

# Constants and $\pi$ -Factors in Radiative Physics:

Exact SI Anchors, Planck's Law, and CTMT-Compatible Provenance

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## Abstract

We present a self-contained derivation of the fundamental constants and  $\pi$ -factors appearing in classical radiative physics, starting exclusively from the exact 2019 SI definitions of  $h$ ,  $k_B$ , and  $c$ . From Planck's law, we derive the Stefan–Boltzmann constant, the radiation constant, and the Wien displacement constants for both wavelength and frequency representations. Special attention is given to the precise mathematical origin of all  $\pi$ -factors, including angular measures, mode counting, and thermal integrals involving the Riemann  $\zeta$ -function. All numerical values are reproduced directly from exact SI anchors using high-precision arithmetic, without empirical fitting. The presentation is compatible with Chronotopic Metric Theory (CTMT) reductions, where classical radiative results emerge as uniform-coherence limits of information-geometric transport.

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## 1 Exact SI Anchors

Following the 2019 redefinition of the International System of Units, the following constants are exact by definition:

$$h = 6.626\,070\,15 \times 10^{-34} \text{ J s}, \quad k_B = 1.380\,649 \times 10^{-23} \text{ J K}^{-1}, \quad c = 299\,792\,458 \text{ m s}^{-1}.$$

The Planck constant  $h$  and Boltzmann constant  $k_B$  are fixed numerically, while the speed of light  $c$  is fixed by the definition of the metre. These definitions eliminate experimental uncertainty from all derived constants considered in this work.

*Remark 1.1.* Throughout, SI base units are used. Angles are measured in radians and are dimensionless.

## 2 Planck's Law and Total Radiant Exitance

The spectral radiance of an ideal black body at temperature  $T$  is given, in frequency representation, by

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_B T)} - 1},$$

and equivalently, in wavelength representation, by

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_B T)} - 1}.$$

The total radiant exitance  $M(T)$  is obtained by integrating the spectral radiance over all frequencies and over the outward hemisphere. The angular integration contributes a factor of  $\pi$  via Lambert's cosine law, yielding

$$M(T) = \pi \int_0^\infty B_\nu(\nu, T) d\nu.$$

## 3 The $\pi^4/15$ Integral and the Stefan–Boltzmann Constant

Introducing the dimensionless variable  $x = h\nu/(k_B T)$ , the frequency integral reduces to

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \Gamma(4) \zeta(4) = \frac{\pi^4}{15},$$

where  $\Gamma$  is the Gamma function and  $\zeta$  the Riemann zeta function. Combining this result with the angular factor yields the Stefan–Boltzmann law

$$M(T) = \sigma T^4, \quad \sigma = \frac{2\pi^5 k_B^4}{15h^3 c^2}.$$

**Numerical value.** Using exact SI anchors,

$$\sigma = 5.670\,374\,419\,184\,429 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4},$$

in agreement with CODATA tabulations (rounded representation).

## 4 Radiation Energy Density

The radiation energy density  $u(T)$  inside a blackbody cavity is

$$u(T) = aT^4, \quad a = \frac{4\sigma}{c}.$$

From the value of  $\sigma$  above,

$$a = 7.565\,733\,250\,28 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4},$$

with rounding shown.

## 5 Wien Displacement Laws

### 5.1 Wavelength peak

Maximization of  $B_\lambda(\lambda, T)$  leads to the transcendental equation

$$5(1 - e^{-x_\lambda}) = x_\lambda, \quad x_\lambda = \frac{hc}{\lambda_{\max} k_B T}.$$

The unique positive solution is

$$x_\lambda \approx 4.965114231744 \dots,$$

yielding the Wien displacement constant

$$b_\lambda = \frac{hc}{k_B x_\lambda} = 2.897\,771\,955\,185\,173 \times 10^{-3} \text{ m K}.$$

### 5.2 Frequency peak

Maximization of  $B_\nu(\nu, T)$  yields

$$3(1 - e^{-y_\nu}) = y_\nu, \quad y_\nu \approx 2.821439372122 \dots,$$

so that

$$\nu_{\max} = \frac{k_B T}{h} y_\nu.$$

The difference between wavelength and frequency peaks reflects the non-linear relation  $\nu = c/\lambda$  and is not a physical inconsistency.

