UNIT - III

CHOPPERS

* Control Strategies of Choppers:

the average o/p Voltage "V_0" is given by

$$V_0 = \alpha V_s$$

* this average value of o/p voltage "V_0" Can be Controlled through "α" by opening & closing the Semiconductor Switch periodically.

* the Various Control Strategies for Varying duty cycle "α" are as follows.

* time ratio control [TRC]:

* As the Name Suggests, in this Control Scheme, time ratio T_{on}/T is Varied.
* this is realized in two different Strategies Called Constant frequency System & Variable frequency System.

* Constant frequency System:

* in this Scheme the "on" time "T_{on}" is Varied but the Chopping frequency "f" (or) Chopping period "T" is Kept Constant.

* Variation of "T_{on}" Means adjustment of pulse-width Modulation as Such this Scheme is also Called pulse-width Modulation Scheme.

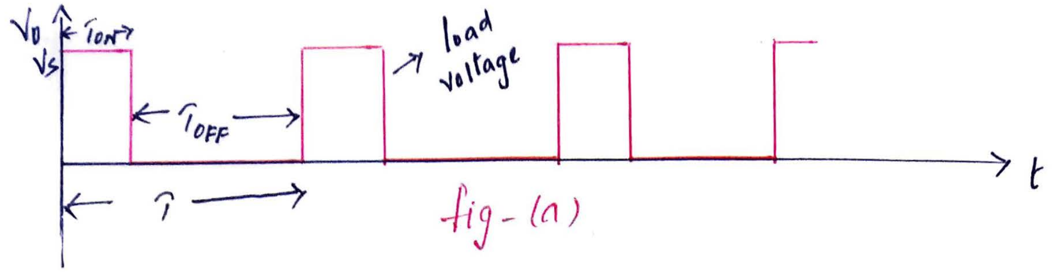


fig-(a)

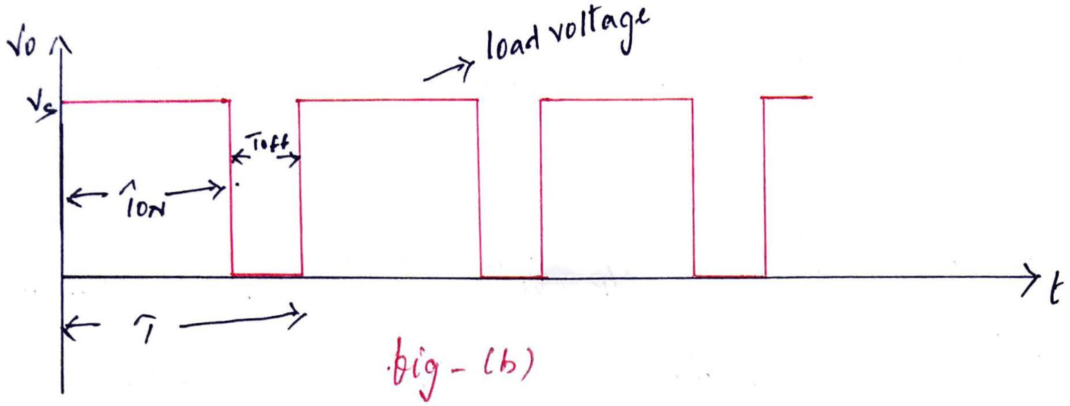


fig-(b)

* the principle of Pulse width Modulation is illustrated by using the above figure.
* here the chopping period "T" is constant and in fig: (a) $T_{ON} = 1/4\,T$, so that $\alpha = 0.25$ (or) $\alpha = 25\%$
* where as in fig: (b): $T_{ON} = 3/4\,T$, so that $\alpha = 0.75$ (or) $\alpha = 75\%$
* ideally "α" can be varied from zero to unity. ∴ o/p voltage "V_0" can be varied between zero & source voltage "V_s".

* Variable frequency System :-

* in this scheme, the chopping frequency "f" (or) chopping period "T" is varied & either "T_{ON}" (or) "T_{OFF}" is kept constant.
* this method of controlling "α" is also called frequency modulation scheme.
* from the fig:-(a) $T_{ON} = 3/4\,T$ so that $\alpha = 0.75$
* from the fig:-(b) T_{OFF} is kept constant & T is varied.

* in this Control strategy the "on" and "off" of Chopper Circuit is guided by the previous set values of the load Current.

* this two set values are Maximum load Current "I_{omx}" & minimum load Current "I_{omn}".

* When the load Current reaches the upper limit "I_{omx}" Chopper is switched off. Now load Current freewheels & began to decay Exponentially.

