CTMT: Stationary Phase, Coherence Dynamics, and Admissibility

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Abstract

We formalize the stationary-phase regime and coherence dynamics in the *Chronotopic Metric Theory* (CTMT), a geometry-first framework in which distance, identifiability, and admissibility emerge from phase-bearing transport rather than being postulated *a priori*. Using a complex kernel with a bona fide phase channel, we connect stationary-phase structure to Fisher information geometry and derive operational acceptance and rejection criteria under dimensional closure. We prove that phase-linked observables improve Fisher isotropy, that Gauss–Newton updates are admissible only when subordinated to stable information geometry, and that coherence rupture manifests jointly as robust change points and Fisher collapse. Continuity of acceptance under vanishing causal violations establishes falsifiability. The framework is grounded in classical results on Fisher information, nonlinear least squares, robust statistics, and phase metrology, and is illustrated by elemental-scale validation.

Contents

In CTMT, geometry is not assumed, time is not postulated, and causality is not inferred from fit quality; all three are diagnosed operationally through the stability of Fisher information under phase-bearing transport.

1 Kernel, Stationary Phase, and Observables

CTMT assumes a complex kernel $\mathcal{K} : \mathcal{T} \times \mathcal{T} \to$ of the form

$$\mathcal{K}(t, t') = A(t, t') \exp\left(\mathrm{i}\, \Phi(t, t')\right), \tag{1}$$

with amplitude channel $A \geq 0$ and phase channel $\Phi \in \mathbb{R}$. No background geometry is assumed; coherence and admissibility arise operationally from data and Jacobians [7, 8].

Assumption 1 (Phase sensitivity)**.** The phase is measurably sensitive in the second argument: there exists $\epsilon > 0$ such that the set $\{(t, t') : |\partial_{t'} \Phi(t, t')| > \epsilon\}$ has non-zero measure.

1.1 Stationary Phase as an Operational Criterion

CTMT does not rely on asymptotic limits (e.g. $\omega \to \infty$) to invoke stationary-phase arguments. Instead, stationarity is treated as an *operational property of the kernel on the measurement grid.* Specifically, CTMT tests whether phase variations are slow relative to sampling and noise scales by finite-difference estimates of $\partial_{t'}\Phi$ and $\partial_t\Phi$.

A regime is deemed *stationary-phase admissible* if the stationary locus Λ persists under: (i) mild grid refinement, (ii) moderate noise inflation, and (iii) reweighting of amplitude-dominated observables.

Failure of these tests indicates coherence leakage rather than mere sampling error.

Definition 1 (Stationary-phase locus)**.** A locus $\Lambda \subset \mathcal{T}^2$ is stationary when $|\partial_{t'}\Phi(t, t')| \leq \delta$ and $|\partial_t\Phi(t, t')| \leq \delta$ for a tolerance $\delta > 0$ tied to sampling. The *effective curvature* κ_Φ is recovered from second differences of Φ along t and t'.

Proposition 1 (Stationary-phase contribution)**.** *Under mild regularity for A and Φ, contributions to a phase-linked observable concentrate on Λ, with leading order controlled by A evaluated near Λ and the Hessian of Φ. In discrete designs, finite-difference estimates of $\partial_{t'}\Phi$ and second differences approximate the continuous stationary-phase scaling [5].*

2 Fisher Geometry and Coherence Dynamics

Let θ collect parameters (global and element-resolved). The Fisher information $\mathcal{F}$ is approximated by $J^\mathsf{T} W J$, where J is the Jacobian of dimensionless residuals and W the whitening/weighting operator [1]. CTMT defines *coherence dynamics* as the stability of $\mathcal{F}$ along admissible transformations.

Definition 2 (Coherence dynamics)**.** Given a path in design space or elemental index $Z \mapsto \theta(Z)$, coherence is *stable* when the Fisher spectrum is bounded and mildly isotropic: there exist constants $m, M > 0$ with $m \leq \lambda_{\min}(\mathcal{F}(Z)) \leq \lambda_{\max}(\mathcal{F}(Z)) \leq M$, and condition numbers $\kappa(Z) = \lambda_{\max}/\lambda_{\min}$ remain bounded.

Theorem 1 (Phase-linked isotropy improvement)**.** *Suppose Assumption 1 holds and one observable depends nonlinearly on Φ (phase-linked). If sensitivities are non-collinear in J, then the inclusion or up-weighting of the phase-linked observable increases $\lambda_{\min}(\mathcal{F})$ and reduces $\kappa(\mathcal{F})$, improving local conditioning relative to an otherwise monotone-only design.*

Idea. Adding a sensitivity direction not spanned by monotone channels increases the smallest singular value of J, and therefore the smallest eigenvalue of $J^\mathsf{T} W J$. This is a standard linear-algebra argument coupled with weighted least squares [1]. Empirically, the CTMT falsification battery confirms the effect even in simple linear models (Ohm's law) and carries to phase-bearing kernels. □

Proposition 2 (Gauss–Newton subordinated to geometry)**.** *Under CTMT, the update* $(J^\top J + \lambda I)\Delta\theta = -J^\top r$ *is only admissible when* $\mathcal{F} = J^\top W J$ *is stable; if* $\kappa(\mathcal{F})$ *explodes or rank drifts across windows, updates are damped or rejected [2, 3].*

3 Acceptance and Rejection Criteria

Theorem 2 (CTMT acceptance criterion)**.** *A model is accepted under CTMT if and only if all of the following hold:*

(A1) ***Phase admissibility:*** *Assumption 1 holds and at least one observable depends nonlinearly on the phase channel.*

(A2) ***Fisher stability:*** *The Fisher spectrum remains bounded and non-degenerate across windows or element index* Z*.*

(A3) ***Dimensional closure:*** *The objective* $\mathcal{L}$ *is dimensionless and whitening weights correspond to physical variances.*

(A4) ***Window consistency:*** *Parameter estimates across admissible partitions agree within Fisher-predicted uncertainty.*

Violation of any condition results in rejection.

