Exercises Week 5

Econometrics

1. Suppose that the idiosyncratic errors in the errors-component model, $V_{i,t}$, are serially uncorrelated with constant variance, σ_V^2.

i. Show that the correlation between adjacent differences, $\Delta V_{i,t}$ and $\Delta V_{i,t+1}$ is -0.5.

ii. Construct the autocorrelation matrix for the first differences of the errors, $\Delta V_{i,t}$, and show that it is Toeplitz

Hint: A matrix is Toeplitz if each descending diagonal from left to right is constant.

2. **Exercise 7.24 in ETM**: The between-groups estimator is obtained by running OLS in the equation:

$$\bar{Y}_i = \bar{X}_i\beta + \bar{\mu}_i + \bar{V}_i$$

where $\bar{Z}_i = (1/T)\sum_{t=1}^{T} Z_{i,t}$. Let μ_i have variance σ_μ^2 and the $V_{i,t}$ have variance σ_V^2. Given these assumptions, show that the variance of the error terms in the regression above is $\sigma_\mu^2 + \sigma_V^2/T$.

Use this development to obtain another estimate for the variances needed to estimate random effects.

Hint: $\bar{Z}_i = P_D Z$, where D is the dummy variables matrix defined in the lecture.

3. In a random effects model, define the composite error $U_{i,t} = \mu_i + V_{i,t}$, where μ_i is uncorrelated with $V_{i,t}$, $Var(\mu_i) = \sigma_\mu^2$, and the $V_{i,t}$ have constant variance σ_V^2 and are serially uncorrelated. Define $e_{i,t} = U_{i,t} - \lambda\bar{U}_i$, where $\lambda = 1 - \left[\frac{\sigma_V^2}{\sigma_V^2 + T\sigma_\mu^2}\right]^2$.

   i. Show that $E(e_{i,t}) = 0$.
   ii. Show that $Var(e_{i,t}) = \sigma_V^2,\ t = 1, \cdots, T$.
   iii. Show that for $t \neq s$, $Cov(e_{i,t}, e_{i,s}) = 0$.

This is the weighting scheme used in the random effects estimator.

4. **Exercise 10.1 in AGME**: Consider the following simple panel data model:

$$y_{i,t} = x_{i,t}\beta + \alpha_i^* + \varepsilon_{i,t}, \quad i = 1, \cdots, N, \quad t = 1, \cdots, T,$$

where β is one-dimensional, and where it is assumed that

$$\alpha_i^* = \bar{x}_i\lambda + \alpha_i, \quad \text{with} \quad \alpha_i \sim NID(0, \sigma_\alpha^2), \quad \varepsilon_{i,t} \sim NID(0, \sigma_\varepsilon^2),$$

mutually independent and independent for all $x_{i,t}$, where $\bar{x}_i = (1/T)\sum_{t=1}^{T} x_{i,t}$.

The parameter β can be estimated by the fixed effects (or within) estimator given by

$$\hat{\beta}_{FE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{i,t} - \bar{x}_i)(y_{i,t} - \bar{y}_i)}{\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{i,t} - \bar{x}_i)^2}.$$

As an alternative, the correlation between the error term $\alpha_i^* + \varepsilon_{i,t}$ and $x_{i,t}$ can be handled by an instrumental variables approach.

i. Give an expression for the IV estimator $\hat{\beta}_{IV}$ using $x_{i,t} - \bar{x}_i$ as an instrument for $x_{i,t}$. Show that $\hat{\beta}_{IV}$ and $\hat{\beta}_{FE}$ are identical.

Another way to eliminate the individual effects α_i^* from the model is obtained by taking first differences. This results in

$$y_{i,t} - y_{i,t-1} = (x_{it} - x_{i,t-1})\beta + (\varepsilon_{i,t} - \varepsilon_{i,t}), \quad i = 1, \cdots, N, \quad t = 2, \cdots, T.$$

ii. Denote the OLS estimator based on the equation just above by $\hat{\beta}_{FD}$. Show that $\hat{\beta}_{FD}$ is identical to $\hat{\beta}_{IV}$ and $\hat{\beta}_{FE}$ if $T = 2$. This identity does not longer hold for $T > 2$.

5. For this exercise, you are going to use the Rental dataset attached in the repository. The data contains information on rental prices and other variables for college towns for the years 1980 and 1990. The idea is to see whether a stronger presence of students affects rental rates.

The unobserved effects model is:

$$\log(rent_{it}) = \beta_0 + \delta_0 y90_t + \beta_1 \log(pop_{it}) + \beta_2 \log(avginc_{it}) + \beta_3 pctstu_{it} + a_i + u_{it},$$

where $y90_t$ is a dummy for year 90, *pop* is city population, *avginc* is average income, and *pctstu* is student population as a percentage of city population (during the school year).

i. Estimate the equation by pooled OLS and report the results in standard form. What do you make of the estimate on the 1990 dummy variable? What do you get for $\hat{\beta}_{pctstu}$?

ii. Are the standard errors you report in part (i) valid? Explain.

iii. Now, difference the equation and estimate by OLS. Compare your estimate of β_{pctstu} with that from part (i). Does the relative size of the student population appear to affect rental prices?

iv. Estimate the model by fixed effects to verify that you get identical estimates and standard errors to those in part *iii*.