Measuring the horizontal distance travelled by a tennis ball after falling from a slope at different angles and heights.

Internal Assessment

Physics SL

1. Introduction

1.1. Rationale

Ballistics is an rather interesting field of physics. It allows for precise prediction of movement object based on only setup when it is launched. Furthermore it is connected with optimalization, which I am obsessed with, and allows for usage of very advanced mathematical apparatus. This branch of physics has many interesting uses ranging from sports and games through warfare to spaceship crafting. It was used hundreds of years ago to calculate the angles at which to launch the cannonballs for the missiles to cause the most destruction to the target and now it is used to calculate the trajectories of rockets and for optimalization of the form of kicking or throwing balls and other objects by sportsmen. It has even found its uses in video games, for example in Minecraft, where some player found and optimal angle for usage of fireworks, which serve as fuel, during the flight using elytra – wings.

The problem I chose is a more advanced version of the very old problem "at which angle should I throw the ball for it to fly the furthest?", which then evolved into many other problems, some of including that the thrower is standing on a elevated ground, while other include the ground being sloped or not even. My version of the problem is including a slope, in which the higher the angle, the faster the higher the kinetic energy of the ball, but also the ratio of horizontal part to the vertical part of the speed vector of the ball is smaller, this is interesting, as this poses an challenge to find the optimal angle – the one at which the greatest horizontal displacement it reached.

1.2. Description of the research problem

The analysed model consists of a hollow sphere being thrown off a ramp placed a certain height above ground at a certain angle, as shown in Figure 1. Gravitational acceleration, radius and mass of the ball do not have any effect on the result of the experiment in purely theoretical setting, which will be proven later in the paper. The independent, dependent and control variables are as follows:

Independent variables:

1. Slope angle – labelled "α"
2. Slope elevation from the ground – labelled "h"

Dependent variables:

1. Horizontal distance travelled by the ball from the end of the slope – labelled "d"

Control variables:

