Parul® University

Basic Electrical Engineering-203106101

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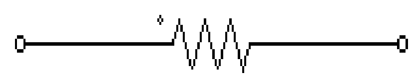
CHAPTER-1

DC Circuit

Electrical circuit elements

Resistance : “The opposition offered by a substance to the flow of electric current is called resistance .”

When the potential difference is applied to a conductor , the current start to flow or the free electrons start moving.
While moving , the free electrons collide with the atoms and molecules of the conductors .
Because of collision the rate of flow of electrons or current is restricted .



Symbolic representation of resistor

Resistance measured in ohm (Ω) and denoted with (R, r)

Image source : Google

Factors affecting to Resistor

1) Length of conductor : when length of conductor is increase resistance increase . In other word we can say that resistance is directly proportional to length of the conductor.

2) Area of conductor : When area of conductor is change weather increase or decrease resistance of conductor can change with decrease and increase. In simple world we can say that resistance is inversely proportional to the area .

3) Temperature : Resistance can increase if temperature is increase and can decrease as the temperature decrease

Mathematically, $R = \rho\, l/a \quad \Omega$

Inductance

Inductance is the one kind of property of material which can store the energy in the form of magnetic energy. In circuit when the current is change with time at that time only inductor can exhibited.

Inductance is the nature of the coil by which it opposes any small change of direction of the current when it flows through the coil. When the current is passing in to the coil it creates magnetic field around it and if the any change in the current magnitude, magnetic field also change according that induced emf also change .

So we can understand that inductance of the coil depends upon the rate of change of current.

Image source : Google

Image source : Google

Inductance Conti...

Mathematically,

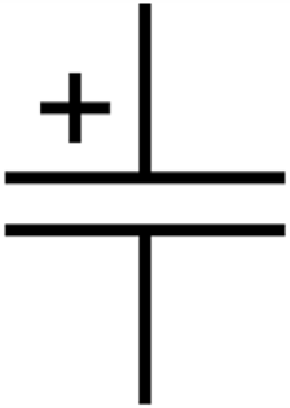
V = L(dI/dt) volt

Where V is the voltage, L is the inductance, I is the current and t is the time period.

Inductance, ‘L’, is measured in Henrys, named after Joseph Henry, the American scientist who discovered electromagnetic induction.

Capacitance

Capacitance is a two-terminal element that has the capability of charge storage and so we can say that it can stored energy in the form of voltage.



Mathematically, the *capacitance* of a conductor is defined by

C= Q/V (farad)

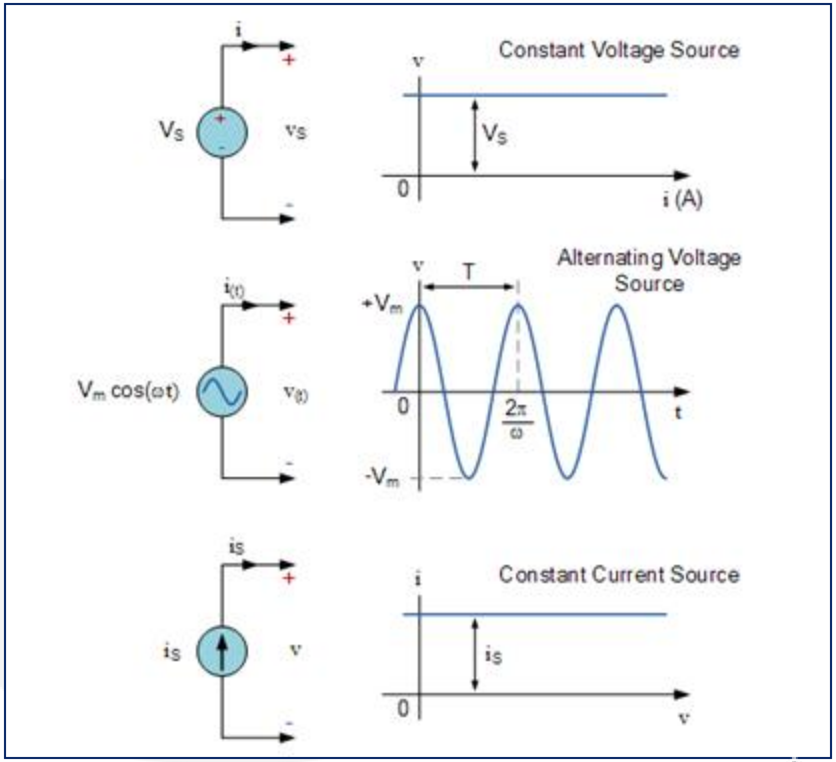
Where Q charges and V is the potential difference.
The current through the capacitor is proportional to the derivative of voltage across capacitor and is given by expression

I = C dv/dt amp

Image source : Google

Electrical Sources

Electrical sources, both as a voltage source or a current source can be classed as being either independent (ideal) or dependent, (controlled) that is whose value depends upon a voltage or current elsewhere within the circuit, which itself can be either constant or time-varying.



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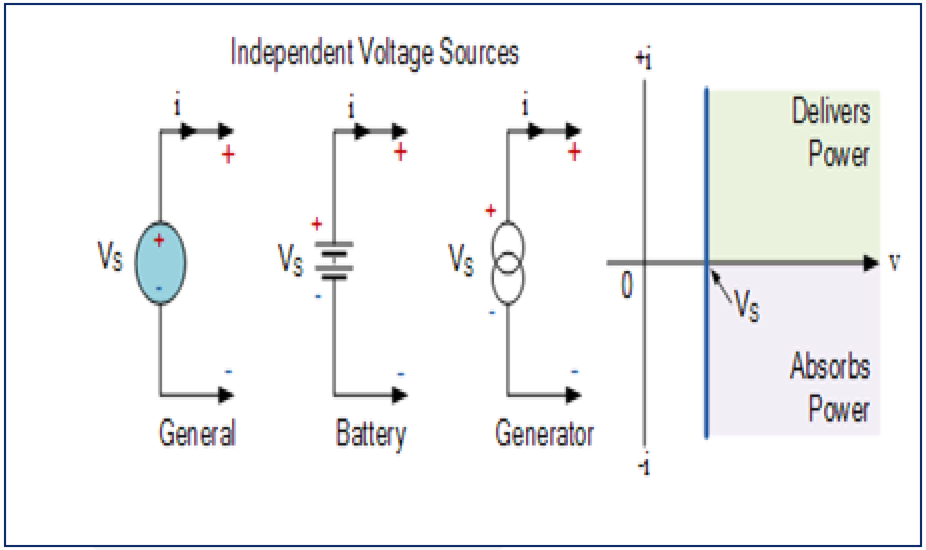
Ideal voltage source

➢An ideal voltage source is defined as a two terminal active element that is capable of supplying and maintaining the same voltage, (v) across its terminals regardless of the current, (i) flowing through it.

➢In other words, an ideal voltage source will supply a constant voltage at all times regardless of the value of the current being supplied producing an I-V characteristic represented by a straight line.

➢An ideal voltage source is also known as an Independent Voltage Source as its voltage does not depend on either the value of the current flowing through the source or its direction



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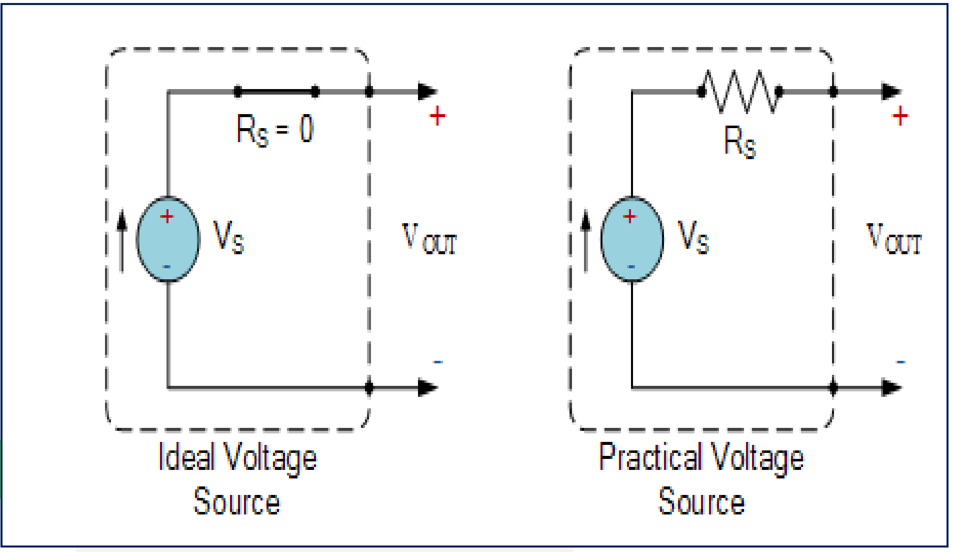
Ideal and practical voltage source

- Ideal voltage source having series resistance value is very low near to zero.

- Practical voltage source having some series resistance value

- if the series source resistance is low, the voltage source is ideal. When the source resistance is infinite, the voltage source is open-circuited.



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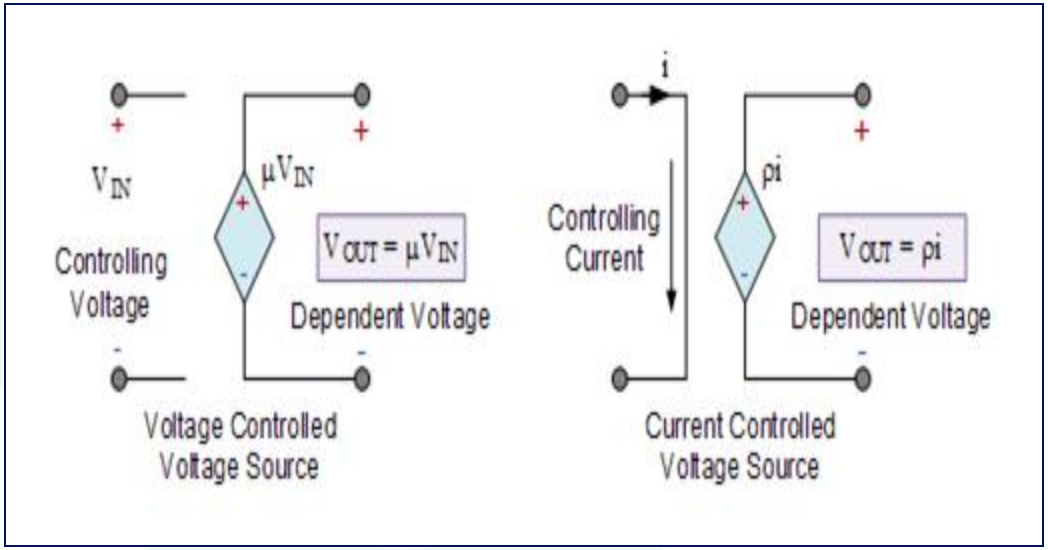
Dependent Voltage Source

- Unlike an ideal voltage source , a controlled or dependent voltage source changes its terminal voltage depending upon the voltage across, or the current through.
- Dependent voltage source that depends on a voltage input is generally referred to as a **Voltage Controlled Voltage Source** or **VCVS**. A voltage source that depends on a current input is referred too as a **Current Controlled Voltage Source** or **CCVS**.



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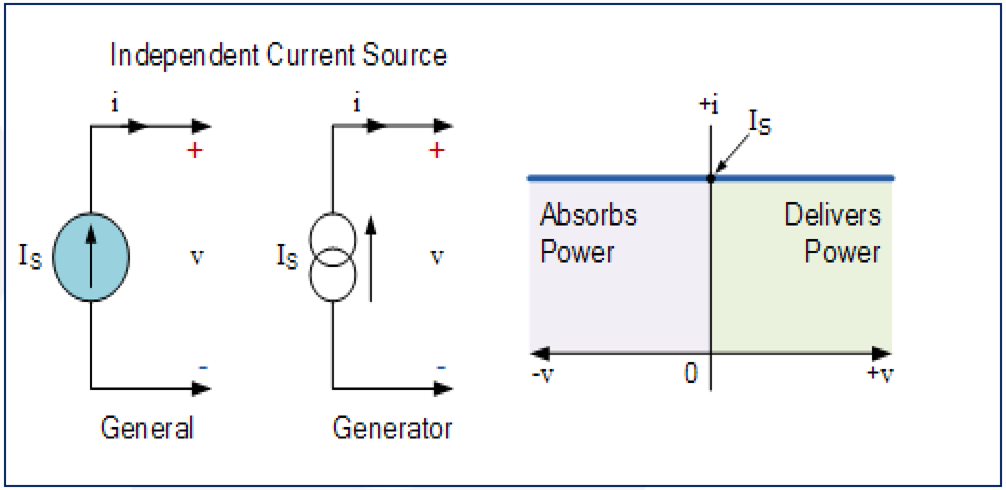
Ideal Current source

- A Current Source is an active circuit element that is capable of supplying a constant current flow to a circuit regardless of the voltage developed across its terminals
- An ideal current source is called a “constant current source” as it provides a constant steady state current independent of the load connected to it producing an I-V characteristic represented by a straight line



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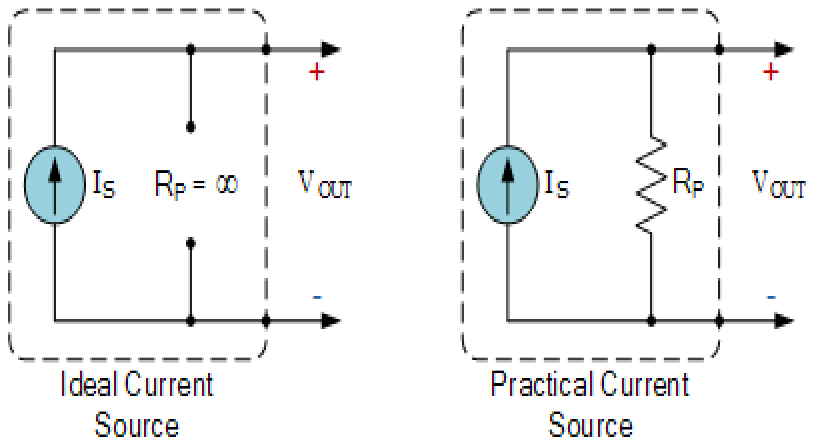
Practical current source

- In ideal current source provide constant current as its internal resistance is infinite.

- Practical current source having some value of internal resistance.



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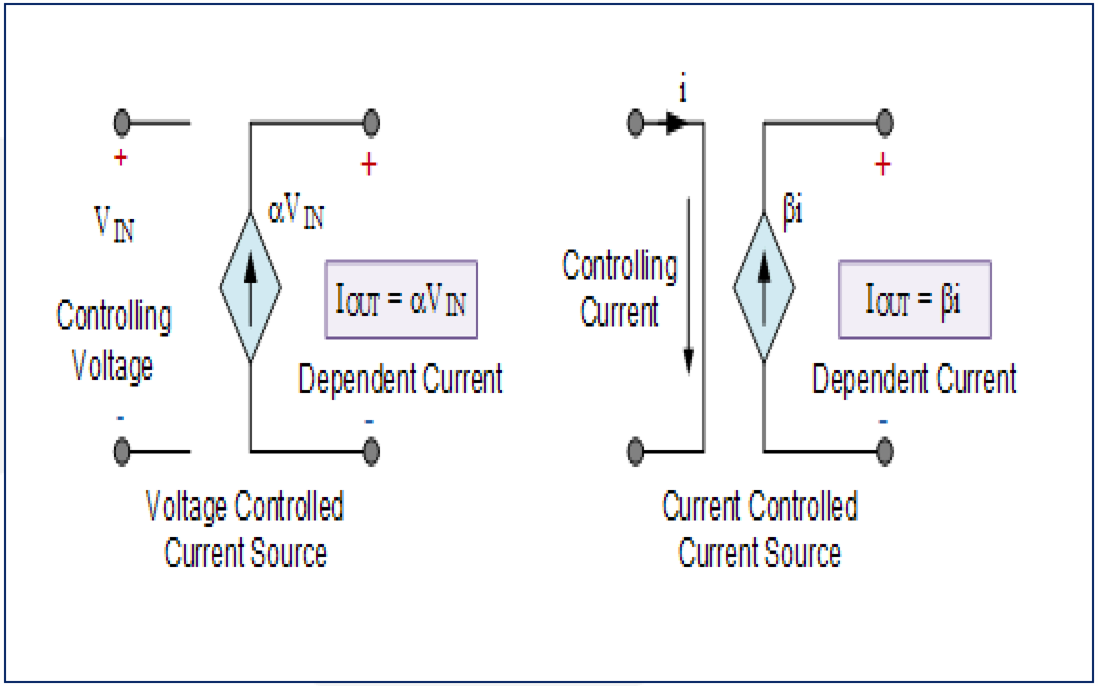
Dependent current source

➢A controlled or dependent current source changes its current depending upon the voltage across, or the current through, some other element connected to the circuit.

➢A current source that depends on a voltage input is generally referred to as a Voltage Controlled Current Source or VCCS. A current source that depends on a current input is generally referred too as a Current Controlled Current Source or CCCS.



