# Unit - II

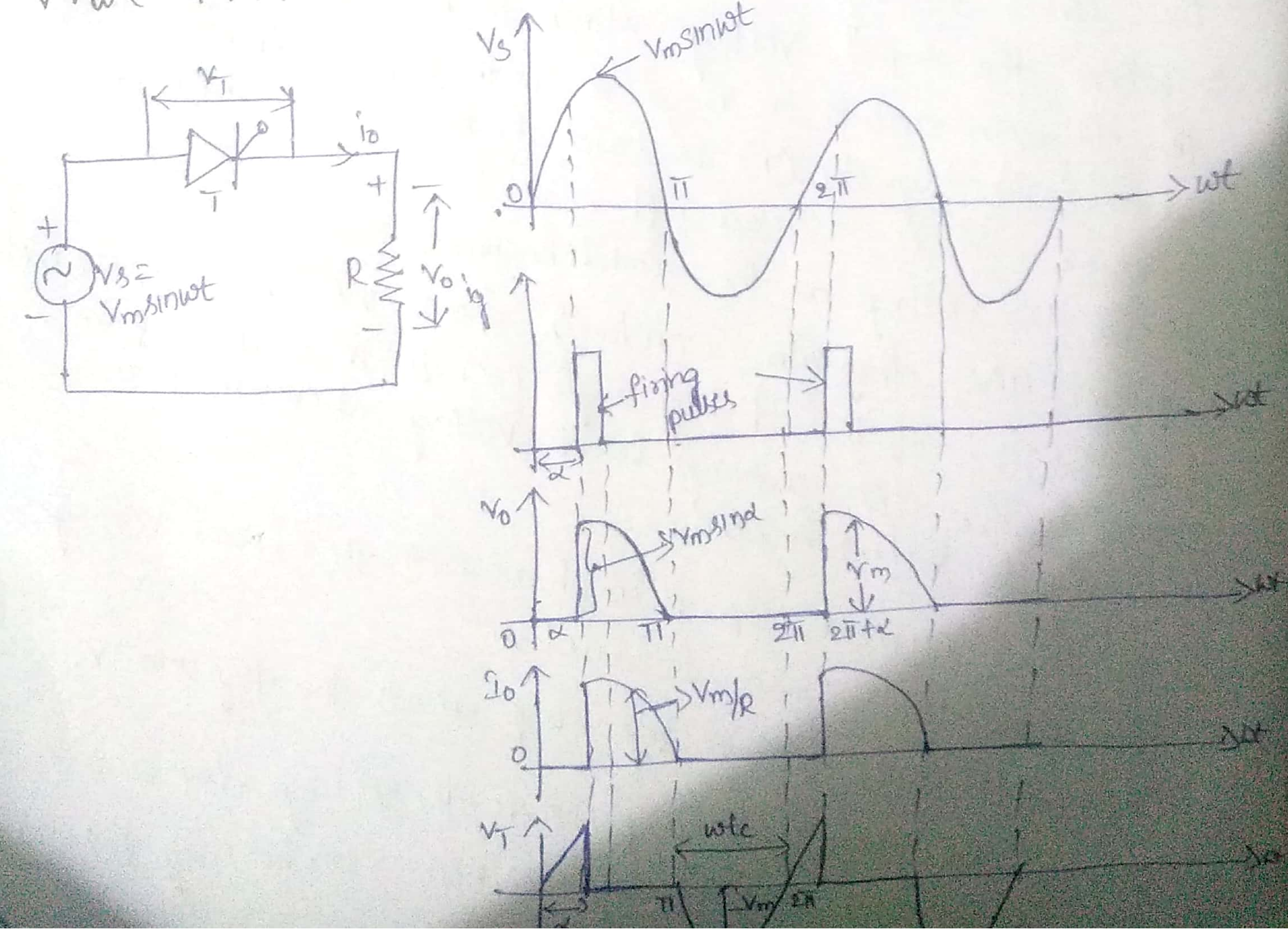
## phase controlled Rectifiers

**Rectifier:-** A rectifier is also known as ac-dc converter

**Applications:-**

a) steel- rolling mills, paper mills
b) Traction system working on dc
c) Electrochemical & electrometallurgical processes
d) Magnet power supplies
e) portable hand tool drives
f) High- voltage dc transmission

**Single phase Half wave circuit with R-Load**



→ fig. shows the 1-ϕ half wave controlled converter using single thyristor feeding resistive load. it consists of a single thyristor feeding DC power to resistive load

→ let us consider a source voltage $V_s = V_m \sin\omega t$ is impressed to the circuit

→ During the positive half cycle of input voltage, the thyristor anode is positive with respect to its cathode hence the thyristor is said to be forward biased

→ when the thyristor is triggered at some delay angle $\omega t = \alpha$, then thyristor starts conducting and full voltage is appears across the load as $V_0$.

→ At the instant of delay angle (or) firing angle $\alpha$, the load voltage $V_0$ rises from zero to $V_m \sin\alpha$. at $\omega t = \pi$,

→ when the input voltage starts to be negative with respect to its cathode the thyristor anode is negative with respect to its cathode causes the thyristor (T) is said to be reversed biased therefore, it is turned off.

Hence the voltage at the load becomes zero

→ from, the thyristor conducts from $\omega t = \alpha$ to $\pi$, $2\pi + \alpha$ to $3\pi$ and so on. if the firing angle ($\alpha$) is increased from zero to $\pi$, the average load voltage decreases from the largest value to zero

→ The load current, $i_0$ is in phase with the load voltage $V_0$ because the load is resistive

→ the waveform of voltage drop across the thyristor $V_T$

→ During the conducting periods the voltage drop across the thyristor is zero and during the non-conducting periods the voltage drop in the thyristor is appeared and is represented as the wave form

→ The thyristor is reverse biased for π radians, the ckt turn-off time is given by

$$t_c = \frac{\pi}{\omega} \text{ sec.}$$

where $\omega = 2\pi f$ and $f$ is the supply frequency in Hz

→ the circuit turn-off time $t_c$ must be more than the SCR turn-off time $t_q$ as specified by the manufacturers

→ Average voltage $V_0$ across load R.

Average voltage $V_0$ across load R in terms of firing angle for the 1-φ half wave ckt in terms of firing angle α is given by

$$V_0 = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \cdot d(\omega t)$$

$$= \frac{V_m}{2\pi}(1+\cos\alpha)$$

where α - firing angle

~~Avera~~ Maximum output voltage, $V_0$ occurs at $\alpha = 0°$

$$V_{0 \cdot max} = \frac{V_m}{2\pi} \cdot 2 = \frac{V_m}{\pi}$$

Normalizing output voltage, $V_0 = \frac{V_{0 \cdot max}}{2}[1+\cos\alpha]$

Average load current, $i_0 = \frac{V_0}{R} = \frac{V_{0 \cdot max}}{2\pi R}[1+\cos\alpha]$

circuit turn-off time is given

In some types of loads, one may be interested in rms value of load voltage $V_{or}$. Ex. of such loads are electric heating and incandescent lamps.

→ RMS voltage $V_{rms}$ in such cases is given by

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t \cdot d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha) + \tfrac{1}{2}\sin 2\alpha\right]^{1/2}$$

→ The value of rms current $I_{rms}$ is

$$I_{rms} = \frac{V_{rms}}{R}$$

power delievered to resistive load $= (V_{rms}) \times (I_{rms})$

$$= \frac{V_{rms}^2}{R} = I_{rms}^2 \cdot R$$

Input volt-amps $= (V_S)$ (total rms line current)

$$= V_S \cdot I_{rms} = \frac{\sqrt{2}\, V_s^2}{2R\sqrt{\pi}}\left[(\pi-\alpha) + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

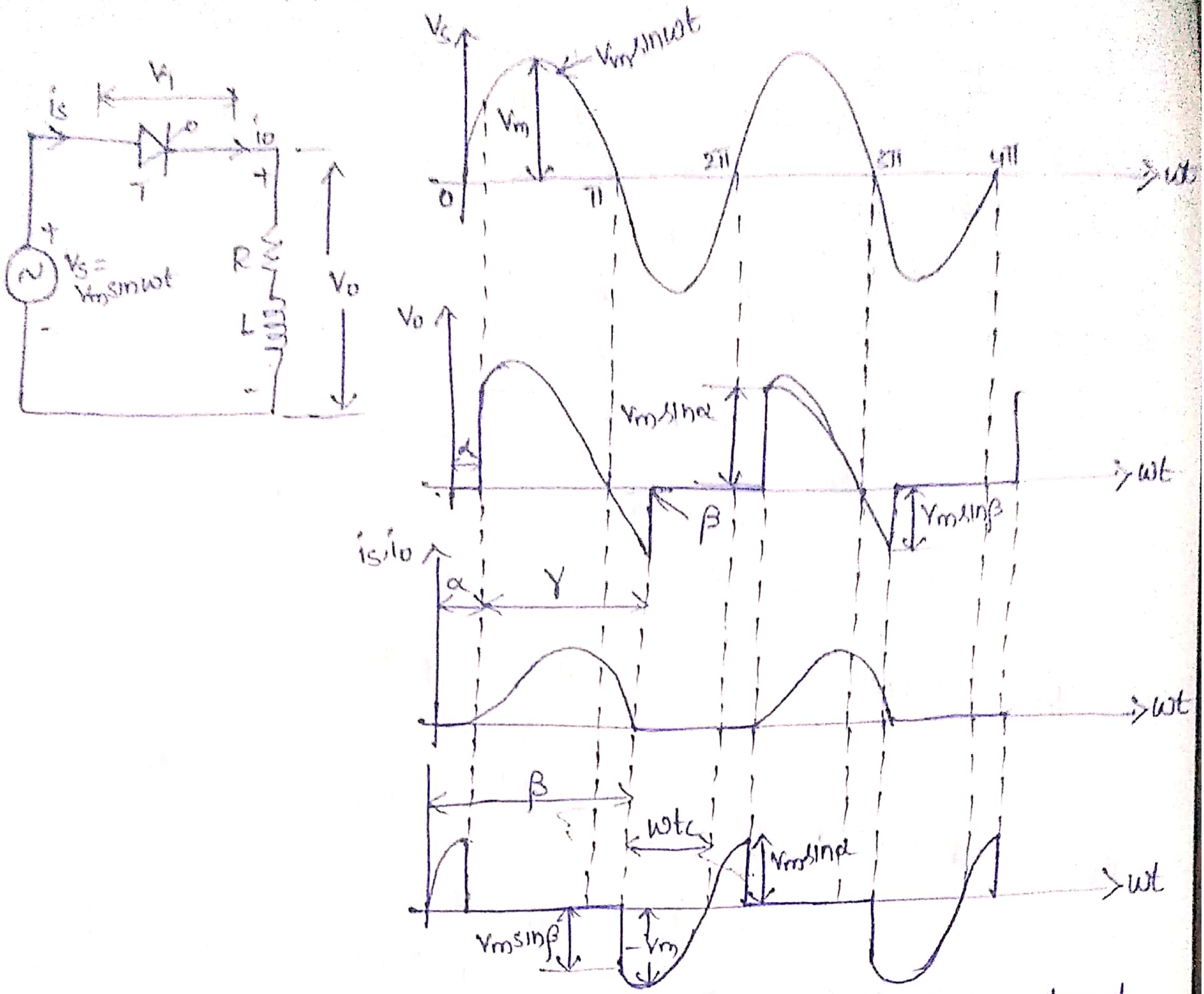
$$\text{input power factor} = \frac{\text{power delievered to load}}{I/p\ VA}$$

$$= \frac{V_{rms} \cdot I_{rms}}{V_S \cdot I_{rms}} = \frac{V_{rms}}{V_S}$$

$$\text{input power factor} = \frac{1}{\sqrt{2\pi}}\left[(\pi-\alpha) + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

Scanned with CamScanner

## ⇒ Single-phase Half wave ckt with RL load:




→ A single phase half wave thyristor ckt with RL load line voltage $V_s$. At $\omega t = \alpha$, thyristor is turned on by gating signal.

→ The line voltage $V_0$ at once becomes equal to source voltage $V_s$.

→ But the inductance L forces the load, or output, current $i_0$ to rise gradually. after sometime, $i_0$ reaches maximum value and then begins to decrease.

→ At $\omega t = \pi$, $V_0$ is zero, but $i_0$ is not zero because of the load inductance L.

→ After $\omega t = \pi$, SCR is subjected to reverse anode voltage but it will not be turned off as load current $i_0$ is

The thyristor

not less than the holding current, At some angle $\beta > \pi$, $i_0$ reduces to zero and SCR is turned off as it is already reverse biased.

→ At $\omega t = \beta$, $V_0 = 0$ and $i_0 = 0$. At $\omega t = 2\pi + \alpha$, SCR is triggered again, $V_0$ is applied to the load and load current develops as before.

→ Angle $\beta$ is called the extinction angle and $(\beta - \alpha) = \gamma$ is called conduction angle.

→ The waveform of voltage across thyristor T reveals that when $\omega t = \alpha$, $V_T = V_m \sin \alpha$, from $\omega t = \alpha$ to $\beta$, $V_T = 0$ and at $\omega t = \beta$, $V_T = V_m \sin \beta$,

→ As $\beta > \pi$, $V_T$ is negative at $\omega t = \beta$, ∴ thyristor is reverse biased from $\omega t = \beta$ to $2\pi$.

Thus, the ckt, turn-off time,

$$t_c = \frac{2\pi - \beta}{\omega} \text{ sec.}$$

The voltage equ$^n$ for the ckt, when T is on,

$$V_m \sin \omega t = R i_0 + L \frac{di_0}{dt}$$

→ The load current $i_0$ consists of two components one steady-state component $i_s$ and the other transient component $i_t$, Here $i_s$ is given by

$$i_s = \frac{V_m}{\sqrt{R^2 + X^2}} \sin(\omega t - \phi)$$

where $\phi = \tan^{-1}\frac{X}{R}$ and $X = \omega L$

The transient component, it can be obtained from force - free equ$^n$

$$Ri_t + L\frac{di_t}{dt} = 0$$

it solution gives, $i_t = Ae^{-(R/L)t}$

$$i_0 = i_s + i_t = \frac{V_m}{Z}\sin(\omega t - \phi) + A^{-(R/L)t} \rightarrow (1)$$

where $Z = \sqrt{R^2 + X^2}$

Constant A canbe obtained from the boundary condition at $\omega t = \alpha$

As this time $t = \frac{\alpha}{\omega}$, $i_0 = 0$

then, $0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A \cdot e^{-R\alpha/L\omega}$

$$A = -\frac{V_m}{Z}\sin(\alpha - \phi)\, e^{R\alpha/\omega L}$$

sub 'A' in equ (1)

$$i_0 = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{V_m}{Z}\sin(\alpha - \phi)\exp\left\{\frac{R}{\omega L}(\omega t - \alpha)\right\} \rightarrow (2)$$

for $\alpha < \omega t < \beta$

it also seen from the wave form of $i_0$

when $\omega t = \beta$, load current $i_0 = 0$, substituting this

$$\sin(\beta - \phi) = \sin(\alpha - \phi) \cdot \exp\left\{-\frac{R}{\omega L}(\beta - \alpha)\right\}$$

The transcendental equ$^n$ canbe solved the value of extinction angle $\beta$, average load voltage $V_0$ is given by

$$V_0 = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Avg. load current, $I_0 = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta)$

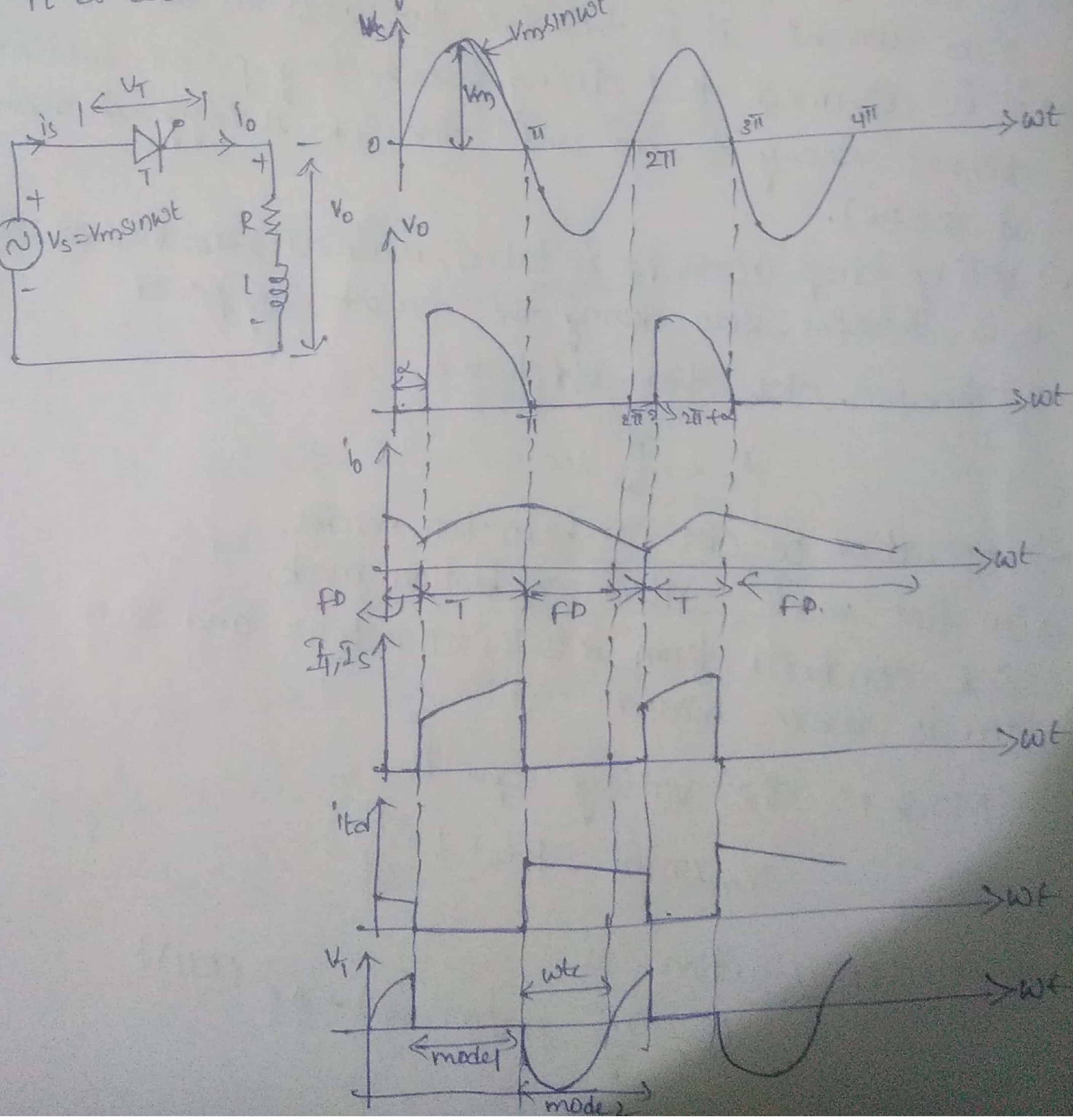
RMS load voltage, $V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\beta} V_m^2 \sin^2\omega t \cdot d(\omega t)\right]^{1/2}$

$$= \frac{V_m}{2\sqrt{\pi}}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{1/2}$$

## Single phase halfwave ckt with RL load & free wheeling diode:

→ 1-ϕ half wave ckt with RL load, the wave form of load current is not continous and due to this the operating power factor of the system becomes very poor which may gives rise to poor performance of the ckt

→ In order to improve the wave form of load current $i_o$ and power factor of the ckt a diode is connected across the load is called free wheeling diode. it is also called By-pass (or) commutating diode.



→ At $\omega t = 0$, Source voltage is becoming positive.
At some delay angle $\alpha$, forward biased SCR is triggered and Source voltage $V_s$ appears across load as $V_o$

→ At $\omega t = \pi$, source voltage $V_s$ is zero and just after this instant, as $V_s$ tends to reverse, freewheeling diode FD is forward biased through the conducting SCR.

→ As a result, load current $i_o$ is immediately transferred from SCR to FD as $V_s$ tends to reverse.

→ At same time, SCR is subjected to reverse voltage and zero current, it is therefore turned off at $\omega t = \pi$.

→ it is assumed that during freewheeling period, load current does not decay to zero until the SCR is triggered again at $(2\pi + \alpha)$.

→ Voltage drop across FD is taken as almost zero, the load voltage $V_o$ is, therefore, zero during the free wheeling period.

Therefore, ckt turn-off time is

$$t_c = \frac{\pi}{\omega} \text{ sec.}$$

→ operation of ckt can be in two modes.

→ In first mode called conduction mode.
SCR conducts from $\alpha$ to $\pi$, $2\pi + \alpha$ to $3\pi$ and so on and is reverse biased

Mode 1: the voltage eqn is

$$V_m \sin \omega t = R i_o + L \frac{di_o}{dt}$$

its solution is

$$i_o = \frac{V_m}{Z} \cdot \sin(\omega t - \phi) + A e^{-(R/L)t}$$

At $\omega t = \alpha$, $i_0 = I_0$, at $t = \alpha/\omega$, $i_0 = I_0$

$$\therefore A = \left[I_0 - \frac{V_m}{Z}\sin(\alpha-\phi)\right\} e^{R\alpha/\omega t}$$

$$\therefore i_0 = \frac{V_m}{Z}\sin(\omega t-\phi) + \left[I_0 - \frac{V_m}{Z}\sin(\alpha-\phi)\right\}$$
$$\exp\left\{-R/L\left(t-\frac{\alpha}{\omega}\right)\right\}$$

$\therefore$ at Mode 1, $\alpha \le \omega t \le \pi$

Mode 2: This mode is called free wheeling mode, extends from $\pi$ to $2\pi+\alpha$, $3\pi$ to $4\pi+\alpha$ and so on. In this mode SCR is reverse biased from $\pi$ to $2\pi$, $3\pi$ to $4\pi$ as by voltage wave form $v_1$.

