Chap 2

Context free grammar

Context free grammar is a formal grammar which is used to generate all possible strings in a given formal language.

Context free grammar G can be defined by four tuples as:

1. G= (V, T, P, S)

   Where,

   G describes the grammar

   T describes a finite set of terminal symbols.

   V describes a finite set of non-terminal symbols

   P describes a set of production rules

   S is the start symbol.

   In CFG, the start symbol is used to derive the string. You can derive the string by repeatedly replacing a non-terminal by the right hand side of the production, until all non-terminal have been replaced by terminal symbols.

   Production rules:

1. S → aSa
2. S → bSb
3. S → c

   Now check that abbcbba string can be derived from the given CFG.

1. S ⇒ aSa
2. S ⇒ abSba
3. S ⇒ abbSbba
4. S ⇒ abbcbba

   By applying the production S → aSa, S → bSb recursively and finally applying the production S → c, we get the string abbcbba.

Derivation

Derivation is a sequence of production rules. It is used to get the input string through these production rules. During parsing we have to take two decisions. These are as follows:

- We have to decide the non-terminal which is to be replaced.
- We have to decide the production rule by which the non-terminal will be replaced.

We have two options to decide which non-terminal to be replaced with production rule.

Left-most Derivation

In the left most derivation, the input is scanned and replaced with the production rule from left to right. So in left most derivatives we read the input string from left to right.

Example:

Production rules:

1. S = S + S
2. S = S - S
3. S = a | b |c

Input:

```
a - b + c
```

The left-most derivation is:

1. S = S + S
2. S = S - S + S
3. S = a - S + S
4. S = a - b + S
5. S = a - b + c

Right-most Derivation

In the right most derivation, the input is scanned and replaced with the production rule from right to left. So in right most derivatives we read the input string from right to left.

Example:

1. S = S + S
2. S = S - S
3. S = a | b |c

Input:

```
a - b + c
```

The right-most derivation is:

1. S = S - S
2. S = S - S + S
3. S = S - S + c
4. S = S - b + c
5. S = a - b + c

Parse tree

- Parse tree is the graphical representation of symbol. The symbol can be terminal or non-terminal.
- In parsing, the string is derived using the start symbol. The root of the parse tree is that start symbol.
- It is the graphical representation of symbol that can be terminals or non-terminals.
- Parse tree follows the precedence of operators. The deepest sub-tree traversed first. So, the operator in the parent node has less precedence over the operator in the sub-tree.

The parse tree follows these points:

- All leaf nodes have to be terminals.
- All interior nodes have to be non-terminals.
- In-order traversal gives original input string.

Example:

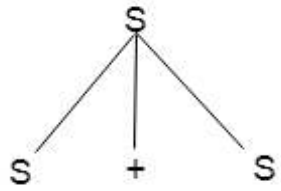
Production rules:

1. T= T + T | T * T
2. T = a|b|c

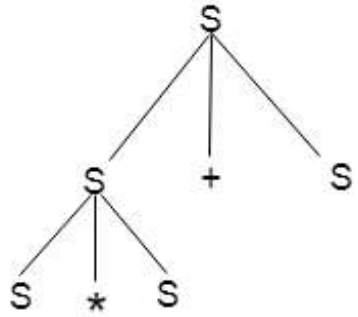
Input:

```
a * b + c
```

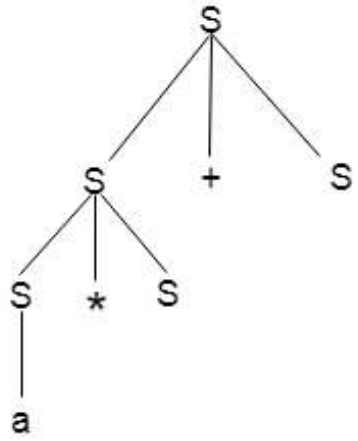
Step 1:



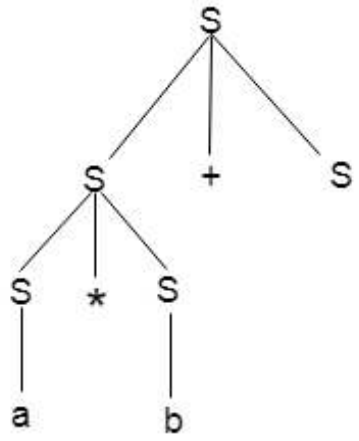
Step 2:



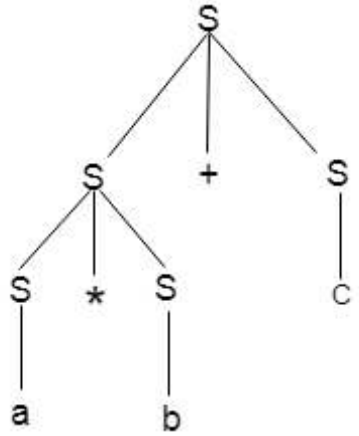
Step 3:



Step 4:



Step 5:



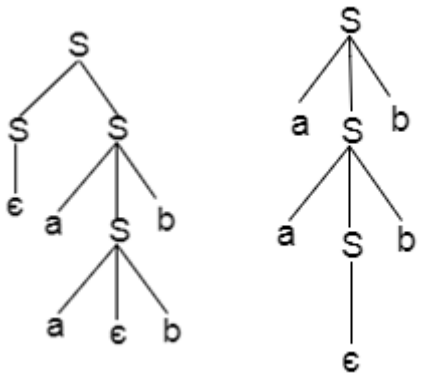
Ambiguity

A grammar is said to be ambiguous if there exists more than one leftmost derivation or more than one rightmost derivative or more than one parse tree for the given input string. If the grammar is not ambiguous then it is called unambiguous.

