

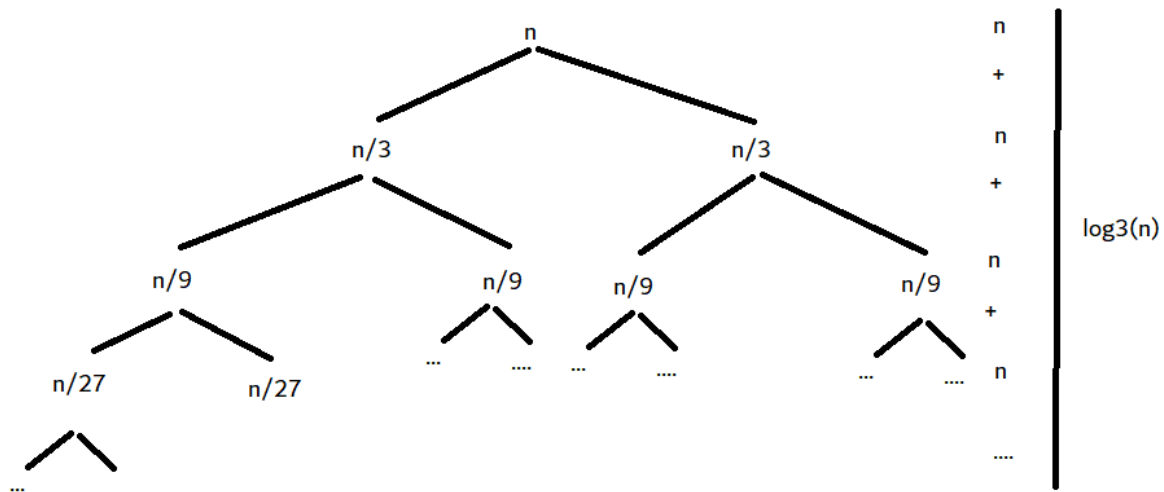
// NOME: Mateus Kenzo lochimoto TIA: 32216289

2. Dado os algoritmos recursivos abaixo, apresente suas funções de complexidade de tempo.

a)

```
void f(int n)
{
    if (n > 1){
        for (int i = 1; i <= n; i++)
        {
            printf("%d\n",n);
        }
        f(n/3);
    }
}
```

$$T(n) = T(n/3) + n$$



$$T(n) = T(n/3) + n$$

$$T(n) = [T(n/3) + n/3] + n$$

$$T(n) = T(n/3^2) + n + n$$

$$T(n) = [T(n/3^2) + n/3^2] + n + n$$

$$T(n) = T(n/3^3) + 3n$$

⋮

$$T(n) = T(n/3^k) + kn$$

Quando $n/3^k = 1$, temos $n = 3^k$ e $k = \log_3 n$

$$T(n) = T(n/3^k) + kn$$

$$= 3^{\log_3 n} \cdot T(1) + n \log_3 n$$

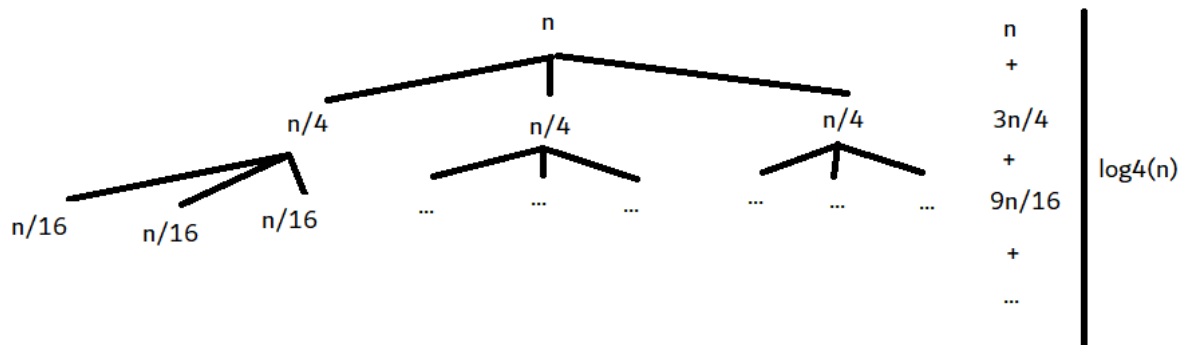
$$= n + n \log_3 n$$

$$\therefore T(n) = O(n \log_3 n)$$

b)

```
void f(int n)
{
    if (n > 1){
        for (int i = 1; i <= n; i++)
        {
            printf("%d\n",n);
        }
        f(n/4);
        f(n/4);
        f(n/4);
    }
}
```