1. Length of the slope – labelled "l".

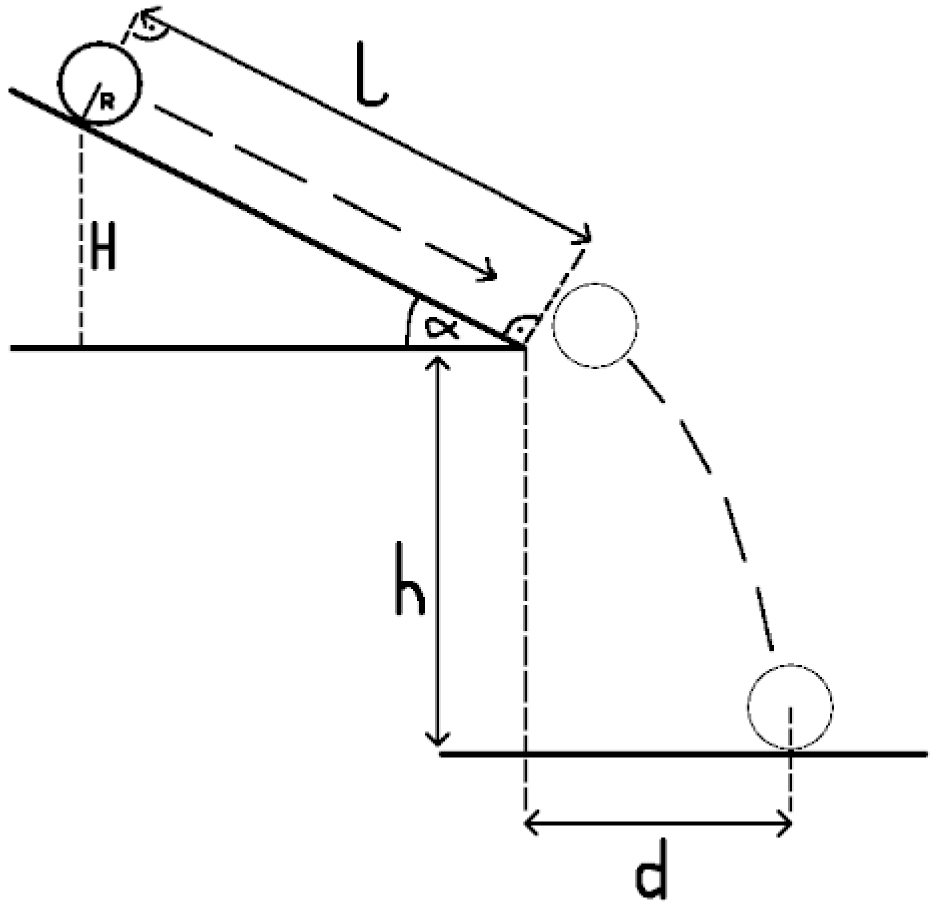


Figure 1 drawing of the model

The goal of the research is to find optimal angle of the slope (α) at which the greatest horizontal displacement is achieved (d) for varying height (h).

2. Theoretical predictions

2.1. Formulas

Formula for the speed of an object rolling down a slope:

$$E_k = \frac{\mathrm{Mv}^2 + \mathrm{I}\omega^2}{2}$$

$$E_p = hMg$$

Relationship between angular speed and linear velocity in rolling without slipping movement:

$$v = \omega \mathrm{R}$$

Moment of inertia formulas:

$$Moment\ of\ inertia\ of\ a\ hollow\ sphere\ (I) = \frac{2}{3}\mathrm{MR}^2$$

Distance travelled :

$$d = v_h * t$$

Components of velocity vector:

$$Horizontal\ component\ of\ the\ velocity\ vector\ (v_h) = \mathrm{v} * \cos\alpha$$

$$Vertical\ component\ of\ the\ velocity\ vector\ (v_v) = \mathrm{v} * \sin\alpha$$

2.2. Finding the formula for horizontal displacement of the ball

To find d the time of free fall of the ball after losing contact with the slope and vertical component of the velocity vector are needed. Both depend only on the angle and magnitude of the velocity vector after losing contact with the slope. The angle of the vector is obviously α. The magnitude can be calculated in the following way:

$$E_p = \frac{\mathrm{Mv}^2 + \mathrm{I}\omega^2}{2}$$

$$\mathrm{Mgl} * \sin\alpha = \frac{\mathrm{Mv}^2 + \frac{2}{3}\mathrm{MR}^2\omega^2}{2}$$

$$\mathrm{gl} * \sin\alpha = \frac{\mathrm{v}^2 + \frac{2}{3}\mathrm{v}^2}{2}$$

$$\mathrm{gl} * \sin\alpha = \frac{5}{6}v^2$$

$$\mathrm{v} = \sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}}$$

$$v_v = \sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}} * \sin\alpha \qquad v_h = \sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}} * \cos\alpha$$

After the magnitude is calculated, the time of the fall is given as:

$$\frac{t^2 g}{2} + t v_v - h = 0$$

And from quadratic equation, since t has to be positive, follows:

$$t = \frac{-v_v + \sqrt{v_v^2 + 2gh}}{g}$$

Substituting the value of v_v:

$$t = \frac{-\sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}} * \sin\alpha + \sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}\sin^2\alpha + 2gh}}{g}$$

From the formula $d = tv_h$ comes:

$$d = \frac{-\sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}} * \sin\alpha + \sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}\sin^2\alpha + 2gh}}{g} * \sqrt{\frac{6\mathrm{gl} * \sin\alpha}{5}} * \cos\alpha$$

Which can be shortened to:

$$\begin{aligned} d &= \left(-\sqrt{\frac{6\mathrm{l} * \sin^3\alpha}{5}} + \sqrt{\frac{6\mathrm{l} * \sin^3\alpha}{5} + 2h}\right)\sqrt{\frac{6\mathrm{l} * \sin\alpha}{5}} * \cos\alpha \\ &= \cos\alpha\left(\sqrt{\frac{36l^2\sin^4\alpha}{25} + \frac{60lh\sin\alpha}{25}} - \frac{6\mathrm{l} * \sin^2\alpha}{5}\right) \\ &= \frac{6l}{5}\cos\alpha\left(\sqrt{\sin^4\alpha + \frac{5h\sin\alpha}{3l}} - \sin^2\alpha\right) \end{aligned}$$

2.3. Conclusion from theoretical calculations

From the final formula it can be deduced that the optimal angle of the slope (the one with highest horizontal displacement) depends only on the ratio of height divided by length. Unfortunately the derivative of this formula is a very complicated function and will not clearly show the optimal angle. On the other side it can be printed by a calculator, giving a great approximation, which can be seen in figure 2.

