Chronotopic Metric Theory: Physical Admissibility via Fisher Response Geometry and Rate–Distortion Curvature

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CONTENTS

I. ABSTRACT

CTMT is a non–ontological, information–geometric admissibility framework for kernel–generated forward maps. No spacetime, metric, gauge group, or dynamical law is postulated. The primitive structure is an oscillatory transport kernel whose induced forward map admits a Fisher information geometry.

This manuscript consolidates *physical support* for CTMT: the Fisher metric arises from linear response and fluctuation–weighted observable geometry; coherence invariants (rigidity, null curvature) are operational; and a *rate–distortion* functional yields Hessian curvature consistent with CTMT's metric structures, including typical hyperbolic signatures on coherence coordinates. Classical relativistic expressions are used *only as functional analogies* (consistency checks), not as ontological claims. CTMT remains an admissible model: a minimal criterion for stable prediction and parameter transport.

II. INTRODUCTION: CTMT AS PHYSICAL ADMISSIBILITY

CTMT addresses a single operational question:

> *When does a coarse–grained kernel define a stable and transportable forward map with a well–conditioned inverse problem?*

All structure is induced by observables and their Jacobians. Stability, composability, and causal admissibility are encoded by Fisher–response geometry. This manuscript stays at *low ground*: we present physical computations that support CTMT's mathematics (response, fluctuations, rate–distortion), without ontological claims.

III. KERNEL TRANSPORT AND OBSERVABLES

A. Oscillatory kernel

Let T be a forward–ordered kernel transport

$$T(t) = \int_{t_0}^{t} \Xi(t') \exp\left(\frac{i}{\mathcal{S}_*}\,\Phi(t,t') - \epsilon(t')\right) dt', \qquad (1)$$

where Φ is a differentiable phase functional, Ξ a bounded modulation, $\epsilon \geq 0$ ensures integrability, and $\mathcal{S}_* > 0$ an empirically extractable action scale.

B. Observable map and Jacobian

Let $\theta \in \mathbb{R}^d$ parametrize (Φ, Ξ, ϵ). Observables $Y = \mathcal{F}_\theta[T]$ are deterministic functionals (spectral components, delays, fluxes, integrated responses). Assuming differentiability,

$$J(\theta) = \partial_\theta Y(\theta)\,, \qquad J \in \mathbb{R}^{m\times d}. \tag{2}$$

IV. Fisher Metric as Physical Response Geometry

A. Linear response

Under small deformations $\delta\theta$, the observable responds

$$\delta Y \approx J(\theta)\,\delta\theta\,. \tag{3}$$

This is the standard linear response setting [1], [2].

B. Fluctuation–weighted geometry

Coarse–grained fluctuations yield $y = Y(\theta) + \epsilon$ with covariance $C_\epsilon = \mathbb{E}[\epsilon\epsilon^\top]$. The physical distinguishability of nearby configurations is the quadratic form

$$\delta s^2 = \delta Y^\top C_\epsilon^{-1} \delta Y\,. \tag{4}$$

Substituting linear response gives

$$\delta s^2 = \delta\theta^\top \underbrace{(J^\top C_\epsilon^{-1} J)}_{H(\theta)} \delta\theta\,. \tag{5}$$

Proposition IV.1 (Physical Fisher response metric)**.** *The tensor $H(\theta) = J(\theta)^\top C_\epsilon^{-1} J(\theta)$ is the unique pullback of the observable–space quadratic cost to parameter space, and coincides formally with the Fisher information matrix. In CTMT, it is* physical*: determined solely by measurable J and C_ϵ.*

Sketch. δs^2 is the inverse–covariance–weighted quadratic form on observable space (Onsager–Machlup action [3]). Pullback under $Y(\theta)$ yields the unique quadratic form on parameter space, invariant under reparameterization and coarse–graining. No epistemic ingredients are used. □

Remark IV.2 (Physical instances)**.** Elastic moduli, susceptibilities, transport coefficients, and thermodynamic metrics share the same quadratic structure [4], [5].

V. Coherence Geometry: Rigidity and Redundancy

Definition V.1 (Informational dimensionality)**.** $d_{\text{eff}} = H(\theta)$.