The voltage equ" for mode 2 is

$$0 = Ri_0 + L\frac{di_0}{dt}$$

its solution is $i_0 = Ae^{-(R/L)t}$

At $\omega t = \pi$, $i_0 = I_{01}$

it gives $A = I_{01}\cdot \exp\left\{-\frac{R}{L}(t-\pi/\omega)\right\}$

$\therefore$ At mode 2, $\pi < \omega t \le (2\pi+\alpha)$

Avg. load voltage $V_0$ is given by

$$V_0 = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d(\omega t).$$

$$= \frac{V_m}{2\pi}(1+\cos\alpha)$$

Average load current,

$$I_0 = \frac{V_0}{R} = \frac{V_m}{2\pi R}\left[1+\cos\alpha\right]$$

The Advantages of free wheeling diode are

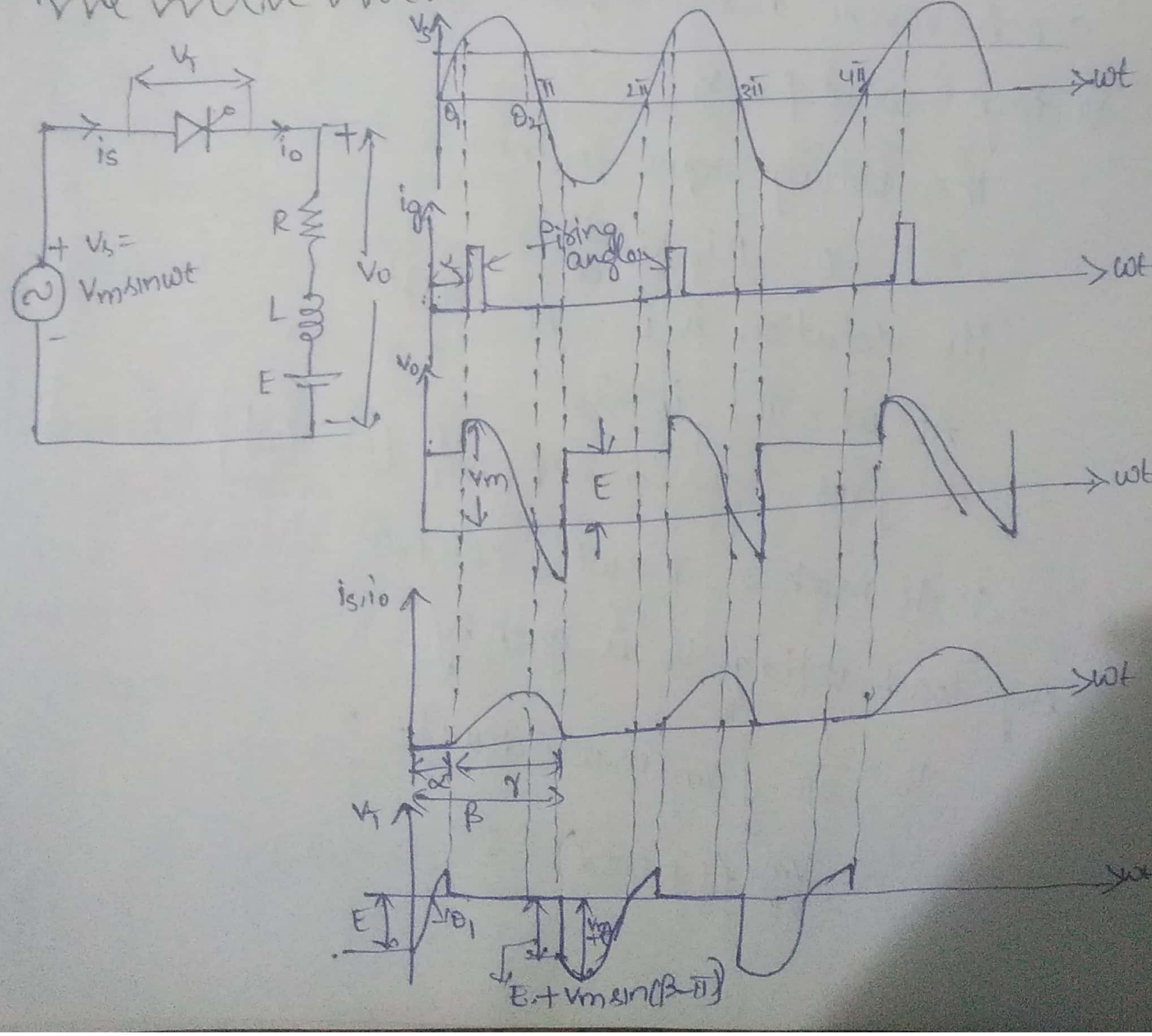
i) input p.f is improved

ii) load current wave-from is improved

iii) load performame is better

iv) As energy stored in L is transferred to R during the free wheeling period, overall converter efficiency improves

⟹ Single phase Half wave circuit with RLE load:-



A 1-$\phi$ halfwave controlled rectifier with RLE load.
The counter emf E in the load maybe due to a battery or a dc motor. The minimum value of firing angle is obtained from the relation $V_m \sin \omega t = E$. This is shown to occur at an angle $\theta_1$ where

$$\theta_1 = \sin^{-1}(E/V_m)$$

→ At Thyristor T is fired at an angle $\alpha < \theta_1$, then $E > V_s$, SCR is reverse biased and therefore it will not turn on.
Similiarly, max. value of firing angle is $\theta_2 = \pi - \theta_1$
→ During the interval load current $i_0$ is zero, load voltage $V_0 = E$ and during the time $i_0$ is not zero, $V_0$ follows $V_s$ curve for the ckt with SCR $T_{on}$, KVL gives the voltage diff equ is

$$V_m \sin \omega t = Ri_0 + L\frac{di_0}{dt} + E$$

The Solution of this equn made up of two components
1. Steady state current component is and
2. Transient current component it

for convenience is maybe thought of as the sum of $i_{s1}$ & $i_{s2}$ where $i_{s1}$ is the steady state current due to ac source voltage acting alone
$i_{s2}$ is that due to dc counter emf E acting alone.

$$\therefore \quad i_{s1} = \frac{V_m}{Z}\sin(\omega t - \phi)$$

if only E were present, then steady state current $i_{s2}$ is

$$i_{s2} = -(E/R)$$

The transient current $i_t$ is $i_t = A \cdot e^{-(R/L)t}$

Then the total current $i_0 = i_{s1} + i_{s2} + i_t$

$$= \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{-(R/L)t}.$$

At $\omega t = \alpha$, $i_0 = 0$, i.e., at $t = \frac{\alpha}{\omega}$, $i_0 = 0$

This gives $A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right] e^{R\alpha/L\omega}$

$$i_0 = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)\exp\left\{-R/\omega L\,(\omega t - \alpha)\right\}\right]$$
$$- \frac{E}{R}\left[1 - \exp\left\{-R/\omega L\,(\omega t - \alpha)\right\}\right]$$

it is applicable for $\alpha \le \omega t \le \beta$.

The extinction angle $\beta$ depends upon load emf $E$, firing angle $\alpha$ and the load impedance angle $\phi$

→ Average voltage across inductance is zero. Thus, average value of load current can be obtained by integrating $(V_m \sin\omega t - E)/R$ between $\alpha$ and $\beta$.

The average load current $I_0$ is given by

$$I_0 = \frac{1}{2\pi R}\left[\int_\alpha^\beta (V_m \sin\omega t - E)\, d(\omega t)\right]$$
$$= \frac{1}{2\pi R}\left[V_m(\cos\alpha - \cos\beta) - E(\beta - \alpha)\right]$$

Here conduction angle $\gamma = \beta - \alpha$, putting $\beta = \gamma + \alpha$ gives

$$I_0 = \frac{1}{2\pi R}\left[V_m(\cos\alpha - \cos(\gamma + \alpha)) - E.\gamma\right]$$

using the trigonometric equ$^n$

$$\cos x - \cos y = 2\sin\frac{x+y}{2}.\sin\frac{y-x}{2}$$

Then $$I_0 = \frac{1}{2\pi R}\left[2V_m \sin\left(\alpha + \frac{\gamma}{2}\right)\sin\frac{\gamma}{2} - E.\gamma\right]$$

Average load voltage is given by

$$V_0 = E + I_0 . R = E + \frac{1}{2\pi}\left[2V_m \sin\left(\alpha + \frac{\gamma}{2}\right)\sin\frac{\gamma}{2} - \gamma . E\right]$$
$$= E\left(1 - \frac{\gamma}{2}\right) + \frac{V_m}{\pi}\sin\left(\alpha + \frac{\gamma}{2}\right)\sin\frac{\gamma}{2}$$

The above expression for the avg load voltage $V_0$ can be for periodically $2\pi$, extending from $\alpha$ to $(2\pi+\alpha)$, we have

$$V_0 = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin(\omega t).d(\omega t) + E(2\pi+\alpha-\beta)\right]$$

$$= \frac{1}{2\pi}\left[V_m(\cos\alpha - \cos\beta) + E(2\pi+\alpha-\beta)\right]$$

If load inductance $L$ is zero, the extinction angle $\beta$ would be equal to $\theta_2 = \pi - \theta_1$. Now $\beta$ would be less than $\pi$

then $\beta = \pi - \theta_1$

$\therefore$ Avg load current $I_0$ is

$$I_0 = \frac{1}{2\pi R}\left[V_m(\cos\alpha - \cos(\pi-\theta_1)) - E(\pi-\theta_1-\alpha)\right]$$

$$= \frac{1}{2\pi R}\left[V_m(\cos\alpha + \cos\theta_1) - E(\pi-(\theta_1+\alpha))\right]$$

RMS value of load current,

$$I_{rms}^2 = \frac{1}{2\pi}\int_{\alpha}^{\beta}\left[\frac{V_m \sin\omega t - E}{R}\right]^2 d(\omega t)$$

$$= \frac{1}{2\pi R^2}\int_{\alpha}^{\beta}(V_m^2 \sin^2\omega t + E^2 - 2V_m E \sin\omega t)\, d(\omega t)$$

its amplification gives

$$I_{rms} = \left[\frac{1}{2\pi R^2}\left\{(V_s^2+E^2)(\beta-\alpha) - \frac{V_s^2}{2}(\sin 2\beta - 2\sin 2\alpha) - 2V_m E(\cos\alpha - \cos\beta)\right\}\right]^{1/2}$$

power delievered to load, $P = I_{rms}^2 \cdot R + I_0 \cdot E$

Supply power factor $= \dfrac{I_{rms}^2 \cdot R + I_0 \cdot E}{V_s \cdot I_{rms}}$

The ckt turnoff time $t_c = \dfrac{2\pi + \theta_1 - \beta}{\omega}$ sec

## Full wave Rectifiers :-

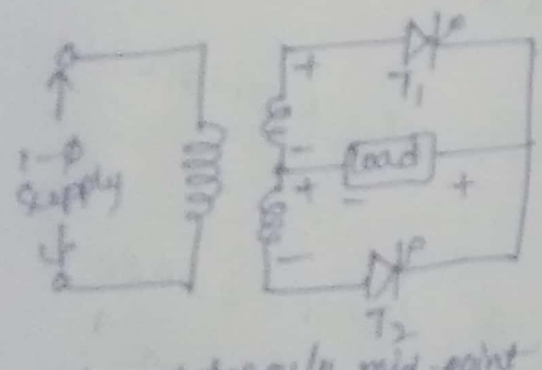
In 1-$\phi$ half wave controlled rectifiers produce only one pulse of load current during one cycle of source voltage, these can therefore be termed as single phase single pulse converters.

→ The disadvantages of 1-$\phi$ half wave, or single phase one-pulse rectifier, are minimised by the use of 1-$\phi$ full wave, or 1-$\phi$ two pulse rectifiers

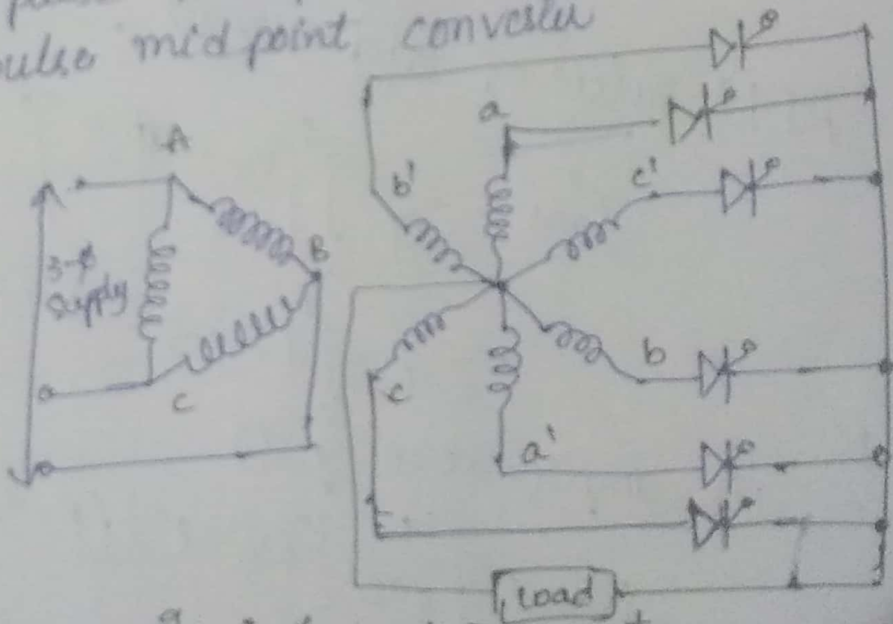
→ There are two basic configurations for full wave controlled converter.

→ One configuration uses an input transformer with two winding for each input phase winding. This is called mid-point converter.

1. A single phase two pulse mid point SCR rectifier
2. A Three phase six pulse mid point converter



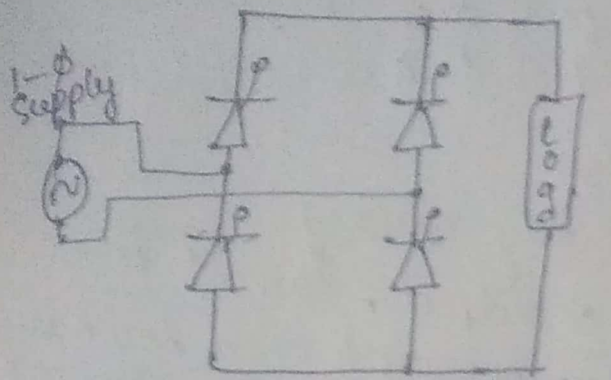

1. 1-$\phi$ two pulse mid-point



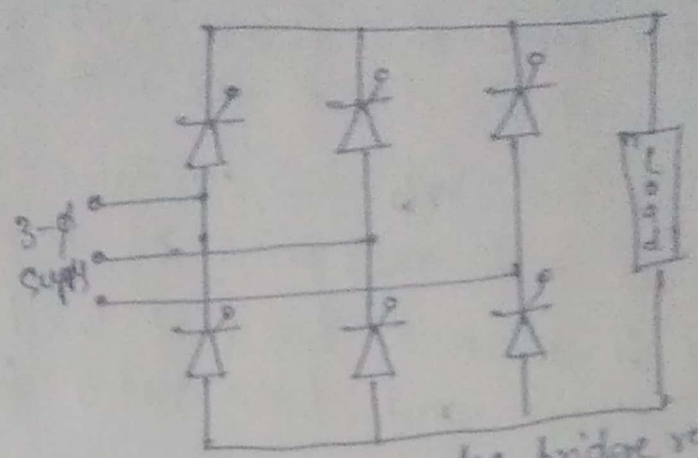

2. 3-$\phi$ 6 pulse mid point

→ Second configuration uses SCRs in the form of bridge circuit

1. 1-$\phi$ full wave, or two-pulse, bridge converter using four SCR's
2. A 3-$\phi$ six pulse bridge converter using six SCR's

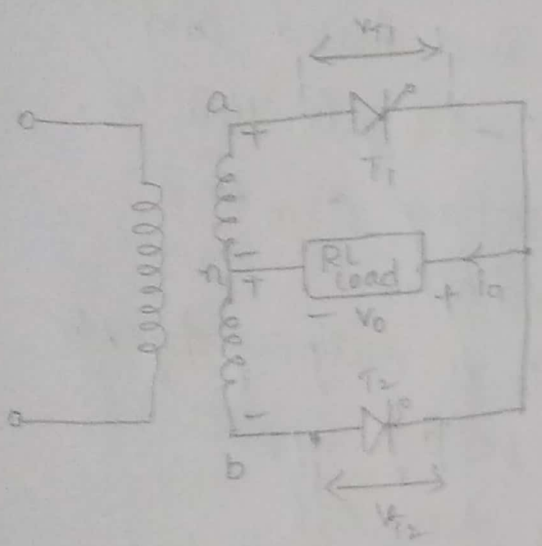



1. 1-φ two pulse bridge Rectifier

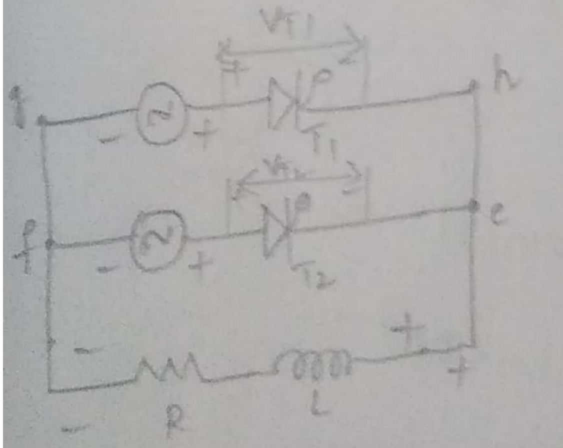


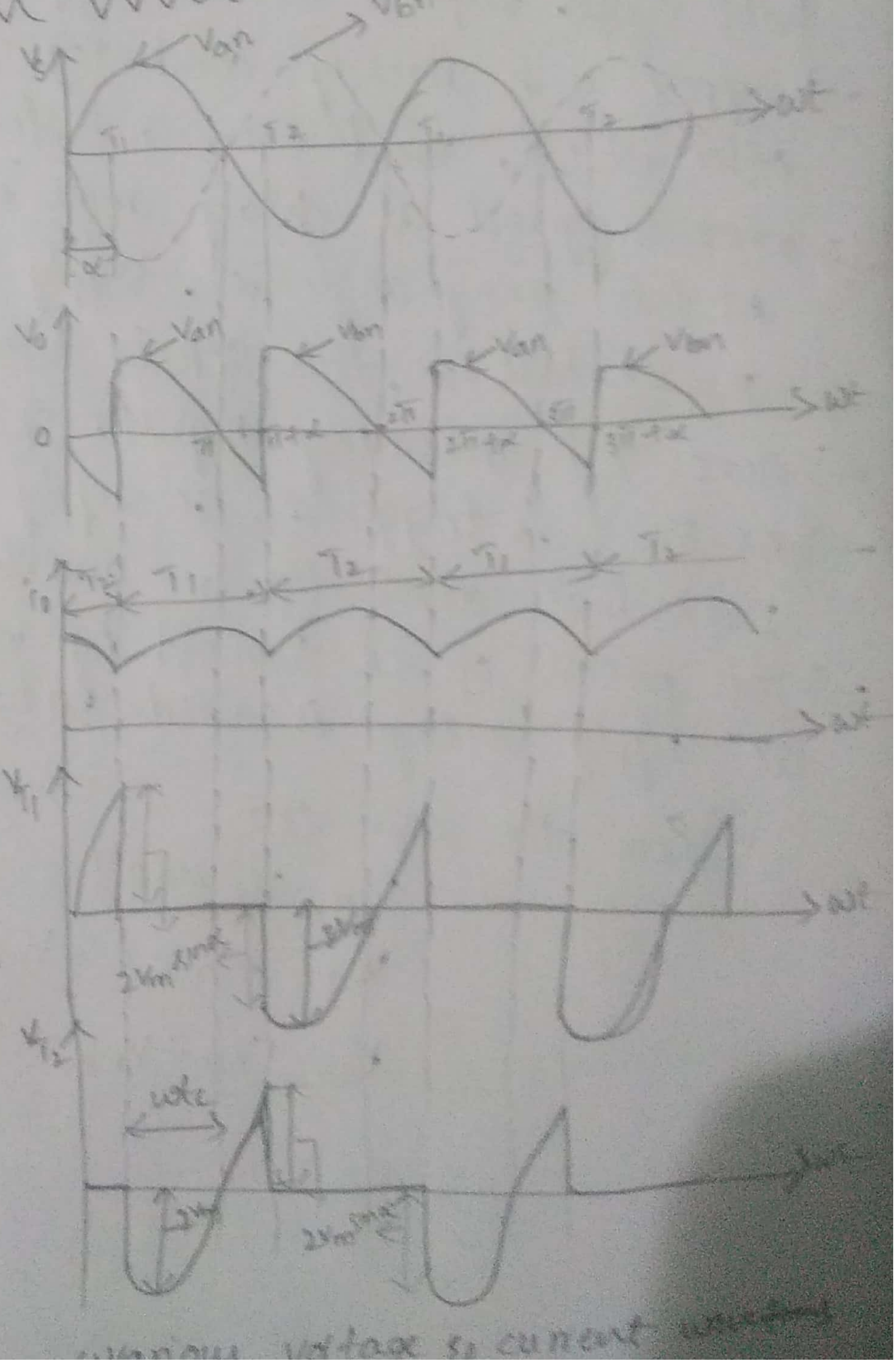

2. 3-φ six pulse bridge rectifier

## Single phase full wave Rectifier :- (midpoint converter)




a) circuit diagram




various voltage & current waveforms

→ 1-$\phi$ full wave rectifier using a center-tapped t/f

→ when terminal a is +ve with respect to n,
terminal n is +ve with respect to b.

