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Deterministic Pushdown Automata and Closure Properties of CFLs
Chapter 3: Grammars

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Introduction

This section explores:

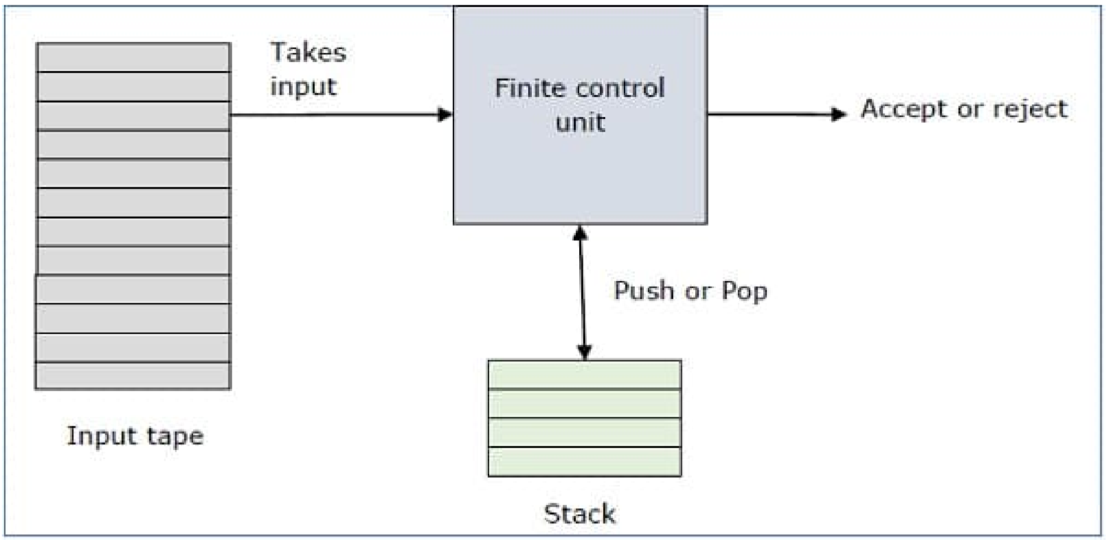
- Focuses on Deterministic Pushdown Automata (DPDA)
- Compares DPDA with Non-Deterministic PDA (NPDA)
- Discusses closure properties of Context-Free Languages (CFLs)

What is a DPDA?

- A pushdown automaton is a way to implement a context-free grammar in a similar way we design DFA for a regular grammar.
- A DFA can remember a finite amount of information, but a PDA can remember an infinite amount of information.
- Basically a pushdown automaton is –
- "Finite state machine" + "a stack"
- A pushdown automaton has three components –
- an input tape,
- a control unit, and
- a stack with infinite size.

What is a DPDA?

- The stack head scans the top symbol of the stack.
- A stack does two operations –
- Push – a new symbol is added at the top.
- Pop – the top symbol is read and removed



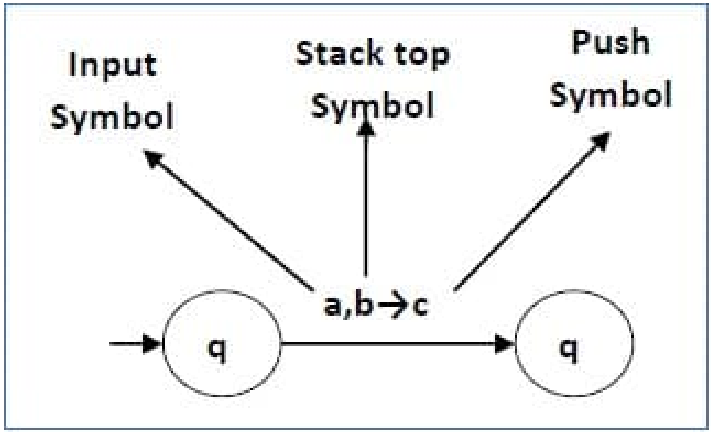
Formal Definition

A PDA can be formally described as a 7-tuple $(Q, \Sigma, S, \delta, q_0, I, F)$ –

- Q is the finite number of states
- Σ is input alphabet
- S is stack symbols
- is the transition function: $Q \times (\Sigma \cup \{\varepsilon\}) \times S \times Q \times S^*$
- q_0 is the initial state ($q_0 \in Q$)
- I is the initial stack top symbol ($I \in S$)
- F is a set of accepting states ($F \in Q$)

