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1) a) $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ com $f(x,y) = sen(x)\cos(y)$

$\frac{\partial f}{\partial x} = \cos(x)\cos(y)$ $\frac{\partial f}{\partial y} = -sen(x)sen(y)$ $\nabla f = (\cos(x).\cos(y), -sen(x).sen(y))$

b) $\Delta f = \nabla.(\nabla f) = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

$\Delta f = \frac{\partial}{\partial x}(\cos(x)\cos(y)) + \frac{\partial}{\partial y}(-sen(x).Sen(y))$

$\Delta f = -sen(x)\cos(y) - sen(x)\cos(y) = -2\,sen(x)\cos(y)$

c) Gráfico junto ao trabalho do solver.

a) $f(x,y) = 3x^2 + 5y^2$ $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) \rightarrow \nabla f = (6x, 10y)$

b) $\Delta f = \frac{\partial(6x)}{\partial x} + \frac{\partial(10y)}{\partial y} \rightarrow \Delta f = 16$

c) Gráfico junto ao trabalho do solver.

2) $f(x,y,z) = \cos(xy).e^{2z}$

a) $\nabla f = (\partial x, \partial y, \partial z) f$

$\partial x f = -y\,sen(xy)e^{2z}$

$\partial y f = -x\,sen(xy)e^{2z}$

$\partial z f = 2\cos(xy)e^{2z}$

Logo $\nabla f = (-y\,sen(xy)e^{2z}, -x\,sen(xy)e^{2z}, 2\cos(xy)e^{2z})$

b) $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$\frac{\partial^2 f}{\partial x^2} = -y^2\cos(xy)e^{2z}$

$\frac{\partial^2 f}{\partial y^2} = -x^2\cos(xy)e^{2z}$

$\frac{\partial^2 f}{\partial z^2} = 4\cos(xy)e^{2z}$

Logo $\Delta f = e^{2z}\cos(xy)(4 - x^2 - y^2)$

c) $\Delta(f)^2 = \Delta(f.f) = \frac{\partial^2}{\partial x^2}(f.f) + \frac{\partial^2}{\partial y^2}(f.f)$

$\frac{\partial(f.f)}{\partial x} = \frac{\partial f}{\partial x}f + \frac{\partial f}{\partial x}f = 2f.\frac{\partial f}{\partial x}$

$\frac{\partial(f\,f)}{\partial y} = 2f.\frac{\partial f}{\partial y}$

$$\Delta(f)^2 = 2\cdot\frac{\partial}{\partial x}\cdot\left(f\cdot\frac{\partial f}{\partial x}\right)+2\frac{\partial}{\partial y}\left(f\cdot\frac{\partial f}{\partial y}\right) \qquad \Delta(f)^2 = 2\left[\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial x}+f\frac{\partial^2 f}{\partial x^2}\right]+2\left[\frac{\partial f}{\partial y}\cdot\frac{\partial f}{\partial y}+f\frac{\partial^2 f}{\partial y^2}\right]$$

$$\Delta(f)^2 = 2\left(\frac{\partial f}{\partial x}\right)^2+2f\frac{\partial^2 f}{\partial x^2}+2\left(\frac{\partial f}{\partial y}\right)^2+2f\frac{\partial^2 f}{\partial y^2} \qquad \Delta(f)^2 = 2.\nabla f.\nabla f+2f.\Delta f$$

$$\nabla f\cdot\nabla f = \left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2+\left(\frac{\partial f}{\partial z}\right)^2 \qquad \nabla f\cdot\nabla f = y^2 sen^2(xy)e^{4z}+x^2 sen^2(xy)e^{4z}+4\cos^2(xy)e^{4z}$$

$$f.\Delta f = e^{4z}\cos^2(xy)(4-x^2-y^2)$$

Logo:

$$\Delta(f)^2 = 2y^2 sen^2(xy)e^{4z}+2x^2 sen^2(xy)e^{4z}+8\cos^2(xy)e^{4z}+8\cos^2(xy)e^{4z}-2x^2\cos^2(xy)e^{4z}-2y^2\cos^2(xy)e^{4z}$$

$$\Delta(f)^2 = 16\cos^2(xy)e^{4z}+2y^2e^{4z}(sen^2(xy)-\cos^2(xy))$$

$$\Delta(f)^2 = 16\cos^2(xy)e^{4z}+2y^2e^{4z}(1-2\cos^2(xy))+2x^2e^{4z}(sen^2(xy)-\cos^2(xy)) \qquad \Big|\ sen^2(xy)=1-\cos^2(xy)$$

$$\Delta(f)^2 = (16-4x^2-4y^2)e^{4z}\cos^2(xy)+(2x^2+2y^2)e^{4z}$$

3) a) $$\frac{\partial P}{\partial t} = \frac{1}{4\pi}\frac{\partial}{\partial t}\left(\frac{1}{t}e^{2t}\cdot e^{-\frac{x^2+y^2}{4t}}\right) \to \frac{\partial P}{\partial t} = \frac{1}{4\pi}\left[\frac{\partial}{\partial t}\left(\frac{e^{2t}}{t}\right)\cdot e^{-\frac{x^2+y^2}{4t}}+\frac{e^{2t}}{t}\frac{\partial}{\partial t}\left(e^{-\frac{x^2+y^2}{4t}}\right)\right]$$

$$\frac{\partial P}{\partial t} = \frac{1}{4\pi}\left[\left[\frac{\partial}{\partial t}\left(\frac{1}{t}\right)\cdot e^{2t}+\frac{1}{t}\frac{\partial}{\partial t}(e^{2t})\right]\cdot e^{-\frac{(x^2+y^2)}{4t}}+\frac{e^{2t}}{t}\cdot e^{-\left(\frac{x^2+y^2}{4t}\right)}\cdot\frac{\partial}{\partial t}\left(\frac{-(x^2+y^2)}{4t}\right)\right]$$

$$\frac{\partial P}{\partial t} = \frac{1}{4\pi}\left\{\left[\frac{-e^{2t}}{t^2}+\frac{2}{t}e^{2t}\right]e^{-\left(\frac{x^2+y^2}{4t}\right)}+\frac{e^{2t}}{t}\cdot e^{-\left(\frac{x^2+y^2}{4t}\right)}\cdot\frac{x^2+y^2}{4t^2}\right\}$$

$$\frac{\partial P}{\partial t} = \frac{-e^{2t}}{4\pi t^2}e^{-\left(\frac{x^2+y^2}{4t}\right)}+\frac{e^{2t}}{2\pi t}e^{-\left(\frac{x^2+y^2}{4t}\right)}+\frac{e^{2t}}{4\pi t}e^{-\left(\frac{x^2+y^2}{4t}\right)}\cdot\left(\frac{x^2+y^2}{4t^2}\right)$$

$$\frac{\partial P}{\partial t} = \frac{-x}{8\pi t^2}e^{2t}e^{-\left(\frac{x^2+y^2}{4t}\right)} \rightsquigarrow \frac{\partial^2 P}{\partial x^2} = \frac{-e^{2t}}{8\pi t^2}\frac{\partial}{\partial x}\left(x\cdot e^{-\left(\frac{x^2+y^2}{4t}\right)}\right) \to \frac{\partial^2 P}{\partial x^2} = \frac{-e^{2t}}{8\pi t^2}\left(e^{-\left(\frac{x^2+y^2}{4t}\right)}-\frac{x^2}{2t}e^{-\left(\frac{x^2+y^2}{4t}\right)}\right) \to$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{-e^{2t}e^{-\left(\frac{x^2+y^2}{4t}\right)}}{8\pi t^2}+\frac{x^2e^{2t}}{16\pi t^3}e^{-\left(\frac{x^2+y^2}{4t}\right)}$$

$$\frac{\partial^2 P}{\partial y^2} = \frac{-e^{2t}e^{-\left(\frac{x^2+y^2}{4t}\right)}}{8\pi t^2}+\frac{y^2e^{2t}}{16\pi t^3}e^{-\left(\frac{x^2+y^2}{4t}\right)}$$

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\partial P = \frac{-e^{2t}}{4\pi t^2}e^{-\left(\frac{x^2+y^2}{4t}\right)}+\frac{e^{2t}}{16\pi t^3}e^{-\left(\frac{x^2+y^2}{4t}\right)}(x^2+y^2)+\frac{e^{2t}}{2\pi t}e^{-\left(\frac{x^2+y^2}{4t}\right)} = \frac{\partial P}{\partial t}$$

b) junto ao trabalho do solver

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4) $r = \sqrt{x^2+y^2}$ e $\theta = tg^{-1}(y/x)$

$$\frac{\partial r}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} = \cos\theta \qquad \frac{\partial \theta}{\partial x} = \frac{\cancel{x}^2}{x^2+y^2}\cdot\left(\frac{-y}{\cancel{x^2}}\right) = \frac{-sen\theta}{r}$$

$$P = P(r,\theta)$$

$$\frac{\partial r}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2}} = sen\theta \qquad \frac{\partial \theta}{\partial y} = \frac{x^2}{x^2+y^2}\cdot\frac{1}{x} = \frac{\cos\theta}{r}$$

