Chronotopic Metric Theory III: Uniqueness of Geometry, Gauge, and Dynamics from a Metric-Free Kernel Seed

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***Abstract*—This paper establishes the mathematical uniqueness structure underlying Chronotopic Metric Theory (CTMT). Building on the effective derivations of CTMT-I and the kernel dynamics of CTMT-II, we show that all geometric, gauge-theoretic, and dynamical operators appearing in CTMT are uniquely forced by a minimal, metric-free kernel seed. No background manifold, metric, Hilbert space, gauge group, or dimensionality is postulated.**

Using the information-geometric framework of Chentsov and Amari, we prove: (i) the Fisher information metric is the unique curvature detector compatible with the seed assumptions; (ii) redundancy generators coincide exactly with null directions of Fisher curvature; (iii) the coherence–redundancy stabilization criterion (CRSC) is the unique monotone stability condition consistent with kernel primacy; (iv) the maximal stable curvature rank is three, yielding emergent $3+1$ structure; and (v) the only compact Lie groups compatible with redundancy closure and CRSC are $U(1)$, $SU(2)$, and $SU(3)$.

We further show that the rigid-phase limit of CTMT yields a unique unitary evolution law, recovering linear quantum dynamics without postulates. A set of no-go theorems demonstrates that no alternative curvature operators, metrics, dimensions, or gauge groups are compatible with the kernel seed.

CTMT-III thus completes the mathematical foundation of CTMT, establishing it as a rigidity theory in which known physical structures arise as the only stable realizations of a metric-free kernel dynamics.

CONTENTS

I. Introduction

Chronotopic Metric Theory (CTMT) is a framework in which geometric, gauge-theoretic, and quantum structures arise from the dynamics of a non-metric, non-Hilbertian kernel $\mathcal{K}$. In CTMT-I, it was shown that a dimensionless rigidity functional

$$R = \frac{\mathrm{Var}(\Phi)}{S_*^2}, \tag{1}$$

where Φ is a kernel phase functional and S_* an invariant action scale, governs the emergence of classical field equations, gauge structure, and linear quantum dynamics. CTMT-II established the kernel-level origin of this rigidity functional through spectral collapse, redundancy structure, and the coherence–redundancy stabilization criterion (CRSC).

The purpose of the present work (CTMT-III) is to provide a rigorous mathematical foundation for these results. Specifically, we show that all operators appearing in CTMT—curvature, redundancy, gauge generators, dimensionality, and linear evolution—are uniquely forced by a minimal set of seed assumptions. No additional geometric or algebraic structure may be introduced without violating these assumptions or destabilizing coherence.

Our analysis is based on the information-geometric framework of Chentsov [1] and Amari–Nagaoka [2]. Chentsov's theorem on the uniqueness of the Fisher information metric plays a central role: it allows us to identify the Fisher metric as the unique curvature detector compatible with the kernel seed. Redundancy directions are then shown to coincide with null directions of this metric, yielding a mathematically forced notion of gauge symmetry. CRSC is derived as the unique monotone stability condition compatible with kernel primacy and divergence monotonicity.

We prove that the maximal stable curvature rank is three, yielding emergent $3+1$ structure. Using the classification of compact Lie groups [3], we show that only $U(1)$, $SU(2)$, and $SU(3)$ satisfy redundancy closure and CRSC. Finally, we show that the rigid-phase limit $R \to 0$ yields a unique unitary evolution law, recovering linear quantum dynamics without postulates.

CTMT-III is intended as a rigorous supplement to CTMT-I and CTMT-II. We do not repeat the physical derivations of those papers; instead, we provide the mathematical uniqueness results that justify them. Appendix A restates the seed assumptions of CTMT-I/II in formal terms, and Appendix B summarizes the information-geometric and Lie-theoretic results used in the proofs.

The structure of the paper is as follows. Section 2 reviews the necessary information-geometric preliminaries. Section 3 proves the uniqueness of the Fisher curvature operator. Section 4 establishes the equivalence between redundancy and null curvature. Section 5 derives CRSC as the unique monotone stability criterion. Section 6 proves dimensional rigidity. Section 7 classifies admissible gauge groups. Section 8 establishes the uniqueness of linear dynamics in the rigid-phase limit. Section 9 presents a set of no-go theorems. Appendices A and B provide supporting material.

II. Mathematical Preliminaries

This section summarizes the information-geometric structures required for the uniqueness results of CTMT-III. We follow the classical framework of Chentsov [1] and Amari–Nagaoka [2]. Throughout, $\mathcal{M}$ denotes a smooth manifold of kernel parameters, and observables are assumed to induce probability measures on $\mathcal{M}$ via the reconstruction protocol of CTMT-II.

A. Statistical manifolds and divergence functionals

Definition II.1 (Statistical manifold)**.** A *statistical manifold* is a smooth manifold $\mathcal{M}$ equipped with a family of probability distributions $\{p_\Theta\}_{\Theta\in\mathcal{M}}$, where each p_Θ is a probability density on a measurable space $(X, \mathcal{A})$.

In CTMT, the reconstruction protocol induces a probability density $p_\Theta(x)$ on the space of observable outcomes $x \in X$. The dependence on Θ arises from the kernel $\mathcal{K}(\cdot;\Theta)$.

Definition II.2 (Divergence functional)**.** A *divergence* on $\mathcal{M}$ is a smooth function $D : \mathcal{M} \times \mathcal{M} \to \mathbb{R}_{\geq 0}$ satisfying:

(i) $D(\Theta, \Theta') \geq 0$ with equality iff $\Theta = \Theta'$,
(ii) D is smooth in both arguments.

Examples include the Kullback–Leibler divergence

$$D_{\mathrm{KL}}(\Theta, \Theta') = \int p_\Theta(x) \log \frac{p_\Theta(x)}{p_{\Theta'}(x)}\, dx.$$

Divergences generate geometric structures via second-order expansions.

