Single - Phase AC Circuits

1. Equation for generation of alternating induce EMF

- An AC generator uses the principle of Faraday's electromagnetic induction law. It states that when current carrying conductor cut the magnetic field then emf induced in the conductor.
- Inside this magnetic field a single rectangular loop of wire rotes around a fixed axis allowing it to cut the magnetic flux at various angles as shown below figure 2.1.

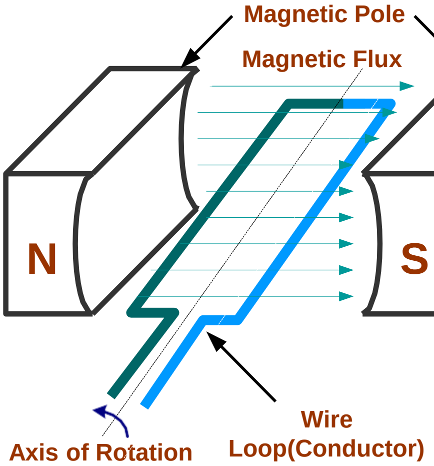

Figure 2.2.1 Generation of EMF

Where,
N =No. of turns of coil
A = Area of coil (m^2)
ω=Angular velocity (radians/second)
ϕ_m= Maximum flux (wb)

- When coil is along XX' (perpendicular to the lines of flux), flux linking with coil= ϕ_m. When coil is along YY' (parallel to the lines of flux), flux linking with the coil is zero. When coil is making an angle θ with respect to XX' flux linking with coil, $\phi = \phi_m \cos\omega t$ [$\theta = \omega t$].

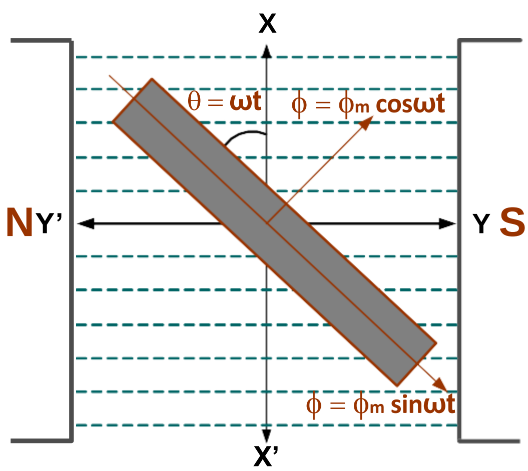

Figure 2.2 Alternating Induced EMF

- According to Faraday's law of electromagnetic induction,

$$e = -N\frac{d\phi}{dt}$$

$$e = -N\frac{d(\phi_m \cos\omega t)}{dt}$$

$$e = -N\phi_m(-\sin\omega t)\times\omega$$

$$e = N\phi_m\omega\sin\omega t$$

$$e = E_m \sin\omega t$$

Where,

$$E_m = N\phi_m\omega$$

$N =$ no. of turns of the coil

$$\phi_m = B_m A$$

$B_m =$ Maximum flux density (wb/m^2)

$A =$ Area of the coil (m^2)

$$\omega = 2\pi f$$

$$\therefore e = N B_m A 2\pi f \sin \omega t$$

- Similarly, an alternating current can be express as

  $i = I_m \sin \omega t$ Where, I_m = Maximum values of current

- Thus, both the induced emf and the induced current vary as the sine function of the phase angle $\omega t = \theta$. Shown in figure 2.3.

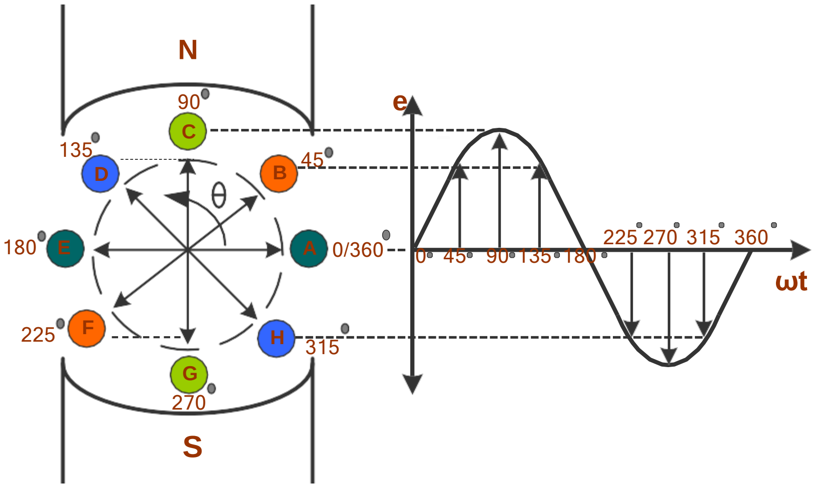

Figure 2.3 Waveform of Alternating Induced EMF

Phase angle	Induced emf $e = E_m \sin \omega t$
$\omega t = 0^0$	$e = 0$
$\omega t = 90^0$	$e = E_m$
$\omega t = 180^0$	$e = 0$
$\omega t = 270^0$	$e = -E_m$
$\omega t = 360^0$	$e = 0$

2.2 Definitions

- **Waveform**

  It is defined as the graph between magnitude of alternating quantity (on Y axis) against time (on X axis).

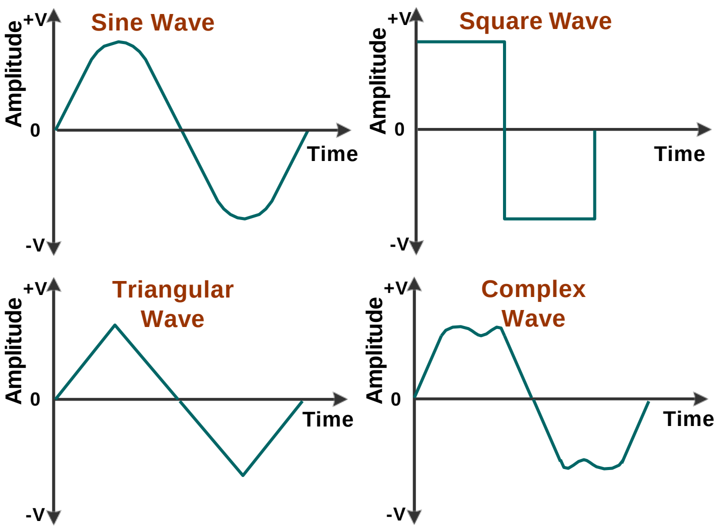

Figure 2.4 A.C. Waveforms

- **Cycle**

  It is defined as one complete set of positive, negative and zero values of an alternating quantity.

2 A.C. Circuits

- **Instantaneous value**

  It is defined as the value of an alternating quantity at a particular instant of given time. Generally denoted by small letters.

  e.g. i= Instantaneous value of current

  v= Instantaneous value of voltage

  p= Instantaneous values of power

- **Amplitude/ Peak value/ Crest value/ Maximum value**

  It is defined as the maximum value (either positive or negative) attained by an alternating quantity in one cycle. Generally denoted by capital letters.

  e.g. I_m= Maximum Value of current

  V_m= Maximum value of voltage

  P_m= Maximum values of power

- **Average value**

  It is defined as the average of all instantaneous value of alternating quantities over a half cycle.

  e.g. V_{ave} = Average value of voltage

  I_{ave} = Average value of current

- **RMS value**

  It is the equivalent dc current which when flowing through a given circuit for a given time produces same amount of heat as produced by an alternating current when flowing through the same circuit for the same time.

  e.g. V_{rms} =Root Mean Square value of voltage

  I_{rms} = Root Mean Square value of current

- **Frequency**

  It is defined as number of cycles completed by an alternating quantity per second. Symbol is f. Unit is Hertz (Hz).

- **Time period**

  It is defined as time taken to complete one cycle. Symbol is T. Unit is seconds.

- **Power factor**

  It is defined as the cosine of angle between voltage and current. Power Factor = pf = $\cos\phi$, where ϕ is the angle between voltage and current.

- **Active power**

  It is the actual power consumed in any circuit. It is given by product of rms voltage and rms current and cosine angle between voltage and current. (VI $\cos\phi$).

  Active Power= P= I^2R = VI $\cos\phi$.

  Unit is Watt (W) or kW.

- **Reactive power**

  The power drawn by the circuit due to reactive component of current is called as reactive power. It is given by product of rms voltage and rms current and sine angle between voltage and current (VI sinϕ).

  Reactive Power = Q= I^2X = VIsinϕ.

  Unit is VAR or kVAR.

- **Apparent power**

  It is the product of rms value of voltage and rms value of current. It is total power supplied to the circuit.

  Apparent Power = S = VI.

  Unit is VA or kVA.

- **Peak factor/ Crest factor**

  It is defined as the ratio of peak value (crest value or maximum value) to rms value of an alternating quantity.

  Peak factor = Kp = 1.414 for sine wave.

- **Form factor**

  It is defined as the ratio of rms value to average value of an alternating quantity. Denoted by K_f. Form factor K_f = 1.11 for sine wave.

- **Phase difference**

  It is defined as angular displacement between two zero values or two maximum values of the two-alternating quantity having same frequency.

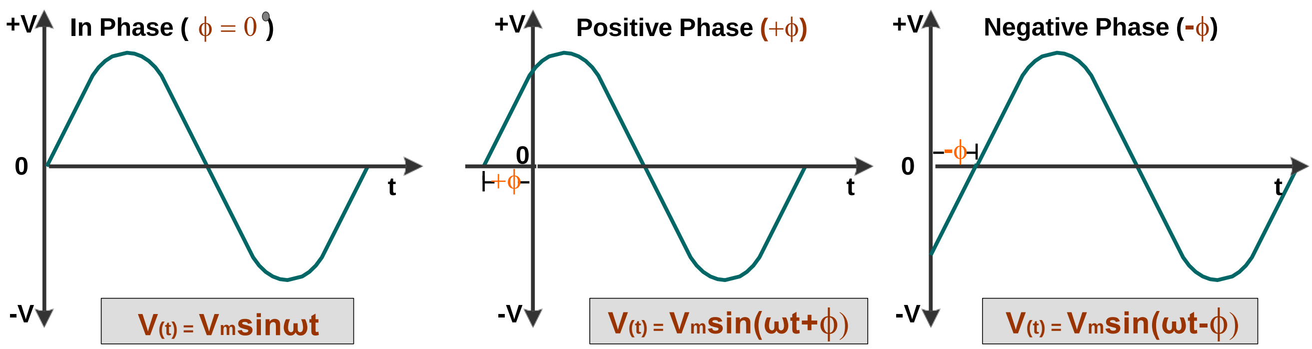

Figure 2.5 A.C. Phase Difference

- **Leading phase difference**

  A quantity which attains its zero or positive maximum value before the compared to the other quantity.

- **Lagging phase difference**

  A quantity which attains its zero or positive maximum value after the other quantity.