$$T(n) = 3T(n/4) + n$$



$$T(n) = 3T(n/4) + n$$

$$T(n) = 3[T(n/4) + n/4] + n$$

$$T(n) = 3^2T(n/4^2) + n + n$$

$$T(n) = 3^2[T(n/4^2) + n/4^2] + n + n$$

$$T(n) = 3^3T(n/4^3) + 3n$$

⋮

$$T(n) = 3^kT(n/4^k) + kn$$

Quando $n/4^k = 1$, temos $n = 4^k$ e $k = \log_4 n$

$$T(n) = 3^kT(n/4^k) + kn$$

$$= 3^{\log_4 n} \cdot T(1) + n \log_4 n$$

$$= n + n \log_4 n$$

$$\therefore T(n) = O(n \log_4 n)$$

c)

```
void f(int n)
{
```

```
    if (n > 1)
```

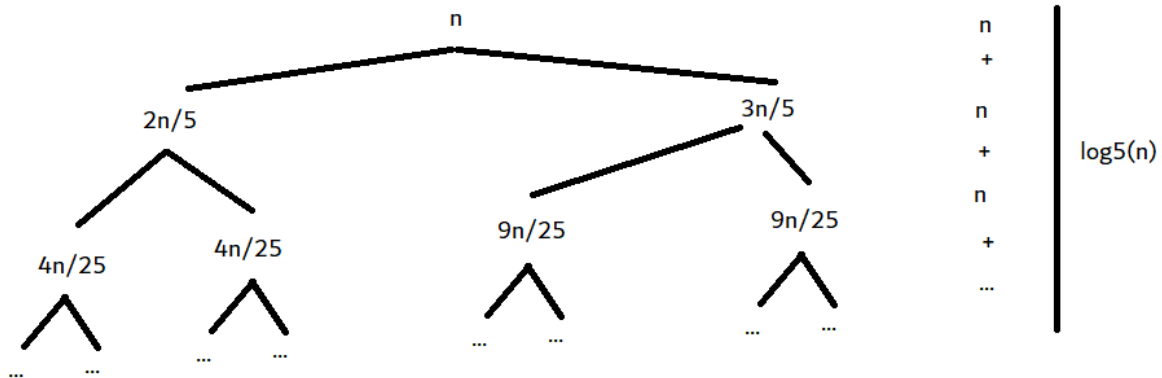
```
    {
```

```

    for (int i = 1; i <= n; i++)
    {
        printf("%d\n",n);
    }
    f(2*n/5);
    f(3*n/5);
}

```

$$T(n) = T(2n/5) + T(3n/5) + n$$



$$T(n) = T(2n/5) + T(3n/5) + n$$

$$T(n) = [T(2n/5) + 2n/5] + [T(3n/5) + 3n/5] + n$$

$$T(n) = T(2^2n/5^2) + T(3^2n/5^2) + n + n$$

$$T(n) = [T(2^2n/5^2) + 2^2n/5^2] + [T(3^2n/5^2) + 3^2n/5^2] + n$$

$$T(n) = T(2^3n/5^3) + T(3^3n/5^3) + 3n$$

⋮

$$T(n) = T(2^k n/5^k) + T(3^k n/5^k) + kn$$

$$\text{Quando } (2n/5)^k = 1, \text{ temos } n = 5^k/2^k \text{ e } k = \log_{5/2} n$$

$$\text{Quando } (3n/5)^k = 1, \text{ temos } n = 5^k/3^k \text{ e } k = \log_{5/3} n$$

$$T(n) = T(3n/5)^k + T(2/5)^k + kn$$

$$= 5^{\log_{5/2} n} \cdot T(1) + 5^{\log_{5/3} n} \cdot T(1) + n \log_5 n$$

$$= n + n \log_5 n$$

$$\therefore T(n) = O(n \log_5 n)$$

d)

```

void f(int n)
{
    if (n > 1)
    {
        for (int i = 1; i*i <= n; i++)
        {
            printf("%d\n",n);
        }
    }
}

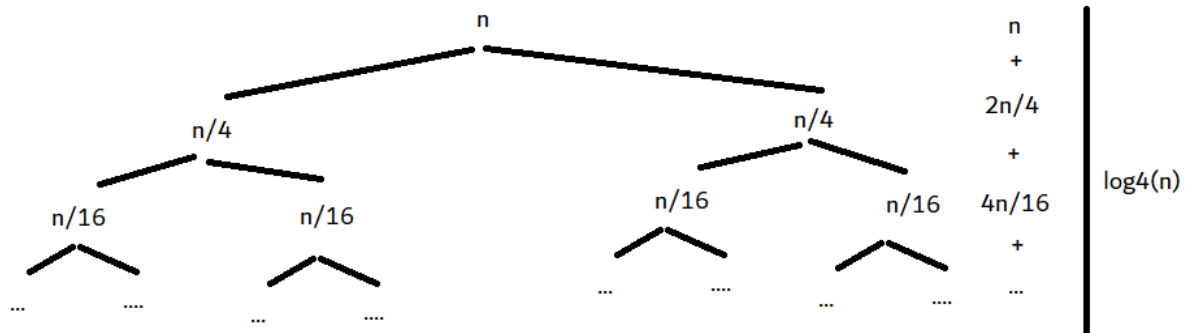
```

```

    }
    f(n/4);
    f(n/4);
    }
}

