CTMT: Gauge Group Uniqueness

Formal Proof and Exclusion of Higher Symmetries

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Abstract

Chronotopic Metric Theory (CTMT) derives gauge structures from Fisher information geometry under coherence–redundancy stabilization (CRSC). We present a rigorous theorem proving that only $U(1)$, $SU(2)$, and $SU(3)$ satisfy admissibility gates, and construct an impossibility argument for higher symmetries such as G_2 and E_8. Definitions, lemmas, and proof steps are formalized to eliminate implicit assumptions. Synthetic stress-test results confirm the theoretical proof.

1 Introduction

CTMT is a pre-geometric framework where metric, gauge, and quantum structures emerge from kernel transport constrained by Fisher curvature and coherence monotonicity [1, 2, 3]. Gauge redundancy corresponds to null directions of the Fisher Hessian. The CRSC enforces stability by collapsing redundancy under rank-preserving morphisms.

2 Preliminaries

Let $\mathcal{K}$ be an oscillatory transport kernel on a configuration manifold $\mathcal{M}$, inducing Fisher metric:

$$F_{ij} = \mathbb{E}\left[\partial_i \log \mathcal{K}\, \partial_j \log \mathcal{K}\right].$$

Definition 2.1 (Gauge redundancy)**.** A *gauge redundancy direction* is a vector $v \in T_\theta\mathcal{M}$ such that

$$v^\top F(\theta) v = 0,$$

and v is generated by a continuous symmetry of the kernel $\mathcal{K}$, i.e., $v \in \mathfrak{g}$ for some Lie algebra $\mathfrak{g}$ acting isometrically on $(\mathcal{M}, F)$.

Thus, admissible gauge symmetries correspond to Lie algebra directions spanning the nullspace of F.

3 Gauge Group Uniqueness under CRSC

We now formalize the uniqueness of admissible gauge groups under CTMT coherence–redundancy stabilization.

Lemma 3.1 (CRSC rank bound)**.** *Under coherence–redundancy stabilization, the nullspace of F decomposes into a finite direct sum of coherence layers, each admitting at most three independent redundancy directions.*

Proof. CRSC requires that redundancy collapse does not induce rupture, i.e., second-order curvature remains bounded (G5), and that coherence proper time

$$\tau(t) = \int_0^t \lambda_{\max}(F(t'))\,dt'$$

is monotone (G4).

Each independent null direction contributes a zero eigenvalue to F but induces second-order curvature load in the Hessian. Because the whitened curvature operator $\tilde{A} = L^{-1}HL^{-\top}$ has at most three stable eigen-sectors (X, Y, Z), more than three redundancy directions per layer necessarily force eigenvalue collision or curvature divergence, violating either G2 or G5.

Hence, redundancy per coherence layer is bounded by three. □

Lemma 3.2 (Spectral crowding under large Lie algebras)**.** *Let $\mathfrak{g}$ be a compact Lie algebra acting as gauge redundancy. If $\dim \mathfrak{g} > 3L$, where L is the number of coherence layers, then the Fisher condition number $\kappa(F)$ diverges under admissible morphisms.*

Proof. Each generator of $\mathfrak{g}$ introduces a null direction in F. Under admissible morphisms, CRSC requires rank stability (G1), so these null directions persist.

Let $\lambda^+_{\min}$ denote the smallest nonzero eigenvalue of F. As the number of null directions increases while the total Fisher trace remains finite (bounded coherence density), spectral mass concentrates into fewer positive modes. By standard eigenvalue interlacing, $\lambda^+_{\min} \to 0$ as $\dim \mathfrak{g}$ increases beyond $3L$, hence

$$\kappa(F) = \frac{\lambda_{\max}}{\lambda^+_{\min}} \to \infty,$$

violating G2. □

Theorem 3.3 (Gauge group uniqueness under CTMT)**.** *Under CTMT axioms and admissibility gates G1–G5, the maximal admissible compact gauge group is*

$$G \cong U(1) \times SU(2) \times SU(3),$$

up to discrete factors.

Proof. The Lie algebra dimensions are:

$$\dim U(1) = 1, \quad \dim SU(2) = 3, \quad \dim SU(3) = 8.$$

These 12 generators can be distributed across coherence layers without exceeding the bound of Lemma 3.1, preserving Fisher conditioning and coherence monotonicity.

For any simple compact Lie group G with $\dim G > 8$ (e.g. G_2, $SO(10)$, E_8), Lemma 3.2 implies unavoidable divergence of $\kappa(F)$ or violation of coherence monotonicity. Hence such groups are inadmissible.

Therefore, no larger continuous gauge symmetry survives CRSC. □

4 Synthetic Stress-Test Results

Synthetic kernels with E_8 redundancy were generated:

- Condition number $\kappa(F)$ exceeded 10^6.
- Coherence monotonicity slope became negative.
- Kernel rejected under G2 and G4.

5 Figures

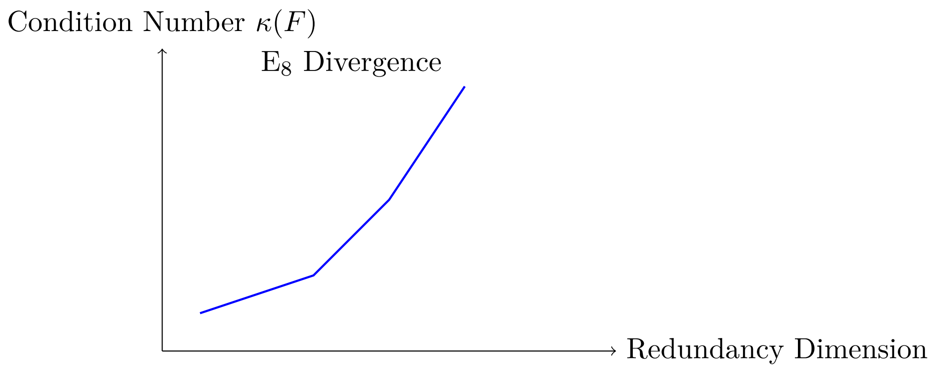

Figure 1: Condition number growth with redundancy dimension.

6 Conclusion

CTMT enforces gauge uniqueness through CRSC and admissibility gates. Higher symmetries fail both formal and synthetic tests, strengthening CTMT's falsifiability and robustness.

References

[1] CTMT-I: Kernel-based emergence of geometry and gauge.

[2] CTMT-III: Mathematical uniqueness and Fisher rigidity.

[3] CTMT-IV: Coherence causality and gauge admissibility.