$\therefore V_{an} = V_{nb} \Rightarrow$ (or) $V_{an} = -V_{bn}$

as 'n' is mid point of secondary winding.

→ it is assumed here that load or output, current is continous and turns ratio from primary to each secondary is unity

→ Thyristors $T_1$ & $T_2$ are forward biased during +ve and -ve half cycles respec. these are triggered accordingly.

→ Suppose $T_2$ is already conducting.
After $\omega t = 0$, $V_{an}$ is +ve, $T_1$ is forward biased and

→ when triggered at delay angle '$\alpha$' → $T_1$ gets Turn on.
At this firing angle '$\alpha$', supply voltage $2V_m \sin\alpha$ reverse bias $T_2$, this SCR is turned off.

→ Here $T_1$ is incoming thyristor & $T_2$ is outgoing thyristor

→ As the $T_1$ is triggered, ac supply voltage applies reverse bias across the $T_2$. and its turns off

→ This process of SCR turn off by natural reversal of ac supply voltage is called natural (or) line commutation

from the equ. ckt

$$V_{an} = V_m \sin\omega t$$

$$V_{bn} = -V_{nb} = -V_m \sin\omega t$$

$$V_{ab} = V_{an} + V_{nb} = 2V_m \sin\omega t$$

→ when $\omega t = \alpha$, $T_1$ is triggered, SCR $T_2$ is reverse voltage
$V_{ab} = 2V_m \sin\alpha$, current transferred from $T_2$ to $T_1$
& $T_2$ is turned off

→ The magnitude of reverse voltage across $T_2$ can also be obtained by applying KVL to the loop efghe at the instant $T_1$ triggered, thus

$$V_{T_2} - V_{bn} + V_{an} - V_{T_1} = 0$$

$$V_{T_2} = V_{bn} - V_{an} + V_{T_1}$$

with $T_1$ conducting, $V_{T_1} = 0$, ∴ the voltage across $T_2$

$$V_{T_2} = -V_m \sin\alpha - V_m \sin\alpha$$

$$= -2V_m \sin\alpha$$

As the above exp $T_2$ is reverse biased by voltage $2V_m \sin\alpha$ and it is turns off.

→ Thyristor $T_1$ conducts from $\alpha$ to $\pi+\alpha$

→ After $\omega t = \pi$, $T_1$ is reverse biased, but it will continue conducting as the forward biased SCR $T_2$

→ At $\omega t = \pi+\alpha$, $T_2$ is triggered, $T_1$ is reverse biased by voltage of magnitude $2V_m \sin\alpha$; current transferred from $T_1$ to $T_2$, $T_1$ is turned off

→ The turn off time provided by the circuit to SCR $T_2$ is ~~at~~ $t_2$

At $t_2$ is turns off & reverse bias from $\omega t = \alpha$ to $\pi$

$$t_c = \frac{\pi - \alpha}{\omega} \text{ sec}$$

→ Thyristor $T_1$ is turned off at $\omega t = \pi + \alpha$ & reverse voltage from $\omega t = \pi+\alpha$ to $\omega t = 2\pi$

$$t_c = \frac{2\pi - (\pi+\alpha)}{\omega} = \frac{\pi - \alpha}{\omega}$$

The average value of output voltage is

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin\omega t \cdot d(\omega t)$$

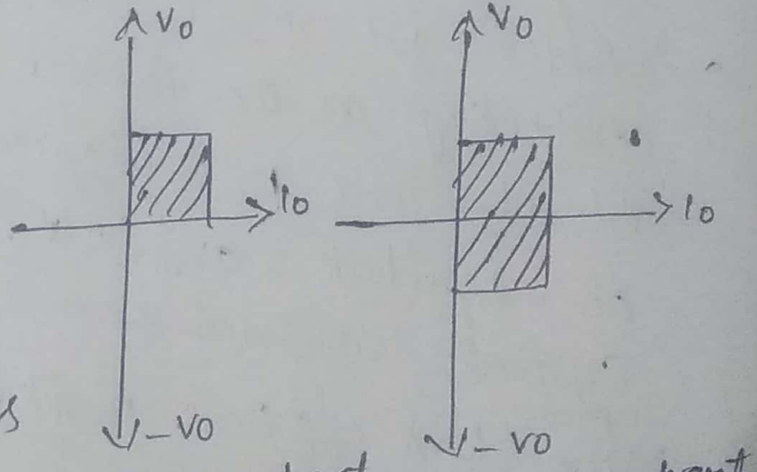
$$= \frac{2V_m}{\pi}\cos\alpha$$

$\Longrightarrow$ for mid- point rectifier observations are

1. when commutation of an SCR is desired, it must be reversed biased and the incoming SCR must be forward biased
2. when incoming SCR is on, current is transferred from outgoing SCR to incoming SCR.
3. The circuit turn-off time is greater than the SCR turn off time

$\Longrightarrow$ Single phase full- bridge Rectifiers

$\rightarrow$ 1-$\phi$ full converters are primarily three types.
they are
1. uncontrolled converters
2. half controlled converters
3. fully controlled converters




a) one quadrant b) Two quadrant

$\rightarrow$ An uncontrolled converter uses only diodes and the level of dc o/p voltage cannot be controlled

$\rightarrow$ A half controlled converter uses a mixture of diodes & thyristors and there is a limited control over the level of dc output voltage.

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→ A fully controlled converter uses thyristors only & there is a wider control over the level of dc output voltage.

→ A semi converter is a one quadrant converter. it has one polarity of dc o/p voltage and current at its output terminals.

→ A two quadrant converter is one in which voltage polarity can reverse but current direction cannot reverse because of the unidirectional nature of thyristors

Single phase fullwave Bridge converter :-

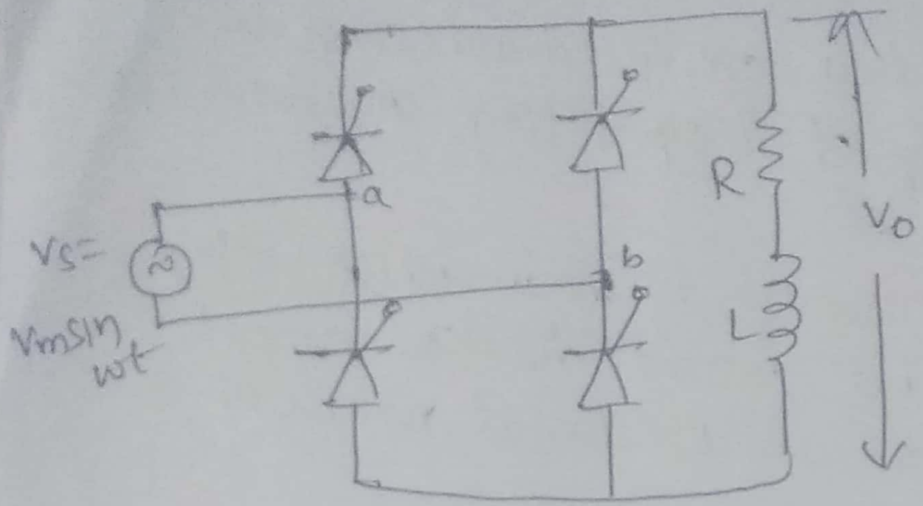
Generally for fullwave rectifier we will take it as a two modes

1. Continous conduction mode
2. Discontinous conduction mode

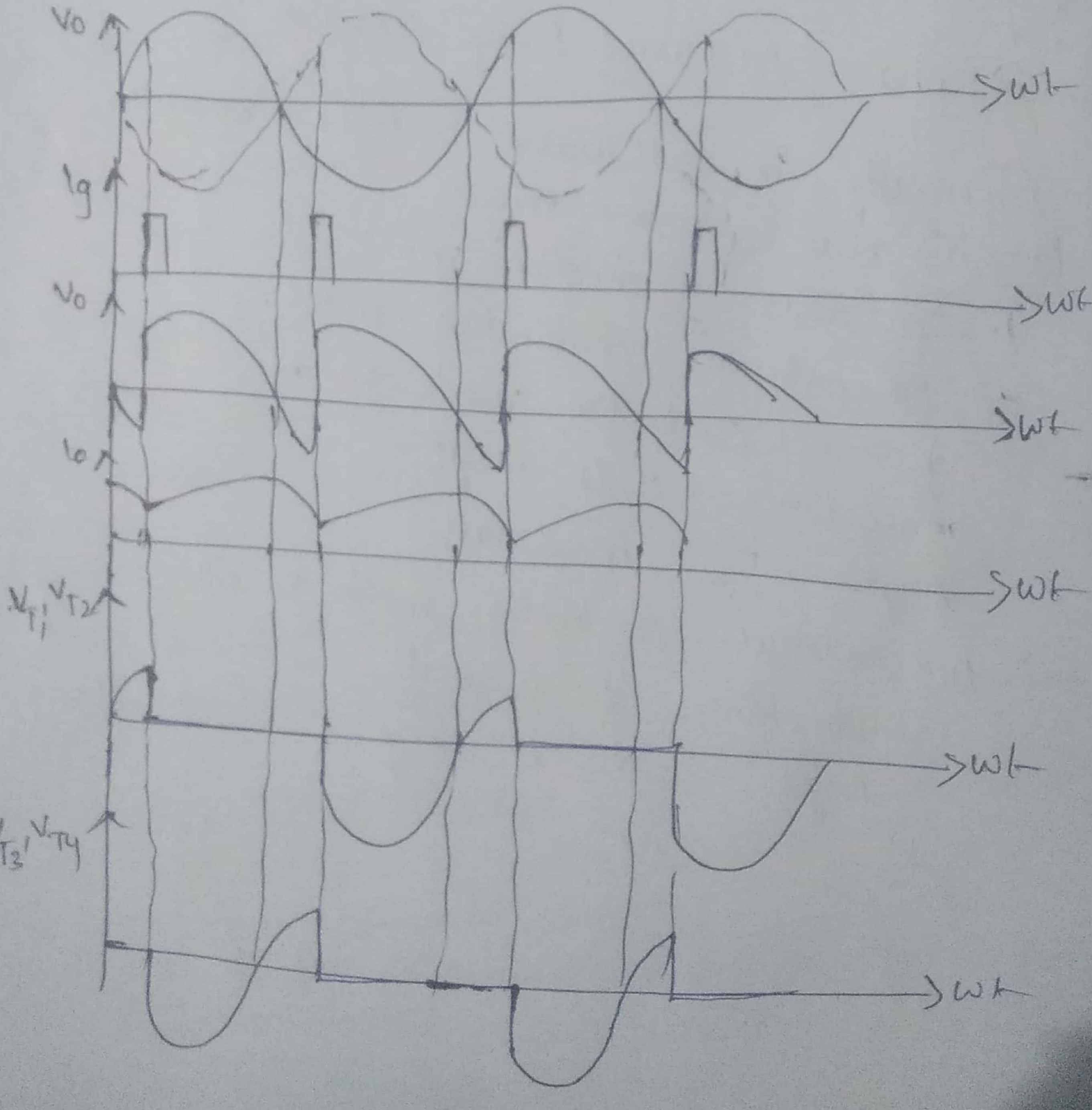
→ continous conduction mode means
inductance L is large
Angle 'α' is small

→ Discontinous conduction mode means
Inductance L is small
Angle α is large

# 1-ϕ full controller bridge rectifier with RL load continous conduction!




$V_{ab} = V_{ba}$



→ The ckt consists of Thysistors $T_1$, $T_2$, $T_3$, & $T_4$

→ Voltage Source $V_s$, & RL load

→ The o/p wave forms for 1-ϕ fullwave bridge rectifier with RL - load & firing angle $\alpha$

→ inductance 'L' is large, angle 'α' is small operation is continous conduction mode

→ inductance 'L' is small, angle $\alpha$ is large operation is discontinous conduction mode

→ consider $\alpha = 30^\circ$, during the +ve halfcycle of the i/p voltage, $T_1$ & $T_2$ are forward bias, it doesnot conduct uptill gate signal is applied to them

→ when gate pulse is applied $T_1$ & $T_2$ at $\alpha = 30^\circ$, it gets turns on & begins to conduct.

→ when i/p voltage applied to the load at $\omega t = \alpha$, the current through the load $I_L$ ~~will insert~~ will flow slowly because of L presence

∴ $I_L$ increases slowly & reaches max. during -ve half cycle

i.e., at $\omega t = \pi$ ⇒ $T_3$ & $T_4$ are forward, $T_1$ & $T_2$ are reverse bias

current at $T_1$ & $T_2 \neq 0$ ∴ It does not turns off

conducts small in -ve half cycle, when $\omega t = \pi + \alpha$ gate pulses are $T_3$ & $T_4$. $T_3$, $T_4$ are turns on the load current $I_L$ shifts path from $T_1$ & $T_2$ to $T_3$ & $T_4$, then $T_1$ & $T_2$ turns off at $\omega t = \pi + \alpha$

→ from $T_1$ & $T_2$ is off state, $T_3$ & $T_4$ are appearing when $\pi + \alpha$, voltage appears

$V_{T3}$, $T_4$ is in not appearing are in offstate, appearing at $T_1$ & $T_2$

$V = \frac{area}{Base}$

Avg voltage

$$V_{oavg} = \int_{\alpha}^{\pi+\alpha} \frac{V_m \sin\omega t}{\pi} \cdot d(\omega t)$$

$$= \frac{V_m}{\pi} \int_{\infty}^{\pi+\alpha} \sin\omega t \cdot d(\omega t)$$

$$= \frac{V_m}{\pi} \left[-\cos\omega t\right]_{\infty}^{\pi+\infty} = \frac{-V_m}{\pi}\left[\cos(\pi+\infty) - \cos\alpha\right]$$

$$\therefore \cos(\pi+\alpha) = -\cos\alpha$$

$$= \frac{-V_m}{\pi}(-\cos\alpha - \cos\alpha)$$

$$V_{oavg} = \frac{-V_m}{\pi} \times -2\cos\alpha = \frac{V_m}{\pi} 2\cos\alpha = \frac{2V_m}{\pi}\cos\alpha$$

$$V_{rms} = \sqrt{\int_{\alpha}^{\pi+\alpha} \frac{V_m^2 \sin^2\omega t}{\pi} \cdot d(\omega t)} = \frac{V_m}{\sqrt{\pi}} \sqrt{\int_{\alpha}^{\pi+\alpha} \frac{1-\cos 2\omega t}{2} \cdot d(\omega t)}$$

$$= \frac{V_m}{\sqrt{\pi \cdot 2}} \sqrt{\int_{\alpha}^{\pi+\alpha} (1-\cos 2\omega t)\, d(\omega t)} = \frac{V_m}{\sqrt{\pi}} \sqrt{(\omega t)_{\infty}^{\pi+\infty} - \left(\frac{\sin 2\omega t}{2}\right)_{\infty}^{\pi+\alpha}}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{(\pi+\infty-\alpha) - \frac{1}{2}\left[\sin 2(\pi+\alpha) - \sin 2\alpha\right]}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\pi - \frac{1}{2}(\sin(2\pi+2\alpha) - \sin 2\infty)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\pi - \frac{1}{2}(\sin 2\alpha - \sin 2\alpha)} = \frac{V_m}{\sqrt{2} \times \cancel{\sqrt{\pi}}} \times \cancel{\sqrt{\pi}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

1-$\phi$ fully controller bridge rectifier with RL load:-
(with Discontinous conduction)

→ Suppose $\alpha = 90°$, during the +ve half cycle, thyristors $T_1, T_2$ are foward biased, $T_3, T_4$ are reverse biased at $\alpha = 90°$, we are giving at gate pulses to $t_1$ & $t_2$ for the supply voltage $V_s$ across the RL load

→ During the +ve half cycle $T_1$ & $T_2$ conducts so the o/p current increases slowly at $wt = \pi$ due to the presence of inductor L, the current slowly reaches the zero value
So, ∴ a small -ve voltage appears across the ~~voltage~~ load

→ During the -ve half cycle $T_1$ & $T_2$ are reverse bias $T_3$ & $T_4$ are foward biased at $wt = \pi + \alpha$ firing pulses are given to $T_3, T_4$ so this voltage $V_{ba}$ appearing across the load,

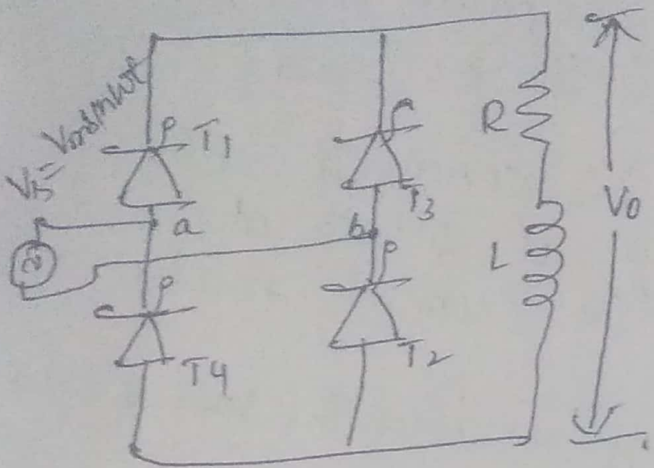
→ here also load increases slowly at $wt = 2\pi$ due to the presence of inductor output current decreases slowly to zero. So a -ve voltage appears across the load $(T_3 T_4)$, so here the load current $I_c$ is Discontinous,

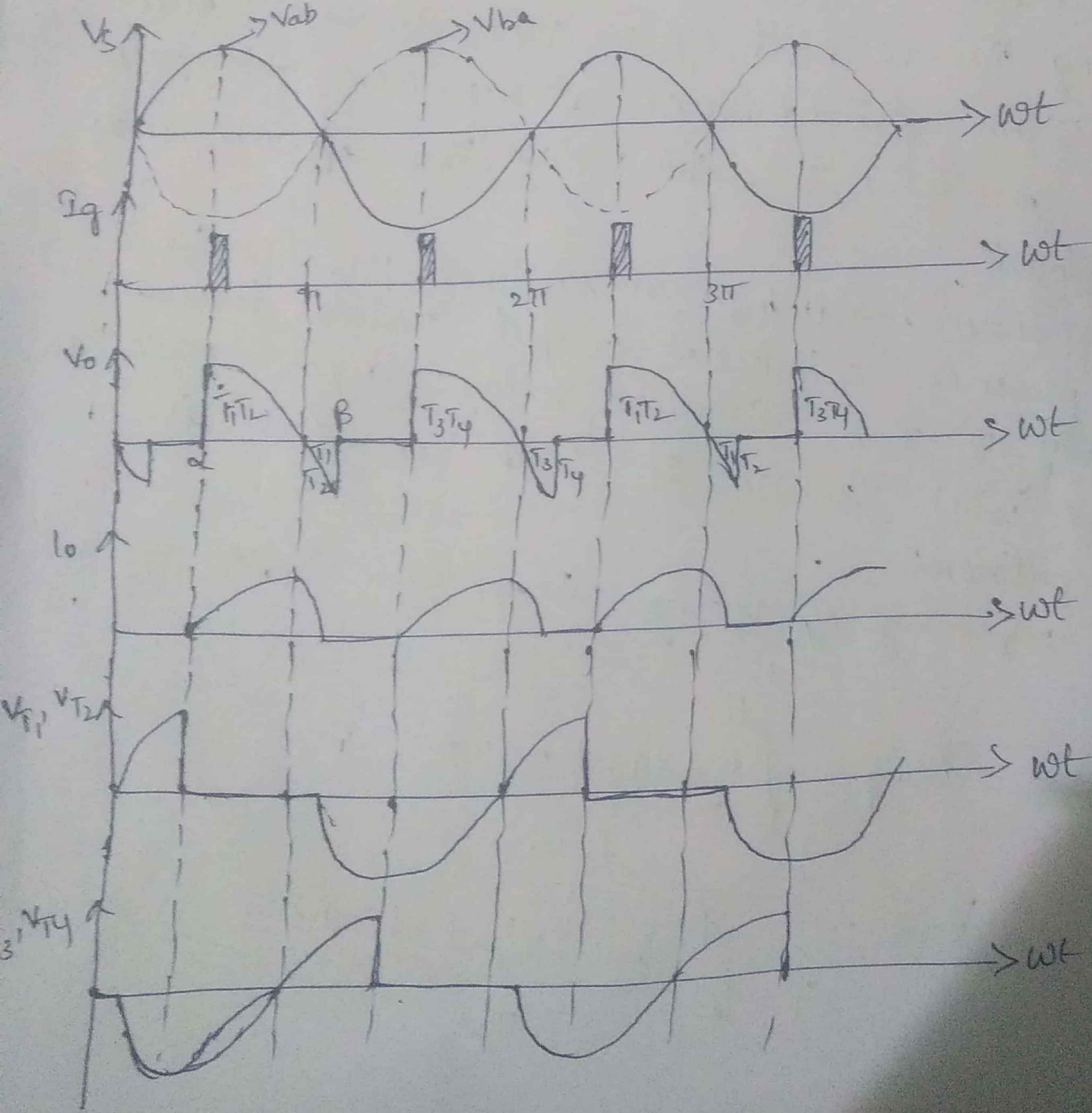
→ By referring o/p. voltage we can draw $V_{T_1}, V_{T_2}, V_{T_3}$ & $V_{T_4}$

$$V_{o\,avg} = \frac{\int_{\alpha}^{\beta} V_m \sin wt \cdot dwt}{\pi}$$

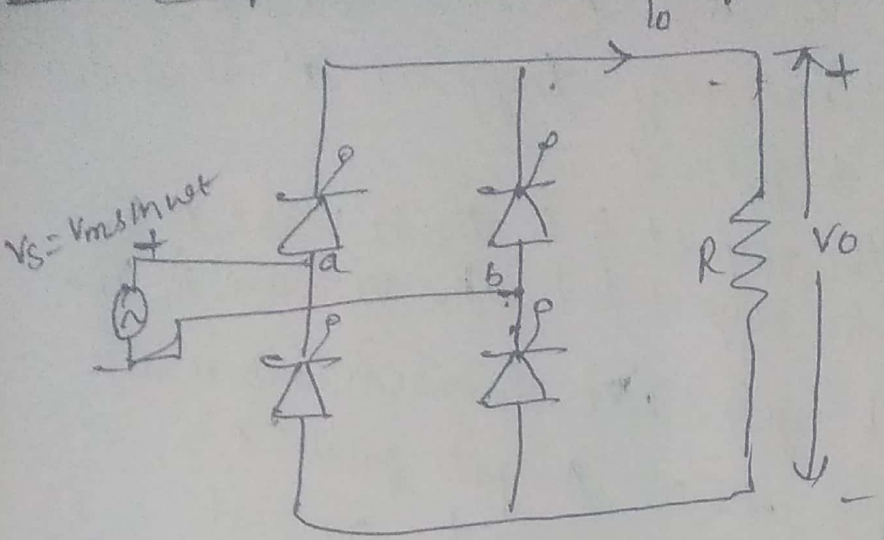
$$V_{o\,rms} = \sqrt{\frac{\int_{\alpha}^{\beta} V_m^2 \sin^2 wt \cdot d(wt)}{\pi}}$$

