CTMT: A Universal Causal Transport Law for Energy

Unifying Quantum, Classical, Thermal, Mechanical, Radiative, Chemical, Biological, and Gravitational Regimes

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Abstract

We formulate and analyze a *universal causal transport law* for energy central to the Chronotopic Metric Theory (CTMT): for spacetime point $x = (\boldsymbol{x}, \tau)$, the local energy (or power) density is a causal convolution of a source $\mathcal{S}$ with a transport kernel $\mathcal{T}$ over the invariant measure:

$$p(x) = \int_{\mathcal{M}} \mathcal{T}(x, x')\, \mathcal{S}(x')\, d\mu(x') \, .$$

We show that standard models in heat conduction, continuum mechanics, radiative transfer, open quantum systems, quantum field theory, chemical/biological energetics, and gravitation are *instances* of this structure when their kernels are the appropriate (retarded) Green functions or Lindbladian propagators. We prove (i) existence and uniqueness of causal kernels for linear, causal PDEs (via Duhamel/Green methods); (ii) a quantum-to-classical diffusion limit under strong decoherence; and (iii) the *emergence of mass–energy equivalence* $E = mc^2$ from CTMT under Lorentz covariance, local energy–momentum conservation, and dimensional closure.

Contents

1 Statement of the Universal Energy Transport Law

Let $\mathcal{M}$ denote spacetime with invariant measure $d\mu(x')$ and proper time τ.

Definition 1.1 (CTMT Universal Energy Law)**.** For a physical system with source density $\mathcal{S}(x')$ the local energy (or power) density $p(x)$ is given by

$$p(x) = \int_{\mathcal{M}} \mathcal{T}(x, x')\, \mathcal{S}(x')\, d\mu(x')\ , \tag{1}$$

where the transport kernel $\mathcal{T}$ obeys strict causal support $\mathcal{T}(x, x') = 0$ for $\tau < \tau'$.

Dimensional Closure (CTMT axiom). *All* measurement protocols and unit choices are fixed so that $\mathcal{S}$ carries units of energy (or power) density, $\mathcal{T}$ carries the inverse spacetime units required for closure, and p has energy or power density units in every regime. No extra postulates are required beyond causality, conservation and dimensional consistency.

Remark 1.2 (Conservation)*.* For closed conservative transport one may impose the normalization $\int_{\Sigma_\tau} \mathcal{T}(x, x')\, d^3\boldsymbol{x} = 1$, which is exactly satisfied for probability-like diffusion kernels and approximated for energy transport when there are no sinks beyond $\mathcal{S}$; in field theories, $\mathcal{T}$ instead solves a causal Green function problem associated with the governing operator.

2 Source Burn, Dissipation, and Medium Propagation

The universal energy transport law separates naturally into three structurally distinct components: source activation, transport through a medium, and dissipative attenuation. These components were implicit in earlier formulations; here we make them explicit.

2.1 Burn Factor as Source Activation

Let $\mathcal{S}_0(x)$ denote the raw energy availability (chemical, nuclear, mechanical, biochemical, etc.). Not all available energy is injected into the transport channel. We therefore define a burn factor $B(x) \in [0, 1]$ such that

$$\mathcal{S}(x) = B(x)\,\mathcal{S}_0(x).$$

The burn factor encodes reaction completeness, duty cycle, or activation efficiency. It is a *source-side* quantity and must not depend on future states or transport outcomes, ensuring causal admissibility.

Typical examples include fuel burnup in combustion, metabolic efficiency in biological systems, or conversion efficiency in power electronics.

2.2 Dissipation and Attenuation (Fisk Factor)

Energy transport through real systems is attenuated by scattering, friction, decoherence, viscosity, or radiative loss. These effects are encoded as an exponential or multiplicative damping of the transport kernel:

$$\mathcal{T}(x, x') \;\rightarrow\; \mathcal{T}(x, x')\,\exp[-\Phi(x, x')],$$

where $\Phi(x, x') \geq 0$ is an integrated dissipation functional.

This term generalizes frictional loss, optical depth, decoherence rate, or viscous damping. It acts *only* on the kernel amplitude and never on the source, preserving Fisher rank stability and window consistency.

2.3 Medium-Dependent Propagation

The geometry and support of the transport kernel depend on the properties of the intervening medium, represented collectively as $\mathcal{M}(x)$ (e.g. density, conductivity, refractive index, elastic moduli).