* when it falls to lower limit "I_{omn}", Chopper is switched "on" & load Current began to rise as shown in the figure.

* profile of load Current shows that it began's to fluctuates between "I_{omx}" & "I_{omn}", ∴ Can not be dis Continious.

* switching frequency of Chopper Can be Controlled by setting "I_{omx}" & "I_{omn}".

disadvantages of Current limit Control:-

* Current limit Control involves feedback loop, the trigger Circuit for the Chopper is ∴ More Complex.

DC Chopper :

* A Chopper is a static Circuit that Converts fixed DC i/p Voltage to Variable DC o/p voltage directly.
* a Chopper may be thought of as DC Equivalent of an AC transformer ∵ they behave in an identical Manner.

Step - down Chopper :-

* a Chopper is a high- Speed ON / off Semiconductor Switch. it Connects Source to load & disconnects the load from Source at a fast Speed.
* in this Manner a chopped load voltage which is shown in the figure is obtained from a Constant DC Supply of Magnitude "V_s".
* in the figure the Chopper is represented by a Switch "SW" inside a dotted rectangle, which May be turn-on or turned - off as desired.
* During the period "T_{on}" chopper is in "on" & load voltage is Equal to Source Voltage "V_s".
* During the interval "T_{off}" chopper is "off", load current flows through the freewheeling diode "FD".
* As a result, load terminals are short Circuited by FD & load voltage is therefore zero during "
* in this Manner a Chopped DC Voltage is prod the load terminals.

has been in wide use

* the load Current as Shown in the fig: is Continious.

* during "T_{ON}", load Current rises where as during "T_{off}" load Current decays.

* the average load Voltage "V_0" is given by

$$V_0 = \frac{T_{ON}}{T_{ON}+T_{off}} \cdot V_s = \frac{T_{ON}}{T} \cdot V = \alpha . V_s$$

$$\boxed{\therefore \quad V_0 = \alpha V_s}$$

* Where T_{on} = on - time $\quad T_{off}$ = off - time

$T = T_{on} + T_{off}$ = Chopping period.

$\alpha = T_{on}/T$ = duty cycle.

* thus the load voltage Can be Controlled by Varying duty cycle "α".

* from the "V_0" Equation it is Clear that the load voltage is independent of load Current $\therefore$ "V_0" Can be also Expressed as.

$$\left[\because \alpha = T_{on}/T \Rightarrow \frac{1}{T} = f \right]$$

$$\boxed{V_0 = f . T_{on} . V_s}$$

where $f = 1/T$ = chopping frequency.

Step - up Chopper :

* for the step - down chopper, the average o/p Voltage "V_0" is less than the i/p voltage "V_s".
* however the average o/p voltage "V_0" greater than "V_s" Can be obtained by a Chopper Called Step - up chopper.
* in this Chopper, a large inductor "L" in series with the Source voltage "V_s" is essential as shown in the figure. (a)
* when the chopper "CH" is on, the closed Current path is shown in fig: (b) & inductor stores Energy during "T_{ON}" period.
* When the Chopper "CH" is off, as the inductor Current cannot decrease down instantaneously, this Current is forced to flow through the diode & load for a time T_{OFF} shown in fig: (c).
* As the Current tends to decrease, polarity of the Em induced in "L" is reversed.
* as a result voltage across the load, given by

$$V_L = V_S + L\left[\frac{di}{dt}\right]$$

Exceeds the source voltage "V_s".

* in this Manner the Circuit act's as step-up Chopper & th Energy stored in "L" is released to the load.

* When "CH" is on, Current through the inductance "L" would increase from I_1 to I_2 as shown in fig: (d).
* When "CH" is off, Current whould fall from I_2 to I_1.
* With "CH" on, source voltage is applied to "L" i.e, $V_L = V_s$.
* When "CH" off, KVL for fig (c) gives

$$V_L - V_0 + V_S = 0 \quad (\text{or}) \quad V_L = V_0 - V_S$$

* here "V_L" is voltage across "L".
* Variation of source voltage "V_s", Source Current "i_s", load voltage "V_0", load current "i_0" is shown in fig: (d).
* Assuming linear variation of o/p current, the Energy i/p to inductor from the source during the period "T_{on}" is

$$W_{in} = [\text{voltage across } L][\text{average Current through } L]\, T_{on}$$

$$= V_s \cdot \left[\frac{I_1 + I_2}{2}\right] \cdot T_{on}$$

* during the time T_{off}, when chopper is off, the Energy released by inductor to the load is

$$W_{off} = [\text{voltage across } L]\,[\text{average Current through } L]\, T_{off}$$

$$= [V_0 - V_s]\,\left[\frac{I_1 + I_2}{2}\right] \cdot T_{off}$$

* Considering the system to be loss less, this two Energies should be Equal.

