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Properties of regular languages, pumping lemma for regular languages

Chapter - 2: Regular languages and finite automata

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Introduction

- Regular languages are the simplest type of languages recognized by finite automata
- We study:
  - ✅ Closure properties
  - ✅ Decidability properties
  - ✅ Pumping lemma — a method to prove a language is not regular

Properties of Regular Languages

Regular languages are closed under the following operations :

Operation	Meaning
Union	If L_1 and L_2 are regular, $L_1 \cup L_2$ is regular
Concatenation	$L_1 \cdot L_2$ is regular
Kleene Star	L^* is regular
Intersection	$L_1 \cap L_2$ is regular
Complementation	$\neg L$ is regular
Difference	$L_1 - L_2$ is regular
Reversal	L^R is regular
Homomorphism	$h(L)$ is regular

Example – Closure Under Union

Let:

- L_1 = strings over {a, b} with even number of a's
- L_2 = strings that end with b

Then $L_1 \cup L_2$ is also regular ✅ Construct DFA for L_1, DFA for L_2, and combine them using product construction

Decidability Properties

Problem	Decision Method
Emptiness	Check for reachable final state
Finiteness	Check for cycles in FA
Membership ($w \in L$?)	Simulate input in DFA
Equivalence of two DFAs	Minimize and compare
Subset ($L_1 \subseteq L_2$?)	Use difference and emptiness test

Pumping Lemma – Formal Statement

Theorem:

If L is a regular language, then there exists a constant $p > 0$ (pumping length), such that any string $s \in L$ with $|s| \geq p$, can be divided into 3 parts:

$s = xyz$, such that:

1. $|y| > 0$ ($y \neq \varepsilon$)
2. $|xy| \leq p$
3. For all $i \geq 0$, $x \cdot y^i \cdot z \in L$

Pumping Lemma – Example

Proving a Language is Not Regular:

Language:

$L = \{ a^n b^n \mid n \geq 0 \}$

Claim: L is not regular

Proof by contradiction using Pumping Lemma:

1. Assume L is regular
2. Let p be the pumping length
3. Choose s = a^p b^p ∈ L
4. s = xyz, with |xy| ≤ p → x and y contain only a's
5. Pump y (repeat i times): $x \cdot y^i \cdot z$ = a^(p+i·|y|) b^p
6. Unequal number of a's and b's ⇒ not in L

✅ Contradiction → L is not regular

Key Takeaways

- Regular languages are closed under many operations
- Decidability is strong for regular languages
- Pumping Lemma is a powerful proof technique
    - Proves that a language is not regular
    - Works by contradiction
    - Focuses on the inability to pump a string without violating language rules

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