2.3 Derivation of average value and RMS value of sinusoidal AC signal

➢ **Average Value**

Graphical Method

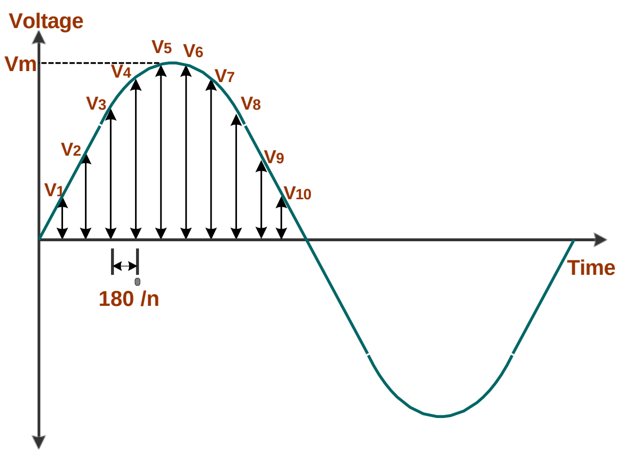

Figure 2.6 Graphical Method for Average Value

$$V_{ave} = \frac{Sum\ of\ All\ \ Instantaneous\ \ Values}{Total\ \ No.\ of\ \ Values}$$

$$V_{ave} = \frac{v_1 + v_2 + v_3 + v_4 + v_5 + ...+ v_{10}}{10}$$

Analytical Method

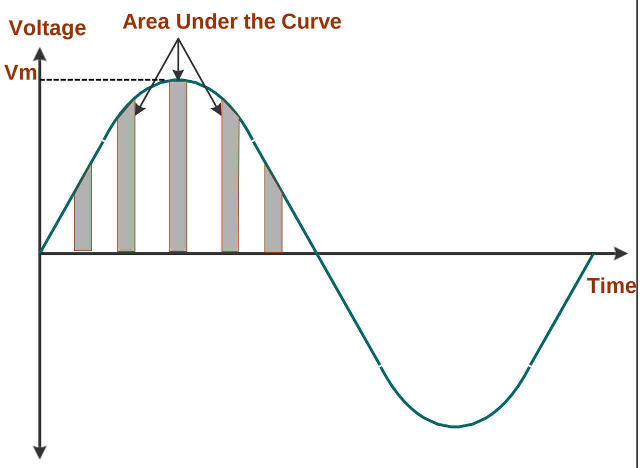

Figure 2.7 Analytical Method for Average Value

$$V_{ave} = \frac{Area\ \ Under\ \ the\ Curve}{Base\ \ of\ the\ \ Curve}$$

$$V_{ave} = \frac{\int_0^{\pi} V_m\ Sin\ \ \omega t\ \ d\omega t}{\pi}$$

$$V_{ave} = \frac{V_m}{\pi}\left(-\cos\omega t\right)_0^{\pi}$$

$$V_{ave} = -\frac{V_m}{\pi}\left(\cos\pi - \cos 0\right)$$

$$V_{ave} = \frac{2V_m}{\pi}$$

$$V_{ave} = 0.637\ V_m$$

- **RMS Value**

Graphical Method	**Analytical Method**
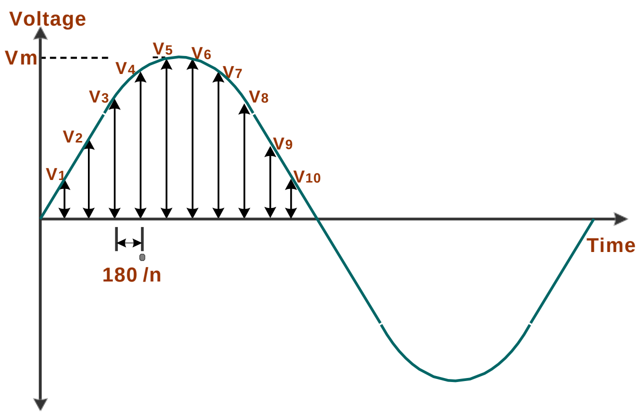	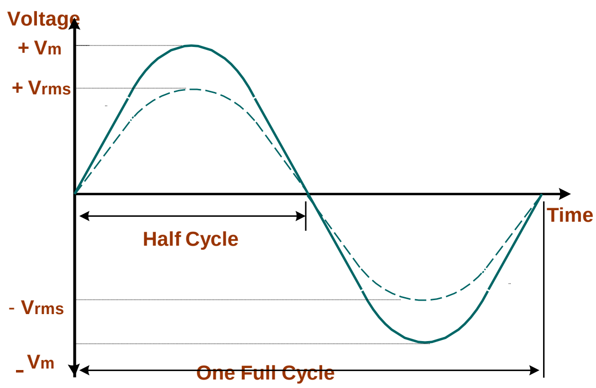
Figure 2.8 Graphical Method for RMS Value	*Figure 2.9 Analytical Method for RMS Value*
$V_{rms} = \sqrt{\dfrac{Sum\ \ of\ \ all\ \ sq.\ of\ instantaneous\ values}{Total\ \ No.\ \ of\ \ Values}}$ $V_{rms} = \sqrt{\dfrac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + \ldots\ldots + v_{10}^2}{10}}$	$V_{rms} = \sqrt{\dfrac{Area\ \ under\ \ the\ sq.\ curve}{Base\ of\ the\ curve}}$ $V_{rms} = \sqrt{\dfrac{\int_0^{2\pi} V_m^2\ Sin^2\ \omega t\ \ d\omega t}{2\pi}}$ $V_{rms} = \sqrt{\dfrac{V_m^2}{2\pi}\int_0^{2\pi}\dfrac{(1-\cos 2\omega t)}{2}d\omega t}$ $V_{rms} = \sqrt{\dfrac{V_m^2}{4\pi}\left[\left[\omega t\right]_0^{2\pi} - \left[\dfrac{(\sin 2\omega t)}{2}\right]_0^{2\pi}\right]}$ $V_{rms} = \sqrt{\dfrac{V_m^2}{4\pi}(2\pi - 0)}$ $V_{rms} = \dfrac{V_m}{\sqrt{2}}$ $V_{rms} = 0.707\ \ V_m$

2.4 Phasor Representation of Alternating Quantities

- Sinusoidal expression given as: $v_{(t)} = V_m \sin(\omega t \pm \Phi)$ representing the sinusoid in the time-domain form.
- Phasor is a quantity that has both "Magnitude" and "Direction".

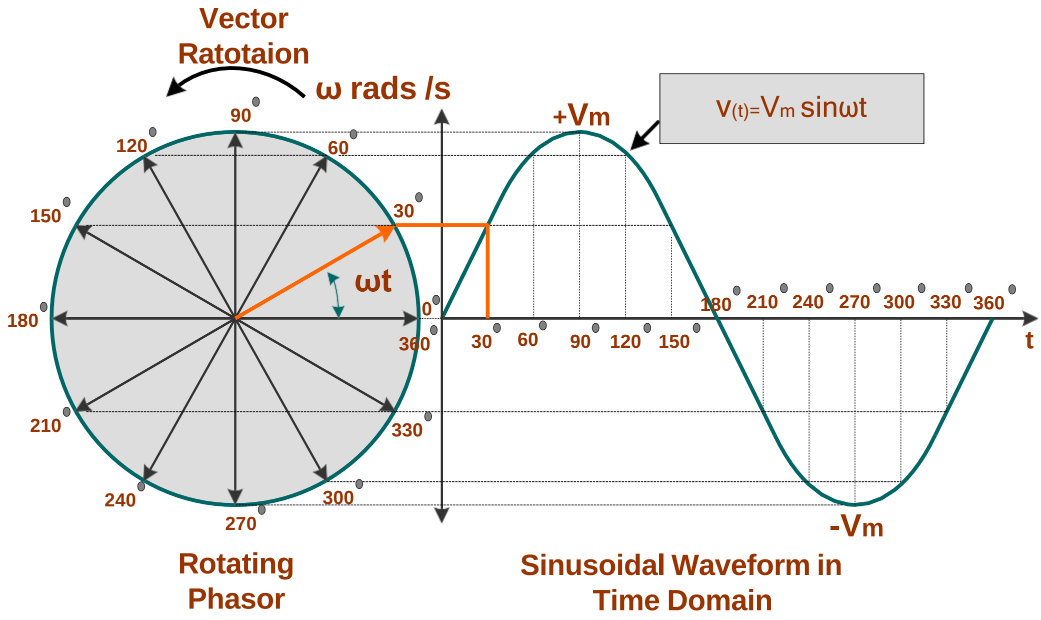

Figure 2.10 Phasor Representation of Alternating Quantities

Phase Difference of a Sinusoidal Waveform

- The generalized mathematical expression to define these two sinusoidal quantities will be written as:

$$v = V_m \, Sin \;\; \omega t$$

$$i = I_m \, sin \;\; (\omega t - \varphi)$$

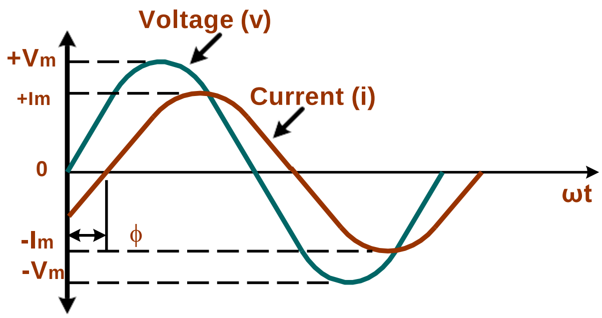

Figure 2.11 Wave Forms of Voltage & Current

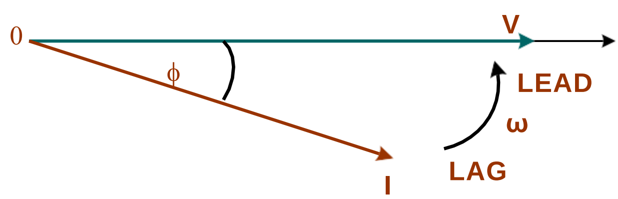

Figure 2.12 Phasor Diagram of Voltage & Current

- As show in the above voltage and current equations, the current, i is lagging the voltage, v by angle ϕ .
- So, the difference between the two sinusoidal quantities representing in waveform shown in Fig. 2.11 & phasors representing the two sinusoidal quantities is angle ϕ and the resulting phasor diagram shown in Fig. 2.12.

2.5 Purely Resistive Circuit

- The Fig. 2.13 an AC circuit consisting of a pure resistor to which an alternating voltage $v_t = V_m \sin\omega t$ is applied.

Circuit Diagram

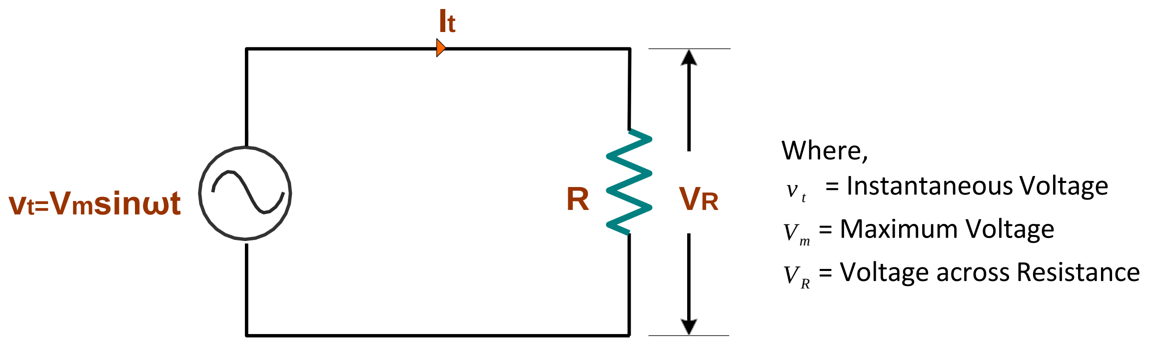

Figure 2.13 Pure Resistor Connected to AC Supply

Equations for Voltage and Current

- As show in the Fig. 2.13 voltage source

$$v_t = V_m \, Sin \;\; \omega t$$

- According to ohm's law

$$i_t = \frac{v_t}{R}$$

$$i_t = \frac{V_m \sin\omega t}{R}$$

$$i_t = I_m \sin\omega t$$

- From above equations it is clear that current is in phase with voltage for purely resistive circuit.

Waveforms and Phasor Diagram

- The sinewave and vector representation of $v_t = V_m \; Sin\,\omega t$ & $i_t = I_m \; sin\,\omega t$ are given in Fig. 2.14 & 2.15.