Accordingly, the kernel is written as

$$\mathcal{T}(x, x'; \mathcal{M})$$

with causal support restricted by the propagation speed and coherence structure induced by $\mathcal{M}$. In diffusive media this yields Gaussian kernels; in ballistic or wave-like regimes, oscillatory or retarded kernels arise.

2.4 Combined Universal Law

With these components made explicit, the admissible energy transport law takes the form

$$\rho(x) = \int \mathcal{T}(x, x'; \mathcal{M})\,\exp[-\Phi(x, x')]\,B(x')\,\mathcal{S}_0(x')\,d\mu(x').$$

Each factor has a distinct causal and physical role:

- $\mathcal{S}_0$ supplies available energy,
- B activates or consumes it,
- $\mathcal{T}$ propagates it through the medium,
- Φ attenuates it via dissipation,
- ρ is the resulting stored or observable energy density.

This separation is required for dimensional closure, Fisher rigidity, and causal ordering, and recovers all standard transport laws as special cases.

3 Kernel Existence via Causal Green Functions

Theorem 3.1 (Causal Kernel Representation for Linear, Causal PDEs)**.** *Consider a linear, time-translation invariant operator $\mathcal{L}$ on fields $\phi(x)$ with causal evolution and appropriate growth conditions. Then for the inhomogeneous problem $\mathcal{L}\phi = \mathcal{S}$ there exists a unique* retarded Green function G^R *such that*

$$\phi(x) = \int_{\mathcal{M}} G^R(x, x')\, \mathcal{S}(x')\, d\mu(x')\ , \qquad G^R(x, x') = 0 \textit{ for } \tau < \tau'. \tag{2}$$

Consequently, energy or power densities $p(x)$ depending linearly on ϕ take the CTMT form (1) *with $\mathcal{T}$ a (possibly composed) retarded kernel.*

Sketch. By standard Green/Duhamel constructions for diffusion, wave, Poisson/Helmholtz, and related operators one constructs causal G^R satisfying $\mathcal{L}G^R = \delta$. Convolution yields the unique causal solution under the given growth/initial data. □

4 Canonical Realizations of the CTMT Kernel

4.1 Thermal Systems (Heat Conduction)

For thermal diffusivity $\alpha = k/(\rho c_p)$, the inhomogeneous heat equation $\partial_\tau T - \alpha\nabla^2 T = \frac{1}{\rho c_p} p$ has the causal heat kernel $G_{\text{heat}}(\boldsymbol{r}, \Delta\tau) = (4\pi\alpha\Delta\tau)^{-3/2} \exp\left[-|\boldsymbol{r}|^2/(4\alpha\Delta\tau)\right]$ on $\mathbb{R}^3$, giving $T = \frac{1}{\rho c_p} G_{\text{heat}} * p$. This is the diffusive kernel often appearing as the classical limit of quantum transport (Sec. 5.3).

4.2 Mechanical Dissipation (Continuum Mechanics)

In small-strain viscoelastic/elastic media, *power density* is $p = \sigma_{ij}\dot{\epsilon}_{ij}$ and momentum balance reads $\rho\, \ddot{u}_i = \nabla_j \sigma_{ij} + f_i$. The displacement u solves a (possibly damped) wave/elliptic PDE with causal G^R_{elast}, hence $u = G^R_{\text{elast}} * f$ and p follows from constitutive laws.

4.3 Radiative Transfer

The radiative energy deposition is $p(\boldsymbol{x}, \tau) = \int_0^\infty \int_{4\pi} I_\nu(\boldsymbol{x}, \hat{\Omega}, \nu, \tau)\, \kappa_\nu(\boldsymbol{x}, \nu)\, d\hat{\Omega}\, d\nu$, with transport governed by the (linear) radiative transfer equation whose integral solutions are kernel forms (discrete ordinates/invariance principles).

4.4 Chemical & Biological Systems

At mesoscopic scales, $p(\boldsymbol{x}, \tau) = \sum_j v_j\, \Delta G_j$, where fluxes v_j propagate by diffusion–advection with kernel G^R_{chem}; compartmental exchange appears as a matrix of causal kernels. (This matches nonequilibrium thermodynamics/linear response when near equilibrium; see Sec. 7.)