Ckt & waveforms

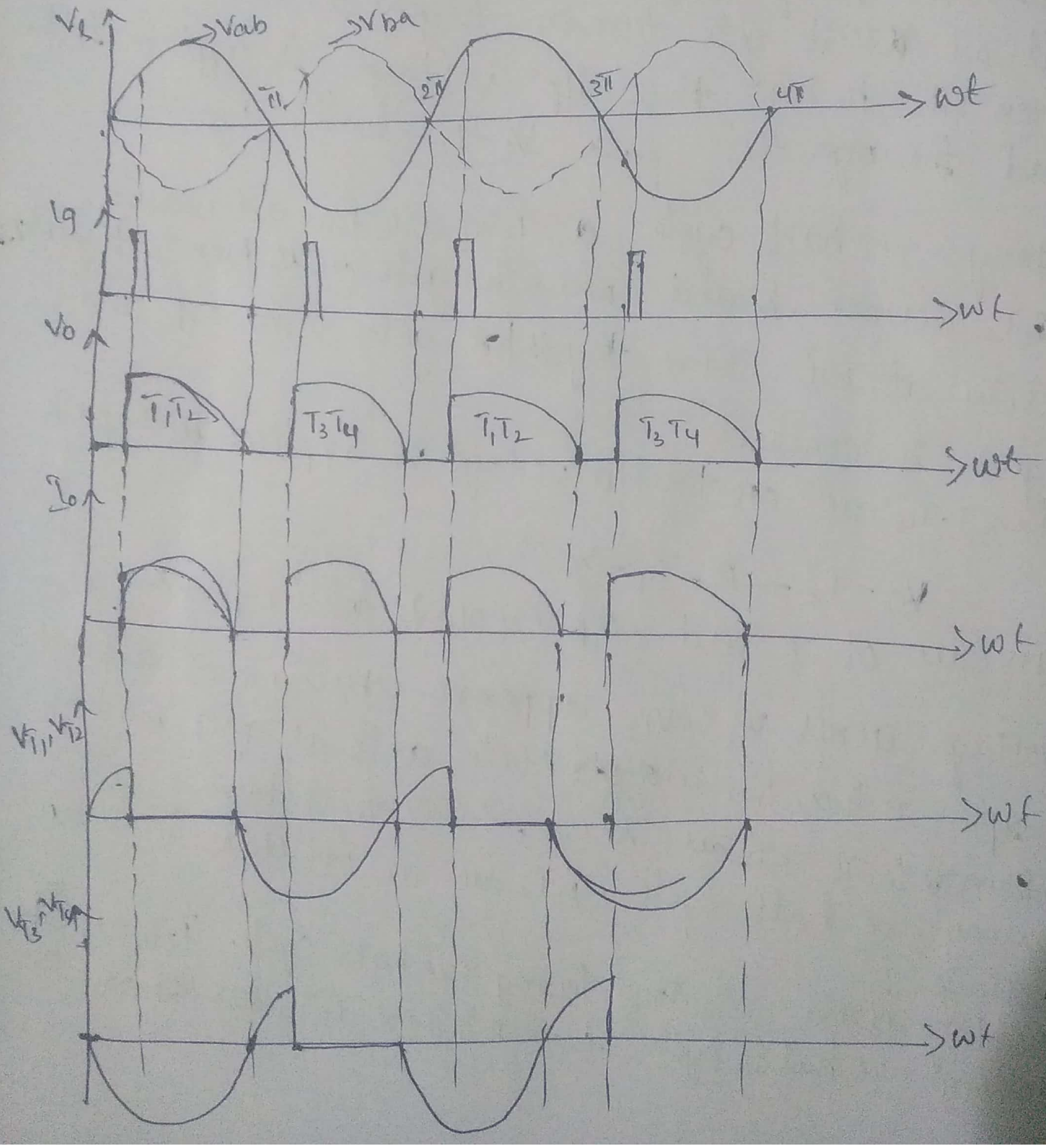


# 1-Ø fully controlled bridge rectifier with R-load :-




$V_{ab} = -V_{ba}$



→ Ckts consists of 4 thyristors $T_1, T_2, T_3$ & $T_4$ at a voltage source $V_s$ and Resistive load R

→ $T_1$ & $T_2$ are simultaneously triggered at 'π' radians later, $T_3$ & $T_4$ are triggered

→ when 'a' is +ve with respect to b the $V_s$ wave is $V_{ab}$
b is +ve with respect to b the $V_s$ wave is $V_{ba}$

→ During the +ve half cycle $T_1$ is forward $T_3 T_4$ is reverse
consider firing angle $\alpha = 30$ for gate pulses to $T_1, T_2$

→ at $wt = \alpha$ it gets turn on Begins to conduct
when $T_1 T_2$ is on this i/p voltage is applied to the load through the path $V_s - T_1 -$ load(R) $- T_2 - V_s$.

→ During -ve half cycle $T_3 T_4$ are forward biased $T_1 T_2$ reverse biased when the gate pulse given to thyristors $T_3 T_4$ at $wt = \pi + \alpha$ thyristors gets turns on and begins to conduct
when $T_3 T_4$ are on the i/p voltage $V_s$ applied to the path

$V_S - T_3 - R - T_4 - V_s$

→ In case of R load o/p V & o/p I are in phase.

→ Voltage across $V_{T_1}$ & $V_{T_2}$ appears during +ve half cycle upto 0 to α, $T_1 T_2$ in off state acts as open switch
from α to π $T_1 T_2$ are S.C., So Zero voltage

→ During -ve half cycle $T_1 T_2$ are in off state in reverse

→ voltage across $V_{T_3}, V_{T_4}$ during +ve half cycle $T_3 T_4$ are off
During -ve half cycle from π to π+α $T_3 T_4$ are ON

$$V_{avg} = \int_{\alpha}^{\pi} \frac{V_m \sin\omega t \, d(\omega t)}{\pi}$$

$$= \frac{V_m}{\pi} \int_{\alpha}^{\pi} \sin\omega t \cdot d(\omega t)$$

$$= \frac{V_m}{\pi} \left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= -\frac{V_m}{\pi} \left[\cos\pi - \cos\alpha\right]$$

$$= -\frac{V_m}{\pi} \left[-1 - \cos\alpha\right] = \frac{V_m}{\pi} \left[1 + \cos\alpha\right]$$

$$I_{avg} = \frac{V_{avg}}{R}$$

$$V_{Rms} = \sqrt{\int_{\alpha}^{\pi} \frac{V_m^2 \cdot \sin^2\omega t \cdot d(\omega t)}{\pi}} = \frac{V_m}{\sqrt{\pi}} \sqrt{\int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\int_{\alpha}^{\pi} (1 - \cos 2\omega t) \cdot d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{(\omega t)_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi}}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{(\pi - \alpha) - \frac{1}{2}\left\{\sin 2\pi - \sin 2\alpha\right\}} \qquad \therefore \sin 2\pi = 0$$

$$V_{rms} = \frac{V_m}{\sqrt{2\pi}} \sqrt{(\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$I_{rms} = \frac{V_{Rms}}{R}$$

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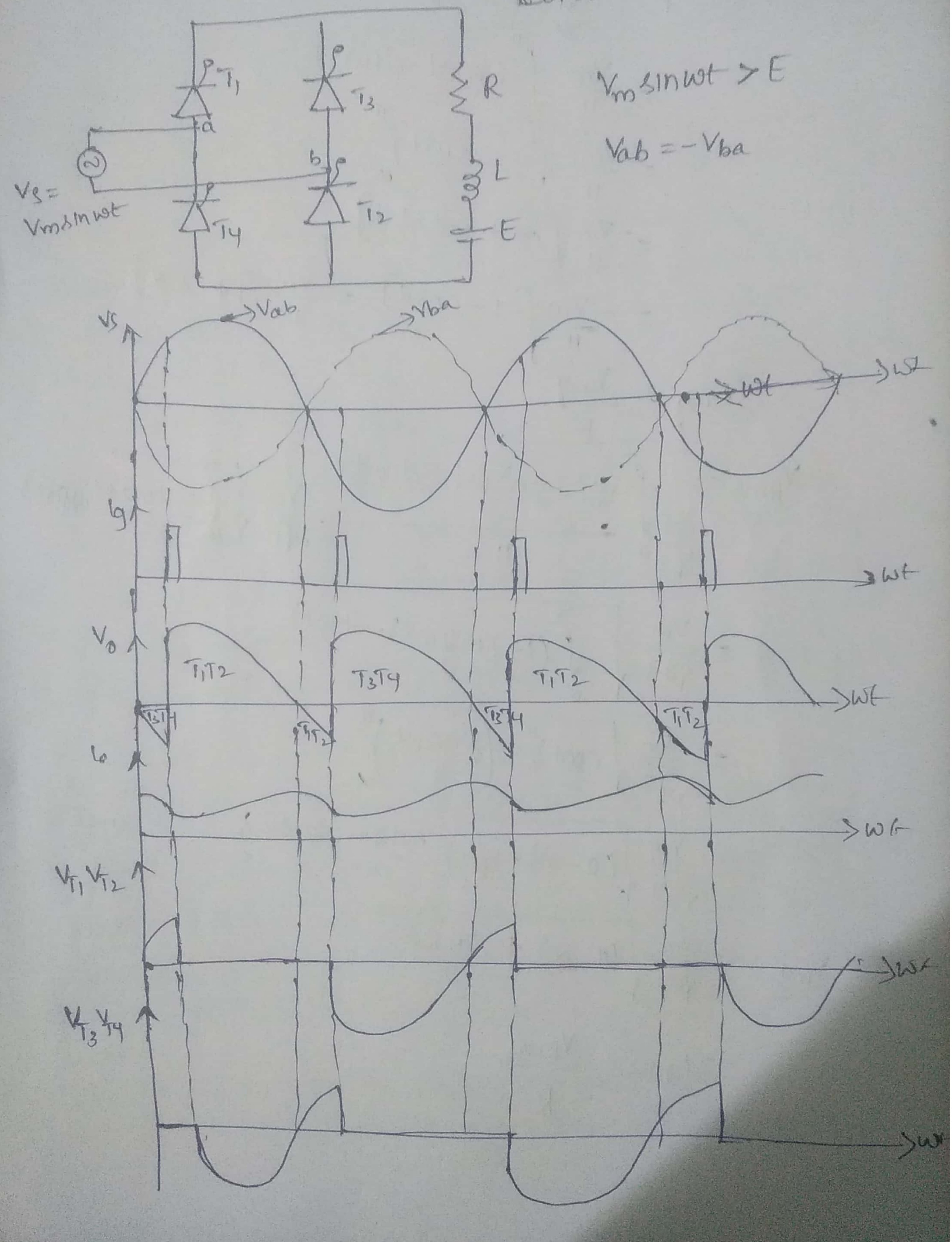
# 1-ϕ fully controlled bridge Rectifier with RLE load continous conduction




$V_m \sin wt > E$

$V_{ab} = -V_{ba}$

→ During positive half cycle the thyristors $T_1 T_2$ are forward biased, $T_3 T_4$ are reverse bias

→ $\omega t = 30$, firing pulses are applying at $T_1, T_2$

∴ $\omega t = \alpha$ $T_1 T_2$ are turned on. and the i/p voltage $V_s$ is applied to the load.

And here current gradually increases

→ at $\omega t = \pi$ i.e., during the -ve half cycle $T_3 T_4$ are forward bias, $T_1 T_2$ are reverse biased but the the current at the thyristors $T_1 T_2$ are not zero ~~during~~ due the presence of inductor L

→ $\pi$ to $\pi + \alpha$ the current through the inductor L begins to $V_K$ to 0 and $T_1 T_2$ conducts for a small duration of -ve half cycle

→ $\omega t = \pi + \alpha$ firing pulses are given to $T_3 T_4$ $T_3 T_4$ are turns on, & the load current shifted to $T_3 T_4$ from $T_1 T_2$, current increases gradually.

→ when 0 to $\alpha$ ~~$T_4$~~, $T_1 T_2$ are off, then $V_{T_1}, V_{T_2}$ are appearing across the thyristor.

From $\alpha$ to $\pi + \alpha$ $T_1 T_2$ are ON, so, $V_{T_1}, V_{T_2}$ are zero

From $\pi + \alpha$ to $2\pi + \alpha$ $T_1 T_2$ are in OFF state

→ when 0 to $\alpha$ $T_3 T_4$ are ON, ∴ $V_{T_3} V_{T_4}$ are zero

$\alpha$ to $\pi + \alpha$ $T_3 T_4$ are OFF, $V_{T_3} V_{T_4}$ are ~~off state~~ reverse

~~$\pi + \alpha$ to $2\pi + \alpha$~~

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$$V_{avg} = \int_{\alpha}^{\pi+\alpha} \frac{V_m \sin\omega t \cdot d(\omega t)}{\pi}$$

$$= \frac{V_m}{\pi} \int_{\alpha}^{\pi+\alpha} \sin\omega t \cdot d(\omega t)$$

$$= \frac{V_m}{\pi} \left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha} = -\frac{V_m}{\pi} \left[\cos(\pi+\alpha) - \cos\alpha\right]$$

$$= -\frac{V_m}{\pi} \left[-\cos\alpha - \cos\alpha\right] = \frac{2V_m}{\pi} \cos\alpha$$

$$V_{avg} = \frac{2V_m}{\pi} \cos\alpha$$

$$I_{avg} = \frac{V_{avg}}{R}$$

$$V_{rms} = \sqrt{\int_{\alpha}^{\pi+\alpha} \frac{V_m^2 \sin^2\omega t \cdot d(\omega t)}{\pi}} = \frac{V_m}{\sqrt{\pi}} \sqrt{\int_{\alpha}^{\pi+\alpha} \left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\int_{\alpha}^{\pi+\alpha} (1-\cos 2\omega t) \cdot d(\omega t)} = \frac{V_m}{\sqrt{2\pi}} \sqrt{(\omega t)_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi+\alpha}}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{(\pi+\alpha-\cancel{\alpha}) - \tfrac{1}{2}\left(\sin(2\pi+2\alpha)\} - \sin 2\alpha\right)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\pi - \tfrac{1}{2}(\cancel{\sin 2\alpha} - \cancel{\sin 2\alpha})} = \frac{V_m}{\sqrt{2\pi}} \times \sqrt{\pi} - \frac{V_m}{\sqrt{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{V_{rms}}{R}$$

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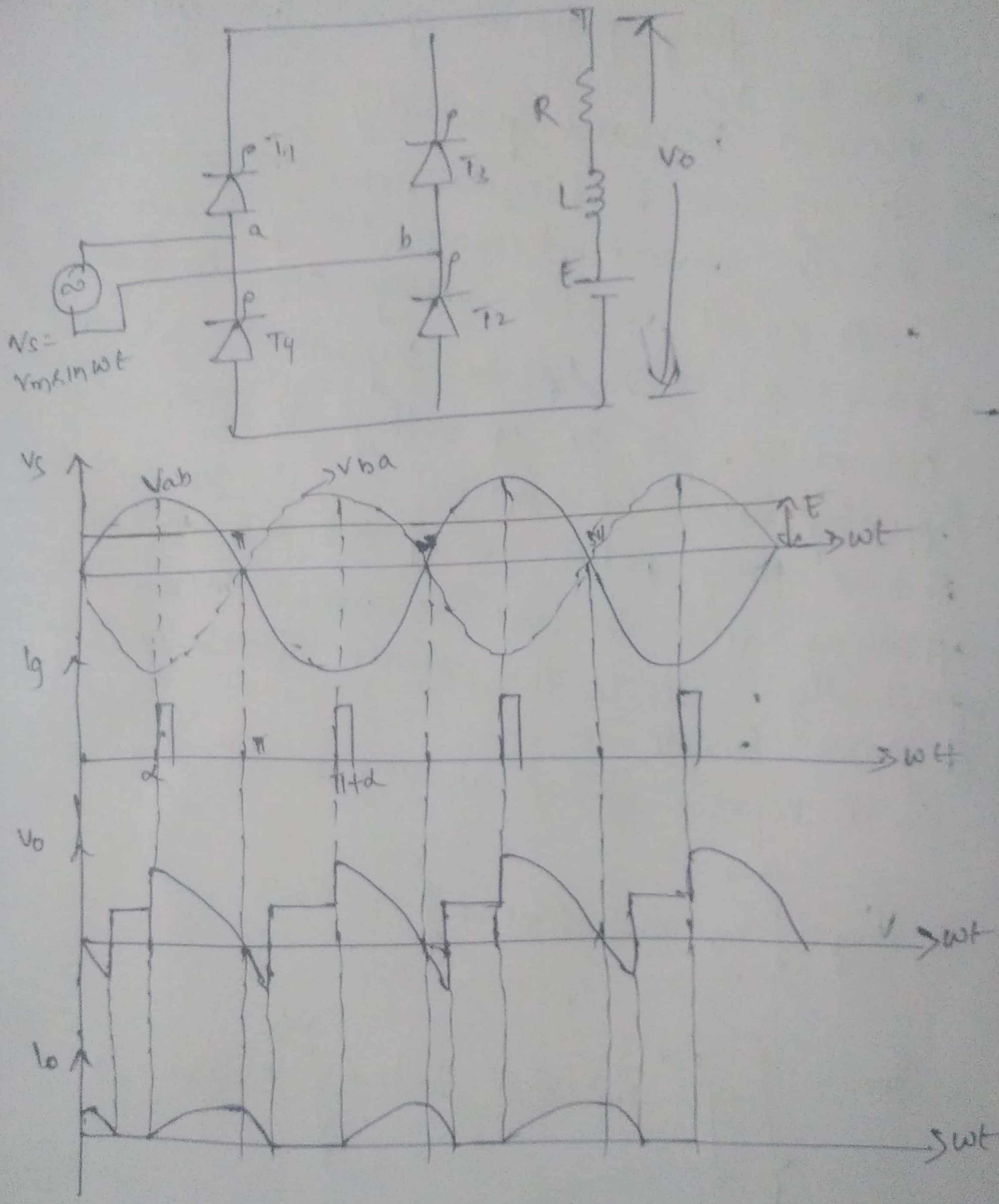
Discontinous conduction:     $\alpha = 90°$

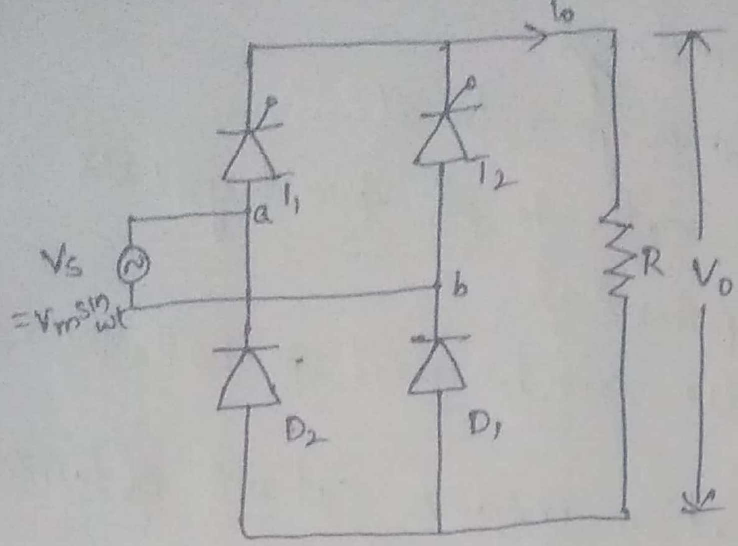


→ At $\alpha = 90°$, during +ve half cycle ~~$T_1 T_2$ are~~
$T_1 T_2$ are forward, $T_3 T_4$ are reverse
$\omega t = \pi$, firing pulses are given to $T_1 T_2$ are turns on
across the $V_s$.
current increases gradually
→ during -ve half cycle, due the presence of
inductor current decreases slowly to zero
∴ A small negative voltage appear the load
→ when $\beta$ to $\pi + \alpha$ All the thyristors $T_1 T_2 T_3 T_4$ are
off state. So, the o/p voltage $V_0 = E$
So, $V = E$ appearing across the load.
→ At $\pi + \alpha$ firing pulses are given to $T_3 T_4$ so $V_{ba}$
appearing across the load so current increases
gradually. due to the presence of inductor current
decreases to zero.
So, A negative voltage appearing across the load.