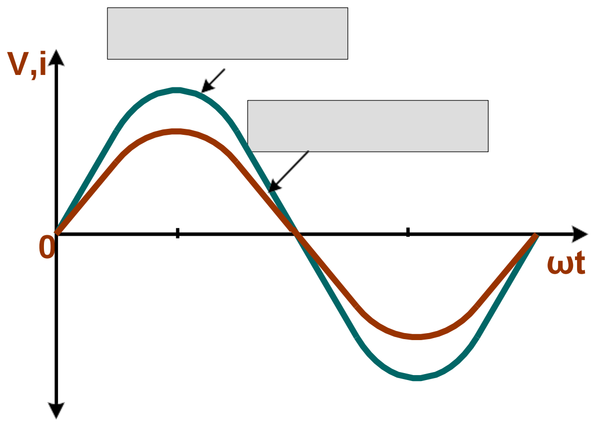

Figure 2.14 Waveform of Voltage & Current for Pure Resistor

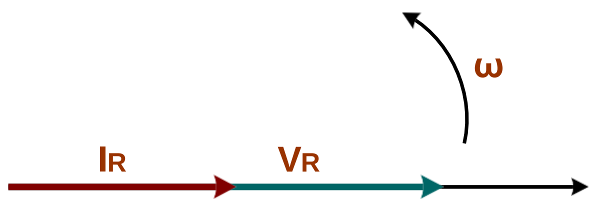

Figure 2.15 Phasor Diagram of Voltage & Current for Pure Resistor

Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

Instantaneous power

$$p_{(t)} = v \times i$$

$$p_{(t)} = V_m \sin\omega t \times I_m \sin\omega t$$

$$p_{(t)} = V_m I_m \sin^2 \omega t$$

$$p_{(t)} = \frac{V_m I_m (1 - \cos 2\omega t)}{2}$$

Average Power

$$P_{ave} = \frac{\int_0^{2\pi} \frac{V_m I_m (1 - \cos 2\omega t)}{2} d\omega t}{2\pi}$$

$$P_{ave} = \frac{V_m I_m}{4\pi} \left[[\omega t]_0^{2\pi} - \left[\frac{(\sin 2\omega t)}{2} \right]_0^{2\pi} \right]$$

$$P_{ave} = \frac{V_m I_m}{4\pi} \left[[2\pi - 0] - [0 - 0] \right]$$

$$P_{ave} = \frac{V_m I_m}{2}$$

$$P_{ave} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$P_{ave} = V_{rms} \quad I_{rms}$$

$$P_{ave} = VI$$

- The average power consumed by purely resistive circuit is multiplication of Vrms & I_{rms} .

2.6 Purely Inductive Circuit

- The Fig. 2.16 an AC circuit consisting of a pure Inductor to which an alternating voltage $v_t = V_m \sin\omega t$ is applied.

Circuit Diagram

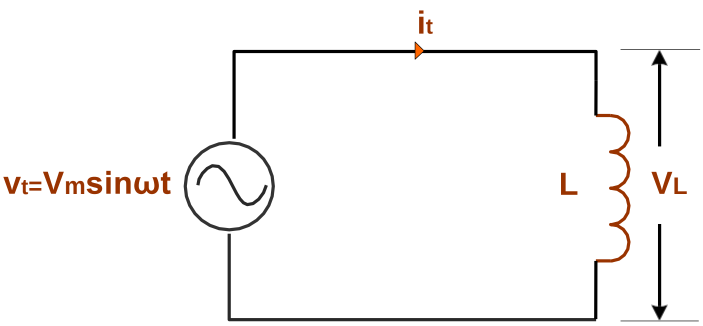

Figure 2.16 Pure Inductor Connected to AC Supply

Equations for Voltage and Current

- As show in the Fig. 2.16 voltage source

$$v_t = V_m \, Sin \; \omega t$$

- Due to self-inductance of the coil, there will be emf indued in it. This back emf will oppose the instantaneous rise or fall of current through the coil, it is given by

$$e_b = -L \frac{di}{dt}$$

- As, circuit does not contain any resistance, there is no ohmic drop and hence applied voltage is equal and opposite to back emf.

$$v_t = -e_b$$

$$v_t = -\left(-L \frac{di}{dt}\right)$$

$$v_t = L \frac{di}{dt}$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m \sin \omega t \;\; dt}{L}$$

- Integrate on both the sides,

$$\int di = \frac{V_m}{L} \int \sin \omega t \;\; dt$$

$$i_t = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$i_t = -\frac{V_m}{\omega L} \cos \omega t$$

$$i_t = I_m \sin\left(\omega t - 90^\circ\right) \qquad \left(\because \frac{V_m}{\omega L} = I_m\right)$$

- From the above equations it is clear that the current lags the voltage by 90⁰ in a purely inductive circuit.

Waveform and Phasor Diagram

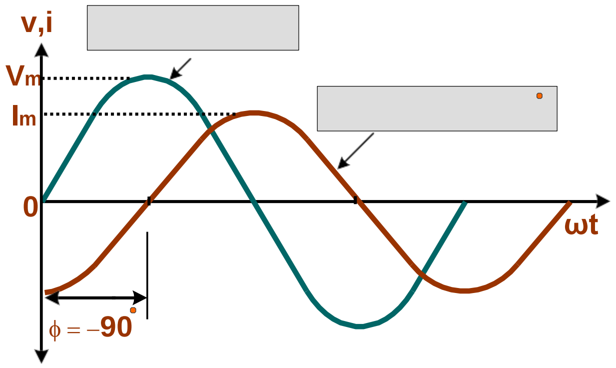

Figure 2.17 Waveform of Voltage & Current for Pure Inductor

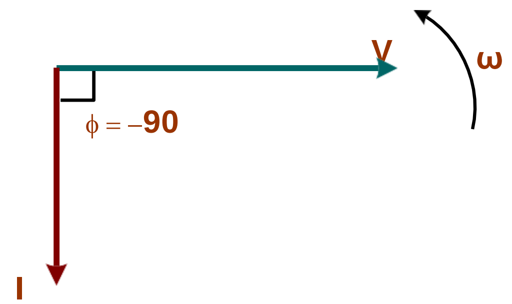

Figure 2.18 Phasor Diagram of Voltage & Current for Pure Inductor

Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

Instantaneous Power

$$p_t = v \times i$$

$$p_t = V_m \sin \omega t \times I_m \sin \; (\omega t - 90^\circ)$$

$$p_t = V_m \sin \omega t \times (-\, I_m \cos \omega t\,)$$

$$p_t = \frac{-\,2V_m \; I_m \sin \omega t \;\; \cos \omega t}{2}$$

$$p_t = -\frac{V_m I_m}{2} \sin 2\omega t$$

Average Power

$$P_{ave} = \frac{\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t}{2\pi} \, d\omega t$$

$$P_{ave} = -\frac{V_m I_m}{4\pi} \left[\frac{-\cos 2\omega t}{2} \right]_0^{2\pi}$$

$$P_{ave} = \frac{V_m I_m}{8\pi} [\cos 4\pi - \cos 0]$$

$$P_{ave} = 0$$

- The average power consumed by purely inductive circuit is zero.

2.7 Purely Capacitive Circuit

- The Fig. 2.19 shows a capacitor of capacitance C farads connected to an a.c. voltage supply $v_t = V_m \sin\omega t$.

Circuit Diagram

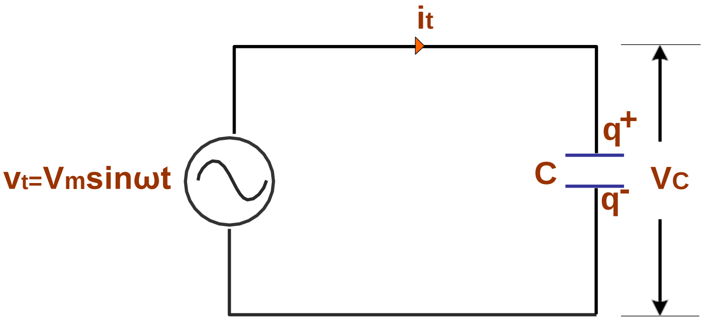

Figure 2.19 Pure Capacitor Connected AC Supply

Equations for Voltage & Current

- As show in the Fig. 2.19 voltage source

$$v_t = V_m \, Sin \; \omega t$$

- A pure capacitor having zero resistance. Thus, the alternating supply applied to the plates of the capacitor, the capacitor is charged.
- If the charge on the capacitor plates at any instant is ‘q’ and the potential difference between the plates at any instant is ‘v_t’ then we know that,

$$q = C v_t$$

$$q = C V_m \sin\omega t$$

- The current is given by rate of change of charge.

$$i_t = \frac{dq}{dt}$$

$$i_t = \frac{d C V_m \sin\omega t}{dt}$$

$$i_t = \omega C V_m \sin \omega t$$

$$i_t = \frac{V_m}{1/\omega C} \cos \omega t$$

$$i_t = \frac{V_m}{X_c} \cos \omega t$$

$$i_t = I_m \sin(\omega t + 90^o) \qquad \left(\because \frac{V_m}{X_c} = I_m\right)$$

- From the above equations it is clear that the current leads the voltage by 90^0 in a purely capacitive circuit.

Waveform and Phasor Diagram

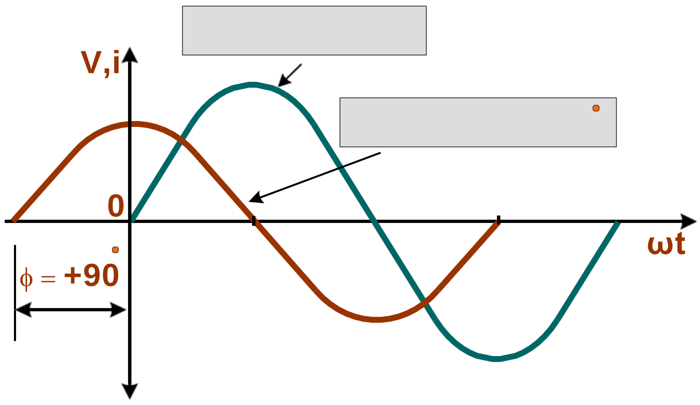

Figure 2.20 Waveform of Voltage & Current for Pure Capacitor

Figure 2.21 Phasor Diagram of Voltage & Current for Pure Capacitor

Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

Instantaneous Power

$$p_{(t)} = v \times i$$

$$p_{(t)} = V_m \sin \omega t \times I_m \sin\left(\omega t + 90^\circ\right)$$

$$p_{(t)} = V_m \sin \omega t \times I_m \cos \omega t$$

$$p_{(t)} = V_m I_m \sin \omega t \cos \omega t$$

$$p_{(t)} = \frac{2V_m I_m \sin \omega t \cos \omega t}{2}$$

$$p_{(t)} = \frac{V_m I_m}{2} \sin 2\omega t$$

Average Power

$$P_{ave} = \frac{\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t}{2\pi} \, d\omega t$$

$$P_{ave} = \frac{V_m I_m}{4\pi}\left[\frac{-\cos\omega t}{2}\right]_0^{2\pi}$$

$$P_{ave} = \frac{V_m I_m}{8\pi}\left[-\cos 4\pi + \cos 0\right]$$

$$P_{ave} = 0$$

- The average power consumed by purely capacitive circuit is zero.

2.8 Series Resistance-Inductance (R-L) Circuit

- Consider a circuit consisting of a resistor of resistance R ohms and a purely inductive coil of inductance L henry in series as shown in the Figure 2.22.

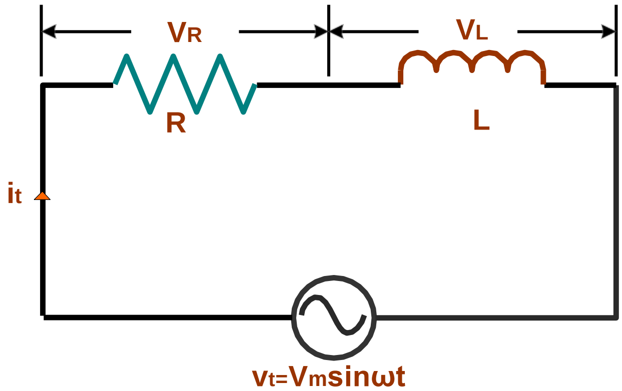

Figure 2.22 Circuit Diagram of Series R-L Circuit

- In the series circuit, the current i_t flowing through R and L will be the same.
- But the voltage across them will be different. The vector sum of voltage across resistor V_R and voltage across inductor V_L will be equal to supply voltage v_t.

Waveforms and Phasor Diagram

- The voltage and current waves in R-L series circuit is shown in Fig. 2.23.