```

$$T(n) = 2(n/4) + n$$



$$T(n) = 2T(n/4) + n$$

$$T(n) = 2 [T(n/4) + n/4] + n$$

$$T(n) = 2^2 T(n/4^2) + n + n$$

$$T(n) = 2^3 [T(n/4^3) + n/4^3] + n + n$$

$$T(n) = 2^3 T(n/4^3) + 3n$$

⋮

$$T(n) = 2^k T(n/4^k) + kn$$

Quando $n/4^k = 1$, temos $n = 4^k$ e $k = \log_4 n$

$$T(n) = 2^k T(n/4^k) + kn$$

$$= 2^{\log_4 n} \cdot T(1) + n \log_4 n$$

$$= n + n \log_4 n$$

$$\therefore T(n) = O(n \log_4 n)$$

3. Resolva as recorrências.

(a) $T(n) = T(n - 1) + 1$

$a = 1$

caso 2 do teorema mestre

$$T(n) = O(n * 1) = O(n)$$

(b) $T(n) = T(n - 10) + 1$

$a = 1$

caso 2 do teorema mestre

$$T(n) = O(n * 1) = O(n)$$

(c) $T(n) = T(n - 1) + n$

$a = 1$

caso 2 do teorema mestre

$$T(n) = O(n * n) = O(n^2)$$

$$(d) T(n) = 2T(n - 1) + 1$$

$$a = 2$$

caso 3 do teorema mestre

$$T(n) = O(2^{n-1} * 1) = O(2^n)$$

$$(e) T(n) = 2T(n - 1) + n$$

$$a = 2$$

caso 3 do teorema mestre

$$T(n) = O(2^{n-1} * n) = O(n2^n)$$

$$(f) T(n) = 2T(n - 1) + n^2$$

$$a = 2$$

caso 3 do teorema mestre

$$T(n) = O(2^{n/1} * n^2) = O(n^2 * 2^n)$$

$$(g) T(n) = T(n/2) + 1$$

$$a = 1, b = 2, k = 0$$

caso 2 do teorema mestre

$$T(n) = O(n^0 * \log_2 n) = O(\log n)$$

$$(h) T(n) = T(n/2) + n$$

$$a = 1, b = 2, k = 1$$

caso 3 do teorema mestre

$$T(n) = O(n)$$

$$(i) T(n) = T(n/2) + n^2$$

$$a = 1, b = 2, k = 2$$

caso 3 do teorema mestre

$$T(n) = O(n^2)$$

$$(j) T(n) = 2T(n/2) + 1$$

$$a = 2, b = 2, k = 0$$

caso 1 do teorema mestre

$$T(n) = O(n^{\log_2 2})$$

$$(k) T(n) = 2T(n/2) + n$$

$$a = 2, b = 2, k = 1$$

caso 2 do teorema mestre

$$T(n) = O(n^1 * \log n) = O(n \log n)$$

$$(l) T(n) = 2T(n/2) + n^2$$

$$a = 2, b = 2, k = 2$$

caso 3 do teorema mestre

$$T(n) = O(n^2)$$

$$(m) T(n) = 4T(n/2) + n$$

$$a = 4, b = 2, k = 1$$

caso 1 do teorema mestre

$$T(n) = O(n^{\log_4 4}) = O(n^2)$$

$$(n) \quad T(n) = 3T(n/2) + n$$

$$a = 3, b = 2, k = 1$$

caso 1 do teorema mestre

$$T(n) = O(n^{\log_2 3})$$

$$(o) \quad T(n) = 3T(n/4) + n$$

$$a = 3, b = 4, k = 1$$

caso 3 do teorema mestre

$$T(n) = O(n)$$

$$(p) \quad T(n) = 7T(n/4) + \log n$$

$$a = 7, b = 4, k = 0$$

caso 1 do teorema mestre

$$T(n) = O(n^{\log_4 7})$$