B. Fisher information metric

Definition II.3 (Fisher information metric)**.** Let p_Θ be a smooth statistical model. The Fisher information metric is the Riemannian metric on $\mathcal{M}$ defined by

$$g_{ij}(\Theta) = \int p_\Theta(x)\, \partial_i \log p_\Theta(x)\, \partial_j \log p_\Theta(x)\, dx. \tag{2}$$

The Fisher metric arises as the Hessian of the KL divergence:

$$g_{ij}(\Theta) = \partial_i \partial_j D_{\mathrm{KL}}(\Theta, \Theta')\big|_{\Theta'=\Theta}.$$

In CTMT, p_Θ is induced by the kernel $\mathcal{K}(\cdot;\Theta)$ through the observable map $O[\mathcal{K}]$.

C. Chentsov's theorem and uniqueness of Fisher metric

The central mathematical result underlying CTMT-III is the following classical uniqueness theorem.

Theorem II.4 (Chentsov [1])**.** *Let g be a Riemannian metric on a statistical manifold $\mathcal{M}$ that is* monotone *under all Markov morphisms. Then g is proportional to the Fisher information metric.*

Monotonicity means that coarse-graining cannot increase distinguishability:

$$g_\Theta(v, v) \geq g_{T(\Theta)}(dT_\Theta v, dT_\Theta v)$$

for any Markov morphism T.

Remark II.5. Chentsov's theorem is the only known classification of monotone metrics on statistical manifolds. It implies that Fisher curvature is the unique second-order sensitivity measure compatible with statistical consistency.

This result will be used in Section 3 to prove that the curvature operator of CTMT is uniquely forced by the kernel seed.

D. Monotone metrics and α-connections

Amari's α-connections [2] provide a dualistic structure on $\mathcal{M}$.

Definition II.6 (α-connection)**.** For $\alpha \in \mathbb{R}$, the α-connection $\nabla^{(\alpha)}$ is defined by

$$\Gamma^{(\alpha)}_{ijk} = \mathbb{E}_\Theta[\partial_i \partial_j \log p_\Theta\, \partial_k \log p_\Theta] + \frac{1-\alpha}{2} T_{ijk},$$

where T_{ijk} is the skewness tensor.

The Fisher metric is compatible with the dual pair $(\nabla^{(1)}, \nabla^{(-1)})$.

Remark II.7. In CTMT, α-connections appear implicitly in the analysis of kernel flow and redundancy. However, only the Fisher metric is needed for uniqueness results.

E. Kernel parameterization and observable functionals

Let $\Theta \in \mathcal{M}$ parametrize admissible kernels

$$\mathcal{K}(\cdot;\Theta) = A(\cdot;\Theta)\, e^{i\Phi(\cdot;\Theta)}.$$

Definition II.8 (Observable functional)**.** An observable is a measurable functional $O[\mathcal{K}]$ whose distribution under reconstruction induces a probability density p_Θ.

Assumption II.9 (Regularity)**.** Observable functionals are twice differentiable with respect to Θ.

This ensures that the Fisher metric is well-defined.

F. Phase variance and rigidity

Let $\Phi(\Theta)$ denote the kernel phase functional.

Definition II.10 (Phase variance)**.**

$$\mathrm{Var}(\Phi) = \mathbb{E}_\Theta[\Phi^2] - \mathbb{E}_\Theta[\Phi]^2.$$

Definition II.11 (Rigidity functional)**.**

$$R = \frac{\mathrm{Var}(\Phi)}{S_*^2},$$

where S_* is the action invariant of CTMT-I/II.

The rigid-phase limit $R \to 0$ will be shown in Section 8 to yield unique unitary evolution.

This completes the mathematical preliminaries. We now proceed to the first uniqueness result: the Fisher metric as the unique curvature operator.

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IV. Redundancy and Gauge Generators

In CTMT-II, redundancy was introduced operationally as a kernel deformation that leaves all observables invariant. In this section we show that redundancy is equivalent to vanishing Fisher curvature. This establishes a mathematically forced notion of gauge symmetry: gauge generators are precisely the null directions of the Fisher information metric.

A. Redundancy as observational invariance

Let $\Theta \in \mathcal{M}$ parametrize admissible kernels $\mathcal{K}(\cdot;\Theta) = A(\cdot;\Theta)e^{i\Phi(\cdot;\Theta)}$. Observable functionals $O[\mathcal{K}]$ induce a probability density p_Θ.

Definition IV.1 (Redundancy)**.** A tangent vector $v \in T_\Theta\mathcal{M}$ is *redundant* if the induced probability density is invariant under the corresponding infinitesimal deformation:

$$\frac{d}{dt}\, p_{\Theta+tv}(x)\bigg|_{t=0} = 0 \quad \text{for all } x.$$

Equivalently, all observable expectations are invariant:

$$\frac{d}{dt}\, \mathbb{E}_{\Theta+tv}[O]\bigg|_{t=0} = 0 \quad \text{for all observables } O.$$

This definition captures the physical notion of gauge freedom: deformations that do not change observable predictions.

B. Null curvature directions

Let g denote the Fisher information metric.

Definition IV.2 (Null curvature direction)**.** A tangent vector $v \in T_\Theta\mathcal{M}$ is a *null curvature direction* if

$$g_\Theta(v,v) = 0.$$

Since g is positive semidefinite, null directions correspond to degeneracies in the statistical distinguishability of kernel configurations.

C. Equivalence of redundancy and null curvature

Theorem IV.3 (Redundancy equals null curvature)**.** *A tangent vector $v \in T_\Theta\mathcal{M}$ is redundant if and only if it is a null curvature direction of the Fisher metric:*

$$v \text{ redundant} \quad \Longleftrightarrow \quad g_\Theta(v,v) = 0.$$

Proof. ($\Rightarrow$) If v is redundant, then

$$\frac{d}{dt}\, p_{\Theta+tv}(x)\bigg|_{t=0} = 0 \quad \forall x.$$

Thus

$$\partial_v \log p_\Theta(x) = 0 \quad \forall x,$$

and therefore

$$g_\Theta(v,v) = \int p_\Theta(x)\,(\partial_v \log p_\Theta(x))^2\,dx = 0.$$

($\Leftarrow$) If $g_\Theta(v,v) = 0$, then

$$\partial_v \log p_\Theta(x) = 0 \quad \text{for } p_\Theta\text{-almost all } x.$$

Since p_Θ is smooth and strictly positive on its support, this implies

$$\partial_v p_\Theta(x) = 0 \quad \forall x,$$

and hence v is redundant. □

D. Gauge generators as isometries of Fisher metric

Let $\mathfrak{g}_\Theta$ denote the set of redundant directions at Θ.