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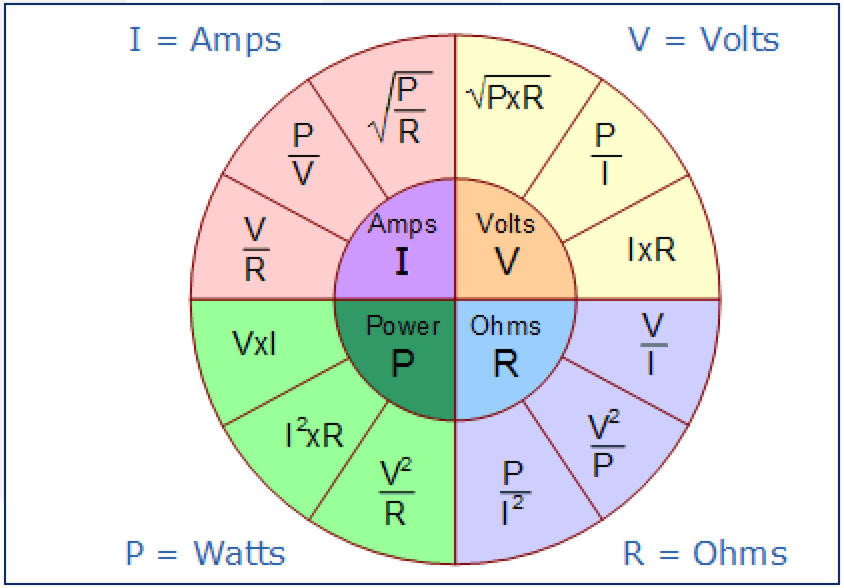
Ohm's Law

Georg Ohm found that, "**at a constant temperature, the electrical current flowing through a fixed linear resistance is directly proportional to the voltage applied across it, and also inversely proportional to the resistance**".

$$Current\ I = \frac{Voltage(V)}{Resistance(R)}\ in\ Ampers(A)$$

Ohm’s Law Pie Chart

By knowing any two values of the Voltage, Current or Resistance quantities we can use Ohms Law to find the third missing value. Ohms Law is used extensively in electronics formulas and calculations so it is “very important to understand and accurately remember these formulas”.



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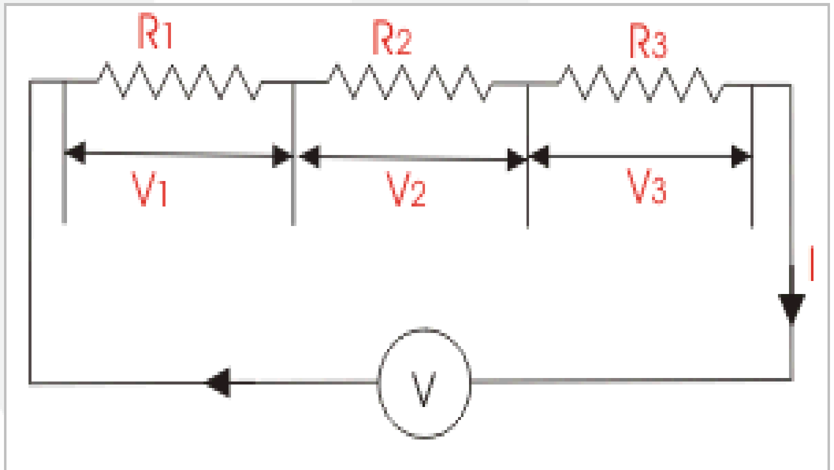
Series circuit

- When the resistors are connected end to end, so that they form only one path for the flow of current, then resistors are said to be connected in series and such circuits are known as series circuits

Now according to ohm's law
Voltage drop across resistors R1, R2,R3 are V1= IR1, V2= IR2, V3=IR3.
Total voltage V = IR1 + IR2 + IR3
= I (R1 + R2 +R3)
V/R = R1 + R2 +R3
According to ohm's law V/I gives the whole circuit resistance, R ohm.
So Total resistance of the circuit ,
R = R1 + R2 +R3



Series Circuit

Image source : Google

Voltage Divider Equation

- Suppose we want to find voltage V1, V2, and V3 in terms of voltage in above figure with out knowing the value of current we can apply voltage divider equation.

- Total current in the circuit , $I = V/ R1+R2+R3$ or $I = V/Req$ A

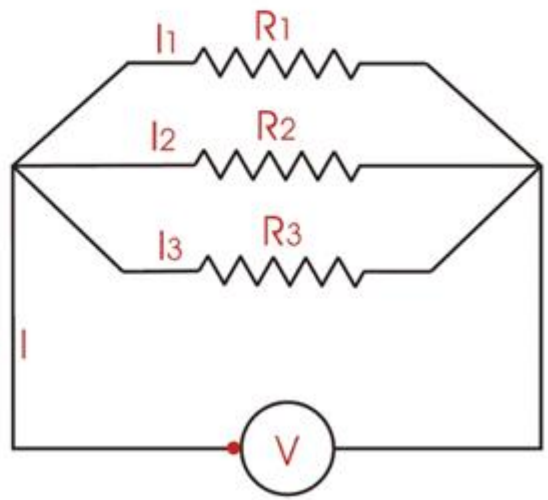
- By ohm's law $V1 = IR1$ volt or $V1= (R1/ Req) V$

Like $V2 = (R2/ Req) V$ and $V3 = (R3/ Req) V$

Parallel Circuit

When two or more electrical components are connected in a way that one end of each component is connected to a common point and the other end is connected to another common point, then the electrical components are said to be connected in parallel, and such an electrical DC circuit is referred as a **parallel DC circuit**

In this circuit every component will have the same voltage drop across them, and it will be exactly equal to the voltage which occurs between the two common points where the components are connected.



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Parallel Circuit Conti..

Suppose three resistors R_1, R_2 and R_3 are connected in parallel across a voltage source of V (volt) as shown in the figure. Let I (Ampere) be the total circuit current which is divided into current I_1, I_2 and I_3 flowing through R_1, R_2 and R_3 respectively. Now according to Ohm's law:

Voltage drop across resistor R_1, $V = I_1.R_1$

Voltage drop across resistor R_2, $V = I_2.R_2$

Voltage drop across resistor R_3, $V = I_3.R_3$

Voltage drop across whole parallel DC circuit,

V = Voltage drop across resistor R_1 = voltage drop across resistor R_2 = voltage drop across resistor R_3

$\Rightarrow V = I_1.R_1 = I_2.R_2 = I_3.R_3$

Parallel Circuit Conti..

$$Therefore,\ I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2},\ and\ I_3 = \frac{V}{R_3}$$

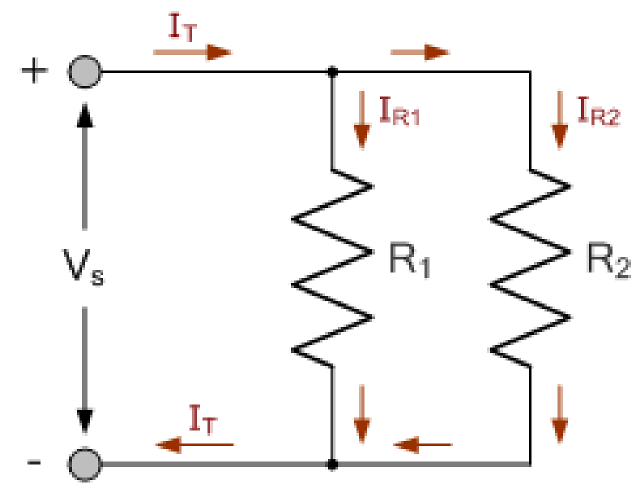
$$I = I_1 + I_2 + I_3\ and\ as\ per\ Ohm's\ law,\ I = \frac{V}{R}\ hence,$$

$$Therefore,\ I = \frac{V}{R} = I_1 + I_2 + I_3 = \frac{V}{R_1} = \frac{V}{R_2} = \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$Therefore\ from\ above\ expression\ we\ get,\ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Current divider Equation

Here this basic current divider circuit consists of two resistors: R_1, and R_2 in parallel which splits the supply or source current I_S between them into two separate currents I_{R1} and I_{R2} before joining together again and returning back to the source.



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Current divider Equation

Without knowing the value of applied voltage we can find the every branch current by applying current divider equation

$$I_T = I_{R1} + I_{R2}$$

$$I_{R1} = \frac{V}{R_1} \quad \text{and} \quad I_{R2} = \frac{V}{R_2}$$

$$I_T = \frac{V}{R_1} + \frac{V}{R_2} = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore\ V = I_T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1} = I_T\left[\frac{R_1 R_2}{R_1 + R_2}\right]$$

Current divider Equation

Solving for I_{R1} gives

$$I_{R1} = \frac{V}{R_1} = I_T\left[\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}}\right]$$

$$\therefore I_{R1} = I_T\left(\frac{R_2}{R_1+R_2}\right)$$

Solving for IR2 gives

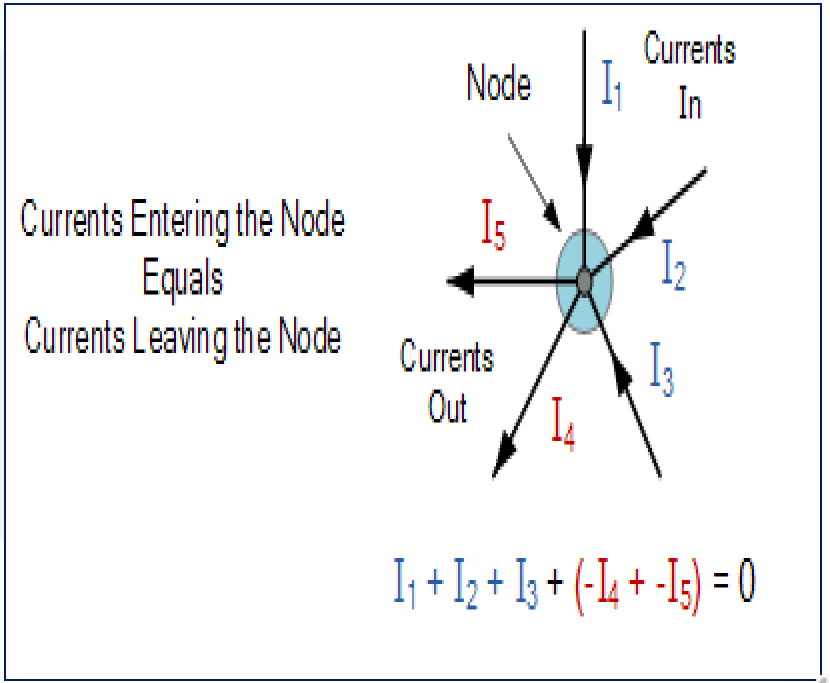
$$I_{R2} = \frac{V}{R_2} = I_T\left[\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}}\right]$$

$$\therefore I_{R2} = I_T\left(\frac{R_1}{R_1+R_2}\right)$$

Kirchhoff"s current law

Kirchhoffs Current Law or KCL, states that the "total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node". In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

$I_{(exiting)} + I_{(entering)} = 0$



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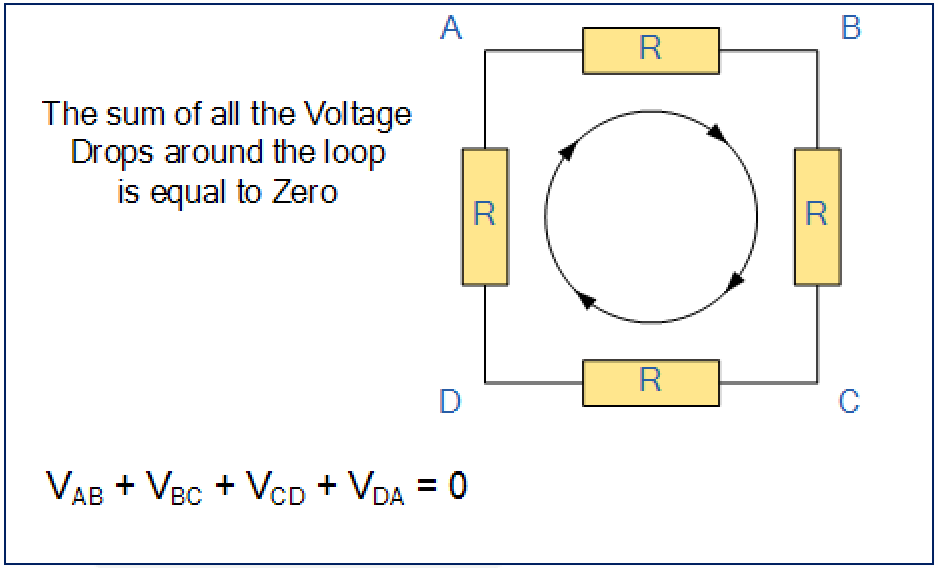
Kirchhoff"s Voltage law

Kirchhoffs Voltage Law or KVL, states that ***"in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop"*** which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero.



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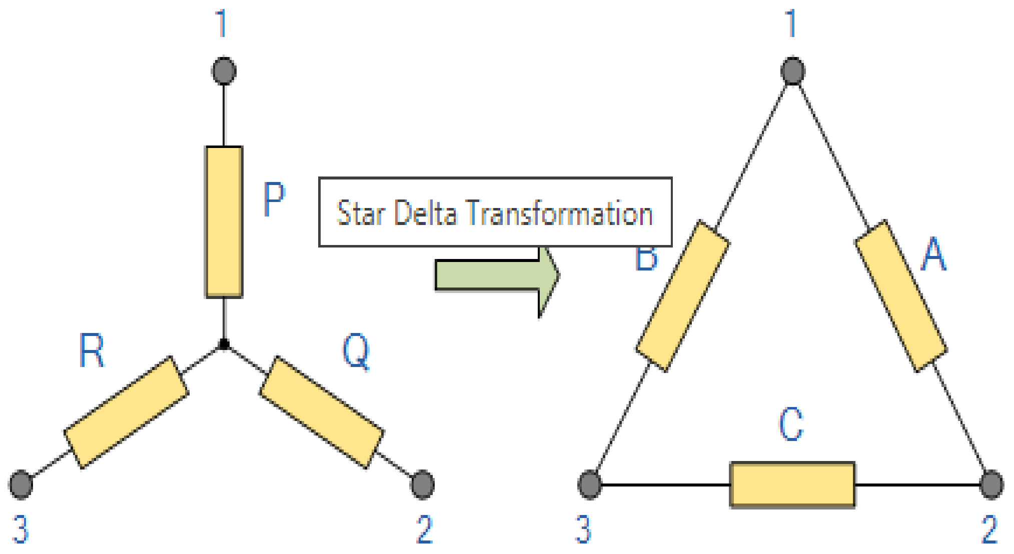
Star-Delta transformation

The value of the resistor on any one side of the delta, Δ network is the sum of all the two-product combinations of resistors in the star network divide by the star resistor located "directly opposite" the delta resistor being found. For example, resistor A is given as:



Star-Delta transformation

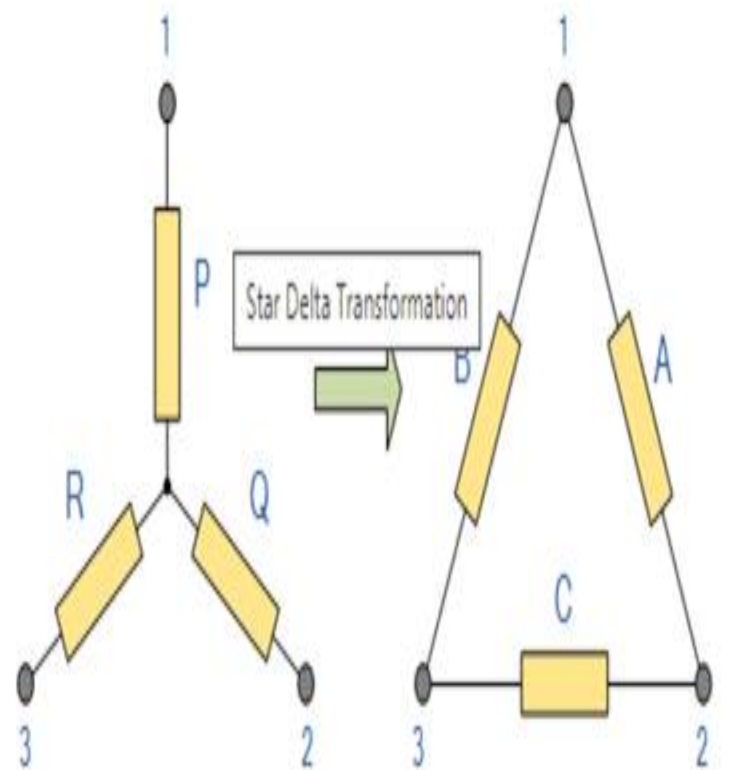
$$A = \frac{PQ + QR + RP}{R}$$

with respect to terminal 3 and resistor B is given as:

$$B = \frac{PQ + QR + RP}{Q}$$

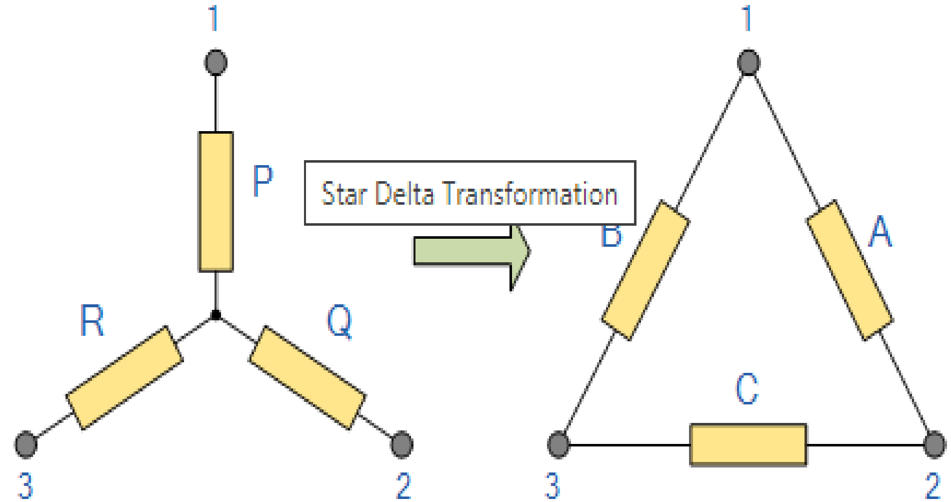
with respect to terminal 2 with resistor C given as:

$$C = \frac{PQ + QR + RP}{P}$$



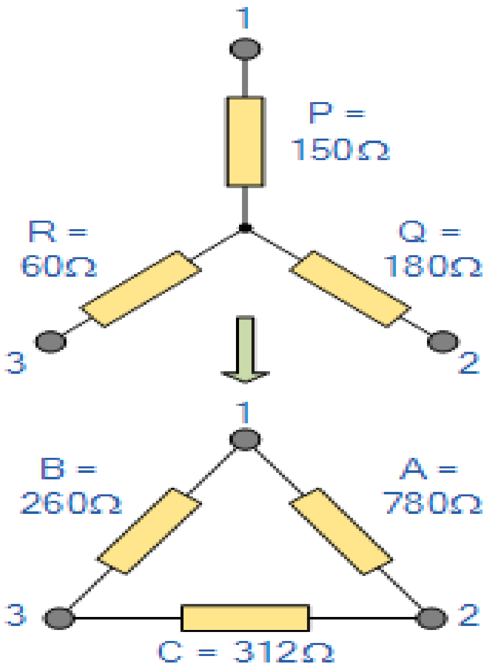
Star-Delta transformation

$$A = \frac{PQ}{R} + Q + P \qquad B = \frac{RP}{Q} + P + R \qquad C = \frac{QR}{P} + Q + R$$