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial P}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} \rightarrow \frac{\partial P}{\partial x} = \cos\theta\cdot\frac{\partial P}{\partial r} - \frac{sen\theta}{r}\cdot\frac{\partial P}{\partial \theta}$$

$$\frac{\partial P}{\partial y} = \frac{\partial P}{r}\cdot\frac{\partial r}{\partial y} + \frac{\partial P}{\partial \theta}\cdot\frac{\partial \theta}{\partial y} \rightarrow \frac{\partial P}{\partial y} = sen\theta\frac{\partial P}{\partial r} + \frac{\cos\theta}{r}\cdot\frac{\partial P}{\partial \theta}$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{\partial \cos\theta}{\partial x}\cdot\frac{\partial P}{\partial r} + \cos\theta\left(\frac{\partial^2 P}{\partial r^2}\cdot\frac{\partial r}{\partial x} + \frac{\partial^2 P}{\partial\theta\partial r}\cdot\frac{\partial\theta}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{sen\theta}{r}\right)\frac{\partial P}{\partial\theta} - \frac{sen\theta}{r}\left(\frac{\partial^2 P}{\partial r\partial\theta}\cdot\frac{\partial r}{\partial x} + \frac{\partial^2 P}{\partial\theta^2}\frac{\partial\theta}{\partial x}\right)$$

$$\frac{\partial^2 P}{\partial x^2} = -sen\theta\cdot\frac{\partial\theta}{\partial x}\cdot\frac{\partial P}{\partial r} + \cos\theta\frac{\partial^2 P}{\partial r^2}\cdot\frac{\partial r}{\partial x} + \cos\theta\frac{\partial^2 P}{\partial\theta\partial r}\cdot\frac{\partial\theta}{\partial x} - \left(\frac{\frac{\partial sen\theta}{\partial x}\cdot r - \frac{\partial r}{\partial x}sen\theta}{r^2}\right)\frac{\partial P}{\partial\theta} - \frac{sen\theta}{r}\frac{\partial^2 P}{\partial r\partial\theta}\cdot\frac{\partial r}{\partial x}$$
$$-\frac{sen\theta}{r}\cdot\frac{\partial^2 P}{\partial\theta^2}\cdot\frac{\partial\theta}{\partial x}$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{sen^2\theta}{r}\cdot\frac{\partial P}{\partial r} + \cos^2\theta\frac{\partial P}{\partial r} + \cos^2\theta\frac{\partial P}{\partial r} - \frac{sen\theta\cos\theta}{r}\frac{\partial^2 P}{\partial\theta\partial r} - \frac{1}{r^2}\frac{\partial P}{\partial\theta}\left(\cos\theta\, r\cdot\frac{\partial\theta}{\partial x} - sen\theta\cos\theta\right) -$$
$$\frac{sen\theta.\cos\theta}{r}\frac{\partial^2 P}{\partial r\partial\theta} + \frac{sen^2\theta}{r^2}\cdot\frac{\partial^2 P}{\partial\theta^2}$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{sen^2\theta}{r}\frac{\partial P}{\partial r} + \cos^2\theta\frac{\partial^2 P}{\partial r^2} - 2\frac{sen\theta\cos\theta}{r}\frac{\partial^2 P}{\partial r\partial\theta} + \frac{sen^2\theta}{r^2}\frac{\partial^2 P}{\partial\theta^2} + \frac{2\,sen\theta\cos\theta}{r^2}\frac{\partial P}{\partial\theta} \quad (\mathrm{I})$$

$$\frac{\partial^2 P}{\partial y^2} = \frac{2sen\theta}{2y} + \frac{\partial P}{\partial r} + sen\theta\left(\frac{\partial^2 P}{\partial r^2}\cdot\frac{\partial r}{\partial y} + \frac{\partial^2 P}{\partial\theta\partial r}\cdot\frac{\partial\theta}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\cos\theta}{r}\right)\cdot\frac{\partial P}{\partial\theta} + \frac{\cos\theta}{r}\cdot\left(\frac{\partial^2 P}{\partial r\partial\theta}\cdot\frac{\partial r}{\partial y} + \frac{\partial^2 P}{\partial\theta^2}\frac{\partial\theta}{\partial y}\right)$$

$$\frac{\partial^2 P}{\partial y^2} = \cos\theta\frac{\partial\theta}{\partial y}\cdot\frac{\partial P}{\partial r} + sen\theta\left(sen\theta\frac{\partial^2 P}{\partial r^2} + \frac{\cos\theta}{r}\frac{\partial^2 P}{\partial\theta\partial r}\right) + \left(\frac{\frac{\partial\cos\theta}{\partial y}\cdot r - \cos\theta\frac{\partial r}{\partial y}}{r^2}\right)\frac{\partial P}{\partial\theta} + \frac{\cos\theta}{r}\left(sen\theta\frac{\partial^2 P}{\partial r\partial\theta} + \frac{\cos\theta}{r}\frac{\partial^2 P}{\partial\theta^2}\right)$$

$$\frac{\partial^2 P}{\partial y^2} = \frac{\cos^2\theta}{r}\frac{\partial P}{\partial r} + sen^2\theta\frac{\partial^2 P}{\partial r^2} + \frac{sen\theta\cos\theta}{r}\frac{\partial^2 P}{\partial r\partial\theta} + \frac{sen\theta\cos\theta}{r}\frac{\partial^2 P}{\partial r\partial\theta} + \frac{\cos^2\theta}{r^2}\frac{\partial^2 P}{\partial\theta^2} + \frac{1}{r^2}\frac{\partial P}{\partial\theta}\left(-sen\theta\frac{\partial\theta}{\partial y}\cdot r - \cos\theta\frac{\partial r}{\partial y}\right)$$

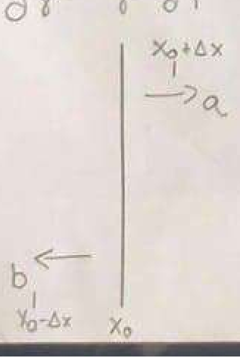
$$\frac{\partial^2 P}{\partial y^2} = \frac{\cos^2\theta}{r}\frac{\partial P}{\partial r} + sen^2\theta\frac{\partial^2 P}{\partial r^2} + \frac{2\,sen\theta\cos\theta}{r}\frac{\partial^2 P}{\partial r\partial\theta} + \frac{\cos^2\theta}{r^2}\frac{\partial^2 P}{\partial\theta^2} - \frac{2\,sen\theta\cos\theta}{r^2}\frac{\partial P}{\partial\theta} \quad (\mathrm{II})$$

(I) + (II):

$$\Delta P = \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 P}{\partial\theta^2}}$$ Simetria radial

$$\frac{\partial P}{\partial t} = D\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right)$$

5)



$$J\Delta t = \left(-a\,P_{(x_0+\Delta x,t)} + b\,P_{(x_0-\Delta x,t)}\right)\Delta x \qquad P_{(x_0+\Delta x,t)} = P_{(x_0,t)} + \frac{\partial P_{(x_0,t)}}{\partial x}\Delta x \qquad P_{(x_0-\Delta x,t)} = P_{(x_0,t)} - \frac{\partial P_{(x_0,t)}}{\partial x}\Delta x$$

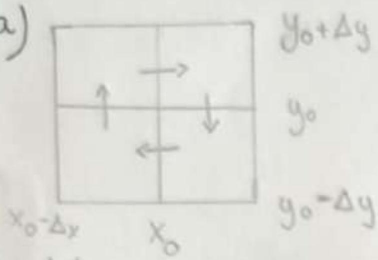
$$J\Delta t = \left(-a\,P - a\Delta x\frac{\partial P}{\partial x} + bP - b\Delta x\frac{\partial P}{\partial x}\right)\Delta x \qquad J = \frac{-(a-b)\Delta x}{\Delta t}P - \frac{(a+b)\Delta x^2}{\Delta t}\frac{\partial P}{\partial x}$$

$$\frac{\partial P}{\partial t} = -\frac{\partial J}{\partial x} \rightarrow \frac{\partial P}{\partial t} = \frac{+(a-b)\Delta x}{\Delta t}\frac{\partial P}{\partial x} + \frac{(a+b)\Delta x^2}{\Delta t}\frac{\partial^2 P}{\partial x^2}$$

$$\boxed{\frac{\partial P}{\partial t} - \underbrace{\frac{(a-b)\Delta x}{\Delta t}}_{V}\frac{\partial P}{\partial x} = \underbrace{\frac{+(a+b)\Delta x^2}{\Delta t}}_{D}\frac{\partial^2 P}{\partial x^2}}$$

Temos problemas quando $a=1$ e $b=0$ ou $a=0$ e $b=1$ (temos em que não existe difusão.)