Example of PDA

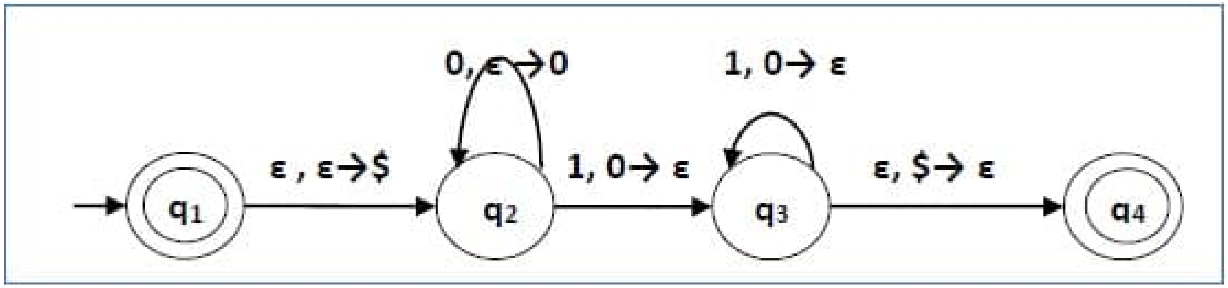
The following diagram shows a transition in a PDA from a state q1 to state q2, labeled as a,b → c –



This means at state q1, if we encounter an input string a and top symbol of the stack is b, then we pop b, push c on top of the stack and move to state q2.

Example of PDA

Construct a PDA that accepts L = {0n 1n | n ≥ 0}



PDA for L= $\{0^n 1^n \mid n \geq 0\}$

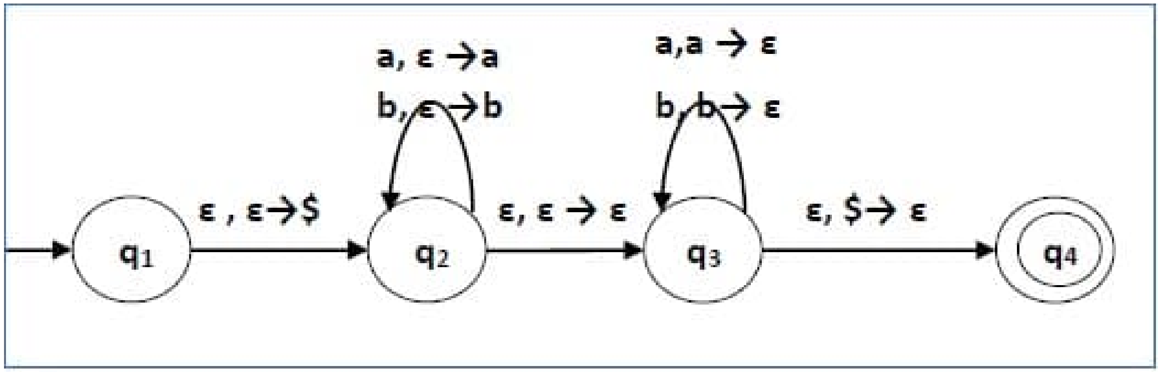
- Initially we put a special symbol $ into the empty stack.
- Then at state q2, if we encounter input 0 and top is Null, we push 0 into stack. This may iterate. And if we encounter input 1 and top is 0, we pop this 0.

Example of PDA

- Then at state q3, if we encounter input 1 and top is 0, we pop this 0. This may also iterate. And if we encounter input 1 and top is 0, we pop the top element.
- If the special symbol $ is encountered at top of the stack, it is popped out and it finally goes to the accepting state q4.

Example of PDA

Construct a PDA that accepts L = { wwR | w = (a+b)* }



PDA for $L = \{ww^R \mid w = (a+b)^*\}$

- Initially we put a special symbol $ into the empty stack.
- At state q_2, the w is being read.
- In state q_3, each 0 or 1 is popped when it matches the input. If any other input is given, the PDA will go to a dead state. When we reach that special symbol $, we go to the accepting state q_4.

Difference Between NPDA and DPDA

Attribute	**Deterministic PDA**	**Non-Deterministic PDA**
Transition Function	Single next state for each input symbol	Multiple possible next states for each input symbol
Acceptance	Accepts input if it reaches an accepting state	Accepts input if any computation path reaches an accepting state
Complexity	Less expressive but easier to analyze	More expressive but harder to analyze
Determinism	Determined by input and current state	Non-deterministic choices made during computation

Properties of Context Free Languages

1. Union

If L and M are two context-free languages, then their union L ∪ M is also a CFL.

2. Concatenation

If L and M are CFLs, then their concatenation LM is also a CFL.

3. Kleene Closure

If L is a CFL, then its Kleene closure L* (zero or more repetitions of strings in L) is also a CFL.

Properties of Context Free Languages

4. Reversal

- If L is a CFL, then its reversal L^R (where each string is reversed) is also a CFL.

5. Homomorphism

- A homomorphism is a function that replaces each symbol in a string with another string.
- If L is a CFL and h is a homomorphism, then h(L) is also a CFL.

6. Inverse Homomorphism

- If L is a CFL and h is a homomorphism, then $h^{-1}(L)$ is also a CFL.

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