Star – Delta Example

Convert the following Star Resistive Network into an equivalent Delta Network



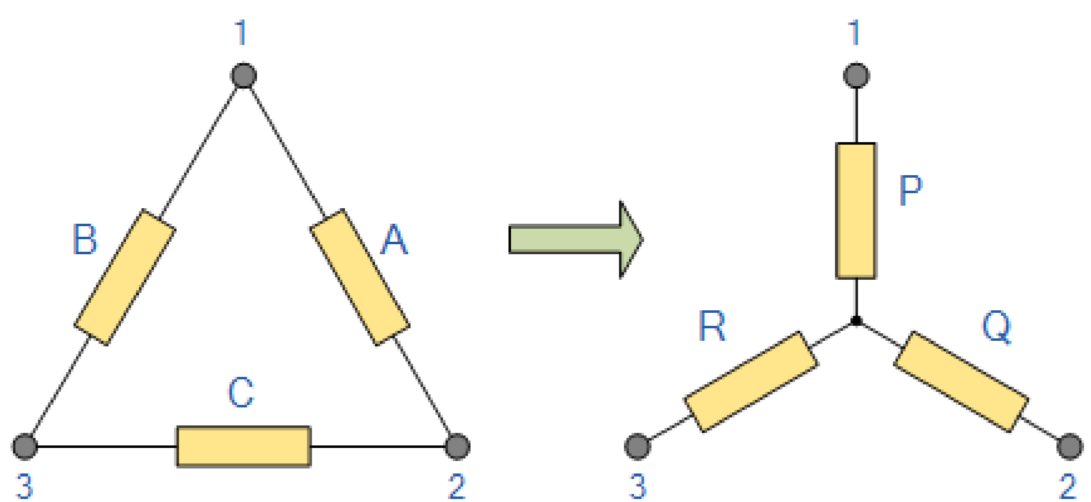
$$A = \frac{QP}{R} + Q + P = \frac{180 \times 150}{60} + 180 + 150 = 780\Omega$$

$$B = \frac{RP}{Q} + R + P = \frac{60 \times 150}{180} + 60 + 150 = 260\Omega$$

$$C = \frac{QR}{P} + Q + R = \frac{180 \times 60}{150} + 180 + 60 = 312\Omega$$

Delta-Star transformation

To convert a delta network to an equivalent star network we need to derive a transformation formula for equating the various resistors to each other between the various terminals. Consider the circuit below.



Delta-Star transformation

From which gives us the final equation for resistor P as:

$$P = \frac{AB}{A+B+C}$$

Then to summarize a little about the above maths, we can now say that resistor P in a Star network can be found as Equation 1 plus (Equation 3 minus Equation 2) or Eq1 + (Eq3 – Eq2).

Delta-Star transformation

Similarly, to find resistor Q in a star network, is equation 2 plus the result of equation 1 minus equation 3 or Eq2 + (Eq1 – Eq3) and this gives us the transformation of Q as:

$$Q = \frac{AC}{A + B + C}$$

and again, to find resistor R in a Star network, is equation 3 plus the result of equation 2 minus equation 1 or Eq3 + (Eq2 – Eq1) and this gives us the transformation of R as:

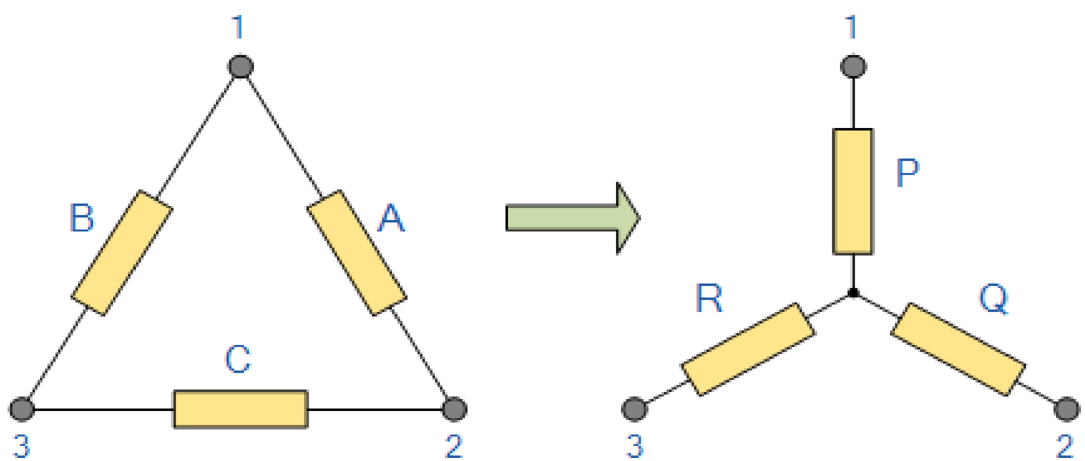
$$R = \frac{BC}{A + B + C}$$

Delta-Star transformation

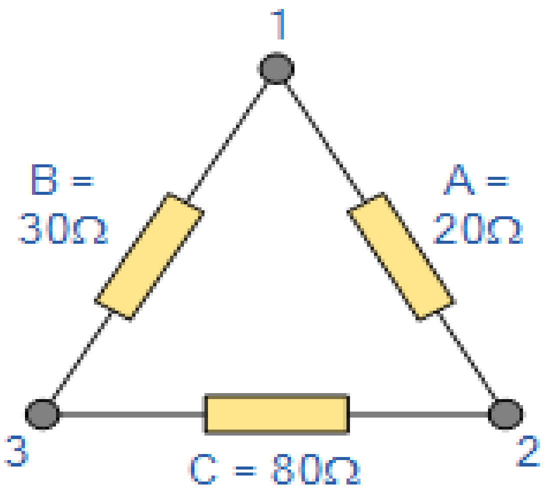
When converting a delta network into a star network the denominators of all of the transformation formulas are the same: A + B + C, and which is the sum of ALL the delta resistances. Then to convert any delta connected network to an equivalent star network we can summarized the above transformation equations as:

Delta-Star transformation

$$P = \frac{AB}{A+B+C} \quad Q = \frac{AC}{A+B+C} \quad R = \frac{BC}{A+B+C}$$



Delta-Star Example



$$Q = \frac{AC}{A+B+C} = \frac{20\times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20\times 30}{130} = 4.61\Omega$$

$$R = \frac{BC}{A+B+C} = \frac{30\times 80}{130} = 18.46\Omega$$

Superposition Theorem

Statement

- Superposition theorem states that in any linear, active, bilateral network with more than one source, the response across any component is the sum of the responses obtained from each source considered separately and their internal resistance is replaced by all other sources.

- **Limitation of Superposition theorem**

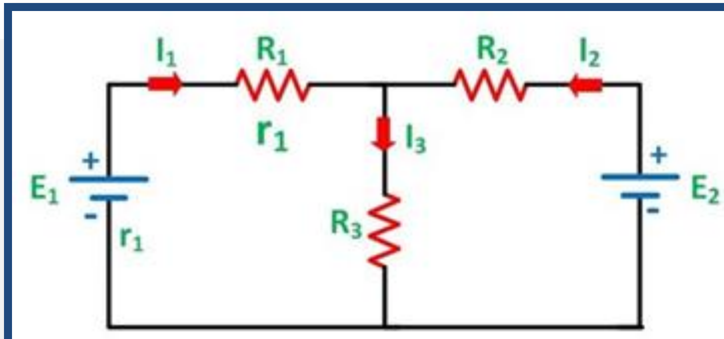
The theorem of superposition is used to resolve the network where two or more sources exist and are connected.

Steps for solving Theorem

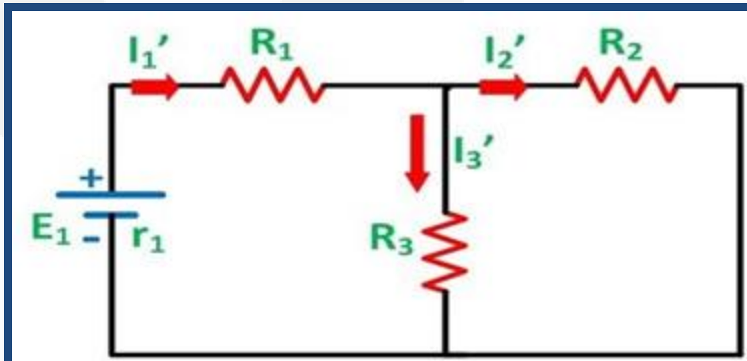
Step 1 – Take only one independent source of voltage or current and deactivate the other sources.

Step 2 – In the circuit diagram B shown above, consider the source E_1 and replace the other source E_2 by its internal resistance. If its internal resistance is not given, then it is taken as zero and the source is short-circuited.

Step 3 – If there is a voltage source than short circuit it and if there is a current source then just open circuit it.



Circuit Diagram A



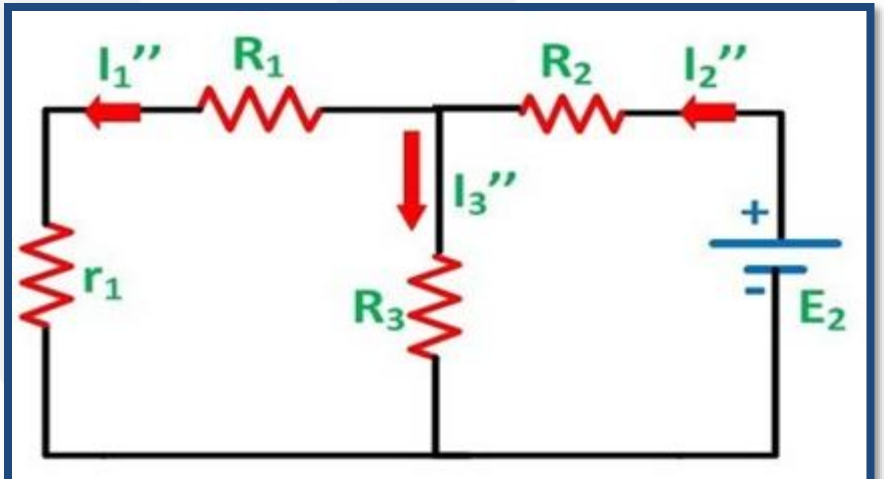
Circuit Diagram B

Steps for solving Theorem

Step 4 – Thus, by activating one source and deactivating the other source find the current in each branch of the network. Taking the above example find the current I_1', $I_2'$and I_3'.

Step 5 – Now consider the other source E_2 and replace the source E_1 by its internal resistance r_1 as shown in the circuit diagram C.

Step 6 – Determine the current in various sections, I_1'', I_2'' and I_3''.



Circuit Diagram C

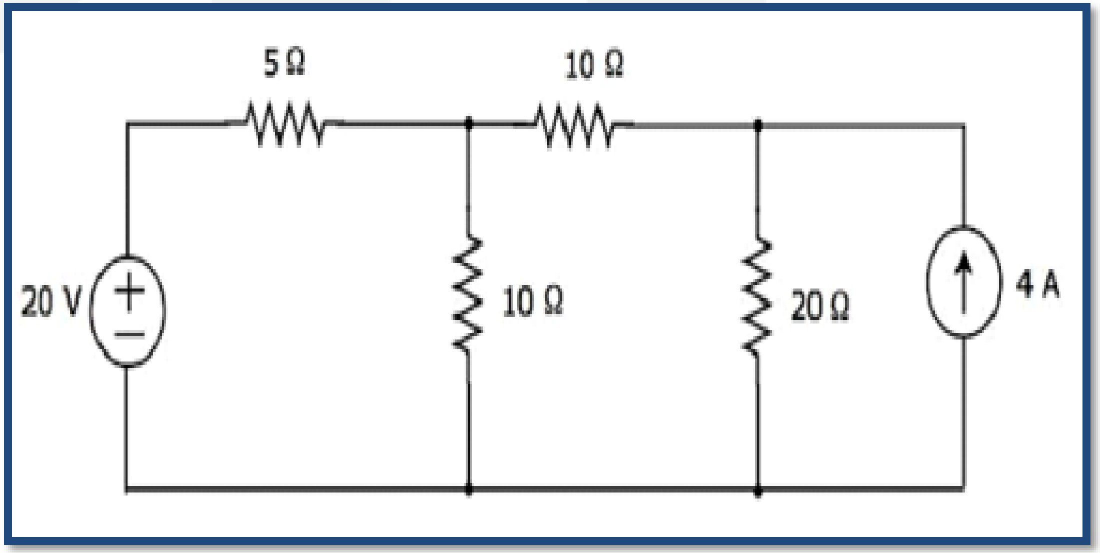
Steps for solving Theorem

Step 7 – Now to determine the net branch current utilizing the superposition theorem, add the currents obtained from each individual source for each branch.

Step 8 – If the current obtained by each branch is in the same direction then add them and if it is in the opposite direction, subtract them to obtain the net current in each branch.

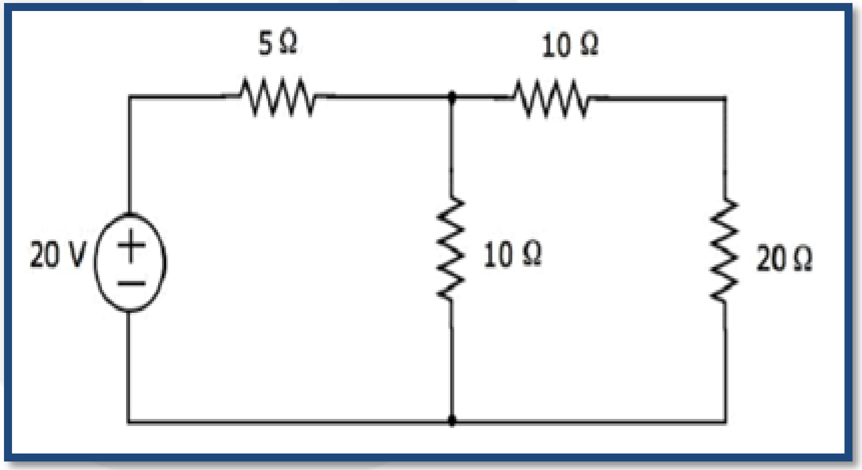
Examples of Superposition Theorem

- Find the current flowing through 20 Ω resistor of the following circuit using **superposition theorem**.