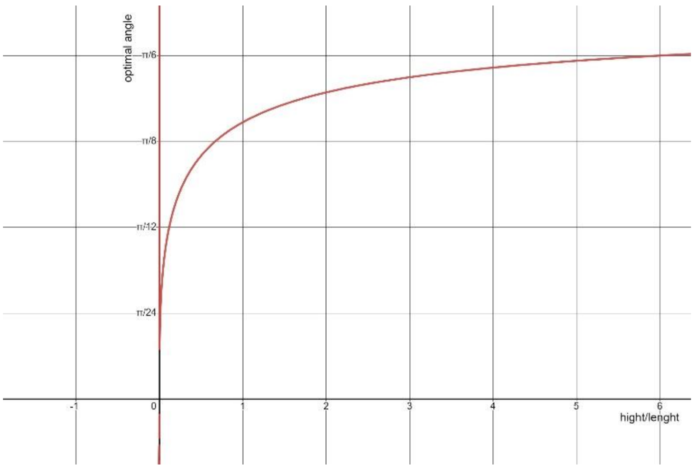


Figure 2 relation of height and length to the optimal angle
Source: https://www.desmos.com/calculator

3. The Experiment

3.1. The procedure

The experiment was conducted by constructing a setup based on Figure 1. A wooden plank with a mark on 70 cm interval (every measurement was from this slope length) from its base was placed on a moveable desk and taped to it at one side. Then using a level a point on the floor directly under the end of the slope was found and from it A4 paper stripped with lines at 1 cm interval, with one in every 5 lines highlighted for easier reading, was taped to the floor parallelly to the plank. Then books were put under the slope for it to achieve a certain angle, which was calculated using trigonometry – the 70 cm mark had to be a certain distance above the desk.

List of equipment used:

- desk with adjustable height
- one meter plank
- measuring tape
- level
- tennis ball
- A4 paper
- books
- sticky tape
- slow motion camera
- marker and a pen

After the setup was created, the author dropped the ball from the slope 75 times and recorded every drop on a slow motion camera. Then the footage was reviewed and distances were input into an excel file. The ball was dropped from 3 different heights: 0.8, 0.9 and 1.0 m, and at 5 different angles: 10, 20, 30, 40 and 50 degrees, totalling 15 combinations, at which 5 measurements were made for precision.

3.2. The results

Height	0.80m			Measurement		
		1	2	3	4	5
Angle						
10°		42	43	42	42	43
20°		56	52	54	58	56
30°		55	57	58	56	57
40°		53	52	54	54	54
50°		45	45	46	46	47

Height	0.90m			Measurement		
		1	2	3	4	5
Angle						
10°		46	45	45	45	44
20°		61	62	62	60	60
30°		63	62	61	61	61
40°		57	57	58	59	58
50°		48	50	49	49	49

Height	1.00m			Measurement		
		1	2	3	4	5
Angle						
10°		46	47	48	46	47
20°		60	61	62	59	61
30°		64	64	64	65	64
40°		57	57	57	56	55
50°		57	53	54	54	55

Measurement error: ±0.03 m

Figure 3 results of the experiment, the results are in centimetres.

The results of the experiment are shown in figure 3.

4. Analysis

4.1. Refined data

Collected data now has to be processed. First of all, the averages of each five measurements have to be calculated, as they will be used in graphing and analysis. Second thing to be done is the calculation of the uncertainty of the measurement. The measuring instrument used should cause 0.01 m uncertainty, but there are other factors, such as slight misplacement of the ball or slight imprecision in the angle. From all the measurements, the highest difference between singular one and the average of five was 2.8 cm, so assuming uncertainty of the measurement of 0.03 m is a reasonable and safe choice. The processed data is presented in figure 4. Also the averages for each angle at each height and vice versa were added.

Uncertanity: ±0.03 m					Angle				Avarge
			10°	20°	30°	40°	50°		
	0.80 m		42	55	57	53	46		51
Height	0.90 m		45	61	62	58	49		55
	1.00 m		47	61	64	56	55		57
	Avarge		44,667	59	61	55,667	50		

Figure 4 refined results

4.2. Interpretation of the results

Figure 5 shows the basic graphical representation of the results. Uncertanities were not added because the graph would be hard to read.