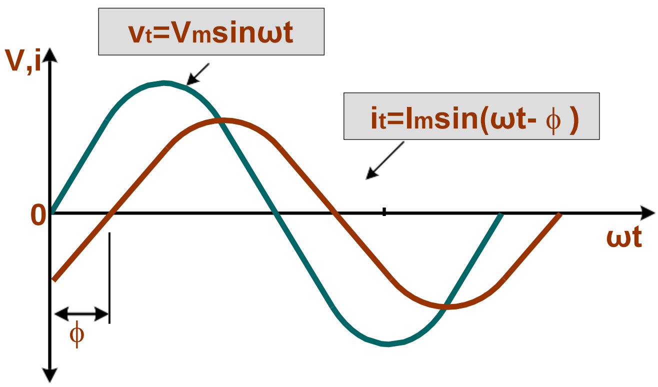

Figure 2.23 Waveform of Voltage and Current of Series R-L Circuit

- We know that in purely resistive the voltage and current both are in phase and therefore vector V_R is drawn superimposed to scale onto the current vector and in purely inductive circuit the current I lag the voltage V_L by 90^o.
- So, to draw the vector diagram, first I taken as the reference. This is shown in the Fig. 2.24. Next V_R drawn in phase with I. Next V_L is drawn 90^o leading the I.
- The supply voltage V is then phasor Addition of V_R and V_L.

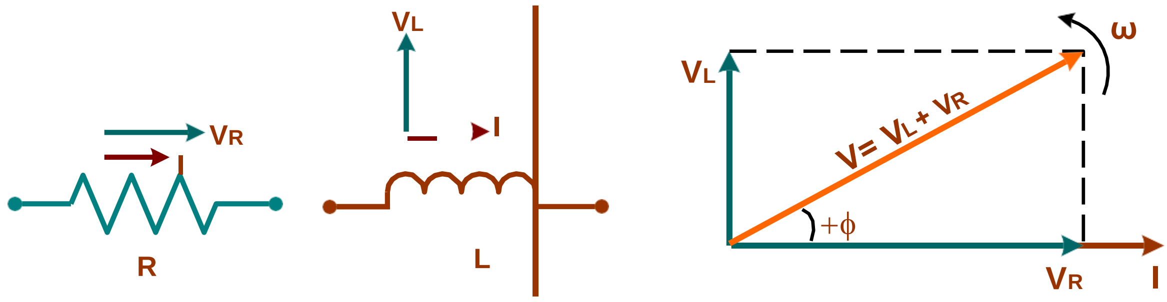

Figure 2.24 Phasor Diagram of Series R-L Circuit

- Thus, from the above, it can be said that the current in series R-L circuit lags the applied voltage V by an angle ϕ. If supply voltage

$$v = V_m \, Sin \;\; \omega t$$

$$i = I_m \, sin \;\; (\omega t - \phi) \qquad \text{Where } I_m = \frac{V_m}{Z}$$

Voltage Triangle

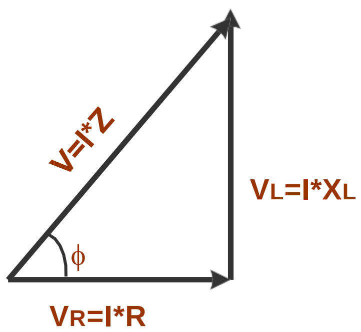

Figure 2.25 Voltage Triangle Series R-L Circuit

$$V = \sqrt{V_R^2 + V_L^2}$$
$$= \sqrt{(IR)^2 + (IX_L)^2}$$
$$= I \sqrt{R^2 + X_L^2}$$
$$= IZ$$
$$where, \;\; Z = \sqrt{R^2 + X_L^2}$$

Impedance Triangle

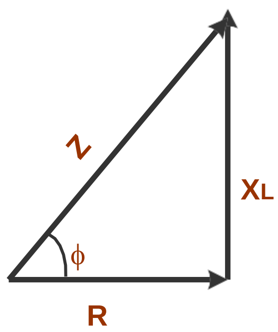

Figure 2.26 Impedance Triangle Series R-L Circuit

$$Z = \sqrt{R^2 + X_L^2}$$

$$\phi = tan^{-1} \frac{X_L}{R}$$

Power Triangle

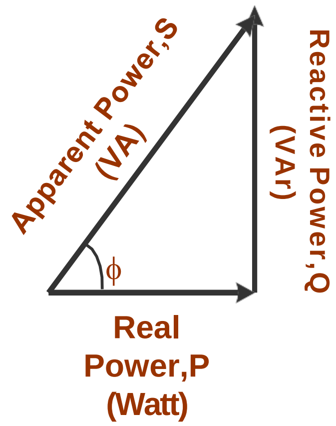

Figure 2.27 Power Triangle Series R-L Circuit

$$Real \;\; Power \;\; P = V \; I \;\; \cos\phi$$
$$= I^2 R$$
$$Reactive \;\; Power \;\; Q = V \; I \;\; \sin\phi$$
$$= I^2 X_L$$
$$Apparent \;\; Power \;\; S = V \; I$$
$$= I^2 Z$$

Power Factor

$$Power \;\; factor = \cos\varphi = \frac{R}{Z}$$
$$= \frac{P}{S}$$

2 A.C. Circuits

Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

Instantaneous power

$$p_t = v \times i$$

$$p_t = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$p_t = V_m \; I_m \sin \omega t \times \sin(\omega t - \phi)$$

$$p_t = \frac{2 \; V_m \; I_m \sin \omega t \times \sin(\omega t - \phi)}{2}$$

$$p_t = \frac{V_m I_m}{2} \left[\cos \phi - \cos(2\omega t - \phi) \right]$$

- Thus, the instantaneous values of the power consist of two components.
- First component is constant w.r.t. time and second component vary with time.

Average Power

$$P_{ave} = \int_0^{2\pi} \frac{V_m I_m}{2} \left[\cos \phi - \cos(2\omega t - \phi) \right] d\omega t$$

$$P_{ave} = \frac{V_m I_m}{2\pi} \int_0^{2\pi} \frac{1}{2} \left[\cos \phi - \cos(2\omega t - \phi) \right] d\omega t$$

$$P_{ave} = \frac{V_m I_m}{4\pi} \left[\int_0^{2\pi} \cos \phi \, d\omega t - \int_0^{2\pi} \cos(2\omega t - \phi) \, d\omega t \right]$$

$$P_{ave} = \frac{V_m I_m}{4\pi} \left[\cos \phi (\omega t)_0^{2\pi} - \left\{ \frac{\sin(2\omega t - \phi)}{2} \right\}_0^{2\pi} \right]$$

$$P_{ave} = \frac{V_m I_m}{4\pi} \left[2\pi \cos \phi \right] - \frac{V_m I_m}{8\pi} \left[\sin(4\pi - \phi) - \sin(-\phi) \right]$$

$$P_{ave} = \frac{V_m I_m}{2} \left[\cos \phi \right] - \frac{V_m I_m}{8\pi} \left[-\sin \phi + \sin \phi \right]$$

$$P_{ave} = \frac{V_m I_m}{2} \cos \phi - 0$$

$$P_{ave} = \frac{V_m I_m}{2} \cos \phi$$

$$P_{ave} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi$$

$$P_{ave} = VI \cos \phi$$

2.9 Series Resistance-Capacitance Circuit

- Consider a circuit consisting of a resistor of resistance R ohms and a purely capacitive of capacitance farad in series as in the Fig. 2.28.

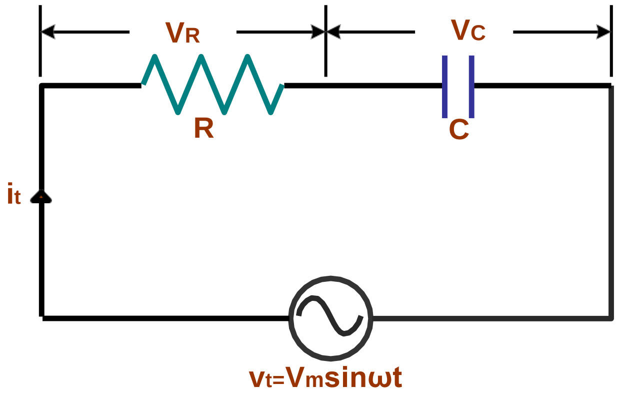

Figure 2.28 Circuit Diagram of Series R-C Circuit

- In the series circuit, the current i_t flowing through R and C will be the same. But the voltage across them will be different.
- The vector sum of voltage across resistor V_R and voltage across capacitor V_C will be equal to supply voltage v_t.

Waveforms and Phasor Diagram

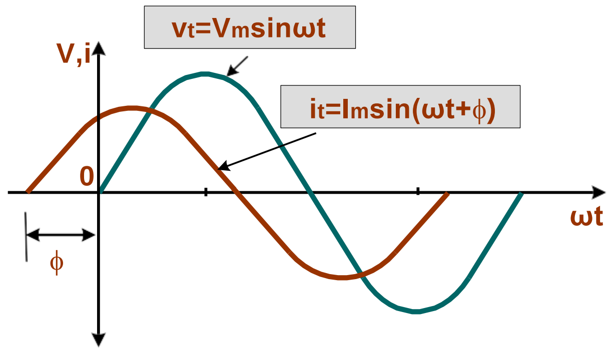

Figure 2.29 Waveform of Voltage and Current of Series R-C Circuit

- We know that in purely resistive the voltage and current in a resistive circuit both are in phase and therefore vector V_R is drawn superimposed to scale onto the current vector and in purely capacitive circuit the current I lead the voltage V_C by 90^o.
- So, to draw the vector diagram, first I taken as the reference. This is shown in the Fig. 2.30. Next V_R drawn in phase with I. Next V_C is drawn 90^o lagging the I. The supply voltage V is then phasor Addition of V_R and V_C.

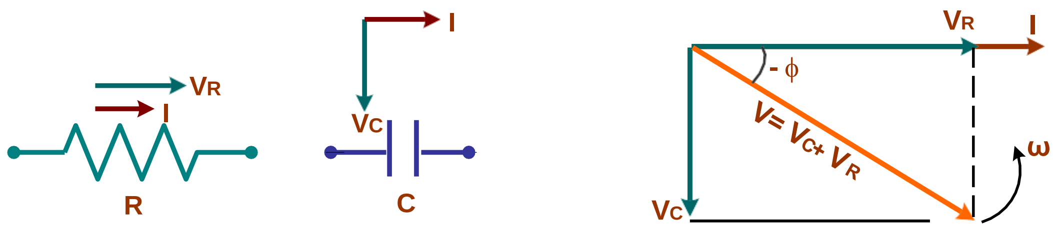

Figure 2.30 Phasor Diagram of Series R-C Circuit

- Thus, from the above equation it is clear that the current in series R-C circuit leads the applied voltage V by an angle ϕ. If supply voltage

$$v = V_m \, Sin \, \omega t$$

$$i = I_m \, sin(\omega t + \phi) \qquad \text{Where, } I_m = \frac{V_m}{Z}$$

Voltage Triangle

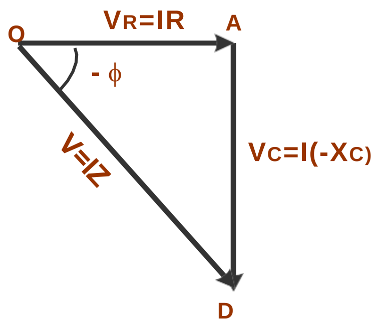

Figure 2.31 Voltage Triangle of Series R-C Circuit

$$V = \sqrt{V_R^2 + V_C^2}$$
$$= \sqrt{(IR)^2 + (IX_C)^2}$$
$$= I \sqrt{R^2 + X_C^2}$$
$$= IZ \qquad where, \; Z = \sqrt{R^2 + X_C^2}$$

Impedance Triangle

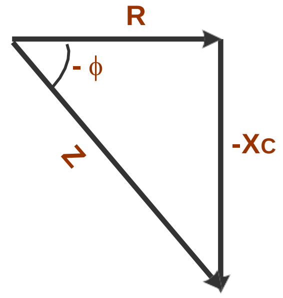

Figure 2.32 Impedance Triangle Series R-L Circuit

$$Z = \sqrt{R^2 + X_C^2}$$
$$\phi = tan^{-1} \frac{-X_C}{R}$$

Power Triangle

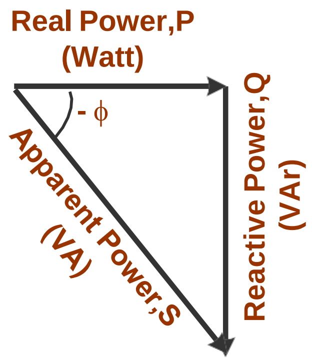

Figure 2.33 Power Triangle Series R-L Circuit

$$Real \;\; Power, \;\; P = V\,I \;\; cos\phi$$
$$= I^2 R$$
$$Reactive \;\; Power, \; Q = V\,I \;\; sin\phi$$
$$= I^2 X_L$$
$$Apparent \;\; Power, S = V \;\; I$$
$$= I^2 Z$$

Power Factor

$$p.f. = cos\phi = \frac{R}{Z} or \frac{P}{S}$$

Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

Instantaneous power

$$p_t = v \times i$$

$$p_t = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$p_t = V_m \; I_m \sin \omega t \times \sin(\omega t + \phi)$$

$$p_t = \frac{2 \, V_m \; I_m \sin \omega t \;\; \times \;\; sin \;\; (\omega t + \phi)}{2}$$

$$p_t = \frac{V_m I_m}{2} [cos \;\; \phi - cos(2\omega t + \phi)]$$

- Thus, the instantaneous values of the power consist of two components. First component remains constant w.r.t. time and second component vary with time.