6) a)



Vamos supor a probabilidade de movimento em cada sentido como $\frac{1}{2}$.

$$J_x = \frac{1}{2}\left(-P_{(x_0+\Delta x,t)} + P_{(x_0-\Delta x,t)}\right)\frac{\Delta x}{\Delta t} \qquad P_{(x_0+\Delta x,t)} = P + \frac{\partial P}{\partial x}\Delta x \qquad P_{(x_0-\Delta x,t)} = P - \frac{\partial P}{\partial x}\Delta x$$

$$J_x = -\left(\frac{\Delta x^2}{2\Delta t}\right)\frac{\partial P}{\partial x}$$

$$J_y = \frac{1}{2}\left(-P_{(y_0+\Delta y,t)} + P_{(y_0-\Delta y,t)}\right)\frac{\Delta y}{\Delta t} \qquad P_{(y_0+\Delta y,t)} = P + \frac{\partial P}{\partial y}\Delta y \qquad P_{(y_0-\Delta y,t)} = P - \frac{\partial P}{\partial y}\Delta y$$

$$J_y = -\left(\frac{\Delta y^2}{2\Delta t}\right)\frac{\partial P}{\partial y}$$

Logo:

$$\frac{\partial P}{\partial t} = -\nabla\cdot\vec{J} \qquad \frac{\partial P}{\partial t} = -\left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y}\right) \qquad \frac{\partial P}{\partial t} = +\left[\underbrace{\left(\frac{\Delta x^2}{2\Delta t}\right)}_{D_x}\frac{\partial^2 P}{\partial x^2} + \underbrace{\left(\frac{\Delta y^2}{2\Delta t}\right)}_{D_y}\frac{\partial^2 P}{\partial y^2}\right]$$

Supondo um meio isotrópico $\Delta x = \Delta y \rightarrow D_x = D_y = D$

$$\frac{\partial P}{\partial t} = D\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right)$$

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6) b)
$$\begin{cases} X = ax + by & \times b \\ Y = -bx + ay & \times a \end{cases} \to \begin{cases} bX = abx + b^2 y \\ aY = -abx + a^2 y \end{cases} \to (a^2+b^2) y = bX + aY \to y = bX + aY$$

$$\begin{cases} X = ax + by & \times a \\ Y = -bx + ay & \times -b \end{cases} \to \begin{cases} aX = a^2 x + aby \\ -bY = b^2 x + aby \end{cases} \to (a^2+b^2) x = aX - bY \to x = aX - bY$$

Transformações inversas
$$\begin{cases} x = aX - bY \to x = x(X,Y) \\ y = bX + aY \to y = y(X,Y) \end{cases}$$

$$\Delta_{XY} u = \frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2} \quad \text{mas } u = u(x,y) = u(x(X,Y), y(X,Y))$$

$$\frac{\partial u}{\partial X} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial X} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial X} = \frac{\partial u}{\partial x}.a + \frac{\partial u}{\partial y}.b$$

$$\frac{\partial^2 u}{\partial X^2} = \left(\frac{\partial^2 u}{\partial x^2}\frac{\partial x}{\partial X} + \frac{\partial^2 u}{\partial y \partial x}\cdot\frac{\partial y}{\partial X}\right) a + \left(\frac{\partial^2 u}{\partial x \partial y}\frac{\partial x}{\partial X} + \frac{\partial^2 u}{\partial y^2}\cdot\frac{\partial y}{\partial X}\right) b$$

$$\frac{\partial^2 u}{\partial X^2} = \left(\frac{\partial^2 u}{\partial x^2}.a + \frac{\partial^2 u}{\partial y \partial x}.b\right) a + \left(\frac{\partial^2 u}{\partial x \partial y}.a + \frac{\partial^2 u}{\partial y^2} b\right) b$$

$$\frac{\partial^2 u}{\partial X^2} = a^2\frac{\partial^2 u}{\partial x^2} + b^2\frac{\partial^2 u}{\partial y^2} + ab\frac{\partial^2 u}{\partial y \partial x} + ab\frac{\partial^2 u}{\partial x \partial y} \quad \text{I}$$

$$\frac{\partial u}{\partial Y} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial Y} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial Y} = -b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y}$$

$$\frac{\partial^2 u}{\partial Y^2} = -b\left(\frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial x}{\partial Y} + \frac{\partial^2 u}{\partial y \partial x}\cdot\frac{\partial y}{\partial Y}\right) + a\left(\frac{\partial^2 u}{\partial x \partial y}\cdot\frac{\partial x}{\partial Y} + \frac{\partial^2 u}{\partial y^2}\cdot\frac{\partial y}{\partial Y}\right)$$

$$\frac{\partial^2 u}{\partial Y^2} = -b\left(-b\frac{\partial^2 u}{\partial x^2} + a\frac{\partial^2 u}{\partial y \partial x}\right) + a\left(-b\frac{\partial^2 u}{\partial x \partial y} + a\frac{\partial^2 u}{\partial y^2}\right)$$

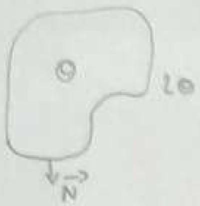
$$\frac{\partial^2 u}{\partial Y^2} = b^2\frac{\partial^2 u}{\partial x^2} + a\frac{\partial^2 u}{\partial y^2} - ab\frac{\partial^2 u}{\partial y \partial x} - ab\frac{\partial^2 u}{\partial x \partial y} \quad \text{II}$$

$$\text{I} + \text{II}: \frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2} = (a^2+b^2)\frac{\partial^2 u}{\partial x^2} + (a^2+b^2)\frac{\partial^2 u}{\partial y^2}$$

$$\Delta_{XY} u = \Delta_{xy} u$$

c) Como foi feito em ⓐ de forma geral, vê-se que devemos assumir que a difusividade é igual em cada direção. Isso vem com a suposição de que $\Delta x = \Delta y$, ou seja, a população percorre a mesma distância no mesmo tempo em cada direção.

7) Considere o domínio Θ com borda $\partial\Theta$ e normal exterior N. O termo de reação é $f(t,x,P$



Supondo que o movimento ocorre em uma direção preferencial $\vec{\gamma} \in \mathbb{R}^N$.

Temos:

$$\underbrace{\frac{d}{dt}\left(\int_\Theta P_{(t,x)}\,dx\right)}_{\substack{\text{Variação temporal}\\ \text{dentro de }\Theta}} = \underbrace{\int_\Theta f_{(t,x,P)}\,dx}_{\substack{\text{Quantidade}\\ \text{gerada ou}\\ \text{criada em }\Theta}} - \underbrace{\int_{\partial\Theta} \vec{J}_{(t,x)}\cdot N_{(x)}\,ds}_{\substack{\text{Quantidade}\\ \text{que entra ou}\\ \text{sai de }\Theta}}$$

O fluxo ocorre de acordo com a direção preferencial a uma velocidade V, daí:

$$\vec{J} = \overbrace{vP_{(t,x)}\vec{\gamma}}^{\to\text{Advecção}} - \underbrace{D\vec{\nabla}P}_{\hookrightarrow\text{Difusão}}$$

$$\frac{d}{dt}\left(\int_\Theta P\,dx\right) = -\int_{\partial\Theta}(vP\vec{\gamma} - D\nabla P)\vec{N}\cdot dS + \int_\Theta f\,dx$$

De acordo com o Teorema de Gauss:

$$\int_{\partial\Theta}(vP\vec{\gamma} - D\nabla P)\vec{N}\,dS = \int_\Theta \vec{\nabla}(vP\vec{\gamma} - D\nabla P)\,dx$$

Assumindo que Θ está bem definido, pode-se entrar com $\frac{d}{dt}$ para dentro da integral:

$$\int_\Theta \frac{\partial P}{\partial t}\,dx = -\int_\Theta \vec{\nabla}(vP\vec{\gamma} - D\nabla P)\,dx + \int_\Theta f\,dx$$

Como Θ é geral, temos:

$$\frac{\partial P}{\partial t} = -\vec{\nabla}(vP\vec{\gamma}) + D\underbrace{\vec{\nabla}(\nabla P)}_{\Delta P} + f$$

Supondo que v é constante:

$\vec{\nabla}(vP\vec{\gamma}) = v\vec{\nabla}\cdot(P\vec{\gamma}) \to v\vec{\nabla}\circ(P\vec{\gamma}_1, \dots, P\vec{\gamma}_n) = v\left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \dots, \frac{\partial P}{\partial w^n}\right)\vec{\gamma}$

