Observing the Normal Zeeman Effect and Estimating e/m for Orbital Electrons

Mateusz and Gustavo

January 2024

1 Introduction

The Zeeman effect was first observed in 1896 by the Dutch physicist Pieter Zeeman. Although at first there was no clear explaination for the results, the experiment was a cornerstone in proving the existence of electron Spin and magentic moments.

The Zeeman effect is the splitting of spectral lines in the presence of a magnetic field. The effect is caused by the interaction of the magnetic field with the magnetic moment of the electron.

It is important to point out that the Zeeman effect does not occur in all cases. For example, a simple case where it does not occur is in the ground state of the Helium atom, where the electrons are paired and the total magnetic moment is zero, as both atoms occupy the 1s orbital, and have opposite spin (remember also that the s orbitals do not have any orbital angular momentum, becuase the only possible l quantum number is l=0).

In this experiment, we will be using a Hg-Cd lamp to observe the Zeeman effect. The electron for Mercury (Hg) is given by: $[Xe]4f^{14}5d^{10}6s^2$, and the configuration for Cadmium (Cd) is given by: $[Kr]4d^{10}5s^2$

All of the orbitals of these are filled, the spin magnetic moments inside the orbitals will cancel out, and we can safely ignore the effect of spin in spectral line separation. The report will then be scoped to the **Normal Zeeman Effect**.

In order to set the foundation for the experiment, we will have to go back to electromagnetism. We can consider the electron as a point charge, with a charge of $-e$, and a mass of m_e travlling around the charged nuclus.

A torque given by $\tau = \vec{\mu} \times \vec{B}$ will be applied to the electron, where $\vec{\mu}$ is the magnetic moment of the electron, and is given by $\vec{\mu} = \gamma \vec{L}$, where $\gamma = -\frac{e}{2m_e}$ is the gyromagnetic ratio, and $\vec{L}$ is the orbital angular momentum of the electron.

Given this information, we can now calculate the energy shift of the electron in the presence of a magnetic field.

$$\Delta E = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{L} \cdot \vec{B} = -\gamma L_z B \tag{1}$$

Where L_z is the z component of the orbital angular momentum (assuming the field B is applied in the z direction). We also know that $L_z = m_l \hbar$, where m_l is the magnetic quantum number, and $\hbar$ is the reduced Planck constant. We can rewite the above expression as:

$$\Delta y = -\gamma m_l \hbar B \tag{2}$$

According to this, and keeping in mind that m_l can take intager values from $-l$ to l, we can see that the energy levels of the electron are spaced by $\Delta E = \gamma \hbar B$. This is the energy difference between the Zeeman levels.

The Zeeman effect can be used to estimate the value of $\frac{e}{m}$.

The change in wavelength of the spectral lines is given by:

$$\Delta\lambda = \frac{\Delta S}{\Delta A} \frac{\lambda^2}{2d} \frac{2}{\sqrt[2]{\eta^2 - 1}} \tag{3}$$

Where ΔS is the separation of the zeeman split, ΔA is the separation of successive interference lines, λ is the wavelength of the light, d is the distance between the grating and the screen, and ν is the refractive index of the medium. Figure 1 helps build a better understanding of the variables involved in the equation.

We are given that the refractive index of the Lummer plate is $\nu = 1.4567$, the wavlength of the red line in cadmimum is $\lambda = 643.8nm$, and the thickness of the Lummer plate is $d = 4.04mm$.

Further, it can also be obtained that the change in frequency when a field is applied is given by:

$$\Delta\nu = \frac{eB}{4\pi m} \tag{4}$$

Where e is the charge of the electron, B is the magnetic field, and m is the mass of the electron.

