Chronotopic Metric Theory II: Kernel Dynamics, Redundancy, and the Origin of Rigidity

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***Abstract*—In a companion paper, Chronotopic Metric Theory (CTMT) was formulated at an effective level, where a dimensionless rigidity functional $R = (\Phi)/S_*^2$ controls the emergence of linear quantum dynamics, classical field equations, and geometric structure. In the present work we derive this effective law from underlying kernel dynamics, without introducing new physical postulates.**

We formulate the evolution of a non-metric, non-Hilbertian kernel $\mathcal{K}$ on a configuration manifold equipped with an information-geometric structure. Redundancy directions appear as null modes of the Fisher information Hessian, while curvature modes undergo spectral flow driven by a coherence–redundancy stabilization criterion (CRSC). We show that generic kernel flows exhibit spectral collapse to low-rank configurations, and that phase fluctuations in this regime are governed by a single macroscopic control parameter whose leading-order behavior coincides with $R = (\Phi)/S_*^2$.

The Maxwell, Newtonian, Schrödinger, and Dirac limits obtained in the effective theory are recovered here as fixed points of the kernel flow. Hilbert space, gauge structure, and spacetime dimensionality thus appear as emergent features of stabilized kernel dynamics rather than fundamental assumptions.

CONTENTS

I. Introduction

The companion paper *Chronotopic Metric Theory I* introduced an effective description in which geometry, gauge structure, and quantum linearity arise once a dimensionless rigidity functional

$$R = \frac{(\Phi)}{S_*^2} \tag{1}$$

falls below a critical threshold. Here Φ is a kernel phase functional and S_* an invariant action scale.

In that formulation, (1) was taken as a phenomenological scalar control parameter. The present work removes that shortcut and derives the rigidity functional from underlying kernel dynamics. No new physical axioms are introduced; instead, we make explicit how rigidity emerges from spectral properties of kernel flow driven by redundancy and coherence constraints.

From a methodological standpoint, this paper should be read as a contribution to information-geometric dynamics rather than as a foundational replacement program. Kernel flows, spectral collapse, and rank selection are well-studied in inverse problems, learning theory, and information geometry. The novelty of CTMT lies not in the mathematical tools employed, but in the physical interpretation of stabilized low-rank kernel phases as geometric, gauge, and quantum structures.

II. Chronotopic kernel dynamics

A. Kernel domain and configuration space

Definition II.1 (Chronotopic kernel)**.** Physical configurations are encoded by a complex-valued kernel

$$: \mathrm{Anch} \times \mathrm{Topo} \times \mathbb{R} \to \mathbb{C},$$

where Anch denotes a discrete anchor set (sources, detectors, interaction loci) and Topo a purely topological neighborhood structure. No metric or dimensional structure is assumed.

Let Θ denote a (possibly infinite-dimensional) set of kernel parameters. We write

$$(\cdot;\Theta) = A(\cdot;\Theta)\, e^{i\Phi(\cdot;\Theta)},$$

with real amplitude A and phase functional Φ.

Definition II.2 (Kernel configuration manifold)**.** The admissible kernel parameters Θ form a configuration manifold , equipped with an information structure induced by observables.

Definition II.3 (Observable functionals)**.** Given a reconstruction protocol, observable quantities are functionals

$$O = O[]$$

of the kernel. Expectation values and fluctuations are computed with respect to an induced measure P on .

III. Redundancy, Fisher curvature, and CRSC

A. Redundancy operators and Fisher structure

Definition III.1 (Redundancy operators)**.** A redundancy operator R_α is a transformation on such that

$$(\cdot; R_\alpha\Theta) = (\cdot;\Theta)$$

for all observables in a given class. The index α labels independent redundancy directions.

Definition III.2 (Fisher information Hessian)**.** Given a likelihood functional $\mathcal{L}(\Theta)$ capturing the fit of kernel predictions to data, define

$$H_{ij} = \partial_i\partial_j \log \mathcal{L}(\Theta),$$

with indices i, j running over coordinates on .

Remark III.3. Null eigenvectors of H correspond to redundancy directions; non-null eigenvectors define curvature axes. The spectrum of H therefore encodes the effective number and stiffness of independent curvature directions supported by the kernel.

B. Coherence–redundancy stabilization

Definition III.4 (Coherence density and -volume)**.** Let ϕ denote phases sampled from Φ under a reconstruction protocol. The local coherence density is

$$\rho_{\mathrm{coh}} = \left|\left[e^{i\phi}\right]\right|,$$

and the global coherence measure is

$$= \int \rho_{\mathrm{coh}}\, d\mu,$$

where $d\mu$ is a topology-compatible measure on kernel support.

Definition III.5 (Redundancy rank)**.** Given a maximal independent set $\{R_\alpha\}$ of redundancy generators, the redundancy rank is

$$r = \dim \mathrm{span}\{R_\alpha\}.$$

Definition III.6 (Coherence–redundancy stabilization criterion (CRSC))**.** A kernel configuration is dynamically stable under reconstruction if

$$\frac{\partial}{\partial r} \geq 0.$$

Configurations with $\partial/\partial r < 0$ are driven away by the dynamics; redundancy then degrades, rather than enhances, coherence.

