Chronotopic Metric Theory: Emergent Geometry, Gauge Structure, and Quantum Linearity from Kernel Coherence

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***Abstract*—Chronotopic Metric Theory (CTMT) is a metric-free, kernel-based framework in which spacetime geometry, gauge structure, and quantum linearity emerge from coherence transport and Fisher-curvature stability. No background manifold, metric, or Hilbert space is assumed. Instead, dimensionality, null propagation, gauge redundancy, and linear quantum evolution arise as dynamically selected configurations of a chronotopic kernel.**

In this focused contribution, we present explicit structural derivations showing that: (i) null propagation and Maxwell invariants arise from rank-one Fisher sectors, (ii) Newtonian gravity emerges from curvature minimization under coherence sourcing, (iii) Schrödinger-type dynamics appear at a rigidity threshold where kernel expectations linearize, and (iv) $U(1)$, $SU(2)$, and $SU(3)$ gauge groups arise from redundancy closures in kernel phase labeling. No adjustable dimensionless parameters are introduced; known constants appear only as empirical rigidity thresholds. The results place CTMT within the broader class of emergent spacetime and information-geometric approaches [1]–[5].

I. Foundational Axioms

Definition I.1 (Chronotopic kernel)**.** Physical configurations are described by a complex-valued chronotopic kernel

$$\mathcal{K} : \mathrm{Anch} \times \mathrm{Topo} \times \mathbb{R} \to \mathbb{C},$$

where Anch encodes discrete anchors (sources, detectors, interaction loci) and Topo encodes purely topological neighborhoods of kernel support. No metric or dimensional structure is assumed on these sets.

Axiom 1 (Kernel primacy). There is no fundamental background spacetime. All observable geometry arises from coherence relations and curvature of $\mathcal{K}$.

Axiom 2 (Action invariant). All physical processes preserve the dimensionally closed invariant

$$S_* = \frac{E}{\nu},$$

where E is an effective energy scale and ν a characteristic frequency. In particular, for photons $E = h\nu$ gives $S_* = h$.

Axiom 3 (No fundamental Hilbert space). Linear state spaces, inner products, and superposition are not fundamental. They emerge only in a rigid-phase regime, once the kernel spectrum stabilizes under coherence flow.

II. Kernel Calculus and Coherence

Definition II.1 (Observable field)**.** Given an anchor configuration κ and kernel $\mathcal{K}$, an induced observable field is

$$\psi(x,t) = \int \mathcal{K}(x,x';t)\,\kappa(x')\,dx'.$$

Definition II.2 (Coherence density)**.** Let ϕ denote kernel phases sampled under a reconstruction protocol. The local coherence density is

$$\rho_{\mathrm{coh}} = \left|\mathbb{E}\left[e^{i\phi}\right]\right|.$$

Definition II.3 (-volume)**.** The global coherence measure

$$= \int \rho_{\mathrm{coh}}\,d\mu$$

quantifies kernel stability, where $d\mu$ is a topology-compatible measure on the support of $\mathcal{K}$.

No metric appears in these definitions; all effective geometry is encoded in coherence, curvature, and transport properties of $\mathcal{K}$.

III. Fisher Curvature and Rank Attraction

Definition III.1 (Fisher information Hessian)**.** Let Θ denote kernel parameters. The Fisher information Hessian of a likelihood functional $\mathcal{L}$ is

$$H_{ij} \equiv \nabla^2_{\Theta} \log \mathcal{L},$$

with indices i, j running over admissible kernel directions.

Remark III.2. Coherent eigenspaces of H define Fisher curvature sectors. Their rank controls the number of independent curvature directions that can be stably supported.

Proposition III.3 (Rank attraction)**.** *CTMT dynamics preferentially collapse onto low-rank coherent eigenspaces of H.*

Sketch. Sustaining coherence along N independent curvature directions requires simultaneous phase stability across N modes. Noise and rupture scale superlinearly with N, while $\rho_{\mathrm{coh}} \le 1$. Higher-rank sectors therefore lose coherence faster, causing eigenvalue collapse and dynamical reduction of effective rank. □

IV. Null Propagation and Maxwell Structure

We now show explicitly how null propagation and Maxwell invariants arise from a minimal kernel phase.

Definition IV.1 (Minimal coherent phase)**.** Consider a phase field

$$\Phi(x,t) = \omega t - kx,$$

with constant coherence density $\rho_{\mathrm{coh}} = \rho_0$.

