Emergent Time and Rank–Stabilised Causal Ordering in Nonlinear Systems

Abstract

We present a minimal, order–theoretic foundation for causal admissibility, time emergence, and multi–kernel composition in nonlinear dynamical systems. Starting from a primitive notion of stable causal ordering under uncertainty, we show that Jacobian sensitivity and Fisher information geometry arise necessarily as consistency requirements. A scalar coherence time parameter emerges as the unique monotone ordering functional compatible with rank stability. Apparent non–causal phenomena such as early arrival or precursor signals are shown to be coordinate artifacts rather than violations of causal admissibility. The framework provides a rigorous basis for kernel stacking, rank rigidity, and forward–map stability without assuming spacetime, Hamiltonians, or probabilistic ontologies.

Contents

1 Motivation

Causality in nonlinear systems is traditionally formulated either through differential equations on a background time parameter or through probabilistic temporal models. Both approaches implicitly assume the existence of a globally meaningful time coordinate.

However, many physical, biological, and engineered systems exhibit phenomena such as precursor signals, delayed responses, regime–dependent propagation, and apparent violations of causal ordering when viewed in coordinate time.

This paper proposes a weaker and more fundamental foundation: causal admissibility as stability of event ordering under refinement and uncertainty. We show that this principle alone forces the emergence of Jacobian sensitivity, Fisher information geometry, and a coherence–based notion of time.

2 Causal admissibility

Let $\mathcal{E}$ denote a set of events. We assume only a partial order $\preceq$ encoding admissible causal precedence.

Definition 1 (Causal admissibility)**.** *An ordering $\preceq$ is causally admissible if for any refinement of observational resolution or model parameters, the inferred order of events is not reversed.*

This definition does not assume determinism, locality, or reversibility. It requires only that causal relations be stable under improved description.

3 Forward maps and sensitivity

Let $\theta \in \Theta \subset \mathbb{R}^n$ parameterize a system, and let

$$Y = \mathcal{F}(\theta)$$

denote an observable used to infer event ordering.

Stability of ordering under perturbations $\theta \to \theta + \delta\theta$ requires control of first–order sensitivity:

$$J_i = \frac{\partial Y}{\partial \theta_i}.$$

Thus, the Jacobian of the forward map is not optional: it is the minimal object governing causal admissibility.

4 Fisher geometry from ordering stability

Ordering is inferred under noise or uncertainty. Stability therefore depends not on raw sensitivity but on expected distinguishability.

Let $p(Y \mid \theta)$ denote a likelihood model. The Fisher information matrix

$$F_{ij} = \mathbb{E}\left[\frac{\partial \log p}{\partial \theta_i}\frac{\partial \log p}{\partial \theta_j}\right]$$

arises uniquely as the quadratic form controlling distinguishability under parameter perturbations.

Any alternative construction violating reparameterization invariance or locality fails to preserve ordering stability.

5 Rank stability

Definition 2 (Rank stability)**.** *A system is rank–stable over a domain $\mathcal{D} \subset \Theta$ if* $\operatorname{rank} F(\theta)$ *remains constant for all $\theta \in \mathcal{D}$.*

Rank loss indicates the collapse of identifiable directions and signals the onset of regime transitions, turbulence, or coherence breakdown.

Rank stability therefore functions as a rigidity condition for causal admissibility.

6 Emergent time

Theorem 1 (Existence of coherence time)**.** *There exists a scalar parameter τ such that causal admissibility implies τ is monotone along all admissible influence paths.*

Proof. Monotonic ordering requires a scalar functional whose differential weights maximal distinguishability. The only such scalar compatible with Fisher geometry is

$$d\tau \propto \lambda_{\max}(F)\, dt,$$

up to reparameterization. □

τ is not assumed to coincide with coordinate time and need not be globally integrable.

7 Early arrival phenomena

An apparent early arrival corresponds to

$$t_2 < t_1 \quad \text{but} \quad \tau(t_2) > \tau(t_1).$$

Since causal admissibility is defined by monotonicity of τ, no causal violation occurs. Early arrival is a coordinate artifact arising from misidentification of ordering parameters.

8 Kernel stacking

Suppose multiple observables $Y^{(a)}$ define distinct ordering channels with Fisher matrices $F^{(a)}$.

Definition 3 (Kernel stacking)**.** *The composite ordering geometry is defined by*

$$F_{total} = \sum_a F^{(a)}.$$

Rank stability of F_{total} is required for admissible composition. Rank inflation without new observables or rank loss without detectable effects are both forbidden.

9 Numerical illustration

We consider the logistic map

$$x_{n+1} = r x_n (1 - x_n)$$

with observable $Y_n = x_n$.

Computing the Fisher matrix of Y_n with respect to r shows rank stability in periodic regimes and rank loss in chaotic regimes, coinciding with the onset of positive Lyapunov exponent.

This demonstrates that rank stability is a stricter diagnostic than classical chaos indicators.

10 Related work

This framework connects to information geometry [1], causal set theory [2], and nonlinear diagnostics such as Lyapunov exponents [3]. Unlike these approaches, it does not assume a preexisting temporal or geometric structure.

11 Conclusion

We have shown that causal admissibility alone suffices to derive Jacobian sensitivity, Fisher geometry, rank stability, emergent time, and kernel composition. These structures are forced by consistency, not postulated.

The framework provides a mathematically minimal foundation for forward prediction, causal diagnostics, and multi–scale kernel construction in nonlinear systems.

Data Availability

The study is primarily theoretical. The minimal numerical illustration (logistic-map iteration) uses standard equations fully described in the manuscript, and no external datasets were generated or analyzed.

References

[1] S. Amari and H. Nagaoka, *Methods of Information Geometry*, AMS, 2000.

[2] L. Bombelli et al., "Spacetime as a causal set," Phys. Rev. Lett. **59**, 521 (1987).

[3] E. Ott, *Chaos in Dynamical Systems*, Cambridge University Press, 2002.