$\cdot\, v\nabla P\vec{\gamma} \rightsquigarrow$ Gradiente

$$\frac{\partial P}{\partial t} = -v\nabla P\vec{\gamma} + D\Delta P + f \to \boxed{\frac{\partial P}{\partial t} + v\nabla P\cdot\vec{\gamma} = D\Delta P + f} \rightsquigarrow \text{Equação Advecção, Difusão, reação}$$

No caso $1D \to \vec{\gamma} = \hat{\imath}$.

$$\frac{\partial P}{\partial t} + v\frac{\partial P}{\partial x} = D\frac{\partial^2 P}{\partial x^2} + f$$

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8)

Equações
$$\begin{cases} A \xleftrightarrow{K_1} X \\ B+Y \xleftrightarrow{K_2} Y+D \\ 2X+Y \xleftrightarrow{K_3} 3X \\ X \xleftrightarrow{K_4} E \end{cases}$$

Suponha que A e B são "alimentados" de modo que sua concentração constante. Escreva em sistemas de equações de reação-difusão para X e Y. Considere dx e dy os coeficientes de difusão de X e Y, respectivamente.

$$\frac{\partial X}{\partial t} = dx\,\Delta X + K_1 A + K_3 X^2 Y - K_4 X$$

$$\frac{\partial Y}{\partial t} = dy\,\Delta Y - K_2 BY + K_2 BY - K_3 X^2 Y = dy\,\Delta Y - K_3 X^2 Y$$

9) a)

$C(0,x) = 0$ → inicialmente, apenas água pura.

$C(t,0) = C_1$

$C(t,L) = C_2$

b) Queremos uma função linear $f(x) = ax + b$ tal que $f(0) = C_3$ e $f(L) = 0$.

$f(0) = a.0 + b = C_3 \rightarrow b = C_3$

$f(L) = a.L + C_3 = 0 \rightarrow a = \frac{-C_3}{L}$

então, $f(x) = \frac{-C_3}{L} x + C_3$

$C(0,x) = \frac{-C_3}{L} x + C_3$

$\frac{\partial c}{\partial t}(t,0) = 0$; tubo selado em $x = 0$.

$C(t,L) = 0$; concentração constante em $x = L$.

Matheus Muniz Damasco
Parte 1 - Lista de Exercícios

1 - c) Os códigos usados no Python para gerar os gráficos das duas superfícies são exibidos nas figuras a seguir:

Gráfico 1:

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

x = np.linspace(-2, 2, 40)
y = x
x, y = np.meshgrid(x, y)
z = np.sin(x) * np.cos(y)

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
surf = ax.plot_surface(x, y, z, cmap='viridis')

ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.set_title('sin(x)cos(y)')

plt.show()
```

Gráfico 2:

```
x = np.linspace(-2, 2, 40)
y = x[:, np.newaxis]
z = 3 * x**2 + 5 * y**2

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
surf = ax.plot_surface(x, y, z, cmap='viridis')

ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.set_title('3x^2 + 5y^2')

plt.show()
```

Os resultados foram as superfícies no espaço cartesiano das figuras abaixo:

Gráfico 1:

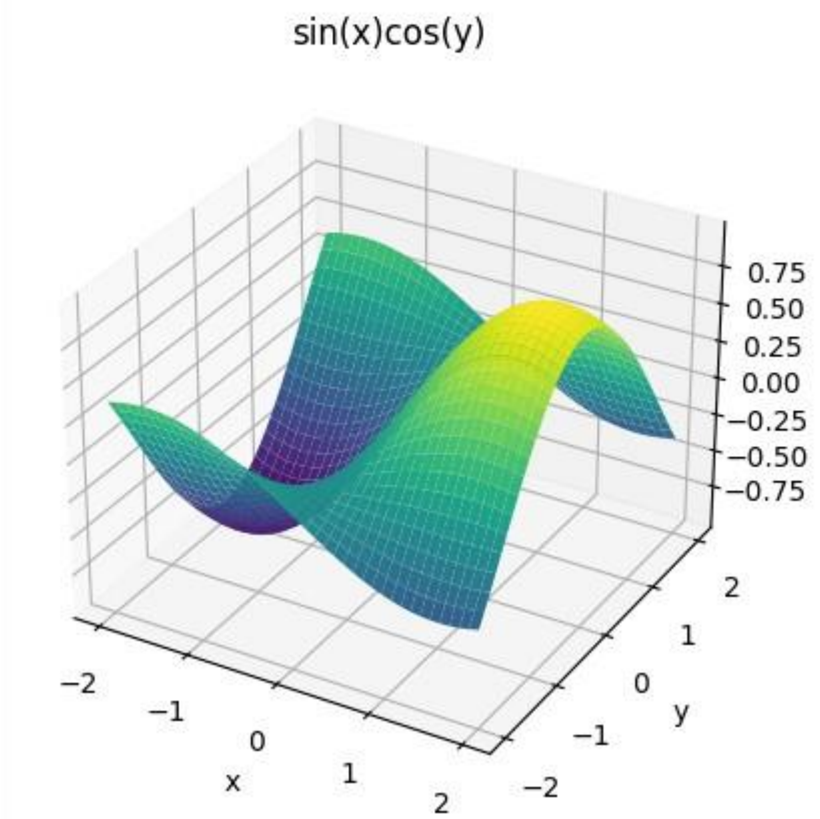
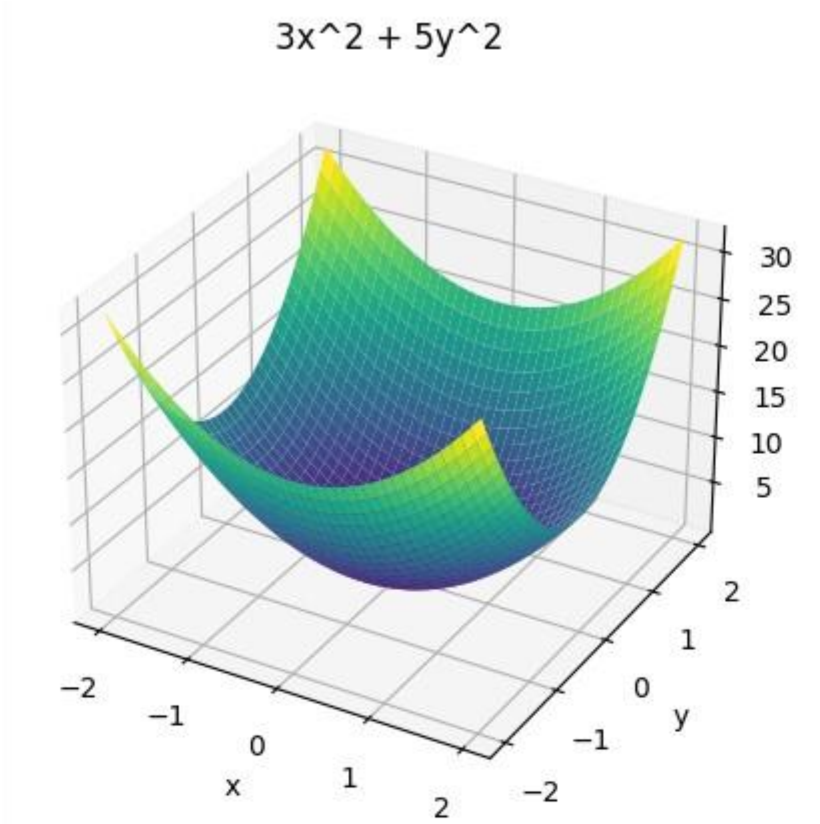


Gráfico 2:



Como esperado, a primeira superfície é ondulada, já a segunda é um parabolóide elíptico.

Gráfico 3:

```
x = np.linspace(-8, 8, 100)
y = np.linspace(-8, 8, 100)
X, Y = np.meshgrid(x, y)
Z = np.sin(X) * np.cos(Y)

plt.contour(X, Y, Z)
plt.title("Curvas de Nível para sin(x)cos(y)")
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```