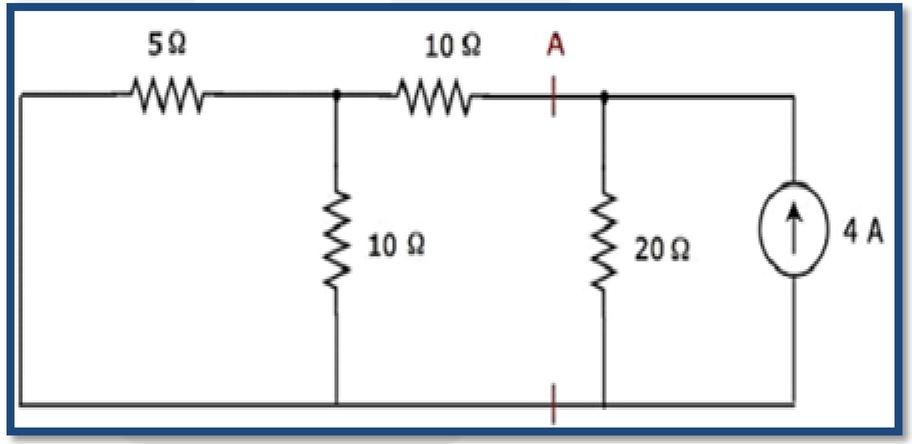
Examples of Superposition Theorem

Let us find the current flowing through 20 Ω resistor by considering only **20 V voltage source**. In this case, we can eliminate the 4 A current source by making open circuit of it.



Examples of Superposition Theorem

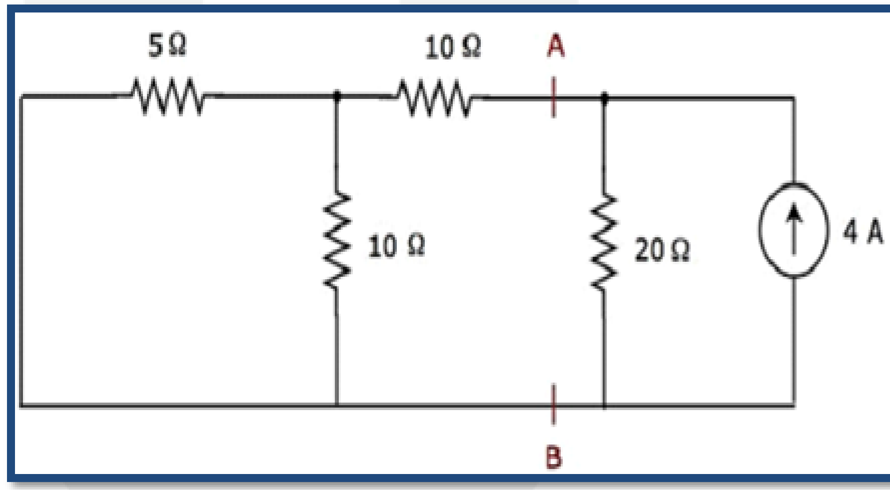
Let us find the current flowing through 20 Ω resistor by considering only **4 A current source**. In this case, we can eliminate the 20 V voltage source by making short-circuit of it.



Examples of Superposition Theorem

Let us find the current flowing through 20 Ω resistor by considering only **4 A current source**. In this case, we can eliminate the 20 V voltage source by making short-circuit of it.

There are three resistors to the left of terminals A & B. We can replace these resistors with a single **equivalent resistor**. Here, 5 Ω & 10 Ω resistors are connected in parallel and the entire combination is in series with 10 Ω resistor.

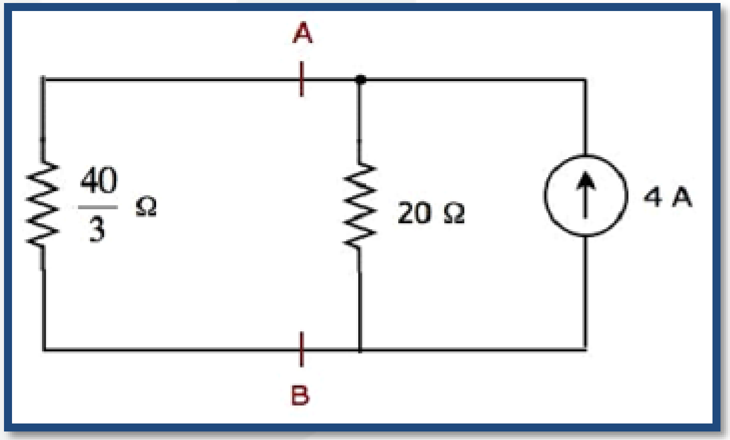


$$R_{AB}=((5\times10)/(5+10))+10$$
$$=10/3+10=40/3\Omega$$

Examples of Superposition Theorem

We can find the current flowing through 20 Ω resistor, by using **current division principle**

the current flowing through 20 Ω resistor is 1.6 A, when only 4 A current source is considered.



Thevenin’s theorem:

Statement :

Thevenin’s Theorem is that any linear active network consisting of independent or and elements can be replaces by voltage source in series with resistance.

Where the voltage source being the open-circuited voltage across the open-circuited load terminals and the resistance being the internal resistance of the source.

Thevenin’s theorem:

In other words, the current flowing through a resistor connected across any two terminals of a network by an equivalent circuit having a voltage source Eth in series with a resistor Rth.
Where ;

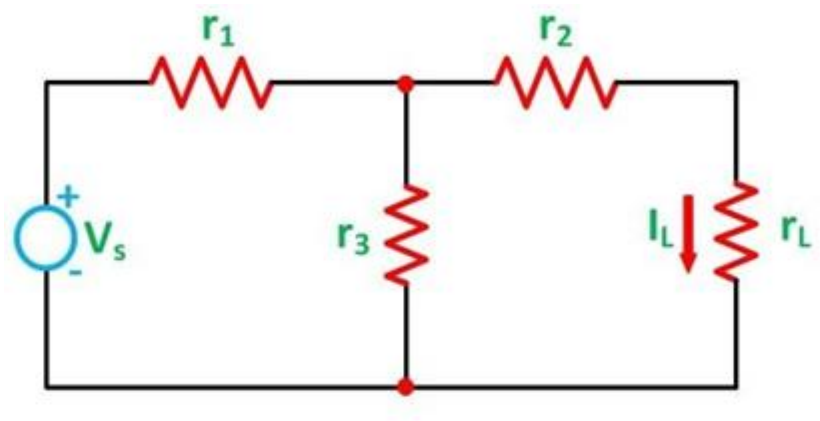
Eth is the open-circuit voltage between the required two terminals called the Thevenin voltage

And

Rth is the equivalent resistance of the network as seen from the two-terminal with all other sources replaced by their internal resistances called Thevenin resistance.

Thevenin’s theorem :

Let us consider a simple DC circuit as shown in the figure above, where we have to find the load current IL by the Thevenin’s theorem.



Steps for solving Thevenin's circuit for finding IL:

Step 1 – First of all remove the load resistance rL of the given circuit.

Step 2 – Replace all the sources by their internal resistance.

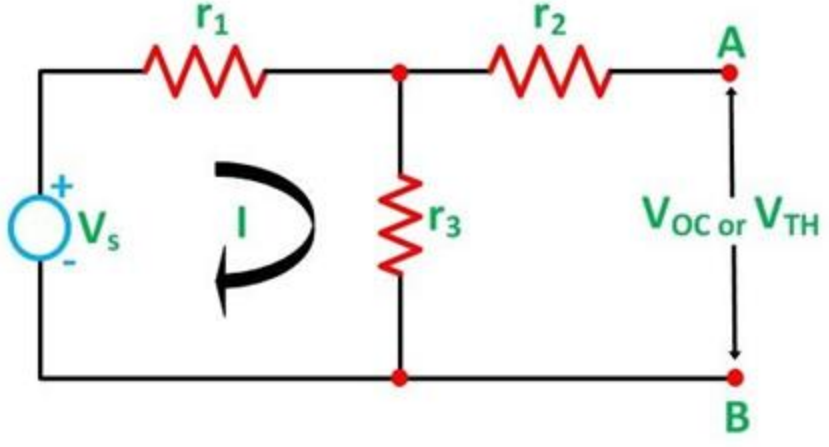
Step 3 – If sources are ideal then short circuit the voltage source and open circuit the current source.

Step 4 – Now find the equivalent resistance at the load terminals, known as Thevenin's Resistance (RTH).

Step 5 – Draw the Thevenin's equivalent circuit by connecting the load resistance and after that determine the desired response.

Steps for solving Thevenin's circuit for finding :

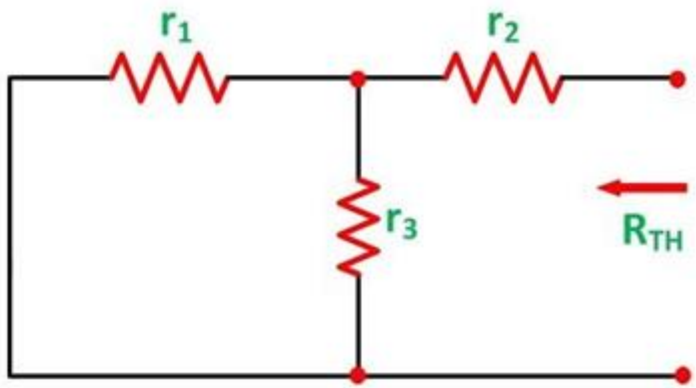
In order to find the equivalent voltage source, rL is removed from the circuit as shown in the figure below and Voc or VTH is calculated.



$$V_{OC} = I\, r_3 = \frac{V_S}{r_1 + r_3}\, r_3$$

Steps for solving Thevenin’s circuit :

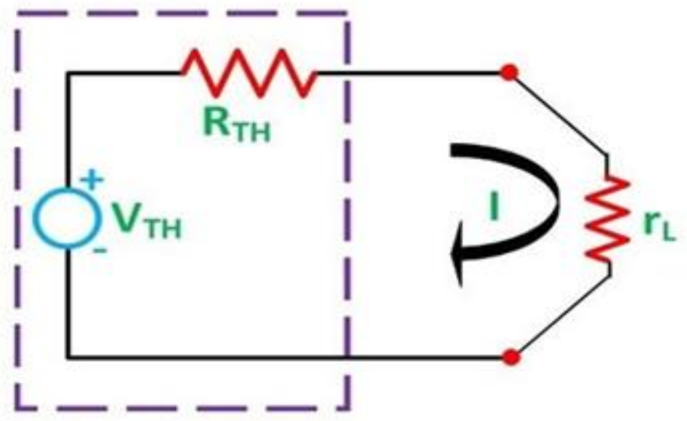
Now, to find the internal resistance of the network (Thevenin’s resistance or equivalent resistance) in series with the open-circuit voltage VOC , also known as Thevenin’s voltage VTH, the voltage source is removed or we can say it is deactivated by a short circuit (as the source does not have any internal resistance) as shown in the figure below:



$$R_{TH} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$

Steps for solving Thevenin's circuit :

As per Thevenin's Statement, the load current is determined by the circuit shown above and the equivalent Thevenin's circuit is obtained.
The load current IL is given as:



Steps for solving Thevenin's circuit :

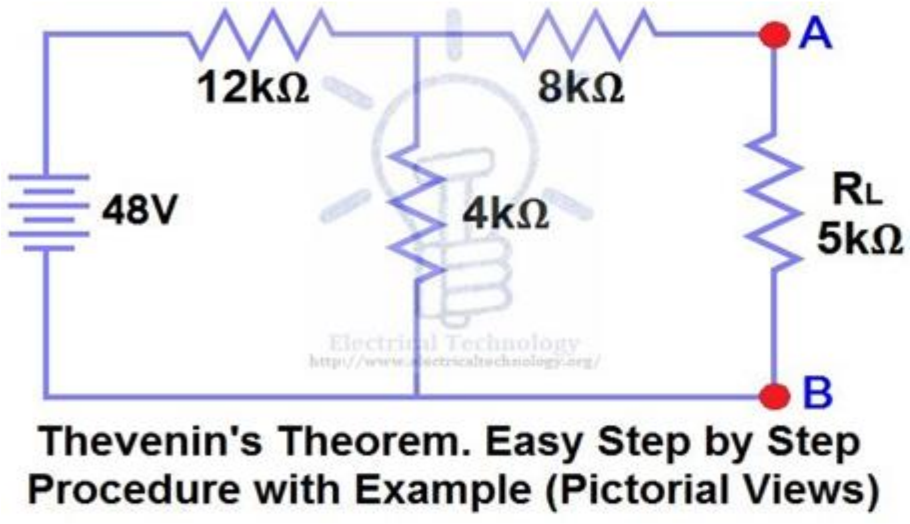
The load current IL is given as:

$$I_L = \frac{V_{TH}}{R_{TH} + r_L}$$

Where,
VTH is the Thevenin's equivalent voltage. It is an open circuit voltage across the terminal AB known as load terminal.
RTH is the Thevenin's equivalent resistance, as seen from the load terminals.
rL is the load resistance

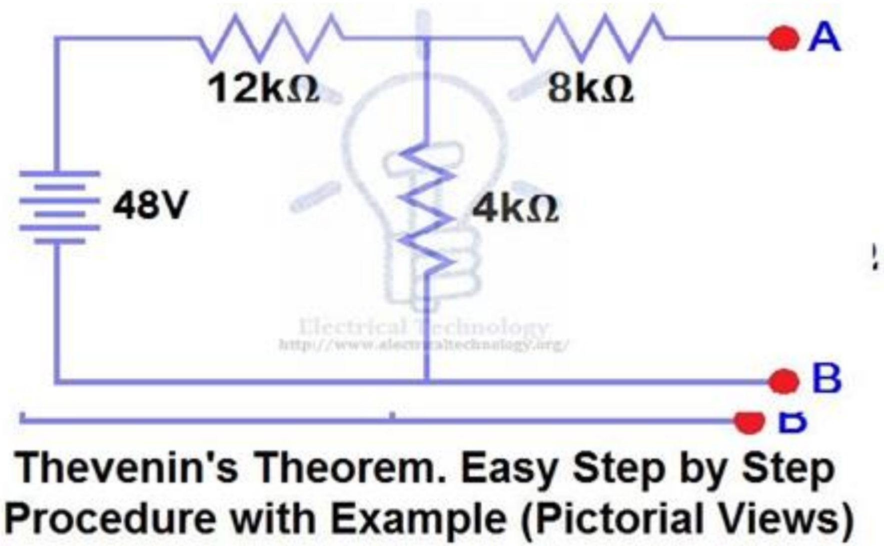
Thevenin’s theorem example :

Find VTH, RTH and the load current IL flowing through and load voltage across the load resistor in fig (1) by using Thevenin’s Theorem.



Thevenin's theorem example :

Step 1. Open the 5kΩ load resistor



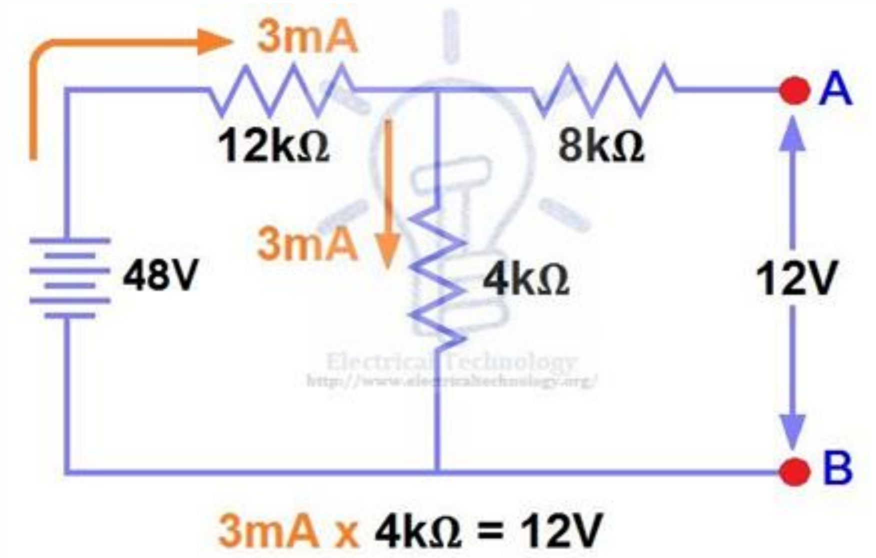
Thevenin's theorem example :

Step 2.
Calculate / measure the open circuit voltage. This is the Thevenin Voltage (VTH)
Now we have to calculate the Thevenin's Voltage.

Since 3mA current flows in both 12kΩ and 4kΩ resistors as this is a series circuit and current will not flow in the 8kΩ resistor as it is open.

Total current is = 16k-ohm/48 v = 3 mA

Thevenin's theorem example :



Since 3mA current flows in both 12kΩ and 4kΩ resistors as this is a series circuit and current will not flow in the 8kΩ resistor as it is open.

Thevenin's theorem example :

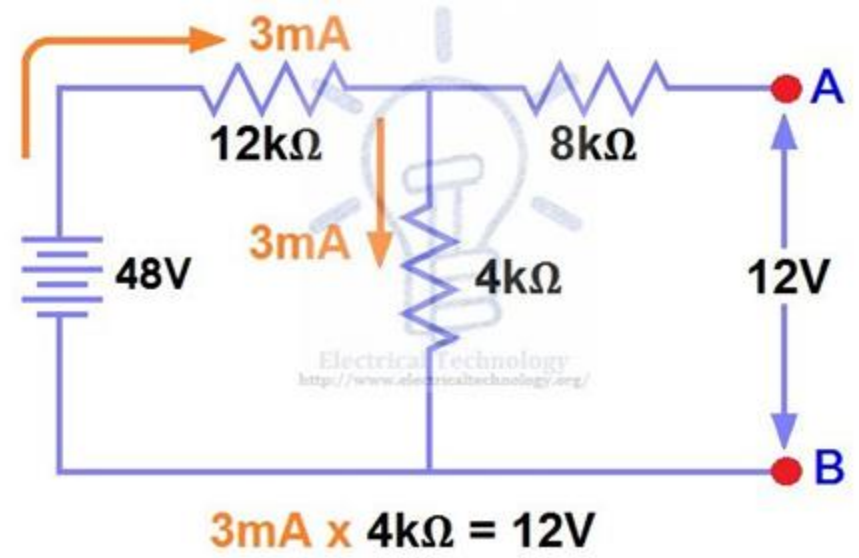
This way, 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is an open circuit, but the 8kΩ resistor is in parallel with 4k resistor.