$$\therefore V_s \left[\frac{I_1 + I_2}{2}\right] \cdot T_{on} = [V_0 - V_s]\left[\frac{I_1 + I_2}{2}\right] \cdot T_{off}$$

$$\Rightarrow V_s \cdot T_{on} = V_0 T_{off} - V_s \cdot T_{off}$$

$$\Rightarrow V_0 T_{off} = V_s [T_{on} + T_{off}] = V_s \cdot T$$

* One of the earliest circuits which has been in wide use

(or) $V_0 = V_S \cdot T/T_{OFF} = V_S \cdot T/T - T_{on}$

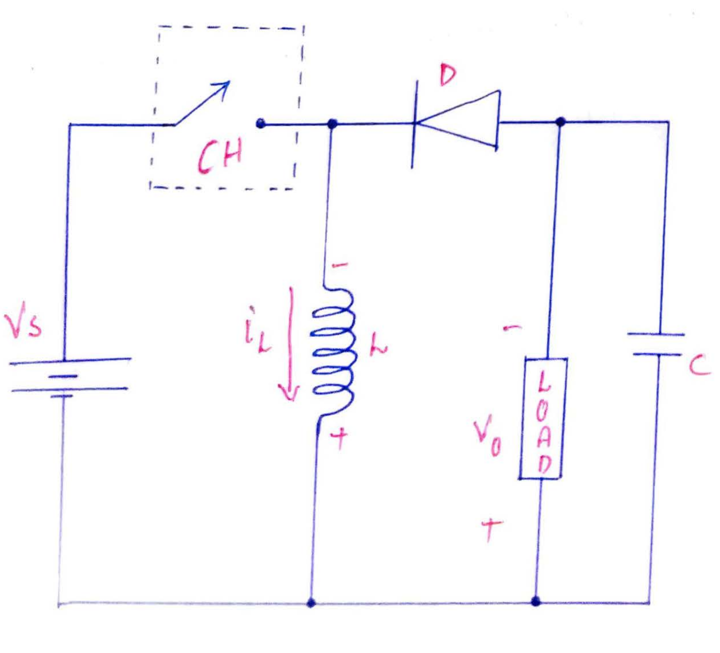
$$\Rightarrow \boxed{V_0 = V_S \cdot \frac{1}{1-\alpha}}$$

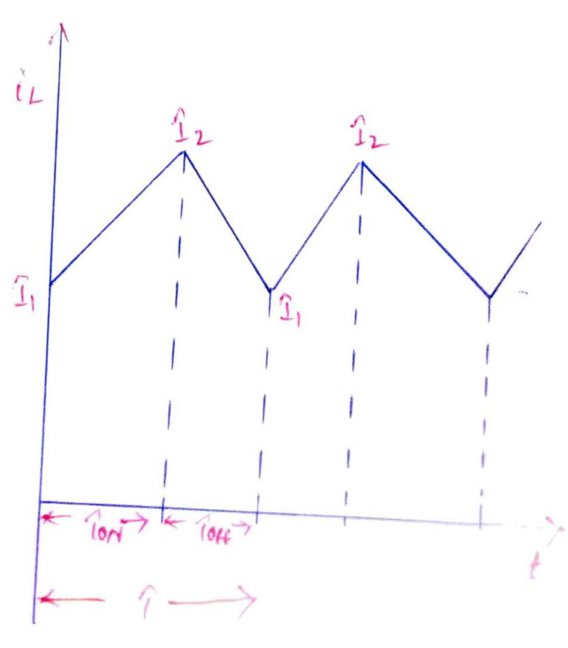
* From the above Equation the average o/p voltage can be step-up by varying the duty cycle.

Note: * if the chopper is always off $\alpha = 0$ & $V_0 = V_S$
* if the chopper is always ON $\alpha = 1$ & $V_0 = \infty$

* Buck / Boost Converter :

* Circuit configurations of step-up & step-down choppers are different from each other.
* there is however one circuit topology that can operate in both step-up & step-down modes as its duty cycle is varied continiously from zero to unity.
* figure shows the power circuit diagram of a step-up & step down chopper.
* it consists of DC voltage source V_s, chopper switch CH, inductor L, diod D, capacitor C & load.





e function of "L" is to store Energy & release it when required. the Capacitor "C" tends to Maintain load voltage constant.

It would be observed from the circuit that load voltage polarity is opposite to source voltage polarity.

Current waveforms of this chopper will vary linearly.

When chopper "CH" is on, current flows from source "V_s" to CH – L and back to V_s.