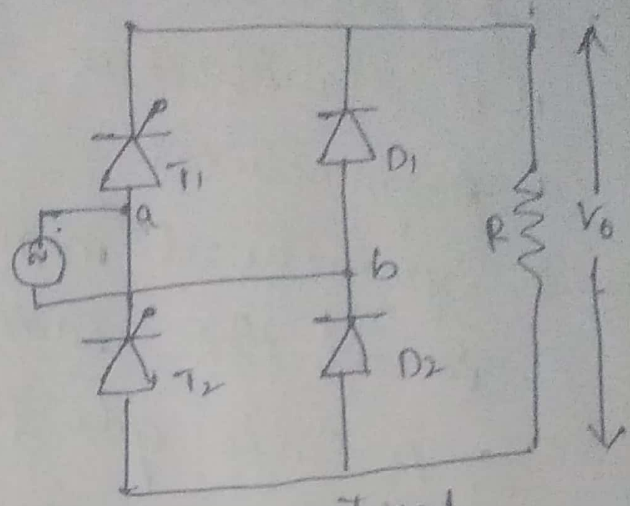
$$V_{avg} = \frac{\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \cdot d(\omega t) + \int_{\beta}^{\pi+\alpha} E \cdot d(\omega t)}{\pi}$$

$$V_{rms} = \sqrt{\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2\omega t \cdot d(\omega t) + \int_{\beta}^{\pi+\alpha} E^2 \cdot d(\omega t)}$$

## ⟹ Single phase semi-controlled Bridge Rectifier with R-load:-

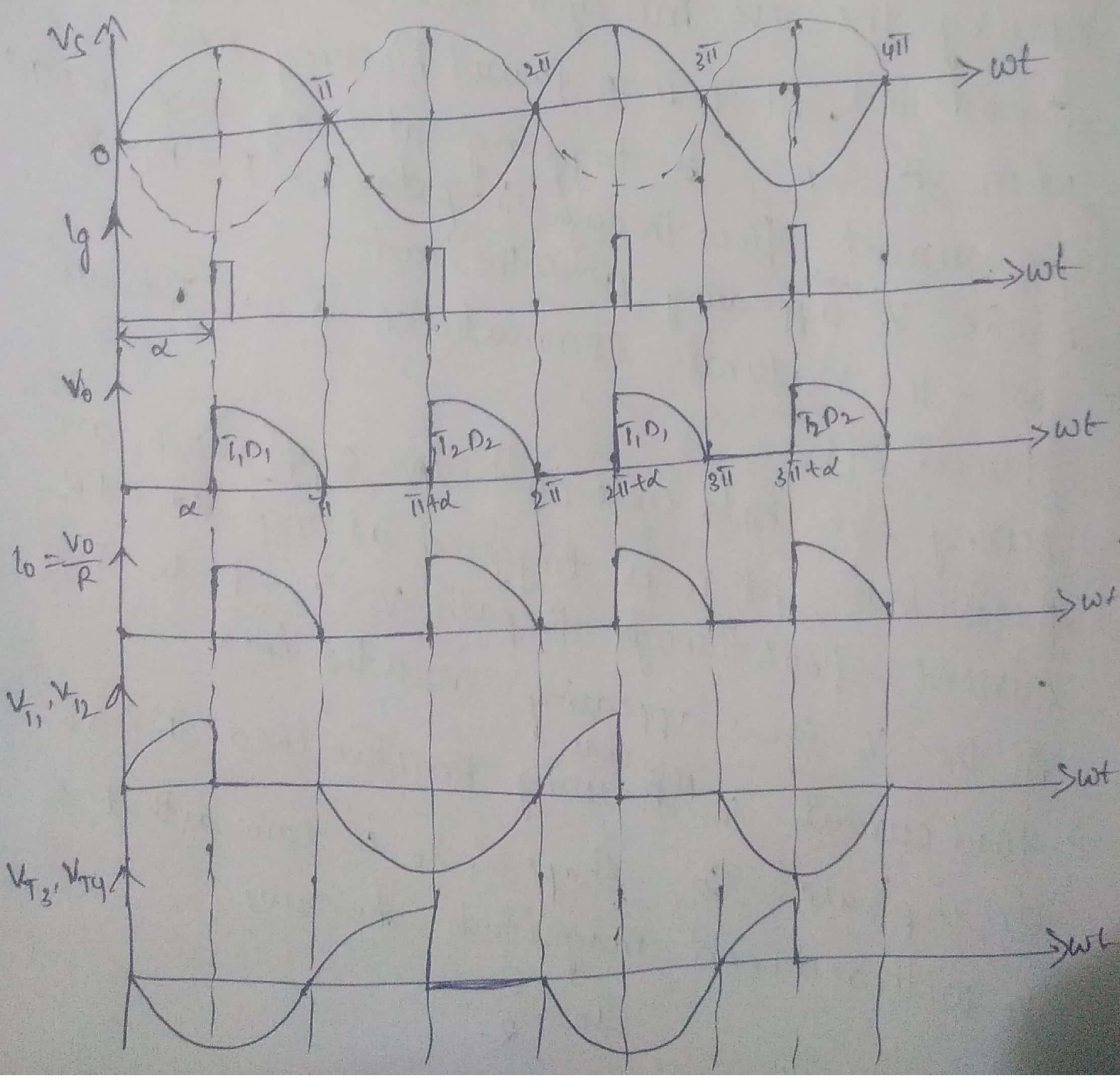



Symmetrical




Asymmetrical

1-ø semi controlled rectifier with symmetrical configuration & R-Load



→ A 1-$\phi$ semi controlled converter is also known as 1-$\phi$ halfwave rectifier
it is classified as two ways
1. symmetrical  2. Asymmetrical
→ Symmetrical ~~means~~ cathode of two thyristors are at the same potential
∴ A single gate pulse can be used for both the SCR's
→ Asymmetrical configuration means cathode of two SCRs are at the different potential.
→ During the +ve half cycle of the AC supply $T_1 D_1$ are forward bias. At $T_1$ is in forward blocking mode when SCR $T_1$ is triggering at firing angle $\alpha = 90°$. the current flow through the path $V_s - T_1 - R - D_1 - V_s$
At the $V_s (V_{ab})$ appearing across the load
→ $wt = \pi$, natural commutation occurs $T_1 D_1$ are turned off
→ During -ve half cycle of the AC supply $T_2 D_2$ are forward bias, At $T_2$ is triggered at angle $\pi + \alpha$ current flows through the path, $V_s - T_2 - R - D_2 - V_s$
At the $V_s (V_{ba})$ appearing across the load
→ Load current $I_0$, Nope, wave using Resistive load $I_0$ & $V_0$ are in phase, So, shape of $I_0$ is same as that of $V_0$ with reduced magnitude because
$$I_0 = \frac{V_0}{R}.$$

V across $T_1 D_1$

→ from 0 to $\alpha$ $T_1 D_1$ are in off state, ∴ $V_{ab}$ appears across the switch $T_1 D_1$

from $\alpha$ to $\pi$ $T_1 D_1$ are S.C, i.e., on no voltage appear across the switches

from $\pi$ to $2\pi+\alpha$ $T_1 D_1$ are off state. i.e., supply voltage appear across the switches $T_1 D_1$

Similiarly V across $T_2 D_2$

from 0 to $\pi+\alpha$ $T_2 D_2$ are in off state, ∴ $V_{ba}$ appearing across the switches

$$\Rightarrow V_{avg} = \int_{\alpha}^{\pi} \frac{V_m \sin\omega t \cdot d(\omega t)}{\pi}$$

$$= \frac{V_m}{\pi} \int_{\alpha}^{\pi} \sin\omega t \cdot d(\omega t) = \frac{V_m}{\pi} \left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{-V_m}{\pi} \left[+\cos\pi - \cos\alpha\right] = -\frac{V_m}{\pi}\left[-1-\cos\alpha\right]$$

$$\therefore V_{avg} = \frac{V_m}{\pi} \cos\alpha$$

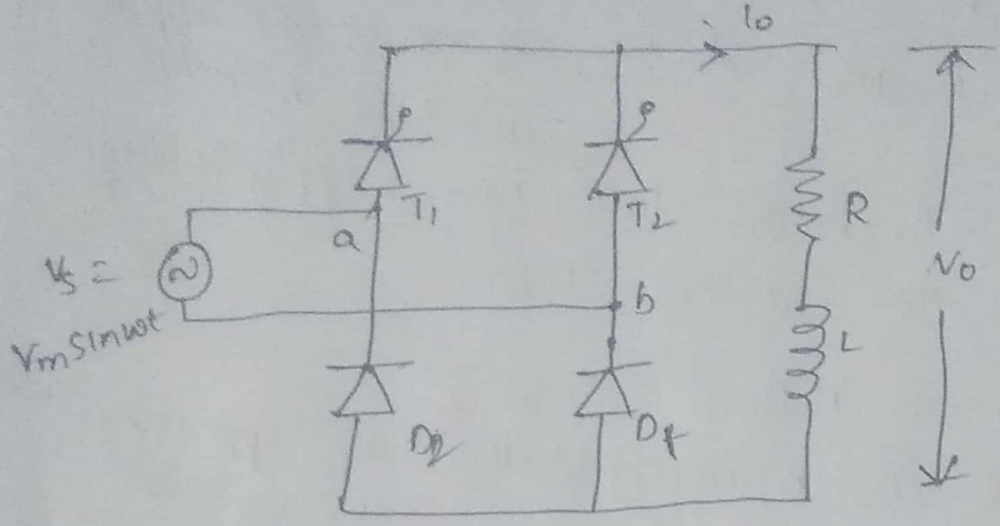
$$V_{Rms} = \sqrt{\int_{\alpha}^{\pi} \frac{V_m^2 \sin^2\omega t\, d(\omega t)}{\pi}} = \sqrt{\frac{V_m^2}{\pi} \int_{\alpha}^{\pi} \left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\int_{\alpha}^{\pi} (1-\cos 2\omega t)\, d(\omega t)} = \frac{V_m}{\sqrt{2\pi}} \sqrt{(\omega t)_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi}}$$

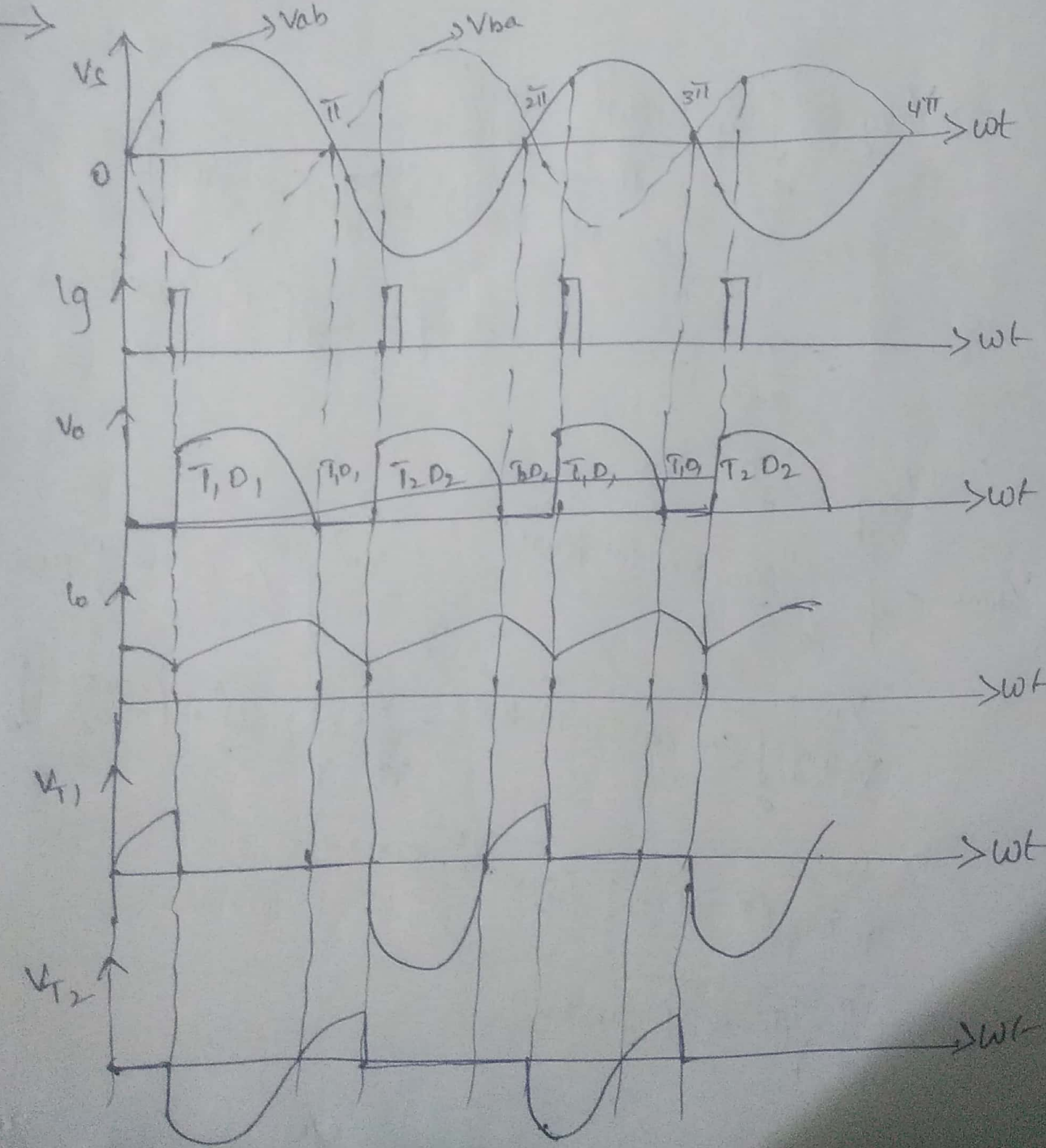
$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{(\pi-\alpha) - \frac{1}{2}(\cancel{\sin 2\pi}^{0} - \sin 2\alpha)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{(\pi-\alpha) + \frac{\sin 2\alpha}{2}}$$

## ⇒ 1-ϕ Semi converter (Symmetrical) with RL-Load :-




→

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→ Ckts consists of two SCRs $T_1, T_2$ Diodes $D_1$ $D_2$ Voltage Source $V_s$ along with RL load

→ During +ve half i/p voltage $V_s$ the SCR $T_1 D_1$ are forward bias but thyristor $T_1$ does not conduct until a gate is applied to $T_1$

At $wt = \alpha = 30$, A gate pulse is given $T_1$, turns on $T_1$ and begins to conduct $V_s$ appears across the load. due to the presence of 'L' the current increases gradually

→ During -ve half cycle $T_2 D_2$ are forward bias due to the presence of 'L' the '$T_1$' is forcefully in the ON state

from $\pi$ to $\pi + \alpha$ $T_2 D_2$ conducts, so it acts as S.C $[V_o = 0]$

~~Io shifted to~~ [$I_o$ shifted to $T_2 D_2$] from $\pi$ to $\pi + \alpha$

So, current decreases slowly.

→ when a gate pulse is given $T_2$ at $wt = \pi + \alpha$ $T_2$ get turned on and begins conduct

when $T_2 D_2$ on the voltage $V_{ba}$ appears across the load

$T_2 D_2$ on current increases gradually.

→ During +ve half cycle $T_1 D_1$ are forward bias due to presence of 'L' the $T_2$ is forcefully in the ON state

from $2\pi$ to $2\pi + \alpha$ $T_2 D_1$ conducts so it acts as S.C $[V_o = 0]$

[$I_o$ shift to $T_1 D_1$] from $2\pi$ to $2\pi + \alpha$

So, current decreases slowly.

from 0 to $\alpha$, $T_1$ is in off state, Vab appearing across the switch $T_1$, $\pi+\alpha$ to $2\pi$, $T_1$ is in off state Vab appearing across the ~~Vab appearing across the~~ ~~acts as open switch~~

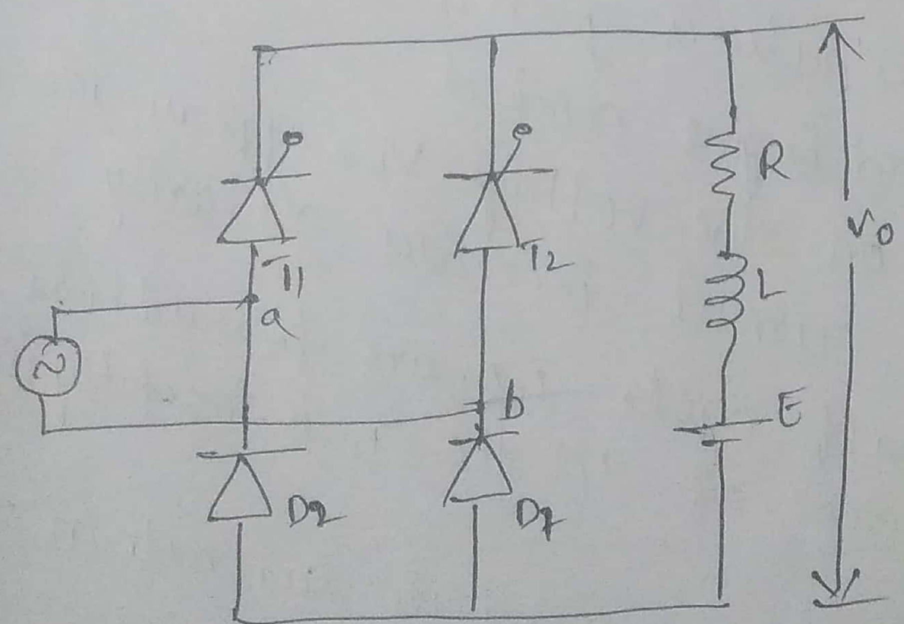
Ily $V_{T_2}$ 0 to $\alpha$ $T_2$ is in off state, $\alpha$ to $\pi+\alpha$ $T_2$ is in ON state, acts as open switch

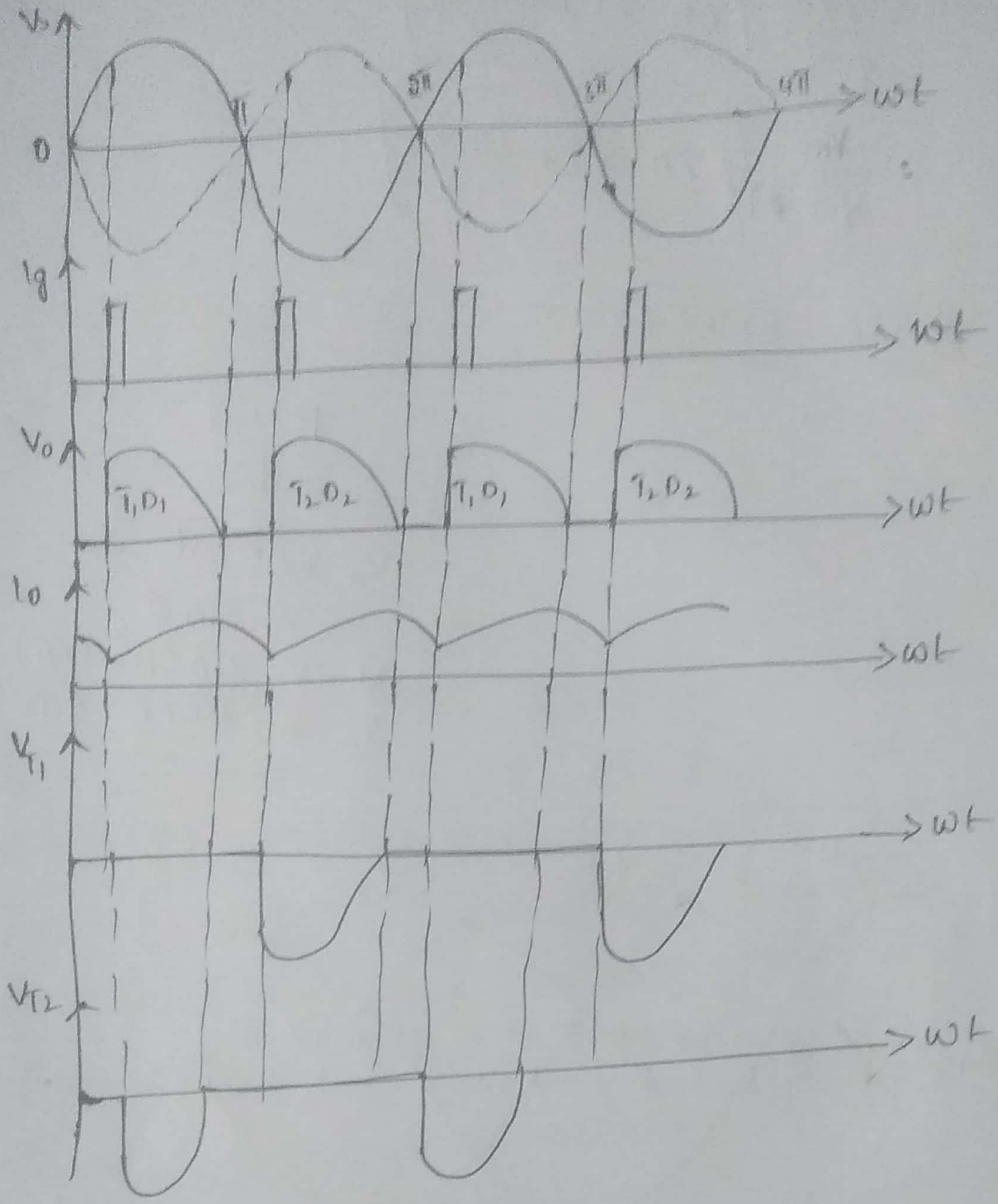
$$V_{avg} = \int_{\alpha}^{\pi} \frac{V_m \sin wt \cdot d(wt)}{\pi}$$

$$V_{avg} = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

$$V_{Rms} = \frac{V_m}{\sqrt{2\pi}}\sqrt{(\pi-\alpha) + \frac{\sin 2\alpha}{\alpha}}$$