Remark III.7. CRSC plays a role analogous to stability conditions in variational and renormalization-group flows, selecting physically realized configurations from a larger kinematic space.

IV. Kernel flow and spectral collapse

A. Kernel flow equation

We model kernel evolution on via

$$\frac{d\Theta}{d\tau} = -\nabla_\Theta [] + [\Theta], \tag{2}$$

where:

- [] is a spectral functional penalizing curvature variation and loss of coherence,
- $[\Theta]$ represents bounded fluctuations arising from finite sampling, reconstruction error, or environmental coupling,
- τ is an abstract evolution parameter (not physical time).

Definition IV.1 (Spectral functional)**.** Let $\{\lambda_i\}$ be eigenvalues of $H(\Theta)$. Let $f : \mathbb{R}^+ \to \mathbb{R}$ be a smooth convex function with $f'(x) > 0$ for $x > 0$. Define

$$[] = \sum_i f(\lambda_i).$$

Proposition IV.2 (Spectral collapse)**.** *Under the flow (2) with convex f, high-rank curvature spectra are dynamically unstable: generic initial conditions are driven toward low-rank plateaus in which most λ_i either vanish or lie in a small, well-separated set of robust nonzero values.*

Sketch. Gradient flow of a convex spectral functional decreases [] by redistributing spectral weight from many comparable eigenvalues into a small set of dominant ones. Noise destabilizes small positive eigenvalues, pushing them to zero (redundancy) or into the dominant cluster. This is closely analogous to spectral concentration phenomena in low-rank matrix recovery and information geometry, where regularization and noise jointly favor low-rank representations. □

Remark IV.3. CRSC restricts admissible flows to those for which coherence is not destroyed by growing redundancy rank. The combined effect of CRSC and spectral collapse is to select a small number of curvature axes (the X/Y/Z sectors of CTMT-I) from a much larger kinematic space.

V. Emergent rigidity

A. Phase fluctuations from spectral data

Let $\Phi(\Theta)$ be the phase functional associated with $(\cdot;\Theta)$. Near a low-rank plateau of the spectrum, phase fluctuations may be expanded as

$$\Phi(\Theta) = \Phi_0 + \sum_{i\in\mathcal{I}} \delta\Phi_i(\Theta),$$

where $\mathcal{I}$ indexes eigen-directions associated with surviving curvature modes and $\delta\Phi_i$ denotes the fluctuation contribution of each mode.

Under mild regularity assumptions, a linear-response argument gives

$$(\Phi) \sim \sum_{i\in\mathcal{I}} g(\lambda_i),$$

for some decreasing function g that encodes suppression of fluctuations along stiff (large-λ) directions.

B. Microscopic rigidity and effective scalar reduction

Definition V.1 (Microscopic rigidity)**.** Define the microscopic rigidity functional

$$_{\mathrm{micro}}[] = \frac{(\Phi[])}{S_*^2},$$

where S_* is the invariant action scale of CTMT-I.

Theorem V.2 (Effective rigidity law)**.** *Under CRSC-driven spectral collapse, $_{\mathrm{micro}}$ reduces, at the level of macroscopic behavior, to the effective scalar rigidity functional*

$$R = \frac{(\Phi)}{S_*^2} \tag{3}$$

used in CTMT-I, with the same regime structure:

- $R \gg 1$*: classical averaging, destructive interference dominates;*
- $R \sim 1$*: nonlinear coherence, significant mode coupling;*
- $R \ll 1$*: rigid phase, small fluctuations and emergent linearity.*

Sketch. After spectral collapse, the set $\{\lambda_i\}_{i\in\mathcal{I}}$ is small and well-separated. The variance (Φ) therefore becomes a smooth scalar function of a few stiffness parameters. CRSC forbids flows that increase redundancy at the expense of coherence, so further evolution cannot re-populate high-rank sectors. Thus (Φ) behaves effectively as a single scalar order parameter governing the strength of phase fluctuations relative to the action scale S_*. Matching to the effective theory identifies this scalar with R. □

Remark V.3. Rigidity is therefore not an independent postulate but a macroscopic summary of how phase fluctuations are constrained by low-rank kernel dynamics.

VI. Recovery of CTMT-I limits

We now indicate how the principal limits of CTMT-I arise as fixed points of the kernel flow (2) in the rigid regime $R \ll 1$.

