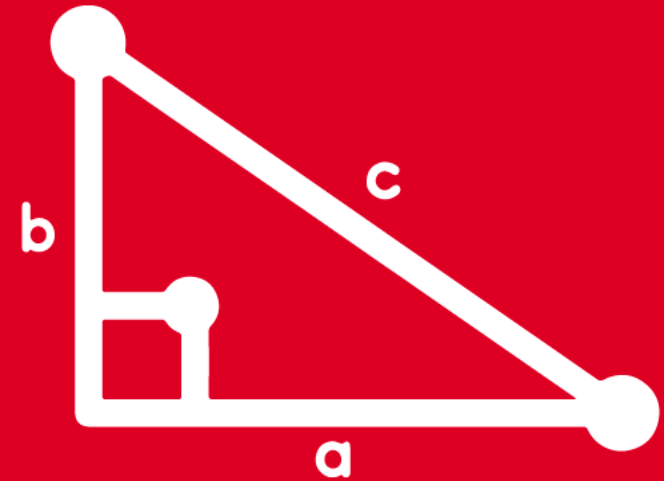
TRIGONOMETRY

Chapter 5

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SAN MARCOS

**Identidades Trigonométricas
Fundamentales**

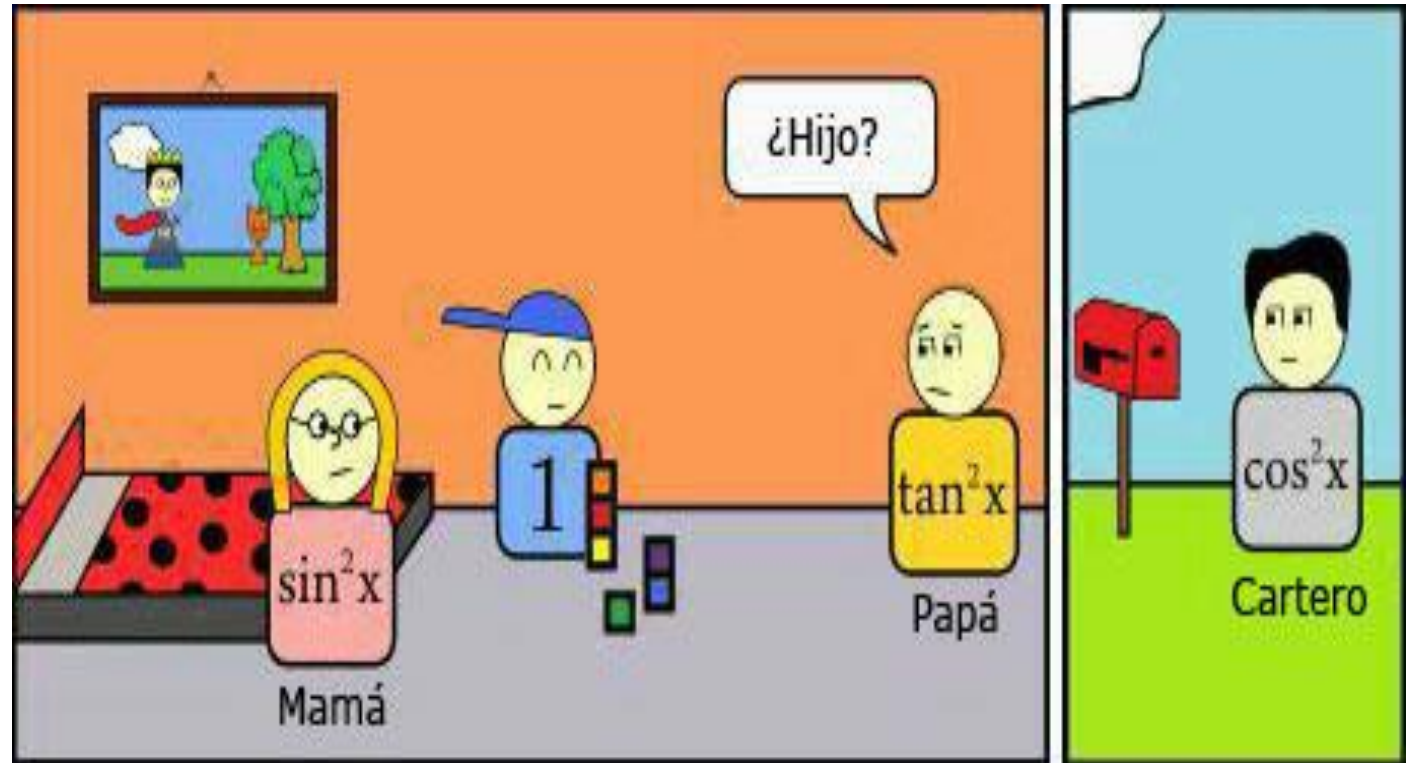


SACO OLIVEROS



En este capítulo, el uso adecuado de las identidades trigonométricas, nos permitirá simplificar las expresiones trigonométricas.

¿Cuál es tu comentario acerca del dibujo? 😁





IDENTIDADES TRIGONOMÉTRICAS

Una identidad trigonométrica es una igualdad entre expresiones trigonométricas, las cuales se verifican para todo valor permitido de la variable.

1° Identidades por División

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta$$

2° Identidades Recíprocas

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

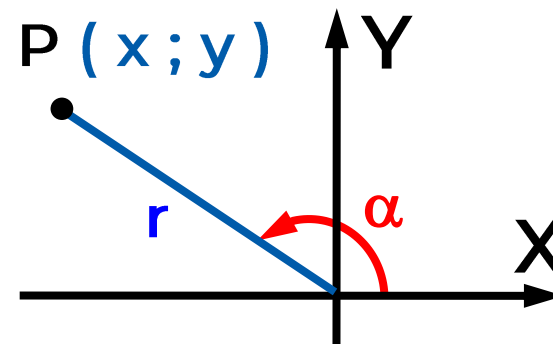




3° Identidades Pitagóricas

$\sin^2 \alpha + \cos^2 \alpha =$	$\sin^2 \alpha = 1 - \cos^2 \alpha$
	$\cos^2 \alpha = 1 - \sin^2 \alpha$
$\sin^2 \alpha - \cos^2 \alpha =$	$\sin^2 \alpha = 1 - \cos^2 \alpha$
	$\cos^2 \alpha = 1 - \sin^2 \alpha$
$\sin^2 \alpha + \cos^2 \alpha =$	$\sin^2 \alpha = 1 - \cos^2 \alpha$
	$\cos^2 \alpha = 1 - \sin^2 \alpha$

ALGUNAS DEMOSTRACIONES



$$r = \sqrt{x^2 + y^2}$$

Usando definiciones:

$$\sin \alpha = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\cos \alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\sin^2 \alpha + \cos^2 \alpha = \left(\frac{y}{\sqrt{x^2 + y^2}}\right)^2 + \left(\frac{x}{\sqrt{x^2 + y^2}}\right)^2 = \frac{y^2 + x^2}{x^2 + y^2} = 1$$





1. Reduzca:

$$K = \frac{\tan x + \sec x}{1 + \sin x}$$

A) $\tan x$

C) $\sin x$

B) $\cot x$

~~D) $\sec x$~~



Recordar las

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

RESOLUCIÓN

Dato: $= \frac{\quad + \quad}{\quad + \quad}$

$\Rightarrow = \frac{\text{---} + \text{---}}{\quad + \quad}$

$\Rightarrow = \frac{\text{---} + \text{---}}{\quad + \quad}$

$\Rightarrow = \underline{\hspace{2cm}}$

$\therefore K = \sec x$





2. Reduzca:

$$E = (\sec x + \csc x) \sin x - \tan x$$

~~A) 1~~

B) 2

C) $\csc x$

D) $\cot x$

Recordar las



$$\sin x = \frac{\text{opuesto}}{\text{hipotenusa}}$$

$$\cos x = \frac{\text{adyacente}}{\text{hipotenusa}}$$

$$\tan x = \frac{\text{opuesto}}{\text{adyacente}}$$

RESOLUCIÓN

Dato:
$$= \frac{\sec x + \csc x}{\sin x} - \tan x$$

$$\Rightarrow = \frac{\sec x + \csc x}{\sin x} - \tan x$$

$$\Rightarrow = \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\sin x} - \tan x$$

