Numerical Methods

Report

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Exercise: **Theory of Errors**

Group: 2, Team:

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Task 1 – Calculate the Lagrange remainder

Data:

$$n = 2, \qquad a = 0.5, \qquad x_0 = 1, \qquad b = 1.5$$

$$f(x) = \frac{1}{2\sin(x)}$$

Calculations:

$$r_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x - x_0)^{n+1}$$

$$r_2(x) = \frac{f^{(3)}(z)}{3!}(x-1)^3$$

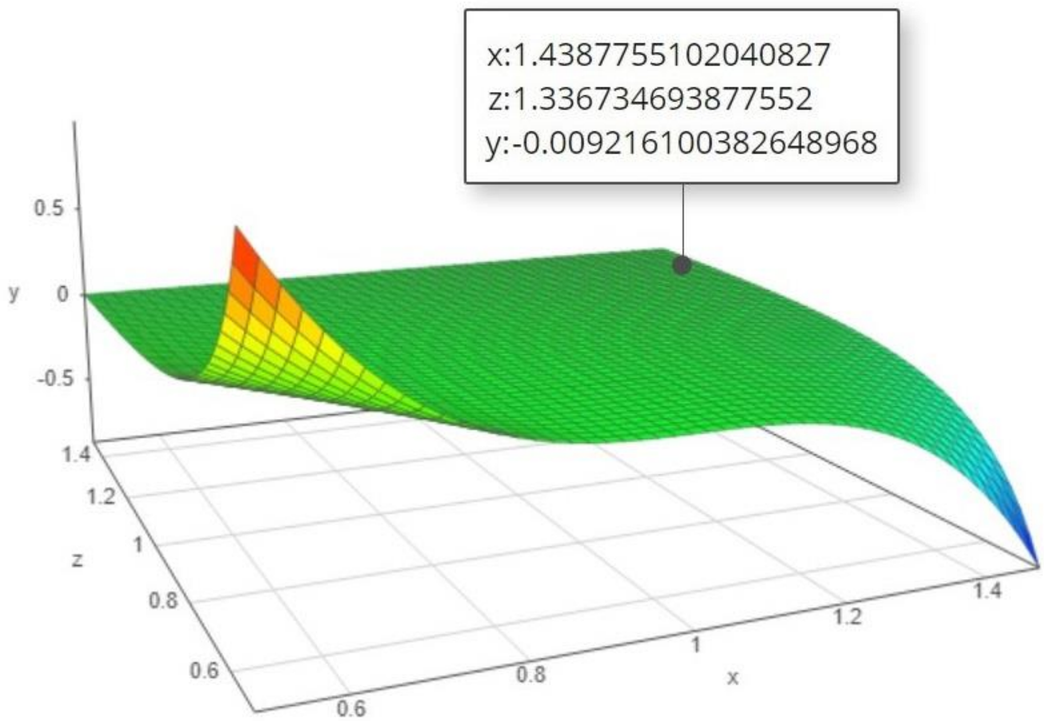
$$f'(x) = -\frac{\cos(x)}{2sin^2(x)}$$

$$f''(x) = \frac{1}{2\sin(x)} + \frac{\cos^2(x)}{sin^3(x)}$$

$$f'''(x) = -\frac{5\cos(x)}{2sin^2(x)} - \frac{3\cos^3(x)}{sin^4(x)}$$

$$\boldsymbol{r_2}(\boldsymbol{x}, \boldsymbol{z}) = -\frac{5\cos(z)}{2sin^2(z)} - \frac{3\cos^3(z)}{sin^4(z)} * \frac{1}{6} * (x-1)^3$$

Conclusion:



For the second degree Taylor Polynomial of $\mathbf{1/2\sin(x)}$ on the interval $\mathbf{0.5 \leq x \leq 1.5}$, centered at $\boldsymbol{x_0 = 1}$, the Lagrange remainder is dependent on the argument of the function $\boldsymbol{x}$ and the value $\boldsymbol{z}$, which sits between $\boldsymbol{x_0}$ and $\boldsymbol{x}$.