Example:

1. S = aSb | SS
2. S = ∈

For the string aabb, the above grammar generates two parse trees:



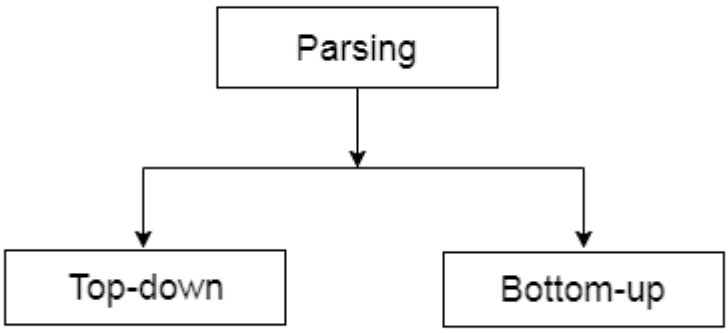
If the grammar has ambiguity then it is not good for a compiler construction. No method can automatically detect and remove the ambiguity but you can remove ambiguity by re-writing the whole grammar without ambiguity.

Parser

Parser is a compiler that is used to break the data into smaller elements coming from lexical analysis phase.

A parser takes input in the form of sequence of tokens and produces output in the form of parse tree.

Parsing is of two types: top down parsing and bottom up parsing.

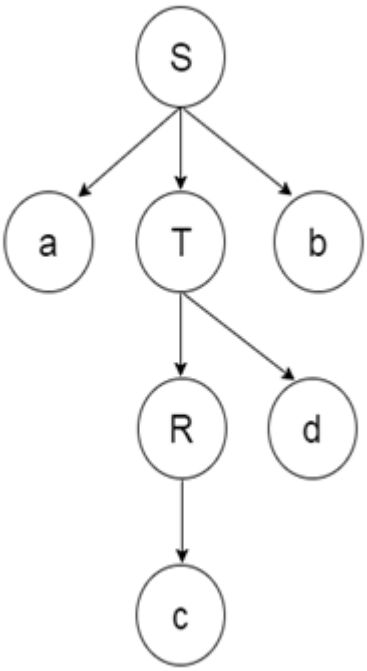


Top down paring

- The top down parsing is known as recursive parsing or predictive parsing.
- Bottom up parsing is used to construct a parse tree for an input string.

- In the top down parsing, the parsing starts from the start symbol and transform it into the input symbol.

Parse Tree representation of input string "acdb" is as follows:



Bottom up parsing

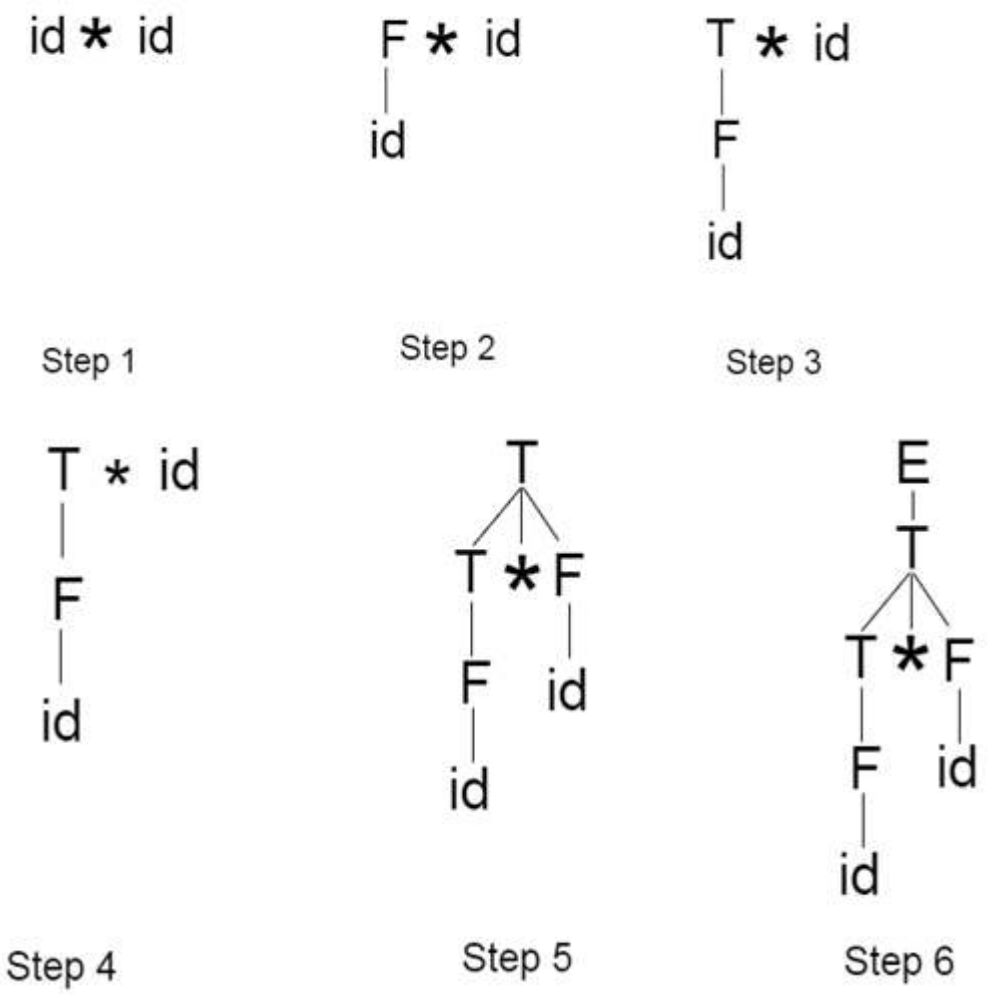
- Bottom up parsing is also known as shift-reduce parsing.
- Bottom up parsing is used to construct a parse tree for an input string.
- In the bottom up parsing, the parsing starts with the input symbol and construct the parse tree up to the start symbol by tracing out the rightmost derivations of string in reverse.