5 Quantum Systems and Quantum–Classical Transition

5.1 Open Quantum Dynamics (GKSL form)

For Markovian open quantum systems, density operator $\hat{\rho}$ evolves by the GKSL/Lindblad equation

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \sum_k \Big(\hat{L}_k \hat{\rho} \hat{L}_k^\dagger - \tfrac{1}{2}\{\hat{L}_k^\dagger \hat{L}_k, \hat{\rho}\}\Big), \tag{3}$$

which is the general generator of completely positive trace-preserving semigroups. Local quantum energy density is $p(\boldsymbol{x}, \tau) = \mathrm{Tr}[\hat{\rho}(\tau)\, \hat{\mathcal{H}}(\boldsymbol{x})]$.

5.2 Retarded Green/Keldysh Structure in QFT

Real-time transport and causal response use the *retarded* propagator G^R (or Schwinger–Keldysh formalism), not the time-symmetric Feynman propagator, when enforcing strict causality. In linear response, observables obey Kubo-type relations with G^R.

5.3 Quantum → Classical Diffusion Limit

Under strong decoherence or weak coupling to a broad thermal bath (e.g., Caldeira–Leggett class) the reduced dynamics exhibits momentum diffusion and friction; after coarse-graining, spatial probability and energy flows converge to *classical diffusion* with kernel

$$\mathcal{T}_{\text{diffusive}}(\boldsymbol{r}, \Delta\tau) = \frac{1}{(4\pi\alpha\Delta\tau)^{3/2}} \exp\Big(- \frac{|\boldsymbol{r}|^2}{4\alpha\Delta\tau} \Big).$$

This provides a CTMT bridge:

$$\text{QFT} \longrightarrow \text{QM (open)} \longrightarrow \text{thermal diffusion.}$$

6 Relativistic Fields and Gravitation

6.1 Stress–Energy and Local Conservation

In relativistic field theory, the stress–energy tensor $T^{\mu\nu}$ encodes energy/momentum densities/fluxes; local covariant conservation reads $\nabla_\mu T^{\mu\nu} = 0$. Global conserved charges require Killing symmetries; otherwise only local conservation holds.

6.2 Causal Kernels: Retarded Propagators

Relativistic transport of fields (scalar, Dirac, gauge) uses *Lorentz-covariant retarded* propagators $G^R(x - x')$ as kernels; gauge interactions couple by $D_\mu = \partial_\mu + \frac{iq}{\hbar} A_\mu$.

6.3 Newtonian and Linearized Gravity

In the Newtonian limit, the Poisson kernel $G(\boldsymbol{r}) \propto 1/|\boldsymbol{r}|$ is the causal Green function for instantaneous potential theory; in GR, gravitational radiation and linearized perturbations propagate with *retarded* kernels on the light cone, consistent with $\nabla_\mu T^{\mu\nu} = 0$.

7 Noether Symmetry, Kubo Response, and CTMT

7.1 Noether's Theorem

Time-translation invariance ⇒ energy conservation, space-translation invariance ⇒ momentum conservation, rotations ⇒ angular momentum—grounding the CTMT conservation axiom in variational mechanics and field theory.

7.2 Linear Response and Transport Kernels

Kubo response relates causal susceptibilities to equilibrium correlation functions; the retarded response function is (schematically) $\chi_{AB}(t) = \frac{i}{\hbar}\Theta(t)\,\langle[A(t), B(0)]\rangle$, which is exactly the kernel mapping sources to observables in the CTMT sense.

8 Emergence of $E = mc^2$ from CTMT

8.1 Setup: Lorentz Covariance, Local Conservation, Dimensional Closure

Assume (i) Lorentz-covariant transport kernels $\mathcal{T}$ (hence p transforms as T^{00} in local inertial frames), (ii) local conservation $\nabla_\mu T^{\mu\nu} = 0$, and (iii) dimensional closure so that the measured p integrates to total energy E in the rest frame.