Task 2 a) - Find least upper bound of absolute error of object volume

Data:

$V_1 = ab^2$

$V_2 = \pi r^2 h$

$a = 2.37cm \ \pm 0.008cm$

$b = 3.15cm \ \pm 0.002cm$

$r = 0.85cm \ \pm 0.001cm$

$h = 5.10cm \ \pm 0.03cm$

$\pi = 3.14cm \ \pm 0.0016cm$

Calculations:

$$V_1 = ab^2$$

$$\frac{\partial V_1}{\partial a} = b^2 = 9.9225$$

$$\frac{\partial V_1}{\partial b} = 2ab = 14.931$$

$$\Delta V_1 = \left|\frac{\partial V_1}{\partial a}\right| * |\Delta a| + \left|\frac{\partial V_1}{\partial b}\right| * |\Delta b| = 9.9225 * 0.008 + 14.931 * 0.002 = 0.109cm^3$$

$$V_1 = ab^2 = 23.516 \pm 0.109cm^3$$

$$V_2 = \pi r^2 h$$

$$\frac{\partial V_2}{\partial \pi} = r^2 = 3.685$$

$$\frac{\partial V_1}{\partial r} = 2\pi r h = 27.224$$

$$\frac{\partial V_1}{\partial h} = \pi r^2 = 2.269$$

$$\Delta V_2 = \left|\frac{\partial V_2}{\partial \pi}\right| * |\Delta \pi| + \left|\frac{\partial V_2}{\partial r}\right| * |\Delta r| + \left|\frac{\partial V_2}{\partial h}\right| * |\Delta h| = 0.101 cm^3$$

$$V_1 = ab^2 = 11.57 \pm 0.101 cm^3$$

$$\boldsymbol{V = V_1 + V_2 = 35.086 \pm 0.209 cm^3}$$

The total volume of this geometric object is equal to $\mathbf{35.086 \pm 0.209} \boldsymbol{cm^3}$**.**

Task 2 b) - Find interval in which the volume of the object is contained

Data:

$$V_1 = ab^2$$

$$V_2 = \pi r^2 h$$

$$\underline{a} = 2.31, \qquad \overline{a} = 2.62$$

$$\underline{b} = 3.15, \qquad \overline{b} = 3.49$$

$$\underline{r} = 0.54, \qquad \overline{r} = 0.67$$

$$\underline{h} = 4.93, \qquad \overline{h} = 5.22$$

$$\underline{\pi} = 3.09, \qquad \overline{\pi} = 3.24$$

Calculations:

$$V_1 = ab^2$$

$$\underline{b^2} = 9.9225, \qquad \overline{b^2} = 12.1801$$

$$\underline{V_1} = \underline{a} * \underline{b^2} = 22.921, \qquad \overline{V_1} = \overline{a} * \overline{b^2} = 31.912$$

$$V_2 = \pi r^2 h$$

$$\underline{r^2} = 0.292, \qquad \overline{r^2} = 0.449$$

$$\underline{V_2} = \underline{\pi} * \underline{r^2} * \underline{h} = 4.448, \qquad \overline{V_2} = \overline{\pi} * \overline{r^2} * \overline{h} = 7.594$$

$$\boldsymbol{\underline{V} = \underline{V_1} + \underline{V_2} = 27.369cm^3}$$

$$\boldsymbol{\overline{V} = \overline{V_1} + \overline{V_2} = 39.506cm^3}$$

The total volume of this geometric object is contained within the interval **[27.369;39.506].**

Task 2 c) – Find the inverse error

$$V = ab^2 + \pi r^2 h$$

$$a = 2cm$$

$$b = 4cm$$

$$\pi = 3.14cm$$

$$r = 1cm$$

$$h = 5cm$$

$$\frac{\partial V}{\partial r} = 2\pi r h = 31.4cm^3$$

$$\frac{\partial V}{\partial h} = \pi r^2 = 3.14cm^3$$

$$\frac{\partial V}{\partial \pi} = \pi r^2 = 3.14cm^3$$

$$\frac{\partial V}{\partial a} = b^2 = 16cm^3$$

$$\frac{\partial V}{\partial b} = 2ab = 16cm^3$$

$$n = 5$$

$$\Delta x = \frac{\Delta V}{n|\frac{\partial f}{\partial x_i}|}$$

$$\Delta a = 0.00125$$

$$\Delta b = 0.00125$$

$$\Delta \pi = 0.004$$

$$\Delta r = 0.00063694267$$

$$\Delta h = 0.00636942675$$