So the same voltage 12V will appear across the 8kΩ resistor as well as 4kΩ

resistor. Therefore 12V will appear across the AB terminals.

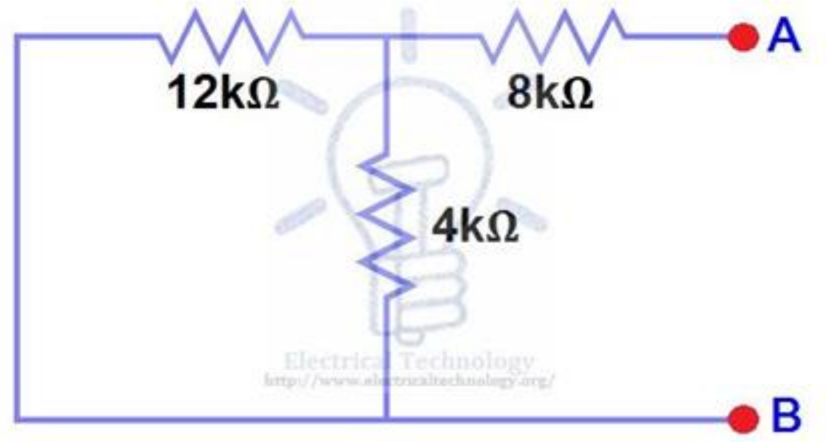
VTH = 12V

Thevenin’s theorem



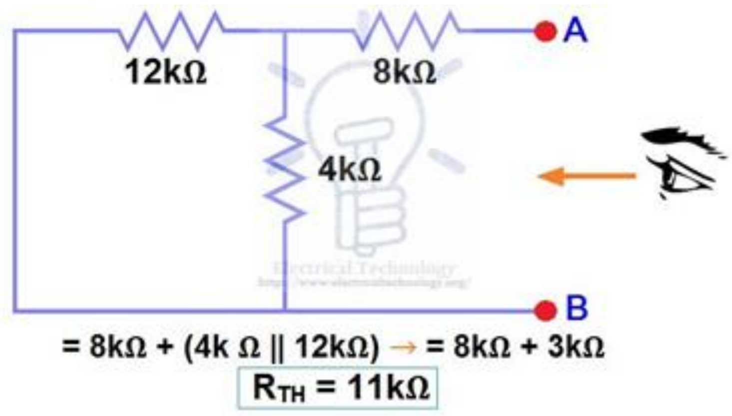
Thevenin’s theorem example :

Step.3 Open current sources and short voltage sources as shown below Calculate / measure the open circuit resistance. This is the Thevenin Resistance (RTH)



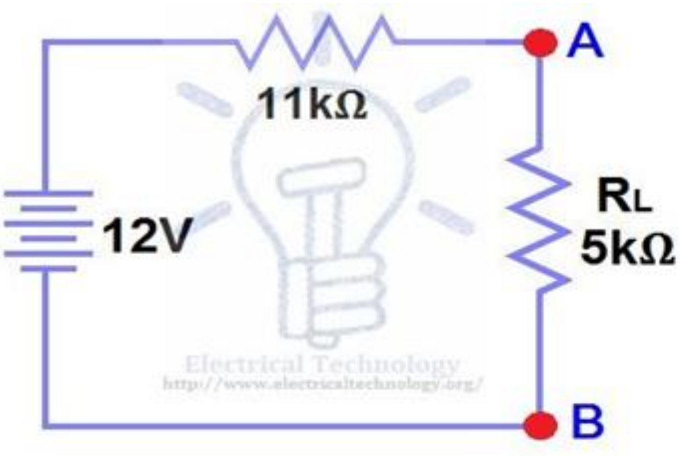
Thevenin’s theorem example :

Step.4 We have removed the 48V DC source to zero as equivalent i.e. 48V DC source has been replaced with a short in step 3.
We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor



Thevenin's theorem example :

Connect the RTH in series with Voltage Source VTH and re-connect the load resistor. This makes Thevenin circuit with load resistor. This the Thevenin's equivalent circuit.



Thevenin's theorem example :

step.5 Now apply the last step i.e Ohm's law . Calculate the total load current and load voltage as shown in fig 6.

$IL = VTH / (RTH + RL)$

$IL = 12V / (11k\Omega + 5k\Omega) \rightarrow = 12/16k\Omega$ $IL = 0.75mA$

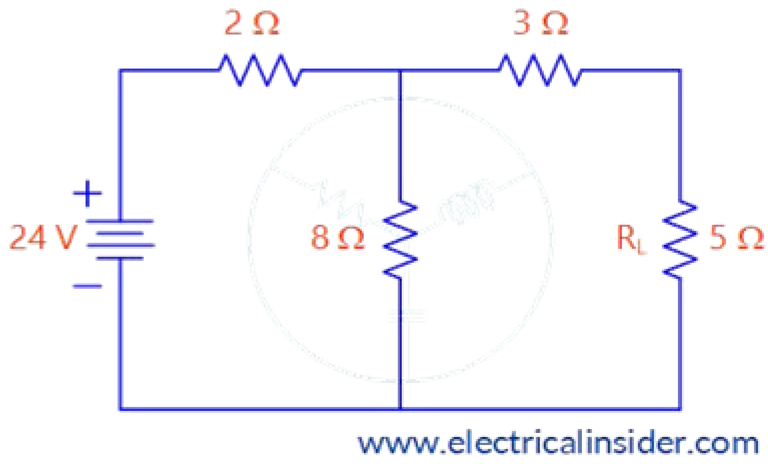
And

$VL = IL \times RL$

$VL = 0.75mA \times 5k\Omega$

Thevenin's theorem example :

Example.1 Find the load current and power delivered to the load, using thevenin's theorem.



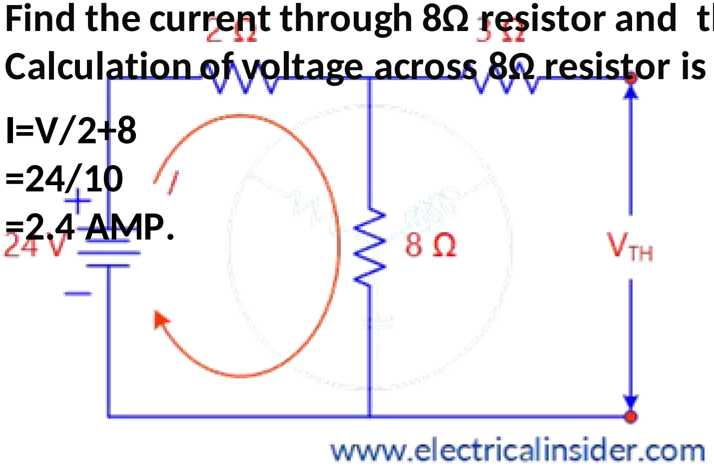
Thevenin’s theorem example :

Step 1

Open the load resistor (5Ω) and find the voltage across the load terminals.

Find the current through 8Ω resistor and then calculate the Thevenin’s voltage. Calculation of voltage across 8Ω resistor is given below.

I=V/2+8
=24/10
=2.4 AMP.

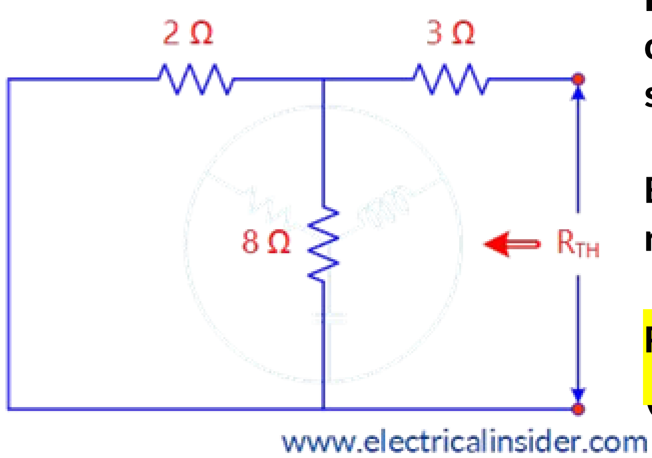


VTH=VOC=2.4*
=19.2 VOLT

Thevenin’s theorem example :

Step 2

Find the Thevenin’s equivalent resistance of the network which is seen from the load terminals. Here replace the 24V voltage source by a short circuit to find the equivalent resistance.



In the diagram, 8Ω and 2Ω resistors are connected in parallel and this combination is in series with 3Ω resistor.

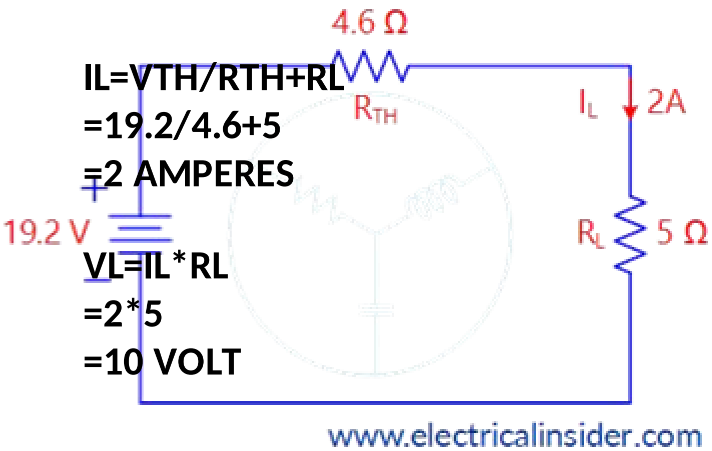
By network reduction techniques, the equivalent resistance is calculated as follows.

RTH=Req= 3+
=4.6
ohm

Thevenin’s theorem example :

Step 3

Now draw the thevenin’s equivalent circuit for the given circuit.
Draw the thevenin’s voltage in series with thevenin’s resistance and add the load resistor in series with the circuit.



IL=VTH/RTH+RL
=19.2/4.6+5
=2 AMPERES

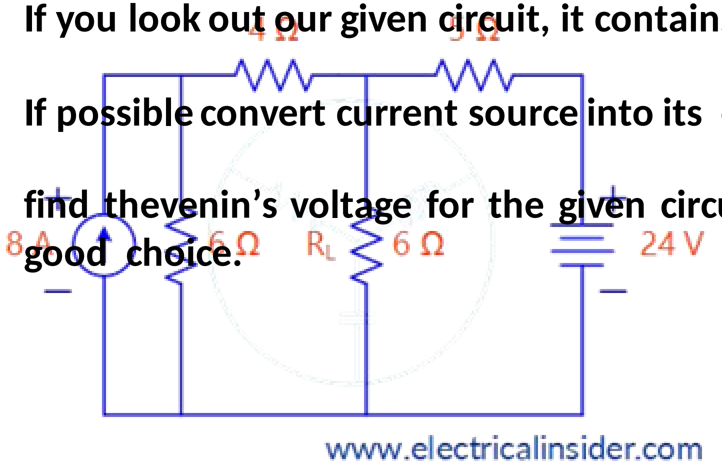
VL=IL*RL
=2*5
=10 VOLT

Thevenin’s theorem example :

EXAMPLE:2 Calculate the current through 6Ω load resistor using thevenin’s theorem.

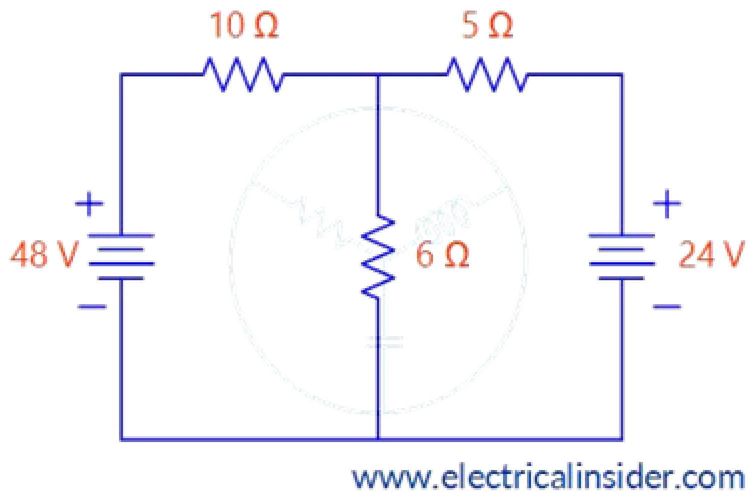
If you look out our given circuit, it contains a current source.

If possible convert current source into its equivalent voltage source. Since we need to find thevenin’s voltage for the given circuit, having a voltage source in our circuit is a good choice.



Thevenin’s theorem example :

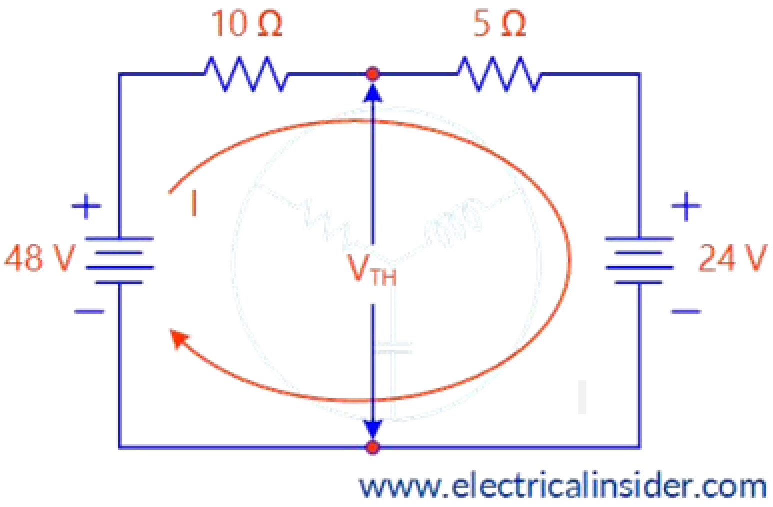
So, the simplified circuit with the voltage source is given below.



Thevenin’s theorem example :

Step 1

To find thevenin’s voltage, remove the load resistor (6Ω) and find the voltage across the terminal AB.



Thevenin's theorem example :

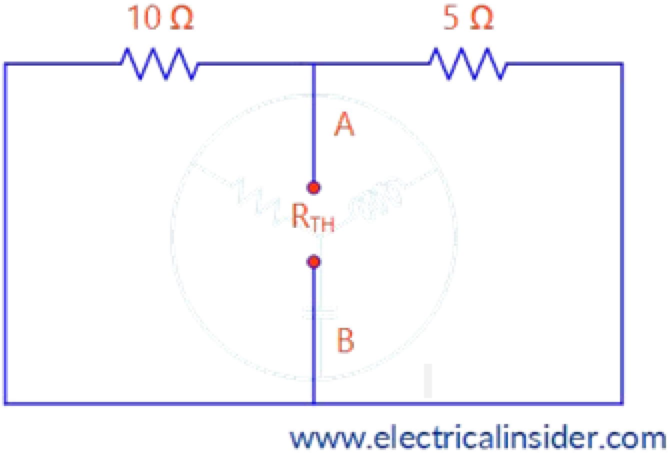
- The voltage at terminal AB will be the subtraction of voltage drop occurs at 10Ω resistor from the 48V voltage source.
- By solving mesh equations, you will get the current flows in the circuit. From the current you can calculate the voltage drop at 10Ω resistor.

VTH=VAB=48-
=32 VOLT

- The thevenin voltage calculation by mesh analysis is given below. 48-10I-5I-24=0
- 15I=24 I=24/15

Thevenin’s theorem example :

Step 2
Remove the load resistor and find the equivalent resistance of the network seen from the open circuited terminals.
In order to perform the calculation, short the 48V, and 24V voltage sources and then calculate the resistance.



Thevenin's theorem example :

Here the 10Ω and 5Ω resistors are connected in parallel. So the effective resistance will be as given below.

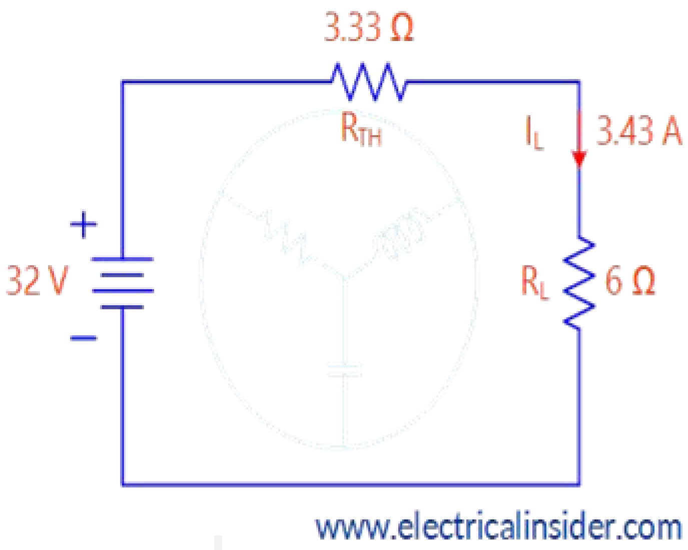
RTH=10*5/10+
5
=3.33 OHM

Thevenin’s theorem example :

Step 3

Now, determine the Thevenin’s equivalent circuit with thevenin’s voltage and thevenin’s resistance along with the load resistor.