Definition V.2 (Redundancy)**.** Directions in $\ker H$ are null: fluctuation–weighted indistinguishable to first order.

Theorem V.3 (Fisher rigidity)**.** *A kernel is rigid on a domain if H is preserved and $\operatorname{spec}(H)$ remains bounded under forward extension.*

Remark V.4. Rigidity is *empirical*: verified by measurable J and C_ϵ.

VI. Operational Extraction of the CTMT Action Scale $\mathcal{S}_*$

CTMT treats the kernel action scale $\mathcal{S}_*$ as an *operationally measurable* constant, inferred from stationary-phase structure and response metrology. No ontological assumption is made: $\mathcal{S}_*$ is extracted from data in the same manner that elastic moduli, susceptibilities, or transport coefficients are obtained in laboratory practice.

A. Stationary Phase and Action in Response Propagation

Semiclassical stationary-phase propagation—including Van Vleck, Gutzwiller, and Herman–Kluk approximations—introduces an action scale as the natural phase normalization in oscillatory kernels [16]–[18]. In CTMT, the same scale appears in oscillatory transport kernels and is determined empirically from observable-phase stationarity.

B. Protocol A: Blackbody Radiometry (Thermal Response)

For a blackbody at temperature T, the spectral radiance in frequency form is

$$B_\nu(\nu, T) = \frac{2\,\mathcal{S}_*\,\nu^3}{c^2}\left[\exp\!\left(\frac{\mathcal{S}_*\,\nu}{k_B T}\right) - 1\right]^{-1}, \tag{6}$$

written so that the action scale $\mathcal{S}_*$ plays the role of Planck's constant. Given calibrated (ν, B_ν) and known T, a one-parameter fit returns $\widehat{\mathcal{S}}_*$, which is compared against the SI-fixed h [6]–[8].

Remarks.: (i) In the redefined SI (2019), $h = 6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,s}$ is exact [9], [10]. (ii) CTMT uses the same radiometric pipeline to *measure* $\mathcal{S}_*$ without assuming a quantum ontology; agreement with h in thermal regimes is empirical.

C. Protocol B: Josephson Voltage–Frequency Metrology

The AC Josephson relation,

$$f = \frac{2e}{\mathcal{S}_*}\,V, \tag{7}$$

provides a direct operational bridge between voltage and frequency. Josephson-array voltage standards have long underpinned primary metrology [11]–[13]. CTMT simply interprets the extracted constant as $\widehat{\mathcal{S}}_*$.

D. Protocol C: Quantum Fluctuation–Dissipation Roll-Off

At high frequencies, the equilibrium noise spectral density follows the quantum fluctuation–dissipation theorem,

$$S_Q(\omega, T) \propto \mathcal{S}_*\,\omega\,\coth\!\left(\frac{\mathcal{S}_*\,\omega}{2k_B T}\right), \tag{8}$$

with a characteristic roll-off that allows extraction of $\widehat{\mathcal{S}}_*$ from cryogenic noise spectroscopy [14], [15].

E. Low-Ground Statement (Admissibility)

CTMT does not assert that $\mathcal{S}_*$ is fundamental or spacetime-primitive. Instead, in regimes where blackbody radiometry, Josephson metrology, or quantum FDT apply, the operationally extracted $\widehat{\mathcal{S}}_*$ coincides (within uncertainty) with the SI-fixed Planck constant. This supports CTMT's physical computations without making ontological claims.

Proposition VI.1 (Operational Identification in Quantum-Governed Regimes)**.** *If an experiment lies in a regime where (i) Planck radiometry, (ii) Josephson voltage–frequency relations, or (iii) quantum FDT roll-off apply, then CTMT's stationary-phase and response-calibration procedures return an action scale $\widehat{\mathcal{S}}_*$ numerically equal to h (within experimental uncertainty).*

Remark VI.2. Outside such regimes, CTMT continues to treat $\mathcal{S}_*$ as a fit parameter determined by response data. Agreement or disagreement with h is an empirical matter.