Average Power

$$P_{ave} = \int_0^{2\pi} \frac{V_m I_m}{2}\left[\cos\ \phi - \cos(2\omega t+\phi)\right] d\omega t$$

$$P_{ave} = \frac{V_m I_m}{2\pi} \int_0^{2\pi} \frac{1}{2}\left[\cos\ \phi - \cos(2\omega t+\phi)\right] d\omega t$$

$$P_{ave} = \frac{V_m I_m}{4\pi}\left[\int_0^{2\pi} \cos\phi d\omega t - \int_0^{2\pi} \cos(2\omega t+\phi)\, d\omega t\right]$$

$$P_{ave} = \frac{V_m I_m}{4\pi}\left[\cos\phi(\omega t)_0^{2\pi} - \left\{\frac{\sin(2\omega t+\phi)}{2}\right\}_0^{2\pi}\right]$$

$$P_{ave} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi - 0)\right] - \frac{V_m I_m}{8\pi}\left[\sin(4\pi+\phi) - \sin(\phi)\right]$$

$$P_{ave} = \frac{V_m I_m}{2}\left[\cos\phi\right] - \frac{V_m I_m}{8\pi}\left[\sin\phi - \sin\phi\right]$$

$$P_{ave} = \frac{V_m I_m}{2}\cos\phi - 0$$

$$P_{ave} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\ \phi$$

$$P_{ave} = VI\cos\ \phi$$

2.10 Series RLC circuit

- Consider a circuit consisting of a resistor of R ohm, pure inductor of inductance L henry and a pure capacitor of capacitance C farads connected in series.

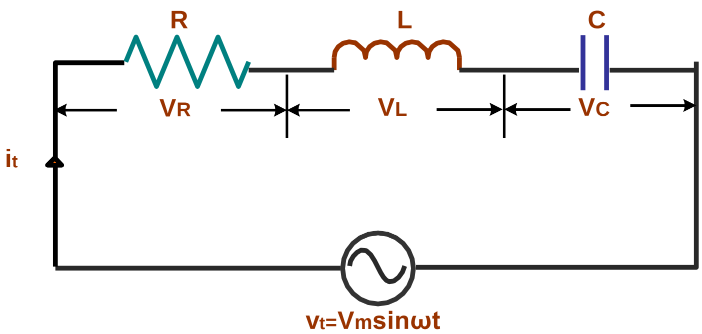

Figure 2.34 Circuit Diagram of Series RLC Circuit

Phasor Diagram

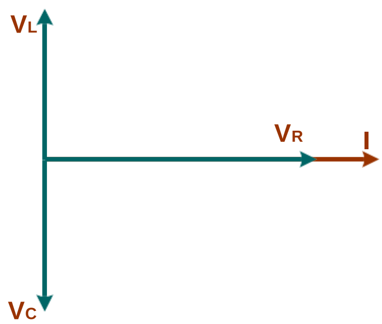

Figure 2.35 Phasor Diagram of Series RLC Circuit

Current I is taken as reference.
V_R is drawn in phase with current,
V_L is drawn leading I by 90^0,
V_C is drawn lagging I by 90^0

- Since V_L and V_C are in opposition to each other, there can be two cases:
  (1) $V_L > V_C$
  (2) $V_L < V_C$

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Case-1

When, $V_L > V_C$, the phasor diagram would be as in the figure 2.36

Phasor Diagram

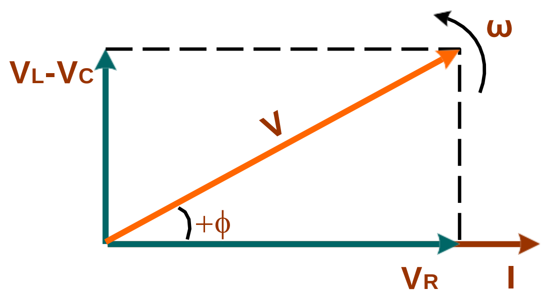

Figure 2.36 Phasor Diagram of Series R-L-C Circuit for Case VL > VC

$$V = \sqrt{V_R^2 + \left(V_L - V_C\right)^2}$$
$$= \sqrt{(IR)^2 + I\left(X_L - X_C\right)^2}$$
$$= I\sqrt{R^2 + \left(X_L - X_C\right)^2}$$
$$= IZ \quad where,\ Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

- The angle ϕ by which V leads I is given by

$$tan\phi = \frac{\left(V_L - V_C\right)}{R}$$
$$\therefore \phi = tan^{-1}\frac{I\left(X_L - X_C\right)}{IR}$$
$$\therefore \phi = tan^{-1}\frac{\left(X_L - X_C\right)}{R}$$

- Thus, when $V_L > V_C$ the series current I lags V by angle ϕ.
  If $v_t = V_m\ Sin\ \omega t$
  $i_t = I_m\ Sin\ (\omega t - \phi)$
- Power consumed in this case is equal to series RL circuit $P_{ave} = VI\cos\phi$.

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Case-2

When, $V_L < V_C$, the phasor diagram would be as in the figure 2.37

Phasor Diagram

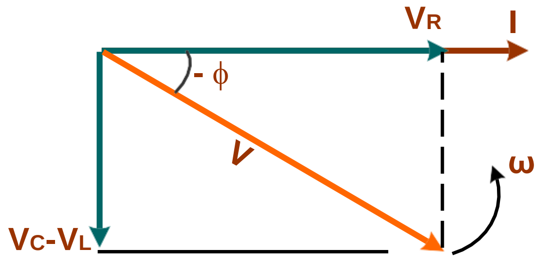

Figure 2.37 Phasor Diagram of Series R-L-C Circuit for Case VL < VC

$$V = \sqrt{V_R^2 + \left(V_C - V_L\right)^2}$$
$$= \sqrt{(IR)^2 + I\left(X_C - XL\right)^2}$$
$$= I\sqrt{R^2 + \left(X_C - X_L\right)^2}$$
$$= IZ \quad where,\ Z = \sqrt{R^2 + \left(X_C - X_L\right)^2}$$

- The angle ϕ by which V lags I is given by

$$tan\phi = \frac{\left(V_C - V_L\right)}{R}$$
$$\therefore \phi = tan^{-1}\frac{I\left(X_C - X_L\right)}{IR}$$
$$\therefore \phi = tan^{-1}\frac{\left(X_C - X_L\right)}{R}$$

- Thus, when $V_L < V_C$ the series current I leads V by angle ϕ.
  If $v_t = V_m\ Sin\ \omega t$
  $i_t = I_m\ Sin\ (\omega t + \phi)$
- Power consumed in this case is equal to series RC circuit $P_{ave} = VI\cos\phi$.

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2.11 Series resonance RLC circuit

- Such a circuit shown in the Fig. 2.38 is connected to an A.C. source of constant supply voltage V but having variable frequency.

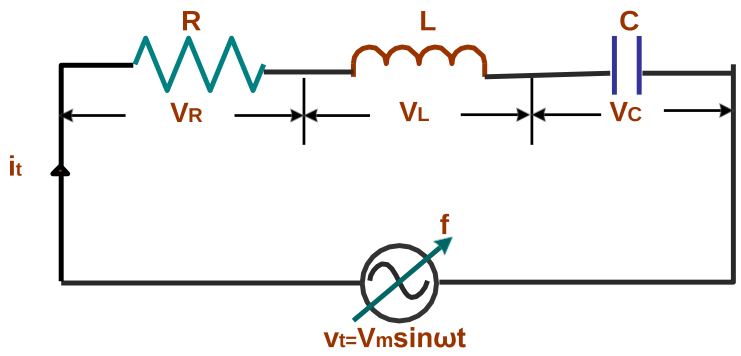


Figure 2.38 Circuit Diagram of Series Resonance RLC Circuit

- The frequency can be varied from zero, increasing and approaching infinity. Since X_L and X_C are function of frequency, at a particular frequency of applied voltage, X_L and X_C will become equal in magnitude and power factor become unity.

  Since $X_L = X_C$,

  $\therefore \quad X_L - X_C = 0$

  $$\therefore \quad Z = \sqrt{R^2 + 0} = R$$

- The circuit, when $X_L = X_C$ and hence Z = R, is said to be in resonance. In a series circuit since current I remain the same throughout we can write,

  $IX_L = IX_C$ i.e. $V_L = V_C$

Phasor Diagram

- Shown in the Fig. 2.39 is the phasor diagram of series resonance RLC circuit.

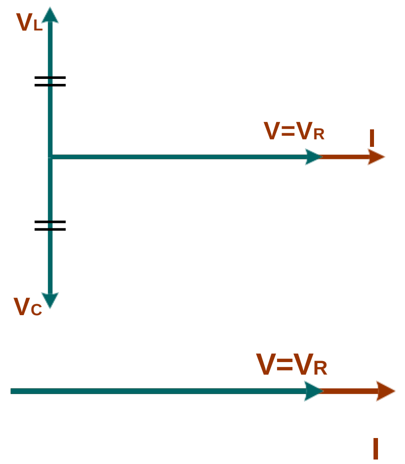


Figure 2.39 Phasor Diagram of Series Resonance RLC Circuit

- So, at resonance V_L and V_C will cancel out of each other.

$\therefore$ The supply voltage

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\therefore \quad V = V_R$$

- i.e. the supply voltage will drop across the resistor R.

Resonance Frequency

- At resonance frequency $X_L = X_C$

$$\therefore \quad 2\pi f_r L = \frac{1}{2\pi f_r C} \qquad (f_r \text{ is the resonance frequency})$$

$$\therefore \quad f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

Q- Factor

- The Q- factor is nothing but the voltage magnification during resonance.
- It indicates as to how many times the potential difference across L or C is greater than the applied voltage during resonance.
- Q- factor = Voltage magnification

$$Q - Factor = \frac{V_L}{V_S}$$

$$= \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$= \frac{\omega_r L}{R}$$

$$= \frac{2\pi f_r L}{R} \qquad But \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad Q - Factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Graphical Representation of Resonance

- **Resistance (R)** is independent of frequency. Thus, it is represented by straight line.
- **Inductive reactance (X_L)** is directly proportional to frequency. Thus, it is increases linearly with the frequency.

$$\because X_L = 2\pi fL$$

$$\therefore X_L \propto f$$

- **Capacitive reactance(X_C)** is inversely proportional to frequency. Thus, it is show as hyperbolic curve in fourth quadrant.

$$\because X_C = \frac{1}{2\pi fC}$$

$$\therefore X_C \propto \frac{1}{f}$$

- **Impedance (Z)** is minimum at resonance frequency.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For, $f = f_r, Z = R$

- **Current (I) is** maximum at resonance frequency.

$$\because I = \frac{V}{Z}$$

For $f = f_r, I = \frac{V}{R}$ is maximum, I_{MAX}

- **Power factor** is unity at resonance frequency.

