Exercises Week Simultaneous Equations

Econometrics

1. **GMM and 3SLS equivalence**: You are going to show that the GMM estimator is equivalent to 3SLS. That is, show that

$$(P_W X_\bullet)'(\Sigma^{-1} \otimes I) = X_\bullet'(\Sigma^{-1} \otimes P_W),$$

   where $P_W X_\bullet$ is a block-diagonal matrix with blocks given by $P_W X_i$.

Hint: Write $P_W X_\bullet$ as a Kroenecker product.

2. **Exercises 12.29-30 in ETM**: Consider the demand-supply model

$$q_t = \beta_{1,1} + \beta_{2,1} x_{t2} + \beta_{3,1} x_{t3} + \gamma_{21} p_t + u_{t1},$$

$$q_t = \beta_{1,2} + \beta_{4,2} x_{t4} + \beta_{5,2} x_{t5} + \gamma_{22} p_t + u_{t2},$$

   where q_t is the log of quantity, p_t is the log of price, x_{t2} is the log of income, x_{t3} is a dummy variable that accounts for regular demand shifts, and x_{t4} and x_{t5} are the prices of inputs. Thus the first equation is a demand function and the second equation is a supply function.

   i. For this model, precisely what is the vector $\beta_\bullet$ defined in the lecture?

   ii. How many overidentifying restrictions are there for each equation?

   iii. The file *demand_supply.csv* contains 120 observations generated by the model. Estimate this model by 2SLS.

   iv. Test the overidentifying restrictions for each equation.

   v. Now estimate the model using 3SLS.

3. The dataset *smoke.csv* contains information on smoking habits and other variables for a random sample of single adults from the United States. It contains 807 observations in 10 variables:

Variable	Description
educ	years of schooling
cigpric	state cig. price, cents/pack
white	1 if white
age	in years
income	annual income
cigs	cigs. smoked per day

Variable	Description
restaurn	1 if restaurant smk. restrictions
lincome	log(income)
agesq	age squared
lcigpric	log(cigprice)

We use the data to estimate a demand function for daily cigarette consumption.

i. A model to estimate the effects of smoking on annual income (perhaps through lost work days due to illness, or productivity effects) is

$$log(income) = \beta_0 + \beta_1 cigs + \beta_2 educ + \beta_3 age + \beta_4 age^2 + u_1.$$

How do you interpret β_1?

ii. To reflect the fact that cigarette consumption might be jointly determined with income, a demand for cigarettes equation is

$$cigs = \gamma_0 + \gamma_1 log(income) + \gamma_2 educ + \gamma_3 age + \gamma_4 age^2 + \gamma_5 log(cigpric) + \gamma_6 restaurn + u_2.$$

Assuming these are exogenous to the individual, what signs would you expect for γ_5 and γ_6?

iii. Estimate the income equation by OLS and discuss the estimate of β_1.

iv. Estimate the reduced form for *cigs*. (Recall that this entails regressing *cigs* on all exogenous variables.) Are $log(cigpric)$ and *restaurn* significant in the reduced form?

v. Now, estimate the income equation by 2SLS. Discuss how the estimate of β_1 compares with the OLS estimate.

vi. Do you think that cigarette prices and restaurant smoking restrictions are exogenous in the income equation?