Example

Production:

1. E → T
2. T → T * F
3. T → id
4. F → T
5. F → id

Parse Tree representation of input string "id * id" is as follows:



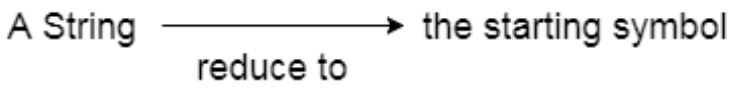
Bottom up parsing is classified in to various parsing. These are as follows:

1. Shift-Reduce Parsing
2. Operator Precedence Parsing
3. Table Driven LR Parsing

a. LR(1)
b. SLR(1)
c. CLR (1)
d. LALR(1)

Shift reduce parsing

- Shift reduce parsing is a process of reducing a string to the start symbol of a grammar.
- Shift reduce parsing uses a stack to hold the grammar and an input tape to hold the string.



- Sift reduce parsing performs the two actions: shift and reduce. That's why it is known as shift reduces parsing.
- At the shift action, the current symbol in the input string is pushed to a stack.
- At each reduction, the symbols will replaced by the non-terminals. The symbol is the right side of the production and non-terminal is the left side of the production.

Example:

Grammar:

1. S → S+S
2. S → S-S
3. S → (S)
4. S → a

Input string:

a1-(a2+a3)

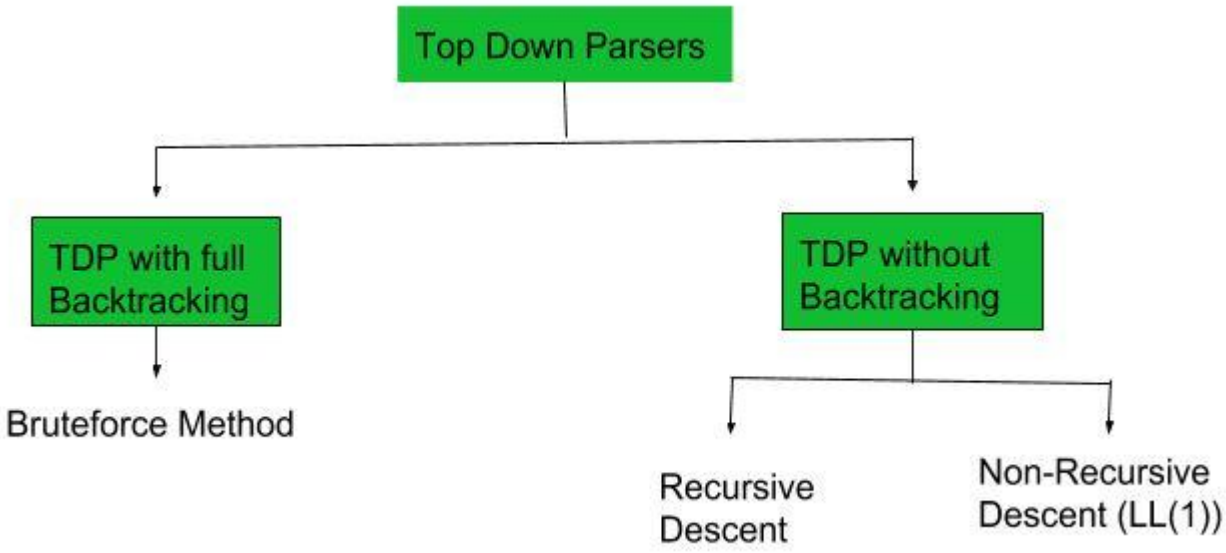
Parsing table:

Stack contents	Input string	Actions
$	a1-(a2+a3)$	shift a1
$a1	-(a2+a3)$	reduce by S → a
$S	-(a2+a3)$	shift -
$S-	(a2+a3)$	shift (
$S-(	a2+a3)$	shift a2
$S-(a2	+a3)$	reduce by S → a
$S-(S	+a3) $	shift +
$S-(S+	a3) $	shift a3
$S-(S+a3	) $	reduce by S → a
$S-(S+S	) $	shift)
$S-(S+S)	$	reduce by S → S+S
$S-(S)	$	reduce by S → (S)
$S-S	$	reduce by S → S-S
$S	$	Accept

CHAP 2

LL(1) Parsing:

Top-Down Parsers without backtracking can further be divided into two parts:



In this article, we are going to discuss Non-Recursive Descent which is also known as LL(1) Parser.