## 6 Structural Origin of $\pi$ -Factors

All  $\pi$ -factors appearing in radiative physics originate from one of three sources:

1. angular integration over rotationally invariant measures;
2. Fourier duality between conjugate variables;
3. thermal mode sums yielding  $\zeta$ -function values.

No  $\pi$  arises from arbitrary convention; each reflects an underlying symmetry or measure choice.

## 7 Relation to CTMT Reductions

In Chronotopic Metric Theory (CTMT), geometry and transport are induced by information curvature rather than postulated background structure. When CTMT is reduced to classical radiative thermodynamics under uniform coherence, the constants and relations derived above re-emerge without modification. All  $\pi$ -factors coincide with their textbook values because they arise from measure-theoretic and spectral properties preserved under CTMT reduction.

## Fisher Rank and Thermal Fixed Points

In CTMT, the exponent of the dimensionless thermal variable in the Planck integral reflects the effective Fisher rank of the transport manifold. For blackbody radiation, convergence of the total radiant exitance requires rank stabilization at four effective degrees of freedom, yielding an  $x^3$  spectral weight.

Lower-rank configurations fail to transport sufficient entropy, while higher-rank configurations are dynamically unstable under coherence loss. The Stefan–Boltzmann law thus corresponds to a rank–4 information fixed point, with the  $\pi^4/15$  factor arising as the unique stable integral measure under isotropic Fisher transport.

*Law 1* (CTMT Radiative–Coherence Law). Assume:

[(i)]isotropy and stationarity of the radiative field, yielding a Lambertian hemispherical angular measure; bosonic thermal occupancy with a finite, flat Fisher information block (uniform coherence and full rank); a declared Fourier convention for temporal and spectral variables; and exact SI anchors ( $h, k_B, c$ ).

Then the equilibrium spectral radiance is

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_B T)} - 1},$$

and the total radiant exitance obeys the Stefan–Boltzmann law

$$M(T) = \sigma T^4, \quad \sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2}.$$

The radiation energy density is

$$u(T) = aT^4, \quad a = \frac{4\sigma}{c},$$

and the Wien displacement constants are

$$b_\lambda = \frac{hc}{k_B x_\lambda}, \quad x_\lambda \simeq 4.965114, \quad \nu_{\max} = \frac{k_B T}{h} y_\nu, \quad y_\nu \simeq 2.821439.$$

All  $\pi$ -factors arise uniquely from: (i) the hemispherical angular measure ( $\pi$ ), (ii) the convergent thermal integral  $\int_0^\infty \frac{x^3}{e^x - 1} dx = \Gamma(4)\zeta(4) = \pi^4/15$ , and (iii) the chosen Fourier normalization ( $2\pi$ ).

## 8 Reproducibility

All numerical values were computed using arbitrary-precision arithmetic from exact SI definitions. A minimal Python listing is provided below.

```
import mpmath as mp
mp.mp.dps = 50

c = mp.mpf('299792458')
h = mp.mpf('6.62607015e-34')
kB = mp.mpf('1.380649e-23')

sigma = 2*mp.pi**5 * kB**4 / (15*h**3*c**2)
a = 4*sigma/c

f = lambda x: 5*(1-mp.e**(-x)) - x
x_lambda = mp.findroot(f, 5)
```

```

b_lambda = (h*c)/(kB*x_lambda)

fnu = lambda y: 3*(1-mp.e**(-y)) - y
y_freq = mp.findroot(fnu, 3)

```

## 9 Conclusion

Starting solely from exact SI definitions, we have derived the principal constants of radiative physics and documented the precise mathematical origin of every  $\pi$ -factor involved. The results reproduce standard values exactly and provide a transparent provenance compatible with information-geometric formulations such as CTMT. This work may serve as a reference for both metrological consistency and theoretical reductions.

## References

- [1] NIST CODATA, Planck constant.
- [2] NIST CODATA, Boltzmann constant.
- [3] M. Planck, Ann. Phys. (1901).