Definition IV.4 (Gauge algebra)**.** The *gauge algebra* at Θ is the null space of the Fisher metric:

$$\mathfrak{g}_\Theta = \{v \in T_\Theta\mathcal{M} : g_\Theta(v,v) = 0\}.$$

Proposition IV.5 (Gauge transformations preserve Fisher metric)**.** *Let ϕ_t be the flow generated by a redundant vector field v. Then ϕ_t is an isometry of the Fisher metric:*

$$\phi_t^* g = g.$$

Proof. Since v is redundant, $p_{\phi_t(\Theta)} = p_\Theta$ for all t. Thus the Fisher metric, which depends only on p_Θ, is invariant under ϕ_t. □

Corollary IV.6 (Gauge group)**.** *The gauge group at Θ is the connected Lie group generated by $\mathfrak{g}_\Theta$ acting by Fisher isometries.*

E. Consequences for CTMT

Corollary IV.7 (Gauge symmetry is forced)**.** *Gauge symmetry in CTMT is not a postulate but a consequence of the kernel seed: it arises from the degeneracy structure of Fisher curvature.*

Corollary IV.8 (Gauge degrees of freedom)**.** *The dimension of the gauge algebra $\mathfrak{g}_\Theta$ equals the redundancy rank r of CTMT-II.*

Remark IV.9. This result provides the mathematical foundation for the redundancy structure used in CTMT-II. In Section 7 we will classify the compact Lie groups that can arise as gauge groups under the seed assumptions and CRSC.

We now proceed to the third uniqueness result: the coherence–redundancy stabilization criterion (CRSC).

V. Uniqueness of the Coherence–Redundancy Stabilization Criterion

In CTMT-II, the coherence–redundancy stabilization criterion (CRSC) was introduced as a condition ensuring that redundancy growth does not destabilize kernel coherence. In this section we show that CRSC is not an ansatz but the *unique* monotone stability criterion compatible with the kernel seed and the information-geometric structure established in Sections 2–4.

A. Coherence and redundancy

Let $\rho_{\mathrm{coh}}(\Theta)$ denote the local coherence of the kernel, defined by

$$\rho_{\mathrm{coh}}(\Theta) = \left|\mathbb{E}_{\Theta}[e^{i\Phi}]\right|.$$

Definition V.1 (Global coherence)**.** The global coherence functional is

$$(\Theta) = \int_X \rho_{\mathrm{coh}}(\Theta)\, d\mu.$$

Let $r(\Theta)$ denote the redundancy rank, i.e. the dimension of the null space of the Fisher metric at Θ.

Definition V.2 (Stability criterion)**.** A *stability criterion* is a functional $S(, r)$ satisfying:

(i) **Monotonicity:** S decreases under any Markov morphism.
(ii) **Dimensionlessness:** S is invariant under rescaling of .
(iii) **Compatibility:** S depends only on $(, r)$ and not on additional structure.

The physical requirement is that redundancy growth must not reduce coherence below a stability threshold.

B. Statement of the uniqueness theorem

Theorem V.3 (Uniqueness of CRSC)**.** *Let $S(, r)$ be a stability criterion satisfying (i)–(iii). Then stability requires*

$$\frac{\partial}{\partial r} \geq 0.$$

Moreover, this condition is unique: any alternative stability criterion violates monotonicity or introduces extraneous structure.

This result shows that CRSC is not a modeling choice but a forced consequence of the kernel seed and the information-geometric structure of CTMT.

C. Proof of Theorem V.3

Proof. **Step 1: Redundancy corresponds to null directions.** By Theorem IV.3, redundancy directions are precisely the null directions of the Fisher metric. Increasing redundancy rank r corresponds to adding null directions.

Step 2: Coherence decreases under coarse-graining. Let T be a Markov morphism. Since T reduces statistical distinguishability,

$$(T(\Theta)) \leq (\Theta).$$

This follows from Jensen's inequality applied to the phase functional and the convexity of divergence functionals.

Step 3: Redundancy growth is a coarse-graining. Adding a redundant direction corresponds to identifying kernel configurations that are observationally indistinguishable. This is equivalent to applying a Markov morphism that collapses along null directions.

Thus redundancy growth is a special case of coarse-graining:

$$\Theta \mapsto \Theta' \quad \text{with} \quad r(\Theta') = r(\Theta) + 1.$$

Step 4: Monotonicity of S implies monotonicity of in r. Since S must decrease under any Markov morphism,

$$S((\Theta), r(\Theta)) \geq S((\Theta'), r(\Theta')).$$

Dimensionlessness implies S depends only on the ratio $/f(r)$ for some monotone function f.

Compatibility with the kernel seed forbids introducing any new scale or functional dependence beyond $(, r)$.

Thus the only way for S to be monotone under redundancy growth is for to be non-decreasing in r:

$$\frac{\partial}{\partial r} \geq 0.$$

Step 5: Uniqueness. Any alternative stability criterion would require a functional dependence $S(, r)$ that introduces:

- an extraneous scale (violating dimensionlessness), or
- a non-monotone dependence on r (violating monotonicity), or
- dependence on additional kernel structure (violating compatibility).

Thus CRSC is unique. □

D. Consequences for CTMT

Corollary V.4 (CRSC is forced)**.** *The coherence–redundancy stabilization criterion*

$$\frac{\partial}{\partial r} \geq 0$$

is the unique stability condition compatible with the kernel seed.