As can be seen balls dropped from 0.8 meter had the least horizontal displacement. Surprisingly balls dropped from 1.0 m travelled less distance than ones dropped from 0.9 m at certain angles, but overall with increase of height the distance increased. The authors first explanation for this would be a phenomena called Magnus effect, which causes a rotating body moving in fluid to be accelerated in the direction of the spin due to difference of pressure of in this case air on both sides of the object, but that does not make much sense as the ball would have to change the direction of the horizontal part its velocity in last 10 cm of the fall. Furthermore at 50 degrees incline the ball travelled significantly longer distance at 1.0 meter than at 0.9 m, which is contractionary with such explanation, as the ball would spin even faster and the pull coming from Magnus effect would be even higher in such case. Due to that fact, the explanation for such result has to be measurement error.

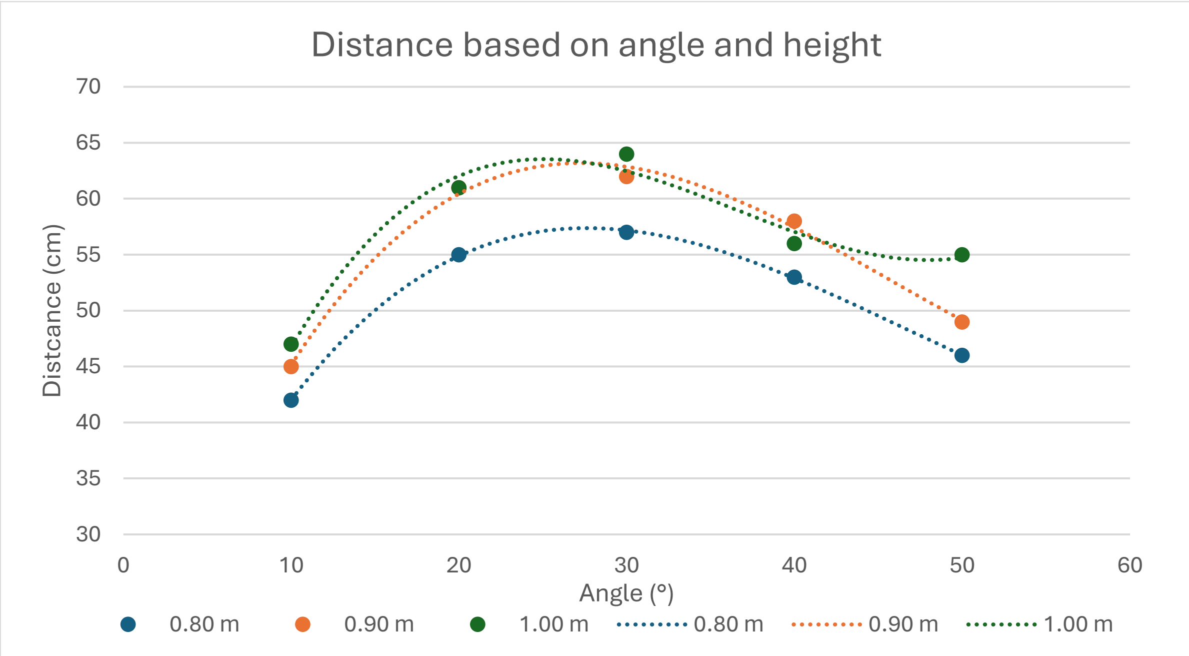


Figure 5 distance based on angle and height

The highest distances were for 20 and 30 degree angles at all heights, indicating that the optimal angle is between those two for most heights, which is also confirmed by the averages for those angles.

4.3. Comparation to theoretical predictions

Figure 6 shows theoretical predictions for the results based on formula calculated in 2.3.

					Angle		
			10°	20°	30°	40°	50°
	0.8		45	55	55	49	41
Lenght	0.9		48	59	59	54	44
	1.0		51	63	63	57	48

Figure 6 theoretical predictions of the results

Following graphs (Figures 7, 8 and 9) compare the results of the experiment and its polynomial best fit line with theoretical predictions of the results for each height. At low angles the distance turned out to be shorter than in predictions, this can be attributed to air resistance, because the ball is moving at a degree close to horizontal and the drag force is acting backwards on it. Similarly at greater angles the distance is higher due to this same force increasing the time of the fall (indeed, the angle of the velocity vector to the horizontal plane quickly increases with the time of fall, making the drag force act upwards).