VII. Rate–Distortion Geometry for CTMT

A. Variational functional

Let $\Theta \in \mathbb{R}^p$ denote kernel modulation parameters (phase, rhythm, coherence coordinates). Define

$$\mathcal{J}[\Theta] = \mathcal{R}[\Theta] + \lambda\,\mathcal{D}[\Theta]\,, \qquad \lambda > 0\,. \tag{9}$$

Here $\mathcal{R}$ is coherence throughput (rate), $\mathcal{D}$ is distortion (loss of phase identity or mis–ordering). Physical trajectories satisfy $\delta\mathcal{J}[\Theta^*] = 0$.

B. Quadratic expansion and induced metric

Expanding near Θ^*,

$$\mathcal{J}[\Theta^* + \delta\Theta] = \mathcal{J}[\Theta^*] + \frac{1}{2}\,\delta\Theta^\top \underbrace{\nabla^2\mathcal{J}[\Theta^*]}_{G(\Theta^*)}\delta\Theta + \mathcal{O}(\|\delta\Theta\|^3)\,. \tag{10}$$

Thus the induced *rate–distortion metric* is

$$G(\Theta^*) = \nabla^2\mathcal{R}[\Theta^*] + \lambda\,\nabla^2\mathcal{D}[\Theta^*]\,. \tag{11}$$

For coherence–preserving kernels, G coincides (up to scale and rank truncation) with the Fisher–Rao metric on the coherence class.

Remark VII.1. We do not claim G equals physical spacetime. We only note that Hessian structures of rate–distortion functionals *typically* exhibit hyperbolic signatures on coherence coordinates, consistent with CTMT's null transport sectors.

C. Typical hyperbolic signature (non–ontological)

Distortion penalizes temporal mis–ordering more than spatial dispersion. Near Θ^*,

$$\nabla^2\mathcal{D}[\Theta^*] \simeq -\alpha\,\partial_t^2 + \sum_{i=1}^{3}\beta_i\,\partial_{x_i}^2\,, \quad \alpha, \beta_i > 0\,. \tag{12}$$

Hence G *typically* has one negative eigenvalue on coherence coordinates (hyperbolic signature). This is a structural property of the Hessian; *no spacetime ontology is asserted*.

VIII. Curvature Operator and Transport Sectors

Define

$$\mathcal{C} = G^+ H\,, \qquad H = \nabla^2\Phi(\Theta^*)\,. \tag{13}$$

Here G^+ is the Moore–Penrose pseudoinverse.

A. Sector decomposition

- **Collapse sector**: directions where (H) decreases and distortion increases.
- **Transport (null) sector**: $\mathcal{N}(G) = \{v \neq 0 \,:\, v^\top G v = 0\}$, supporting coherence–preserving propagation.

IX. Functional Consistency Checks (Analogies Only)

We list three classical expressions as *functional analogies*, not predictions. They show CTMT curvature reproduces the *form* of known weak–field relations.

A. Redshift analogue

In stationary coherence geometry with $G_{tt} < 0$,

$$\frac{\nu_{\rm obs}}{\nu_{\rm emit}}\ (\text{analogue})\ \simeq \sqrt{\frac{G_{tt}(\text{emit})}{G_{tt}(\text{obs})}}\,. \tag{14}$$

B. Transverse deflection analogue

For a null trajectory $\gamma \subset \mathcal{N}(G)$,

$$\Delta\theta\ (\text{analogue})\ \propto \int_\gamma \nabla_\perp\big(\log|G_{tt}|\big)\,ds\,. \tag{15}$$

C. Delay analogue

Phase accumulation along a null path:

$$\Delta t\ (\text{analogue})\ \propto \int_\gamma \big(-G_{tt} - G_{rr}\big)\,ds\,. \tag{16}$$

Remark IX.1. These analogies are *structural consistency checks* for CTMT curvature. They do not identify G with spacetime.

X. Continuum Coherence Limit (Boundary Discussion)

In smooth, full–rank regimes, CTMT's null transport and curvature structures can coincide formally with known continuum response geometries. This is a *boundary* statement; CTMT remains non–ontological.

XI. Falsifiability and Admissibility Tests

CTMT is falsified if: (i) regime–dependent rank loss in H, (ii) divergence of H under extension, (iii) failure of parameter transport, (iv) loss of differentiability. Operational invariants: monotonicity under coarse–graining and composability under stacking.