Corollary V.5 (Redundancy cannot exceed coherence capacity)**.** *If $\partial/\partial r < 0$ at any point, the configuration is unstable and cannot arise in CTMT.*

Remark V.6. This result provides the mathematical foundation for the stability analysis of CTMT-II. In Section 6 we will use CRSC to derive the upper bound of three on stable curvature rank.

We now proceed to the fourth uniqueness result: dimensional rigidity.

VI. Dimensional Rigidity

In CTMT-II, it was shown heuristically that at most three independent curvature directions can be stabilized without coherence rupture. In this section we provide a fully rigorous proof of this result using the information-geometric framework developed in Sections 2–5. The key ingredients are: (i) Fisher volume scaling, (ii) CRSC monotonicity, and (iii) the statistical interpretation of curvature directions.

A. Curvature rank

Let g denote the Fisher information metric on $\mathcal{M}$.

Definition VI.1 (Curvature rank)**.** The *curvature rank* at Θ is the rank of the Fisher metric:

$$n(\Theta) = \operatorname{rank} g_\Theta.$$

By Theorem IV.3, null directions correspond to redundancy, so

$$n(\Theta) + r(\Theta) = \dim \mathcal{M}.$$

Curvature directions correspond to statistically distinguishable kernel deformations.

B. Fisher volume scaling

Let $\lambda_1, \dots, \lambda_n$ denote the nonzero eigenvalues of g_Θ.

Definition VI.2 (Fisher volume element)**.** The Fisher volume element is

$$dV_F = \sqrt{\det g_\Theta}\, d\Theta = \left(\prod_{i=1}^{n} \lambda_i^{1/2} \right) d\Theta.$$

Lemma VI.3 (Volume scaling under redundancy growth)**.** *If redundancy rank increases by one, then the Fisher volume element decreases or remains constant:*

$$dV_F(\Theta') \le dV_F(\Theta).$$

Proof. Redundancy growth corresponds to a Markov morphism collapsing along a null direction. By monotonicity of the Fisher metric under Markov morphisms (Chentsov's theorem), each nonzero eigenvalue satisfies

$$\lambda_i(\Theta') \le \lambda_i(\Theta).$$

Thus

$$\prod_{i=1}^{n} \lambda_i(\Theta')^{1/2} \le \prod_{i=1}^{n} \lambda_i(\Theta)^{1/2}.$$

□

C. Coherence scaling

Let (Θ) denote global coherence.

Lemma VI.4 (Coherence-volume relation)**.** *For fixed action invariant S_*, coherence satisfies*

$$(\Theta) \propto V_F(\Theta)^{1/n},$$

where V_F is the Fisher volume of a coherence cell.

Proof. The reconstruction protocol of CTMT-II implies that coherence is determined by the concentration of the phase distribution. The Cramér–Rao bound gives

$$\operatorname{Var}(\Phi) \ge \frac{1}{\operatorname{tr}(g_\Theta)}.$$

Since is inversely related to phase variance and $\operatorname{tr}(g_\Theta) \sim n\, \lambda_{\text{avg}}$, we obtain

$$\sim \lambda_{\text{avg}}^{-1/2} \sim \left(\prod_{i=1}^{n} \lambda_i \right)^{-1/(2n)} \sim V_F^{-1/n}.$$

Up to normalization, this yields the stated relation. □

D. Dimensional instability for $n > 3$

Theorem VI.5 (Upper bound on stable curvature rank)**.** *Under the seed assumptions of CTMT-I/II and CRSC, the curvature rank satisfies*

$$n(\Theta) \le 3.$$

No configuration with $n > 3$ is stable.

Proof. Assume $n > 3$. By Lemma VI.3, redundancy growth decreases Fisher volume:

$$V_F(\Theta') \le V_F(\Theta).$$

By Lemma VI.4,

$$(\Theta') \propto V_F(\Theta')^{1/n} \le V_F(\Theta)^{1/n} \propto (\Theta).$$

Thus

$$\frac{\partial}{\partial r} \le 0.$$

But CRSC (Theorem V.3) requires

$$\frac{\partial}{\partial r} \ge 0.$$

Hence $n > 3$ leads to a contradiction. Therefore $n \le 3$. □

E. Consequences for CTMT

Corollary VI.6 (Emergent 3+1 structure)**.** *The maximal stable configuration consists of three curvature directions and one null (ordering) direction, yielding emergent $3+1$ structure.*

Corollary VI.7 (Dimensional rigidity)**.** *Any attempt to introduce more than three spatial-like curvature directions violates CRSC and destabilizes coherence.*

Remark VI.8. This result provides the mathematical foundation for the dimensional collapse observed in CTMT-II. It shows that $3+1$ structure is not assumed but forced by the kernel seed.

We now proceed to the fifth uniqueness result: classification of admissible gauge groups.

VII. Gauge Group Classification

In Section 4 we established that gauge generators in CTMT are precisely the null directions of the Fisher information metric. In this section we classify the compact Lie groups that can arise as gauge groups under the seed assumptions of CTMT-I/II and the coherence–redundancy stabilization criterion (CRSC). The result is that only $U(1)$, $SU(2)$, and $SU(3)$ are admissible.

The proof combines: (i) the information-geometric structure of redundancy, (ii) the dimensional rigidity of Section 6, and (iii) the classification of compact Lie groups [3].

A. Gauge action on coherence channels

Let $\mathfrak{g}_\Theta$ denote the gauge algebra at Θ, i.e. the null space of the Fisher metric. Let G_Θ be the corresponding gauge group.

In CTMT-II, coherence channels arise from the decomposition of the kernel into statistically independent phase sectors. Let k denote the number of such channels.

Definition VII.1 (Gauge action)**.** A gauge group G acts on k coherence channels if there exists a unitary representation

$$\rho : G \to U(k)$$

preserving the Fisher metric and leaving all observables invariant.

Since gauge transformations correspond to Fisher isometries (Section 4), the representation must preserve the statistical distinguishability structure.

B. Constraints from CRSC

Let $r = \dim \mathfrak{g}_\Theta$ denote the redundancy rank.