A. Maxwell fixed point

Consider a configuration in which spectral collapse has left a single dominant null curvature sector (X-axis),

$$\lambda_X \approx 0, \qquad \lambda_Y, \lambda_Z > 0,$$

and let the effective phase along this sector be

$$\Phi(x,t) = \omega t - kx$$

in an emergent coordinate chart. Define

$$A_\mu = \partial_\mu \Phi, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Proposition VI.1 (Null-sector Maxwell fixed point)**.** *In the rigid regime $R \ll 1$, kernel flow restricted to the null X-sector admits fixed points for which:*

$$\partial_\nu F^{\mu\nu} = J^\mu, \qquad \partial_\mu J^\mu = 0,$$

with J^μ the coherence current. These equations coincide with Maxwell's equations in an emergent spacetime description.

Sketch. CRSC and spectral collapse enforce coherence-preserving transport along the null direction, leading to a Lorenz-type condition and a hyperbolic equation for Φ. Quadratic functionals of $F_{\mu\nu}$ are the only local, gauge-invariant functionals compatible with this structure, and their Euler–Lagrange equations yield the Maxwell form. □

B. Newtonian fixed point

For static configurations with negligible temporal variation and non-null curvature in spatial directions, kernel flow minimizes an effective functional of the form

$$\int \left[(\nabla\Phi)^2 - \alpha\rho_{\mathrm{coh}}\Phi\right] d^3x,$$

subject to CRSC.

Proposition VI.2 (Poisson fixed point)**.** *Static fixed points of the kernel flow satisfy*

$$\nabla^2\Phi = \alpha\rho_{\mathrm{coh}},$$

reproducing the Poisson equation of CTMT-I in the weak-field limit, with ρ_{coh} identified structurally with mass–energy density.

Sketch. Stationarity of the spectral functional under fixed ρ_{coh} gives the Euler–Lagrange equation for Φ, which takes Laplacian form. Dimensional analysis and the role of ρ_{coh} in curvature variations identify it with mass–energy density in the Newtonian limit. □

C. Schrödinger fixed point

Let $O[]$ be an observable functional and consider the kernel expectation

$$\langle O\rangle = \int \Xi(\Theta)\, e^{i\Phi(\Theta)/S_*}\, \mathcal{D}\Theta,$$

as in CTMT-I. Under kernel flow, $\Theta(\tau)$ evolves by (2), and $\Phi(\Theta(\tau))$ becomes a stochastic process on .

In the rigid regime,

$$(\Phi) \ll S_*^2,$$

so that

$$e^{i\Phi/S_*} \approx 1 + \frac{i}{S_*}\Phi$$

to leading order.

Proposition VI.3 (Rigid-phase linearization)**.** *In the limit $(\Phi)/S_*^2 \to 0$, the kernel expectation obeys*

$$\partial_t\langle O\rangle = \frac{i}{S_*}\langle[\hat{H}, O]\rangle,$$

where $\hat{H}$ is the generator of phase flow on .

Sketch. The leading-order expansion of $e^{i\Phi/S_*}$ converts the kernel expectation into a linear functional of Φ. Time translation is generated by kernel flow acting on Θ, which induces a linear generator $\hat{H}$ on observables. The commutator structure follows from interpreting the infinitesimal action of $\hat{H}$ as an inner derivation on the observable algebra. □

Corollary VI.4 (Schrödinger fixed point)**.** *When $\hat{H}$ is self-adjoint with respect to the flow-induced inner product on kernel amplitudes, the evolution reduces to*

$$i\hbar\,\partial_t\psi = \hat{H}\psi$$

with S_* identified with $\hbar$ in the rigid regime. This reproduces the Schrödinger dynamics derived phenomenologically in CTMT-I.

D. Dirac fixed point

When two curvature sectors (X and Y) survive spectral collapse with a torsional coupling, the kernel flow induces coupled phase evolution of the form

$$\partial_t \begin{pmatrix}\Phi_X\\ \Phi_Y\end{pmatrix} = \mathbf{M}\begin{pmatrix}\Phi_X\\ \Phi_Y\end{pmatrix} + \cdots,$$

where $\mathbf{M}$ has an antisymmetric component encoding torsional exchange. Defining a complex amplitude $\psi = \Phi_X + i\Phi_Y$ and passing again to the rigid limit, the effective evolution acquires a Clifford-like structure and reduces to a Dirac-type equation as in the deconstructed derivation of CTMT-I. In this view, spinor structure is a fixed point of torsional kernel flow.

VII. Conclusion

This paper establishes the kernel-level origin of rigidity in CTMT. Spectral collapse under coherence–redundancy stabilization yields a unique macroscopic control parameter governing phase fluctuations relative to an action invariant. The effective rigidity functional $R = (\Phi)/S_*^2$ of CTMT-I is thus seen to be a coarse-grained summary of underlying kernel dynamics, rather than an independent postulate.

The Maxwell, Newtonian, Schrödinger, and Dirac limits of CTMT-I reappear here as fixed points of the chronotopic kernel flow in the rigid regime. Hilbert space, gauge structure, and 3+1-dimensional geometry are emergent features of stabilized low-rank kernel configurations.

Together, CTMT-I and CTMT-II place CTMT within the broader class of emergent spacetime and information-geometric frameworks, while maintaining explicit falsifiability and structural constraints.

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