* during the on period "T_{on}" of chopper, current through "L" rises from I_1 to I_2 & "V_s" appears across "L".

∴ Energy stored in the inductor during T_{on},

$$W_{in} = V_s \left[\frac{I_1 + I_2}{2}\right] . T_{on}$$

* When chopper "CH" is turned off, inductor current tends to decrease. as a consequence, polarity of the Emf induced in inductor "L" gets reversed.

* Emf Induced in "L" forward biase diode "D" & as a result inductor stored Energy now discharges through the path Load – D – L during the time chopper "CH" is off.

∴ Energy released by Inductor during T_{off}, $W_{off} = V_0\left[\frac{I_1+I_2}{2}\right]T_{off}$

* Assume the system is loss less, the Energy balance Equation

$$\Rightarrow V_s\left[\frac{I_1+I_2}{2}\right] . T_{on} = V_0\left[\frac{I_1+I_2}{2}\right] T_{off}$$

$$\Rightarrow V_0 = V_s \cdot \frac{T_{on}}{T_{off}} = V_s \cdot \frac{T_{on}}{T - T_{on}}$$

$$= V_s \cdot \frac{1}{T/T_{on} - 1} = V_s \cdot \frac{1}{1/\alpha - 1} \Rightarrow \boxed{V_0 = V_s \cdot \frac{\alpha}{1-\alpha}}$$

* one of the earliest circuits which has been in wide use is the voltage - Commutated chopper.
* this chopper is generally used in high-power circuits where load fluctuation is Not very larger.
* this chopper is also called as $||^{le}$ - Capacitor turn off chopper, impulse Commutated chopper (or) Classical chopper.
* the power circuit diagram for this type of chopper is shown figure, where thyristor "T_1" is the Main power switch.
* Commutation circuit for this chopper is Made up of an auxilary thyristor "T_A", Capacitor c, diode D & inductor L. FD is the freewheeling diode Connected across the RLE type load.

* Working of this chopper can start only if the capacitor "c" is charged with polarities as Marked in the ckt. this can be achieved in one of the two ways as below.
  1) Close switch "s" so that capacitor gets charged with polarities as shown in fig: to voltage "V_s" through source V_s, C, S & charging resistor R_c. Switch "s" is then opened.
  2) Auxiliary thyristor "T_A" is triggered so that "C" get's charged through source V_s, C, T_A & the load. the charging current through capacitor "c" decays & as it reaches zero, $V_c = V_s$ & T_A is turned off.
* With capacitor "c" charged with the polarities as shown the chopper ckt is ready for operation. the current i_c, i_{T_1}, i_{fd} & i_0 are taken as positive in the arrow directions Marked.
* $|||^{ly}$, the voltages V_c, V_{T_1}, V_{TA} & V_0 across C, T_1, T_A & load are taken as positive with the polarities Marked.

nplifying assumptions for this chopper are.
load current is constant 2) thyristors & diodes are
ideal elements.

the chopper operation is explained in certain modes.

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Mode: I * the main thyristor is triggered at $t=0$ & RLE
load gets connected across source "V_s" so that load voltage
$V_0 = V_s$.

during this mode, there are two current paths as shown in
fig: (a). load current I_0, assumed constant, constitute one
path & commutation current "i_c" the other path.

* load current "I_0" flows through source V_s - T_1 - load, whereas
the current "i_c" flows through the oscillatory circuit formed by
$C - T_1 - L - D$.

* the capacitor current first rises from zero to a maximum
value when voltage across C is zero at : $t = t_1/2$. As "i_c"
decreases to zero, capacitor is charged to $-V_s$ as shown
at $t = t_1$.

* the capacitor current changes sinusoidally where as the
capacitor voltage cosinusoidally from $t=0$ to $t=t_1$. this voltage
is held constant at $-V_s$ by diode D.

* Voltage across "T_A" is $(-V_s)$ at $t=0$, zero at $t_1/2$ & V_s at t_1,
this variation is shown as cosine wave in fig:. the thyristor
current "i_{T_1}" has a peak at $t_1/2$, because $i_{T_1} = i_c + I_0$ b/w $t=0$
& $t=t_1$.

* At the end of Mode I i.e, at t_1, $i_c = 0$, $i_{T_1} = I_0$, $V_c = -V_s$, $V_{TA} = V_s$
$V_0 = V_s$ as shown in fig:

Mode: II * the conditions existing at "t_1" continue during M

* In other words, for $t_1 \le t \le t_2$, $i_{T_1} = I_0$, $i_c = 0$, $V_c = -V_s$
$V_{TA} = V_s$, $V_0 = V_s$, $i_D = 0$

* during this mode only main SCR "T_1" is conducting.