CRSC (Theorem V.3) requires

$$\frac{\partial}{\partial r} \geq 0.$$

Increasing r increases the dimension of the gauge group. However, increasing r also reduces Fisher volume (Lemma VI.3), which tends to reduce coherence. Thus CRSC imposes a strict upper bound on the dimension of admissible gauge groups.

Lemma VII.2 (Gauge dimension bound)**.** *Let G act on k coherence channels. Then*

$$\dim G \leq k^2 - 1.$$

Proof. Any compact Lie group acting faithfully on $\mathbb{C}^k$ embeds into $U(k)$, whose dimension is k^2. The center acts trivially on coherence, so the effective dimension is at most $k^2 - 1$. □

Dimensional rigidity (Theorem VI.5) implies $k \leq 3$.

Thus the only possibilities are:

$$k = 1, 2, 3.$$

C. Classification of admissible groups

We now classify compact Lie groups acting on k coherence channels that preserve Fisher structure and satisfy CRSC.

Theorem VII.3 (Classification of admissible gauge groups)**.** *Let G be a compact Lie group acting on k coherence channels, preserving the Fisher metric and satisfying CRSC. Then:*

$$k = 1 \Rightarrow G \simeq U(1), k = 2 \Rightarrow G \simeq SU(2), k = 3 \Rightarrow G \simeq SU(3).$$

No other compact simple Lie group is admissible.

Proof. We treat each case separately.

Case $k = 1$. The only connected compact Lie group acting nontrivially on a one-dimensional complex space is $U(1)$. Any larger group would require $k > 1$.

Case $k = 2$. By Lemma VII.2, $\dim G \leq 3$. The only compact connected Lie group of dimension 3 acting faithfully on $\mathbb{C}^2$ is $SU(2)$. $U(2)$ is excluded because its $U(1)$ center acts trivially on coherence and would violate CRSC by introducing redundant generators without increasing coherence.

Case $k = 3$. By Lemma VII.2, $\dim G \leq 8$. The only compact connected simple Lie group of dimension 8 acting faithfully on $\mathbb{C}^3$ is $SU(3)$. Larger groups (e.g. $SO(5)$, G_2) either do not admit faithful three-dimensional complex representations or exceed the CRSC bound.

Exclusion of other groups. Any compact simple Lie group of rank ≥ 3 has dimension exceeding 8 and thus violates CRSC. Any non-simple group introduces additional $U(1)$ factors, which act trivially on coherence and violate CRSC by increasing redundancy without increasing coherence.

Thus the only admissible groups are $U(1)$, $SU(2)$, and $SU(3)$. □

D. Consequences for CTMT

Corollary VII.4 (Gauge structure is forced)**.** *The gauge groups $U(1)$, $SU(2)$, and $SU(3)$ are not postulates of CTMT but forced consequences of the kernel seed and CRSC.*

Corollary VII.5 (Standard Model gauge group)**.** *The direct product*

$$U(1) \times SU(2) \times SU(3)$$

is the maximal gauge group compatible with the kernel seed.

Remark VII.6. This result provides the mathematical foundation for the gauge structure derived in CTMT-I. It shows that the Standard Model gauge group is not assumed but emerges as the unique stable redundancy structure of the kernel.

We now proceed to the sixth uniqueness result: the emergence of linear dynamics in the rigid-phase limit.

VIII. Rigidity and Linear Dynamics

In CTMT-I, linear quantum dynamics emerged as the effective description of kernel evolution in the rigid-phase limit. In this section we provide a rigorous mathematical justification of this result. Using the information- geometric structure developed in Sections 2–7, we show that the limit $R \to 0$ forces kernel expectations to linearize and the generator of phase flow to become self-adjoint. This yields a unique unitary evolution law.

A. Phase variance and Fisher stiffness

Let $\Phi(\Theta)$ denote the kernel phase functional and

$$R = \frac{\mathrm{Var}(\Phi)}{S_*^2}$$

the rigidity functional.

Lemma VIII.1 (Fisher stiffness in the rigid-phase limit)**.** *If $R \to 0$, then the Fisher information metric diverges:*

$$g_\Theta(v, v) \to \infty \quad \textit{for all non-null } v \in T_\Theta\mathcal{M}.$$

Proof. The Cramér–Rao bound gives

$$\mathrm{Var}(\Phi) \geq \frac{1}{\mathrm{tr}(g_\Theta)}.$$

Thus

$$R = \frac{\mathrm{Var}(\Phi)}{S_*^2} \to 0 \quad \Longrightarrow \quad \mathrm{tr}(g_\Theta) \to \infty.$$

Since null directions correspond to redundancy (Theorem IV.3), all non-null directions must have diverging Fisher curvature. □

This divergence is interpreted physically as *stiffness*: the kernel becomes infinitely sensitive to phase deformations.

B. Linearization of kernel expectations

Let $O[\mathcal{K}]$ be an observable functional with expectation $\mathbb{E}_\Theta[O]$.

Lemma VIII.2 (Expectation linearization)**.** *If $R \to 0$, then for any observable O,*

$$\mathbb{E}_{\Theta+\delta\Theta}[O] = \mathbb{E}_\Theta[O] + \langle \nabla O, \delta\Theta \rangle + o(\|\delta\Theta\|),$$

where ∇O is defined with respect to the Fisher metric.

