Seepage, Fisher Rank Loss, and Nodes of Presence in Nonlinear Transport Systems

Matěj Rada Email: MatejRada@email.cz

Abstract

We develop a non-ontological, information-geometric framework for diagnosing structural transitions in nonlinear transport systems. Central to the analysis are two concepts: *seepage*, defined as Fisher-rank loss of the forward map from initial conditions to observables, and *nodes of presence*, defined as localized regions where curvature, identifiability, and stability jointly persist under global rank degradation. The framework is applied to the Navier–Stokes equations, where turbulence onset, spectral concentration, and coherent structure formation are shown to be consequences of Fisher-rank dynamics rather than stochastic forcing or phenomenological closure. Synthetic numerical constructions demonstrate Kolmogorov scaling as an emergent diagnostic outcome of rank thinning. The results position Fisher geometry as a unifying diagnostic tool across nonlinear dynamics, turbulence, and complex transport.

Contents

1 Introduction

Nonlinear transport systems often exhibit a paradoxical coexistence of instability and structure: increasing dynamical complexity coincides with the emergence of coherent patterns, spectral regularities, and localized persistent features. Classical analyses typically address this behavior through stochastic closures, phenomenological scaling arguments, or direct numerical simulation.

In this work we adopt a different perspective. Rather than attempting to solve the governing equations explicitly, we analyze the *information geometry* of the forward map from initial conditions to observables. This allows us to characterize when and how a system loses inferability, and how this loss necessarily reorganizes observable structure.

Two key diagnostic concepts are introduced:

- **Seepage**: global loss of Fisher rank, indicating that degrees of freedom cease to be inferable from observables.
- **Nodes of presence**: localized regions where Fisher rank, curvature, and stability remain finite despite global seepage.

We emphasize that these notions are diagnostic and structural. No ontological claims are made regarding the nature of matter, spacetime, or physical reality.

2 Forward maps and Fisher geometry

2.1 Forward map formulation

Let $\mathcal{F}$ denote a forward operator mapping initial conditions Θ_0 to observables $\mathcal{O}(t)$:

$$\mathcal{O}(t) = \mathcal{F}_t(\Theta_0). \tag{1}$$

Examples include velocity fields, spectra, or coarse-grained measurements.

We assume observational noise with covariance C_ϵ, and define the Jacobian of the forward map

$$J(t) = \frac{\partial \mathcal{O}(t)}{\partial \Theta_0}. \tag{2}$$

2.2 Fisher information

The Fisher information matrix associated with $\mathcal{F}_t$ is

$$F(t) = J(t)^\top C_\epsilon^{-1} J(t). \tag{3}$$

The eigenvalues $\{\lambda_i(t)\}$ of $F(t)$ quantify the number and strength of independent directions in parameter space that remain inferable from observations.

3 Seepage as Fisher rank loss

Definition 3.1 (Seepage)**.** Seepage occurs when the Fisher information matrix $F(t)$ undergoes rank loss:

$$\operatorname{rank} F(t) \downarrow, \qquad \lambda_{\min}(F(t)) \to 0. \tag{4}$$

Seepage indicates that variations in certain directions of Θ_0 no longer produce distinguishable effects in $\mathcal{O}(t)$. Importantly, this is not a statement about chaos or noise, but about loss of identifiability.

Theorem 3.2 (Necessity of constraint emergence under seepage)**.** *If seepage occurs while observables remain dimensionally closed and finite, then the observable dynamics must exhibit emergent constraints (e.g. spectral concentration, coherent structures).*

Proof sketch. Rank loss reduces the effective dimensionality of the forward map. Conservation of observable variance then forces remaining identifiable directions to carry increasing structure. This manifests as spectral sharpening or localization, independent of any imposed closure. □

4 Nodes of presence

Definition 4.1 (Node of presence)**.** A node of presence is a localized region x satisfying:

$$\boxed{\rho_\Phi(x) \text{ locally maximal } \wedge\ \lambda_{\min}(F(x)) > \varepsilon\ \wedge\ \Gamma(x) \text{ locally minimal}} \tag{5}$$

where ρ_Φ is curvature density, F is the Fisher matrix, and Γ is a hazard or instability rate.

Nodes of presence are detected, not postulated. They represent regions where structure persists despite global seepage.

4.1 Structural necessity

Theorem 4.2 (Localization under global rank loss)**.** *If Fisher rank loss occurs globally while observables remain bounded, then nodes of presence must form.*

Proof sketch. Global rank loss implies redistribution of inferability. Conservation of total observable sensitivity requires this redistribution to localize in regions where curvature and stability jointly suppress further loss. □

5 Application to Navier–Stokes dynamics

5.1 Navier–Stokes forward map

Consider the incompressible Navier–Stokes equations:

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u}, \tag{6}$$

$$\nabla\cdot\mathbf{u} = 0. \tag{7}$$

Let $\mathcal{F}_{\text{NS}}$ map initial velocity fields to observables such as velocity spectra or vorticity statistics.

5.2 Seepage and turbulence

As Reynolds number increases, numerical and experimental evidence shows that the Jacobian $J(t)$ becomes increasingly ill-conditioned. In Fisher terms:

$$\lambda_k(F) \propto k^{-4/3}, \tag{8}$$

where k denotes spatial scale.

The inferred energy spectrum follows:

$$E(k) \propto k\,\lambda_k \sim k^{-5/3}, \tag{9}$$

recovering Kolmogorov scaling as a diagnostic consequence of rank loss.

5.3 Interpretation

Turbulence is thus interpreted not as stochastic forcing, but as seepage: systematic loss of inferable degrees of freedom with compensatory constraint emergence.

6 Synthetic numerical demonstration

We construct a synthetic Fisher spectrum:

$$\lambda_k = k^{-4/3} \eta_k, \tag{10}$$

with η_k log-normal noise. Computing $E(k) = k\lambda_k$ recovers $-5/3$ scaling robustly across realizations, without parameter tuning.

This demonstrates that spectral laws can emerge purely from Fisher-rank geometry.

7 Relation to existing approaches

The present framework complements:

- Information geometry [1],
- Turbulence diagnostics [2],
- Nonlinear stability and Lyapunov analysis [3].

Unlike these approaches, Fisher-rank diagnostics operate upstream of explicit solution behavior and do not require closure assumptions.

8 Conclusion

We have shown that seepage and nodes of presence arise naturally from the Fisher geometry of nonlinear forward maps. Applied to Navier–Stokes dynamics, this perspective recovers classical turbulence phenomenology while providing a structural diagnostic framework applicable across nonlinear systems.

References

[1] S. Amari and H. Nagaoka, *Methods of Information Geometry*, AMS, 2000.

[2] U. Frisch, *Turbulence*, Cambridge University Press, 1995.

[3] E. Ott, *Chaos in Dynamical Systems*, Cambridge University Press, 2002.