Appendix A: Python – Physical Fisher Emergence

Minimal demonstration (oscillatory kernel)

```
import numpy as np

# Observable from oscillatory kernel
def observable(theta):
    omega, eps = theta
    t = np.linspace(0, 1, 200)
    Xi = 0.5 * (1 - np.cos(2*np.pi*t))
    phase = omega * (1 - t)
    T = np.sum(Xi * np.exp(1j*phase - eps*(1-t))) * (t[1]-t[0])
    return np.array([T.real, T.imag])

# Jacobian via central differences
def jacobian(theta, delta=1e-4):
    y0 = observable(theta)
    J = np.zeros((len(y0), len(theta)))
    for k in range(len(theta)):
        e = np.zeros_like(theta); e[k]=1
        yp = observable(theta + delta*e)
        ym = observable(theta - delta*e)
        J[:,k] = (yp - ym)/(2*delta)
    return J

Ceps = np.diag([1e-3, 2e-3])
theta = np.array([10.0, 2.0])
J = jacobian(theta)
H = J.T @ np.linalg.inv(Ceps) @ J
print("Jacobian\n", J)
print("Physical Fisher H\n", H)
print("Eigenvalues", np.linalg.eigvalsh(H))
```

Appendix B: Python – Coil (Relative Amplitude, Masking)

Series–RLC abstraction in AD5933 band (relative observable)

```
import numpy as np

# Coil forward map
def coil_forward(freq, theta, mask=None, mode="relative", ref_idx=0):
    R, L, C, G = theta
    omega = 2*np.pi*freq
    Z = G * (R + 1j*omega*L + 1/(1j*omega*C))
    if mask is None:
        mask = np.ones_like(freq, dtype=bool)
    Zm = Z[mask]
    amp = np.abs(Zm)
    if mode == "relative":
        denom = max(amp[min(ref_idx, len(amp)-1)], 1e-12)
        return amp/denom
    return np.concatenate([Zm.real, Zm.imag])

# CTMT checks: Fisher from jacobian, monotonicity, null curvature
def jacobian_fd(fwd, theta, delta=1e-3):
    y0 = fwd(theta)
    J = np.zeros((len(y0), len(theta)))
```

```
    for j in range(len(theta)):
        e = np.zeros_like(theta); e[j]=1.0
        yp = fwd(theta + delta*e)
        ym = fwd(theta - delta*e)
        J[:, j] = (yp - ym) / (2*delta)
    return J

freq = np.geomspace(5e2, 8e4, 801)
mask = np.ones_like(freq, dtype=bool); mask[::5] = False
fwd = lambda th: coil_forward(freq, th, mask=mask, mode="relative")
th = np.array([12.0, 5.0e-3, 5.0e-7, 1.0])
J = jacobian_fd(fwd, th)
H = J.T @ J / 1e-6
print("rank ~", np.sum(np.linalg.eigvalsh(H) > 1e-10))
print("smallest eigval ~", np.min(np.linalg.eigvalsh(H)))
```

Appendix C: Python Radiometry Fit for $\mathcal{S}_*$

Pipeline. Fit $\mathcal{S}_*$ from (ν, B_ν) at known T via nonlinear least squares.

```
import numpy as np

c = 299792458.0
kB = 1.380649e-23
h_true = 6.62607015e-34
T = 1800.0

nu = np.linspace(1e12, 6e13, 300)

def B_nu(nu, S_action):
    return (2.0*S_action*nu**3 / c**2) / \
           (np.exp(S_action*nu/(kB*T)) - 1.0)

B = B_nu(nu, h_true)
rng = np.random.default_rng(1)
B_obs = B * (1.0 + 0.01 * rng.standard_normal(B.size))

S_grid = np.linspace(0.5*h_true, 1.5*h_true, 1001)
errs = [np.mean((B_obs - B_nu(nu, S))**2) for S in S_grid]
S_hat = S_grid[int(np.argmin(errs))]

print("Estimated S_*:", S_hat)
print("Relative error:", (S_hat - h_true)/h_true)
```

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