Draw the thevenin’s voltage in series with thevenin’s resistance and add the load resistor in series with the circuit as shown below.



Thevenin’s theorem example :

LOAD CURRENT; IL= VTH/RTH+RL
=32/3.33+6
= 3.43 AMPERES

LOAD VOLTAGE VL= IL* RL
=3.43*6
=20.58 VOLT.

NORTON’S THEOREM

NORTON'S THEOREM

STATEMENT: Any linear, active, bilateral dc network having a number of voltage sources and/or current sources with resistances can be replaced by a simple equivalent circuit having single current source (I_N) in parallel with a single resistance (R_N).

Where (I_N) is the known as Norton's equivalent current through the terminal a-b.

(R_N) is the Norton's equivalent resistance viewed back into the network from terminal a-b.

NORTON’S THEOREM

Procedure for converting any circuit into Norton's equivalent circuit

Calculate Norton Current

Step 1: remove the load resistance R_L (through which current is required) and short circuit it. Let terminals of load are labelled as a-b. Therefore a-b is the short circuited.

Step 2: Find the current through the terminal a-b by applying KCL, KVL, Ohm’s law or Superposition principle. This current is the short circuit current and it is known as Nortons equivalent current (I_N).

NORTON'S THEOREM

Calculate Norton Resistance (equal to Thevinin resistance)

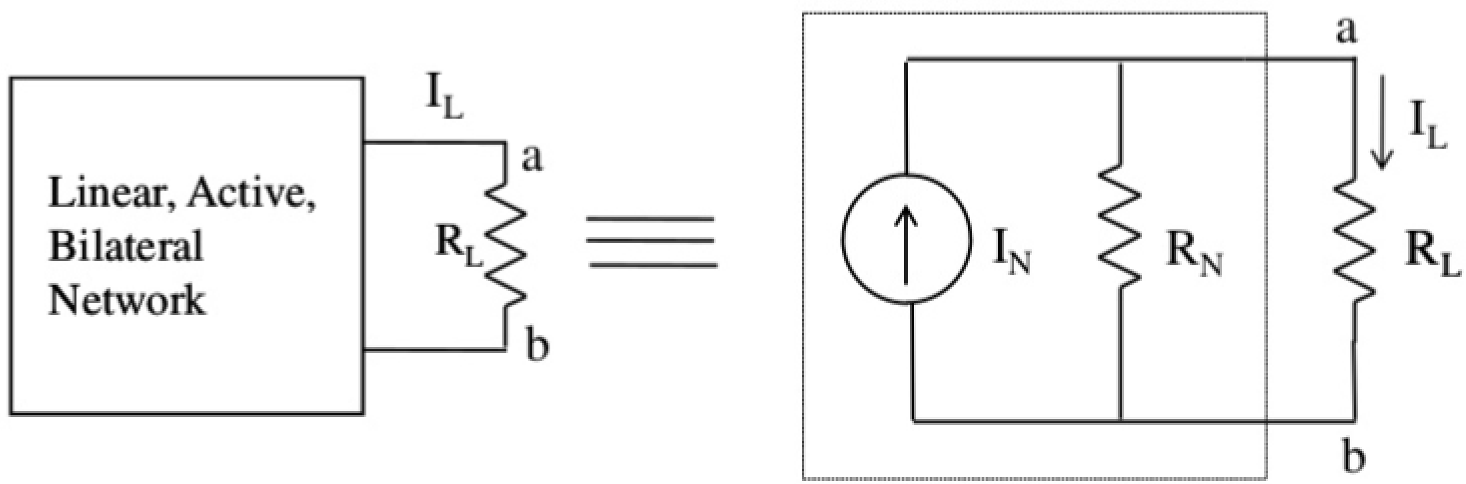
Step 3: Set all Independent voltage Sources as short circuit and Current Sources open circuit. Dependent sources will not be changed

Step 4: Calculate the resistance as "seen" through the terminals a-b into the network. This resistance is known as Norton's equivalent resistance (R_N).

Draw Equivalent Circuit

Step 5: Replace the entire network by Nortons equivalent current (I_N) in parallel with Norton's equivalent resistance (R_N) and connect the load resistance R_L.

NORTON'S THEOREM



Norton's Equivalent Network

NORTON'S THEOREM

Example: Find the current through 3 ohm resistor by Norton's Theorem for the network shown in fig.1a

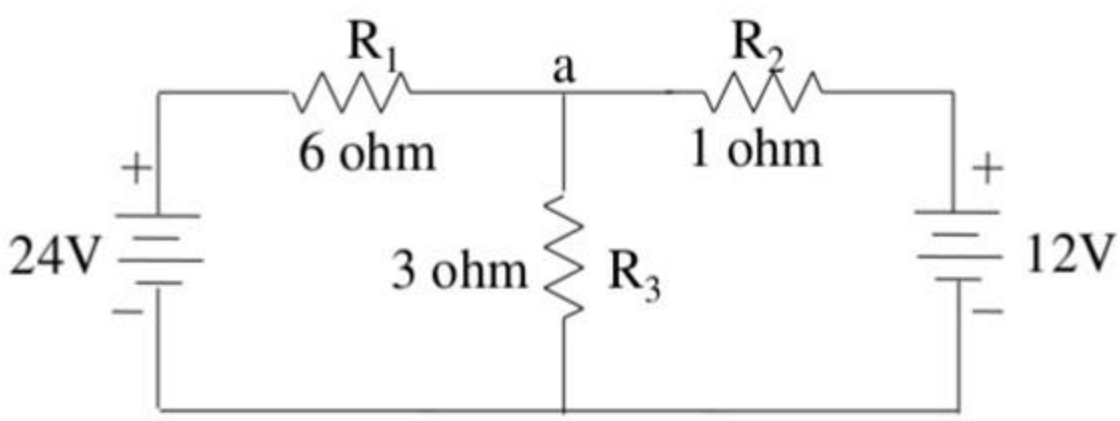


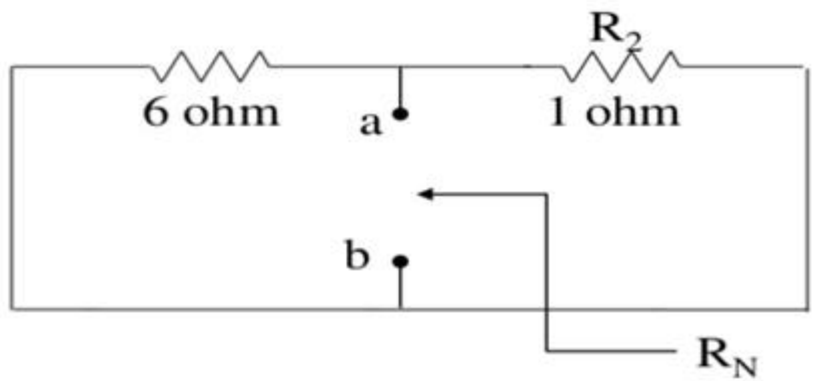
Fig. 1a

SOLUTION:

STEP 1: Calculation of R_N (calculation is same as R_{th}). Redraw the circuit by removing the 3 ohm resistor and short circuit the voltage sources as shown in fig. 1b

NORTON'S THEOREM

Fig. 1b

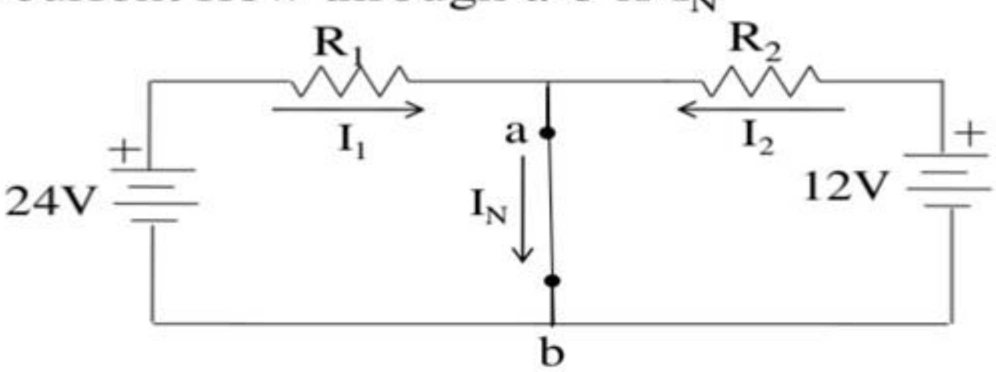


R_1 and R_2 are in parallel

$$R_N = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 1}{6+1} = 0.857\Omega$$

Step2: Calculation of Norton's Current I_N : Short circuit the terminals a-b and the current flow through a-b is I_N

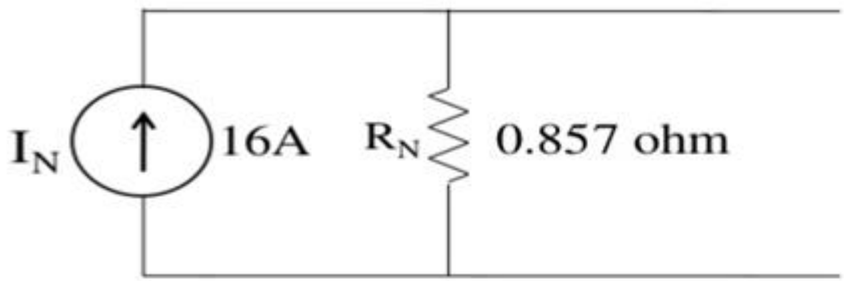
Fig. 1c



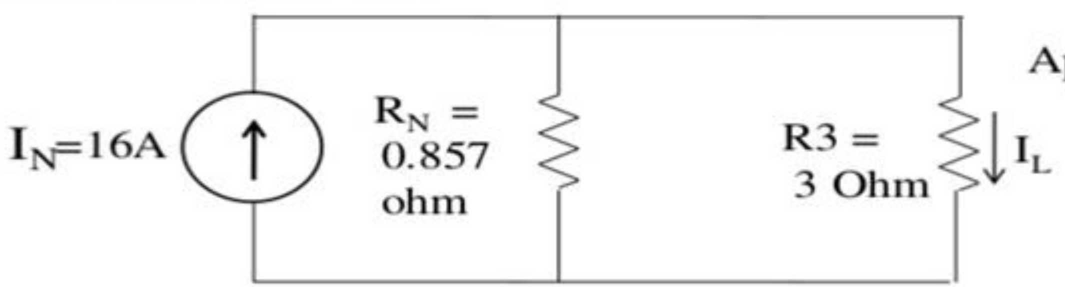
$$I_N = I_1 + I_2 = \frac{24}{6} + \frac{12}{1} = 16A$$

NORTON’S THEOREM

Step2: Draw the Norton’s Equivalent Circuit:



Step3: Calculation of Current through R3, Reconnect R3 to Norton’s Equivalent Circuit (Fig. 1e)



Apply Current divider rule

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

$$I_L = 16 \frac{0.857}{0.857 + 3} = 3.55A$$

NORTON'S THEOREM

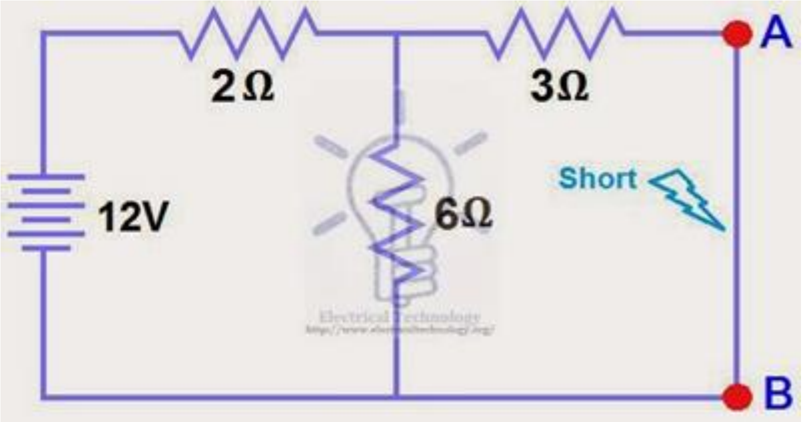
Find the Norton equivalent circuit to the left of terminals A-B for the network shown below. Connect the Norton equivalent circuit to the load and find the current in the 1.5 Ω resistor.

Norton's Theorem. Easy Step by Step Procedure with Example (Pictorial Views)

NORTON’S THEOREM

STEP 1.

Short the 1.5Ω load resistor as shown in Fig .



NORTON'S THEOREM

STEP 2.

Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).

We have shorted the AB terminals to determine the Norton current, $I_{N.}$ The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.

So the Total Resistance of the circuit to the Source is:-
2Ω + (6Ω || 3Ω) ….. (|| = in parallel with).

$R_T = 2\Omega + [(3\Omega \times 6\Omega) / (3\Omega + 6\Omega)] \rightarrow I_T = 2\Omega + 2\Omega = 4\Omega.$

$R_T = 4\Omega$

NORTON'S THEOREM

STEP 2.

$I_T = V / R_T$

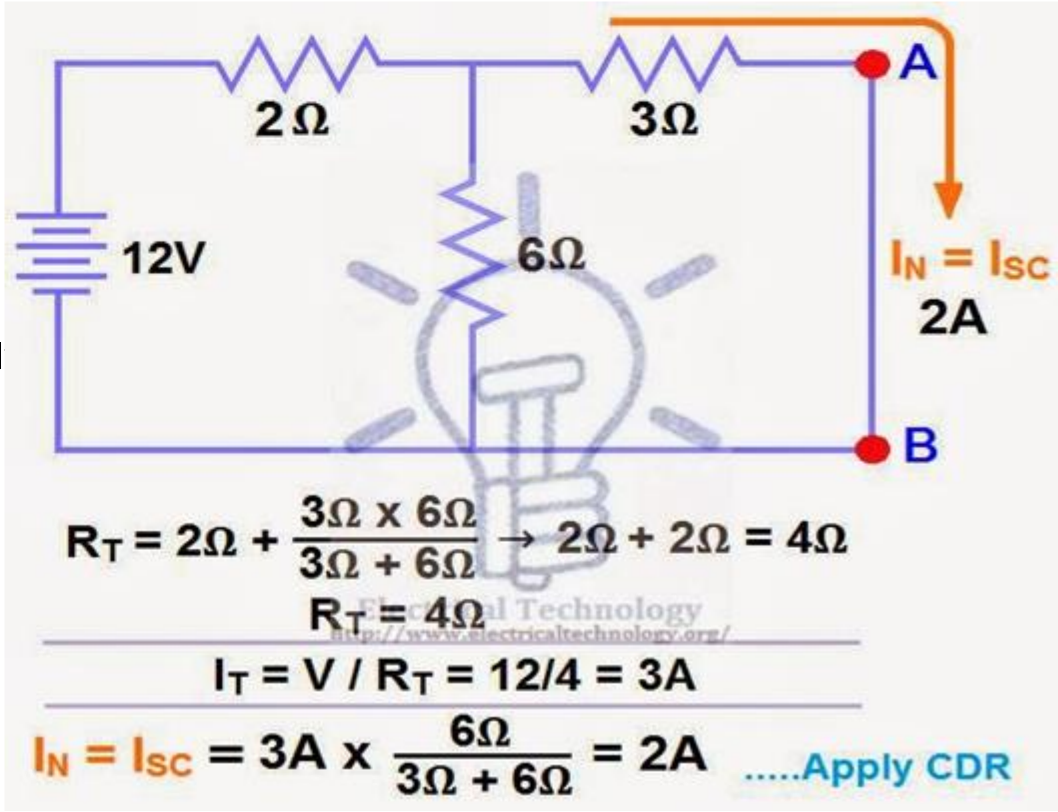
$I_T = 12V / 4\Omega$

$I_T = 3A.$

Now we have to find $I_{SC} = I_N$… Appl

$I_{SC} = I_N = 3A \times [(6\Omega / (3\Omega + 6\Omega)] = 2A.$

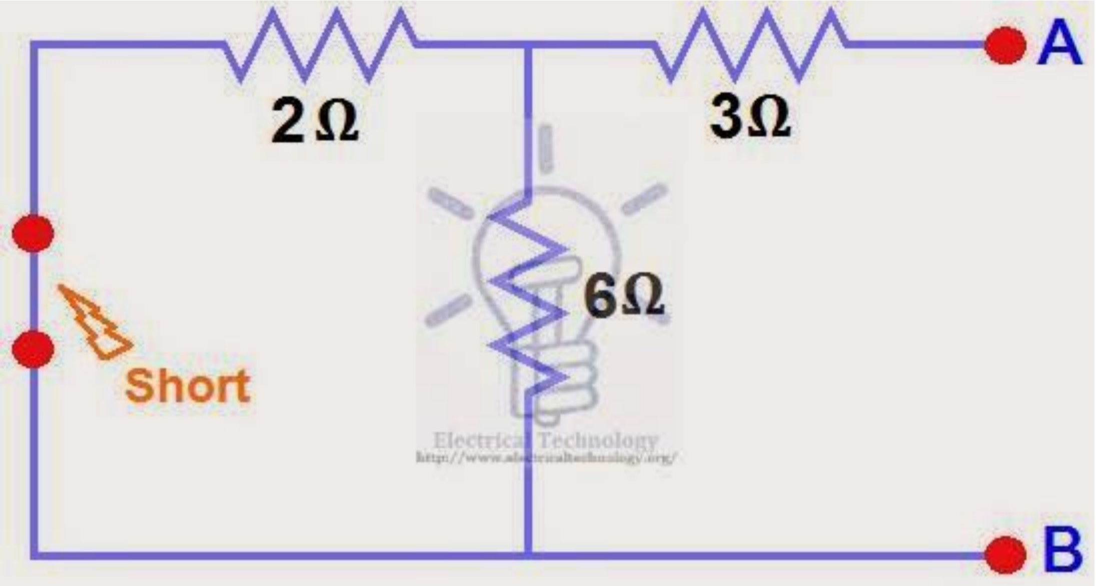
$I_{SC} = I_N = 2A.$



NORTON'S THEOREM

STEP 3.