We can then use the change in frequency to estimate the value of $\frac{e}{m}$ by using $\Delta v = \frac{c}{\lambda^2} \Delta\lambda$:

$$\frac{e}{m} = \frac{4\pi c}{B} \frac{1}{2d \sqrt[2]{\eta^2 - 1}} \frac{\Delta S}{\Delta A} \tag{5}$$

2 Method

The experiment is divided into three sections:

2.1 Part 1: Magnetic field strength and current

In the first part of the experiment, we measure the magnetic field strength as a function of the current supplied to the electromagnet. We do this by measuring the magnetic field strength at the center of the electromagnet using a Hall probe and the current as the current from the power source. We measure the magnetic

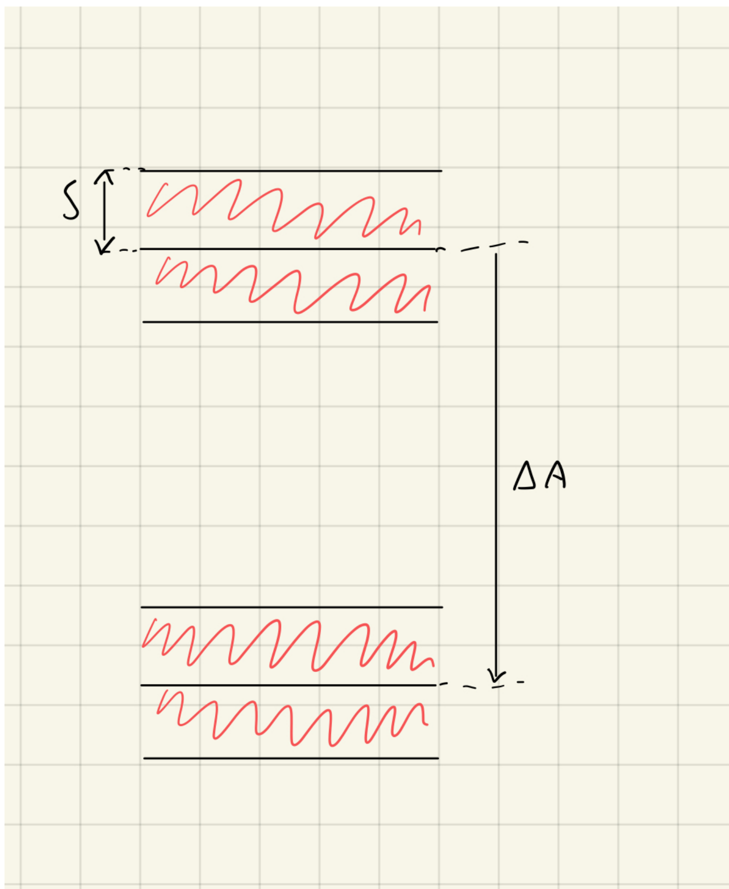

Figure 1: Meaning of variables in change in wavelength of spectral lines equation.

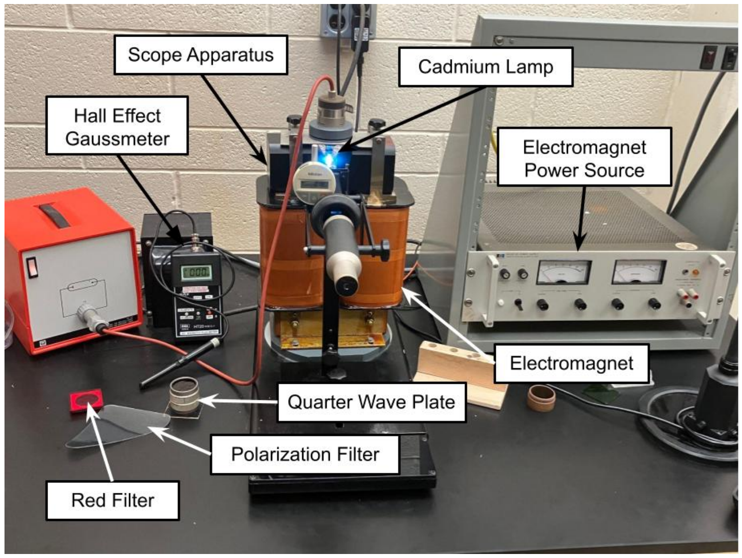

Figure 2: Diagram of Experimental Setup

field strength at 10 different current values, and then fit the data to a linear model to determine the relationship between the current and the magnetic field strength.