Mode - III : * When Main thyristor T_1 is to be turned OFF, auxiliary thyristor "T_A" is triggered at the desired instant "t_2". With the turning on of "T_A" at t_2, Capacitor voltage $-V_s$ appears across T_1, it is therefore reverse biased & turned off.

* As the Capacitor voltage does the required job of commutating the Main thyristor T_1, it is called Voltage Commutated Chopper.

* Current i_{T_1} becomes zero at t_2. After T_1 is turned off, Capacitor "C" & auxiliary SCR "T_A" provide the path for load current I_0 through V_s, C, T_A & the load.

* The load voltage is the sum of source voltage & the voltage across capacitor. ∴ at instant t_2, load voltage is $V_0 = V_s + V_s = 2V_s$ & it decreases linearly as the voltage across capacitor decreases.

* During this Mode, $V_c = V_{T_1}$, because capacitor is directly connected across T_1 through T_A. As the capacitor discharges through the load, V_c & V_{T_1} changes from $-V_s$ to zero at $[t_2 + t_c]$

* Load voltage V_0 changes from $2V_s$ at t_2 to V_s at $[t_2 + t_c]$. After $(t_2 + t_c)$, V_c & V_{T_1} start rising from zero towards "V_s" where as "V_0" starts falling towards zero.

* For Mode-III, $t_2 \le t \le t_3$, Note that V_c & V_{T_1} change linearly from $-V_s$ at t_2 to V_s at t_3, because load current I_0 is assumed constant.

* III'ly V_0 changes linearly from $2V_s$ at t_2 to zero at t_3. Note also that $i_c = -I_0$ & $i_{T_A} = I_0$ during Mode - III.

* Simplifying assumptions for this chopper are

* Mode - IV :

* for this Mode, $t_3 < t < T$ at t_3, $V_c = V_{T_1} = V_s$ & $V_0 = 0$
* in fact, at t_3, Capacitor is Slightly overcharged, free wheeling diode FD, ∴ get forward biased at t_3.
* in the local ckt formed by FD, TA, C & V_s, $V_c > V_s$ at t_3, ∴ Current i_c tends to go negative.
* As i_c or i_{TA}, tends to reverse, auxiliary thyristor TA is turned off Naturally at t_3. thus at time t_3, FD gets turned on & TA is turned OFF.
* As a result, load current after t_3 freewheels through t_3 - load - FD. Note that during freewheeling period from t_3 to T, V_{TA} is Slightly Negative as C is Somewhat over charged.
* During this Mode $i_c = 0$, $i_{T_1} = 0$, $i_{fd} = I_0$, $V_{T_1} = V_s$, $V_c = V_s + \Delta V$, $V_{TA} = -\Delta V$, $V_0 = 0$, $i_{TA} = 0$

* At $t = T$, the Main thyristor T_1 is triggered & the cycle as described from $t = 0$ to $t = T$ repeats.

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$\frac{I}{C}$

$\frac{\,}{t}$

power Circuit diagram for Current - Commutated Chopper
shown in fig:. in this diagram, T_1 is the Main thyristor.
the other Components, namely, auxiliary thyristor TA, Capacitor
C, inductor L, diodes D_1 & D_2 Constitute Commutation ckt.
FD is the freewheeling diode & R_c is the Charging resistor.
Simplifying assumptions for the Chopper are as follows
i) load Current is Constant
2) SCR & diodes are ideal Switches.
3) Charging resistor "R_c" is So large that it can be treated as open Circuit during the Commutation interval.

* Currents i_c, i_{T_1}, i_{fd} & i_0 are treated as positive when they are in the arrow directions Marked. III'y, Voltages V_c, V_{T_1}, V_{TA} & V_0 are taken as positive with the polarities as Marked in the figure.

* like Voltage - Commutated Chopper, Energy for Current - Commutation comes from the Energy stored in a Capacitor. ∴ initially Capacitor C is Charged to a Voltage "V_s" So that Energy for Commutation process is available.

* Capacitor "C" is Charged through V_s - C - R_c to a voltage V_s after this, Main thyristor T_1, is fired at $t=0$ So that load Voltage $V_0 = V_s$ & load Current $i_0 = I_0$ Shown in the fig: up to $t = t_i$

* With the turning on of T_1, Commutation Circuit remains inactive. Initiation of Commutation process begins with the turning-on of thyristor TA. the Commutation process is divided in different Modes.

mplifying assumptions for this chopper are

de - I * At time $t = t_1$, auxiliary thyristor T_A is triggered to commutate main thyristor T_1. With the turning-on of T_A, an oscillatory current $i_c = \frac{V_s}{\omega_0 L} \sin \omega_0 t$ is setup in the circuit consisting of $C - T_A - L$ as shown in the fig.