Open Current Sources, Short Voltage Sources and Open Load Resistor



NORTON'S THEOREM

STEP 4.

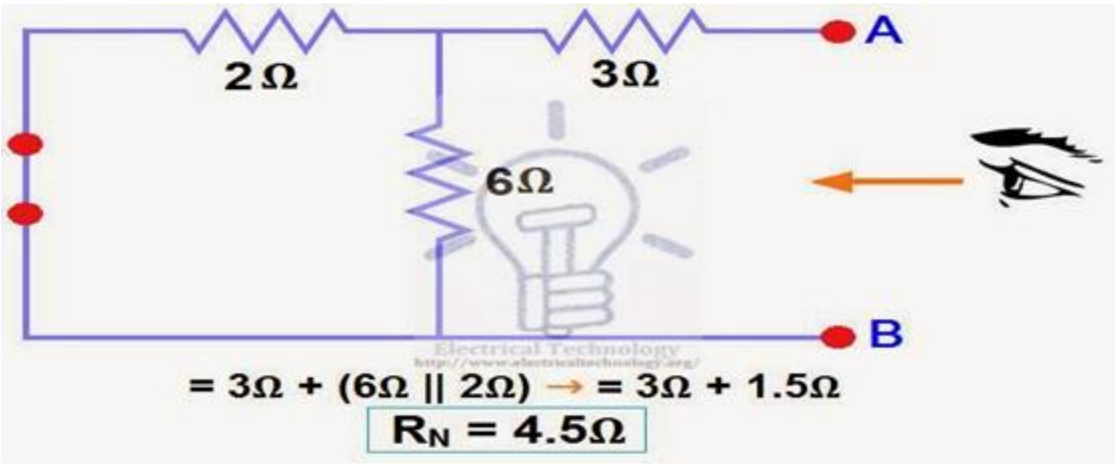
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N)
We have Reduced the 12V DC source to zero is equivalent to replace it with a short in step (3), as shown in figure (4) We can see that 3Ω resistor is in <u>series with a parallel combination</u> of 6Ω resistor and 2Ω resistor. i.e.:

$3\Omega + (6\Omega \,||\, 2\Omega)$ ….. (|| = in parallel with)

$R_N = 3\Omega + [(6\Omega \times 2\Omega) / (6\Omega + 2\Omega)]$

$R_N = 3\Omega + 1.5\Omega$

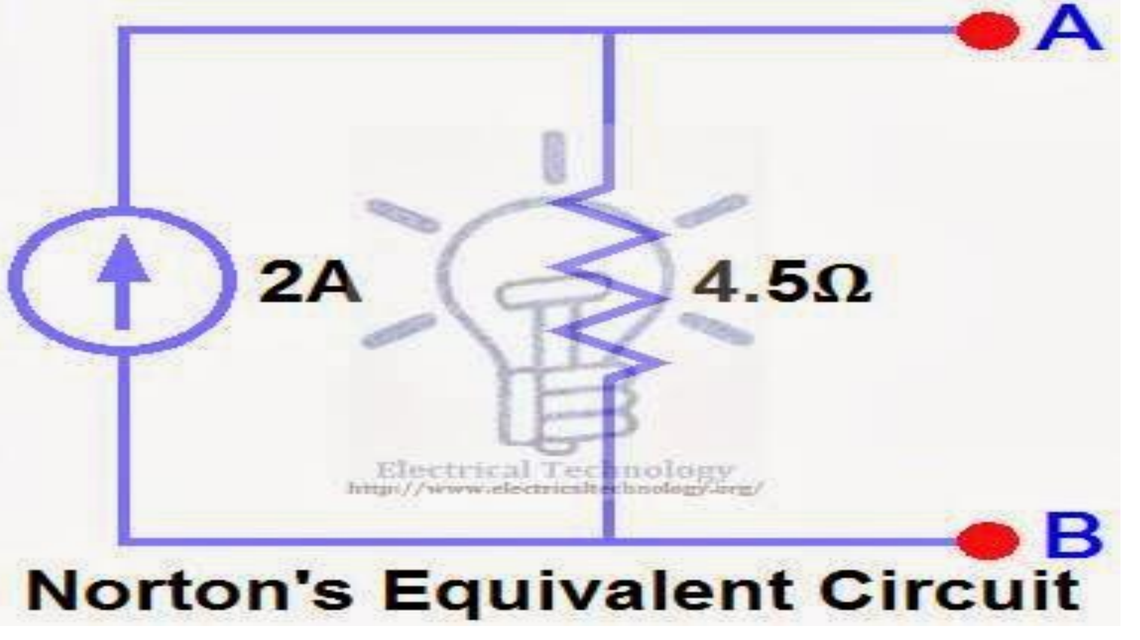
$\mathbf{R_N = 4.5\Omega}$



NORTON'S THEOREM

STEP 5.

Connect the R_N in Parallel with Current Source I_N and reconnect the load resistor. This is shown in fig i.e. Norton Equivalent circuit with load resistor.



NORTON'S THEOREM

STEP 6.

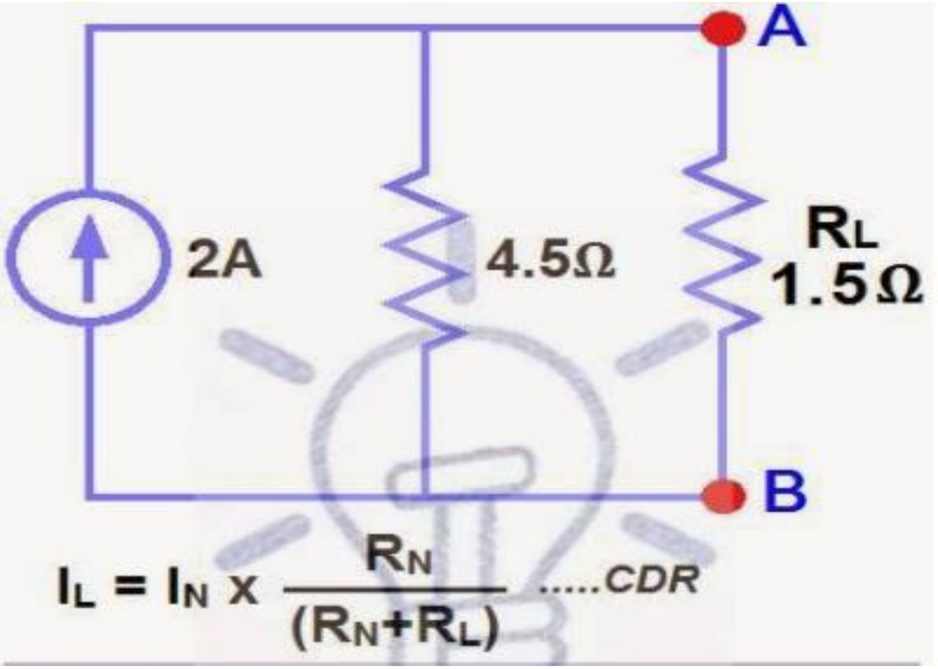
Now apply the last step i.e. calculate the load current through and Load voltage across the load resistor by Ohm's Law as shown in fig .

Load Current through Load Resistor…

$I_L = I_N \times [R_N / (R_N + R_L)]$

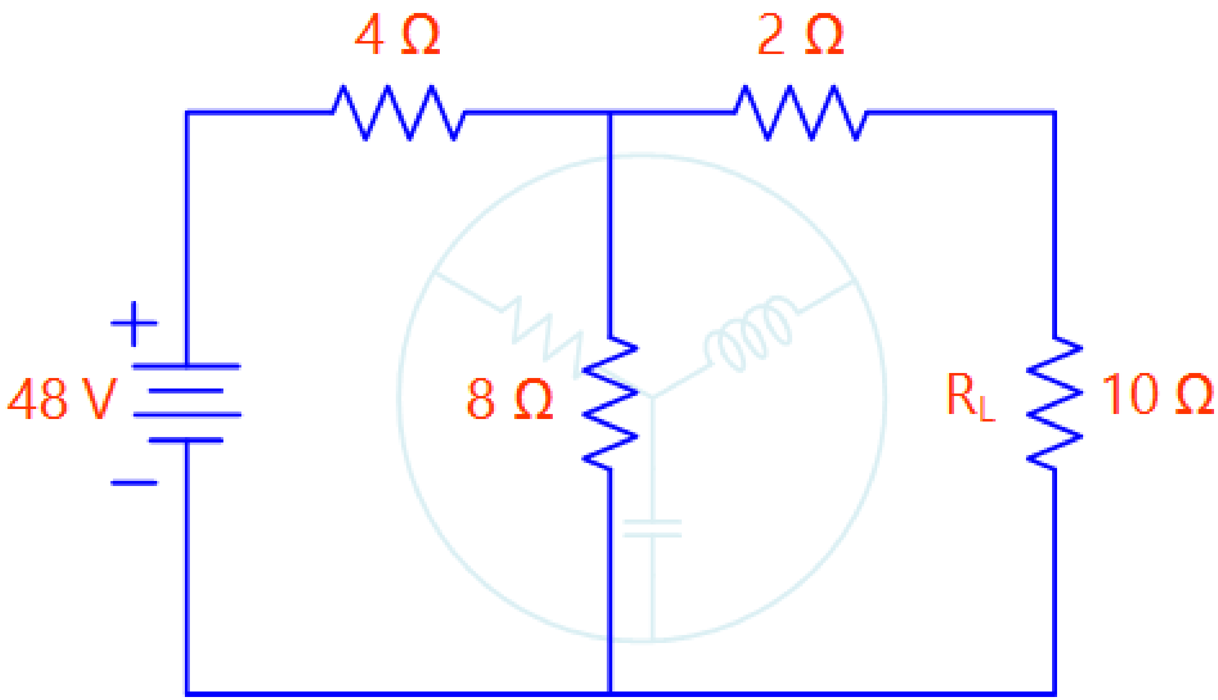
$= 2A \times (4.5\Omega / 4.5\Omega + 1.5\Omega) \rightarrow = 1.5A$

$I_L = 1.5A$



NORTON'S THEOREM

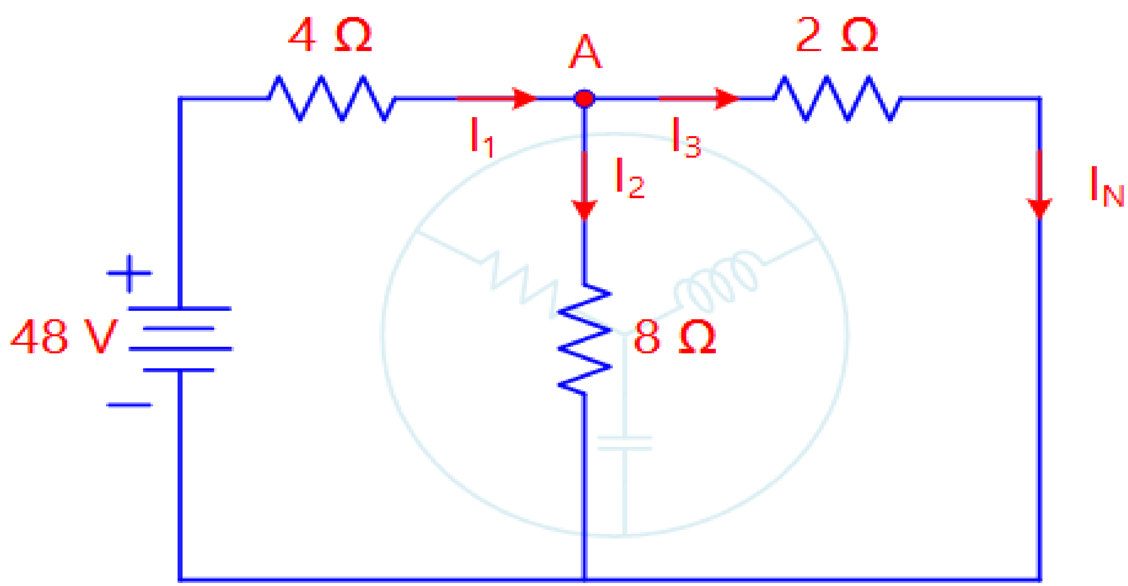
Find the current through 10Ω resistor using Norton's theorem.



NORTON'S THEOREM

Step 1

To find Norton's current, open the load resistor and make a short circuit. Then find the current through the short-circuited terminal by any network analysis method. This short circuit current is called as Norton's current.



NORTON'S THEOREM

The total resistance of the circuit and the total current can be calculated as follows.

$$R_{eq} = 4 + \frac{82}{8+2} = 5.6\,\Omega$$

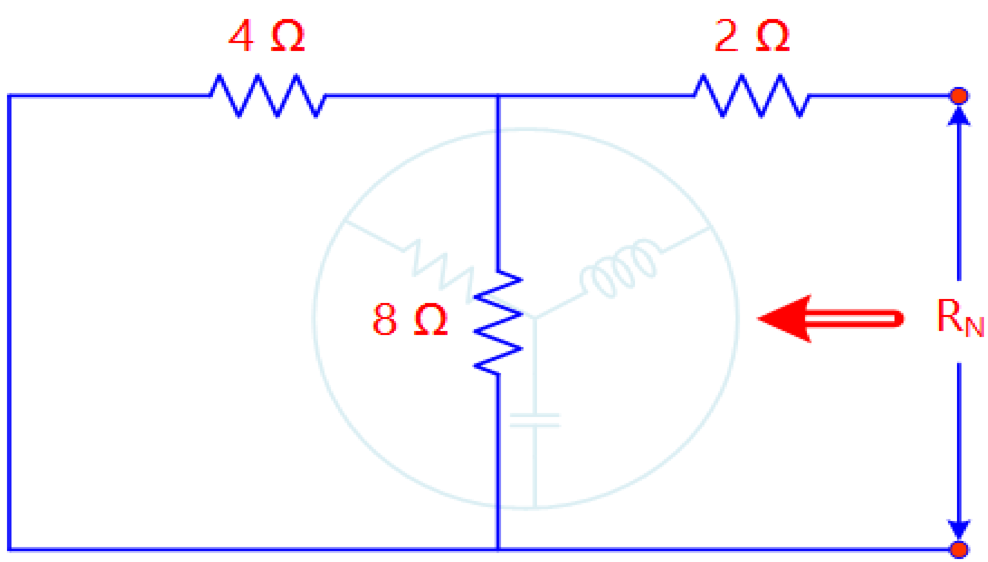
$$I_1 = \frac{V}{Req} = \frac{48}{5.6} = 8.57\,A$$

$$I_N = I_3 = 8.57 \times \frac{8}{8+2} = 6.86\,A$$

NORTON'S THEOREM

Step 2

The next step is to find the norton's resistance of the network. In order to find the resistance, remove the load resistor and replace the 48V voltage source by a short circuit. Then calculate the resistance seen from the open-circuited terminals.