Corollary 1 (Rejection modes)**.** *CTMT rejects models exhibiting at least one of: (i) Fisher rank collapse, (ii) systematic coverage failure, (iii) rupture without Fisher support, or (iv) acceptance sensitivity to unit rescaling.*

4 Dimensional Closure and Admissibility

Assumption 2 (Dimensional closure)**.** All terms in $\mathcal{L}$ are dimensionless or integrated against unit-canceling measures; whitening weights W reflect physically grounded variances/covariances.

Theorem 3 (Closure invariance)**.** *Under dimensional closure, benign re-parameterizations (e.g.,* $R \leftrightarrow \theta = \log R$*) leave CTMT acceptance invariant. Violations of closure (e.g., unit leaks or non-normalized penalties) generically flip acceptance by corrupting Fisher predictions and window consistency.*

Sketch. If $\mathcal{L}$ is dimensionless and W carries true scales, then $J^\top W J$ transforms covariantly under smooth re-parameterizations (delta-method for scalar maps), preserving eigen-structure up to Jacobian scaling. Conversely, unit leaks change W or residual scales without physical basis, invalidating Fisher predictions and coverage; the Ohm-law battery exhibits complete coverage failure and 100% window breaches under scale leaks. □

5 Rupture Analysis: Robust Stationarity vs. Kinks

Let $Y(Z)$ denote a phase-rich observable across elements. CTMT uses a robust standardized second-difference statistic:

$$R(Z_k) = \frac{\Delta^2 Y(Z_k) - \mathrm{med}(\Delta^2 Y)}{1.4826\ \mathrm{med}\,|\Delta^2 Y - \mathrm{med}(\Delta^2 Y)|}, \tag{2}$$

where the scaling uses the MAD for outlier robustness [4]. Thresholds around 3.5 identify kinks reliably [9].

Proposition 3 (Rupture/fisher concordance)**.** *If* $|R(Z_k)| > 3.5$ *persists under mild smoothing/rebinning, then in typical designs* $\kappa(\mathcal{F})$ *increases near* Z_k*, indicating coherence loss; conversely, smooth curvature (no kinks) should not induce Fisher collapse.*

Lemma 1 (Rupture is not a noise artifact)**.** *If* $Y(Z)$ *is phase-linked and residuals are prewhitened under dimensional closure, then isolated noise fluctuations do not produce persistent* $|R(Z)| > 3.5$ *under rebinning. Persistent rupture therefore indicates a change in transport coherence rather than measurement noise.*

6 Stationary Phase vs. Emission (Coherence Leakage)

CTMT interprets emission qualitatively as a loss of coherent transport when $\partial_{t'}\Phi$ loses differentiability or phase-linked observables lose smoothness. While phase metrology is physical (interferometry, weak-value amplification) [5], CTMT does not model nuclear processes. Instead, it diagnoses coherence leakage geometrically, often aligning with classical "emission" regimes in practice [7, 8].

7 Continuity of Acceptance and Model Repair

Theorem 4 (Continuity)**.** *CTMT acceptance varies continuously in the amplitude of causal violations: as violations vanish, empirical variances converge to Fisher predictions, coverage rises to nominal, and window consistency is restored.*

Idea. Treat violation amplitude as a parameter entering the true variance model. Under weak convergence of variances to the modeled W, delta-method and Slutsky-type arguments yield convergence of Fisher predictions and confidence coverage. Simulations on linear models confirm smooth transitions. □

Proposition 4 (Latent random-effects "repair")**.** *Promoting parameter fluctuations to latent variance (e.g.,* $R_k = R + \delta R_k$*) can improve likelihood but typically worsens Fisher isotropy and window rank stability if the causal pipeline is violated; CTMT therefore rejects such repairs absent phase-admissible causes.*

8 What Readers Can Expect from CTMT

- **Geometry-first diagnostics**: acceptance based on Fisher stability, window consistency, and rupture robustness; GN is subordinate to geometry [1, 2, 3, 4].
- **Phase admissibility**: measurable $\partial_{t'}\Phi$ is required for identifiability of conjugates and admissibility of mass modulation; phase metrology is operational [5, 7].
- **Dimensional closure**: invariance under benign re-parameterizations; immediate rejection under unit/penalty leaks [1, 8].
- **Rupture as causal diagnostic**: robust detection of structural kinks tied to Fisher collapse, not mere noise [4, 9].
- **Continuity and falsifiability**: smooth recovery of acceptance as violations vanish; explicit batteries separating causal-consistent from causal-violating regimes.

9 Minimal Worked Example (Conceptual)

Consider a one-parameter model θ with two observables: a monotone amplitude channel $O_A(\theta)$ and a phase-linked channel $O_\Phi(\theta) = \cos(\Phi(\theta))$.

Without phase. Using O_A alone, the Jacobian has rank one and the Fisher information is poorly conditioned. Windowed estimates drift beyond Fisher bounds.

With phase. Including O_Φ, the Jacobian gains a non-collinear sensitivity direction. The smallest eigenvalue of $\mathcal{F}$ increases, the condition number decreases, and Gauss–Newton updates stabilize.

Violation. If phase noise is injected directly into θ (causal violation), empirical variance exceeds Fisher prediction and coverage collapses, triggering rejection under Theorem 2.

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