* for the time interval $[t_2 - t_1]$, i_c & V_c vary sinusoidally through half cycle, during this mode when V_c is zero, i_c is maximum through negative. At t_2, as i_c tends to reverse in the auxiliary thyristor T_A, it gets naturally commutated.

* At t_2, $V_c = -V_s$ as shown in fig, lower plate is +ve & upper plate is -ve. during this mode T_1 remains uneffected, ∴ load current & load voltage remain I_0 & V_s respectively.

Mode - II :-

* As T_A is turned off at t_2, oscillatory current i_c begins to flow through $C - L - D_2 - T_1$ as shown in fig. Note that after t_2, i_c whould flow through T_1 & not through D_1.

* it is because D_1 is reverse biased by a small voltage drop across conducting thyristor T_1. So after t_2, i_c would pass through

*

T_1 & not through D_1.

* In thyristor T_1, i_c is in opposition to load current i_0 so that i_c is in the forward direction through T_1. $i_{T_1} = I_0 - i_c$. Note that i_{T_1} $i_{T_1} = 0$. as a result main SCR T_1 is at t_3, i_c rises to I_0 so that turned off at t_3. Since the oscillatory current through T_1 turns it off, it is called current commutated chopper.

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* during this mode, load voltage remains V_s through T_1. for this mode, $t_2 < t < t_3$.

*

V_1

: d

iii : * As T_1 is turned off at t_3, i_c becomes More
After t_3, i_c Supplies load Current I_0 & the Excess Current
$= i_c - I_0$ is Conducted through diode D_1 as Shown in fig.
he voltage drop in D_1 due to $[i_c - I_0]$ keeps T_1 reverse biased
or $(t_4 - t_3) = t_c$. this is shown in the waveform for V_{T_1}.
at t_4, in Case V_c Exceeds V_s, FD Comes into Conduction, otherwise
Mode IV would follow.

during Mode iii, when i_c is at its peak Value of $I_{cp}\left[= \frac{V_s}{W_0 L}\right]$. upper
$V_c = 0$. After this peak, Capacitor voltage reverses & at t_4,
plate is +ve & lower plate is -ve.

Mode-IV: * At t_4, i_c reduced to I_0, as a result $i_{D_1} = 0$ & diode D_1
is ∴ turned off. After t_4, a Constant Current Equal to I_0
flow through Source V_s - C - L - D_2 - load & therefore Capacitor
C is Charged linearly to Source Voltage V_s at t_5. So during
the time $[t_5 - t_4]$, $i_c = I_0$.

* As D_1 is turned off at t_4, $V_{T_1} = V_{TA} = V_c$, this is shown as ab
in the fig: for V_c, V_{T_1} & V_{TA}. Now the load voltage $V_0 = V_s - V_c$
=> V_s - voltage ab at t_4. this is also marked in the waveform
for V_0 at t_4. At t_5, $V_c = V_s$, therefore load voltage V_0 decreases
to Zero linearly during this interval.

Mode - V :

* At t_5, Capacitor C is actually overcharged to a voltage Some
more than Source voltage V_s. ∴ FD gets forward biased &
Starts to Conduct load Current I_0 at t_5. load Voltage V_0
is reduced to zero at t_5 as Stated in Mode-IV.

As i_c is not zero at t_5, the capacitor C is still conn... load through source V_s, C, L & D_2. As a consequence C is overcharged by the transfer of energy from L to C.

At t_6, $i_c = 0$ & V_c becomes more than source voltage V_s. During $(t_6 - t_5)$, $i_c + i_{fd} = I_0$. With the build up of i_{fd}, i_c decays & finally at t_6, $i_c = 0$ & $i_{fd} = I_0$. Commutation process is completed at t_6. Total turn-off time or commutation interval is $(t_6 - t_1)$.

* At t_6, V_c is shown as equal to xy. As D_2 is turned off at t_6, $V_c = V_{T1A}$ = Voltage xy at t_6.

* From t_6 onwards, i_0 freewheels through FD. As i_c is zero & D_2 is open circuited. C now discharges through R_c for the freewheeling interval of the chopper.

* After t_5, V_{T1} remains constant at V_s, because V_s reaches T_1 terminals through FD. At $t = T$, the main SCR T_1 is again triggered & the cycle repeats.

* **Merits of this Chopper:**

  * Commutation is reliable so long as the load current is less than the peak commutating current I_{cp}.

  * Capacitor is always charged with correct polarity.

  * Auxiliary thyristor is naturally commutated.

