Presek dveh implicitno danih ploskev

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Začetne opombe:
- gradf1, in gradf2 vračata vrstične vektorje.
- testi vzamejo približno 12 sekund.

Contents

1 Konstrukcija G in JG

$$y = x + hF(x)$$
$$v = F(y)$$

$$\begin{aligned} f_1(x) &= 0 \\ f_2(x) &= 0 \\ v \cdot x - v \cdot y &= 0 \end{aligned} \tag{1}$$

$$\begin{aligned} h(x) &= v \cdot x - v \cdot y \\ h_{x_i} = (v \cdot x)_{x_i} - (v \cdot y)_{x_i} &= (v \cdot x)_{x_i} - 0 = \\ &= (v_1 \cdot x_1 + v_2 \cdot x_2 + v_3 \cdot x_3)_{x_i} = v_i \\ gradh(x) &= (v_1, v_2, v_3) \end{aligned}$$

$$G(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ v \cdot x - v \cdot y \end{bmatrix}$$
$$JG(x) = \begin{bmatrix} gradf_1(x)_1 & gradf_1(x)_2 & gradf_1(x)_3 \\ gradf_2(x)_1 & gradf_2(x)_2 & gradf_2(x)_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$$

2 Prevod lažjih funkcij v $F(x) : R^3 \to R, F(x) = 0$:

Prevod enačbe ravnine na $F(x) = 0$:

$$n \cdot x = n \cdot r$$

$$n \cdot x - n \cdot r = 0$$

$$F(x) = n \cdot x - n \cdot r$$

n je normalni vektor ravnine, r je krajevni vektor ene točke na ravnini.

Prevod iz $f(x) : R^2 \to R$ na $F(x) = 0$:
Ravnina x_1, x_3 je domena funkcije.
x_2 predstavlja vrednost $f(x_1, x_3)$
x_2 si predstavljam kot vertikalno dimenzijo.

$$x_2 = f(x_1, x_3)$$

$$0 = f(x_1, x_3) - x_2$$

$$F(x_1, x_2, x_3) := f(x_1, x_3) - x_2$$

$$\Rightarrow F(x) = 0 \Leftrightarrow x_2 = f(x_1, x_3)$$

3 Testi

Test 1:

$$\begin{aligned} n_1 &= [1, 1, 0]^T \\ n_2 &= [-1, 1, 0]^T \\ r_{1,2} &= [0, 0, 0]^T \\ F_1(x) &= x_1 + x_2 \\ F_2(x) &= -x_1 + x_2 \end{aligned} \tag{2}$$

Presek bo premica [0, 0, r]; r e R

Test 2:

$$\begin{aligned} n_1 &= [1, 1, 0]^T \\ n_2 &= [-1, 1, 0]^T \\ r_{1,2} &= [1, 0, 0]^T \\ F_1(x) &= x_1 + x_2 - 1 \\ F_2(x) &= -x_1 + x_2 + 1 \end{aligned} \tag{3}$$

Presek bo premica [1, 0, r]; r e R

Test 3:

$$
\begin{aligned}
f(x_1, x_3) &= x_1^2 + x_3^2 \\
F_1(x) &= x_1^2 + x_3^2 - x_2 \\
\\
x_1^2 + x_3^2 &= C = x_2 \\
x^2 + y^2 &= (r^2) \Rightarrow krog \\
\Rightarrow r = \sqrt{C} &= \sqrt{x_2} = \sqrt{n_2 \cdot r_2} \\
\\
n_2 &= [0, 1, 0]^T \\
r_2 &= [0, 5, 0]^T \\
F_2(x) &= x_2 - 5
\end{aligned} \tag{4}
$$

Če gledamo preseke z x_1, x_3 ravnino, dobimo krožnice. To lahko vidimo, če pogledamo, kdaj ima funkcija $f(x)$ neko konstantno vrednost.
Ravnina $F_2(x)$ je vzporedna ravnini x_1, x_3, saj imata obe normalni vektor $[0, 1, 0]^T$. Iz enačbe $F_2(x) = 0$ pa vidimo, da je x_2 ravno $n_2 \cdot r_2$.
Za našo krivuljo bo torej veljalo, da je skupina vektorjev, kjer veljata enačbi $x_1^2 + x_3^2 = n_2 \cdot r_2$ in $x_2 = n_2 \cdot r_2$

Test 4:

$$
\begin{aligned}
f(x_1, x_3) &= (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 \\
F_1(x) &= (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 - x_2 \\
\\
(\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 &= C = x_2 \\
(\frac{x}{a})^2 + (\frac{y}{b})^2 &= C \Rightarrow elipsa \\
\\
n_2 &= [0, 1, 0]^T \\
r_2 &= [0, 7, 0]^T \\
F_2(x) &= x_2 - 7
\end{aligned} \tag{5}
$$