LL(1) Parsing: Here the 1st **L** represents that the scanning of the Input will be done from Left to Right manner and the second **L** shows that in this parsing technique we are going to use Left most Derivation Tree. And finally, the **1** represents the number of look-ahead, which means how many symbols are you going to see when you want to make a decision.

Essential conditions to check first are as follows:

1. The grammar is free from left recursion.
2. The grammar should not be ambiguous.
3. The grammar has to be left factored in so that the grammar is deterministic grammar.

Consider the Grammar

```
S -> (L) | a
L -> SL'
L' -> )SL' | ε
```

Step1 – The grammar satisfies all properties in step 1

Step 2 – calculating first() and follow()

	First	Follow
S	{ (, a }	{ $,) }
L	{ (, a }	{) }
L'	{) , ε }	{) }

Step 3 – making parser table
Parsing Table:

	(	)	a	$
S	S -> (L)		S -> a	
L	L -> SL'		L -> SL'	
L'	L' ->)SL' L' -> ε			

Here, we can see that there are two productions in the same cell. Hence, this grammar is not feasible for LL(1) Parser. Although the grammar satisfies all the essential conditions in step 1, it is still not feasible for LL(1) Parser. We saw in example 2 that we must have these essential conditions and in example 3 we saw that those conditions are insufficient to be a LL(1) parser.

Operator precedence parsing

Operator precedence grammar is kinds of shift reduce parsing method. It is applied to a small class of operator grammars.

A grammar is said to be operator precedence grammar if it has two properties:

- No R.H.S. of any production has a∈.
- No two non-terminals are adjacent.

Operator precedence can only established between the terminals of the grammar. It ignores the non-terminal.

There are the three operator precedence relations:

a ⋗ b means that terminal "a" has the higher precedence than terminal "b".

a ⋖ b means that terminal "a" has the lower precedence than terminal "b".

a ≐ b means that the terminal "a" and "b" both have same precedence.

Precedence table:

	+	*	(	)	id	$
+	⋗	⋖	⋖	⋗	⋖	⋗
*	⋗	⋗	⋖	⋗	⋖	⋗
(	⋖	⋖	⋖	≐	⋖	X
)	⋗	⋗	X	⋗	X	⋗
id	⋗	⋗	X	⋗	X	⋗
$	⋖	⋖	⋖	X	⋖	X

Parsing Action

- Both end of the given input string, add the $ symbol.
- Now scan the input string from left right until the ⋗ is encountered.
- Scan towards left over all the equal precedence until the first left most ⋖ is encountered.
- Everything between left most ⋖ and right most ⋗ is a handle.
- $ on $ means parsing is successful.

Example

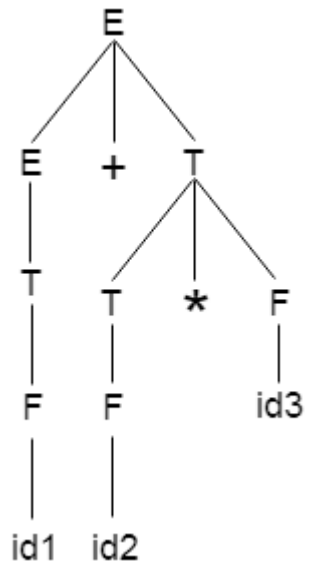
Grammar:

1. E → E+T/T
2. T → T*F/F
3. F → id

Given string:

1. w = id + id * id

   Let us consider a parse tree for it as follows:



   On the basis of above tree, we can design following operator precedence table:

	E	T	F	id	+	*	$
E	X	X	X	X	≐	X	⋗
T	X	X	X	X	⋗	≐	⋗
F	X	X	X	X	⋗	⋗	⋗
id	X	X	X	X	⋗	⋗	⋗
+	X	≐	⋖	⋖	X	X	X
*	X	X	≐	⋖	X	X	X
$	⋖	⋖	⋖	⋖	X	X	X

   Now let us process the string with the help of the above precedence table:

$ ⋖ id1 ⋗ + id2 * id3 $

$ ⋖ F ⋗ + id2 * id3 $

$ ⋖ T ⋗ + id2 * id3 $

$ ⋖ E ≐ + ⋖ id2 ⋗ * id3 $

$ ⋖ E ≐ + ⋖ F ⋗ * id3 $

$ ⋖ E ≐ + ⋖ T ≐ * ⋖ id3 ⋗ $

$ ⋖ E ≐ + ⋖ T ≐ * ≐ F ⋗ $

$ ⋖ E ≐ + ≐ T ⋗ $

$ ⋖ E ≐ + ≐ T ⋗ $

$ ⋖ E ⋗ $

Accept.

LR Parser

LR parsing is one type of bottom up parsing. It is used to parse the large class of grammars.

In the LR parsing, "L" stands for left-to-right scanning of the input.