Gráfico 4:

```
x = np.linspace(-8, 8, 100)
y = np.linspace(-8, 8, 100)
X, Y = np.meshgrid(x, y)
Z = 3 * X**2 + 5 * Y**2

plt.contour(X, Y, Z)
plt.title("Curvas de Nível para 3x^2 + 5y^2")
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```

O produto das compilações desses códigos foram as projeções das curvas de nível para cada função no plano xOy, as figuras a seguir mostram os resultados:

Gráfico 3:

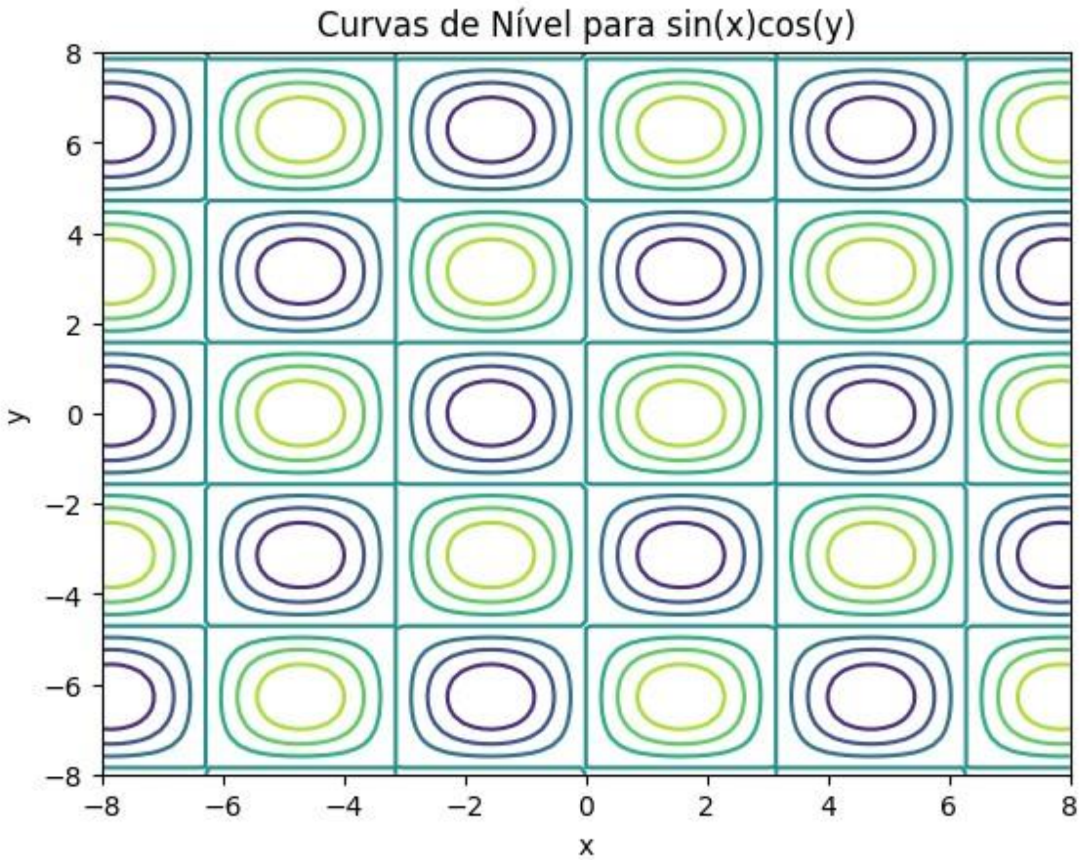
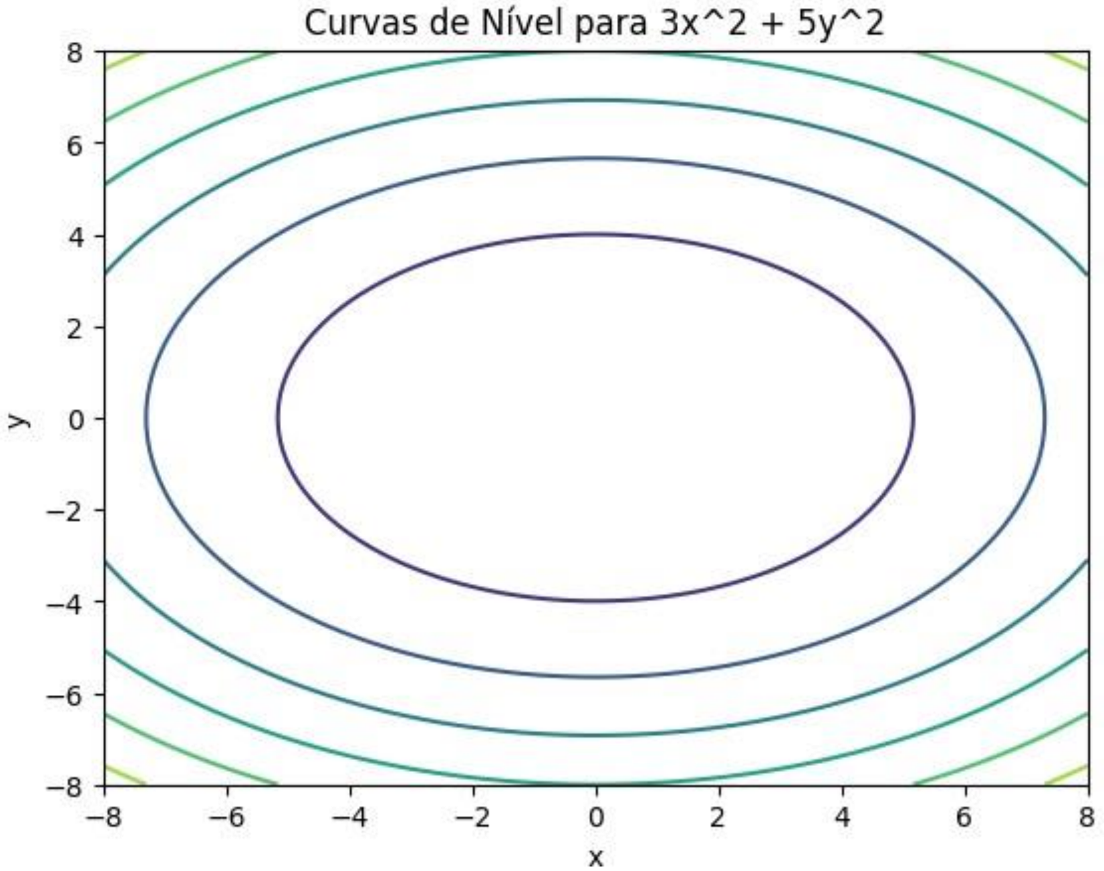


Gráfico 4:



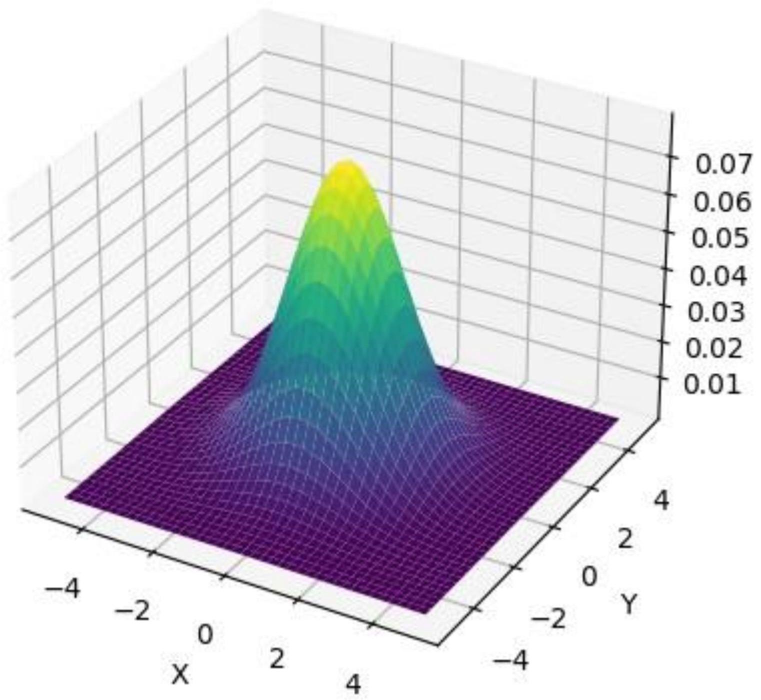
É possível notar nas primeiras curvas de nível os círculos diminuindo nos picos (máximos e mínimos) da onda formada pela função. Já na segunda vemos que as curvas de nível são elipses centradas na origem.