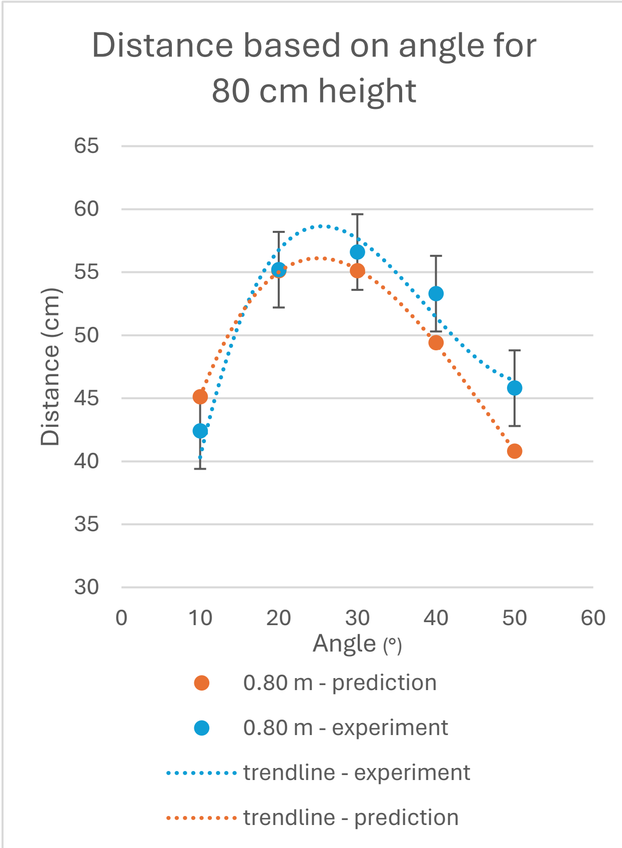
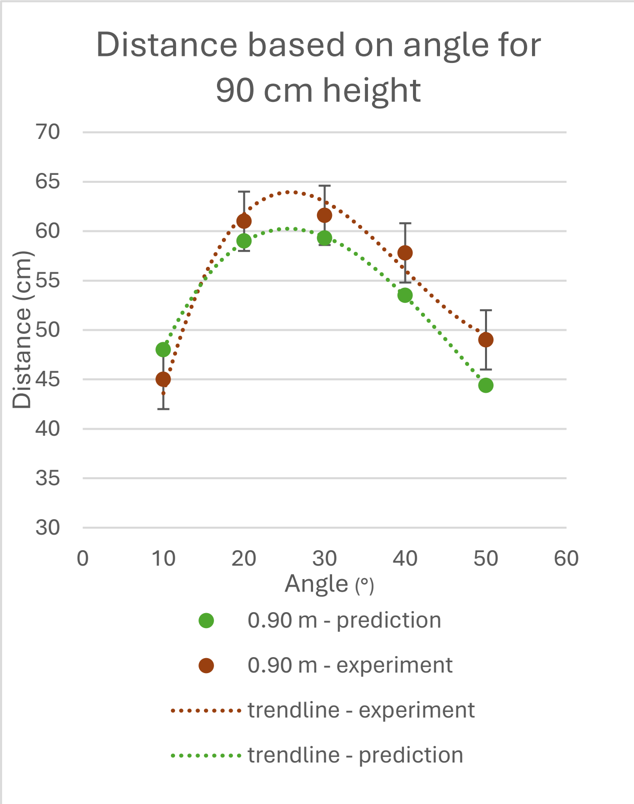


Figure 7 distance based on angle for 80 cm height

Interesting things happen at the hight of 1.0 meter. The sudden drop in distance at 40 degree angle has to be attributed to measurement error, as if it was 3 cm higher, which is in the uncertainty range, it would fit the pattern set by results at other heights and there is no explanation for such an anomaly. Another difference is that the result at 20 degrees is smaller than in theory, which does not occur in the two other cases, this again is connected to drag force and Magnus effect, which become more significant with heigh of drop.

