MATH189R HW1

Q2. a. $X = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$ $Y = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix}$ Q/ find least square estimate

@ $y = \theta^T X$ (Through cramer's rule)

∴ As cramer's rule works only on square matrix so

$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 9 & 29 \end{bmatrix}$

$X^T y = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 18 \\ 56 \end{bmatrix}$

$\underbrace{X^T X}_{A} \underbrace{\theta}_{} = \underbrace{X^T y}_{B}$

$A\theta = B$

$\theta_0 = \dfrac{\begin{bmatrix} 18 & 9 \\ 56 & 29 \end{bmatrix}}{\begin{bmatrix} 4 & 9 \\ 9 & 29 \end{bmatrix}}$ $\theta = 18/35$ $\theta_1 = \dfrac{\begin{vmatrix} 4 & 18 \\ 9 & 56 \end{vmatrix}}{\begin{vmatrix} 4 & 9 \\ 9 & 29 \end{vmatrix}} = \dfrac{62}{35}$

$\begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 18/35 \\ 62/35 \end{bmatrix}$

(b) $\theta = (X^T X)^{-1} X^T y$ Normal Equation

$$\left(\begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 9 \\ 9 & 29 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix}$$

$$\frac{1}{35} \begin{bmatrix} 29 & -9 \\ -9 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix}$$

$$\theta = \frac{1}{35} \begin{bmatrix} 18 \\ 62 \end{bmatrix} = \begin{bmatrix} 18/35 \\ 62/35 \end{bmatrix}$$