Če gledamo preseke z x_1, x_3 ravnino, dobimo elipse. To lahko vidimo, če pogledamo, kdaj ima funkcija $f(x)$ neko konstantno vrednost.
Ravnina $F_2(x)$ je vzporedna ravnini x_1, x_3, saj imata obe normalni vektor $[0, 1, 0]^T$. Iz enačbe $F_2(x) = 0$ pa vidimo, da je x_2 ravno $n_2 \cdot r_2$.
Za našo krivuljo bo torej veljalo, da je skupina vektorjev, kjer veljata enačbi $(\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 = n_2 \cdot r_2$ in $x_2 = n_2 \cdot r_2$

Test 5: isti primer kot test 4, le da bo C oziroma x_2 enak 1.
Tako je graf bolj pregleden, saj se točno vidi polosi.

$$
\begin{aligned}
f(x_1, x_3) &= (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 \\
F_1(x) &= (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 - x_2 \\
\\
n_2 &= [0, 1, 0]^T \\
r_2 &= [0, 1, 0]^T \\
F_2(x) &= x_2 - 1
\end{aligned} \tag{6}
$$

Test 6: Normalni vektor ravnine je 45° glede na x_2, x_3 ravnino. Torej bo $F_2(x)$ glede na ravnino x_2, x_3 ravno $-45°$.

$$\begin{aligned}
f(x_1, x_3) &= (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 \\
F_1(x) &= (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 - x_2 \\
\\
n_2 &= [0, 1, 1]^T \\
r_2 &= [0, 5, 0]^T \\
F_2(x) &= x_2 + x_3 - 5 \\
\\
F_2(x) &= 0 \\
x_2 &= -x_3 + 5 \\
F_1(x) &= 0 \\
(\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 &= x_2 \\
\\
Krivulja \Leftrightarrow (\frac{x_1}{2})^2 + (\frac{x_3}{5})^2 &= -x_3 + 5
\end{aligned} \tag{7}$$

4 Koda za grafe testov

Test 1:

```
F1 = @(X) (X(1) + X(2));
F2 = @(X) (-X(1) + X(2));
gradF1 = @(X) ([1, 1, 0]);
gradF2 = @(X) ([-1, 1, 0]);
Y = presekPloskev(F1, gradF1, F2, gradF2, [0.01; 0.02; 0.4], 0.1, 300, 1e-10,
100);
plot3(Y(1, : ), Y(3, : ), Y(2, : ));
axis equal;
xlabel ("x1");
ylabel ("x3");
zlabel("x2");
```

Test 2:

```
F1 = @(X) (X(1) + X(2) - 1);
F2 = @(X) (-X(1) + X(2) + 1);
gradF1 = @(X) ([1, 1, 0]);
gradF2 = @(X) ([-1, 1, 0]);
Y = presekPloskev(F1, gradF1, F2, gradF2, [0.01; 0.02; 0.4], 0.1, 300, 1e-10,
100);
plot3(Y(1, : ), Y(3, : ), Y(2, : ));
axis equal;
xlabel ("x1");
ylabel ("x3");
zlabel("x2");
```

Test 3:

```
F1 = @(X) (X(1)^2 + X(3)^2 - X(2));
F2 = @(X) (X(2) - 5);
gradF1 = @(X) ([2*X(1), -1, 2*X(3)]);
gradF2 = @(X) ([0, 1, 0]);
Y = presekPloskev(F1, gradF1, F2, gradF2, [0.8; 0.8; 0.2], 0.1, 300, 1e-10,
100);
plot3(Y(1, : ), Y(3, : ), Y(2, : ));
axis equal;
xlabel ("x1");
ylabel ("x3");
zlabel("x2");
```

Test 4:

```
F1 = @(X) ((X(1)/2)^2 + (X(3)/5)^2 - X(2));
F2 = @(X) (X(2) - 7);
gradF1 = @(X) ([X(1)/2, -1, 2*X(3)/25]);
gradF2 = @(X) ([0, 1, 0]);
Y = presekPloskev(F1, gradF1, F2, gradF2, [0.8; 0.8; 0.2], 0.1, 700, 1e-10,
100);
plot3(Y(1, : ), Y(3, : ), Y(2, : ));
axis equal;
xlabel ("x1");
ylabel ("x3");
zlabel("x2");
```

Test 5:

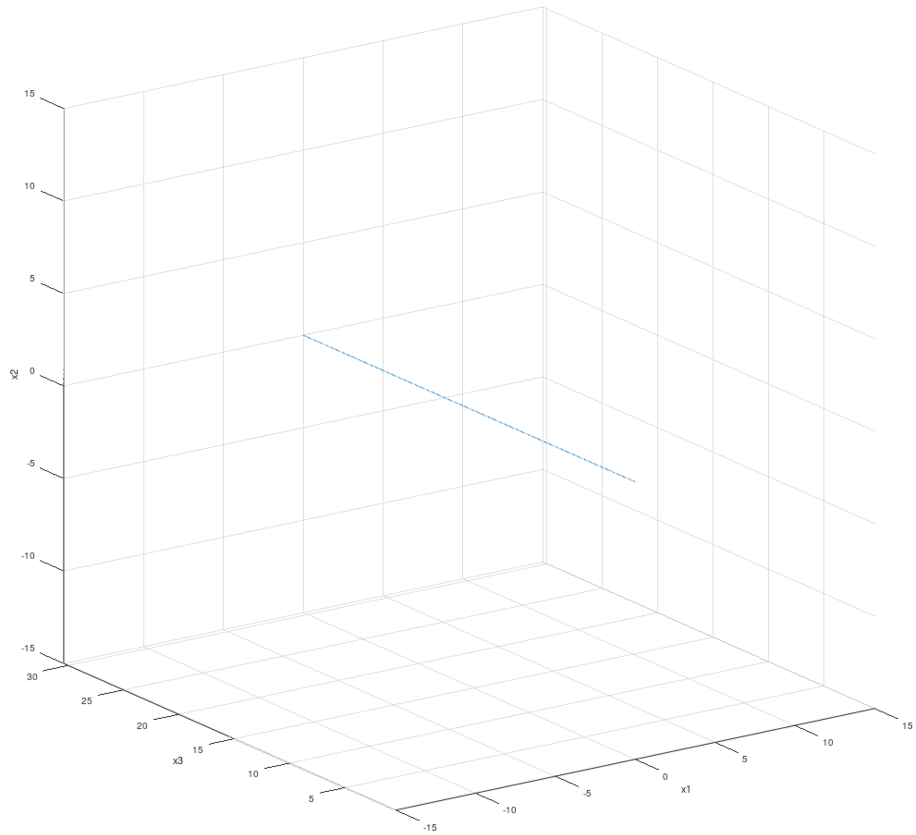
```
F1 = @(X) ((X(1)/2)^2 + (X(3)/5)^2 - X(2));
F2 = @(X) (X(2) - 1);
gradF1 = @(X) ([X(1)/2, -1, 2*X(3)/25]);
gradF2 = @(X) ([0, 1, 0]);
Y = presekPloskev(F1, gradF1, F2, gradF2, [0.8; 0.8; 0.2], 0.1, 300, 1e-10,
100);
plot3(Y(1, : ), Y(3, : ), Y(2, : ));
axis equal;
xlabel ("x1");
ylabel ("x3");
zlabel("x2");
```

Test 6:

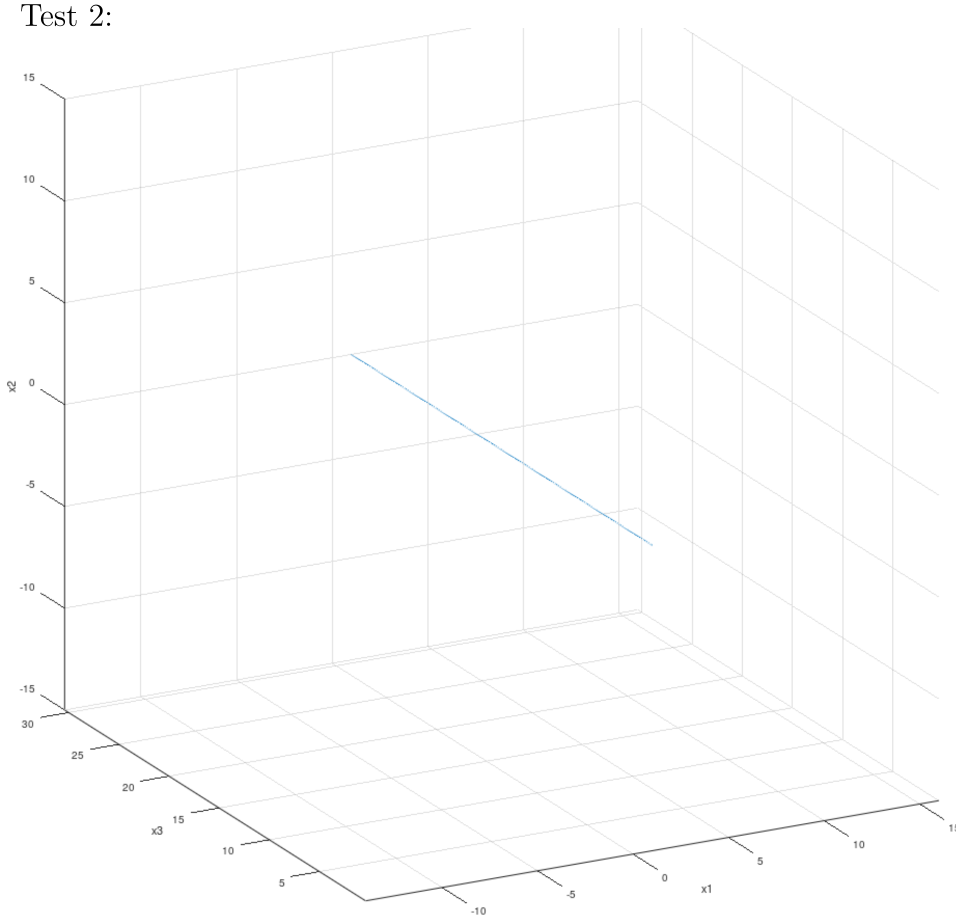
```
F1 = @(X) ((X(1)/2)^2 + (X(3)/5)^2 - X(2));
F2 = @(X) (X(2) + X(3) - 5);
gradF1 = @(X) ([X(1)/2, -1, 2*X(3)/25]);
gradF2 = @(X) ([0, 1, 1]);
Y = presekPloskev(F1, gradF1, F2, gradF2, [0.8; 0.8; 0.2], 0.1, 1200, 1e-10,
100);
plot3(Y(1, : ), Y(3, : ), Y(2, : ));
axis equal;
xlabel ("x1");
ylabel ("x3");
zlabel("x2");
```