The Fisher information Hessian with respect to (t,x) is

$$H_{ij} = \mathbb{E}[\partial_i \Phi \, \partial_j \Phi] = \begin{pmatrix} \omega^2 & -\omega k \\ -\omega k & k^2 \end{pmatrix}.$$

Its determinant vanishes:

$$\det H = \omega^2 k^2 - (\omega k)^2 = 0.$$

Proposition IV.2 (Null Fisher sector and signal speed)**.** *The phase $\Phi(x,t) = \omega t - kx$ defines a null Fisher-curvature direction and a characteristic propagation speed*

$$v \equiv \frac{\omega}{k}.$$

In the rigid-phase limit, this speed coincides operationally with the universal signal speed c. Interpreting Φ as a kernel phase potential and $A_\mu = \partial_\mu \Phi$ as a connection, we obtain a field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

with Lorentz invariants

$$I_1 = F_{\mu\nu} F^{\mu\nu}, \qquad I_2 = \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}.$$

Null coherence propagation corresponds to $I_1 = I_2 = 0$, i.e. vacuum electromagnetic waves in emergent spacetime.

V. Emergent Newtonian Gravity

We next derive the Poisson equation from Fisher-curvature stability.

Definition V.1 (Fisher curvature scalar)**.** The scalar Fisher curvature is

$$\mathcal{K} \equiv \mathrm{Tr}\, H = \sum_i \partial_i^2 \Phi.$$

Consider a homogeneous, isotropic coherence background.

Proposition V.2 (Laplacian from curvature stability)**.** *Rank stability under fixed coherence density ρ_{coh} minimizes*

$$\int \left[(\nabla \Phi)^2 - \alpha \, \rho_{\mathrm{coh}} \, \Phi \right] d^3x.$$

The Euler–Lagrange equation yields

$$\nabla^2 \Phi = \alpha \, \rho_{\mathrm{coh}}.$$

Dimensional analysis of kernel curvature shows

$$[\rho_{\mathrm{coh}}] = \frac{\text{action}}{\text{volume}}.$$

In the rigid regime where action localizes as $E \times t$, we obtain

$$\rho_{\mathrm{coh}} \longrightarrow \frac{E}{V},$$

identifying ρ_{coh} with mass–energy density. In the static, weak-field limit, Φ becomes operationally identifiable with the Newtonian potential Φ_N, so

$$\nabla^2 \Phi_N = 4\pi G \, \rho$$

with $\alpha = 4\pi G$. Thus Newtonian gravity emerges as the curvature stability condition under coherence sourcing.

VI. Rigidity and Emergent Quantum Linearity

We now formalize the rigidity threshold at which linear quantum mechanics emerges.

Definition VI.1 (Rigidity functional)**.** Define the dimensionless rigidity

$$\mathcal{R} = \frac{\mathrm{Var}(\Phi)}{S_*^2},$$

where S_* is the action invariant of Axiom 2.

Three regimes follow:

- **Classical averaging:** $\mathcal{R} \gg 1$, destructive interference dominates.
- **Nonlinear coherence:** $\mathcal{R} \sim 1$, coherent but nonlinear transport.
- **Rigid phase:** $\mathcal{R} \ll 1$, phase fluctuations are small.

Consider the kernel expectation of an observable functional O,

$$\langle O \rangle = \int \Xi(\Theta) \, e^{i\Phi(\Theta)/S_*} \, \mathcal{D}\Theta.$$

In the rigid-phase regime $\mathrm{Var}(\Phi) \ll S_*^2$, expand

$$e^{i\Phi/S_*} \approx 1 + \frac{i}{S_*} \Phi.$$

This yields

$$\langle O \rangle \approx \langle \Xi \rangle + \frac{i}{S_*} \langle \Xi \Phi \rangle.$$

Let $\hat{H}$ denote the generator of kernel phase flow. Time translation of Φ induces

$$\partial_t \langle O \rangle = \frac{i}{S_*} \langle [\hat{H}, O] \rangle,$$

where the commutator expresses the action of phase flow as an infinitesimal generator on observables.

Rigidity enforces self-adjointness of $\hat{H}$ with respect to the induced inner product, so evolution is norm-preserving. Identifying $S_* = \hbar$ in the rigid regime yields

$$i\hbar\,\partial_t\psi = \hat{H}\psi,$$

a Schrödinger-type evolution law. Thus Hilbert space, linear superposition, and unitary time evolution arise as emergent structures at the rigidity threshold, in line with information-geometric and entropic approaches to quantum theory [1]–[3].

VII. Gauge Structure from Kernel Redundancy

Gauge freedom arises from redundancy in kernel phase labeling.

Definition VII.1 (Redundant phase shift)**.** A phase shift

$$\Phi \mapsto \Phi + \chi$$

with $\chi \in \ker H$ lies in a null Fisher sector and leaves observables invariant: $\delta\langle O\rangle = 0$.

A. $U(1)$ from Single-Phase Holonomy

A single dominant null-curvature sector (X-axis) supports a global phase redundancy

$$\Phi_X \mapsto \Phi_X + \theta(x,t).$$

Local closure of such redundancies yields a compact Abelian group with one generator, isomorphic to $U(1)$. This sector corresponds to charge-like holonomy in the X-type null-curvature direction.

