

✓ Proposition.

P	Q	and $P \wedge Q$	or $P \vee Q$	conditional $P \rightarrow Q$	bicond: $P \leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

1) Find the P.CNF & P.DNF of the following.

$$(P \rightarrow (P \wedge Q)) \vee (P \rightarrow R)$$

P	Q	R	$P \wedge Q$	$P \rightarrow (P \wedge Q)$	$P \rightarrow R$	$(A \vee B)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	F	T	T
T	F	F	F	F	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

PDNF :

$$\begin{aligned} & (P \bar{A} Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \\ & \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R) \\ & \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \end{aligned}$$

PCNF :

$$(\neg P \vee Q \vee R)$$

$$((P \wedge Q) \rightarrow \neg R) \wedge \neg Q$$

$$P \vee Q \quad \neg P \rightarrow Q \quad P \vee \neg Q \quad \neg P \rightarrow \neg Q$$

P	Q	R	$P \wedge Q$	$\neg R$	$(P \wedge Q) \rightarrow \neg R$	$\neg Q$	$((P \wedge Q) \rightarrow \neg R) \wedge \neg Q$
T	T	T	T	F	F	F	F
T	T	F	T	T	T	F	F
T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	F	T	F	F
F	T	F	F	T	T	F	F
F	F	T	F	F	T	T	T
F	F	F	F	T	T	T	T

DNF:

$$(P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$$

PCNF:

$$(\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$$

Show that  $\neg P$  follows, logically from.

$$\neg(P \wedge \neg Q), \neg Q \vee R, \neg R$$

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Premises:

$$H_1: \neg(P \wedge \neg Q)$$

$$H_2: \neg Q \vee R$$

$$H_3: \neg R$$

Premises:

Conclusion:

$$\therefore \neg P$$

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Deriv

Rule

Rule P.

$$1. \neg(P \wedge \neg Q)$$

De Morgan's law.  
Rule T.

$$2. \neg P \vee Q$$

2, Rule T.

$$3. P \rightarrow Q$$

Rule P.

$$4. \neg Q \vee R$$

4, Rule T

$$5. Q \rightarrow R$$

3, 5 Rule T Chain rule

$$6. P \rightarrow R$$

$$7. \neg P$$

$\neg (S \vee R)$  logically follows from  $(P \vee A)$ ,  $(P \rightarrow R)$   
and  $(A \rightarrow S)$

Premiss:

$$H_1: P \vee A$$

$$H_2: P \rightarrow R$$

$$H_3: \underline{A \rightarrow S}$$

$$P \rightarrow R$$

$$\underline{\neg R \rightarrow \neg P}$$

Conclude:

$$S = \underline{S \vee R}$$

$$P \rightarrow R$$

$$\neg P \vee R$$

So

Derivation

Reasons

1.

$$P \vee A$$

Rule P

2.

$$P \rightarrow R$$

1, Rule T

3.

$$A \rightarrow S$$

Rule P

4.

$$\neg P \rightarrow S$$

2, 3, Rule T  
chain

5.

$$\neg S \rightarrow P$$

4, Rule T  
Contrapositive

6.

$$P \rightarrow R$$

Rule P

7.

$$\neg S \rightarrow R$$

5, 6 Rule T  
chain

8.

$$S \vee R$$

$$P \rightarrow Q$$

$$\neg P \vee Q$$

~~$$P \rightarrow Q, Q \rightarrow R \rightarrow R$$~~

proofs :-

$$H_1 : P \rightarrow Q$$

$$H_2 : Q \rightarrow \neg R$$

$$H_3 : R$$

$$H_4 : P \vee (\neg R)$$

Conclusion :-

$$S = \neg R$$

Sno.

Derivation

reason,

rule P

1)

$$P \rightarrow Q$$

2)

$$Q \rightarrow \neg R$$

rule P

3)

$$P \rightarrow \neg R$$

2, rule T

4.)

$R \rightarrow NP$

3, Rule T

5.)

1.)  $P \vee Q$ ,  $(P \rightarrow R)$ ,  $(Q \rightarrow S)$

Proof

Premises

$H_1 : P \vee Q$

$H_2 : (P \rightarrow R)$

$H_3 : (Q \rightarrow S)$

Conclusion

$S \vee R$

Sno.	Derivation	Reasons
1.)	$P \vee Q$	Rule P
2.)	$NP \rightarrow Q$ ✓	Rule T (1)
3.)	$P \rightarrow R$ ✓	Rule P
4.)	$Q \rightarrow S$	Rule T
5.	$R \rightarrow S$	(3, 4) rule T chain

1.

2.

$$P \vee Q$$

$$\neg P \rightarrow Q$$

$$Q \rightarrow \perp$$

4.

$$\neg P \rightarrow \perp$$

5.

$$P \rightarrow R$$

6.

$$\neg R \rightarrow \neg P$$

7.

$$\neg R \rightarrow \perp$$

$$R \vee \perp$$

Rule P

Rule T

Rule P

Rule T

Rule T



(If) it is rainy, (then) it is a holiday.