  Power factor=$\cos\phi=\frac{R}{Z}$

  For $f = f_r, p.f. = 1$ (unity)

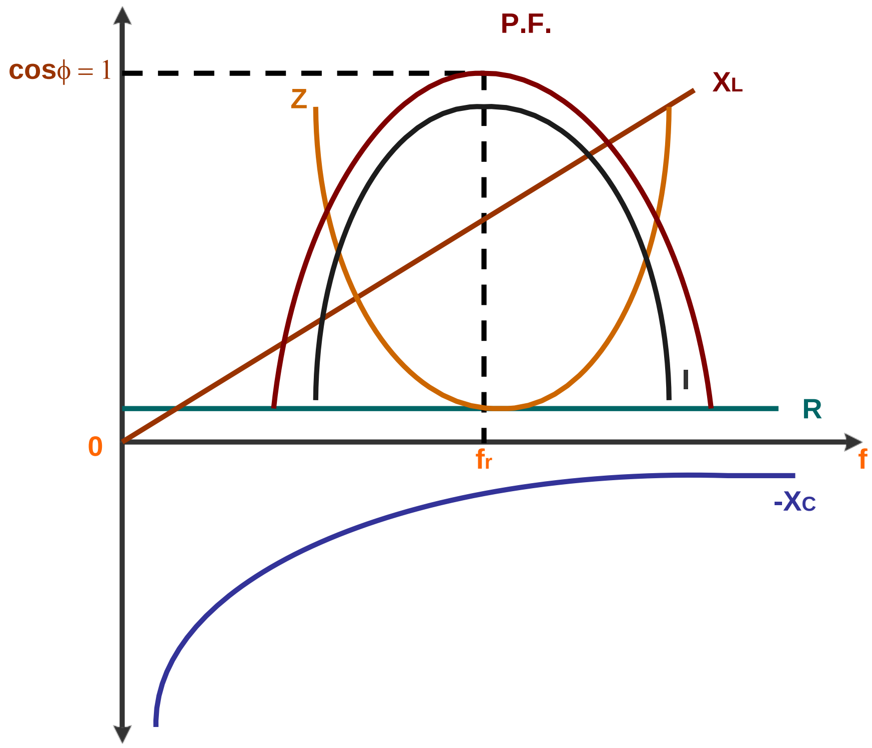


Figure 2.40 Graphical Representation of Series Resonance RLC Circuit

2.11 Parallel Resonance RLC Circuit

- Fig. 2.41 Shows a parallel circuit consisting of an inductive coil with internal resistance R ohm and inductance L henry in parallel with capacitor C farads.

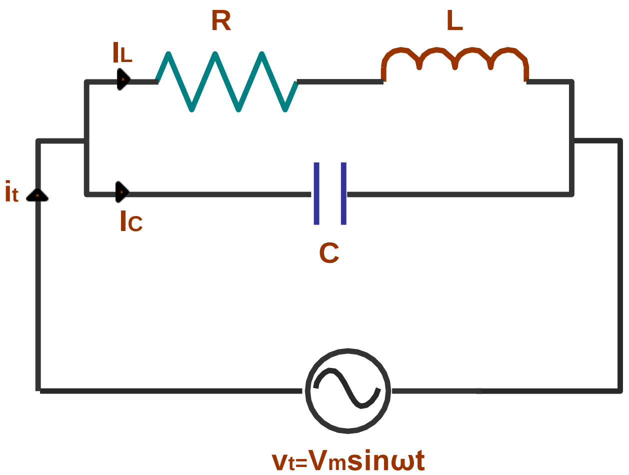


Figure 2.41 Circuit Diagram of Parallel Resonance RLC Circuit

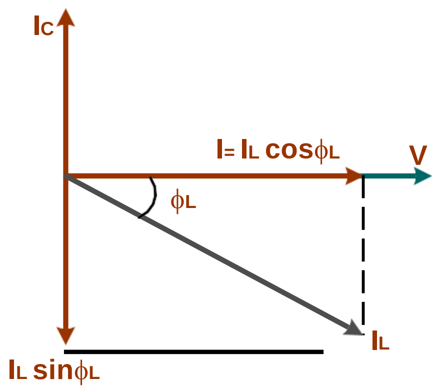


Figure 2.42 Circuit Diagram of Parallel Resonance RLC Circuit

- The current I_C can be resolved into its active and reactive components. Its active component $I_L \cos\phi$ and reactive component $I_L \sin\phi$.

- A parallel circuit is said to be in resonance when the power factor of the circuit becomes unity. This will happen when the resultant current I is in phase with the resultant voltage V and hence the phase angle between them is zero.
- In the phasor diagram shown, this will happen when $I_C = I_L \sin\phi$ and $I = I_L \cos\phi$.

Resonance Frequency

- To find the resonance frequency, we make use of the equation $I_C = I_L \sin\phi$.

$$I_C = I_L \sin \phi$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \frac{X_L}{Z_L}$$

$$Z_L^2 = X_L X_C$$

$$Z_L^2 = 2\pi f_r L \quad \frac{1}{2\pi f_r C} = \frac{L}{C}$$

$$\left(R^2 + \omega_r^2 L^2\right) = \frac{L}{C}$$

$$\omega_r^2 = \frac{L}{C}\left(\frac{1}{L^2}\right) - \frac{R^2}{L^2}$$

$$(2\pi f_r)^2 = \frac{L}{C}\left(\frac{1}{L^2}\right) - \frac{R^2}{L^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

- If the resistance of the coil is negligible,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Impedance

- To find the resonance frequency, we make use of the equation $I = I_L \cos\phi$because, at resonance, the supply current I will be in phase with the supply voltage V.

$$I = I_L \cos\phi$$

$$\frac{V}{Z} = \frac{V}{Z_L} \quad \frac{R}{Z_L}$$

$$Z = \frac{Z_L^2}{R} \qquad But \ Z_L^2 = \frac{L}{C}$$

$$Z = \frac{L}{RC}$$

- The impedance during parallel resonance is very large because of L and C has a very large value at that time. Thus, impedance at the resonance is maximum.

$$I = \frac{V}{Z} \ will \ be \ minimum.$$

Q-Factor

- Q- factor = Current magnification

$$Q-Factor = \frac{I_L}{I}$$

$$= \frac{I_L \sin\phi}{I_L \cos\phi} = \frac{\sin\phi}{\cos\phi}$$

$$= \tan\phi = \frac{\omega_r L}{R}$$

$$= \frac{2\pi f_r L}{R} \qquad But \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad Q-Factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Graphical representation of Parallel Resonance

- **Conductance (G)** is independent of frequency. Hence it is represented by straight line parallel to frequency.
- **Inductive Susceptance (B_L)** is inversely proportional to the frequency. Also, it is negative.

$$B_L = \frac{1}{jX_L} = \frac{1}{j2\pi fL}, \qquad \therefore B_L \propto \frac{1}{f}$$

- **Capacitive Susceptance (B_C)** is directly proportional to the frequency.

$$B_C = \frac{1}{-jX_C} = \frac{j}{X_C} = j2\pi fC, \quad \therefore B_C \propto f$$

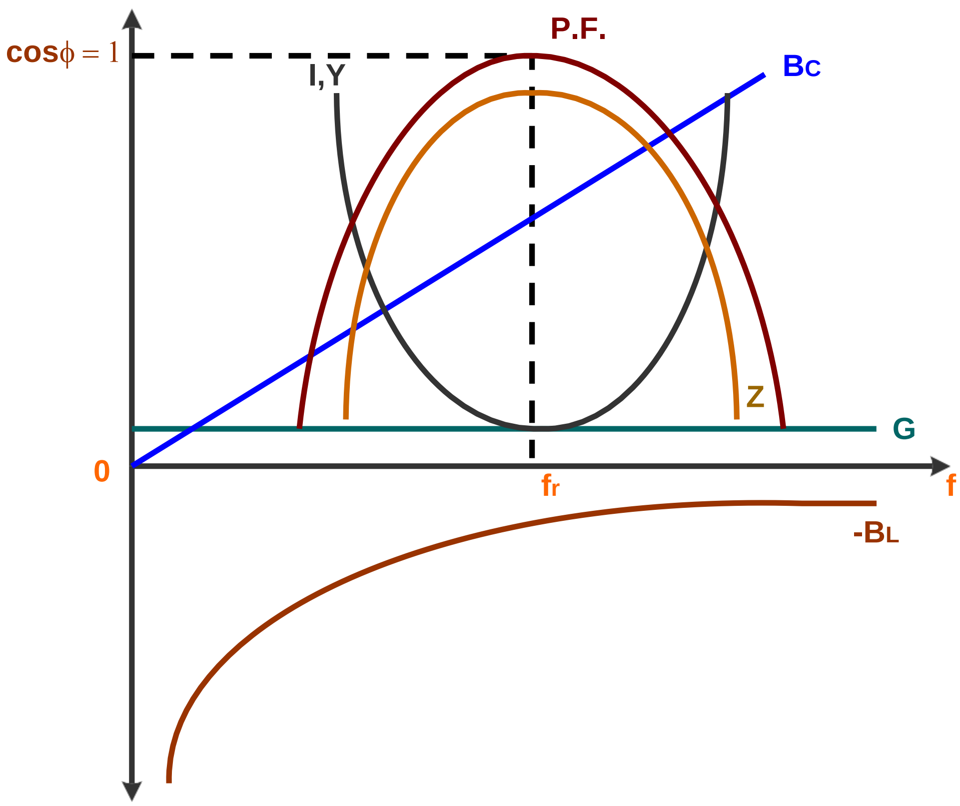


Figure 2.43 Graphical Representation of Parallel Resonance RLC Circuit

- **Admittance (Y)** is minimum at resonance frequency.

  $Y = \sqrt{G^2 + (B_L - B_C)^2}$

  For, $f = f_r, Y = G$

- **Current (I)** is minimum at resonance frequency.

  $\because I = VY$

- **Power factor** is unity at resonance frequency.

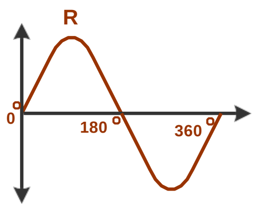
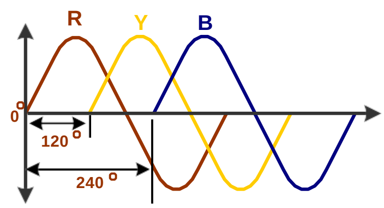
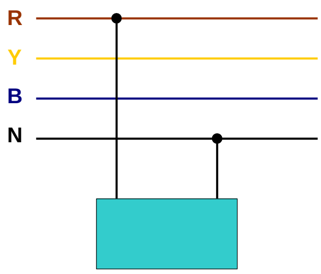
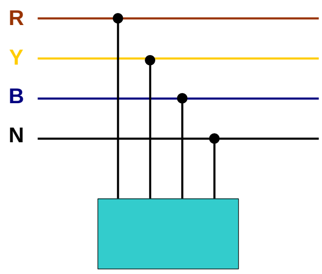
  Power factor=cosϕ= $\frac{G}{Y}$

2.12 Comparison of Series and Parallel Resonance

Sr.No.	Description	Series Circuit	Parallel Circuit
1	Impedance at resonance	Minimum Z = R	Maximum $Z = \frac{L}{RC}$
2	Current	Maximum $I = \frac{V}{R}$	Minimum $I = \frac{V}{L/RC}$
3	Resonance Frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi\sqrt{LC}}$
4	Power Factor	Unity	Unity
5	Q- Factor	$f_r = \frac{1}{R}\sqrt{\frac{L}{C}}$	$f_r = \frac{1}{R}\sqrt{\frac{L}{C}}$
6	It magnifies at resonance	Voltage	Current

<u>Three - Phase AC Circuits</u>

2.13 Comparison between single phase and three phase

Basis for Comparison	**Single Phase**	**Three Phase**
Definition	The power supply through one conductor.	The power supply through three conductors.
Wave Shape		
Number of wire	Require two wires for completing the circuit	Requires four wires for completing the circuit
Voltage	Carry 230V	Carry 415V
Phase Name	Split phase	No other name
Network	Simple	Complicated
Loss	Maximum	Minimum
Power Supply Connection		
Efficiency	Less	High
Economical	Less	More
Uses	For home appliances.	In large industries and for running heavy loads.