"R" stands for constructing a right most derivation in reverse.

"K" is the number of input symbols of the look ahead used to make number of parsing decision.

LR parsing is divided into four parts: LR (0) parsing, SLR parsing, CLR parsing and LALR parsing.

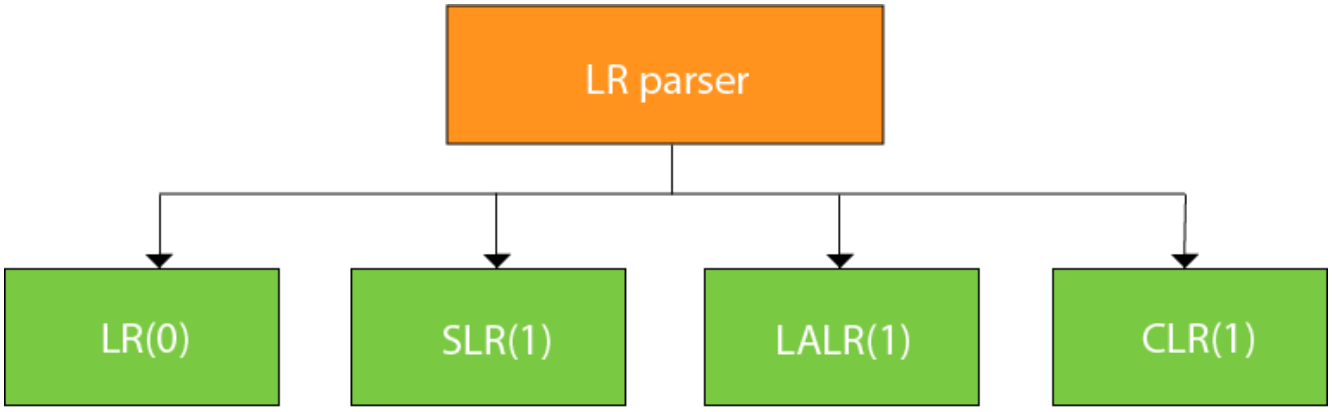


Fig: Types of LR parser

LR algorithm:

The LR algorithm requires stack, input, output and parsing table. In all type of LR parsing, input, output and stack are same but parsing table is different.

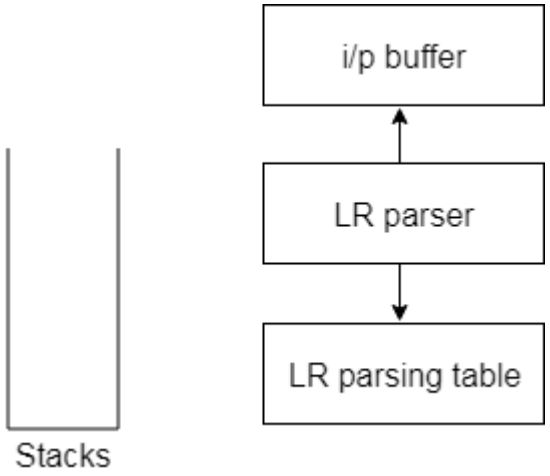


Fig: Block diagram of LR parser

Input buffer is used to indicate end of input and it contains the string to be parsed followed by a $ Symbol.

A stack is used to contain a sequence of grammar symbols with a $ at the bottom of the stack.

Parsing table is a two dimensional array. It contains two parts: Action part and Go To part.

LR (1) Parsing

Various steps involved in the LR (1) Parsing:

- For the given input string write a context free grammar.
- Check the ambiguity of the grammar.
- Add Augment production in the given grammar.
- Create Canonical collection of LR (0) items.
- Draw a data flow diagram (DFA).
- Construct a LR (1) parsing table.

Augment Grammar

Augmented grammar G` will be generated if we add one more production in the given grammar G.

Example

Given grammar

1. S → AA
2. A → aA | b

The Augment grammar G` is represented by

1. S`→ S
2. S → AA
3. A → aA | b

SLR (1) Parsing

SLR (1) refers to simple LR Parsing. It is same as LR(0) parsing. The only difference is in the parsing table.To construct SLR (1) parsing table, we use canonical collection of LR (0) item.

In the SLR (1) parsing, we place the reduce move only in the follow of left hand side.

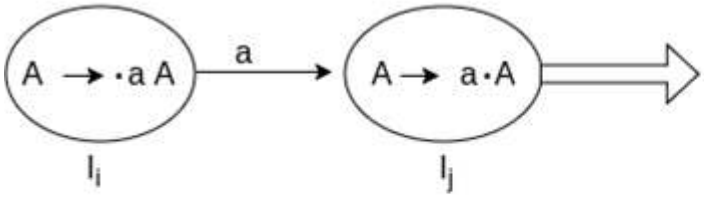
Various steps involved in the SLR (1) Parsing:

- For the given input string write a context free grammar
- Check the ambiguity of the grammar
- Add Augment production in the given grammar
- Create Canonical collection of LR (0) items
- Draw a data flow diagram (DFA)
- Construct a SLR (1) parsing table

SLR (1) Table Construction

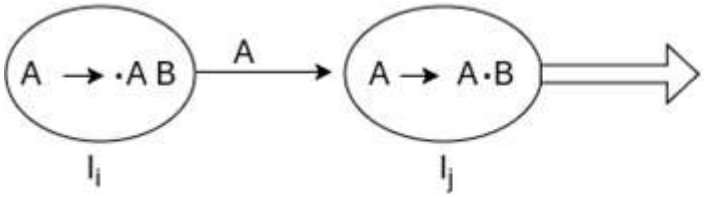
The steps which use to construct SLR (1) Table is given below:

If a state (I_i) is going to some other state (I_j) on a terminal then it corresponds to a shift move in the action part.