NORTON’S THEOREM

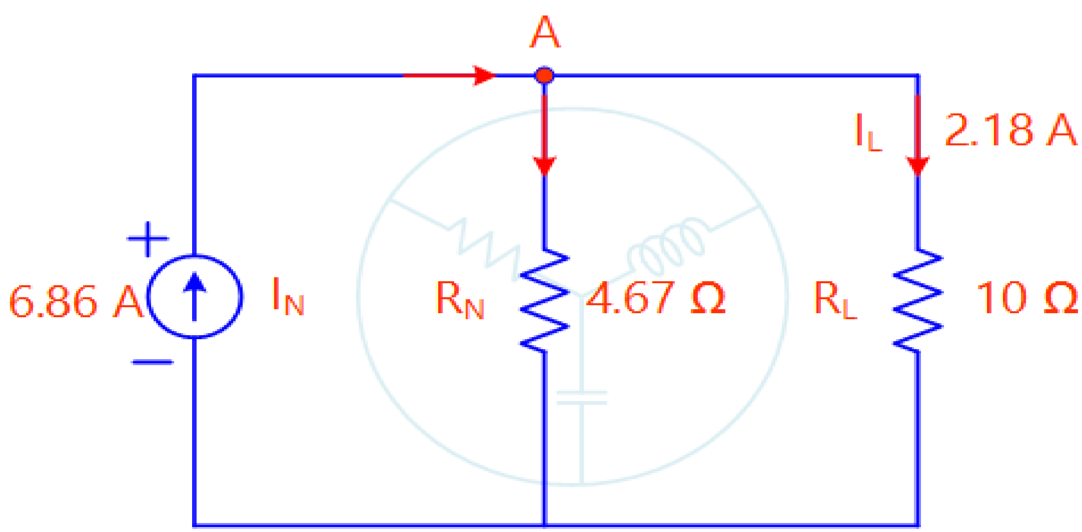
Here 8Ω and 2Ω resistors are connected in parallel and this combination is connected in series with 4Ω resistor. So the thevenin’s resistance is calculated as follows.

$$R_N = \frac{4 \times 8}{4+8} + 2 = 4.67\,\Omega$$

NORTON’S THEOREM

Step 3

Now redraw the circuit with Norton’s current source in parallel with the norton’s resistance. Then add the load resistor in parallel with the above circuit to form norton’s equivalent circuit for the given circuit.



NORTON'S THEOREM

Apply current division rule at node A, and find the load current.

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

$$= 6.86 \times \frac{4.67}{4.67 + 10} = 2.18\ Amperes$$

NORTON'S THEOREM

For the circuit shown in fig.8.10(a), find the current through resistor 1 ohm branch using Norton's theorem.

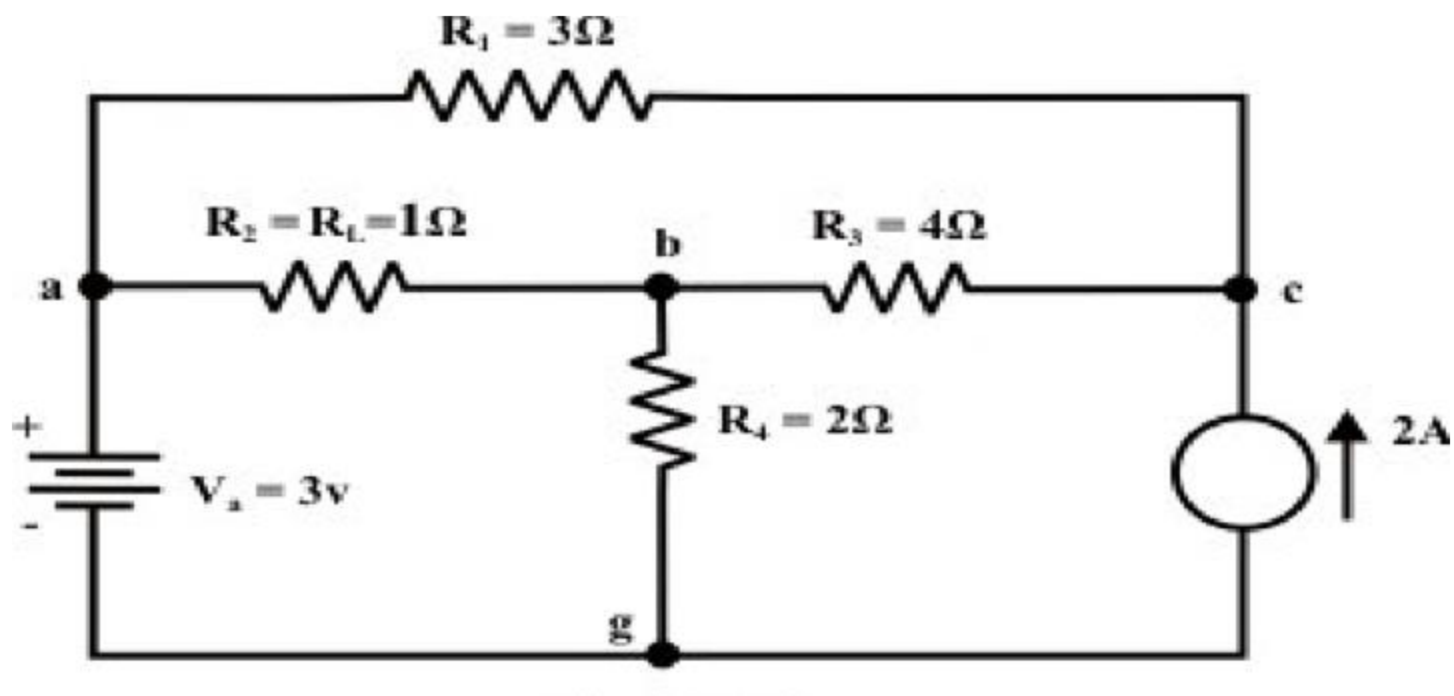


Fig. 8.10(a)

NORTON’S THEOREM

Solution:

Step-1: Remove the resistor through which the current is to be found and short the terminals ‘a’ and ‘b’ (see fig.8.10(b)).

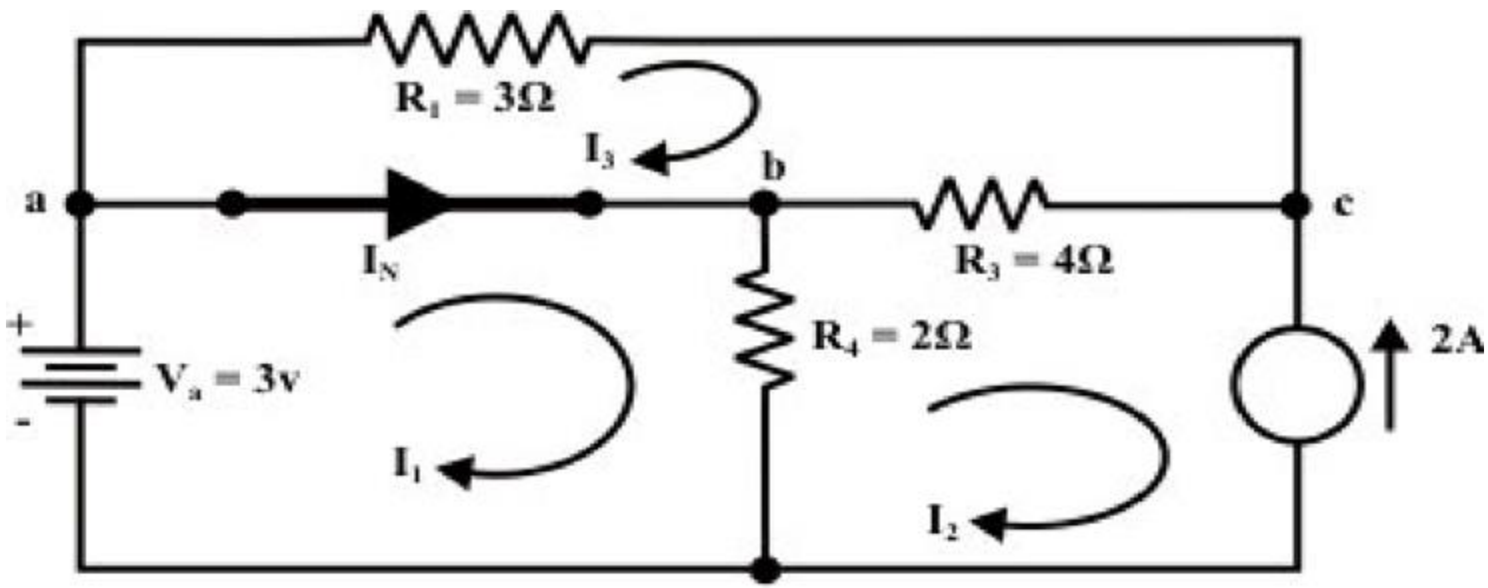


Fig. 8.10(b)

NORTON'S THEOREM

Step-2: Any method can be adopted to compute the current flowing through the a-b branch. Here, we apply 'mesh – current' method.

Loop-1

$3 - R_4(I_1 - I_2) = 0$, where $I_2 = -2A$

$R_4 I_1 = 3 + R_4 I_2 = 3 - 2 \times 2 = -1 \quad \therefore I_1 = -0.5A$

Loop-3

$$-R_1 I_3 - R_3(I_3 - I_2) = 0$$
$$-3I_3 - 4(I_3 + 2) = 0$$
$$-7I_3 - 8 = 0$$
$$I_3 = -\frac{8}{7} =$$
$$\therefore I_N = (I_1 - I_3) = \left(-0.5 + \frac{8}{7}\right) = \frac{-7+16}{14}$$
$$= \frac{9}{14} \text{A (current is flowing from 'a' to 'b')}$$

NORTON'S THEOREM

Step-3: To compute R^N, all sources are replaced with their internal resistances. The equivalent resistance between 'a' and 'b' terminals is same as the value of Thevenin's resistance of the circuit shown in fig.8.3(d).

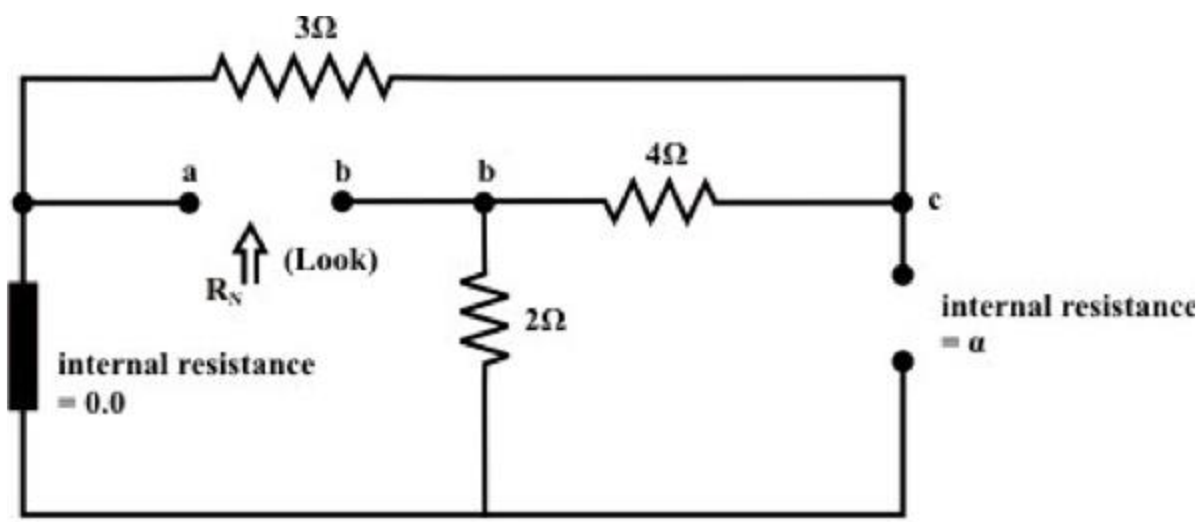


Fig. 8.10(c)

NORTON'S THEOREM

Step-4: Replace the original circuit with an equivalent Norton's circuit as shown in fig.8.10(d).

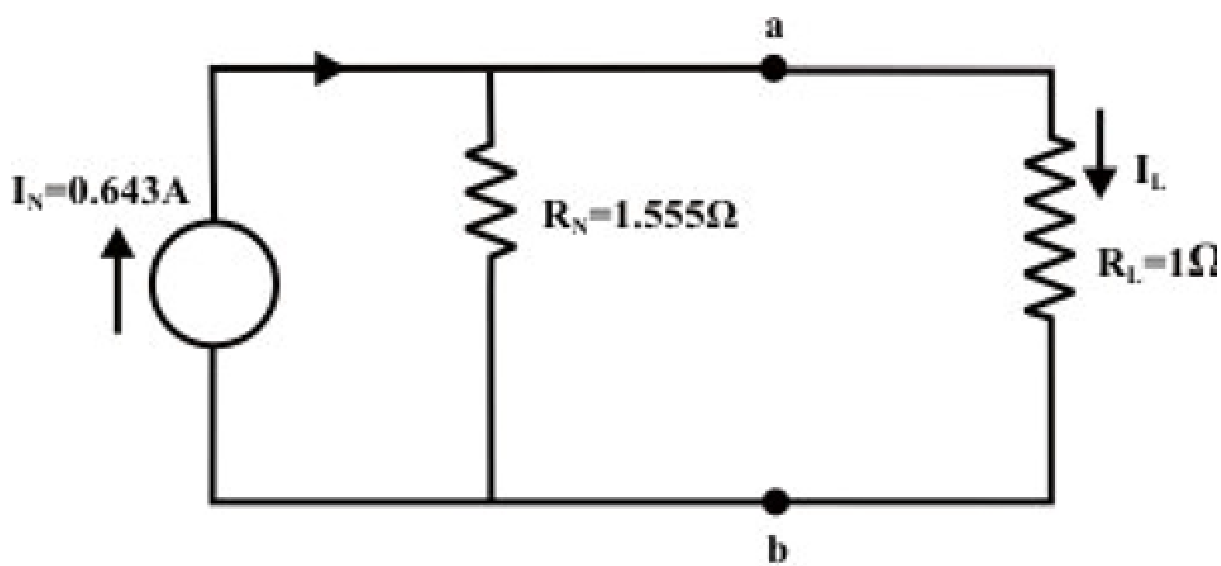


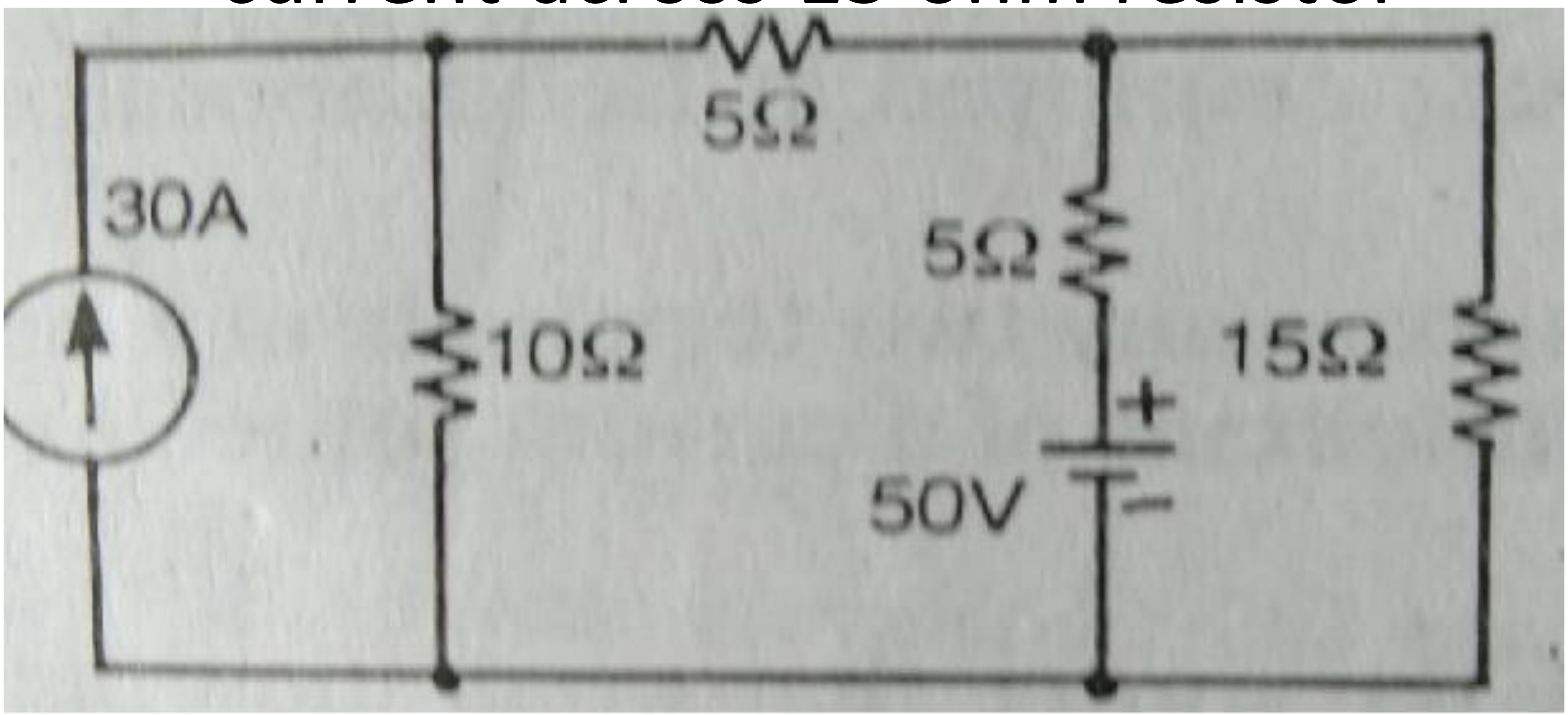
Fig. 8.10(d)

NORTON’S THEOREM

$$I_L = \frac{R_N}{R_N + R_L} \times I_N = \frac{1.555}{1.555 + 1} \times 0.643 = 0.39\text{A (a to b)}$$

In order to calculate the voltage across the current source the following procedures are adopted. Redraw the original circuit indicating the current direction in the load.

Find Norton's current, resistance and load current across 15 ohm resistor



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