



★ Pumping Lemma for RL (RL or NRL?)

For any RL, $L \nexists$ an integer n (is dependent on L)

✓ s.t. $\forall z \in L$ and $|z| \geq n$

Based
on
proof
by
contradiction

i) $z = uvw$

ii) $|uv| \leq |z|$

iii) $|v| \geq 1$ or $|v| \neq 0$

then $uv^i w \in L \quad \forall i \geq 0$ u, v, w, z are words

Examples built using Σ

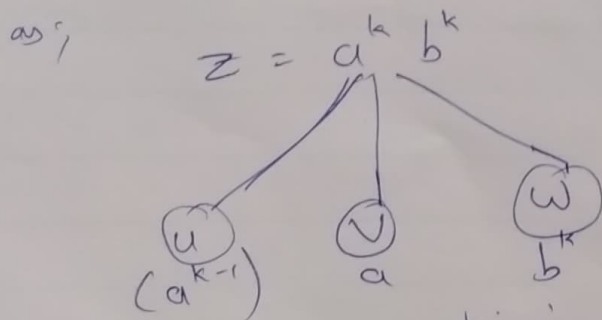
1) $L = \{a^m b^m \mid m \geq 1\}$

Let assume L is a RL for some n

$z \in L$

$z = a^k b^k$; choose k s.t. $|a^k b^k| = 2k \geq n$

Now, we have to divide this ($a^k b^k$) (once we get to know value of n)
as; we can choose k



$z = uvw$
 $\Rightarrow |uv| \Rightarrow k \leq 2k \Rightarrow |uv| \leq |z|$
 $|v| = 1 \geq 1$

As the above satisfies all the condition
 $uv^i w$ must $\in L \quad \forall i \geq 0$

For $i=2$ $uv^2 w \Rightarrow a^{k-1} a^2 b^k = a^{k+1} b^k \Rightarrow$ but no. of a 's \neq no. of b 's must be =
 hence, given L is NRL

2) $L = \{a^m b^n \mid m > n\}$

Let L is a RL
 $z \in L$

As m is always greater than n , then
 Let $z = a^{k+1} b^k$; choose k s.t. $|z| \geq n$
 $2k+1 \geq n$

Now, $z = a^{k+1} b^k$

$$\begin{array}{ccc} & & \\ & \swarrow & \downarrow & \searrow \\ u & & v & & w \\ a^{k+1} & & b & & b^{k-1} \end{array}$$

Let us check whether it satisfies all three conditions or not.

$z = uvw$ (i) — satisfied

$|w| = k+2 \leq 2k+1 \Rightarrow |uv| \leq |z|$ — (ii) — satisfied

$|v| = 1 \neq 0 \Rightarrow |v| \geq 1$

As it satisfies all the conditions,

$uv^i w \in L \quad \forall i \geq 0$, But,

for $i=2 \Rightarrow uv^2 w = a^{k+1} b^2 b^{k-1}$
 $= a^{k+1} b^{k+1} \notin L$
 (Note: $k+1 = k+1$ but $\# a's > \# b's$)

Here $k+1 = k+1$ but $\# a's > \# b's$

Hence L is NRL

3) $L = \{a^{3^n} \mid n \geq 1\}$

Let $z \in L$

$$\begin{array}{ccc} & & \\ & \swarrow & \downarrow & \searrow \\ a^{3^n} & & a & & \epsilon \end{array}$$

$|uv| \leq |z|$

$|v| \neq 0$

It must satisfy

Thus, $uv^i w \notin L \quad i \geq 0$
 For $i=2$

$uv^2 w = a^{3^n-1} a^2 \in L$
 $= a^{3^n+1} \notin L$

Hence L is NRL



* Special Case of pumping lemma when $\Sigma = \{a\}$

Lengths of the string must follow Arithmetic

Progression for a Lang. to be a RL.

Proof: $uv^i w \in L$ let us take $u=a, v=a^n, w=a$

Example

then $i=0, 1, 2, \dots$
 $aa, a^2aa, a^3aaaa, \dots$

length / #a's 2, 4, 6, \dots
 follows AP

$$1) L = \{a^{3n} \mid n \geq 1\}$$

$$L = \{a^3, a^6, a^9, a^{12}, \dots\}$$

$$3, 6, 9, 12, \dots \Rightarrow \text{AP}$$

L is RL

$$2) L = \{a^{2n+1} \mid n \geq 0\}$$

$$L = \{a, a^3, a^5, a^7, \dots\}$$

Length of string $\in L$ is 1, 3, 5, 7, \dots which is an AP

L is RL

$$3) L = \{a^{n^2} \mid n \geq 0\}$$

For $n \geq 0$, length of string;

0, 1, 4, 9, \dots which is not an AP

L is NPL





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$$4) L = \{a^p \mid p \text{ is prime}\}$$

length of strings corresponding to L ;

2, 3, 5, 7, 11, ... \neq AP $\Rightarrow L$ is NRL