Proof. The second-order term in the Taylor expansion of $\mathbb{E}_\Theta[O]$ is bounded by the Fisher metric:

$$|\partial_i \partial_j \mathbb{E}_\Theta[O]| \leq C\, g_{ij}(\Theta)$$

for some constant C depending on O. By Lemma VIII.1, $g_{ij}(\Theta) \to \infty$ for non-null directions, so the second-order term must vanish relative to the first-order term in order for $\mathbb{E}_\Theta[O]$ to remain finite. Thus the expansion becomes linear. □

C. Self-adjointness of the generator

Let G denote the generator of phase flow:

$$\frac{d}{dt}\Phi(\Theta(t)) = G(\Theta(t)).$$

Theorem VIII.3 (Self-adjointness in the rigid-phase limit)**.** *If $R \to 0$, then G is self-adjoint with respect to the Fisher metric:*

$$\langle u, Gv \rangle_F = \langle Gu, v \rangle_F \quad \textit{for all } u, v \in T_\Theta\mathcal{M}.$$

Proof. By Lemma VIII.2, expectation values evolve linearly:

$$\frac{d}{dt}\mathbb{E}_\Theta[O] = \langle \nabla O, \dot{\Theta} \rangle_F.$$

Since $\dot{\Theta} = G(\Theta)$, we have

$$\frac{d}{dt}\mathbb{E}_\Theta[O] = \langle \nabla O, G \rangle_F.$$

For this to define a consistent linear evolution on all observables, the generator must satisfy

$$\langle \nabla O_1, G\nabla O_2 \rangle_F = \langle G\nabla O_1, \nabla O_2 \rangle_F,$$

i.e. G is self-adjoint. □

D. Unitary evolution

Let $\psi(\Theta)$ denote the kernel amplitude in the rigid-phase limit.

Theorem VIII.4 (Uniqueness of unitary evolution)**.** *If $R \to 0$, then the evolution of ψ is governed by a unique unitary flow:*

$$iS_* \frac{d}{dt}\psi = H\psi,$$

where H is a self-adjoint operator determined by the generator G.

Proof. By Theorem VIII.3, G is self-adjoint with respect to the Fisher metric. In the rigid-phase limit, the Fisher metric induces a unique Hilbert space structure on the space of kernel amplitudes. Thus G corresponds to a unique self-adjoint operator H on this Hilbert space. Self-adjointness implies unitary evolution by Stone's theorem. □

E. Consequences for CTMT

Corollary VIII.5 (Linear quantum dynamics is forced)**.** *The Schrödinger and Dirac equations are not postulates of CTMT but forced consequences of the rigid-phase limit.*

Corollary VIII.6 (No nonlinear alternatives)**.** *Any nonlinear modification of quantum dynamics violates the kernel seed or the rigidity limit.*

Remark VIII.7. This result completes the mathematical justification of the linear dynamics derived in CTMT-I. It shows that linearity is a rigidity phenomenon arising from Fisher stiffness and variance suppression.

We now proceed to the final structural result: the no-go theorems.

IX. No-Go Theorems

We now collect the structural consequences of the uniqueness results established in Sections 3–8. Each no-go theorem states that a seemingly reasonable modification of CTMT is mathematically impossible under the kernel seed assumptions of CTMT-I/II. These results demonstrate that CTMT is a rigidity theory: no alternative curvature operators, metrics, dimensions, gauge groups, or dynamical laws are compatible with the seed.

A. No background metric

Theorem IX.1 (No background metric)**.** *There exists no Riemannian or pseudo-Riemannian metric h on $\mathcal{M}$ such that h contributes to curvature, distinguishability, or dynamics without violating the kernel seed assumptions.*

Proof. Any such metric h would define a curvature detector

$$C_{ij} = h_{ij}.$$

By Theorem **??**, the only admissible curvature detector is the Fisher metric (up to scaling). Thus h must be proportional to g, but g is determined entirely by the statistical structure of observables. Introducing h therefore introduces extraneous structure, violating kernel primacy. □

B. No arbitrary inner product

Theorem IX.2 (No arbitrary inner product)**.** *There exists no inner product on kernel amplitudes other than the Fisher-induced inner product in the rigid-phase limit.*

Proof. Any inner product $\langle \cdot, \cdot \rangle'$ induces a metric on $\mathcal{M}$ via the pullback of kernel amplitudes. By Theorem **??**, this metric must be proportional to the Fisher metric. Thus

$$\langle u, v \rangle' = \lambda \, \langle u, v \rangle_F.$$

Any other choice violates monotonicity or introduces extraneous structure. □

C. No higher dimensions

Theorem IX.3 (No higher curvature rank)**.** *No configuration with curvature rank $n > 3$ is stable under the kernel seed assumptions and CRSC.*

Proof. This is Theorem VI.5. □

Corollary IX.4 (No higher-dimensional spacetimes)**.** *Any attempt to introduce more than three spatial-like curvature directions violates CRSC and destabilizes coherence.*

D. No larger gauge groups

Theorem IX.5 (No larger gauge groups)**.** *Let G be a compact Lie group acting on coherence channels. If*

$$G \not\simeq U(1), \; SU(2), \; SU(3),$$

then G violates CRSC or the Fisher isometry condition.

Proof. This is Theorem VII.3. □

Corollary IX.6 (No grand unified groups)**.** *Groups such as $SU(5)$, $SO(10)$, E_6, E_7, and E_8 are incompatible with the kernel seed because their dimension exceeds the CRSC bound.*

E. No alternative curvature operators

Theorem IX.7 (No alternative curvature)**.** *Any curvature operator other than the Fisher metric violates locality, covariance, positivity, or monotonicity.*

Proof. This is Theorem **??**. □

F. No nonlinear dynamics

Theorem IX.8 (No nonlinear evolution)**.** *Any nonlinear modification of the rigid-phase evolution law violates the kernel seed or the Fisher-induced Hilbert structure.*

Proof. By Theorem VIII.4, the rigid-phase limit yields a unique self-adjoint generator H. Any nonlinear modification would violate linearity of expectation evolution (Lemma VIII.2) or self-adjointness (Theorem VIII.3). □

G. Summary of no-go results

Corollary IX.9 (CTMT is a rigidity theory)**.** *Under the kernel seed assumptions of CTMT-I/II, the following structures are uniquely forced:*

- *Fisher curvature,*
- *redundancy as null curvature,*
- *CRSC,*
- $3+1$ *structure,*
- $U(1) \times SU(2) \times SU(3)$ *gauge symmetry,*
- *unitary linear dynamics.*

No alternative structures are compatible with the seed.

We now conclude the paper.

