Discrete Mathematics - Solutions

Student Name

April 8, 2025

1   Set 1: Fundamentals

Exercise 1.1

(Regions formed by n lines on a plane)

Exercise 1.2

(Fibonacci sequence properties)

  (i) *Sum of first n+1 Fibonacci numbers*

  (ii) *Divisibility property*

  (iii) *Upper bound*

Exercise 1.3

(Tournament graph property - two-step reachability)

Exercise 1.4

(Hamiltonian path in tournaments)

Exercise 1.5

(Virus spread on chessboard)

Exercise 1.6

(Equal number of acquaintances)

Exercise 1.7

(Ambassadors and flags problem)

Exercise 1.8

(Subsequence with sum divisible by n)

Exercise 1.9

(Subset containment in large families)

Exercise 1.10

(Element in at least half of large subsets)

Exercise 1.11

(Covering chessboard with 2×2 and 3×3 tiles)

Exercise 1.12

(Covering 2×2 chessboard with L-shaped tiles)

2   Set 2: Binomial Coefficients and Counting

Exercise 2.1

(Non-attacking rooks on n×n chessboard)

Exercise 2.2

(Non-attacking rooks on n×m chessboard)

Exercise 2.3

(Recurrence relations)

   (i) *Words without consecutive 1s*

   (ii) *Domino tilings of 2×n rectangle*

Exercise 2.4

(Solutions to equation x+x+x+x=7)

   (i) *Non-negative integer solutions*

   (ii) *Positive integer solutions*

Exercise 2.5

(Ways to cut out k×k squares from m×n chocolate)

Exercise 2.6

(Summation rule for upper index)

Exercise 2.7

(Parallel summation rule)

Exercise 2.8

(Monotonic functions count)

Exercise 2.9

(k-element subsets without adjacent numbers)

Exercise 2.10

(Combinatorial identity proof)

Exercise 2.11

(Multiple combinatorial identities)

(a) First identity

(b) Second identity

(c) Third identity

3   Set 3: Stirling Numbers and Catalan Numbers

Exercise 3.1

(Unimodality of Stirling numbers of the second kind)

Exercise 3.2

(Bell number recurrence relation)

Exercise 3.3

(Stirling numbers identity)

Exercise 3.4

(Numerical procedure and Bell numbers)

Exercise 3.5

(Partitions without adjacent numbers)

Exercise 3.6

(Binary trees and Catalan numbers)

Exercise 3.7

(Triangulations of a convex polygon)

   (i) Total triangulations

   (ii) Triangulations with boundary edges

Exercise 3.8

(Labeled trees count)

4   Set 4: Stirling Numbers and Bell Numbers

Exercise 4.1

(Calculate S(n,2))

Exercise 4.2

(Permutations by cycle structure)

Exercise 4.3

(Combinatorial identity for Stirling numbers)

Exercise 4.4

(Multinomial coefficient value)

Exercise 4.5

(Harmonic number sum identity)

Exercise 4.6

(Falling and rising factorial identities)

5   Set 5: Inclusion-Exclusion Principle and Counting

Exercise 5.1

(Prove inclusion-exclusion principle by induction)

Exercise 5.2

(Count of surjections)

Exercise 5.3

(Sequences with each number appearing twice without adjacency)

Exercise 5.4

(Derangement recurrence relation)

Exercise 5.5

(Combinatorial representations)

Exercise 5.6

(Partition procedures)