Zad przygotowawcze

zał:

$$u(t) = A \cdot \sin(2\pi f t) \; ; \qquad f = \frac{1}{T_0}$$

$$U_{sk} = \sqrt{\frac{1}{T_0} \int_0^{T_0} u^2(t)\,dt} \quad \Rightarrow \int_0^{T_0} u^2(t)\,dt = \int_0^{T_0} A^2 \sin^2(2\pi f t) = A^2 \int_0^{T_0} \sin^2(2\pi f t)$$

$$\Downarrow$$

$$U_{sk} = \sqrt{\frac{1}{T_0} \cdot \frac{A^2}{2f}} = \left| \frac{A\sqrt{2}}{2} \right|$$

$$= \left| \begin{array}{l} 2\pi f t = x \\ dt = \frac{1}{2\pi f} dx \\ 0 \to 0 \\ T_0 \to 2\pi \end{array} \right| = \frac{A^2}{2\pi f} \int_0^{2\pi} \sin^2 x \, dx = \frac{A^2}{2f}$$

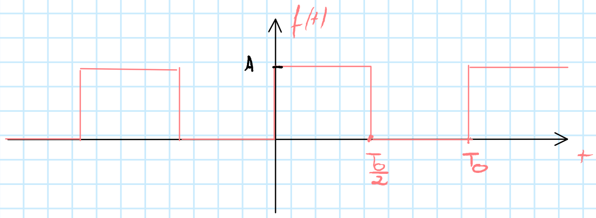
$$\int_0^{2\pi} \sin^2 x \, dx = \pi$$

Zad domowe

Tuesday, October 29, 2024 6:56 PM

Zad.

Przebieg prostokątny



, gdzie:

$$f(t) = \begin{cases} 0 & , \text{dla } t \in \left((2k-1)\cdot \frac{T_0}{2}, k \cdot T_0 \right), & k \in \mathbb{Z} \\ \frac{1}{2} A & ; \text{dla } t = k \cdot \frac{T_0}{2} ; & , k \in \mathbb{Z} \\ A & , \text{dla } t \in \left(k \cdot T_0 , (2k+1) T_0 \right) & , k \in \mathbb{Z} \end{cases}$$

Analityczny wzór na elementy nieskończonego szeregu Fouriera:

1) $$a_0 = \frac{1}{T_0} \int_0^{T_0} f(t)\,dt = \frac{1}{T_0} \cdot At \Big|_0^{\frac{T_0}{2}} = \frac{A}{2}$$

2) $$a_n = \frac{1}{T_0} \int_0^{T_0} f(t) \cos(\omega_0 n t)\,dt = \frac{1}{T_0} \cdot \left(\int_0^{\frac{T_0}{2}} A \cos(\omega_0 n t)\,dt + 0 \right) = \left| \begin{matrix} \omega_0 n t = x \\ dt = \frac{1}{\omega_0 n} dx \\ 0 \to 0 \\ \frac{T_0}{2} \to \pi n \end{matrix} \right| =$$

$$= \frac{1}{\pi n} \int_0^{\pi n} A \cdot \cos(x)\,dx = \frac{A}{\pi n} \sin(x) \Big|_0^{\pi n} = 0$$

3) $$b_n = \frac{1}{T_0} \int_0^{T_0} f(t) \sin(\omega_0 n t)\,dt = \frac{1}{T_0} \cdot \left(\int_0^{\frac{T_0}{2}} A \sin(\omega_0 n t)\,dt + 0 \right) = \left| \begin{matrix} \omega_0 n t = x \\ dt = \frac{1}{\omega_0 n} dx \\ 0 \to 0 \\ \frac{T_0}{2} \to \pi n \end{matrix} \right| =$$

$$= \frac{1}{\pi n} \int_0^{\pi n} A \cdot \sin(x)\,dx = \frac{A}{\pi n} \left[-\cos(x) \right]_0^{\pi n} = \frac{A\left(1 - \cos(\pi n)\right)}{\pi n}$$

Na podstawie punktów 1÷3 otrzymano:

$$f(t) = \frac{1}{2} A + \sum_{n=1}^{\infty} \frac{A}{\pi n} \left(1 - \cos \pi n\right) \cdot \sin n \omega t \quad ; \text{ gdzie } \omega = \frac{2\pi}{T_0}$$

10 pierwszych współczynników obliczonych na podstawie wzorów analitycznych

n	a_n	b_n
1	0	$\frac{2}{\pi} A$
2	0	0
3	0	$\frac{2}{3\pi} A$
4	0	0
5	0	$\frac{2}{5\pi} A$

4	0	0
5	0	$\frac{2}{5\pi}A$
6	0	0
7	0	$\frac{2}{7\pi}A$
8	0	0
9	0	$\frac{2}{9\pi}A$
10	0	0

Wyznaczenie wartości skutecznej:

$$f_{sk} = \sqrt{\frac{1}{T_0} \cdot \int_0^{T_0} f^2(t)\,dt} = \sqrt{\frac{1}{\cancel{T_0}} \cdot A^2 \cdot \frac{\cancel{T_0}}{2}} = \left|\frac{A\sqrt{2}}{2}\right|$$

Wyznaczenie współczynnika THD:

$$S_1 = \sqrt{\frac{1}{T_0} \int_0^{T_0} \frac{4}{\pi^2} A^2 \sin^2 \omega t\,dt} = \left|\frac{A\sqrt{2}}{\pi}\right| \quad ;$$

Dla $n = 2k+1$, $k \in \mathbb{Z}$

$$S_n^2 = \frac{1}{T_0} \cdot \int_0^{T_0} \frac{4A^2}{n^2\pi^2} \sin^2 n\omega t\,dt \text{ , gdzie}$$

$$\int_0^{T_0} \sin^2 n\omega t\,dt = \begin{Bmatrix} \frac{2\pi}{T_0} n t = x \\ dt = \frac{T_0}{2\pi n} dx \\ 0 \to 0 \\ T_0 \to 2\pi n \end{Bmatrix} = \frac{T_0}{2\pi n} \int_0^{2\pi n} \sin^2 x\,dx = \frac{T_0}{\cancel{2\pi n}} \cdot \cancel{\pi n}$$

Zatem:

$$S_n^2 = \begin{cases} \frac{2A^2}{n^2\pi^2} & , \quad n = 2k-1, \quad k \in \mathbb{Z} \\ 0 & , \quad n = 2k, \quad k \in \mathbb{Z} \end{cases}$$

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} S_n^2}}{S_1} = \frac{\cancel{\left|\frac{A\sqrt{2}}{\pi}\right|} \sqrt{\sum_{k=1} \frac{1}{(2k-1)^2}}}{\cancel{\left|\frac{A\sqrt{2}}{\pi}\right|}} = \sqrt{\sum_{k=1}^{\infty} \frac{1}{(2k+1)^2}} = \sqrt{\frac{\pi^2}{8} - 1} \approx 0{,}483$$