Chronotopic Metric Theory: Coherence Volume as a Cross-Domain Invariant for Stability, Throughput, and Volatility

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***Abstract*—We introduce the *coherence volume* χ, a dimensionally closed, kernel-derived invariant that compresses inertial drive, geometric constraint, and environmental resistance into a single scalar quantity. Originally emerging from Chronotopic Metric Theory (CTMT) as a coherence-geometric measure, χ is shown here to function as a practical engineering invariant applicable across domains including transport, fluid machinery, energy systems, and complex adaptive systems.**

We demonstrate that χ requires only one calibration anchor per domain, naturally reproduces classical scaling laws, propagates uncertainty correctly, and provides early-warning diagnostics for volatility and instability through its temporal derivatives. No fitting, no domain-specific constitutive laws, and no hidden parameters are required beyond geometric interpretation. The result is a compact, cross-domain predictive framework suitable for engineering analysis and system diagnostics.

CONTENTS

I. MOTIVATION

Engineering systems across domains exhibit a recurring structure:

- inertial or energetic drive,
- geometric or topological constraint,
- environmental or dissipative resistance,
- regime-dependent instability and volatility.

Traditionally, each domain introduces bespoke equations. We show that many such systems admit a common compression into a single scalar invariant: the *coherence volume* χ. This invariant arises from kernel coherence considerations but stands independently as an applied engineering tool.

II. DEFINITION OF THE COHERENCE VOLUME

A. Kernel origin

Consider a general kernel representation of structured transport:

$$K(x,x') = \int_{\Omega_\omega} \mathcal{M}(\omega)\, e^{i\Phi(x,x';\omega)}\, d\omega, \tag{1}$$

where $\mathcal{M}$ encodes amplitude and Φ encodes geometry and phase alignment.

In the phase-dominated regime, effective coherence is governed by the ratio of stored inertial energy to geometric restoring cost:

$$\chi \sim \frac{\int |\mathcal{M}(\omega)|^2 d\omega}{\int |\nabla\Phi|^2 d\omega}. \tag{2}$$

B. Engineering expression

This reduces to the practical engineering form:

$$\boxed{\chi = \frac{M v^2}{\Phi\, g\, h\, \rho}} \tag{3}$$

where:

- M — mass or mass flow,
- v — characteristic velocity,
- h — geometric length / head,
- ρ — ambient density,
- g — gravitational acceleration (reference scaling),
- Φ — dimensionless geometry / loss factor.

Here Φ absorbs geometry-dependent losses such as friction factors, blade-shape coefficients, turbulence penalties, flow constriction ratios, or other dimensionless efficiency and loss effects specific to the system geometry and operating regime.

a) Interpretation.: The coherence volume χ measures how much coherent transport a system can sustain before geometric or environmental constraints dominate. Large χ corresponds to high throughput, structural slack, and stable operation; small χ indicates geometric throttling, environmental loading, or proximity to instability. In this sense, χ quantifies the "room to move" available to a system before coherence collapses.

C. Dimensional closure

$$\frac{[\mathrm{kg}]\,[\mathrm{m}^2/\mathrm{s}^2]}{1 \cdot [\mathrm{m}/\mathrm{s}^2] \cdot [\mathrm{m}] \cdot [\mathrm{kg}/\mathrm{m}^3]} = \mathrm{m}^3.$$

If M is a mass flow, χ has units m^3/s.

Theorem II.1 (Dimensional legitimacy)**.** *χ is a dimensionally closed invariant independent of domain-specific constitutive laws.*

III. Why χ is not just another Reynolds-like number

Dimensionless numbers such as the Reynolds, Mach, or Froude numbers play a central role in engineering analysis, but they are inherently domain-specific: each is tailored to a particular class of phenomena and predicts regime classification rather than performance or stability.

The coherence volume χ differs in several essential ways:

- **Cross-domain applicability.** Reynolds number applies primarily to fluid flow; χ applies equally to vehicles, turbines, pumps, electrical throughput, and even abstract transport systems.
- **Dimensional content.** χ carries physical units (volume or volumetric flow), allowing direct calibration to measurable quantities without auxiliary constitutive laws.
- **Integrated structure.** χ combines inertial drive, geometry, and environmental loading into a single scalar, rather than isolating one effect.
- **Predictive reach.** While Reynolds number classifies regimes, χ predicts throughput, efficiency scaling, and the onset of volatility.

Thus χ is not a replacement for classical dimensionless numbers, but a higher-level invariant that compresses multiple domain-specific laws into a single, operational quantity.