2.2 Part 2: Observing the Zeeman split perpendicularly to the magnetic field with different field values

In the first part of the expeirment we measure how the Zeeman split changes with different magnetic field values. We do this by measuring the width of the spectral lines of the mercury lamp with and without a magnetic field. We then calculate the Zeeman shift for each trial.

First, without the filed turned on, we measure the position given by the dial gauge for a number of lines. Then, we turn on the magnetic field and measure the upper and lower bounds of each line. Because the lines do not appear to perfectly separate, and only a widening of the lines can be observed, we measure the top most and bottom most points of the line, and calculate the width of the line as the difference between the two.

The expected result is that the shift will be proportional to the field.

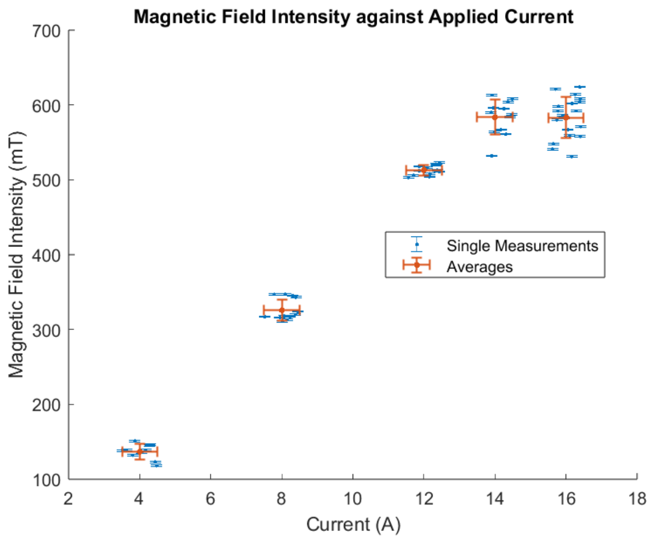

Figure 3: Magnetic Field Strength vs Current Jitter Plot

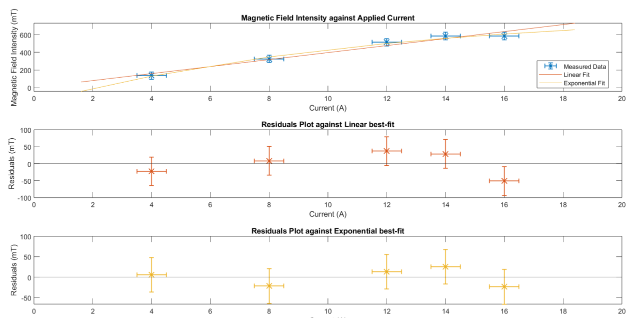

Figure 4: Magnetic Field Strength vs Current Regression Plots for Linear and Exponential fits

Trial:	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Field Value:	151 ± 0.5	146 ± 0.5	145 ± 0.5	139 ± 0.5	139 ± 0.5
Trial:	Trial 6	Trial 7	Trial 8	Trial 9	Average
Field Value:	138 ± 0.5	136 ± 0.5	132 ± 0.5	123 ± 0.5	136.7 ± 10

Trial	Line Position	Upper edge position	Lower edge position	Δ width	Zeeman Shift
1	0.23 ± 0.05	0.02 ± 0.05	-0.08 ± 0.05	0.10 ± 0.05	0.05 ± 0.05
2	0.0 ± 0.05	-0.22 ± 0.05	-0.31 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
3	-0.24 ± 0.05	-0.41 ± 0.05	-0.49 ± 0.05	0.08 ± 0.05	0.04 ± 0.05
4	-0.41 ± 0.05	-0.59 ± 0.05	-0.68 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
5	-0.6 ± 0.05	-0.73 ± 0.05	-0.82 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
6	-0.76 ± 0.05	-0.90 ± 0.05	-0.96 ± 0.05	0.06 ± 0.05	0.03 ± 0.05
Average					0.043 ± 0.01