2.14 Generation of three phase EMF

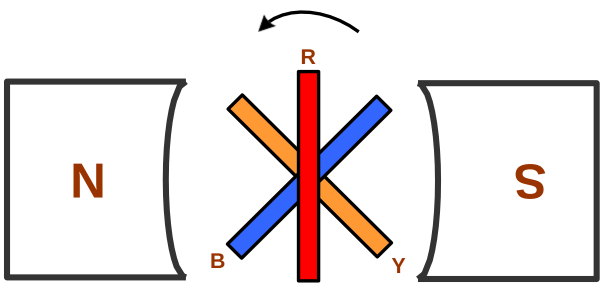


Figure 2.44 Generation of three phase emf

- According to Faraday's law of electromagnetic induction, we know that whenever a coil is rotated in a magnetic field, there is a sinusoidal emf induced in that coil.

- Now, we consider 3 coil C_1(R-phase), C_2(Y-phase) and C_3(B-phase), which are displaced 120^0 from each other on the same axis. This is shown in fig. 2.44.
- The coils are rotating in a uniform magnetic field produced by the N and S pols in the counter clockwise direction with constant angular velocity.
- According to Faraday's law, emf induced in three coils. The emf induced in these three coils will have phase difference of 120^0. i.e. if the induced emf of the coil C_1 has phase of 0^0, then induced emf in the coil C_2 lags that of C_1 by 120^0 and C_3 lags that of C_2 120^0.

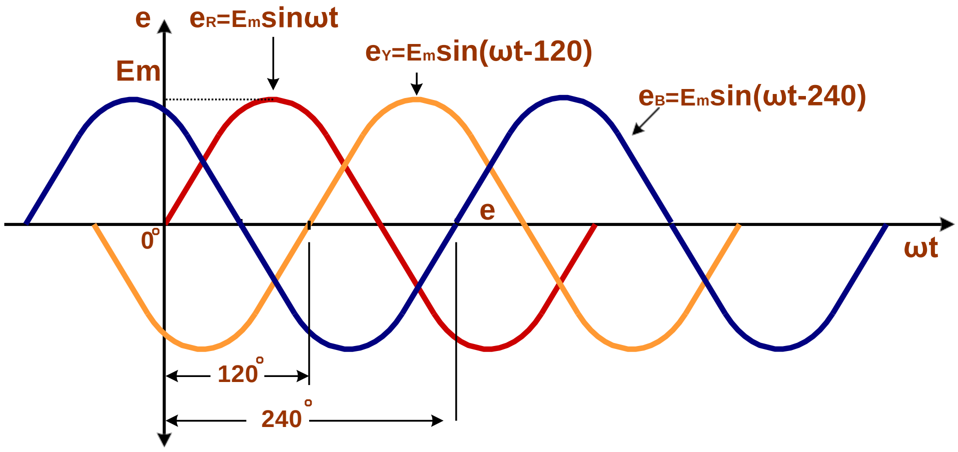


Figure 2.45 Waveform of Three Phase EMF

- Thus, we can write,

$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin\left(\omega t - 120^0\right)$$
$$e_B = E_m \sin\left(\omega t - 240_0\right)$$

- The above equation can be represented by their phasor diagram as in the Fig. 2.46.

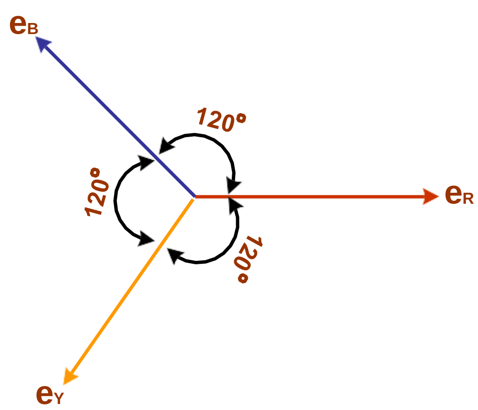


Figure 2.46 Phasor Diagram of Three Phase EMF

2.15 Important definitions

➢ **Phase Voltage**

It is defined as the voltage across either phase winding or load terminal. It is denoted by V_{ph}.

Phase voltage V_{RN}, V_{YN} and V_{BN} are measured between R-N, Y-N, B-N for star connection and between R-Y, Y-B, B-R in delta connection.

- **Line voltage**

  It is defined as the voltage across any two-line terminal. It is denoted by V_L.

  Line voltage V_{RY}, V_{YB}, V_{BR} measure between R-Y, Y-B, B-R terminal for star and delta connection both.

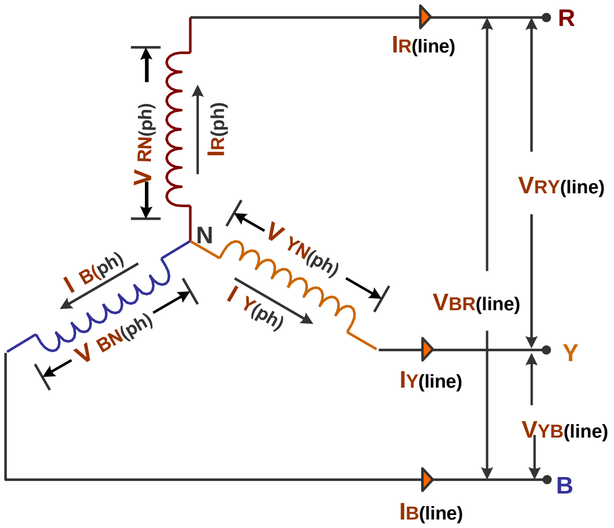


Figure 2.47 Three Phase Star Connection System

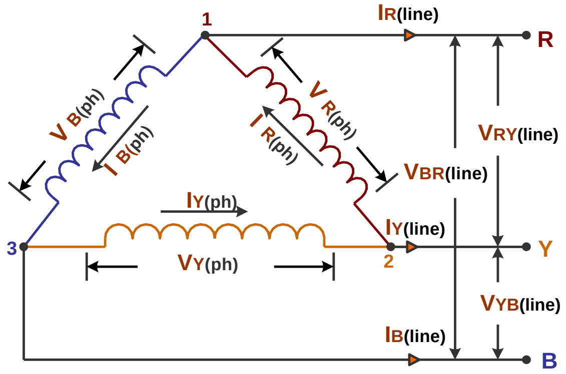


Figure 2.48 Three Phase Delta Connection System

- **Phase current**

  It is defined as the current flowing through each phase winding or load. It is denoted by I_{ph}.

  Phase current $I_{R(ph)}$, $I_{Y(ph)}$ and $I_{B(Ph)}$ measured in each phase of star and delta connection. respectively.

- **Line current**

  It is defined as the current flowing through each line conductor. It denoted by I_L.

  Line current $I_{R(line)}$, $I_{Y(line)}$, and $I_{B((line)}$ are measured in each line of star and delta connection.

- **Phase sequence**

  The order in which three coil emf or currents attain their peak values is called the phase sequence. It is customary to denoted the 3 phases by the three colours. i.e. red (R), yellow (Y), blue (B).

- **Balance System**

  A system is said to be balance if the voltages and currents in all phase are equal in magnitude and displaced from each other by equal angles.

- **Unbalance System**

  A system is said to be unbalance if the voltages and currents in all phase are unequal in magnitude and displaced from each other by unequal angles.

- **Balance load**

  In this type the load in all phase are equal in magnitude. It means that the load will have the same power factor equal currents in them.

- **Unbalance load**

  In this type the load in all phase have unequal power factor and currents.

2.16 Relation between line and phase values for voltage and current in case of balanced delta connection.

➢ **Delta (Δ) or Mesh connection**, starting end of one coil is connected to the finishing end of other phase coil and so on which giving a closed circuit.

Circuit Diagram

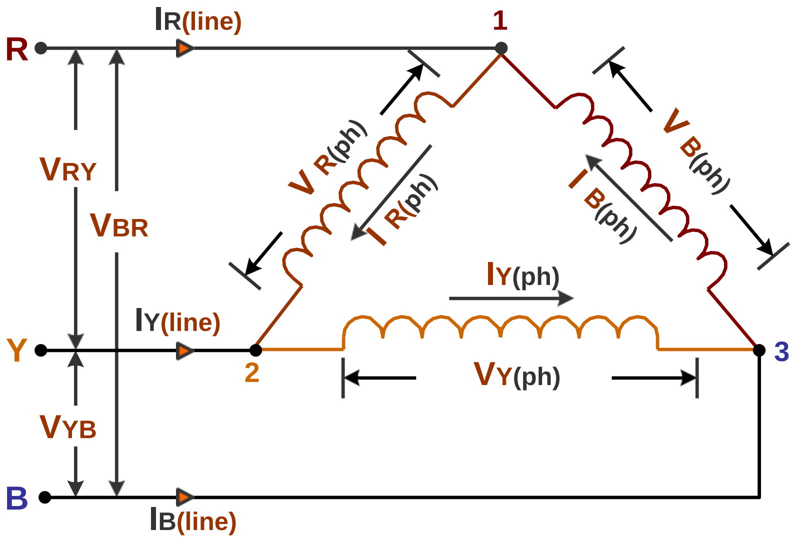


Figure 2.49 Three Phase Delta Connection

- Let,

  Line voltage, $V_{RY} = V_{YB} = V_{BR} = V_L$

  Phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

  Line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

  Phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line and phase voltage

- For delta connection line voltage V_L and phase voltage V_{ph} both are same.

  $V_{RY} = V_{R(ph)}$

  $V_{YB} = V_{Y(ph)}$

  $V_{BR} = V_{B(ph)}$

  $\therefore V_L = V_{ph}$

  Line voltage = Phase Voltage

Relation between line and phase current

- For delta connection,

  $I_{R(line)} = I_{R(ph)} - I_{B(ph)}$

  $I_{Y(line)} = I_{Y(ph)} - I_{R(ph)}$

  $I_{B(line)} = I_{B(ph)} - I_{Y(ph)}$

- i.e. current in each line is vector difference of two of the phase currents.

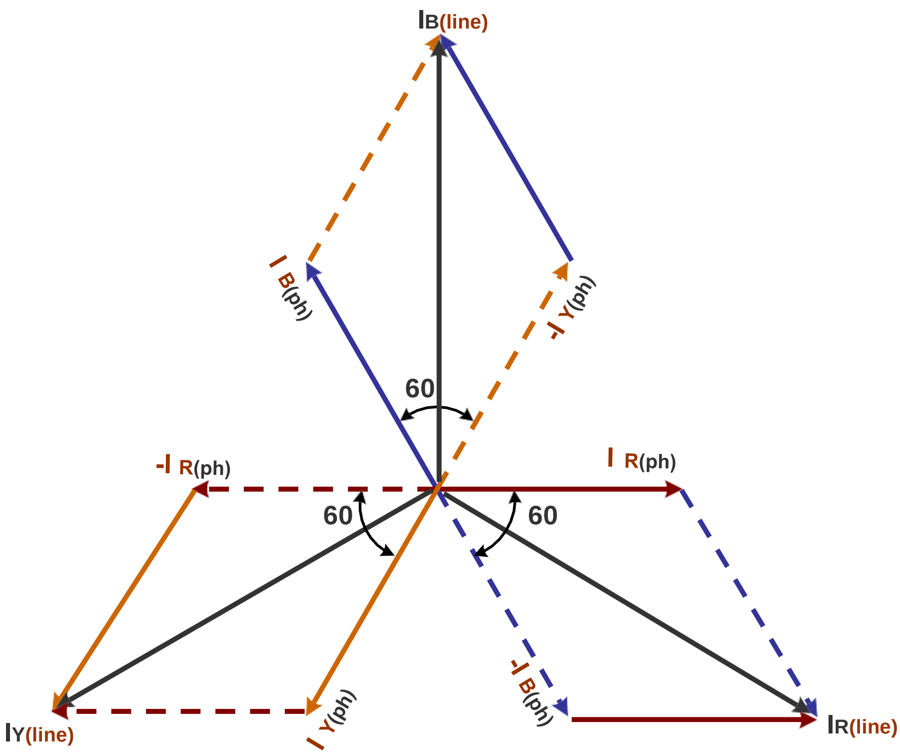


Figure 2.50 Phasor Diagram of Three Phase Delta Connection

- So, considering the parallelogram formed by I_R and I_B.

$$I_{R(line)} = \sqrt{I_{R(ph)}^2 + I_{B(ph)}^2 + 2I_{R(ph)}I_{B(ph)}\cos\theta}$$

$$\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos 60^\circ}$$

$$\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2 \times \left(\frac{1}{2}\right)}$$

$$\therefore I_L = \sqrt{3I_{ph}^2}$$

$$\therefore I_L = \sqrt{3}I_{ph}$$

- Similarly, $I_{Y(line)} = I_{B(line)} = \sqrt{3}\, I_{ph}$
- Thus, in delta connection Line current = $\sqrt{3}$ Phase current

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$

$$P = 3V_{ph}I_{ph}\cos\phi$$

$$P = 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$$

$$\therefore P = \sqrt{3}V_L I_L \cos\phi$$

2.17 Relation between line and phase values for voltage and current in case of balanced star connection.

- In the **Star Connection**, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point.

