Equilibrium Interest Rate

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We derived demand and supply for credit here: Demand and Supply Derivation and Graphs.

We rewrite here the supply curve for credit which is a function of interest rate r:

- $\text{Supply}(R) = Q_s = a - \frac{b}{(1+r)}$

We can also rewrite the demand curve for credit which is a function of interest rate r:

- $\text{Demand}(r) = Q_d = \frac{h}{r^k}$

At equilibrium, demand equals to supply, shown graphically as the intersection point in Demand and Supply Derivation and Graphs.

We can solve for equilibrium by trying out a vector of interest rate points, or using nonlinear solution methods.

Alternatively, although this is not a system of linear equations, we can approximate these equations using first order taylor approximation, then they become a system of linear equations. We can then using *linsolve* to find approximate equilibrium Q and r.

First Order Taylor Approximation

Here, we discussed the formula for First Order Taylor Approximation: Definition of Differentials. Using the formula we have from there:

- $f(x) \approx f(a) + f'(a) \cdot (x - a)$

We approximate the demand and Supply curves. Now x is the interest rate, $f(x)$ is the demand or supply at interest rate x we are interested in. a is the interest rate level where we solve for actual demand or supply. We approximate the $f(x)$ by using information from $f(a)$.

For the problem here, let us approximate around $a = r_0 = 1$, this is 100 percent interest rate.

Note the demand and supply curves are monotonic, and they are somewhat linear for segments of r values. If they are not monotonically increasing or decreasing, we should not use taylor approximation.

Approximate the Supply

The Supply equation comes from Optimal Savings Choice in a 2 period Model with initial Wealth, applying the formula above with $a = r_0 = 1$:

```
clear all
syms a b r
```

```
% Supply equation
S = a - b/(1+r);
% For Approximation, need to get the derivative with respect to R
SDiffR = diff(S, r)
```

SDiffR =

$$\frac{b}{(r+1)^2}$$

```
% Now evaluate S at r = 1 and evaluate S'(r) also at r = 1
SatRis1 = subs(S, r, 1)
```

SatRis1 =

$$a-\frac{b}{2}$$

```
SDiffRris1 = subs(SDiffR, r, 1)
```

SDiffRris1 =

$$\frac{b}{4}$$

```
% We now have an equation that approximates supply
SupplyApproximate = SatRis1 + SDiffRris1*(r-1)
```

SupplyApproximate =

$$a-\frac{b}{2}+\frac{b\,(r-1)}{4}$$

Approximate the Demand

The Demand equation comes from Optimal Borrowing Choice Firm Maximization, Applying the formula above with $a = r_0 = 1$:

```
clear all
syms h k r
% Supply equation
D = h/r^k;
% For Approximation, need to get the derivative with respect to R
DDiffR = diff(D, r)
```

DDiffR =

$$-\frac{h\,k}{r^{k+1}}$$

```
% Now evaluate D at r = 1 and evaluate D'(r) also at r = 1
DatRis1 = subs(D, r, 1)
```

DatRis1 = h

```
DDiffRris1 = subs(DDiffR, r, 1)
```

```
DDiffRris1 = -h k
```

```
% We now have an equation that approximates supply
DemandApproximate = DatRis1 + DDiffRris1*(r-1)
```

DemandApproximate = $h - h\,k\,(r-1)$

Solve approximate Demand and Supply using a System of Linear Equations

Now we have two linear equations with two unknowns, we can rearrange the terms. Note that only r and $Q = Q_d = Q_s$ are unknowns, the other letters are parameters.

Starting with the equations from above:

- $S(r) \approx (a - \frac{b}{2}) + \frac{b}{4}(r-1)$
- $D(r) \approx h - k \cdot h(r-1)$

we end up with this system of two equations and two unknowns (Solving for Two Equations and Two Unknowns):

- $\begin{bmatrix} 1 & -\frac{b}{4} \\ 1 & k \cdot h \end{bmatrix} \cdot \begin{bmatrix} Q \\ r \end{bmatrix} = \begin{bmatrix} a - \frac{3}{4}b \\ h + k \cdot h \end{bmatrix}$

We can plug this into matlab and solve for it

```
syms a b h k r
COEFMAT = [1, -b/4;1, k*h];
OUTVEC = [a-(3*b)/4; h + k*h];
approximateSolution = linsolve(COEFMAT, OUTVEC);
QEquiApproximate = approximateSolution(1)
```

QEquiApproximate =

$$\frac{b\,h + 4\,a\,h\,k - 2\,b\,h\,k}{b + 4\,h\,k}$$

```
REquiApproximate = approximateSolution(2)
```

REquiApproximate =

$$\frac{3\,b - 4\,a + 4\,h + 4\,h\,k}{b + 4\,h\,k}$$

Now we have approximate analytical equations for demand and supply. If our $a = r_0 = 1$ was close to true equilibrium rate, we would have a good approximation of how parameters of the model, the a, b, h, k constants, impact the equilibrium interest rate and quantity demanded and supplied.

See this page for how this is applied to the credit demand and supply example: First Order Taylor Approximation of Demand and Supply for Capital