AC Voltage Controllers.

* 1ϕ AC voltage Controller with R- Load :

* the Circuit diagram is Shown in the fig: it Consists of two thyristors Connected in antiparallel to Each other.
* Here, the load Considered is of purely Resistive. its operation May be Explained in two different Modes.

Mode - I [$0 < \omega t < \pi$] :

* during the positive half cycle of the Supply voltage, thyristor "T_1" is in the forward biased Condition where as thyristor T_2 is in the reverse biased Condition.
* Where ever gate Signal has been given to thyristor T_1 at $\omega t = \alpha$, it Starts Conducting & load Current follows the path as Shown in the fig: . the Circuit Completes its path as

$$E_s^{+} - T_1 - \text{load} - E_s^{-}$$

* during this +ve half cycle of Supply voltage, the power gets delivered to load from Source during the period α to π, both E_0 & I_0 falls to zero.

* At this instant, T_1 is Subjected to reverse bias & therefore it gets turned off Naturally.

- ii ?

During the negative half cycle of the supply voltage, thyristor T_1 is in the reverse biased condition & thyristor T_2 is in the forward biased condition.

when gate signal is given to the thyristor T_2, it gets turned on & load current follows the path as shown

$$E_s^+ - L - R - T_2 - E_s^-$$

* this load current traces the negative half cycle of the supply voltage. At the instant $\omega t = 2\pi$, thyristor T_2 must be turned off as the supply voltage, load voltage is zero & as it gets reverse biased at $\omega t = 2\pi$.

* But it does not get turned off due to the presence of inductance in the load circuit. at $(\pi + \alpha + \gamma)$, $i_{T_2} = 0$ & T_2 gets turned off as it is already in the reverse biased condition.

* At this instant, $E_m \sin(\pi + \alpha + \gamma)$ appears as a forward bias voltage across T_2. From $(\pi + \alpha + \gamma)$ to $(2\pi + \alpha)$ no current exists in the power circuit as both the thyristors are in off state.

* Again at $(2\pi + \alpha)$, T_1 is turned on & current starts build up as before. its associated waveforms are shown in the fig:

* **Average o/p Voltage :-**

$$E_{oavg} = \frac{1}{\pi}\int_{\alpha}^{\beta} E_m \sin \omega t \; d(\omega t) = \frac{E_m}{\pi}[\cos\alpha - \cos\beta]$$

$$= \frac{\sqrt{2} E_s}{\pi}[\cos\alpha - \cos\beta]$$

$$E_{oavg} = \frac{2\sqrt{2}E_s}{\pi} \sin\left(\frac{\gamma}{2}\right) \sin\left(\alpha + \frac{\gamma}{2}\right)$$

* **RMS o/p Voltage :-**

$$E_{oRMS}^2 = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} E_m^2 \sin^2 \omega t \, . \, d(\omega t) + \int_{\pi+\alpha}^{\beta} E_m^2 \sin^2 \omega t \; d(\omega t)\right]$$

$$= \frac{E_m^2}{2\pi}\left[\frac{\pi-\alpha}{2} - \frac{\sin 2\pi - \sin 2\alpha}{4} + \frac{\beta-\pi-\alpha}{2} - \frac{\sin 2\beta + \sin 2(\pi+\alpha)}{4}\right]$$

$$E_{oRMS}^2 = \frac{E_m^2}{2\pi}\left[\frac{\beta-\alpha}{2} + \frac{\sin 2\alpha}{4} - \frac{\sin 2\beta}{4}\right]$$

$$E_{oRMS} = E_s\left[\frac{1}{\pi}\left[(\beta-\alpha) + \frac{\sin 2\alpha}{4} - \frac{\sin 2\beta}{4}\right]\right]^{1/2}.$$

* **RMS o/p Current :-**

$$I_0 = \sqrt{I_{RMS}^2 + I_{RMS}^2} = \sqrt{2}\, I_{RMS}.$$

Ac voltage Controller with RL load :

voltage Controller with RL load Circuit diagram is
own in the fig.

perating principle of Ac Voltage Regulator with RL load
s Explained in two modes.

de - I [$0 < \alpha < \pi$] :

During the +ve half cycle of the Supply Voltage, thyristor T_1,
is in the forward biased Condition. Where as thyristor T_2,
is in the reverse " " .

* When gate pulse is given to thyristor T_1, at $\omega t = \alpha$, it starts
Conducting & the load Current traces the path as follows

$$E_s^+ - T_1 - R - L - E_s^-$$

* At $\omega t = \pi$, Supply voltage & load voltage becomes zero but load
Current does not fall to zero due to the presence of inductance
in the load Circuit.