3 - b) Nesse caso foram elaborados 6 gráficos para instantes de tempo distintos t=1,2,3,4,5 e t=10. Para melhorar o entendimento desse fenômeno de espalhamento, esses gráficos foram elaborados no espaço cartesiano. O código criado e usando t=1 é mostrado a seguir:

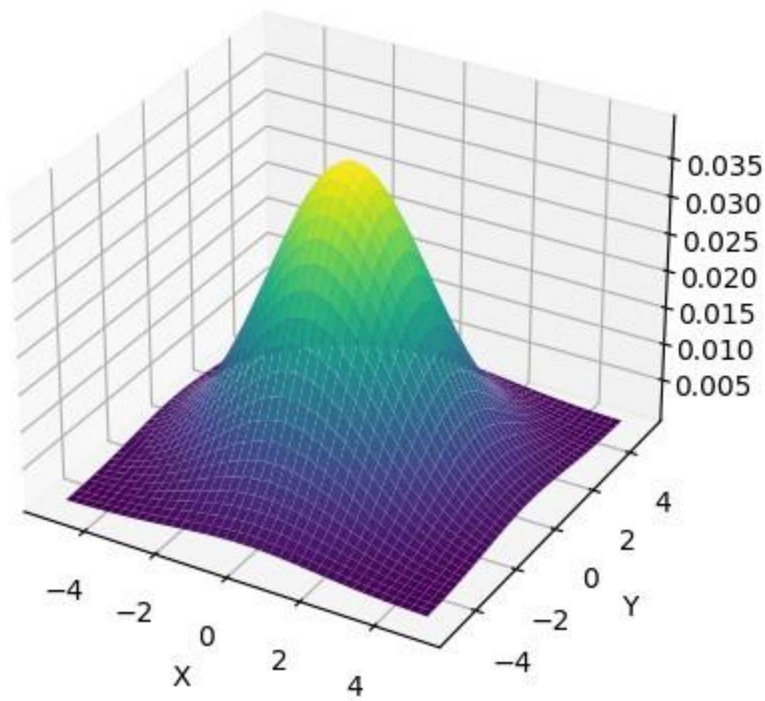
```
X, Y = np.meshgrid(np.arange(-5, 5.25, 0.25), np.arange(-5, 5.25, 0.25))
t = 1
z = (1/(4*np.pi*t)) * (np.exp((-X**2 - Y**2)/(4*t)))
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
surf = ax.plot_surface(X, Y, z, cmap='viridis')
ax.set_title("Espalhamento para t=1")
ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
plt.show()
```

Os resultados são mostrados nas figuras a seguir:

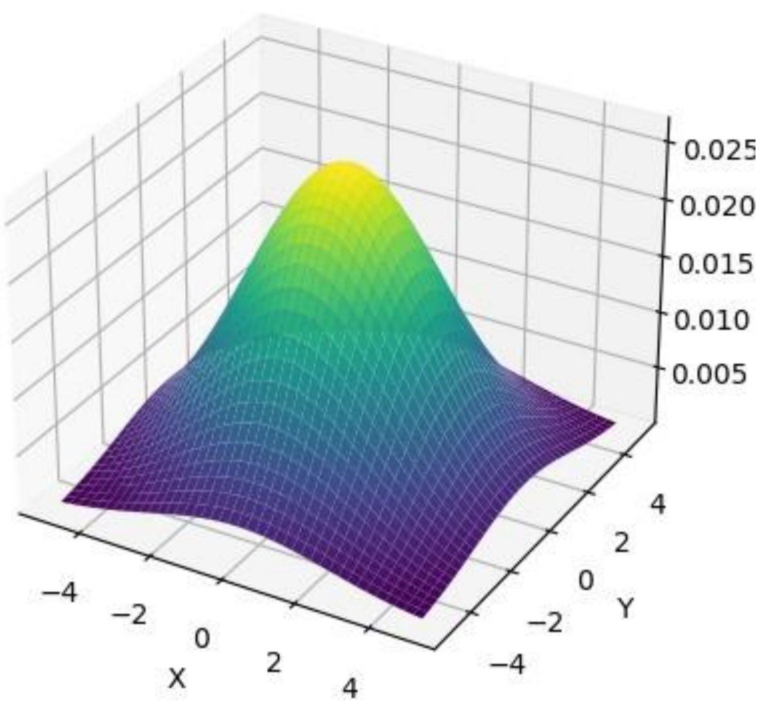
Espalhamento para t=1



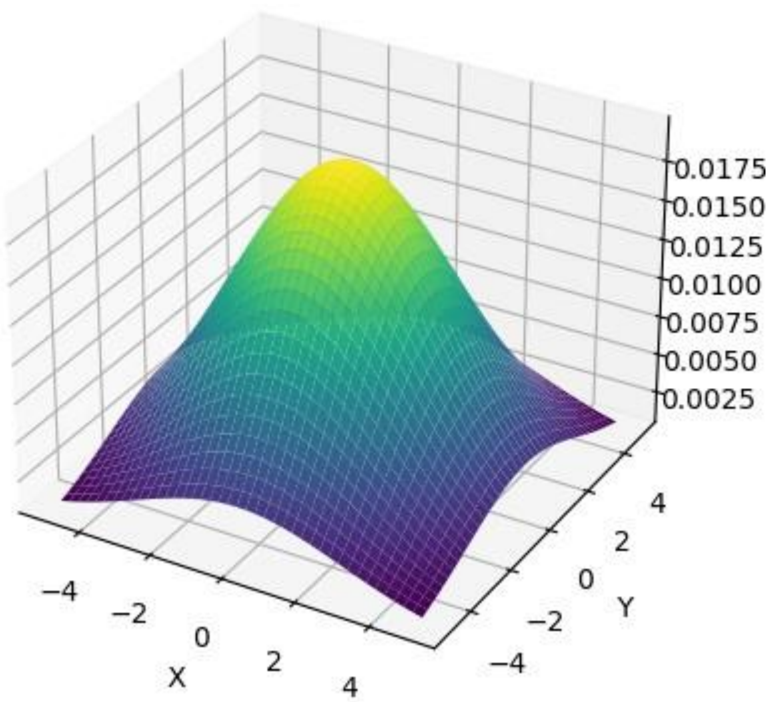
Espalhamento para t=2

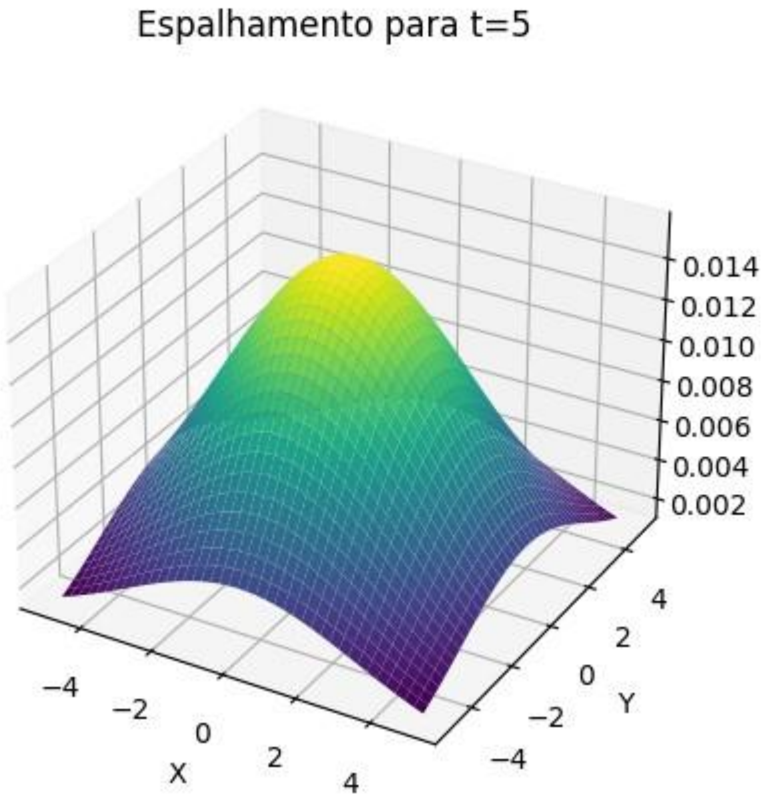


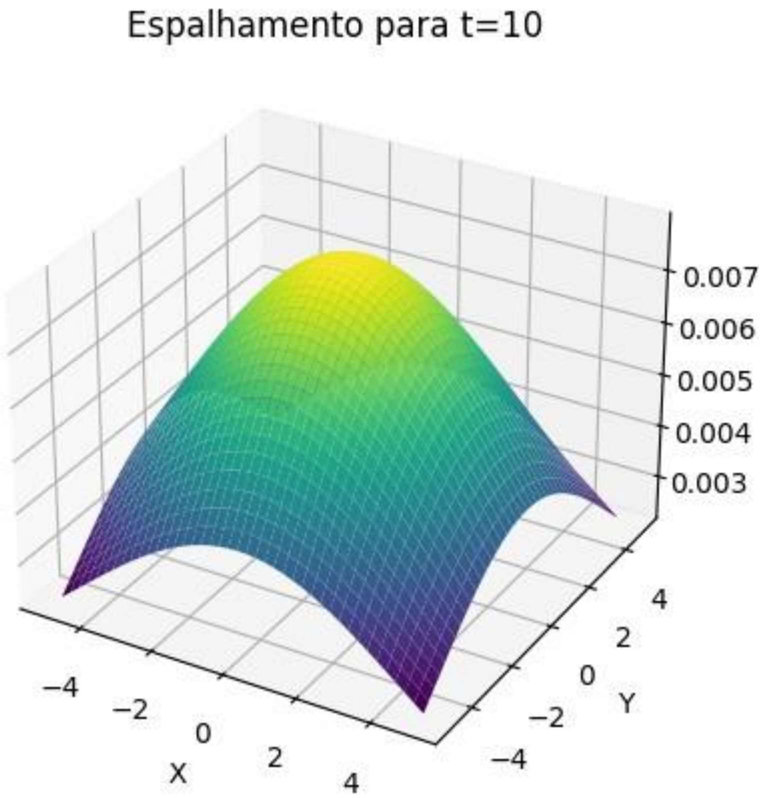
Espalhamento para t=3



Espalhamento para t=4







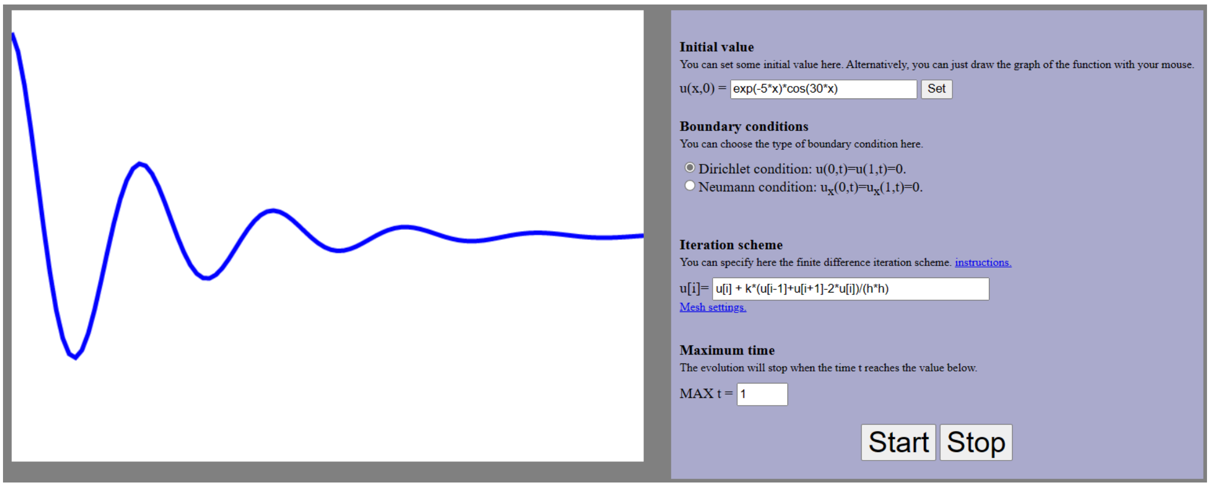
É fácil verificar a característica de fonte da distribuição, devido ao termos exponencial cujo argumento é 2t, vê-se que quando o tempo cresce o espalhamento aumenta de maneira muito rápida.

Parte 2 - Diferenças Finitas e Simulações

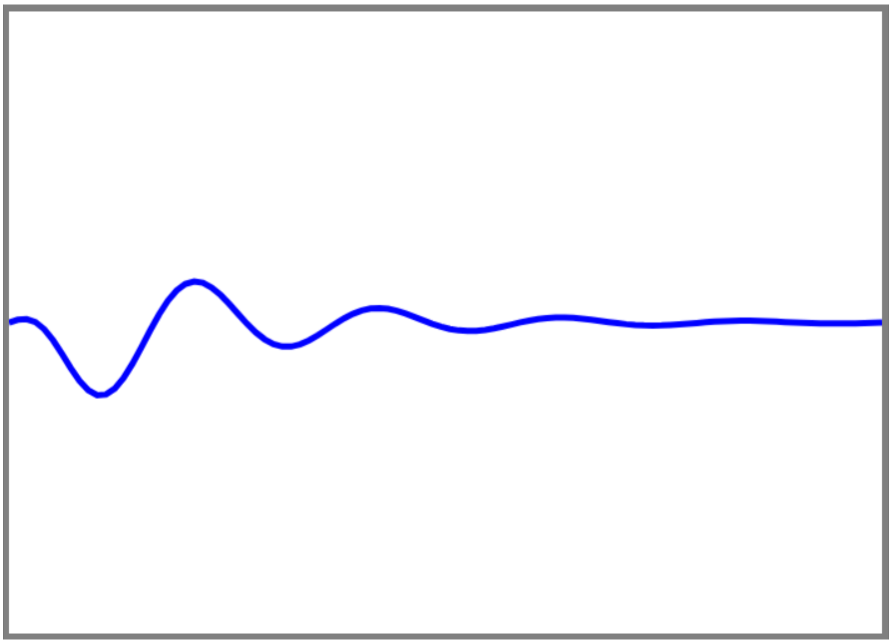
Inicialmente foram deduzidas todas as equações de diferenças finitas para cada modelo. As imagens obtidas em cada uma das simulações são mostradas a seguir (foram usadas apenas condições de Dirichlet):