Write the

contrapositive

converse

inverse

of the above statement.

if  $P$  then  $Q$ .  
 $P \rightarrow Q$

$P \rightarrow Q$  ✓

Contrapositive

$\neg Q \rightarrow \neg P$

converse

$Q \rightarrow P$

inverse

$\neg P \rightarrow \neg Q$

$P$ : it is Sunday

$Q$ : it is a holiday

$$P \rightarrow Q$$

Contrapositive

$$\neg Q \rightarrow \neg P$$

If it is not a holiday then it is not Sunday

converse  $Q \rightarrow P$

inverse  $\neg P \rightarrow \neg Q$

S. no	Der	Rule
1.	$P \rightarrow Q$	Rule P.
2.	$Q \rightarrow R$	Rule P
3.	$P \rightarrow R$	1,2 Rule T chain rule
4.	$\neg R \rightarrow TP$	3, contraposition
5.	$S \rightarrow \neg R$	<u>Rule P.</u>
6.	$S \rightarrow TP$	4,5 chain rule
7.	$\neg S \vee TP$	6, rule T
8.	$\neg(S \wedge P)$	7, De Morgan's
9.	$(P \wedge S)$	rule T
10.	$(F)$	8,9 rule T

Show that the following premises are inconsistent

P: Jack misses many classes due to illness

Q: he fails high school.

R: he is uneducated

S: Reads lot of books

1.  $P \rightarrow Q$

2.  $Q \rightarrow R$

3.  $S \rightarrow \neg R$  ✓

4.  $P \wedge S$

Conclusion: F

$C(x)$ :  $x$  is in this class.

$J(x)$ :  $x$  knows how to write java programs.

$h(x)$ :  $x$  can get high paying job.

Symbolize:

$$H_1: \exists x [C(x) \wedge J(x)]$$

$$H_2: \forall x [J(x) \rightarrow H(x)]$$

$$C: \exists x [C(x) \wedge h(x)]$$

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1.  $\exists x [C(x) \wedge J(x)]$

2.  $C(a) \wedge J(a)$

3.  $C(a)$

4.  $J(a)$

5.  $\forall x [J(x) \rightarrow H(x)]$

6.  $J(a) \rightarrow H(a)$

7.  $H(a)$

rule .

rule P.

rule ES.

2, rule T simplification

2, rule T simplification

rule P

Rule US

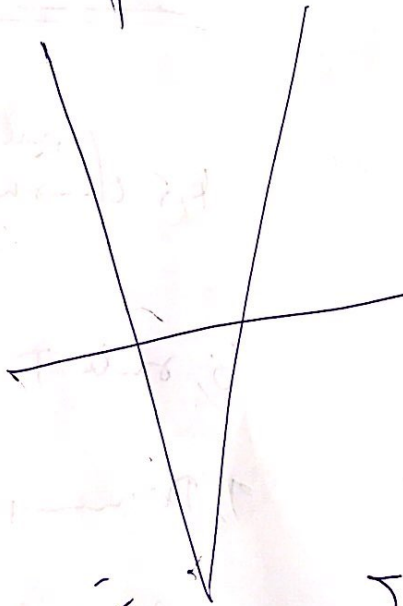
Scanned by CamScanner  
4, 6 rule T simplification

One student in this class knows how to write programs in Java.

Everyone who knows to write program in Java can get a high paying job.

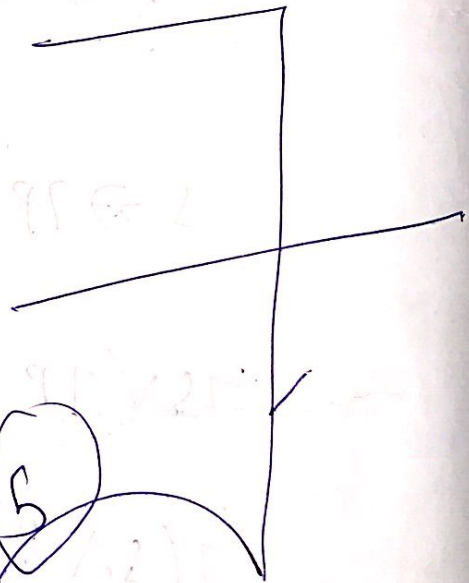
Simply the conclusion  
Someone in this class can get a high paying job.

for all



Universal Quantifier

there exist.



Existential Quantifier

S: Specific

G: General



$C(x)$ :  $x$  is in this class.  
 $J(x)$ :  $x$  knows how to write prog in java  
 $h(x)$ :  $x$  can get hgh payin job.

Symbolizing:

$$H_1: \exists x (C(x) \wedge J(x))$$

$$H_2: \forall x (J(x) \rightarrow h(x))$$

$$C: \exists x (C(x) \wedge h(x))$$

- | steps | deriv                               |
|-------|-------------------------------------|
| 1.    | $\exists x (C(x) \wedge J(x))$      |
| 2.    | $C(a) \wedge J(a)$                  |
| 3.    | $C(a)$                              |
|       | $J(a)$                              |
| 4.    |                                     |
| 5.    | $\forall x (J(x) \rightarrow h(x))$ |
| 6.    | $J(a) \rightarrow h(a)$             |
|       | $h(a)$                              |
| 7.    |                                     |
| 8.    | $C(a) \wedge h(a)$                  |
| 9.    | $\exists x (C(x) \wedge h(x))$      |

rule .

rule P .

1, rule ES

2, rule T, simplification

3, rule T simplification.

rule P

5, rule VS

4, 6 rule T  
modus ponens.

3, 8 rule  
introduction.