$\Rightarrow$ 1-$\phi$ semi converter (symmetrical) with RLE load:






Average o/p voltage

$$V_{avg} = \int_{\alpha}^{\pi} \frac{V_m \sin \omega t \cdot d(\omega t)}{\pi}$$

$$= \frac{V_m}{\pi}\left[1 + \cos\alpha\right]$$

$$V_{Rms} = \sqrt{\int_{\alpha}^{\pi} \frac{V_m^2 \sin^2 \omega t \cdot d(\omega t)}{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}\right]}$$

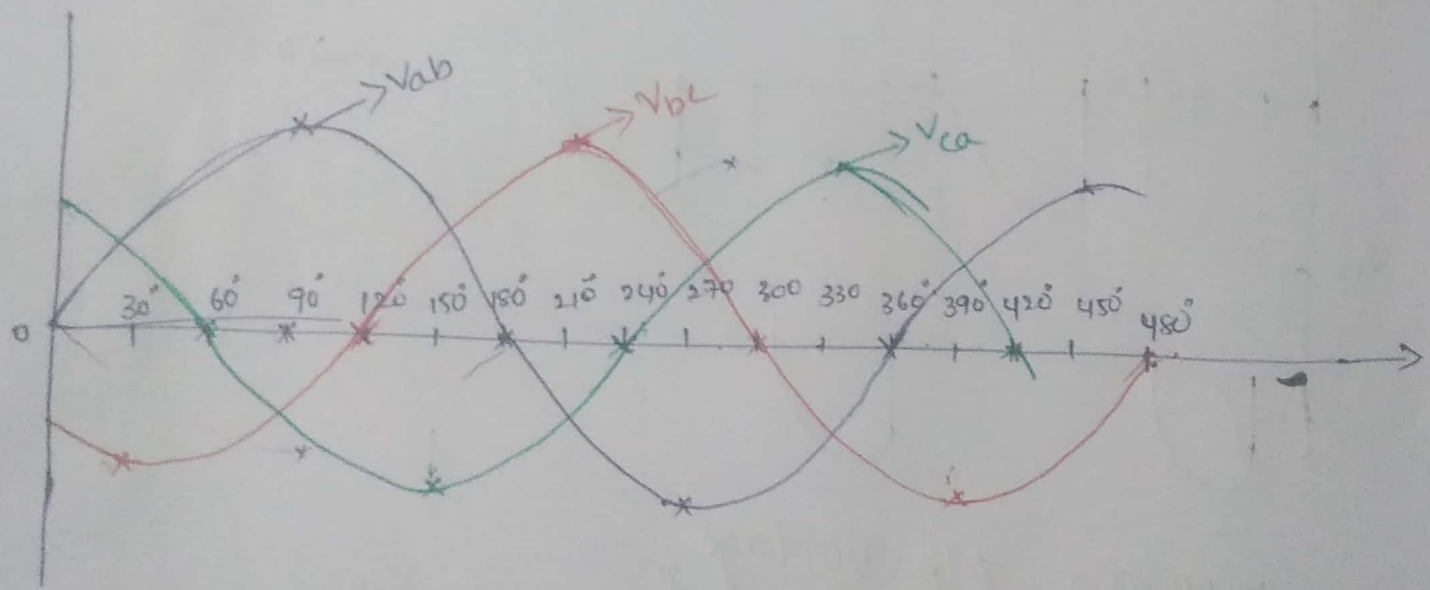
$$= \sqrt{\frac{V_m^2}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

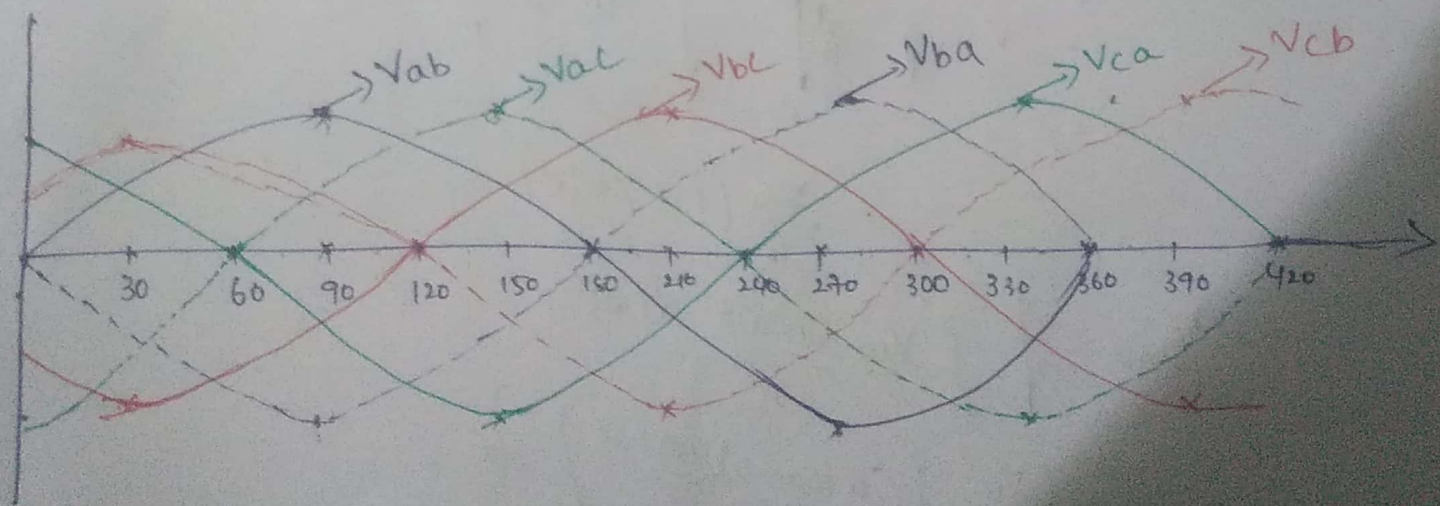
$$= \frac{V_m}{\sqrt{\pi \cdot 2}}\sqrt{\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

## Three phase wave-form:-

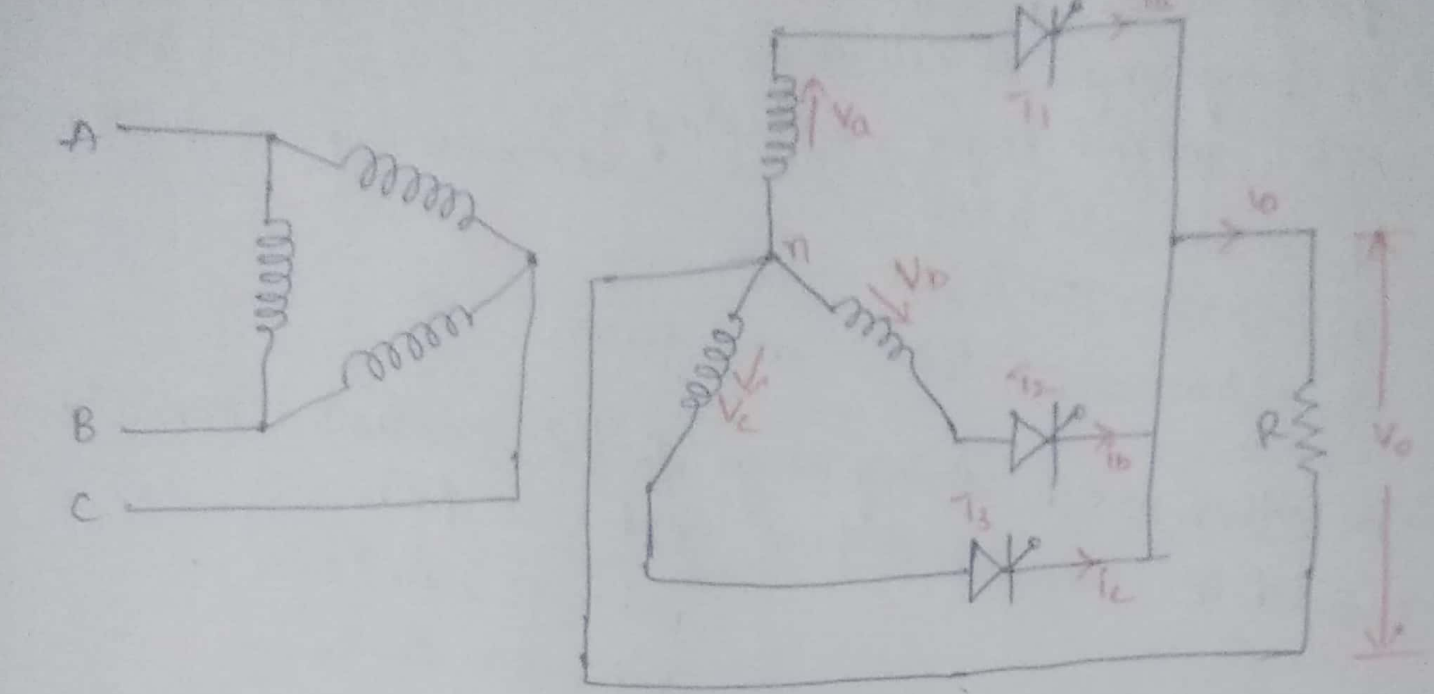
R Y B
a b c

$V_a = V_m \sin \omega t$
$= V_m \sin \theta$
$V_b = V_m \sin(\theta - 120^\circ)$
$V_c = V_m \sin(\theta - 240^\circ)$



# 3-$\phi$ half wave controlled Rectifier with R-load:-




→ A ckt consists of a delta - star transformer connected at 3 thyristors $T_1, T_2, T_3$, which are connected at a R load secondary winding star connected at the

→ In this ckt 3-$\phi$ power supply avaible to the converter through 3-$\phi$ transformer with $\Delta$-primary Y-secondary

→ $V_a, V_b, V_c$ is the phase voltage.
$i_o$, output (or) load current
$V_o$, output (or) load voltage.

→ when $V_a$ is +ve half cycle, $T_1$ becomes forward bias and conducts. During -ve half cycle of ~~+ve~~, $V_a$. $T_1$ turns off. ||ly $T_2, T_3$ conduct only during the +ve cycles of $V_b$ & $V_c$ respec.

→ consider 3-$\phi$ supply voltage

~~from 0 to 30° $v_c$ ...~~

$T_1 \rightarrow 30$ to $150$

$T_2 \rightarrow 150$ to $270$

$T_3 \rightarrow 270°$ to $390°$

→ If firing angle is zero degree. SCR $T_1$ would begin conducting from ~~wt = 30 to 150~~

$T_1 \rightarrow wt = 30°$ to $150°$

$T_2 \rightarrow wt = 150°$ to $270°$

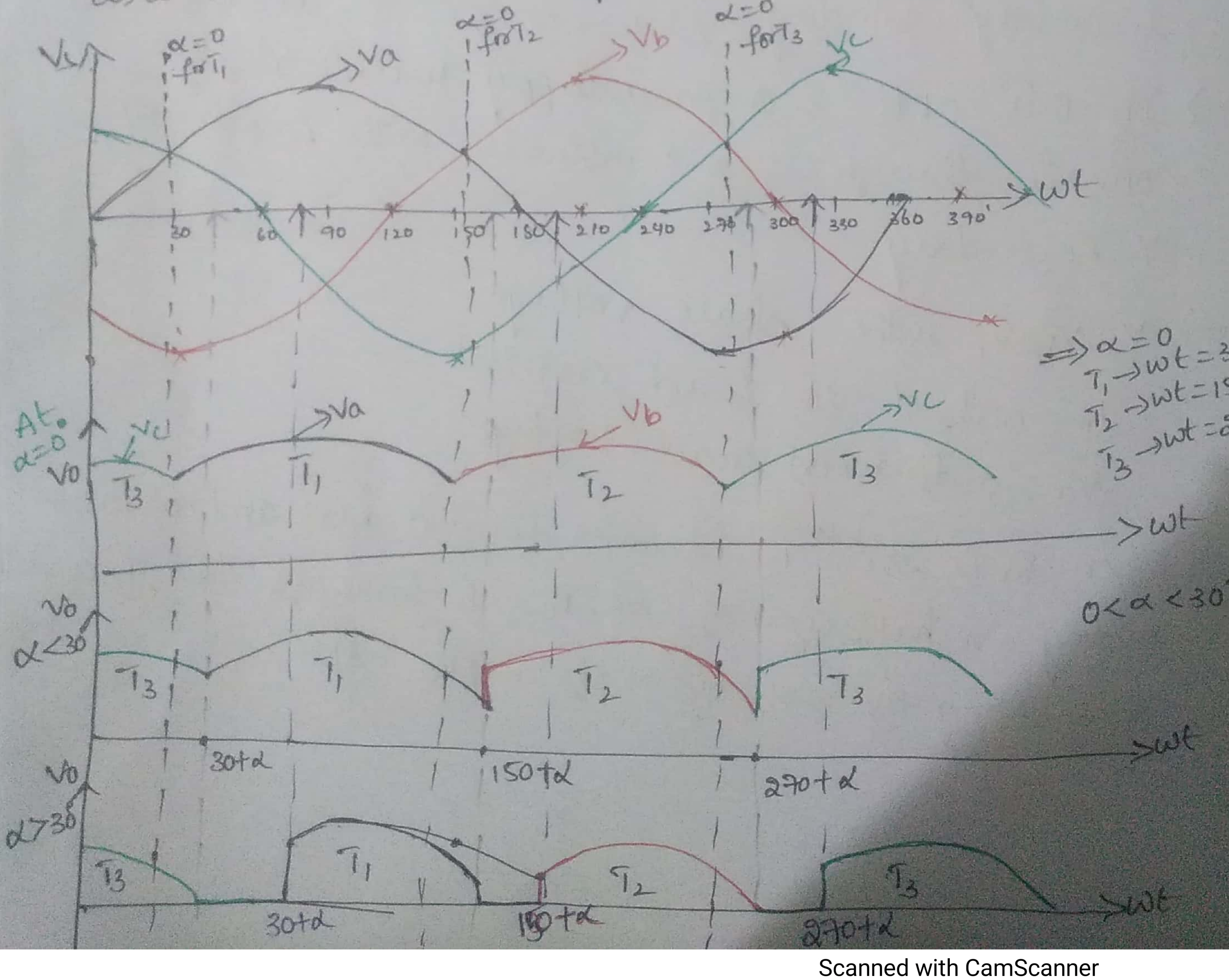
$T_3 \rightarrow wt = 270°$ to $390°$. and so on.

→ In other words, firing angle for this controlled converter would be measured from $wt = 30°$ for $T_1$

$wt = 150°$ for $T_2$

$wt = 270$ for $T_3$.

→ for zero degree firing angle delay, thyristor behaves as a diode and the voltage output waveform $V_0$




$\Rightarrow \alpha = 0$

$T_1 \rightarrow wt = 30°$

$T_2 \rightarrow wt = 150°$

$T_3 \rightarrow wt = 270°$

$0 < \alpha < 30°$

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Case i :- $\alpha < 30^\circ$.

Average value of o/p voltage,

$$V_0 = \int_{30+\alpha}^{150+\alpha} \frac{V_m \sin\omega t \cdot d(\omega t)}{120} = \frac{V_m}{120}\int_{30+\alpha}^{150+\alpha} \sin\omega t \, d(\omega t)$$

$$= \frac{V_m}{2\pi/3}\left[-\cos\omega t\right]_{30+\alpha}^{150+\alpha}$$

$$= \frac{-3V_m}{2\pi}\left[\cos(150+\alpha) - \cos(30+\alpha)\right]$$

$$= \frac{-3V_m}{2\pi}\left[(\cos 150 \cdot \cos\alpha - \sin 150 \cdot \sin\alpha) - \cos 30 \cdot \cos\alpha - \sin 30 \cdot \sin\alpha)\right]$$

$$= \frac{-3V_m}{2\pi}\left[\left(-\sqrt{3}/_2 \cos\alpha - \tfrac{1}{2}\sin\alpha\right) - \left(\tfrac{\sqrt{3}}{2}\cos\alpha - \tfrac{1}{2}\sin\alpha\right)\right]$$

$$= \frac{-3V_m}{2\pi}\left[-\frac{\sqrt{3}}{2}\cos\alpha - \cancel{\frac{1}{2}\sin\alpha} - \frac{\sqrt{3}}{2}\cos\alpha + \cancel{\frac{1}{2}\sin\alpha}\right]$$

$$= -\frac{3V_m}{2\pi}\left[-\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\cos\alpha\right]$$

$$V_0 = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$$

In this ckt, the phase voltage $V_a, V_b, V_c$ across the load

so, $V_0 = \frac{3\sqrt{3}V_{mp}}{2\pi}\cos\alpha$

for line voltage

$V_L = \sqrt{3}V_{ph}$

so, $V_0 = \frac{3V_{ml}\cos\alpha}{2\pi}$

Average load current, $I_0 = \frac{V_0}{R} = \frac{3V_{ml}}{2\pi R}\cos\alpha$

Rms value of output, or load voltage is

$$V_{Rms} = \sqrt{\frac{\int_{30+\alpha}^{150+\alpha} V_m^2 \sin^2\omega t \cdot d(\omega t)}{2\pi/3}}$$

$$= \sqrt{\frac{3V_m^2}{2\pi} \int_{30+\alpha}^{150+\alpha} \left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \int_{30+\alpha}^{150+\alpha} (1-\cos 2\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left\{ (\omega t)_{30+\alpha}^{150+\alpha} - \left[\frac{\sin 2\omega t}{2}\right]_{30+\alpha}^{150+\alpha} \right.}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left\{ \frac{2\pi}{3} - \frac{1}{2}\left[\left(\sin(300^\circ + 2\alpha)\right) - \sin(60+2\alpha)\right] \right\}}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left[ \frac{2\pi}{3} - \frac{1}{2} \left\{ (\sin 300 \cdot \cos 2\alpha + \cos 300 \cdot \sin 2\alpha) - (\sin 60 \cdot \cos 2\alpha + \cos 60 \cdot \sin 2\alpha) \right\} \right]}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left[ \frac{2\pi}{3} - \frac{1}{2} \left\{ \left(-\frac{\sqrt{3}}{2}\cos 2\alpha + \cancel{\frac{1}{2}\sin 2\alpha}\right) - \left(\frac{\sqrt{3}}{2}\cos 2\alpha + \cancel{\frac{1}{2}\sin 2\alpha}\right) \right\} \right]}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left[ \frac{2\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha \right.}$$

$$V_{Rms} = \sqrt{\frac{3V_m^2}{4\pi} \left[ \frac{2\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha \right]}$$

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Case ii :- $\alpha > 30^\circ$

Avg. value of load voltage, $V_0 = \int_{30+\alpha}^{180} \frac{V_m \sin \omega t \cdot d(\omega t)}{120}$

$$= \frac{V_m}{2\pi/3} \int_{30+\alpha}^{180} \sin \omega t \cdot d(\omega t) = \frac{3V_m}{2\pi} \left[-\cos \omega t\right]_{30+\alpha}^{180^\circ}$$

$$= \frac{-3V_m}{2\pi} \left[\cos 180^\circ - \cos(30+\alpha)\right] = \frac{-3V_m}{2\pi} \left[-1 - \cos(30+\alpha)\right]$$

$$V_0 = \frac{3V_m}{2\pi} \left[1 + \cos(30+\alpha)\right]$$

Rms value of o/p voltage, $V_{Rms} = \sqrt{\int_{30+\alpha}^{180} \frac{V_m^2 \sin^2 \omega t \, d(\omega t)}{120}}$

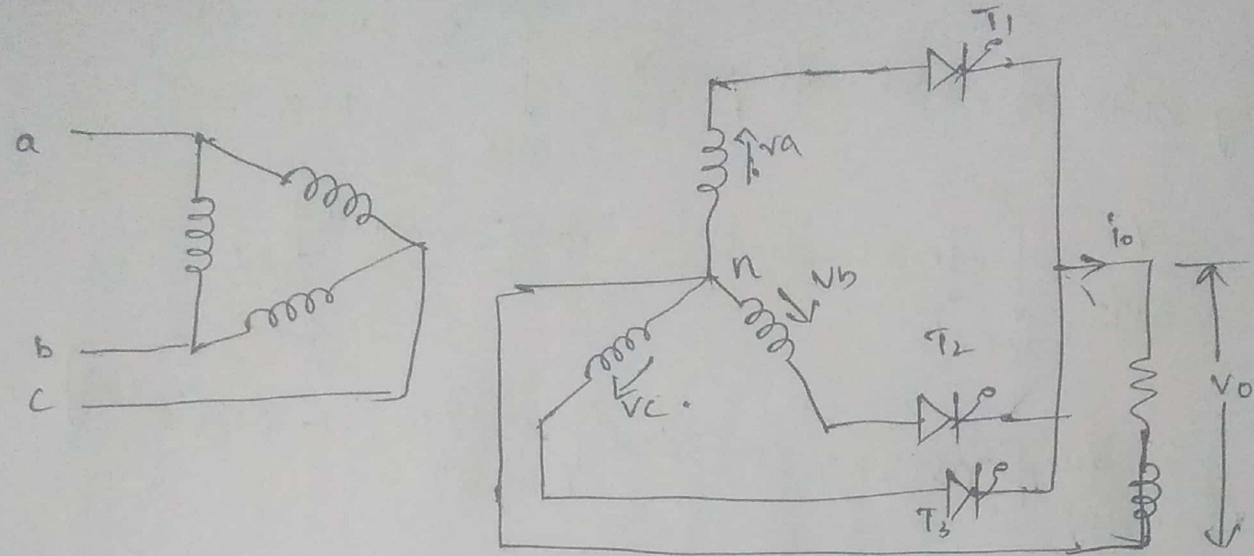
$$= \sqrt{\frac{3V_m^2}{2\pi} \int_{30+\alpha}^{180} \left(\frac{1 - \cos 2\omega t}{2}\right) d(\omega t)} = \sqrt{\frac{3V_m^2}{4\pi} \left[ (\omega t)_{30+\alpha}^{180} - \left(\frac{\sin(2\omega t)}{2}\right)_{30+\alpha}^{180} \right]}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left[ (180 - 30 - \alpha) - \frac{1}{2}\left(\sin(2\times180) - \sin 2(30+\alpha)\right) \right]}$$