1 - Difusão.

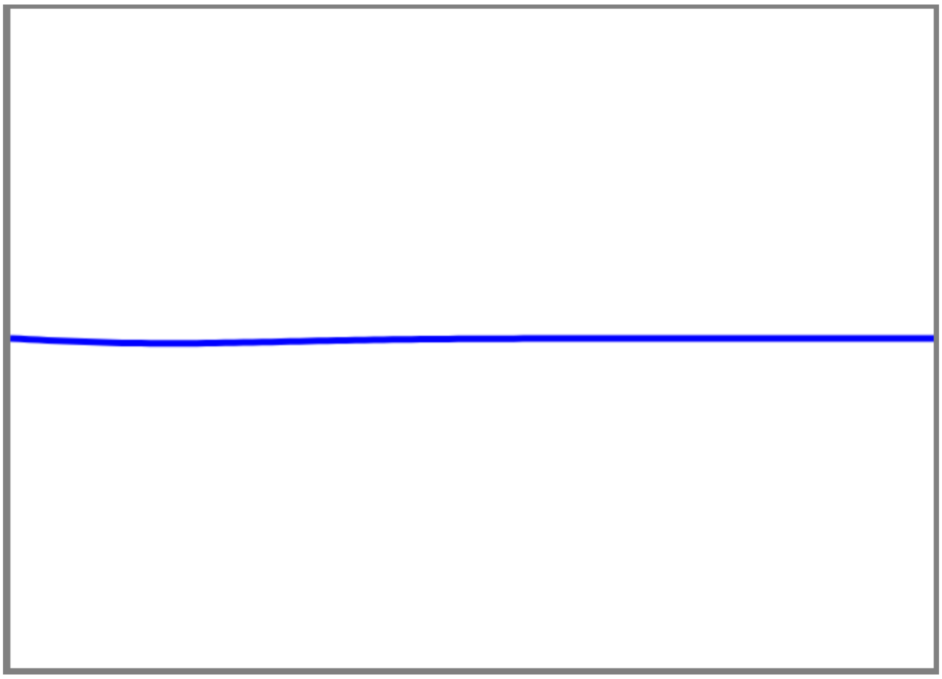
Perfil inicial usado em todas as simulações.



Meio da simulação.

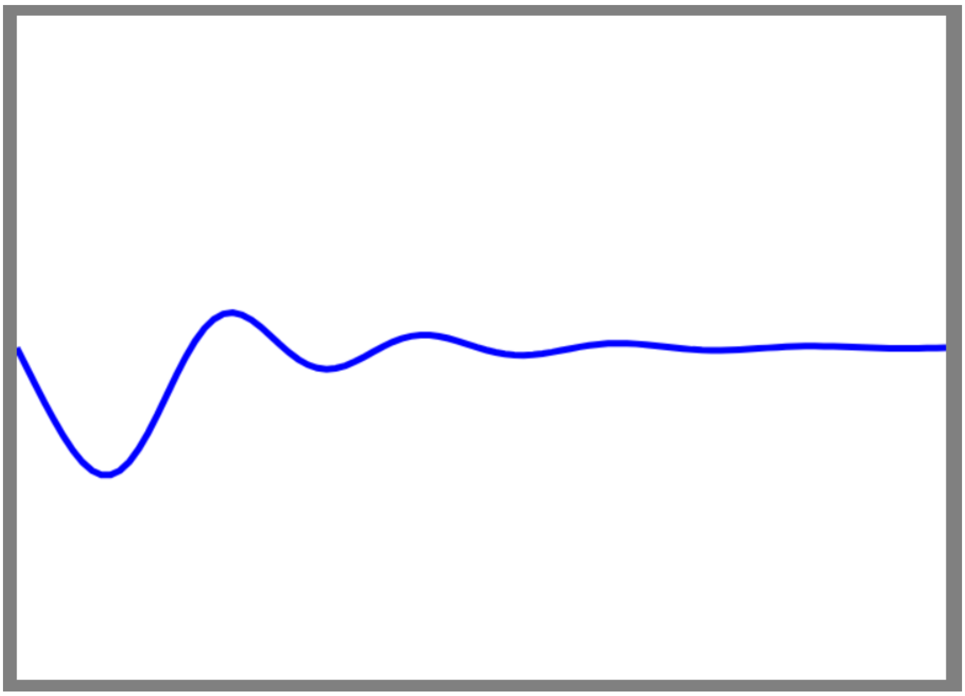


Final da Simulação.



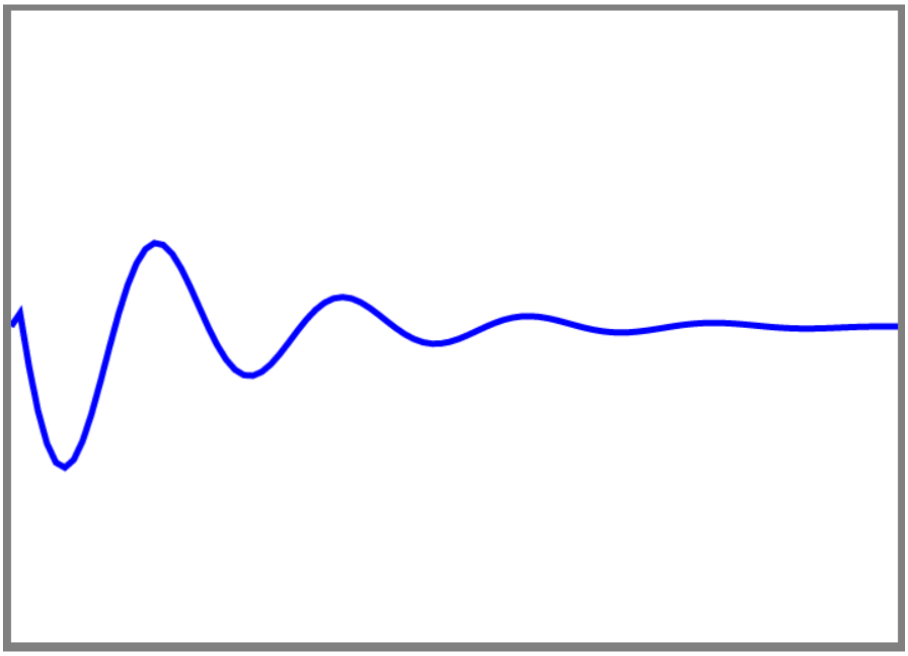
2 - Difusão - Reação.

Meio da simulação.



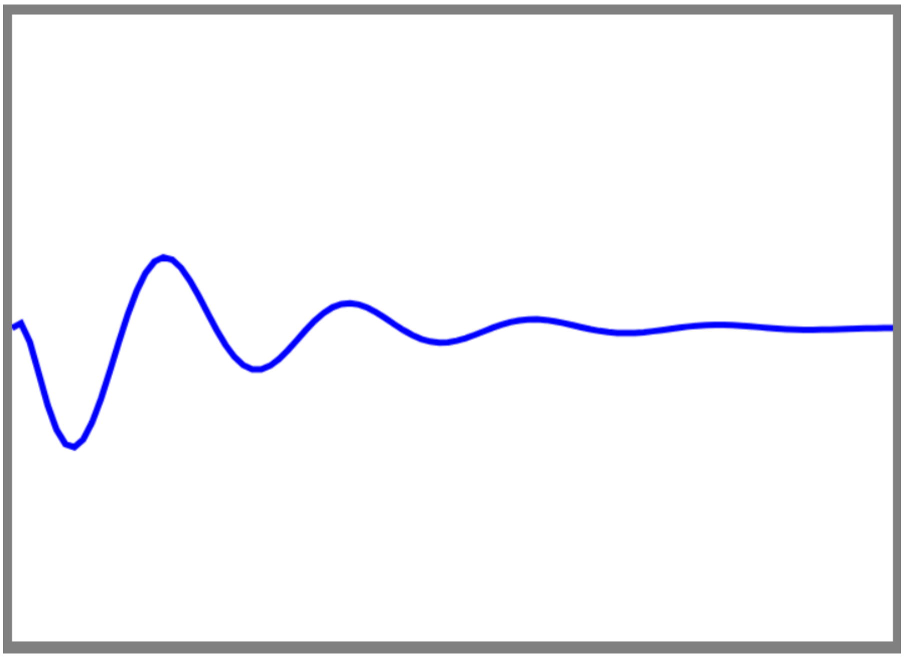
3 - Advecção.

Meio da simulação.



4 - Advecção - Difusão.

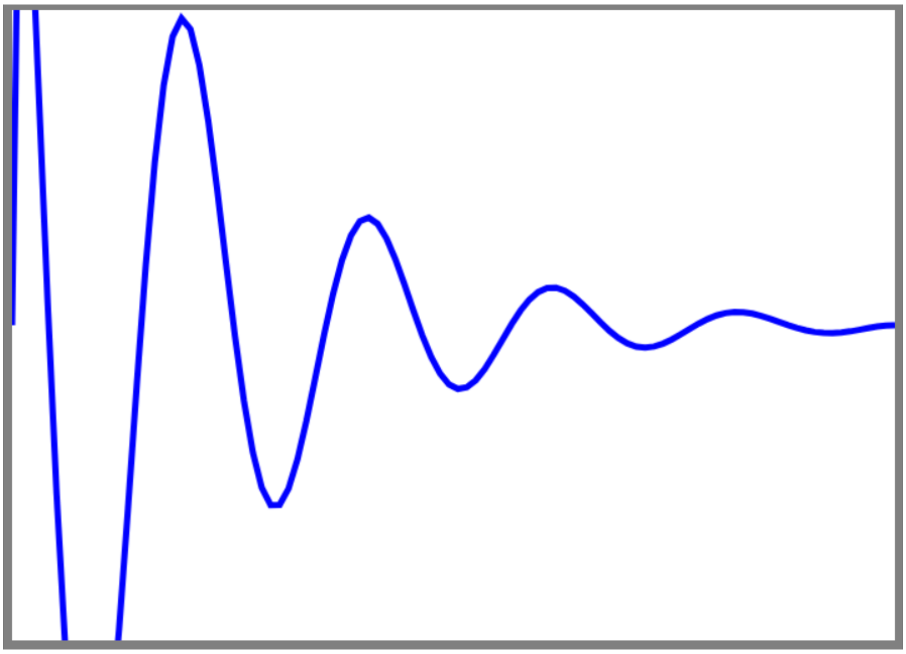
Meio da simulação.



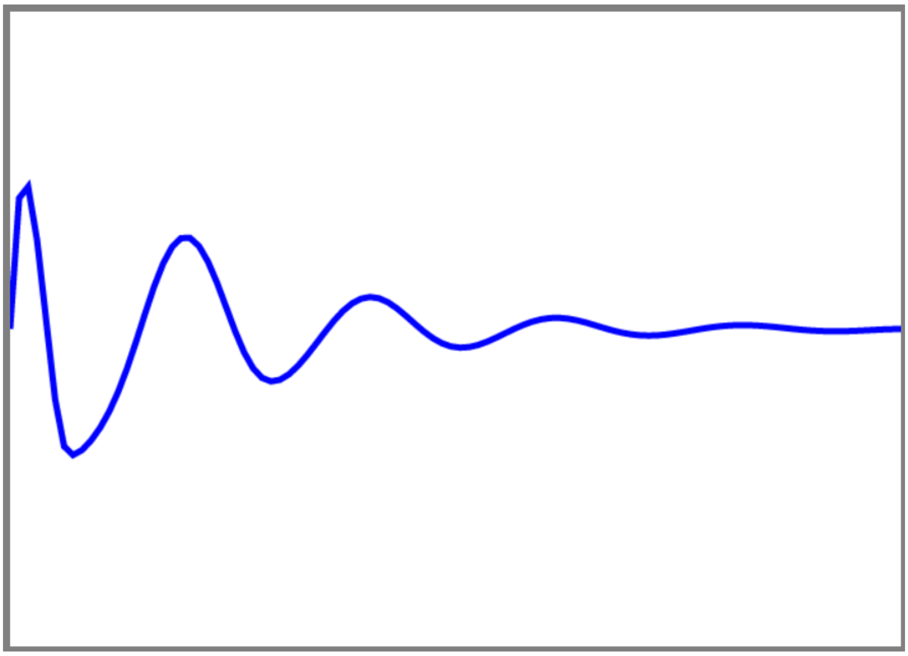
5 - Advecção - Reação.

Meio da simulação.

Como foi escolhido um termo de adição de substâncias, a solução explode quando o tempo tende ao infinito.



6 - Advecção - Reação - Difusão.



[Link dos códigos python com gráficos]()