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Parse Trees, Ambiguity in CFG, Pumping Lemma for CFLs

Chapter 3: Grammars

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Introduction

This section explores:

- Structural representation of derivations using parse trees
- The concept of ambiguity in context-free grammars
- The pumping lemma for proving that certain languages are not context-free

What is a Parse Tree?

- A parse tree visually represents how a string is derived from a CFG.
- Structure:
- Root: Start symbol
- Internal nodes: Non-terminals (variables)
- Leaves: Terminals or ε (empty string)
- Reading the leaves from left to right gives the derived string.

Parse Tree Example

Grammar:

$S \rightarrow aSb$ | ε S→aSb | ε

String: aabb

Derivation:

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aab \Rightarrow aabb$

```
     S
   / | \
  a  S  b
    /|\
   a S b
     |
     ε
```

Each step replaces a non-terminal with its production.

Ambiguity in CFG

- A CFG is ambiguous if there exists at least one string that has:
  - More than one parse tree, or
  - More than one leftmost or rightmost derivation
- Ambiguity leads to confusion in interpretation, especially in compilers.

Example of Ambiguity

Grammar:

E→E+E | E∗E |

String: id + id * id

- Two parse trees:

  1.Interpret as (id + id) * id

  2.Interpret as id + (id * id)

- Grammar does not enforce precedence or associativity.

Removing Ambiguity

- To remove ambiguity:
- Rewrite the grammar to enforce precedence (e.g., * > +)
- Separate productions by level of precedence

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow id$

Pumping Lemma for CFLs

- Used to prove that a language is not context-free
- For every context-free language L, there exists a constant p (pumping length) such that:

Then there is a pumping length n such that any string w εL of length>=n can be written as follows –

|w|>=n

We can break w into 5 strings, w=uvxyz, such as the ones given below

- |vxy| >=n
- |vy| # ε
- For all k>=0, the string $uv^kxy^yz \in L$

How to Use Pumping Lemma

- Assume the language is context-free.
- Choose a string s of length ≥ p.
- Try all valid decompositions s=uvwxys
- Show that for some i, uviwxiy∉ creating a contradiction.

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