Table 1: Combined Data for Part 1 and Zeeman Shift

3 Results

3.1 Part 1: Observing the Zeeman split perpendicularly to the magnetic field

Figure 3 shows the magnetic field strength as a function of the current supplied to the electromagnet, showing the averages and standard deviations at each measured current value. Then, using these averages, figure 4 applies a linear and exponential fit to the data. These fits are given by:

$$B = (1 \pm 51) + (39.6 \pm 4.4)I \ [mT] \quad \chi^2/\nu = 40.0/3 = 13.3$$
$$B = (-1040 \pm 150)exp(-0.089 \pm 0.051 * I) + (860 \pm 260) \ [mT]$$
$$\chi^2/\nu = 17.7/2 = 8.85$$

The linear fit has a reduced chi-squared value of 13.3, which is quite high and the residuals show a clear downards-curving pattern, showing that a linear fit is not. The exponential fit has a lower reduced chi-squared value of 8.85 and the residuals show a more random pattern, indicating that the exponential fit is a better fit to the data.

3.1.1 Trial 1: 4A current

After adjusting the magnet poles, several measurements of the magnetic filed intensity were taken using the Gauss meter. The results are shown in the table below.

These measurements result in an average value of: $B = 136.70 \pm 0.16$ mT.

The field current and pole positions were not changed throughout the experiment, so we assume the magnetic field intensity to be constant.

Trial:	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Field Value:	347 ± 0.5	345 ± 0.5	343 ± 0.5	347 ± 0.5	320 ± 0.5

Trial:	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Field Value:	324 ± 0.5	318 ± 0.5	317 ± 0.5	318 ± 0.5	317 ± 0.5

Trial:	Trial 11	Trial 12	Trial 13	Average
Field Value:	316 ± 0.5	313 ± 0.5	311 ± 0.5	325.85 ± 14

Table 2: Magnetic Field Intensity in mT

Trial	Line Position	Upper edge position	Lower edge position	Δ width	Zeeman Shift
1	0.23 ± 0.05	0.03 ± 0.05	-0.09 ± 0.05	0.12 ± 0.05	0.06 ± 0.05
2	0.0 ± 0.05	-0.21 ± 0.05	-0.33 ± 0.05	0.12 ± 0.05	0.06 ± 0.05
3	-0.24 ± 0.05	-0.41 ± 0.05	-0.50 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
4	-0.41 ± 0.05	-0.57 ± 0.05	-0.66 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
5	-0.6 ± 0.05	-0.73 ± 0.05	-0.83 ± 0.05	0.10 ± 0.05	0.05 ± 0.05
6	-0.76 ± 0.05	-0.87 ± 0.05	-0.96 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
Average					0.051 ± 0.01

Table 3: Combined Data for Part 1 and Zeeman Shift

3.2 Trial 2: 8A current

3.3 Trial 3: 12A current

3.4 Trial 4: 14A current

3.5 Trial 5: 16A current

3.6 Part 1: Observing the linearity of the Zeeman shift with respect to the field strength

Now, we plot the previouslt obtained values in a scatter plot and compare it to a linear fit.

The slope of the linear fit is obtained to be: $1.2660e - 04 \pm 8.2711e - 06 \frac{m}{T}$

Trial:	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Field Value:	512 ± 0.5	523 ± 0.5	521 ± 0.5	520 ± 0.5	518 ± 0.5

Trial:	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Field Value:	516 ± 0.5	513 ± 0.5	511 ± 0.5	508 ± 0.5	506 ± 0.5