B. $SU(2)$ from X/Y Torsional Coupling

The X and Y sectors form a two-dimensional torsional subspace with antisymmetric coupling:

$$[\theta_X, \theta_Y] \neq 0.$$

Closure under local rotations that preserve the Fisher norm defines a compact, simply connected group with three generators: two transverse modes and one torsional generator. This Lie algebra is isomorphic to $\mathfrak{su}(2)$, yielding $SU(2)$ as the minimal non-Abelian redundancy group stabilizing X/Y coherence.

C. $SU(3)$ from Triple-Channel Redundancy

Adding the Z (compressive) sector, redundancy closures must preserve:

- total coherence density,
- null transport in X,
- torsional balance in Y,
- compressive stability in Z.

The minimal simple Lie group acting transitively on three coupled complex coherence channels under these constraints is $SU(3)$. Larger groups such as $SU(4)$ introduce surplus generators that exceed the redundancy capacity allowed by coherence–redundancy stabilization, destabilizing rank coherence and triggering collapse.

VIII. Dimensional Attraction and Failure Modes

Proposition VIII.1 (Rank collapse beyond three spatial curvatures)**.** *Assume four independent spatial Fisher-curvature directions $\{\theta_i\}_{i=1}^4$. The integrated action scales as*

$$\mathcal{A} \sim \rho_{\mathrm{coh}}\, L^4.$$

Stationary-phase coherence requires $\mathcal{A} \sim S_*$, so

$$L \sim \left(\frac{S_*}{\rho_{\mathrm{coh}}}\right)^{1/4}.$$

This scale is incompatible with null-sector transport and torsional closure: curvature eigenvalues split, coherence ruptures, and one direction dynamically collapses. The system relaxes to three spatial-like curvature directions plus a null ordering direction.

Thus $3+1$-dimensional structure and Standard Model-like gauge groups appear as attractors of CTMT dynamics rather than arbitrary choices.

IX. Conclusion

We have presented a focused subset of Chronotopic Metric Theory in which geometry, gauge structure, and quantum linearity emerge from kernel coherence and Fisher curvature stability. Key results include:

- explicit emergence of null propagation and Maxwell invariants from rank-one Fisher sectors;
- derivation of Newtonian gravity as the weak-field limit of kernel curvature stability under coherence sourcing;
- appearance of Schrödinger-type dynamics at a rigidity threshold where kernel expectations linearize;
- emergence of $U(1)$, $SU(2)$, and $SU(3)$ as minimal redundancy closures in kernel phase labeling; and
- dynamical instability of higher spatial rank, leading to $3+1$-dimensional attraction.

No adjustable dimensionless parameters are introduced; known constants enter only as empirical rigidity thresholds. These structural correspondences place CTMT within the broader landscape of emergent spacetime and information-geometric theories, while providing a concrete, kernel-based realization.

Appendix

This appendix collects the structural arguments underlying the gravitational limit, quantum rigidity, gauge-group emergence, and dimensional failure modes.

A. Derivation of the Laplacian from Fisher Curvature Stability

In CTMT, the scalar phase functional $\Phi(\Theta)$ generates observable structure through its Fisher information Hessian

$$H_{ij} = \partial_i\partial_j\Phi. \tag{1}$$

Define the Fisher curvature scalar as the trace over admissible kernel directions:

$$\mathcal{K} \equiv \mathrm{Tr}\, H = \sum_i \partial_i^2\Phi. \tag{2}$$

In a homogeneous, isotropic coherence background, rank stability requires minimizing curvature fluctuations under fixed coherence density ρ_{coh}. This yields the variational condition

$$\delta \int \left[(\nabla\Phi)^2 - \alpha\, \rho_{\mathrm{coh}}\, \Phi \right] d^3x = 0. \tag{3}$$

Taking the Euler–Lagrange equation gives

$$\nabla^2 \Phi = \alpha\, \rho_{\mathrm{coh}}. \tag{4}$$

Thus the Laplacian form is not assumed but follows from Fisher-curvature stability under coherence sourcing. In the weak-field, static, rigid limit, Φ becomes operationally identifiable with the Newtonian potential Φ_N, and ρ_{coh} with mass–energy density, yielding $\alpha = 4\pi G$.

B. Explicit Kernel Toy Model and Null Curvature Sector

Consider again the minimal coherent kernel phase field

$$\Phi(x,t) = \omega t - kx, \tag{5}$$

with constant coherence density

$$\rho_{\mathrm{coh}} = \rho_0. \tag{6}$$

The Fisher information Hessian for translations in (x,t) is

$$H_{ij} = \mathbb{E}[\partial_i \Phi\, \partial_j \Phi] = \begin{pmatrix} \omega^2 & -\omega k \\ -\omega k & k^2 \end{pmatrix}. \tag{7}$$

Its determinant vanishes identically:

$$\det H = \omega^2 k^2 - (\omega k)^2 = 0. \tag{8}$$

This demonstrates the existence of a null Fisher-curvature direction without assuming a metric. Defining the characteristic propagation speed

$$v \equiv \frac{\omega}{k}, \tag{9}$$

the null condition fixes v uniquely.