5 Slike testov

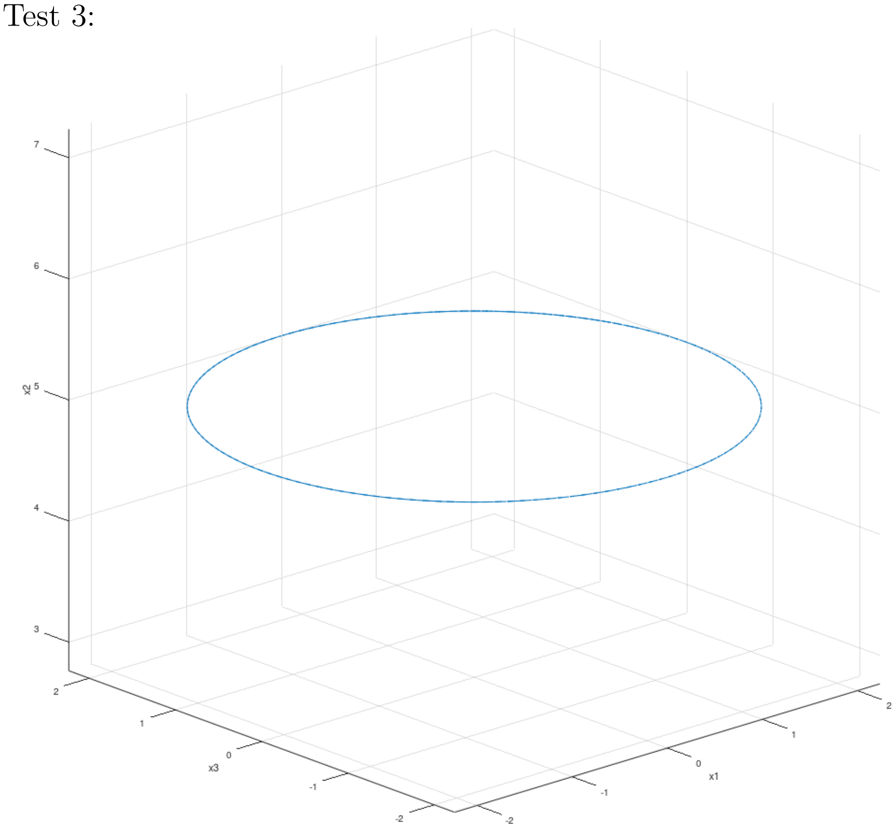
Test 1:

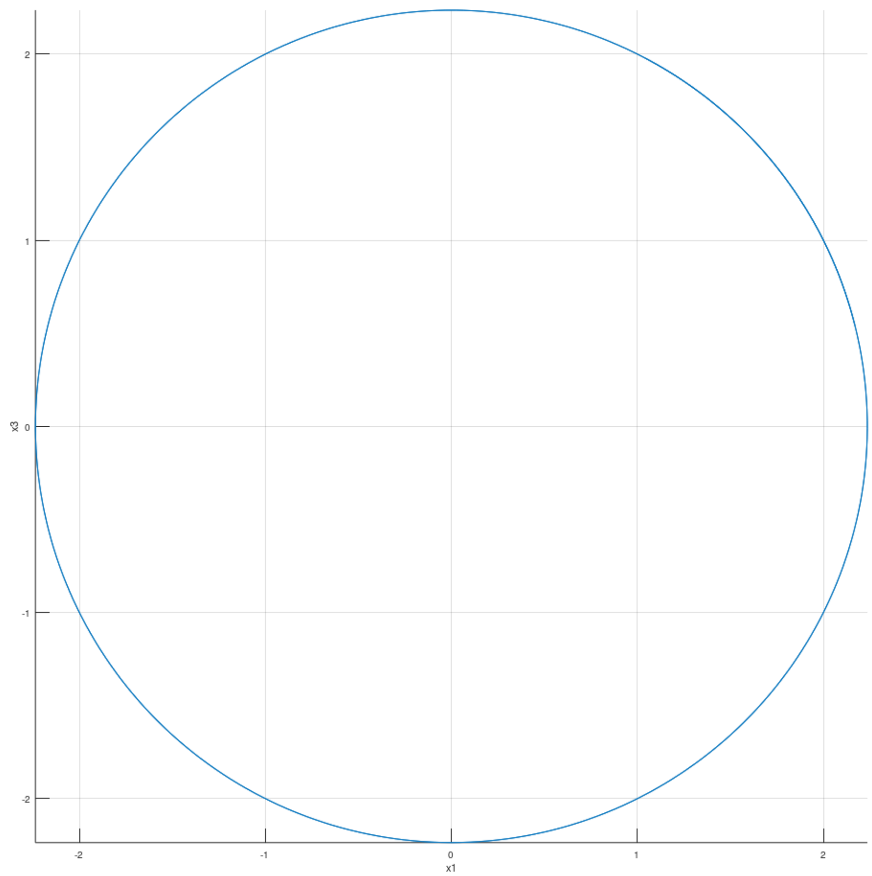


Test 2:



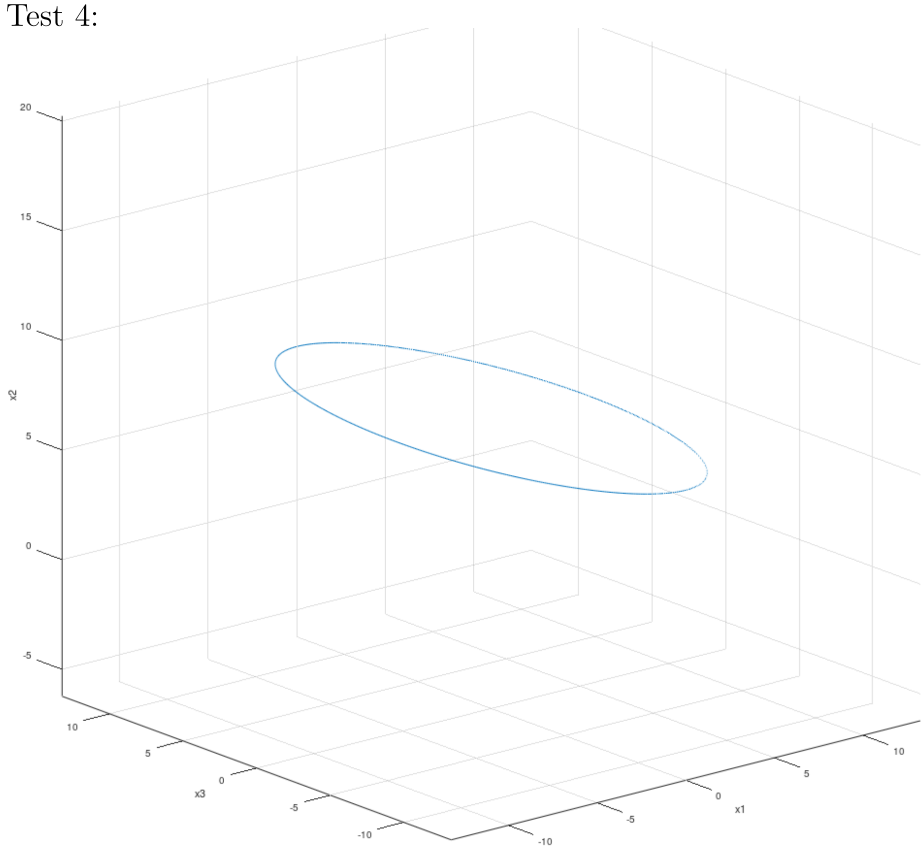
Test 3:

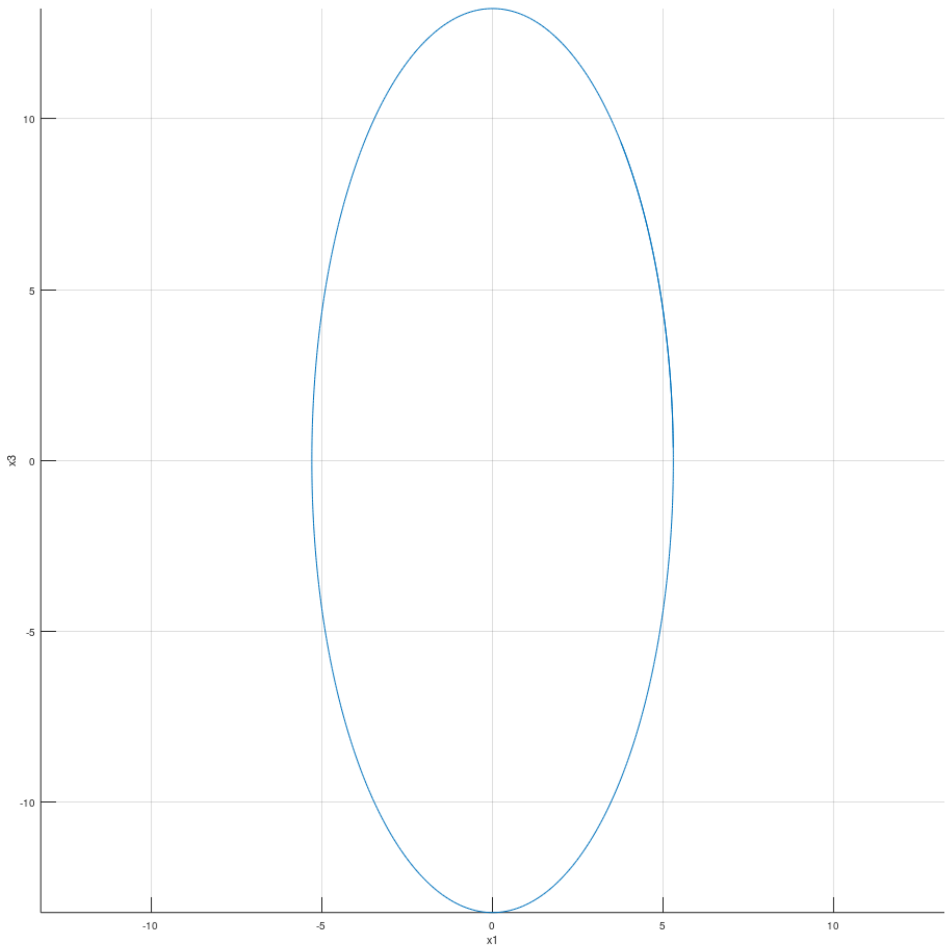




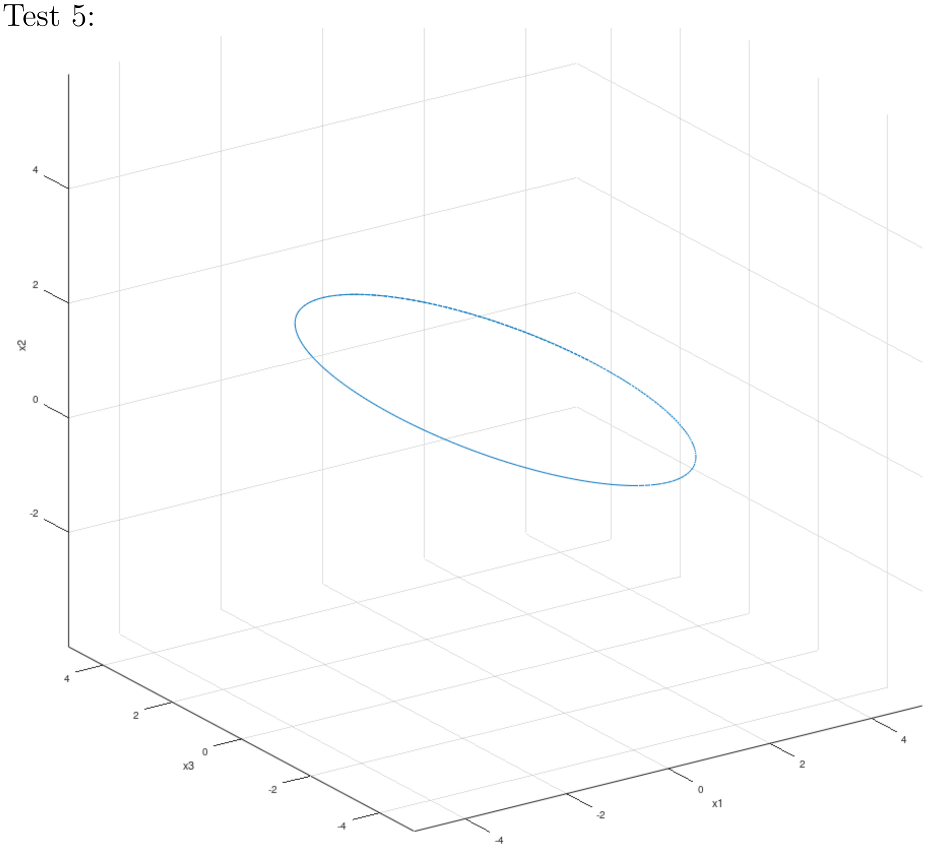
