Proposition.

P	Q	and $P \wedge Q$	or $P \vee Q$	conditional $P \rightarrow Q$.	bicondi $P \leftrightarrow Q$.
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

1) Find the PCNF & PDNF of the following.

$(P \rightarrow (P \wedge Q)) \vee (P \rightarrow R)$

P	Q	R	$P \wedge Q$	A $P \rightarrow (P \wedge Q)$.	B $P \rightarrow R$	$(A \vee B)$
T	T	T	T	T	T	T
T	T	F	T	T	F	T
T	F	T	F	F	T	T
T	F	F	F	F	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

PDNF:

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$

$\vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R)$

$\vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

PCNF:

$(\neg P \vee Q \vee R)$

$((P \wedge Q) \to \neg R) \wedge \neg Q$.

$P \vee Q$ $\quad \neg P \to Q$ $\quad P \vee \neg Q$ $\quad \neg P \to \neg Q$

P	Q	R	$P \wedge Q$	$\sim R$	$(P \wedge Q) \to \sim R$	$\sim Q$	$((P \wedge Q) \to \sim R)$
T	T	T	T	F	F	F	F
T	T	F	T	T	T	F	F
T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	F	T	F	F
F	T	F	F	T	T	F	F
F	F	T	F	F	T	T	T
F	F	F	F	T	T	T	T

PDNF:

$(P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$
$\vee (\sim P \wedge \sim Q \wedge \sim R)$

PCNF:

$(\sim P \vee \sim Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee R)$
$\wedge (P \vee \sim Q \vee \sim R) \wedge (P \vee \sim Q \vee R)$

Show that $\neg P$ follows, logically from:

$\neg(P \wedge \neg Q)$, $\neg Q \vee R$, $\neg R$.

Premises:

H_1: $\neg(P \wedge \neg Q)$

H_2: $\neg Q \vee R$

H_3: $\neg R$

Premies:

Conclusion:

S: $\neg P$

S.No	Der	Rule
1.	$\neg(P \wedge \neg Q)$	Rule P
2.	$\neg P \vee Q$	Demorgan's law. Rule T
3.	~~$\neg P$~~ $P \rightarrow Q$	~~2~~, Rule T
4.	$\neg Q \vee R$	Rule P
5.	$Q \rightarrow R$	4, Rule T
6.	$P \rightarrow R$	3,5 Rule T Chain rule
7.	$\neg R$ ... $\neg P$	Rule

S.T SVR logically follows from $(P \vee Q)$, $(P \to R)$ and $(Q \to S)$

Premises:

H_1 :- $P \vee Q$

H_2 :- $P \to R$

H_3 :- $Q \to S$

$P \to Q$

$\neg Q \to \neg P$

$P \to Q$

$\neg P \vee Q$

Conclusion

$S = S \vee R$

Sno

	Derivation	Reasons
1.	$P \vee Q$	Rule P
2.	$\neg P \to Q$	1, Rule T
3.	$Q \to S$	Rule P
4.	$\neg P \to S$	2,3, Rule T chain rule
5.	$\neg S \to P$	4, Rule T contrapositive
6.	$P \to R$	Rule P
7.	$\neg S \to R$	5,6 Rule T chain rule
8.	$S \vee R$	8 R T

$$P \longrightarrow Q \qquad \neg P \vee Q$$

$\cancel{P \to Q, Q \to \vee R} \to R$

~~persus~~ :-

H_1 :- $P \to Q$

H_2 :- $Q \to \sim R$

H_3 :- R

H_4 :- $P \vee (J \wedge S)$

Conclusion

$\boxed{S = J \wedge S}$

Sno.	Derivation	Reason
1)	$P \to Q$	Rule P
2)	$Q \to \sim R$	Rule P
3)	$P \to \sim R$	2) Rule T

4.)	$R \to \sim P$	3, Rule T
5.)		

1.) $P \vee Q$, $(P \to R)$, $(Q \to S)$

Soln

Premises

H_1 : $P \vee Q$

H_2 : $(P \to R)$

H_3 : $(Q \to S)$

Conclusion

$S \vee R$

Sno.	Derivation	Reasons
1.)	$P \vee Q$	Rule P
2.)	$\sim P \to Q$	Rule T (1)
3.)	$P \to R$	Rule P.
4.)	$Q \to S$	Rule T
5.	$Q \to S$	(3, 4) Rule T chain rule

1.