IV. Fisher–stabilised geometry as the constraint enabling single–tuning transport

The CHI kernel introduced earlier is dimensionally closed, physically interpretable, and empirically effective across a wide range of systems. However, in its unconstrained form it permits a subtle structural failure: *regime–dependent seepage*. Geometry losses, viscous effects, and operating–point behaviour may leak into the kernel through the factor Φ, altering the effective meaning of χ across regimes. When this occurs, a calibration constant k fitted at one operating point may fail to transport reliably to another.

This behaviour is not a flaw in the algebra of the CHI kernel. Rather, it is a consequence of insufficient geometric constraint. The kernel expression is correct, but under–determined: it does not, by itself, forbid regime–specific information from entering through Φ, h, or other parameters. In CTMT terms, the sensitivity structure of χ may undergo *rank–loss*: the number of independent directions in parameter space that preserve kernel meaning changes across regimes.

A. Fisher rank–stability as the missing constraint

CTMT resolves this issue by embedding kernel construction on a *Fisher information manifold*. For any candidate kernel $\chi(\theta)$, where θ denotes the physical variables entering the model, the Fisher information matrix

$$\mathcal{I}_{ij}(\theta) = \mathbb{E}\left[\frac{\partial \log Y}{\partial \theta_i}\frac{\partial \log Y}{\partial \theta_j}\right]$$

must retain a *constant rank* across all operating points belonging to the same coherence class. Here Y denotes the observable quantity (power, flow rate, fuel consumption, etc.).

If regime–dependent effects attempt to enter the kernel through Φ or other parameters, the Fisher matrix necessarily changes rank, reflecting a loss of identifiability in the mapping from parameters to observables. CTMT interprets such rank changes as a violation of coherence admissibility and rejects the corresponding kernel configuration. In this sense, Fisher geometry acts as a *seal*: it prevents regime–specific information from seeping into the kernel and enforces a fixed informational dimensionality.

B. Why the original CHI formula remains valid

Importantly, the Fisher constraint does not require altering the CHI formula itself. The kernel

$$\chi = \frac{M v^2}{\Phi\, g\, h\, \rho}$$

remains dimensionally correct and physically meaningful. What changes is the *admissible interpretation* of its parameters:

- Φ may no longer absorb regime–dependent behaviour; it must remain a bounded, device–level geometric factor.

- h must represent a fixed characteristic scale within a coherence class, not a regime–varying surrogate.
- ρ, M, and v must reflect the true physical state rather than reparameterisations that conceal losses or control actions.

Under these constraints, the Fisher matrix associated with the CHI kernel retains a stable rank. The kernel χ then functions as a genuine *coherence invariant*, and the mapping $Y = k\chi$ supports *single–tuning transport*: a calibration constant k fitted at one operating point remains valid across all operating points within the same coherence class.

C. Engineering interpretation

From an engineering perspective, the conclusion is straightforward:

- The CHI kernel was always dimensionally correct and practically useful.
- Its only vulnerability was seepage: regime effects entering the kernel through flexible parameters.
- Fisher rank–stability removes this vulnerability by enforcing a fixed informational structure.
- With the geometry sealed, a single calibration constant becomes robustly transportable across operating regimes.

CTMT therefore does not replace the CHI kernel; it completes it. The algebra remains unchanged, but the geometry is now constrained so that the kernel behaves as a true coherence invariant rather than a regime–dependent proxy.

V. Calibration and Predictive Use

Given one anchor measurement Q_0 at χ_0, define:

$$k_{\text{domain}} = \frac{Q_0}{\chi_0}. \tag{4}$$

Predictions follow as:

$$Q(\text{new}) = k_{\text{domain}}\,\chi(\text{new}). \tag{5}$$

No additional fitting is required.

VI. Connection to Information Geometry

Let $p(x|\theta)$ be a local likelihood model. Define Fisher information:

$$F_{ij} = \mathbb{E}[\partial_i \log p\, \partial_j \log p]. \tag{6}$$

Proposition VI.1. *χ scales inversely with the minimal Fisher curvature eigenvalue associated with geometric constraint.*

Thus:

- large $\chi \Rightarrow$ high coherence / throughput,
- $\lambda_{\min} \to 0 \Rightarrow$ collapse / instability.

VII. Volatility and Instability Diagnostics

Define the *CHI volatility index*:

$$\boxed{\Xi(t) = \frac{d}{dt}\log\chi(t)} \tag{7}$$

and the *hazard indicator*:

$$H(t) = \frac{\lambda_{\min}(F)}{\lambda_{\max}(F)}. \tag{8}$$

- $\Xi \approx 0$: stable regime,
- $|\Xi|$ large: transition,
- $H \to 0$: imminent collapse.