$$\Rightarrow = \frac{1}{\sin x \cos x} + \frac{1}{\sin^2 x} - \tan x$$

$$\therefore E = 1$$





3. Reduzca:

$$E = (\tan x + \cot x) \sin x \cdot \cos x$$

~~A) 1~~

C) $\cot x$

B) $\tan x$

D) $\sec x$

Recordar las



$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin^2 x + \cos^2 x = 1$$

RESOLUCIÓN

Dato: $\tan x = \frac{\sin x}{\cos x}$ + $\cot x = \frac{\cos x}{\sin x}$

$$\Rightarrow E = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \sin x \cdot \cos x$$

$$\Rightarrow E = \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right) \sin x \cos x$$

$$\Rightarrow E = \left(\underbrace{\sin^2 x + \cos^2 x}_1 \right) \sin x \cos x$$

$$\therefore E = 1$$





4. Reduzca:

$$E = (\sec x - \cos x) \cot x$$

A) 1

C) $\sec x$ B) $\cos x$
~~D) $\sin x$~~


Recordar las

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\sec^2 x = \frac{1}{\cos^2 x}$$

RESOLUCIÓN

Dato: $E = (\sec x - \cos x) \cot x$

$$\Rightarrow E = \left(\frac{1}{\cos x} - \cos x \right) \cot x$$

$$\Rightarrow E = \left(\frac{1 - \cos^2 x}{\cos x} \right) \frac{\cos x}{\sin x}$$

$$\Rightarrow E = \frac{1 - \cos^2 x}{\sin x}$$

$$\Rightarrow E = \frac{\sin^2 x}{\sin x}$$

$$\therefore E = \sin x$$





5. Reduzca:

$$E = (\tan^2 x + 1) \cos x - \tan x \cdot \csc x$$

A) 1

C) $\cot x$ ~~B) 0~~D) $\tan x$

Recordar las



$$\tan^2 + 1 = \sec^2$$

$$\tan = \frac{\sin}{\cos}$$

$$\csc = \frac{1}{\sin}$$

$$\sec = \frac{1}{\cos}$$

$$\sin = \frac{\text{opuesto}}{\text{hipotenusa}}$$

RESOLUCIÓN

Dato: $= \quad + \quad -$

$$\Rightarrow = \frac{\tan^2 + 1}{\cos} - \frac{\tan}{\sin}$$

$$\Rightarrow = \frac{\sin^2 + \cos^2}{\cos \sin} - \frac{\sin}{\sin \cos}$$

$$\Rightarrow = \frac{1}{\cos \sin} - \frac{1}{\cos \sin}$$

$$\therefore E = 0$$





6. Simplifique:

$$H = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} + \frac{\tan x}{\cot x}$$

A) 1

B) $\tan^2 x$ C) $\cot^2 x$
~~D) $\sec^2 x$~~


Recordar las

$\sin^2 x + \cos^2 x = 1$	$\sec^2 x = \frac{1}{\cos^2 x}$	$\csc^2 x = \frac{1}{\sin^2 x}$
$1 + 2 = 3$	$2 + 2 = 4$	

RESOLUCIÓN

Dato: $= \text{---} + \text{---} + \text{---}$

$$\Rightarrow = \frac{\sin^2 x}{\csc^2 x} + \frac{\cos^2 x}{\sec^2 x} + \frac{\tan^2 x}{\cot^2 x}$$

$$\Rightarrow = \frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} + \frac{\tan^2 x}{\cot^2 x}$$

$$\Rightarrow = 1 + \frac{\tan^2 x}{\cot^2 x}$$

$$\therefore H = \sec^2 x$$





7. Reduzca:

$$K = \frac{\csc x - \sen x}{\sec x - \cos x}$$

A) $\tan^3 x$

C) $\sen^2 x$

~~B) $\cot^3 x$~~

D) $\cos^2 x$



Recordar las

$$\frac{\csc x}{\sec x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\sin x \cos x}$$

$$\frac{\sen x}{\cos x} = \frac{\sin x}{\cos x} = \tan x$$

$$\frac{1}{\sin^2 x} = \csc^2 x$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{\tan^2 x}{\sec^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x$$