2. $P \vee Q$

3. $\neg P \to Q$ Rule P

$Q \to S$ Rule T

4. Rule P.

$\neg P \to S$

Rule T

5.

$P \to R$

Rule T

6.

$\neg R \to \neg P$

7. $\neg R \to S$

$R \vee S$

If it is sunday, then it is a holiday.

Write the

~~contrapositive~~

~~converse~~

~~inverse~~

of the above statement.

if P then Q.

$P \longrightarrow Q$

$P \rightarrow Q$

Cotrapo

$\neg Q \rightarrow \neg P$.

converse

$Q \rightarrow P$.

inverse

$\neg P \rightarrow \neg Q$.

P: it is sunday

Q: it is a holiday

$P \to Q$

Contrapositive:

$\neg Q \to \neg P$

If it is not a holiday then it is not sunday

Converse $Q \to P$.

inverse $\neg P \to \neg Q$

S.No	Der	Rule
1.	$P \to Q$	Rule P.
2.	$Q \to R$	Rule P
3.	$P \to R$	1,2 Rule T chain rule
4.	$\neg R \to \neg P$	3, contrapositive
5	$S \to \neg R$	Rule P.
6.	$S \to \neg P$	4,5 chain rule, rule T
7.	$\neg S \vee \neg P$	6, rule T
8.	$\neg(S \wedge P)$	7, Demorgan
9.	$P \wedge S$	rule T
10.	F	8,9, rule T

P

Show that the following premises are inconsistent

P : Jack misses many classes through illness.
Q : he fails high school.
R : he is uneducated
S : Reads a lot of book.

1. $P \to Q$
2. $Q \to R$
3. $S \to \neg R$
4. $P \wedge S$

Conclusion: F

$C(x)$: x is in this class.

$J(x)$: x knows how to write java program.

$h(x)$: x can get high paying job.

symbolize :

H_1 : $\exists x\,[C(x) \wedge J(x)]$

H_2 : $\forall x\,[J(x) \rightarrow H(x)]$

C : $\exists x\,[C(x) \wedge h(x)]$

S. NO	der.	rule.
1.	$\exists x\,[C(x) \wedge J(x)]$	rule P.
2.	$C(a) \wedge J(a)$.	rule ES.
3.	$C(a)$	2, rule T simplication
4.	$J(a)$.	2, rule T simplication
5.	$\forall x\,[J(x) \rightarrow H(x)]$	rule P
6.	$J(a) \rightarrow H(a)$.	Rule US
7.	$H(a)$	4, 6 rule T, modus ponens

One student in this class knows how to write programs in java.

Everyone who knows to write program in java can get a high paying job. ~~tar~~

~~imply~~ the concl

Someone in this class ~~can~~ can get a high paying job.

forall

$\forall$

Universal Quantifier

there exist.

$\exists$

Existential Quantifier

$\Sigma f 3 = 5$

S: specific

G: Gener

$C(x)$: x is in this class.

$J(x)$: x knows how to write prog in java

$h(x)$: x can get high paying job.

Symbolizing:

H_1 : $\exists x\,(C(x) \wedge J(x))$

H_2 : $\forall x\,(J(x) \rightarrow h(x))$:

C : $\exists x\,(C(x) \wedge h(x))$.

s.no	deri	Rule.
		rule P.
1.	$\exists x\,(C(x) \wedge J(x))$	1, rule ES
2.	$C(a) \wedge J(a)$	2, rule T, simplification
3.	$C(a)$	2, rule T simplification.
4.	$J(a)$	
5.	$\forall(x)\,(J(x) \rightarrow h(x))$	rule P
6.	$J(a) \rightarrow h(a)$	5, rule US
7.	$h(a)$	4, 6 rule T modus ponens.
8.	$C(a) \wedge h(a)$	3, 7 rule T addition.
9.	$\exists x\,(C(x) \wedge h(x))$	8, rule EG