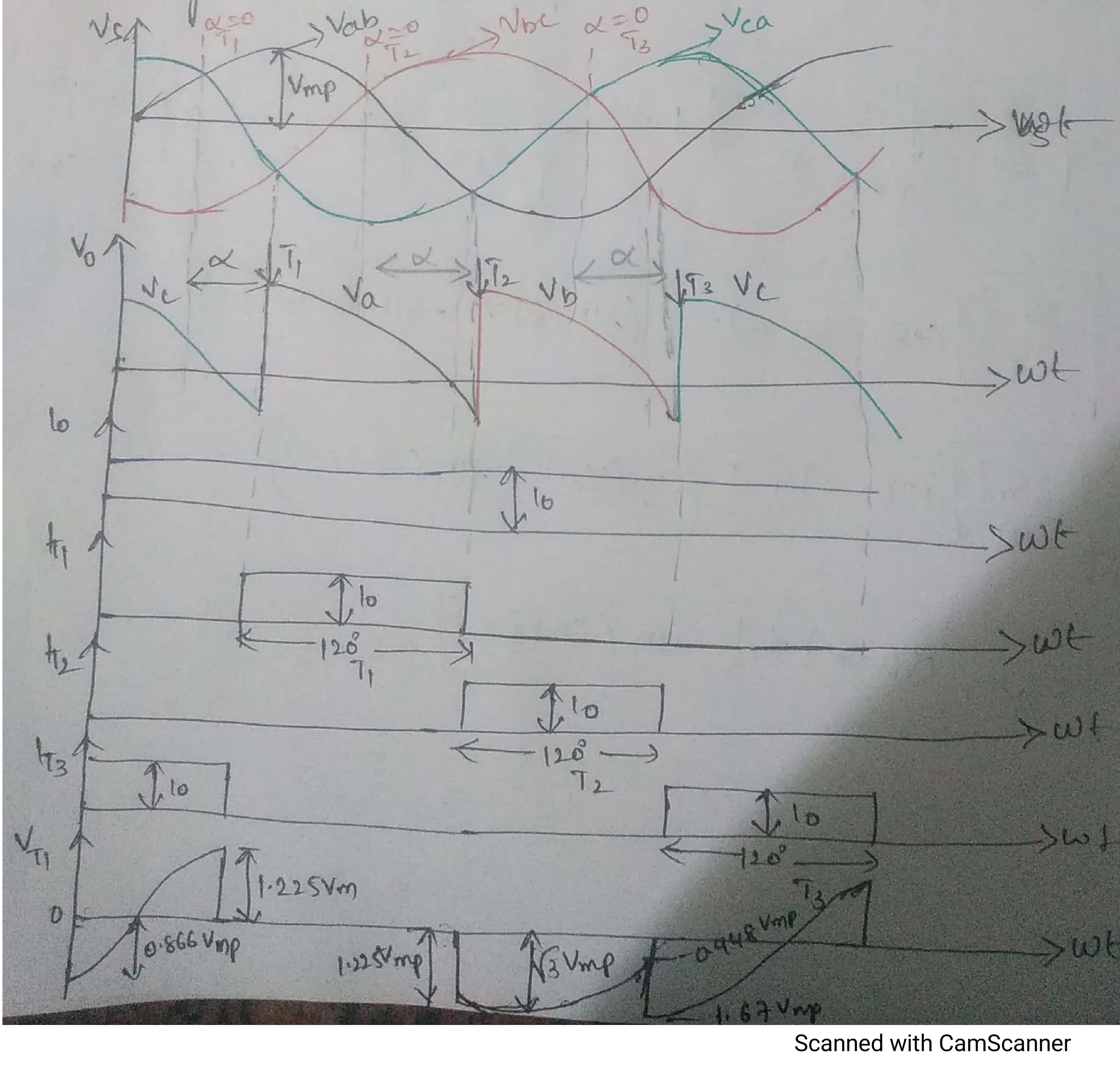
$$= \sqrt{\frac{3V_m^2}{4\pi} \left[ (150-\alpha) - \frac{1}{2}\left[\sin(360)^{\to 0} - \sin(60+2\alpha)\right] \right]}$$

$$= \sqrt{\frac{3V_m^2}{4\pi} \left[ (150-\alpha) + \frac{1}{2}\sin(60+2\alpha) \right]}$$

# 3-ϕ halfwave controlled Rectifiers R-L load!




→ Here we are taking inductance is very high it will not change in the current



→ The load inductance L is large so, that load current is continous and constant at $I_0$.

→ for firing angle range of $30° < \alpha < 90°$ &

$90° < \alpha < 180°$.

Case i:- $30 < \alpha < 90°$

let the firing angle be, say $45°$.

Note that $T_1$ conducts from $30 + \alpha$ to $150 + \alpha$

$T_2 \longrightarrow$ $150 + \alpha$ to $270 + \alpha$

$T_3 \longrightarrow$ $270 + \alpha$ to $390° + \alpha$ & so on

At $\omega t = \pi$, phase voltage $V_0$ is zero,

but $i_{T_1}$ is not zero because of RL load.

$\therefore T_1$ would continue conducting beyond $\omega t = \pi$

As $V_0 = V_a$ goes negative beyond $\omega t = \pi$,

As $V_0 = V_a$ goes negative beyond $\omega t = 150° + \alpha$, load current

→ when $T_2$ is turned on at $\omega t = 150° + \alpha$, load current shifts from $T_1$ to $T_2$ & a voltage

$$V_a - V_b = [V_m \sin(150 + \alpha) - V_m \sin(30 + \alpha)]$$

→ SCR $T_2$ conducts from $(150° + \alpha)$ to $(270 + \alpha)$ & so on.

→ The wave-form $V_{T_1}$ for voltage across $T_1$. on the firing angle $45°$, can be

when $T_1$ is on, $V_{T_1} = V_a - V_a = 0$ from $\omega t = 75°$ to $195°$

$T_2$ is on, $V_{T_1} = V_a - V_b$ from $\omega t = 195°$ to $315°$

$T_3$ is on $V_{T_1} = V_a - V_c$ from $\omega t = 315°$ to $435°$ & so on

→ when $T_2$ is turned on at $wt = 195°$,

$V_{T_1} = V_a - V_b = -V_{mp}\sin 15 - V_{mp}\sin 75°$.

$= -1.225 V_{mp}$

At $wt = 210°$. $V_{T_1} = -1.5 V_{mp}$

$wt = 240°$ $V_{T_1} = \sqrt{3} V_{mp}$

$wt = 270°$ $V_{T_1} = -1.5 V_{mp}$

$wt = 300°$ $V_{T_1} = -V_{mp}\sin 60 - 0 = -0.866 V_{mp}$

$wt = 315°$ $V_{T_1} = -V_{mp}\sin 45° + V_{mp}\sin 15°$

$= -0.448 V_{mp}$

Also At $wt = 315°$, $T_2$ gets turns off, whereas $T_3$ is turned on

$V_{T_1} = V_a - V_c = -V_{mp}\sin 45° + V_{mp}\sin 75°$

$= -1.673 V_{mp}$.

→ At $wt = 315°$, $V_{T_1}$ at once changes from $-0.448 V_{mp}$ to $-1.673 V_{mp}$

At $wt = 330°$, $V_{T_1} = -V_{mp}\sin 30 - V_{mp} = -1.5 V_{mp}$

$wt = 360°$, $V_{T_1} = 0 - 0.866 V_{mp} = -0.866 V_{mp}$

$wt = 390°$, $V_{T_1} = 0.5 V_{mp} - 0.5 V_{mp} = 0$

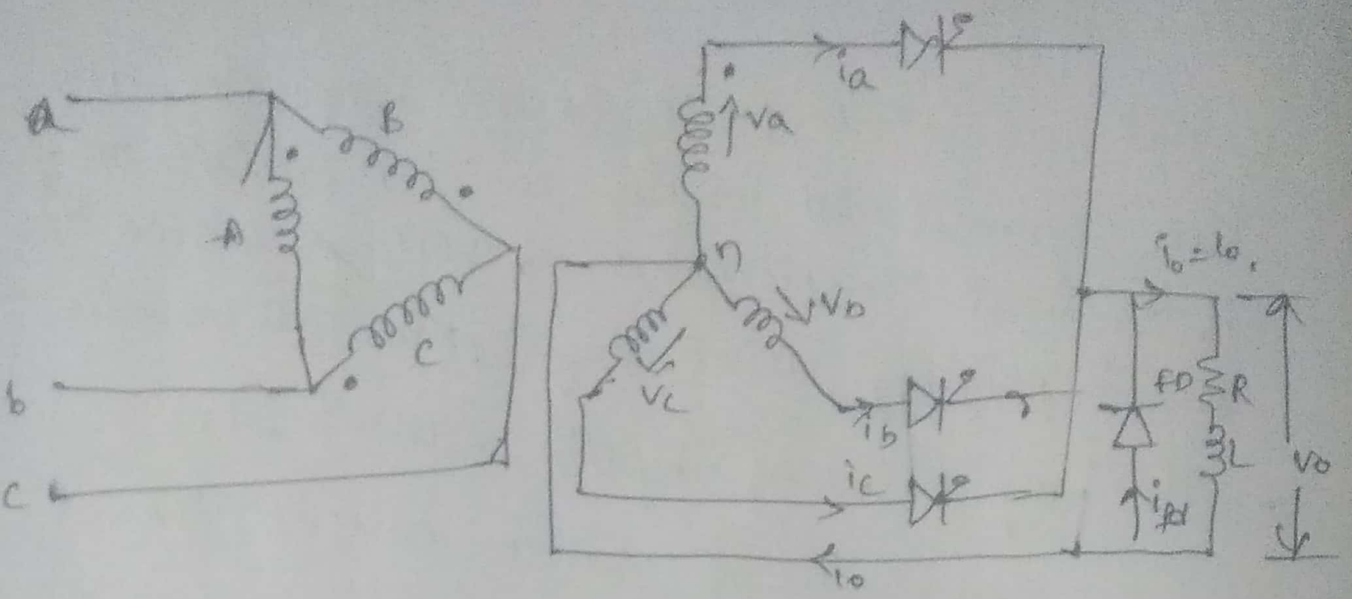
$wt = 420°$, $V_{T_1} = 0.866 V_{mp} - 0 = 0.866 V_{mp}$

$wt = 435°$, $V_{T_1} = V_{mp}\sin 75° + V_{mp}\sin 15° = 1.225 V_{mp}$

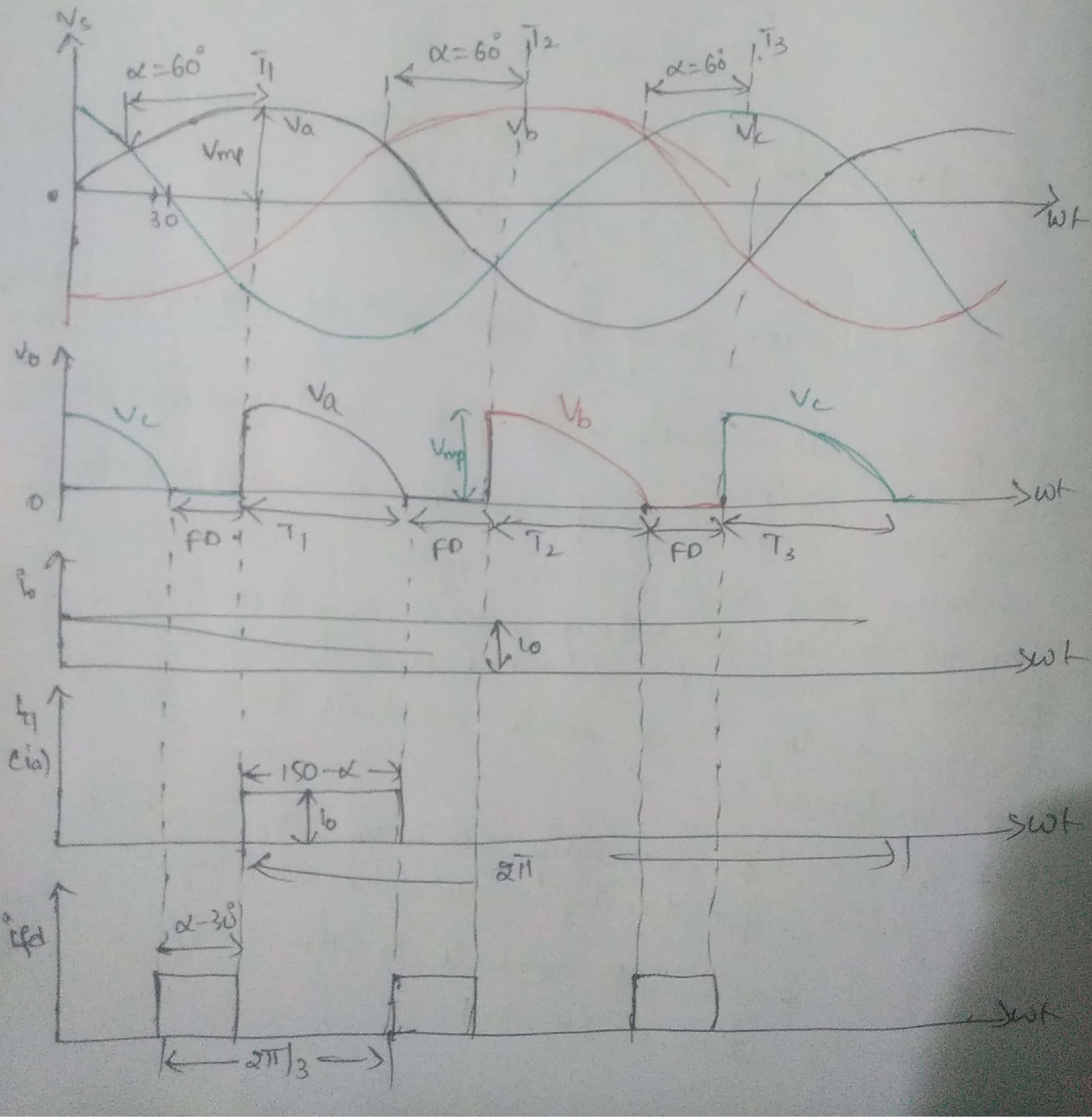
& also $V_{T_1} = V_a - V_a = 0$ & so on.

Avg load o/p voltage, $V_0 = \frac{3\sqrt{3} V_{mp}}{2\pi} \cos\alpha$ (or) $\frac{3 V_{ml} \cos\alpha}{2\pi}$

$V_{Rms} = \sqrt{\frac{3 V_m^2}{4\pi}\left[\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right]}$



A
B
C
b
c
$i_a$
$v_a$
n
$v_b$
$v_c$
$i_b$
$i_c$
$i_o$
$i_o = I_o$
FD
R
L
$i_{fd}$
$V_o$

$V_s$
$\alpha = 60°$
$T_1$
$T_2$
$T_3$
$V_a$
$V_b$
$V_c$
$V_{mp}$
30
$\omega t$
$V_o$
0
FD
$T_1$
$T_2$
$T_3$
$i_o$
$I_o$
$i_{T1}$ ($i_a$)
$150 - \alpha$
$I_o$
$2\pi$
$i_{fd}$
$\alpha - 30°$
$2\pi/3$

Case ii $\alpha > 90°$

$V_0 = -ve \rightarrow \therefore$ 3-$\phi$ 3pulse converter operates as a line commutated inverter which is possible only if the load circuit has a dc voltage source of reverse polarity, as in a single-phase full converter

→ The avg value of thyristor current, $I_{TA}$ = avg value of source current, $I_{SA}$

$$= (I_0 \times 120)/360 = I_0/3$$

→ Rms value of thyristor (or) source current

$$I_{TR} = I_{SR} = \sqrt{\left[\frac{I_0^2 \times 120}{360}\right]} = \frac{I_0}{\sqrt{3}}.$$

⟹ 3-$\phi$ full wave bridge rectifier with RL Load & freewheeling diode:

→ A 3-$\phi$ 3pulse converter feeding RL load & with freewheeling diode across RL load

→ for firing angle $\alpha < 30°$, freewheeling diode does not come into play

→ So, here firing angle is taken, say 60°.

→ At $\omega t = \pi$, as phase voltage $V_0$ tends to go negative, freewheeling diode gets forward biased through $T_1$.

→ $\therefore$ freewheeling diode starts conducts from $\omega t = \pi$ till $T_2$ is turned on.

→ At $\omega t = 150 + \alpha$. Similiarly; when $V_b$ & $V_c$ tends to go negative, freewheeling diode comes into play.

→ Note that each SCR conducts for $(150 - \alpha)$ & freewheeling diode for $(\alpha - 30°)$

Avg. value of o/p voltage,

$$V_0 = \frac{3}{2\pi}\int_{\alpha+\pi/6}^{\pi} V_{mp}\sin\omega t \cdot d(\omega t)$$

$$V_0 = \frac{3V_{mp}}{2\pi}\left[1+\cos(\alpha+\pi/6)\right]$$

$$V_{Rms} = \sqrt{\frac{V_{ml}}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6}-\alpha\right)+\frac{1}{2}\sin(2\alpha+\pi/3)\right]}$$

Avg. thyristor current, $I_{TA} = \frac{I_0\left(\frac{5\pi}{6}-\alpha\right)}{2\pi} = \frac{I_0}{2\pi}\left[\frac{5\pi}{6}-\alpha\right]$

RMS thyristor current, $I_{Tr} = \sqrt{\left[\frac{I_0^2}{2\pi}\left\{\frac{5\pi}{6}-\alpha\right\}\right]}$

$$= I_0\sqrt{\left[\frac{1}{2\pi}\left\{\frac{5\pi}{6}-\alpha\right\}\right]}$$

Avg value of FD current, $I_{FD,A} = \frac{I_0(\alpha-30^\circ)}{2\pi/3} = \frac{3I_0(\alpha-30^\circ)}{2\pi}$

RMS value of FD current, $I_{FD,r} = \sqrt{\left[\frac{I_0^2(\alpha-30^\circ)}{2\pi/3}\right]}$

$$= I_0\sqrt{\frac{3}{2\pi}(\alpha-30^\circ)}$$

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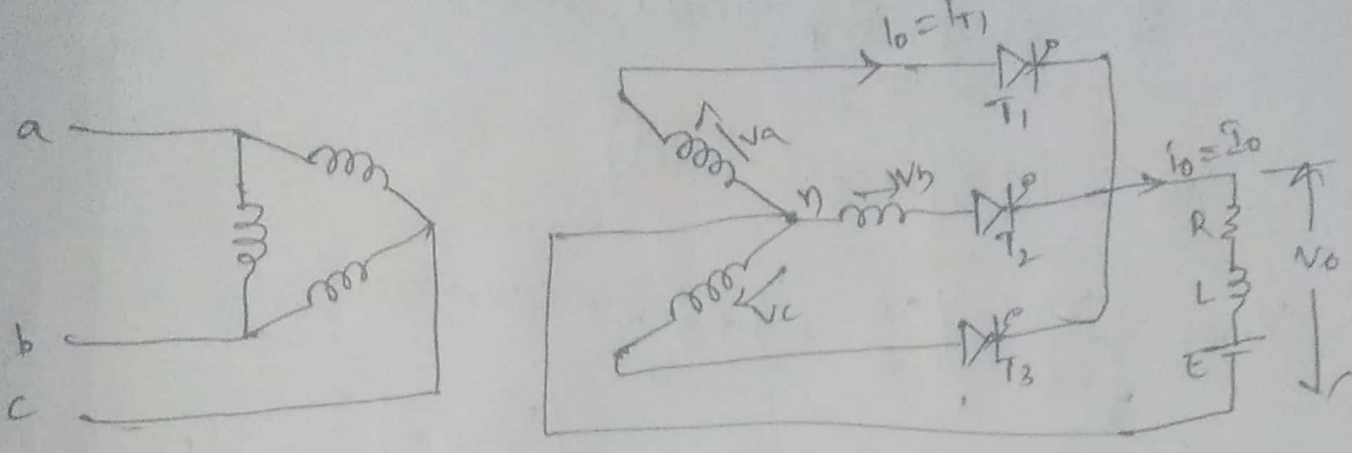
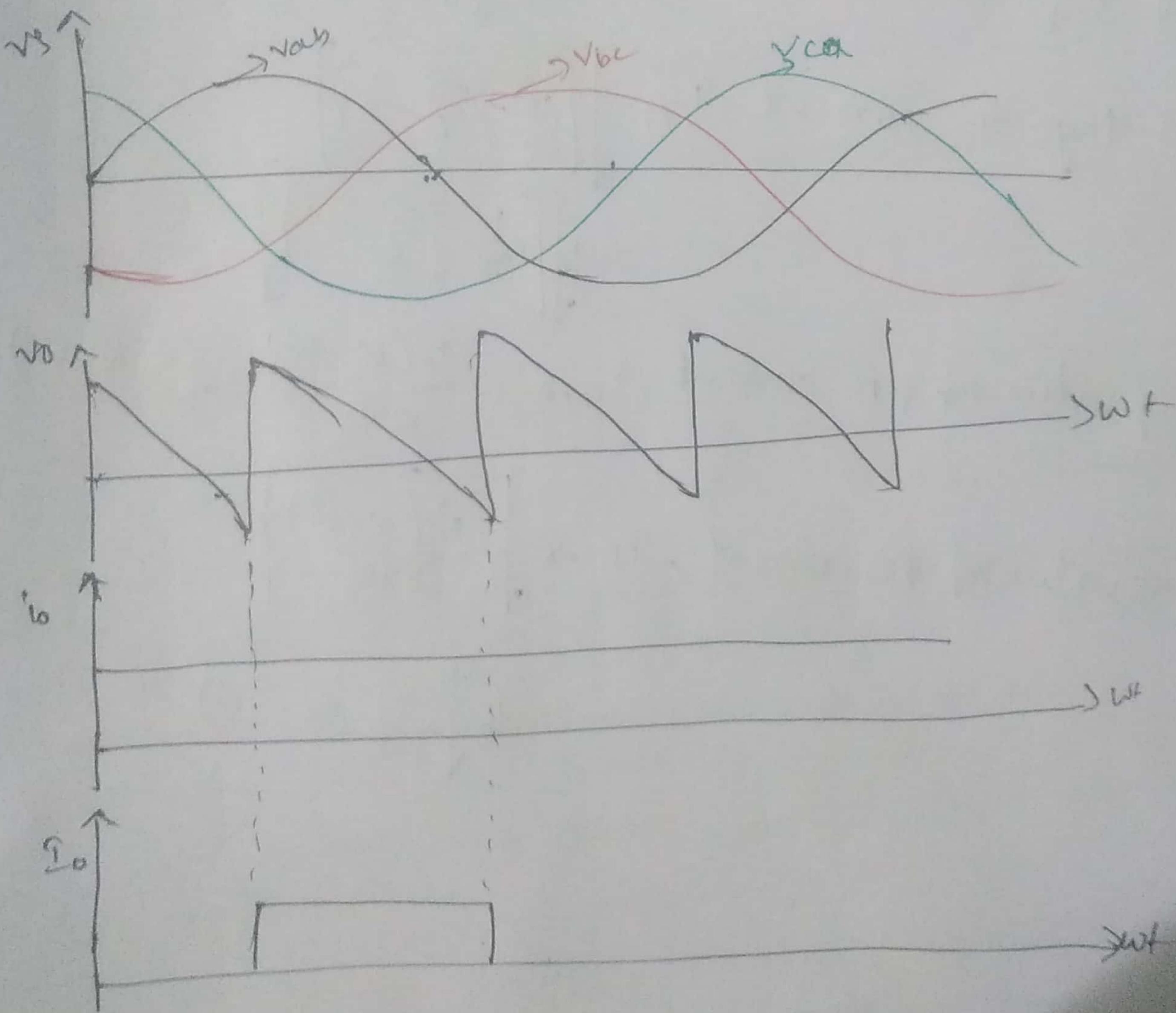
## ⟹ 3-ϕ half wave rectifier with RLE loads :-