Circuit Diagram

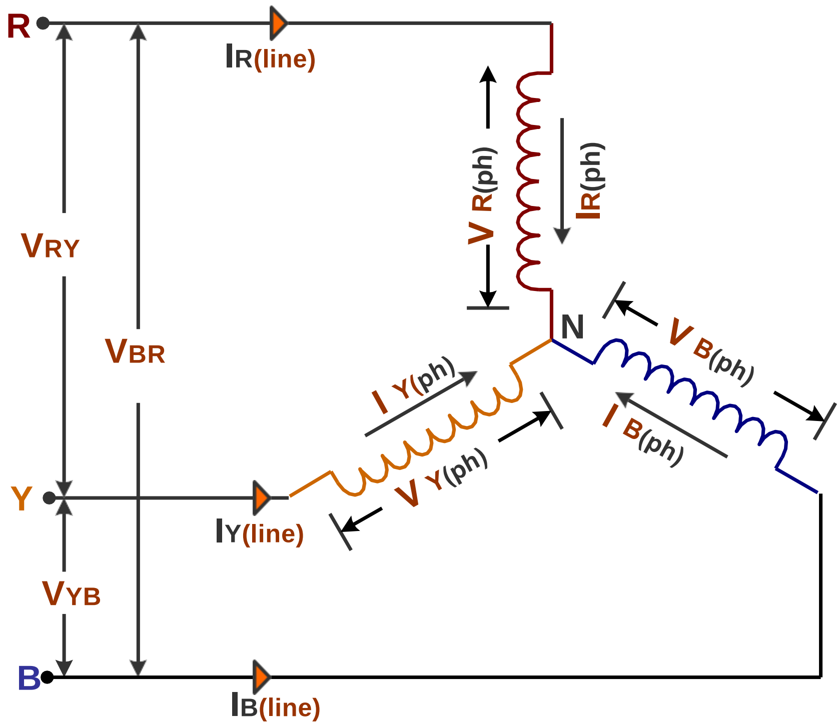


Figure 2.51 Circuit Diagram of Three Phase Star Connection

- Let,

line voltage, $V_{RY} = V_{BY} = V_{BR} = V_L$

phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line and phase voltage

- For star connection, line current I_L and phase current I_{ph} both are same.

$I_{R(line)} = I_{R(ph)}$

$I_{Y(line)} = I_{Y(ph)}$

$I_{B(line)} = I_{B(ph)}$

$\therefore \quad I_L = I_{ph}$

Line Current = Phase Current

Relation between line and phase voltage

- For delta connection,

$$V_{RY} = V_{R(ph)} - V_{Y(ph)}$$
$$V_{YB} = V_{Y(ph)} - V_{B(ph)}$$
$$V_{BR} = V_{B(ph)} - V_{R(ph)}$$

- i.e. line voltage is vector difference of two of the phase voltages. Hence,

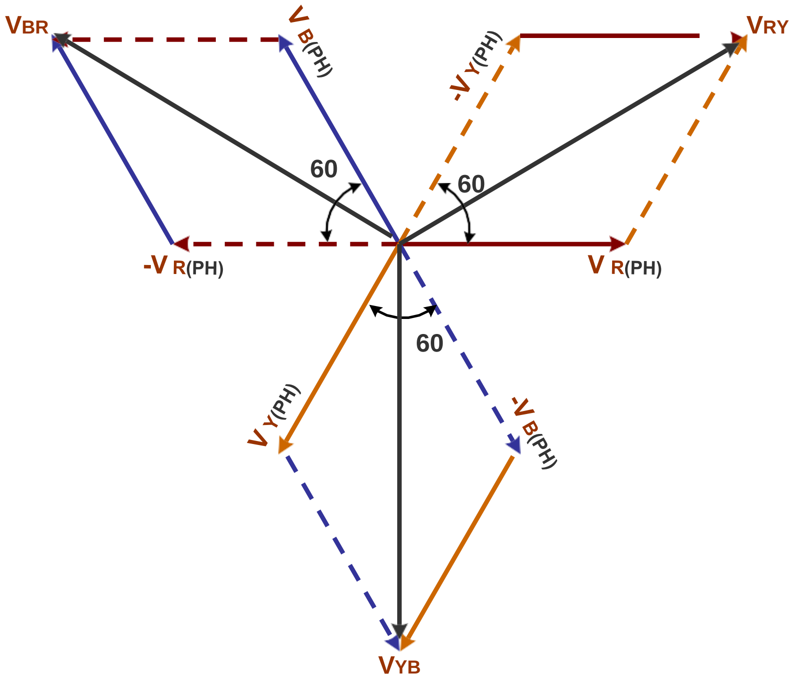


Figure 2.52 Phasor Diagram of Three Phase Star Connection

From parallelogram,

$$V_{RY} = \sqrt{V_{R(ph)}^2 + V_{Y(ph)}^2 + 2V_{R(ph)}V_{Y(ph)}\cos\theta}$$
$$\therefore V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph}\cos 60^\circ}$$
$$\therefore V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \times \left(\tfrac{1}{2}\right)}$$
$$\therefore V_L = \sqrt{3V_{ph}^2}$$
$$\therefore V_L = \sqrt{3}V_{ph}$$

- Similarly, $V_{YB} = V_{BR} = \sqrt{3}\,V_{ph}$
- Thus, in star connection Line voltage = $\sqrt{3}$ Phase voltage

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$
$$P = 3V_{ph}I_{ph}\cos\phi$$
$$P = 3\left(\frac{V_L}{\sqrt{3}}\right)I_L\cos\phi$$
$$\therefore P = \sqrt{3}V_L I_L\cos\phi$$

2.18 Measurement of power in balanced 3-phase circuit by two-watt meter method

- This is the method for 3-phase power measurement in which sum of reading of two wattmeter gives total power of system.

Circuit Diagram

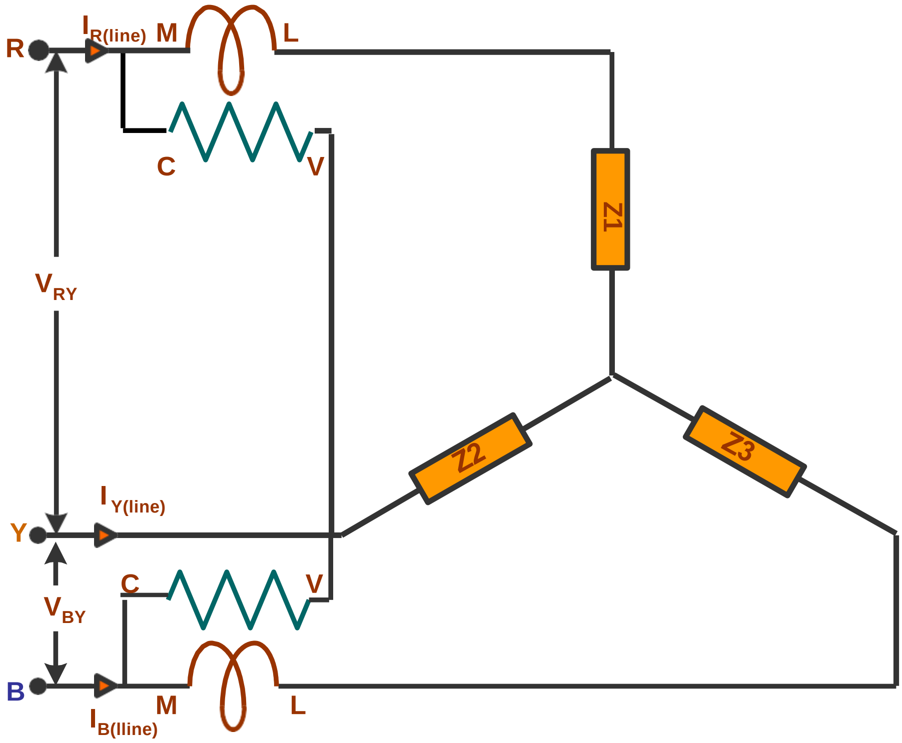


Figure 2.53 Circuit Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

- The load is considered as an inductive load and thus, the phasor diagram of the inductive load is drawn below in Fig. 2.54.

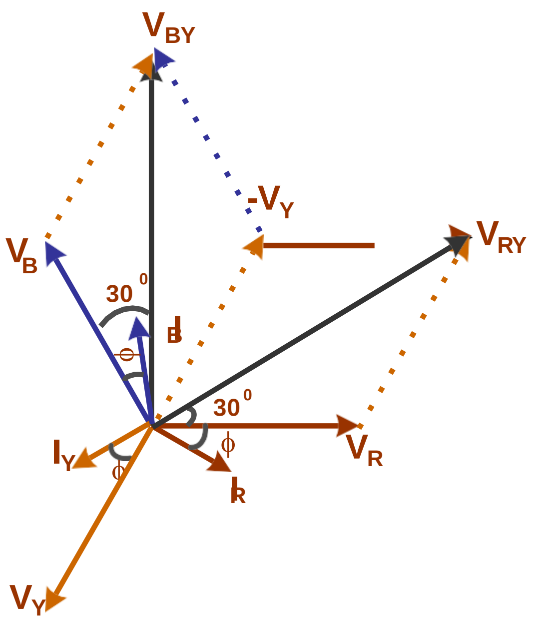


Figure 2.54 Phasor Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

- The three voltages V_{RN}, V_{YN} and V_{BN}, are displaced by an angle of 120^0 degree electrical as shown in the phasor diagram. The phase current lag behind their respective phase voltages by an angle ϕ. The power measured by the Wattmeter, W_1 and W_2.

  Reading of wattmeter, $W_1 = V_{RY} I_R \cos\phi_1 = V_L I_L \cos(30+\phi)$

  Reading of wattmeter, $W_2 = V_{BY} I_B \cos\phi_2 = V_L I_L \cos(30-\phi)$

  Total power, P = W_1+W_2

$$\therefore P = V_L I_L \cos(30+\phi) + V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos(30+\phi) + \cos(30-\phi)\right]$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\cos 30 \cos\phi\right]$$
$$= V_L I_L \left[2\left(\frac{\sqrt{3}}{2}\right)\cos\phi\right]$$
$$= \sqrt{3} V_L I_L \cos\phi$$

- Thus, the sum of the readings of the two wattmeter is equal to the power absorbed in a 3-phase balanced system.

Determination of Power Factor from Wattmeter Readings

- As we know that

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

Now,

$$W_1 - W_2 = V_L I_L \cos(30+\phi) - V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\left(\frac{1}{2}\right)\sin\phi\right] = V_L I_L \sin\phi$$

$$\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\therefore \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

- Power factor of load given as,

$$\therefore \cos\phi = \cos\left(\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right)$$

Effect of power factor on wattmeter reading:

- From the Fig. 2.54, it is clear that for lagging power factor $\cos\phi$, the wattmeter readings are

$$W_1 = V_L I_L \cos(30 + \phi)$$
$$W_2 = V_L I_L \cos(30 - \phi)$$

- Thus, readings W_1 and W_2 will very depending upon the power factor angle ϕ .

p.f	ϕ	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	**Remark**
$\cos\phi = 1$	0^0	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	Both equal and +ve
$\cos\phi = 0.5$	60^0	0	$\frac{\sqrt{3}}{2} V_L I_L$	One zero and second total power
$\cos\phi = 0$	90^0	$-\frac{1}{2} V_L I_L$	$\frac{1}{2} V_L I_L$	Both equal but opposite

****