From the trendlines the optimal angles for each height can be deduced. At 80 cm the highest point on the best fit line is at around 25 degrees, at 90 cm it is around 26 degrees and at 1.0 meter it reaches about 28 degrees. This confirms the prediction that the greater the ratio of height to the length of the slope, the higher the optimal angle will be.



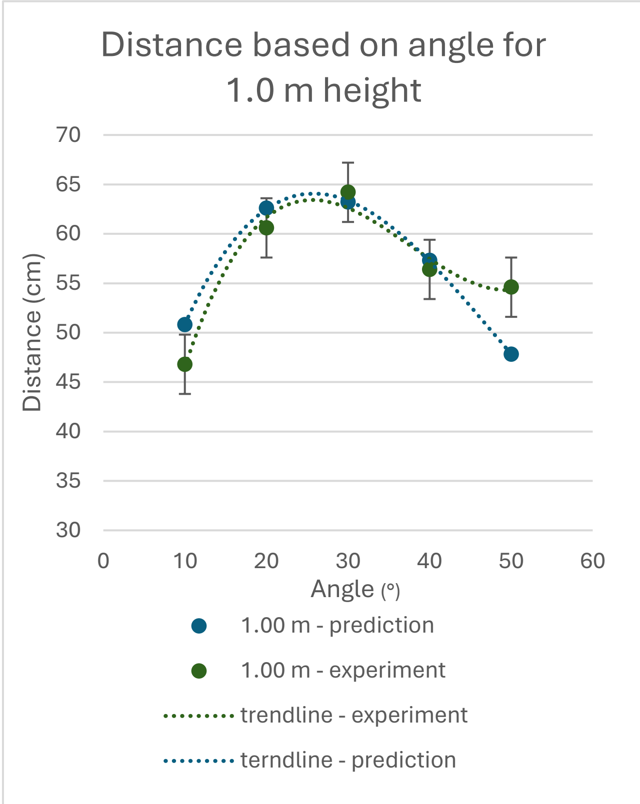


Figure 8 distance based on angle for 80 cm height

Figure 9 distance based on angle for 80 cm height

5. Conclusion

The experiment confirmed the predictions that the distance increases with height and that optimal angle lies between 25 and 30 degrees. There were some differences from the theoretical results that originate from drag force and Magnus effect. Optimal angle increases with the ratio of height of the slope to its length.

6. Evaluation

I believe the experiment was conducted correctly. There was not to much measurement error and the results were generally consistent. Although the results are not matching the predicted ones, the theory does not include many factors such as air resistance. This disproportion of results led the author to investigate the topic further in the search for its causes, which were found in the analysis. The experiment was performed safely, as the author removed all glass objects from the room, furthermore the area where ball dropped was separated by physical barriers from the rest of the room. However, there were small mistakes and space for improvement.

Problem 1: The ball was located on the slope and let off by hand. This could cause change in its trajectory due to accidental contact with it, therefore causing the result to be slightly different.

Improvement 1: Usage of some sort of trigger mechanism would be helpful. It would eliminate the problem mentioned above and furthermore the starting point would be always the same, as placing the ball by hand, even onto the market spot, can never be precise.

Problem 2: Although the author thought that it would be much more interesting to see the results of the experiment performed in everyday conditions and then compare them to theory, those conditions effected the result of the experiment.

Improvement 2: The solution to this problem would be usage of vacuum chamber, or even some smoother ball, as tennis ball is designedly covered with felt to increase effects of air resistance on it, like ping pong ball. The ball could also be heavier (maintaining the same shape), making the forces acting on it less impactful to its trajectory.

Problem 3: The reading of distance after fall, although recorded in 240 FPS, was sometimes difficult as in some cases there were no frames in which the ball touched the tape (then the author took measurement based on last frame before and first after the object touched the ground) or the frame was blurred, this led to slight imperfections in the readings once again.

Improvement 3: Author found two solutions to this problem, first being very expensive and huge overshoot – buying a much better camera. The second solution is much more creative and involves putting some type of powder (for example flour) or some material that will have a mark left on it on the floor and then checking the distance to the first marking made by the ball on the material or powder.

Bibliography

- Tsokos, K. A.. (2014). *Physics for the IB Diploma* (6th ed.). : Cambridge University Press.
- https://www.desmos.com/calculator