Avg. o/p voltage ~~for~~, $V_{avg} = \frac{3V_m}{2\pi} \int_{\pi/6+\alpha}^{150+\alpha} \sin\omega t \cdot d(\omega t)$.

$$= \frac{3V_m}{2\pi} \left[ -\cos(\omega t) \right]_{30+\alpha}^{150+\alpha}$$

$$= \frac{3V_m}{2\pi} \left[ -\cos(150+\alpha) + \cos(30+\alpha) \right]$$

$$= \frac{3V_m}{2\pi} \left[ (-\cos(150)\cos\alpha + \sin 150 \sin\alpha) + (\cos 30 \cos\alpha - \sin 30 \sin\alpha) \right]$$

$$= \frac{3V_m}{2\pi} \left[ 2\cos(30^\circ)\cos\alpha \right]$$

$$= \frac{3V_m}{2\pi} \left[ 2 \times \sqrt{3}/2 \cos\alpha \right]$$

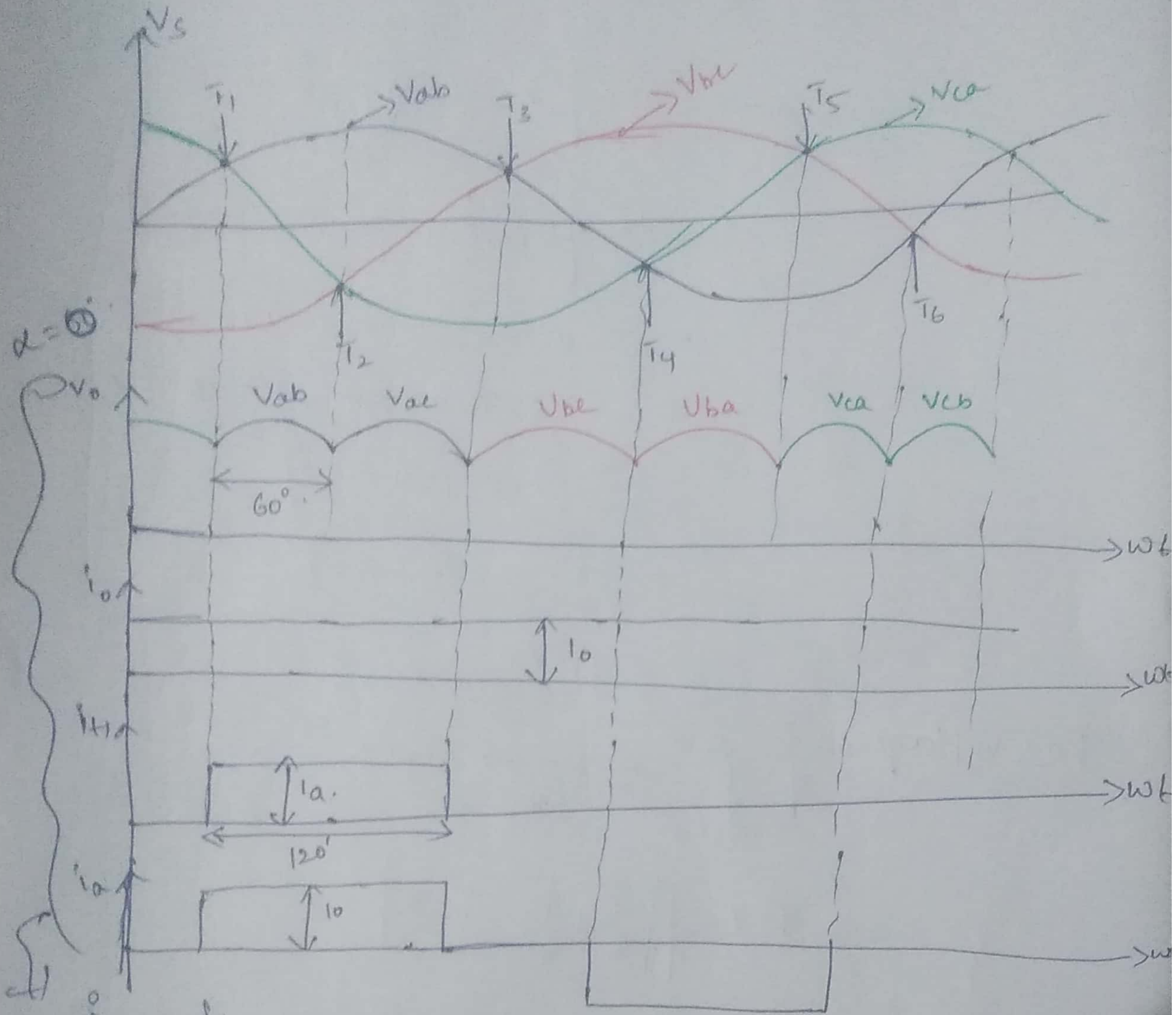
$$= \frac{3\sqrt{3}V_m}{2\pi} \cos\alpha$$

Rms voltage $= \sqrt{\int_{\pi/6+\alpha}^{150+\alpha} \frac{V_m^2 \sin^2\omega t \, d(\omega t)}{2\pi/3}}$

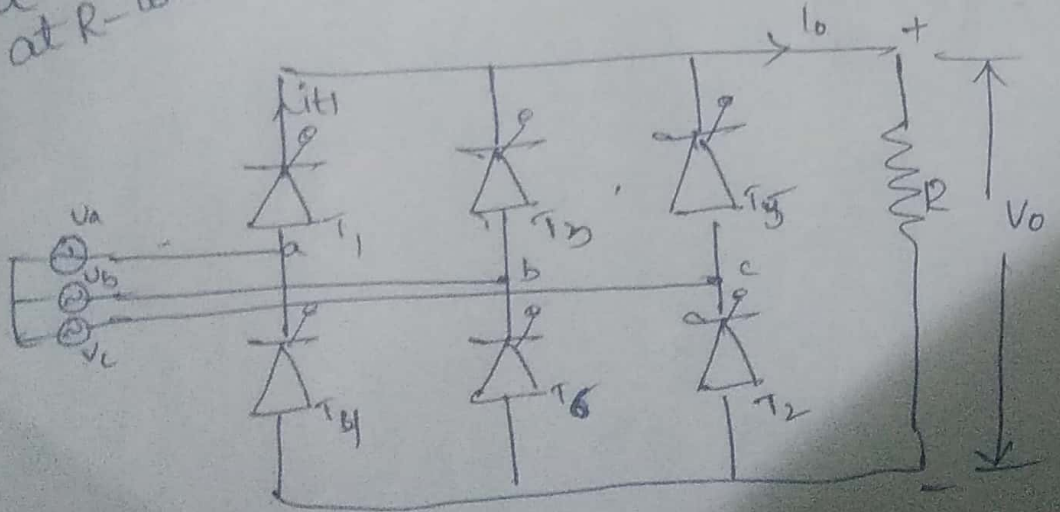
$$= \sqrt{3}V_m \left[ \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha} \right]$$

$$= \sqrt{3}V_m \{$$

# 3-ϕ full wave controlled Rectifier with R-load:




$\alpha = 0°$ at R-load.



→ $T_1$ is triggered at $\omega t = (30+\alpha)$, $T_6$ is already conducting when $T_1$ is turned ON.

→ During the interval $(30+\alpha)$ to $(90+\alpha)$, $T_1$ & $T_6$ conduct together & the o/p load voltage is equal to

$$V_0 = V_{ab} = (V_{an} - V_{bn})$$

→ $T_2$ is triggered at $\omega t = (90+\alpha)$, $T_6$ turns off naturally as it is reverse biased as soon as $T_2$ triggered

→ During the interval $(90+\alpha)$ to $(150+\alpha)$, $T_1$ & $T_2$ conduct together & the o/p load voltage

$$V_0 = V_{ac} = (V_{an} - V_{cn})$$

→ $T_3$ is triggered at $\omega t = (150+\alpha)$, $T_1$ turns off naturally as it is reverse biased as soon as $T_3$ is triggered

→ During the interval $(150+\alpha)$ to $(210+\alpha)$, $T_2$ & $T_3$ conduct together & the o/p load voltage

$$V_0 = V_{bc} = (V_{bn} - V_{cn})$$

→ $T_4$ is triggered at $\omega t = (210+\alpha)$, $T_2$ turns off naturally as it is reverse biased as soon as $T_4$ is triggered

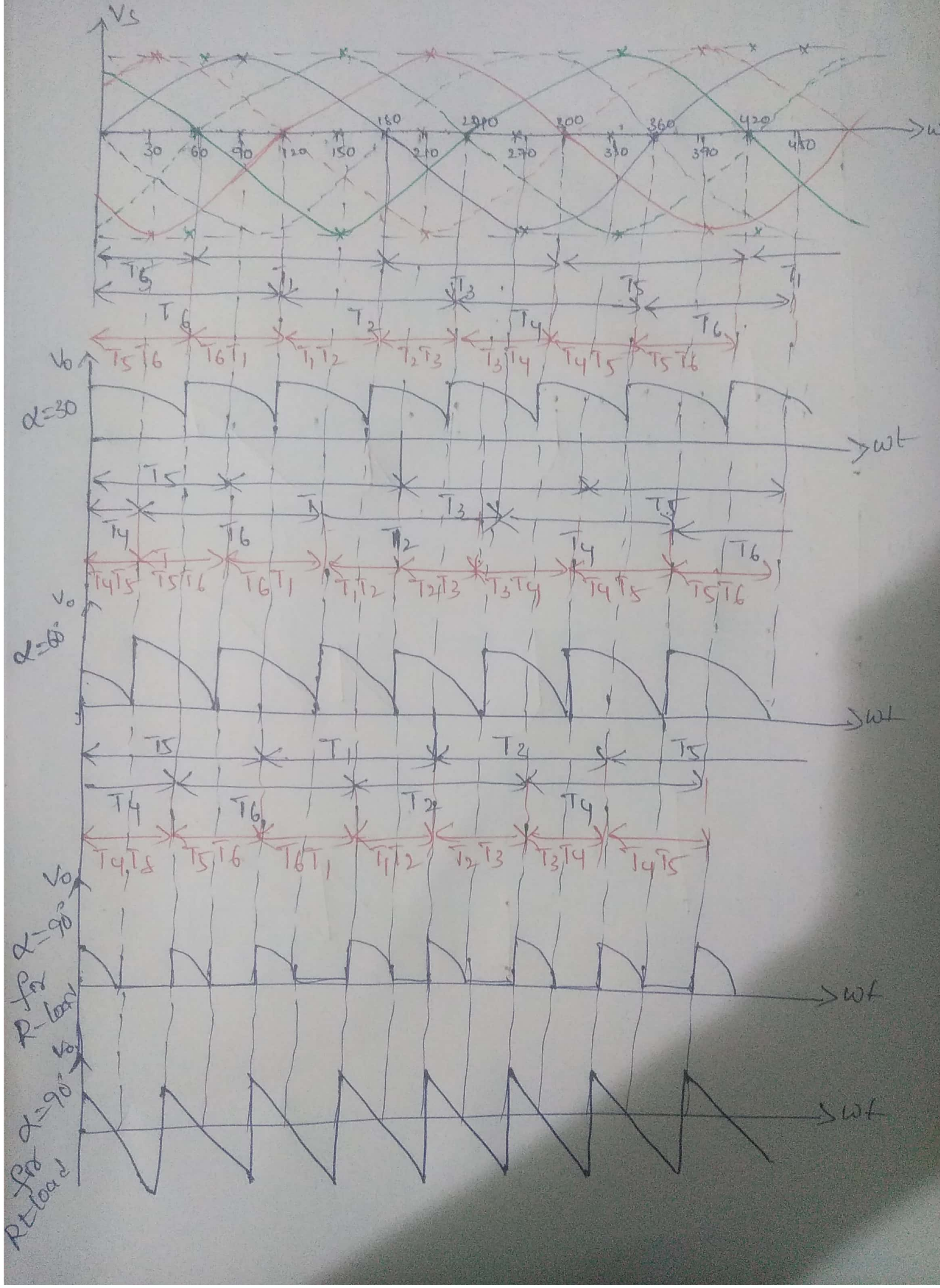
→ During the interval $(210+\alpha)$ to $(270+\alpha)$, $T_3$ & $T_4$ conduct together & the o/p load voltage

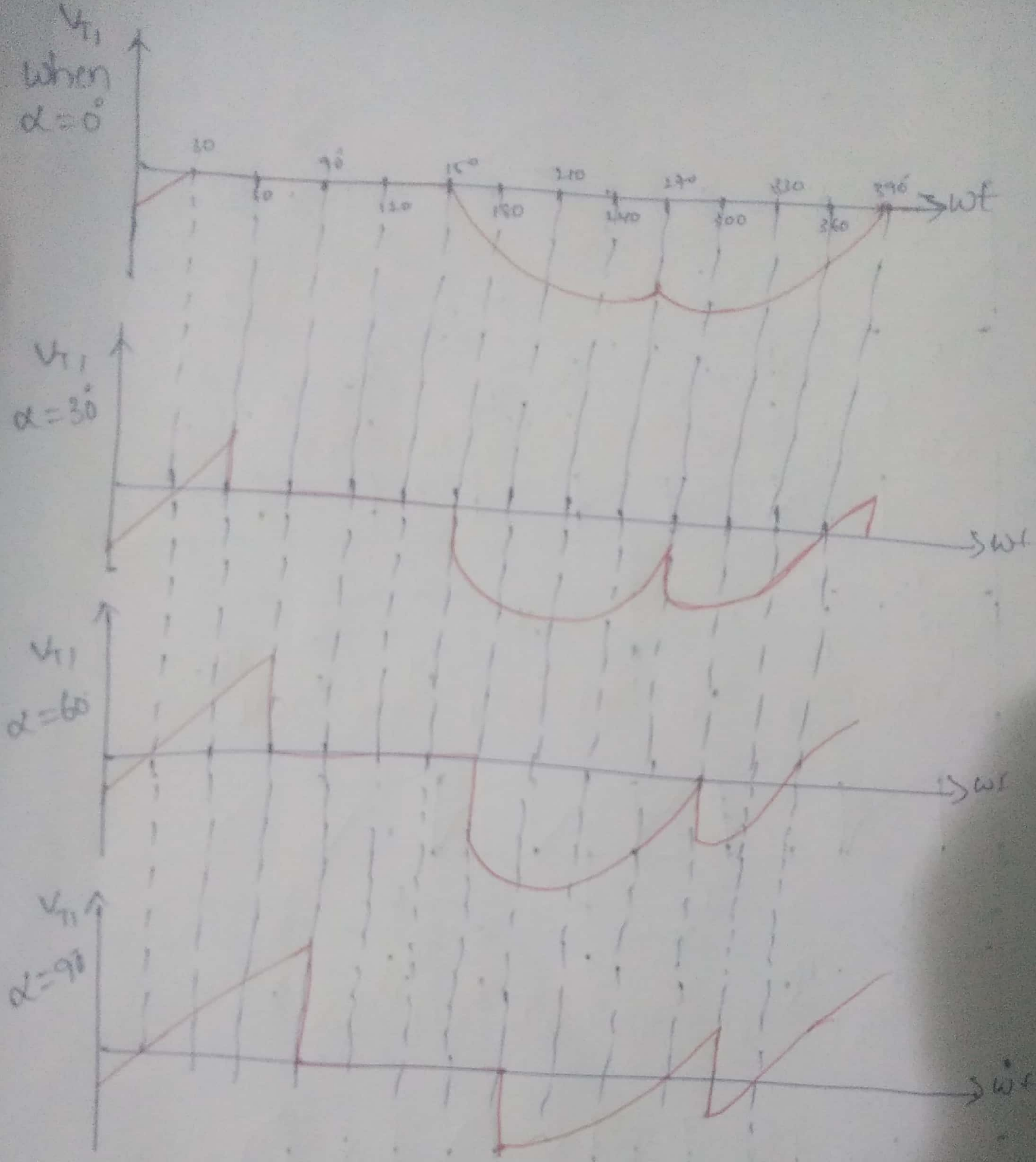
$$V_0 = V_{ba} = (V_{bn} - V_{an})$$

→ $T_5$ is triggered at $\omega t = (270+\alpha)$, $T_3$ turns off naturally as it is reverse biased as soon as $T_5$ is triggered

→ During the interval $(270+\alpha)$ to $(230+\alpha)$, $T_4$ & $T_5$ conduct together. & the o/p load voltage

$$V_0 = V_{ca} = (V_{cn} - V_{an})$$


Vs
ωt
30 60 90 120 150 180 210 240 270 300 330 360 390 420 450
T5 T6 T1 T2 T3 T4 T5 T6
T5T6 T6T1 T1T2 T2T3 T3T4 T4T5 T5T6
Vo
α=30
ωt
T5 T4 T1 T3 T6 T2 T4 T6
T4T5 T5T6 T6T1 T1T2 T2T3 T3T4 T4T5 T5T6
Vo
α=60°
ωt
T5 T1 T2 T5 T4 T6 T2 T4
T4T5 T5T6 T6T1 T1T2 T2T3 T3T4 T4T5
Vo
α=90°
for R-load
ωt
Vo
α=90°
for RL-load
ωt

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Let

$V_{an} = V_m \sin\omega t$ $\qquad$ $V_{ab} = \sqrt{3} V_m \sin(\omega t + \pi/6)$

$V_{bn} = V_m \sin(\omega t - 2\pi/3)$ $\qquad$ $V_{bc} = \sqrt{3} V_m \sin(\omega t - \pi/2)$

$V_{cn} = V_m \sin(\omega t - 4\pi/3)$ $\qquad$ $V_{ca} = \sqrt{3} V_m \sin(\omega t - 7\pi/6)$

The dc component of the o/p voltage & current can be found as

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\pi/2+\alpha} \sqrt{3} V_m \sin(\omega t + \pi/6)\, d(\omega t)$$

$$= \frac{3\sqrt{3} V_m}{\pi} \cos\alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3} V_m}{\pi R} \cos\alpha$$

$$V_{Rms} = \sqrt{\frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} (\sqrt{3} V_m \sin(\omega t + \pi/6))^2\, d(\omega t)}$$

$$V_{Rms} = \sqrt{3} V_m \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha}$$

$$I_{Rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{\sqrt{3} V_m}{\sqrt{(R^2 + (\omega L)^2}} \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha}$$

when $\alpha > 60^\circ$

$$V_o = \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi} \sqrt{3} V_m \sin(\omega t + \frac{\pi}{6})\, d(\omega t)$$

$$= \frac{3\sqrt{3} V_m}{\pi} \cos(\frac{\pi}{3} + \alpha)$$

$$I_o = \frac{V_o}{R} = \frac{3\sqrt{3} V_m}{\pi R} \cos(\frac{\pi}{3} + \alpha)$$

$$V_{Rms} = \sqrt{\frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi} (\sqrt{3} V_m \sin(\omega t + \pi/6))^2 \, d(\omega t)}$$

$$I_{Rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}}$$

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