Trial:	Trial 11	Trial 12	Average
Field Value:	504 ± 0.5	503 ± 0.5	512.92 ± 6.5

Table 4: Magnetic Field Intensity in mT

Trial	Line Position	Upper edge position	Lower edge position	Δ width	Zeeman Shift
1	0.23 ± 0.05	0.06 ± 0.05	-0.13 ± 0.05	0.19 ± 0.05	0.095 ± 0.05
2	0.0 ± 0.05	-0.19 ± 0.05	-0.33 ± 0.05	0.14 ± 0.05	0.07 ± 0.05
3	-0.24 ± 0.05	-0.38 ± 0.05	-0.51 ± 0.05	0.13 ± 0.05	0.065 ± 0.05
4	-0.41 ± 0.05	-0.56 ± 0.05	-0.70 ± 0.05	0.14 ± 0.05	0.07 ± 0.05
5	-0.6 ± 0.05	-0.73 ± 0.05	-0.85 ± 0.05	0.12 ± 0.05	0.06 ± 0.05
6	-0.76 ± 0.05	-0.88 ± 0.05	-0.97 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
Average					0.067 ± 0.01

Table 5: Combined Data for Part 1 and Zeeman Shift

Trial:	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Field Value:	532 ± 0.5	567 ± 0.5	584 ± 0.5	561 ± 0.5	564 ± 0.5
Trial:	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Field Value:	595 ± 0.5	613 ± 0.5	587 ± 0.5	608 ± 0.5	604 ± 0.5
Trial:	Trial 11	Trial 12	Average		
Field Value:	590 ± 0.5	596 ± 0.5	583.42 ± 22		

Table 6: Magnetic Field Intensity in mT

Trial	Line Position	Upper edge position	Lower edge position	Δ width	Zeeman Shift
1	0.23 ± 0.05	0.08 ± 0.05	-0.11 ± 0.05	0.19 ± 0.05	0.095 ± 0.05
2	0.0 ± 0.05	-0.19 ± 0.05	-0.37 ± 0.05	0.18 ± 0.05	0.09 ± 0.05
3	-0.24 ± 0.05	-0.40 ± 0.05	-0.54 ± 0.05	0.14 ± 0.05	0.07 ± 0.05
4	-0.41 ± 0.05	-0.59 ± 0.05	-0.67 ± 0.05	0.08 ± 0.05	0.04 ± 0.05
5	-0.6 ± 0.05	-0.72 ± 0.05	-0.83 ± 0.05	0.11 ± 0.05	0.055 ± 0.05
6	-0.76 ± 0.05	-0.86 ± 0.05	-0.96 ± 0.05	0.10 ± 0.05	0.05 ± 0.05
Average					0.067 ± 0.02

Table 7: Combined Data for Part 1 and Zeeman Shift

Trial:	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Field Value:	580 ± 0.5	592 ± 0.5	624 ± 0.5	571 ± 0.5	567 ± 0.5
Trial:	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Field Value:	559 ± 0.5	548 ± 0.5	558 ± 0.5	583 ± 0.5	621 ± 0.5
Trial:	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15
Field Value:	608 ± 0.5	614 ± 0.5	604 ± 0.5	598 ± 0.5	602 ± 0.5
Trial:	Trial 16	Trial 17	Average		
Field Value:	531 ± 0.5	541 ± 0.5	583.11 ± 26.37		

Table 8: Magnetic Field Intensity in mT

Trial	Line Position	Upper edge position	Lower edge position	Δ width	Zeeman Shift
1	0.23 ± 0.05	0.09 ± 0.05	-0.14 ± 0.05	0.23 ± 0.05	0.115 ± 0.05
2	0.0 ± 0.05	-0.19 ± 0.05	-0.35 ± 0.05	0.16 ± 0.05	0.08 ± 0.05
3	-0.24 ± 0.05	-0.40 ± 0.05	-0.51 ± 0.05	0.11 ± 0.05	0.055 ± 0.05
4	-0.41 ± 0.05	-0.56 ± 0.05	-0.68 ± 0.05	0.12 ± 0.05	0.06 ± 0.05
5	-0.6 ± 0.05	-0.74 ± 0.05	-0.83 ± 0.05	0.09 ± 0.05	0.045 ± 0.05
6	-0.76 ± 0.05	-0.87 ± 0.05	-0.97 ± 0.05	0.10 ± 0.05	0.05 ± 0.05
Average					0.067 ± 0.02

