**习题 1** 对于参数设置 $n \in \mathbb{N}$ 及 $a > 0$, 计算以下积分, 其中你可以尝试直接使用 Riemann 积分中所涉及到的换元方法或者分部积分法.

(1) $\displaystyle\int_0^{2\pi} \frac{\mathrm{d}x}{\sin^4 x + \cos^4 x}$;

(2) $\displaystyle\int_0^{\pi/2} \ln(\sin x)\mathrm{d}x$;

(3) $\displaystyle\int_0^1 x^n(1-x)^\alpha\mathrm{d}x, \ \alpha > 1$;

(4) $\displaystyle\int_0^1 (-\ln x)^n \mathrm{d}x$;

(5) $\displaystyle\int_0^n \frac{1-(1-x/n)^n}{x}\mathrm{d}x$;

(6) $\displaystyle\int_0^1 x^n \ln^n x\,\mathrm{d}x$;

(7) $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{(1+x^2)^n}$;

(8) $\displaystyle\int_1^\infty \frac{\mathrm{d}x}{x^2\sqrt{x^2-1}}$;

(9) $\displaystyle\int_0^\infty \frac{\ln x}{x^2+a^2}\mathrm{d}x$;

(10) $\displaystyle\int_0^\infty \frac{\ln x}{(x^2+a^2)^2}\mathrm{d}x$;

(11) $\displaystyle\int_0^\pi \ln(1+\cos x)\mathrm{d}x$;

(12) $\displaystyle\int_0^\infty \ln\left(x+\frac{1}{x}\right)\frac{\mathrm{d}x}{1+x^2}$.

**习题 2** 判断下列积分是否收敛:

(1) $\displaystyle\int_1^\infty \frac{\ln x}{x\sqrt{x^2-1}}\mathrm{d}x$;

(2) $\displaystyle\int_0^\infty \left(e^{-\frac{a^2}{x^2}} - e^{-\frac{b^2}{x^2}}\right)\mathrm{d}x$;

(3) $\displaystyle\int_a^\infty x^\mu e^{-ax}\mathrm{d}x \ (\mu, a > 0)$;

(4) $\displaystyle\int_0^\infty \frac{x\mathrm{d}x}{\sqrt{e^{2x}-1}}$.

**习题 3** 判断下列积分是否收敛:

(1) $\displaystyle\int_0^\theta \frac{\mathrm{d}\phi}{\sqrt{\cos\phi - \cos\theta}}$;

(2) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{\sqrt[3]{x(e^x-e^{-x})}}$;

(3) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{\ln x}$;

(4) $\displaystyle\int_0^{\pi/2} (\tan x)^p\mathrm{d}x$;

(5) $\displaystyle\int_{-\pi/4}^{\pi/4} \left(\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right)^p \mathrm{d}\theta$.

**习题 4** 判断下列积分是否收敛:

(1) $\displaystyle\int_2^\infty \frac{\mathrm{d}x}{x\ln x}$;

(2) $\displaystyle\int_0^\infty \frac{\sin^2 x}{1+x^2}\mathrm{d}x$;

(3) $\displaystyle\int_0^1 (-\ln x)^a\mathrm{d}x, a \in \mathbb{R}$;

(4) $\displaystyle\int_0^\infty \frac{x\mathrm{d}x}{1+x^2\sin^2 x}$;

(5) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{x^a(-\ln x)^b}, a, b \in \mathbb{R}$.