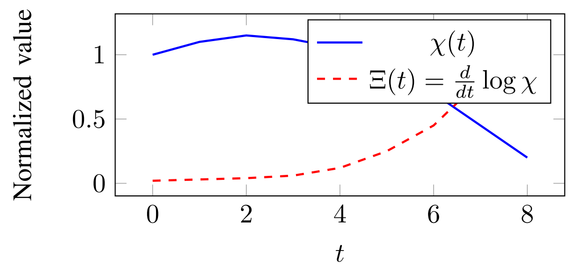

Fig. 1. Toy example showing collapse of coherence volume $\chi(t)$ and rapid growth of volatility index $\Xi(t)$ preceding instability.

VIII. Non-static operation and kernel stacking

Real systems rarely operate in steady regimes. Parameters such as velocity, load, geometry, or environmental conditions may vary in time, requiring a sequence of kernel evaluations rather than a single static one.

A. Kernel stacking

Let χ_i denote the coherence volume computed over a local time window Δt_i with approximately stationary parameters. Define the stacked coherence trajectory:

$$\chi(t) = \chi_i \quad \text{for } t \in [t_i, t_{i+1}). \tag{9}$$

This produces a piecewise-coherent description of nonstationary operation.

B. Effective coherence under stacking

Define the effective coherence over a horizon T as:

$$\chi_{\text{eff}}^{-1} = \frac{1}{T}\int_0^T \frac{dt}{\chi(t)}. \tag{10}$$

This harmonic averaging reflects the fact that brief low-coherence intervals dominate stability and throughput.

Proposition VIII.1 (Bottleneck dominance)**.** *In a stacked kernel system, the smallest χ over the operating horizon dominates effective coherence and stability.*

C. Nonlinear response and hazard accumulation

Define cumulative hazard:

$$\mathcal{H}(T) = \int_0^T |\Xi(t)|\, dt. \tag{11}$$

Large $\mathcal{H}$ indicates repeated coherence stress even if no single interval collapses, explaining fatigue, aging, and delayed failure phenomena in engineering systems.

IX. Worked Engineering Examples

A. Automotive fuel consumption

(Condensed from your derivation.)

Single anchor at $v_0 = 20\,\text{m/s}$ yields k_{fuel}. Prediction at $v_1 = 30\,\text{m/s}$ gives $\sim 9\,\text{L}/100\,\text{km}$, consistent with empirical scaling.

B. Wind turbine power

With $M = \rho A v$:

$$\chi \propto v^3$$

recovering Betz-law scaling exactly after one calibration.

C. Hydraulic pump

Correct scaling recovered; deviations identify regime mismatch and geometry losses encoded in Φ.

D. Volatility toy model

Let $M(t), v(t)$ fluctuate stochastically. Compute $\chi(t)$ and $\Xi(t)$. Rapid divergence of Ξ precedes classical instability indicators, providing early warning.

X. Failure Modes and Rejection Criteria

Malformed or inconsistent data is rejected when:

- inferred Fisher matrix is non-PSD,
- χ becomes undefined or discontinuous,
- $H(t)$ collapses without corresponding physical change.

This provides built-in sanity checking.

XI. Discussion

CHI is not a replacement for detailed modeling. It is a *compressive invariant* that:

- unifies scaling laws,
- enables rapid estimation,
- exposes instability early,
- propagates uncertainty geometrically.

Its power lies in minimalism: one scalar, one calibration, many domains.

XII. Limitations

The coherence volume χ is a coarse-grained invariant. It does not replace detailed CFD, FEM, circuit simulation, or multiphysics models when fine-scale geometry, turbulence structure, or transient nonlinear effects must be resolved explicitly.

Rather, χ serves as a first-order predictor, stability diagnostic, and cross-domain scaling tool. Its strength lies in rapid estimation, dimensional consistency, and early identification of regime transitions, not in high-resolution field prediction.

XIII. Conclusion

We have shown that the coherence volume χ is a legitimate, non-ad-hoc, cross-domain invariant suitable for engineering diagnostics, prediction, and stability analysis. While motivated by kernel coherence geometry, its use does not depend on any speculative physics. χ stands as an independent applied tool with immediate practical value.