States	Action		Go to
	a	$	A
I_i I_j	Sj		

If a state (I_i) is going to some other state (I_j) on a variable then it correspond to go to move in the Go to part.



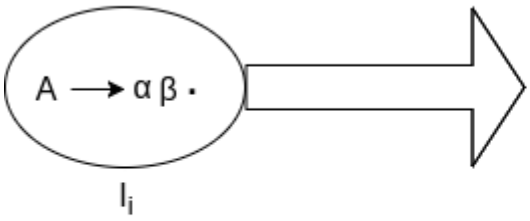
States	Action		Go to
	a	$	A
I_i I_j			j

If a state (I_i) contains the final item like A → ab• which has no transitions to the next state then the production is known as reduce production. For all terminals X in FOLLOW (A), write the reduce entry along with their production numbers.

Example

1. S -> •Aa
2. A->αβ•

1. Follow(S) = {$}
2. Follow (A) = {a}



States	Action			Go to	
	a	b	$	S	A
I_i	r2				

SLR (1) Grammar

S → E
E → E + T | T
T → T * F | F
F → id

Add Augment Production and insert '•' symbol at the first position for every production in G

S` → •E
E → •E + T
E → •T
T → •T * F
T → •F
F → •id

I0 State:

Add Augment production to the I0 State and Compute the Closure

I0 = Closure (S` → •E)

Add all productions starting with E in to I0 State because "." is followed by the non-terminal. So, the I0 State becomes

I0 = S` → •E
E → •E + T
E → •T

Add all productions starting with T and F in modified I0 State because "." is followed by the non-terminal. So, the I0 State becomes.

I0= S` → •E
E → •E + T
E → •T
T → •T * F
T → •F
F → •id

I1= Go to (I0, E) = closure (S` → E•, E → E• + T)
I2= Go to (I0, T) = closure (E → T•T, T• → * F)
I3= Go to (I0, F) = Closure (T → F•) = T → F•
I4= Go to (I0, id) = closure (F → id•) = F → id•
I5= Go to (I1, +) = Closure (E → E +•T)

Add all productions starting with T and F in I5 State because "." is followed by the non-terminal. So, the I5 State becomes

I5 = E → E +•T
T → •T * F
T → •F
F → •id

Go to (I5, F) = Closure (T → F•) = (same as I3)
Go to (I5, id) = Closure (F → id•) = (same as I4)

I6= Go to (I2, *) = Closure (T → T * •F)

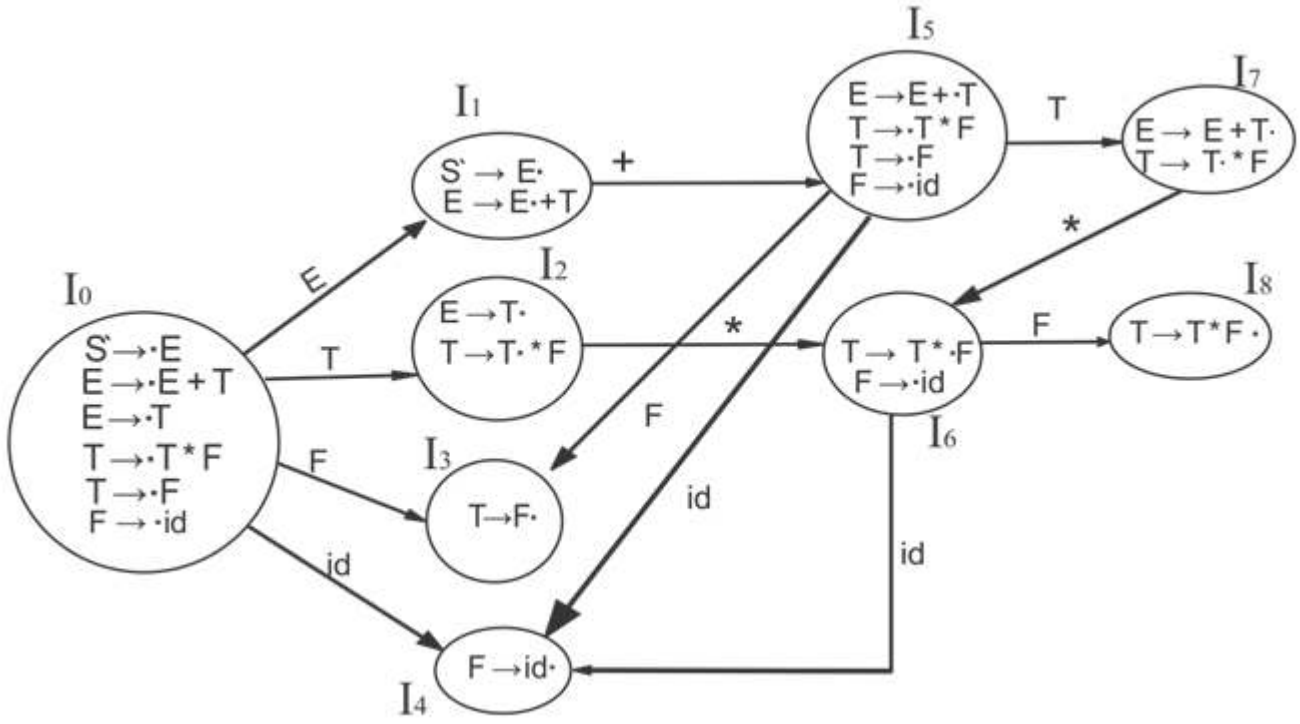
Add all productions starting with F in I6 State because "." is followed by the non-terminal. So, the I6 State becomes