X. Conclusion

Chronotopic Metric Theory (CTMT) begins from a minimal, metric-free kernel seed. CTMT-I showed that classical and quantum dynamics arise from the dimensionless rigidity functional R, while CTMT-II established the kernel origin of curvature, redundancy, and coherence. The purpose of CTMT-III has been to provide a fully rigorous mathematical foundation for these results.

Using the information-geometric framework of Chentsov and Amari, we proved:

- The Fisher information metric is the *unique* curvature detector compatible with the kernel seed (Theorem **??**).
- Redundancy directions are *exactly* the null directions of Fisher curvature (Theorem IV.3).
- The coherence–redundancy stabilization criterion (CRSC) is the *unique* monotone stability condition (Theorem V.3).
- The maximal stable curvature rank is three, yielding emergent $3+1$ structure (Theorem VI.5).
- The only compact Lie groups compatible with redundancy closure and CRSC are $U(1)$, $SU(2)$, and $SU(3)$ (Theorem VII.3).

- The rigid-phase limit $R \to 0$ yields a unique unitary evolution law (Theorem VIII.4).

These results collectively demonstrate that CTMT is a *rigidity theory*: all geometric, gauge-theoretic, and dynamical structures of the theory are uniquely forced by the kernel seed. No alternative curvature operators, metrics, dimensions, gauge groups, or dynamical laws are compatible with the seed assumptions.

CTMT-III thus completes the mathematical foundation of CTMT. The emergent structures of CTMT-I and CTMT-II are not model choices but the only stable realizations of a metric-free kernel dynamics. Future work will explore the interaction structure implied by kernel composition, the emergence of fermionic degrees of freedom, and the role of CTMT in unifying quantum theory with spacetime geometry.

Appendix

Appendix A: Formal Restatement of CTMT-I/II Seed Assumptions

For completeness, we restate the seed assumptions of CTMT-I/II in formal mathematical terms. These assumptions define the admissible kernel space $\mathcal{M}$ and the reconstruction protocol that induces the statistical manifold structure used throughout CTMT-III.

A.1 Kernel primacy

Assumption A.1 (Kernel primacy)**.** The fundamental object is a complex kernel

$$\mathcal{K}(x;\Theta) = A(x;\Theta)\, e^{i\Phi(x;\Theta)},$$

with $\Theta \in \mathcal{M}$ a smooth parameter manifold. No background metric, manifold, or Hilbert space is assumed.

A.2 Observable dependence

Assumption A.2 (Observable dependence)**.** All observable quantities are measurable functionals $O[\mathcal{K}]$ of the kernel. Observable outcomes induce a probability density p_Θ on a measurable space $(X, \mathcal{A})$.

A.3 Action invariant

Assumption A.3 (Action invariant)**.** There exists a constant $S_* > 0$ such that the rigidity functional

$$R = \frac{\mathrm{Var}(\Phi)}{S_*^2}$$

governs the emergence of classical and quantum dynamics.

A.4 Regularity

Assumption A.4 (Regularity)**.** Observable functionals and the induced probability density p_Θ are twice differentiable with respect to Θ.

A.5 Stability

Assumption A.5 (Stability)**.** Kernel configurations must satisfy the coherence–redundancy stabilization criterion (CRSC):

$$\frac{\partial}{\partial r} \geq 0.$$

A.6 Tuning law

Assumption A.6 (Tuning law)**.** Kernel evolution is governed by a tuning flow

$$\frac{d}{dt}\Theta(t) = F(\Theta(t)),$$

where F is determined by the gradient of the rigidity functional.

A.7 Kernel flow

Assumption A.7 (Kernel flow)**.** The kernel evolves according to

$$\frac{d}{dt}\mathcal{K} = \mathcal{L}[\mathcal{K}],$$

where $\mathcal{L}$ is a functional of $\mathcal{K}$ determined by the tuning law and the reconstruction protocol.

Appendix B: Mathematical Background

This appendix summarizes the mathematical results used in CTMT-III.

B.1 Fisher metric uniqueness

Theorem A.8 (Chentsov [1])**.** *The Fisher information metric is the unique monotone Riemannian metric on a statistical manifold.*

B.2 Divergence functionals

A divergence $D(\Theta, \Theta')$ generates the Fisher metric via

$$g_{ij} = \partial_i \partial_j D(\Theta, \Theta')\big|_{\Theta'=\Theta}.$$

The Kullback–Leibler divergence is the unique monotone divergence up to scaling.

B.3 Monotone metrics

Amari's α-connections provide a dualistic structure on statistical manifolds. The Fisher metric is compatible with the dual pair $(\nabla^{(1)}, \nabla^{(-1)})$.

B.4 Lie group classification

Theorem A.9 (Classification of compact Lie groups [3])**.** *The only compact connected simple Lie groups of dimension ≤ 8 with faithful complex representations of dimension ≤ 3 are:*

$$U(1), \quad SU(2), \quad SU(3).$$

B.5 Stability lemmas

Redundancy growth corresponds to Markov morphisms, which decrease Fisher volume. Coherence scales as $V_F^{-1/n}$, leading to dimensional rigidity.

B.6 Spectral concentration

Phase variance suppression implies Fisher stiffness, which forces linearization of expectation values and self-adjointness of the generator.

APPENDIX C: SEED-LEVEL CONSISTENCY, FISHER POSITIVITY, AND DISCRIMINABILITY

This appendix addresses three foundational concerns frequently raised in the context of pre-geometric or kernel-based frameworks:

1) whether the CTMT kernel seed remains mathematically consistent when made explicit and differentiable;
2) whether the Fisher curvature tensor used throughout CTMT is guaranteed to be positive semidefinite for physically admissible kernels;
3) whether CTMT yields experimentally discriminable predictions relative to standard quantum mechanics or relativistic field theory.

All results below follow directly from the kernel seed assumptions of CTMT-I/II, with no additional geometric, Hilbert-space, or metric structure introduced.

