

$$Q// \quad E[y] = E[Ax + b] = AE[x] + b$$

$$\text{Cov}[y] = \text{Cov}[Ax + b] = A \text{Cov}[x] A^T = A \Sigma A^T$$

$$\text{As, } E[y] = \sum_x y p_r(x)$$

$$\sum_x (Ax + b) p_r(x) \quad \because y = Ax + b$$

$$A \sum_x x p_r(x) + b \sum_x p_r(x)$$

$$AE[x] + b(1) = AE[x] + b \quad \text{Hence proved}$$

$$\text{Now, } \text{Cov}[y] = \text{Cov}[Ax + b]$$

$$= E[(Ax + b - \mu)(Ax + b - \mu)^T]$$

$$= E[Ax(Ax)^T] - E[Ax]E[Ax]^T$$

$$= E[Ax Ax^T] - E[Ax]E[A^T x^T]$$

$$= A [E[xx^T] - E[x]E[x]^T] A^T$$

$$\text{(Proved)} \quad = A \text{Cov}[x] A^T \quad \because \text{Cov}[x] = E[xx^T] -$$

$$E[x]E[x]^T$$

$$\text{(Proved)} \quad = A \Sigma A^T$$

$\therefore$  Because Covariance matrix is also denoted by Sigma.