RESOLUCIÓN

Dato:

$$= \frac{\csc x - \sen x}{\sec x - \cos x} \rightarrow = \frac{\frac{1}{\sin x} - \sin x}{\frac{1}{\cos x} - \cos x}$$

m.c.m. en el numerador y denominador:

$$= \frac{\frac{1 - \sin^2 x}{\sin^2 x}}{\frac{1 - \cos^2 x}{\cos^2 x}} = \frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^4 x}{\sin^4 x} = \cot^4 x$$

$$\therefore K = \cot^3 x$$





8. Simplifique la expresión:

$$H = (1 + \tan x)^2 + (1 - \tan x)^2$$

A) $\csc^2 x$

B) $\tan^2 x$

C) 1

 D) $2\sec^2 x$



Identidad de

$$+^2 + -^2 = +^2 + ^2 \dots (*)$$

$$+^2 = ^2$$

RESOLUCIÓN

Dato:

$$H = (1 + \tan x)^2 + (1 - \tan x)^2$$

Usando (*) para: $a = 1$ y $b = \tan x$

$$\Rightarrow H = 2(1^2 + \tan^2 x)$$

$$\Rightarrow H = 2(\underbrace{1 + \tan^2 x}_{\sec^2 x})$$

$$\therefore H = 2\sec^2 x$$





9. Halle el valor de k , de tal manera que se cumpla:

$$(\operatorname{sen} x + \operatorname{cos} x)(\operatorname{tan} x + \operatorname{cot} x) = k \cdot \operatorname{csc} x$$

A) $\operatorname{sen} x$

C) $\operatorname{cos} x$

B) $\operatorname{sec} x - 1$

~~D) $\operatorname{tan} x + 1$~~

Recordar las



$$\operatorname{sen}^2 x + \operatorname{cos}^2 x = 1$$

$$\operatorname{tan} x = \frac{\operatorname{sen} x}{\operatorname{cos} x}$$

$$\operatorname{cot} x = \frac{\operatorname{cos} x}{\operatorname{sen} x}$$

$$\operatorname{csc} x = \frac{1}{\operatorname{sen} x}$$

RESOLUCIÓN

Dato: $(\operatorname{sen} x + \operatorname{cos} x)(\operatorname{tan} x + \operatorname{cot} x) = k \cdot \operatorname{csc} x$

$$\Rightarrow (\operatorname{sen} x + \operatorname{cos} x) \left(\frac{\operatorname{sen} x}{\operatorname{cos} x} + \frac{\operatorname{cos} x}{\operatorname{sen} x} \right) = k \cdot \frac{1}{\operatorname{sen} x}$$

$$\Rightarrow (\operatorname{sen} x + \operatorname{cos} x) \left(\frac{\operatorname{sen}^2 x + \operatorname{cos}^2 x}{\operatorname{sen} x \operatorname{cos} x} \right) = \frac{k}{\operatorname{sen} x}$$

$$\Rightarrow (\operatorname{sen} x + \operatorname{cos} x) \left(\frac{1}{\operatorname{sen} x \operatorname{cos} x} \right) = \frac{k}{\operatorname{sen} x}$$

$$\Rightarrow \operatorname{sen} x + \operatorname{cos} x = k$$

$$\therefore k = \operatorname{tan} x + 1$$





10. Determine el valor de n .

$$\tan^2\theta - \sec^2\theta = n \cdot \tan^2\theta$$

A) 1

C) $2\tan\theta$

~~B) $\sec^2\theta$~~

D) $2\cot\theta$

Recordar las
identidades



$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

RESOLUCIÓN

Dato: $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = 1$$

$$\Rightarrow \frac{1 - \sin^2\theta}{\cos^2\theta} = 1$$

$$\Rightarrow \cos^2\theta (1 - \sin^2\theta) = \cos^2\theta$$

$$\therefore n = 1$$