I6 = T → T * •F
F → •id

Go to (I6, id) = Closure (F → id•) = (same as I4)

I7= Go to (I5, T) = Closure (E → E + T•) = E → E + T•
I8= Go to (I6, F) = Closure (T → T * F•) = T → T * F•

Drawing DFA:



SLR (1) Table

States	Action				Go to		
	id	+	*	$	E	T	F
I_0	S_4				1	2	3
I_1		S_5		Accept			
I_2		R_2	S_6	R2			
I_3		R_4	R_4	R4			
I_4		R_5	R5	R5			
I_5	S4					7	3
I_6	S4						8
I_7		R1	S6	R1			
I_8		R3	R3	R3			

Explanation:

First (E) = First (E + T) ∪ First (T)
First (T) = First (T * F) ∪ First (F)
First (F) = {id}
First (T) = {id}
First (E) = {id}
Follow (E) = First (+T) ∪ {$} = {+, $}
Follow (T) = First (*F) ∪ First (F) = {*, +, $}
Follow (F) = {*, +, $}

- I1 contains the final item which drives S → E• and follow (S) = {$}, so action {I1, $} = Accept
- I2 contains the final item which drives E → T• and follow (E) = {+, $}, so action {I2, +} = R2, action {I2, $} = R2
- I3 contains the final item which drives T → F• and follow (T) = {+, *, $}, so action {I3, +} = R4, action {I3, *} = R4, action {I3, $} = R4
- I4 contains the final item which drives F → id• and follow (F) = {+, *, $}, so action {I4, +} = R5, action {I4, *} = R5, action {I4, $} = R5
- I7 contains the final item which drives E → E + T• and follow (E) = {+, $}, so action {I7, +} = R1, action {I7, $} = R1
- I8 contains the final item which drives T → T * F• and follow (T) = {+, *, $}, so action {I8, +} = R3, action {I8, *} = R3, action {I8, $} = R3.

CLR (1) Parsing

CLR refers to canonical lookahead. CLR parsing use the canonical collection of LR (1) items to build the CLR (1) parsing table. CLR (1) parsing table produces the more number of states as compare to the SLR (1) parsing.

In the CLR (1), we place the reduce node only in the lookahead symbols.

Various steps involved in the CLR (1) Parsing:

- For the given input string write a context free grammar
- Check the ambiguity of the grammar
- Add Augment production in the given grammar
- Create Canonical collection of LR (0) items
- Draw a data flow diagram (DFA)
- Construct a CLR (1) parsing table

LR (1) item

LR (1) item is a collection of LR (0) items and a look ahead symbol.

LR (1) item = LR (0) item + look ahead

The look ahead is used to determine that where we place the final item.

The look ahead always add $ symbol for the argument production.

Example

CLR (1) Grammar

1. S → AA
2. A → aA
3. A → b

Add Augment Production, insert '•' symbol at the first position for every production in G and also add the lookahead.

1. S` → •S, $
2. S → •AA, $
3. A → •aA, a/b
4. A → •b, a/b

I0 State:

Add Augment production to the I0 State and Compute the Closure

I0 = Closure (S` → •S)

Add all productions starting with S in to I0 State because "." is followed by the non-terminal. So, the I0 State becomes

I0 = S` → •S, $
S → •AA, $

Add all productions starting with A in modified I0 State because "." is followed by the non-terminal. So, the I0 State becomes.

I0= S` → •S, $
S → •AA, $
A → •aA, a/b
A → •b, a/b

I1 = Go to (I0, S) = closure (S` → S•, $) = S` → S•, $
I2 = Go to (I0, A) = closure (S → A•A, $)

Add all productions starting with A in I2 State because "." is followed by the non-terminal. So, the I2 State becomes

I2 = S → A•A, $
A → •aA, $
A → •b, $

I3 = Go to (I0, a) = Closure (A → a•A, a/b)

Add all productions starting with A in I3 State because "." is followed by the non-terminal. So, the I3 State becomes

I3= A → a•A, a/b
A → •aA, a/b
A → •b, a/b

Go to (I3, a) = Closure (A → a•A, a/b) = (same as I3)
Go to (I3, b) = Closure (A → b•, a/b) = (same as I4)

I4= Go to (I0, b) = closure (A → b•, a/b) = A → b•, a/b
I5= Go to (I2, A) = Closure (S → AA•, $) =S → AA•, $
I6= Go to (I2, a) = Closure (A → a•A, $)