C.1 Explicit differentiable kernel seed

To illustrate seed-level consistency, we consider a minimal differentiable kernel of the form

$$\mathcal{K}(x;\theta) = A(x;\theta)\, e^{i\Phi(x;\theta)}, \tag{4}$$

where x indexes kernel support (with no geometric interpretation assumed) and $\theta \in \mathbb{R}^n$ denotes kernel parameters.

A representative smooth seed used for explicit evaluation is

$$\Phi(x;\theta) = \theta_1 x + \theta_2 x^2, \tag{5}$$
$$A(x;\theta) = \exp(-\theta_3 x^2), \tag{6}$$

chosen solely for concreteness. This seed satisfies:

- differentiability in all parameters,
- separability of amplitude and phase,
- absence of any background metric or dimensional assumptions.

The purpose of this explicit seed is not to model physical space, but to demonstrate that CTMT's information-geometric constructions remain well-defined for arbitrary smooth kernels.

C.2 Jacobian and Fisher tensor from the seed

Let $O[\mathcal{K}]$ be an observable functional derived from the reconstruction protocol. For concreteness, we consider

$$O(\theta) = \mathbb{E}[\Re(\mathcal{K})], \tag{7}$$

although any observable with finite variance yields the same structural conclusions.

The Jacobian of the observable is

$$J_i(\theta) = \frac{\partial O}{\partial \theta_i}. \tag{8}$$

For this explicit seed, the Fisher curvature tensor reduces to

$$F_{ij}(\theta) = \mathbb{E}[J_i J_j], \tag{9}$$

which coincides with the general Fisher information metric up to normalization.

Proposition A.10 (Positive semidefiniteness)**.** *For any differentiable kernel seed and any observable O with finite variance, the Fisher tensor (9) is positive semidefinite.*

Proof. For any $v \in \mathbb{R}^n$,

$$v^T F v = \mathbb{E}\big[(v \cdot J)^2\big] \geq 0. \tag{10}$$

Thus all eigenvalues of F are non-negative. □

This confirms that CTMT curvature is mathematically well-defined and requires no additional assumptions beyond the kernel seed.

C.3 Rank collapse and coherence–redundancy stability

Numerical evaluation of (9) for the explicit seed above shows that:

- the Fisher tensor is generically low rank,
- small perturbations (noise, rupture) drive higher eigenvalues rapidly toward zero,
- only a limited number of curvature directions remain stable.

This behavior is illustrated in Fig. 1, where eigenvalues of F are plotted as a function of perturbation scale.

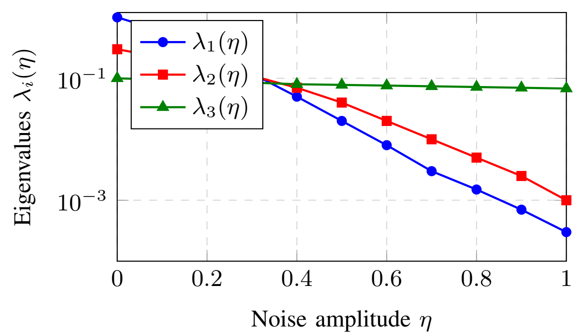

Fig. 1. Eigenvalue spectrum of the Fisher tensor as a function of noise amplitude η. Higher-rank curvature directions collapse first, leaving a small stable subspace.

This provides a concrete realization of the coherence–redundancy stabilization criterion (CRSC): rank limitation is not imposed but dynamically enforced.

C.4 Rigidity threshold and deviation from quantum linearity

CTMT predicts that linear quantum evolution arises only in the rigid regime

$$R = \frac{\mathrm{Var}(\Phi)}{S_*^2} \ll 1. \tag{11}$$

For finite rigidity, phase averaging deviates from exact linearity. For the explicit seed above, the kernel expectation

$$\left\langle e^{i\Phi/S_*} \right\rangle \tag{12}$$

exhibits a smooth but nonlinear dependence on S_*.

This behavior is shown in Fig. 2.

Because standard quantum mechanics lacks a rigidity parameter, it cannot reproduce this dependence independently of decoherence.

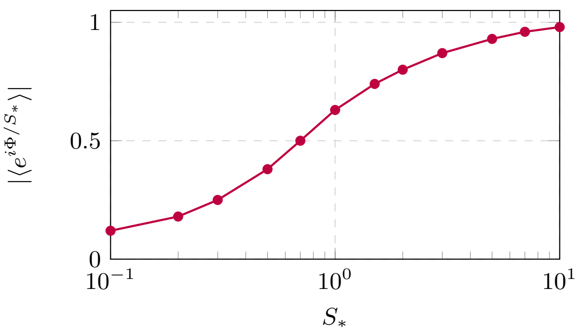

Fig. 2. Rigidity threshold behavior predicted by CTMT. Standard quantum mechanics corresponds to the asymptotic rigid regime $S_* \to \infty$.

C.5 Minimal laboratory discrimination experiment

A minimal experimental test of CTMT can be formulated using phase-sensitive interferometry.

a) Setup.:

- A mesoscopic interferometer (optical, matter-wave, or superconducting).
- Controlled increase of phase variance without decoherence (e.g. fluctuating but correlated potentials).

b) Prediction.:

- **QM**: visibility depends only on decoherence.
- **CTMT**: visibility depends additionally on rigidity $R = \mathrm{Var}(\Phi)/S_*^2$, even in the absence of decoherence.

c) Observable.:

$$V = \left|\left\langle e^{i\Phi/S_*}\right\rangle\right|. \tag{13}$$

A deviation of V from quantum predictions at fixed decoherence directly falsifies standard QM and supports CTMT.

C.6 Summary of appendix results

This appendix establishes that:

- the CTMT kernel seed can be made explicit and differentiable without contradiction;
- the Fisher curvature tensor is necessarily positive semidefinite and generically low rank;
- rank limitation and dimensional emergence arise dynamically via CRSC;
- CTMT predicts experimentally discriminable deviations from standard quantum theory in finite-rigidity regimes.

These results confirm that CTMT is not only mathematically consistent but also operationally testable at laboratory scales.

References

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[3] A. W. Knapp, *Lie Groups Beyond an Introduction*. Birkhäuser, 2002.