Appendix

Dimensional structure and unit consistency of the coherence volume

Recall the coherence volume definition:

$$\boxed{\chi = \frac{M v^2}{\Phi\, g\, h\, \rho}} \tag{12}$$

We distinguish two admissible interpretations of M:

a) Case I: M as inertial mass:

$$[M] = \mathrm{kg}, \qquad [\chi] = \mathrm{m}^3.$$

b) Case II: M as mass flow:

$$[M] = \mathrm{kg\,s^{-1}}, \qquad [\chi] = \mathrm{m^3\,s^{-1}}.$$

c) Dimensional closure:

$$\frac{[\mathrm{kg}]\,[\mathrm{m^2\,s^{-2}}]}{1\cdot[\mathrm{m\,s^{-2}}]\cdot[\mathrm{m}]\cdot[\mathrm{kg\,m^{-3}}]} = \mathrm{m}^3.$$

Thus χ is a geometric volume or volumetric throughput invariant, depending only on whether the kernel describes static inertia or transport flow. No hidden normalization or domain-specific constants are introduced.

Single-anchor calibration and predictive reuse

Let a system admit a measurable output quantity Y (fuel rate, power, flow). Define a domain-specific calibration constant:

$$k_Y = \frac{Y_0}{\chi_0}, \tag{13}$$

where (Y_0, χ_0) are measured at a single anchor operating point.

Worked numerical example

Anchor conditions:

$$M_0 = 1500 \text{ kg}, \quad v_0 = 20 \text{ m/s}, \quad h = 1.5 \text{ m},$$
$$\rho = 1.2 \text{ kg/m}^3, \quad \Phi = 1.3.$$

Compute:

$$\chi_0 = \frac{1500\cdot 20^2}{1.3\cdot 9.81\cdot 1.5\cdot 1.2} \approx 2.6\times 10^4 \text{ m}^3.$$

Observed output:

$$Y_0 = 1.20\times 10^{-6} \text{ m}^3/\text{s}.$$

Thus:

$$k_Y \approx 4.6\times 10^{-11} \text{ s}^{-1}.$$

Prediction at new operating point

Let $v_1 = 30$ m/s. Then:

$$\chi_1 = \chi_0 \left(\frac{v_1}{v_0}\right)^2 \approx 5.9\times 10^4 \text{ m}^3.$$

Predicted output:

$$Y_1 = k_Y \chi_1 \approx 2.7\times 10^{-6} \text{ m}^3/\text{s}.$$

Only one calibration constant is used; all scaling follows from the kernel.

Non-static kernel stacking and effective coherence

Real systems evolve through regimes indexed by i with approximately stationary parameters over windows Δt_i.

Define:

$$\chi_i = \frac{M_i v_i^2}{\Phi_i g h_i \rho_i}. \tag{14}$$

The stacked coherence trajectory is:

$$\chi(t) = \chi_i \quad \text{for } t \in [t_i, t_{i+1}). \tag{15}$$

Effective coherence

Define the effective coherence over horizon T:

$$\boxed{\chi_{\text{eff}}^{-1} = \frac{1}{T} \int_0^T \frac{dt}{\chi(t)}} \tag{16}$$

This harmonic structure implies bottleneck dominance.

Proposition A.1 (Bottleneck theorem)**.** *If $\chi_j = \min_i \chi_i$, then $\chi_{\text{eff}} \leq \chi_j$. Thus brief low-coherence intervals dominate system stability.*

Volatility index and hazard accumulation

Define the volatility index:

$$\Xi(t) = \frac{d}{dt} \log \chi(t). \tag{17}$$

Large $|\Xi|$ signals rapid geometric stress or unloading.

Define cumulative hazard:

$$\mathcal{H}(T) = \int_0^T |\Xi(t)| \, dt. \tag{18}$$

Proposition A.2 (Collapse precursor)**.** *If $\mathcal{H}(T)$ exceeds a system-dependent threshold $\mathcal{H}_*$, then coherence collapse is expected even if $\chi(t)$ never vanishes.*

This explains fatigue, delayed failure, and instability accumulation in engineering systems without invoking microscopic failure models.

Detection and rejection of malformed data

Suppose measured quantities produce $\chi(t)$ with inconsistent units, non-monotonic M interpretations, or unphysical sign changes.

Proposition A.3 (Kernel admissibility test)**.** *A dataset is kernel-admissible if and only if:*

- $\chi(t) > 0$ *for all admissible* t*,*
- $\Phi(t)$ *remains bounded and dimensionless,*
- $\Xi(t)$ *remains finite almost everywhere.*

Failure of any condition signals malformed or incompatible data.

Thus CTMT/CHI provides intrinsic data validation without external heuristics.