In the rigid-phase limit, this speed coincides operationally with the universal signal speed c. This explicitly reproduces:

- null propagation,
- vanishing electromagnetic invariants $I_1 = F_{\mu\nu}F^{\mu\nu} = 0$ and $I_2 = \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = 0$,
- Maxwell-like wave propagation without postulated spacetime structure.

C. Coherence Density as Mass–Energy Density

In CTMT, curvature arises from the second variation of kernel phase under coherent perturbations:

$$\delta^2 \Phi \sim \int \rho_{\mathrm{coh}}\, \delta x^2. \tag{10}$$

Dimensional analysis yields

$$[\rho_{\mathrm{coh}}] = \frac{\text{action}}{\text{volume}}. \tag{11}$$

In the rigid limit where action localizes as energy times time, coherence density reduces to

$$\rho_{\mathrm{coh}} \longrightarrow \frac{E}{V}, \tag{12}$$

identifying ρ_{coh} with mass–energy density. This justifies the identification used in the main text when passing to the Newtonian limit.

D. Rigidity Linearization and Schrödinger Evolution

Consider the kernel expectation

$$\langle O \rangle = \int \Xi(\Theta)\, e^{i\Phi(\Theta)/S_*}\, \mathcal{D}\Theta. \tag{13}$$

In the rigid-phase regime $\mathrm{Var}(\Phi) \ll S_*^2$, expand to first order:

$$e^{i\Phi/S_*} \approx 1 + \frac{i}{S_*}\Phi. \tag{14}$$

Time translation of Φ generates

$$\partial_t \langle O \rangle = \frac{i}{S_*} \langle [\hat{H}, O] \rangle, \tag{15}$$

where $\hat{H}$ is the generator of kernel phase flow and the commutator expresses its action as an infinitesimal generator on observables.

Rigidity enforces self-adjointness of $\hat{H}$ with respect to the induced inner product, yielding unitary evolution. Identifying $S_* = \hbar$, this reduces to Schrödinger-type dynamics

$$i\hbar\, \partial_t \psi = \hat{H}\psi. \tag{16}$$

Thus linear quantum mechanics arises as the unique stable evolution law at the rigidity threshold.

E. Structural Origin of SU(2) and SU(3)

Gauge structure in CTMT emerges from redundancy closure in kernel phase labeling.

a) SU(2) from X/Y Torsional Coupling.: The X and Y sectors form a two-dimensional torsional subspace with antisymmetric coupling:

$$[\theta_X, \theta_Y] \neq 0. \tag{17}$$

Closure under local rotations preserving the Fisher norm defines a compact, simply connected group with three generators: two transverse modes and one torsional generator. This algebra is isomorphic to $\mathfrak{su}(2)$, yielding SU(2) as the minimal non-Abelian redundancy group stabilizing X/Y coherence.

b) SU(3) from Triple-Channel Redundancy.: Including the Z (compressive) sector, redundancy closure must preserve:

- total coherence density,
- null transport in X,
- torsional balance in Y,
- compressive stability in Z.

The minimal simple Lie group acting transitively on three coupled complex coherence channels under these constraints is SU(3). Larger groups introduce surplus generators that destabilize rank coherence.

F. Explicit Failure Mode: Rank Collapse Beyond Three Spatial Curvatures

Assume four independent spatial Fisher-curvature directions $\{\theta_i\}_{i=1}^4$. The integrated action scales as

$$\mathcal{A} \sim \rho_{\mathrm{coh}}\, L^4. \tag{18}$$

Stationary-phase coherence requires $\mathcal{A} \sim S_*$, implying

$$L \sim \left(\frac{S_*}{\rho_{\rm coh}}\right)^{1/4}. \tag{19}$$

This scale is incompatible with null-sector transport and torsional closure: curvature eigenvalues split, coherence ruptures, and one direction dynamically collapses. The system relaxes to three spatial-like curvature directions plus a null ordering direction.

This explicit instability demonstrates that the observed dimensionality and gauge structure are attractors of the CTMT dynamics, not arbitrary choices.

G. *Non-Arbitrariness and Falsifiability*

The framework satisfies the following constraints:

1) invariant tensor structure precedes interpretation;
2) no adjustable dimensionless parameters are introduced;
3) constants appear only as rigidity thresholds;
4) explicit instability modes (rank collapse, coherence loss) are identified.

Violation of any constraint destroys the structural correspondence and serves to falsify the construction.

References

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