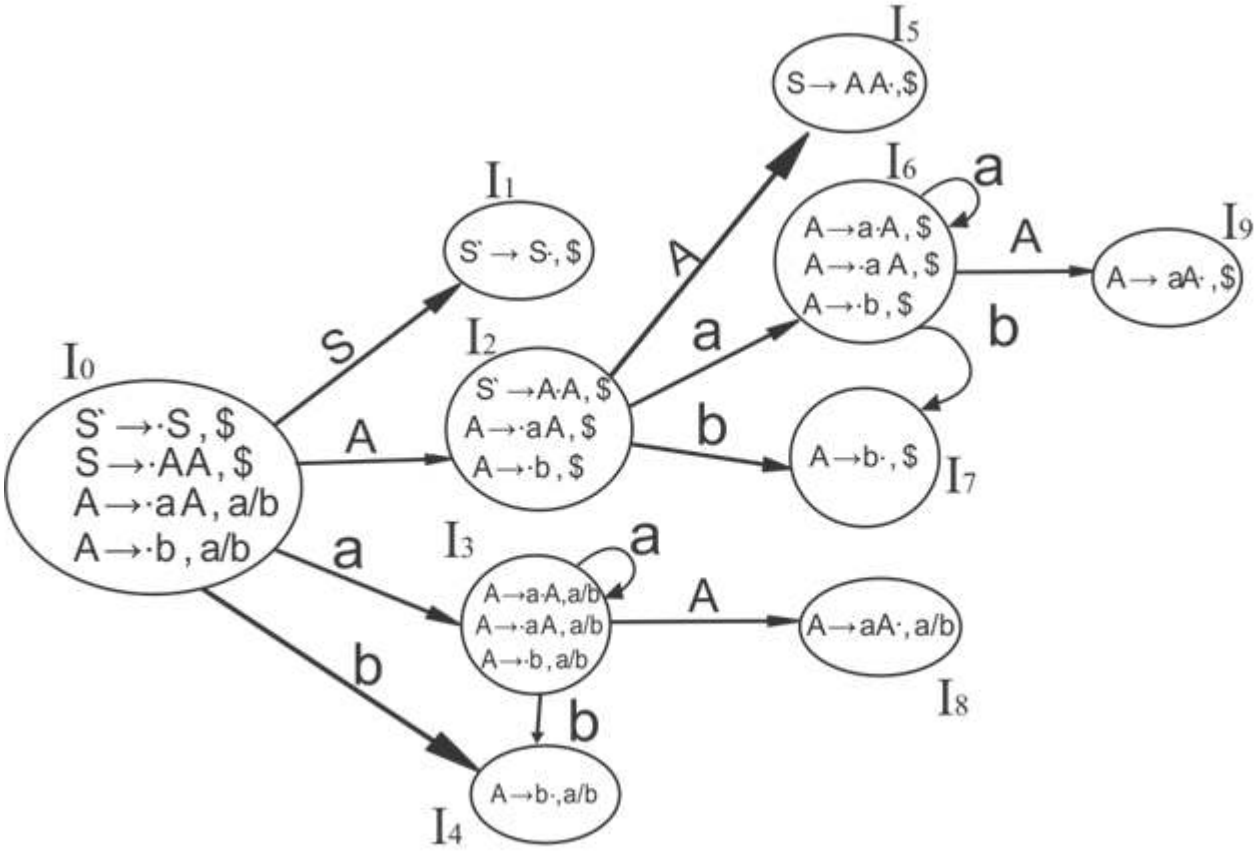
Add all productions starting with A in I6 State because "." is followed by the non-terminal. So, the I6 State becomes

I6 = A → a•A, $
A → •aA, $
A → •b, $

Go to (I6, a) = Closure (A → a•A, $) = (same as I6)
Go to (I6, b) = Closure (A → b•, $) = (same as I7)

I7= Go to (I2, b) = Closure (A → b•, $) = A → b•, $
I8= Go to (I3, A) = Closure (A → aA•, a/b) = A → aA•, a/b
I9= Go to (I6, A) = Closure (A → aA•, $) = A → aA•, $

Drawing DFA:



CLR (1) Parsing table:

States	a	b	$	S	A
I_0	S_3	S_4			2
I_1			Accept		
I_2	S_6	S_7			5
I_3	S_3	S_4			8
I_4	R_3	R_3			
I_5			R_1		
I_6	S_6	S_7			9
I_7			R_3		
I_8	R_2	R_2			
I_9			R_2		

Productions are numbered as follows:

1. S → AA ... (1)
2. A → aA ....(2)
3. A → b ... (3)

The placement of shift node in CLR (1) parsing table is same as the SLR (1) parsing table. Only difference in the placement of reduce node.

I4 contains the final item which drives (A → b•, a/b), so action {I4, a} = R3, action {I4, b} = R3.
I5 contains the final item which drives (S → AA•, $), so action {I5, $} = R1.
I7 contains the final item which drives (A → b•,$), so action {I7, $} = R3.
I8 contains the final item which drives (A → aA•, a/b), so action {I8, a} = R2, action {I8, b} = R2.
I9 contains the final item which drives (A → aA•, $), so action {I9, $} = R2.