* After Some time at the instant $\beta > \pi$, load Current becomes
zero & hence thyristor T_1 gets turned off, which is already
in reverse biased Condition. β is known as Extinction angle

* Now a Voltage of Magnitude $E_m \sin\beta$ appears as reverse bi
across the thyristor T_1, & as forward bias across the thyris
T_2, from β to $\pi + \alpha$, no Current Exists. both the thyristor
are in the off State.

$-\pi \; [\pi < \omega t < 2\pi]$:

During the Negative half cycle of Supply voltage, thyristor ... is in the forward biased Condition & T_1 is in the reverse biased condition.

Now, whenever gate Signal has been given to thyristor T_2, at $\omega t = \pi + \alpha$ immediately it gets turned on & hence Conducts from $(\pi + \alpha)$ to 2π.

T_2 is Subjected to a reverse bias at $\omega t = 2\pi$ & it is therefore line Commutated at this instant. its waveform are shown in the fig:-

* Average o/p voltage :-

$$E_0 = \frac{1}{2\pi}\int_{\alpha}^{\pi} E_m \sin\omega t \, (d\omega t)$$

$$= \frac{E_m}{2\pi}\Big[-\cos\omega t\Big]_{\alpha}^{\pi}$$

$$= \frac{E_m}{2\pi}\Big[-\cos\pi + \cos\alpha\Big]$$

$$= \frac{E_m}{2\pi}\Big[1 + \cos\alpha\Big]$$

RMS o/p voltage :-

$$E_{orms} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} E_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= \left[\frac{E_m^2}{\pi}\int_{\alpha}^{\pi}\left[\frac{1 - \cos 2\omega t}{2}\right]\right]^{1/2}$$

$$= \frac{E_m}{\sqrt{\pi}} \left[\frac{1}{2}(\pi - \alpha) - \frac{1}{2 \times 2} \left(\sin 2\omega t \right)_{\alpha}^{\pi} \right]^{1/2}$$

$$= \frac{E_m}{\sqrt{2\pi}} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right]^{1/2}$$

$$= \frac{E_s}{\sqrt{\pi}} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right]^{1/2} \qquad \left[\because E_s = \frac{E_m}{\sqrt{2}} \right]$$

$$E_{ORMS} = \frac{E_s}{\sqrt{\pi}} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

* RMS load Current given as

$$I_{ORMS} = \frac{E_{ORMS}}{R}$$

* power delivered to the load $= \frac{E_{ORMS}^2}{R}$

* I/P power factor $\cos\phi = \frac{\text{power delivered to load}}{\text{apparent power}}$

$$= \frac{E_0 I_0 \cos\phi_0}{E_s I_s}$$

$$\cos\phi = \frac{E_0 I_0}{E_s I_s}$$

(or)

$$\cos\phi = \frac{\frac{E_s}{\sqrt{\pi}} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right]^{1/2}}{E_s}$$

$$\cos\phi = \frac{1}{\sqrt{\pi}} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right]^{1/2}$$

$$= \frac{E_m}{\sqrt{\pi}}\left[\frac{1}{2}(\pi-\alpha) - \frac{1}{2\times 2}(\sin 2\omega t)_{\alpha}^{\pi}\right]^{1/2}$$

$$= \frac{E_m}{\sqrt{2\pi}}\left[\pi-\alpha+\frac{1}{2}\sin 2\alpha\right]^{1/2}$$

$$= \frac{E_s}{\sqrt{\pi}}\left[\pi-\alpha+\frac{1}{2}\sin 2\alpha\right]^{1/2} \qquad \left[\because E_s = \frac{E_m}{\sqrt{2}}\right]$$

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$$\cos\phi = \frac{E_0 I_0}{E_s I_s}$$

(or)

$$\cos\phi = \frac{\frac{E_s}{\sqrt{\pi}}\left[\pi-\alpha+\frac{1}{2}\sin 2\alpha\right]^{1/2}}{E_s}$$

$$\cos\phi = \frac{1}{\sqrt{\pi}}\left[\pi-\alpha+\frac{1}{2}\sin 2\alpha\right]^{1/2}$$

* Series inverters :-

* inverters in which Commutating Components are permenently Connected in Series with the load are Called Series inverters.
* the Series Circuit so formed Must be underdamped.
* as the Current attains Zero Value due to the Nature of the Series Circuit, Series inverters are also Classified as Self- Commutated or Load Commutated inverters.
* this inverters operates at high frequencies [200 Hz to 100 KHz], the size of the Commutating Elements is therfore Small.
* they are widely used in induction heating, fluorescent lighting e.t.c.

* Basic Series inverter :-