Proposition 8.1 (Energy–Momentum 4-Vector and Rest Energy)**.** *Let $P^\mu = (E/c, \boldsymbol{P})$ be the total 4-momentum obtained by integrating $T^{\mu\nu}$ over a spacelike hypersurface. Lorentz invariance implies $P^\mu P_\mu = m^2c^2$ for an isolated system of invariant mass m. In the rest frame ($\boldsymbol{P} = 0$) one has $E = mc^2$.*

Sketch. By local conservation and Gauss' theorem, P^μ is hypersurface-independent in Minkowski spacetime. The invariant $P^\mu P_\mu$ equals m^2c^2 by definition of invariant mass. In the rest frame, $P^\mu = (E/c, \mathbf{0})$, so $E^2/c^2 = m^2c^2 \Rightarrow E = mc^2$ (choosing $E > 0$). In CTMT, $E = \int p \, d^3\boldsymbol{x}$ with p computed by a causal kernel consistent with Lorentz covariance, so the result is measurement-protocol (unit) closed. □

Theorem 8.2 (CTMT Limit Emergence of $E = mc^2$)**.** *In any CTMT realization whose transport kernel $\mathcal{T}$ is Lorentz covariant and whose energy density $p(x)$ integrates to a rest-frame total invariant E, the low-momentum limit ($\boldsymbol{P} \to \mathbf{0}$) implies $E = mc^2$ for the system's invariant mass m.*

Sketch. Combine: (a) CTMT delivers a scalar rest energy by dimensional closure $E = \int p \, d^3\boldsymbol{x}$; (b) local conservation yields a conserved 4-momentum; (c) Lorentz covariance yields the invariant mass via $P^\mu P_\mu$. In the rest frame $P^i = 0$, hence $E = mc^2$. This proof does not depend on microscopic details: whether p arises from fields (QFT retarded kernels), matter (continuum mechanics kernels), radiation (radiative transfer kernels), or chemical/biological power densities, the same structure applies. □

Remarks on Einstein's 1905 derivation and modern rigor. Einstein's original argument used radiation emission and energy balance. Modern derivations rely on stress–energy and Lorentz symmetry, avoiding circularity critiques; the CTMT route is aligned with the latter.

9 Worked Examples as CTMT Instances

9.1 Heat Deposition by a Pulsed Source

Given $p(\boldsymbol{x}, \tau) = P_0 \, \delta(\boldsymbol{x} - \boldsymbol{x}_0)\delta(\tau - \tau_0)$, $T(\boldsymbol{x}, \tau) = \frac{P_0}{\rho c_p} G_{\text{heat}}(\boldsymbol{x} - \boldsymbol{x}_0, \tau - \tau_0)$ for $\tau > \tau_0$.

9.2 Linear Viscoelastic Damping

With Kelvin–Voigt stress $\sigma = 2\mu \, \epsilon + 2\eta \, \dot{\epsilon}$, the displacement u obeys a damped wave equation with causal G^R_{elast}; the instantaneous dissipation density $p = \sigma : \dot{\epsilon}$ follows.

9.3 Radiative Heating in an Absorbing Medium

Solve the transfer equation along characteristics; integral solutions give I_ν and $p = \int I_\nu \kappa_\nu$.

9.4 Quantum Diffusion from Decoherence

Starting with a particle linearly coupled to a harmonic bath (Caldeira–Leggett), the reduced Wigner function satisfies a Fokker–Planck-type equation whose long-time limit is diffusive with kernel G_{heat}.

10 CTMT Core Machinery and Universality of the Energy Transport Kernel

The universal transport law (1) is not an independent postulate appended to the CTMT framework. Rather, it emerges as the *unique admissible structure* compatible with the geometric, statistical, and causal constraints imposed by coherent inference. This section makes that connection explicit and shows that the kernel form of energy transport is a rigidity result rather than an assumption.

10.1 Fisher Geometry and Rigidity of Transport

Let θ denote the parameters of a candidate transport kernel $\mathcal{T}_\theta$. For each localized data window—defined as a finite subset of observations over which the inverse problem is solved independently—the induced likelihood geometry defines a Fisher information matrix

$$F(\theta) = J(\theta)^\top C^{-1} J(\theta),$$

where J is the Jacobian of observables with respect to θ and C is the observational covariance.

Admissibility of a transport model requires the following conditions:

(i) *Fisher rank stability*: $\operatorname{rank} F(\theta)$ is constant across windows;

(ii) *Bounded coherence density*:
$$\rho_{\text{coh}} = \frac{\operatorname{Tr} F}{V(\Omega)} < \infty;$$

(iii) *Drift decay*: parameter updates satisfy $\|\theta_{k+1} - \theta_k\| \to 0$ under chained inversion.