2
1
0
-1
-2
x3
-2
-1
0
1
2
x1

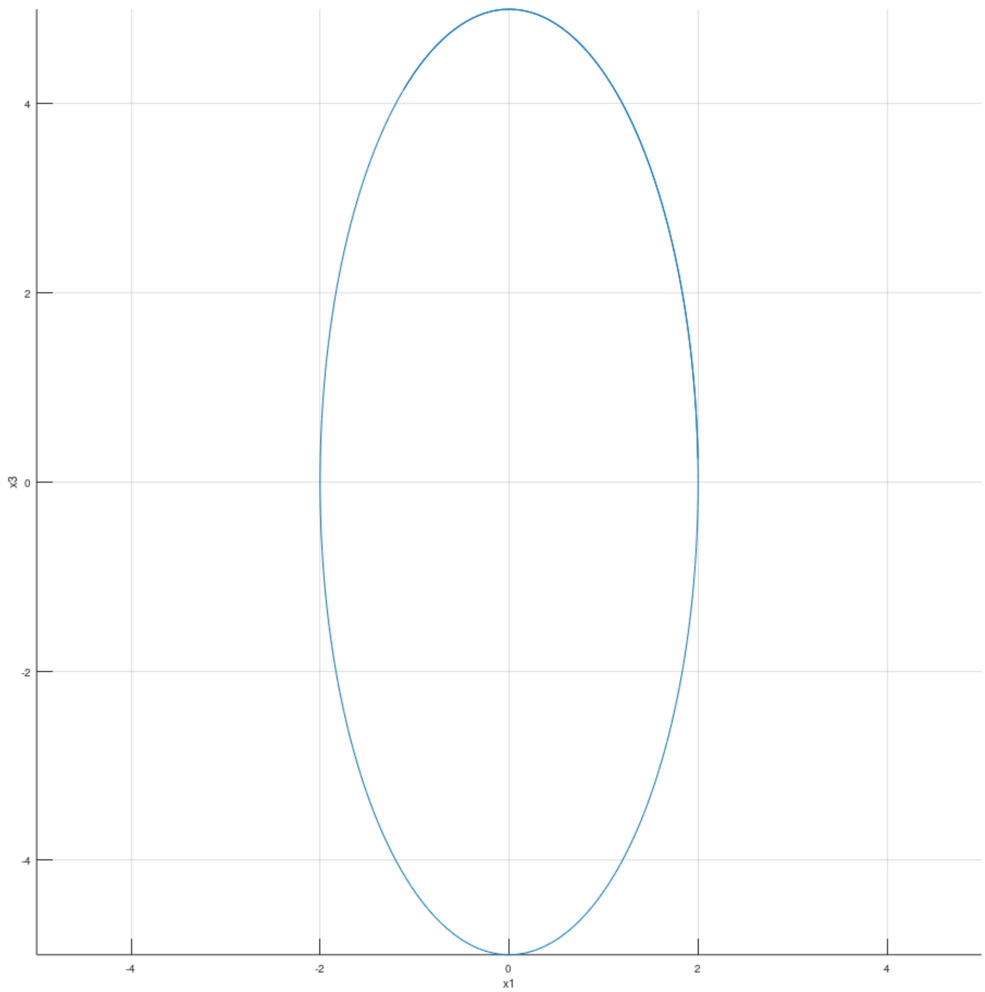
Test 4:




10
5
0
-5
-10
x3
-10
-5
0
5
10
x1

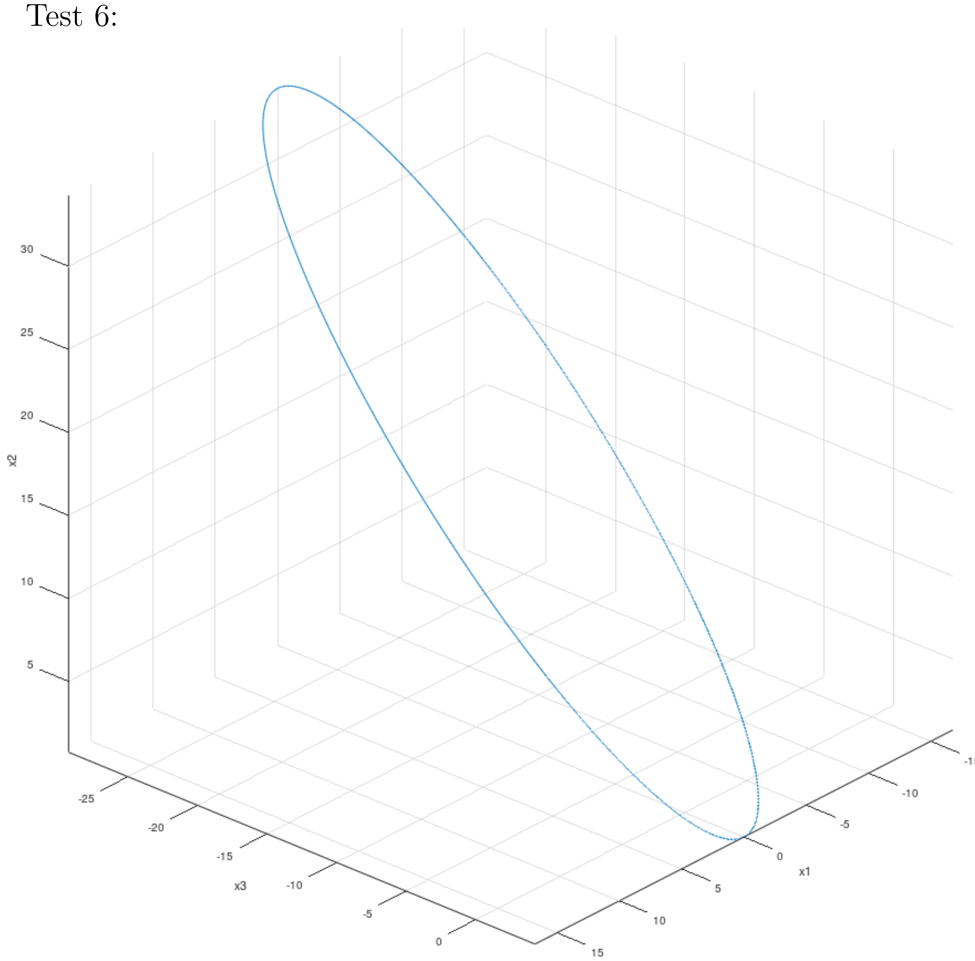
Test 5:

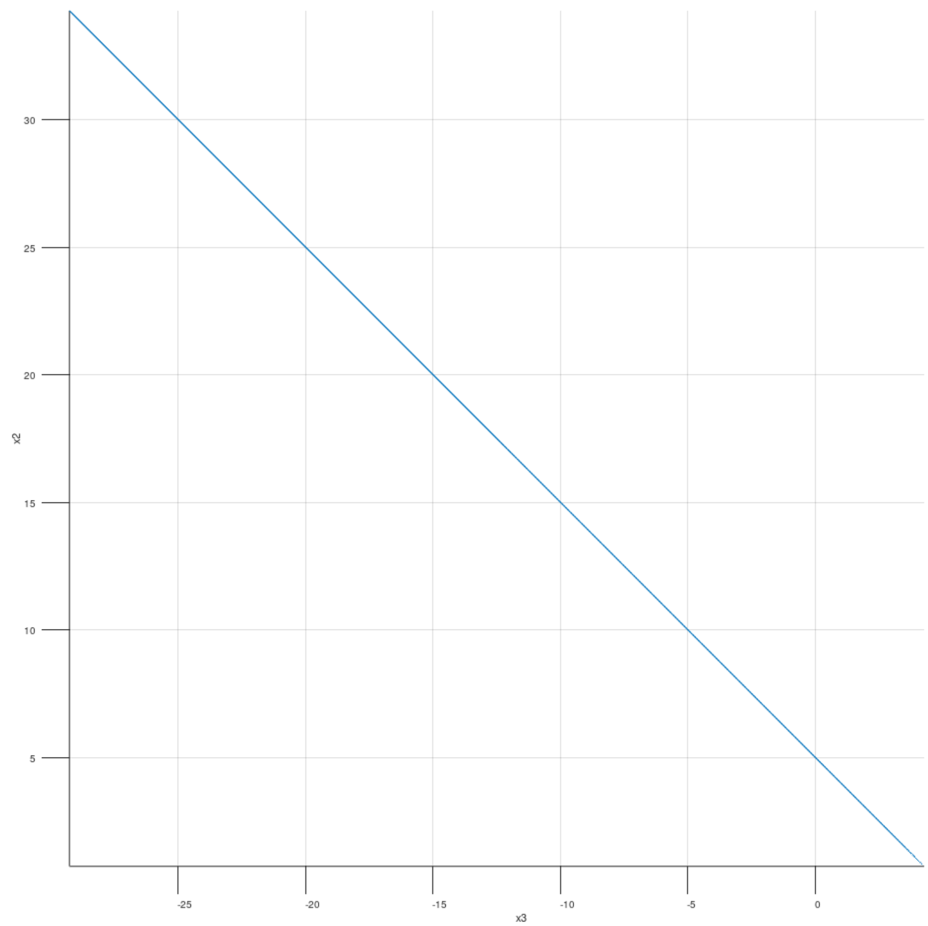




4
2
0
-2
-4
x3
-4
-2
0
2
4
x1

Test 6:




30
25
20
15
10
5
x2
-25
-20
-15
-10
-5
0
x3