

Seat No: A15837

Enrollment No: \_\_\_\_\_

**PARUL UNIVERSITY**  
**FACULTY OF ENGINEERING & TECHNOLOGY**  
**B.Tech. Winter 2023 - 24 Examination**

Semester: 1

Subject Code: 303191101

Subject Name: Mathematics-I

Date: 30/01/2024

Time: 2:00 pm to 4:30 pm

Total Marks: 60

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

**Q.1 Objective Type Questions (All are compulsory) (Each of one mark)**

(15)

1. Which of the following matrix is singular?

a)  $\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$       c)  $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$       d)  $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

2. What is the order of the differential equation  $\frac{d^2y}{dx^2} + 3\left(\frac{d^3y}{dx^3}\right)^4 + y = 0$   
 a) 1      b) 2      c) 3      d) 4

3. Which of the following matrix is in Row Echelon form?

a)  $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$       c)  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$       d)  $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

4. If  $Z = x^2y + y^2x + \cos xy$  then what is the value of  $\left(\frac{\partial z}{\partial x}\right)_{(0,1)} = \underline{\hspace{2cm}}$   
 a) 1      b) 2      c) 3      d) 4

5. Sequence  $a_n = \left\{ \frac{1}{n} \right\}$  is         
 a) Convergent      b) Divergent      c) Oscillatory      d) None of the above

6. Write down the Eigen values of the matrix  $\begin{bmatrix} -1 & 2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & -1 \end{bmatrix}$

7. The Series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is convergent if value of  $p$  is

8. What is the Condition to be a differential equation  $M dx + N dy = 0$  is Exact?

9. Integrating factor of the differential equation is  $\frac{dy}{dx} + P(x)y = Q(x)$  is

10. If  $f(x, y) = y \sin x + y^3$  then What is the value of,  $\lim_{(x,y) \rightarrow (0, \frac{\pi}{2})} f(x, y)$

11. If  $y \sin x = x \cos y$  then find  $\frac{dy}{dx} = \underline{\hspace{2cm}}$

12. An integral  $\int_1^4 \frac{1}{x-3} dx$  is Improper. [True / False]

13. Function  $f(x)$  is odd Function iff  $f(-x) = -f(x)$ . [True / false]

14. Every Square matrix is diagonalizable. [True / False]

15. Every Constant Sequence is Divergent. [True / False]

**Q.2 Answer the following questions. (Attempt any three)**

(15)

- A) If  $u = \ln\left(\frac{x^4+y^4}{x+y}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$

- b) Solve the given linear differential equation  $\frac{dy}{dx} + 2\tan x y = \sin x$ .

- c) Solve the system of linear equations:

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y - z &= -6, \\ 2x - 5y + 5z &= 17 \end{aligned}$$

- D) Test the convergence of the series  $\sum_{n=1}^{\infty} \sqrt{\frac{n}{(n+1)^3}}$

**Q.3** A) Find the Fourier series for the function  $f(x) = (\pi - x); 0 \leq x \leq \pi$  (07)

B) Show that the matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalizable. (08)

**OR**

B) Find the eigenvalues and eigenvectors of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  (08)

**Q.4** A) Find the equation of the tangent plane and normal line to the surface  $z + 8 = xe^y \cos z$  at the point  $(8, 0, 0)$ . (07)

**OR**

A) I) Evaluate the improper integral  $\int_0^\infty e^{-x} dx$  (03)

II) Solve the differential equation  $(x^3 + y^3)dx - xy^2 dy = 0$  (04)

B) I) Verify the Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  and, hence, find  $A^{-1}$ . (04)

II) Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$ . (04)