When these conditions hold, the parameter trajectory collapses to a *rigid limit*, in which $\mathcal{T}_\theta$ becomes unique up to gauge-like reparameterizations that leave observables invariant. In this limit, the only admissible transport structure is a causal convolution of the form (1).

10.2 Coherence Proper Time and Causal Ordering

Coherent inference induces a natural ordering through the *coherence proper time*

$$\tau_{\text{coh}}(t) = \int_0^t \lambda_{\max}(F(t'))\, dt',$$

where $\lambda_{\max}$ is the largest eigenvalue of the Fisher matrix. This quantity measures accumulated inferential curvature.

Admissible kernels are precisely those for which τ_{coh} is monotone and finite. Any transport kernel with support for $\tau < \tau'$ would induce unbounded Fisher curvature, violating the bounded coherence density condition. Consequently, coherence geometry selects causal support as the only stable configuration:

$$\mathcal{T}(x, x') = 0 \quad \text{for} \quad \tau < \tau'.$$

Thus, causal ordering is not imposed externally but arises as a stability requirement of coherent inference.

10.3 Dimensional Closure and Energy Units

Dimensional consistency is enforced by the requirement that all observable channels remain unit-consistent under admissible reparameterizations. For energy transport this implies

$$[\mathcal{S}] = \text{energy density}, \qquad [\mathcal{T}] = [d\mu]^{-1}, \qquad [p] = \text{energy density}.$$

Here $\mathcal{S}(x)$ denotes an energy source density *(energy injected per unit volume per unit coherence proper time), whereas $\rho(x)$ denotes the resulting energy* state density*. Both share compatible dimensions after time integration, but play distinct causal roles in the transport map.*

Any kernel violating this closure induces Fisher rank collapse or persistent parameter drift and is therefore rejected. The dimensional structure of (1) is thus not optional but required by admissibility.

10.4 Window Consistency and Locality of Response

Window consistency requires that residual means and scales remain statistically compatible across overlapping data windows. For transport processes, this implies that the response at x depends only on source terms within its causal past, with weights that remain stable across windows. This locality condition is exactly the structure encoded by the convolutional form (1).

10.5 Rupture Exclusion and Smoothness of Transport

Non-smooth transport laws are detected by the rupture statistic R, defined as a normalized measure of excess curvature in second differences of residual trajectories across windows. Transport kernels with non-causal or discontinuous propagation produce $R \gtrsim 3.5$ and are rejected. Admissible kernels must therefore be both smooth and causally regular.

10.6 Relation to Known Transport Laws

Classical transport equations arise as admissible specializations of the universal kernel under additional structural assumptions. Diffusion corresponds to isotropic kernels with exponential decay; wave propagation corresponds to oscillatory kernels with finite-speed support; and Hamiltonian transport corresponds to unitary kernels with rank-preserving Fisher geometry. The present framework does not replace these laws but explains their validity as rigid limits of the same underlying transport structure.

10.7 Theorem: Forcing of the Universal Transport Law

Theorem 10.1. *Any transport kernel $\mathcal{T}$ satisfying admissibility—causality, dimensional closure, Fisher rank stability, bounded coherence density, window consistency, drift decay, and rupture exclusion—must take the form*

$$p(x) = \int_{\mathcal{M}} \mathcal{T}(x, x')\, \mathcal{S}(x')\, d\mu(x'),$$

with $\mathcal{T}(x, x') = 0$ for $\tau < \tau'$. All physically admissible energy transport laws arise as rigid-limit specializations of this structure.

Remark 10.2. The universal energy transport law is therefore not an assumption but a geometric necessity: admissibility constraints eliminate all other structures.

11 Interpretation and Outlook

CTMT asserts: *energy is neither local nor instantaneous; it is transported causally via kernels determined by symmetries and degrees of freedom.* Quantum coherence, diffusion, dissipation, radiation, chemistry/biology, and gravitation are not separate energy laws but *realizations* of one causal transport structure—distinguished only by the kernel.

Acknowledgments

The author emphasizes CTMT's "dimensional closure" principle across all regimes, i.e., measurement protocols and unit choices close the ontology without regime-specific patches.

References and Notes

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