Table 9: Combined Data for Part 1 and Zeeman Shift

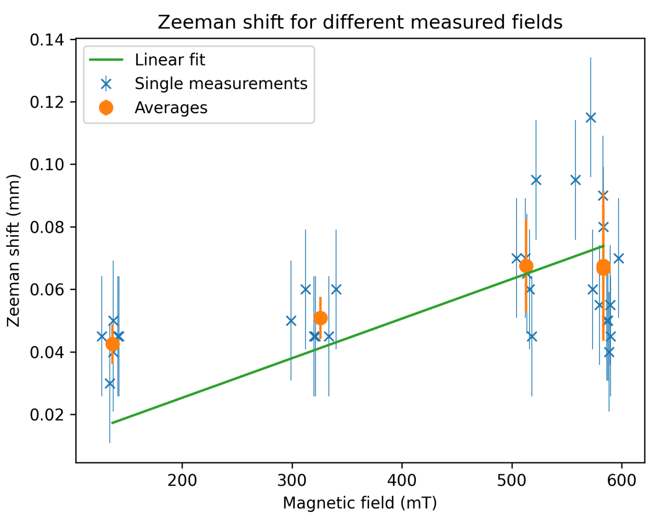

Figure 5: Zeeman Shift vs Magnetic Field Intensity

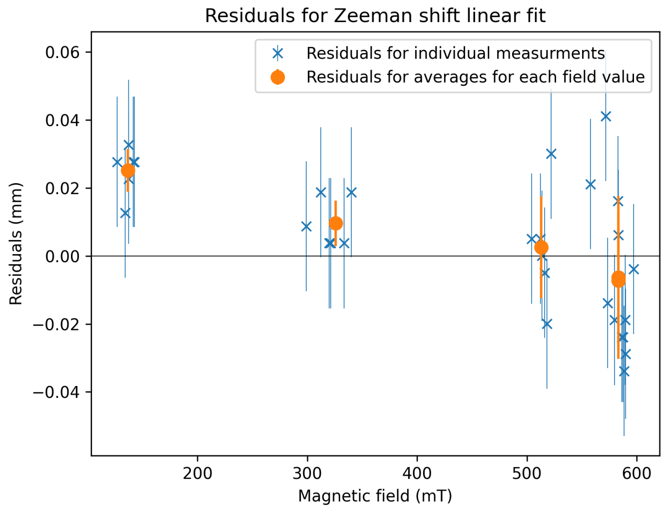

Figure 6: Zeeman Shift vs Magnetic Field Intensity

The Chi squared value is 34.6, and the reduced Chi squared value is 1.19, which indicated a good fit.

3.7 Part 2: measuring the $\frac{e}{m}$ relationship.

Looking back at equation 5, we can see that the value of $\frac{e}{m}$ can be obtained from the slope of the linear fit of the Zeeman shift vs the magnetic field intensity in the previous section.

Before doing this, we must obtain the value of ΔA (the separation of successive interference lines). Because this qunatity is not a property of the line itself, but rather of all of the collection of lines, we obtain all of the separations for the lines without amgnetic field and then use the average of these values as a constant in this equation.

Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
0.23 ± 0.05	0.24 ± 0.05	0.17 ± 0.05	0.19 ± 0.05	0.16 ± 0.05	0.20 ± 0.03

Table 10: Line separation for different line position measurments.

We use the uncertainty propagation formula to obtain the uncertainty to obtain the uncertanties in the $\frac{e}{m}$ value by considering the uncertanties in the slope obtained in the previous section, and the uncertanties in the ΔA value.

We obtain the value of $\frac{e}{m} = (2.7883 \pm 0.4562)e + 07$

4 Conclusion

Conclusion