

# Marginal Product of Labor

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## Marginal Product of Additional Workers (Discrete Workers)

Suppose we can not hire fractions of workers, but have to hire 1, 2, 3, ... units of workers. What is the marginal product of each additional worker?

```
% fixed capital level
K = 1;
% current labor level
L = [1,2,3,4,5,6,7,8,9,10];

% Cobb Douglas Production Parameters
alpha = 0.5;
beta = 1-alpha;

% Output at x0
fx0 = (K^alpha)*(L.^beta);

% a vector of h
h = 1;

% output at fx0plush
x0plush = L+h;
fx0plush = (K^alpha)*((x0plush).^beta);

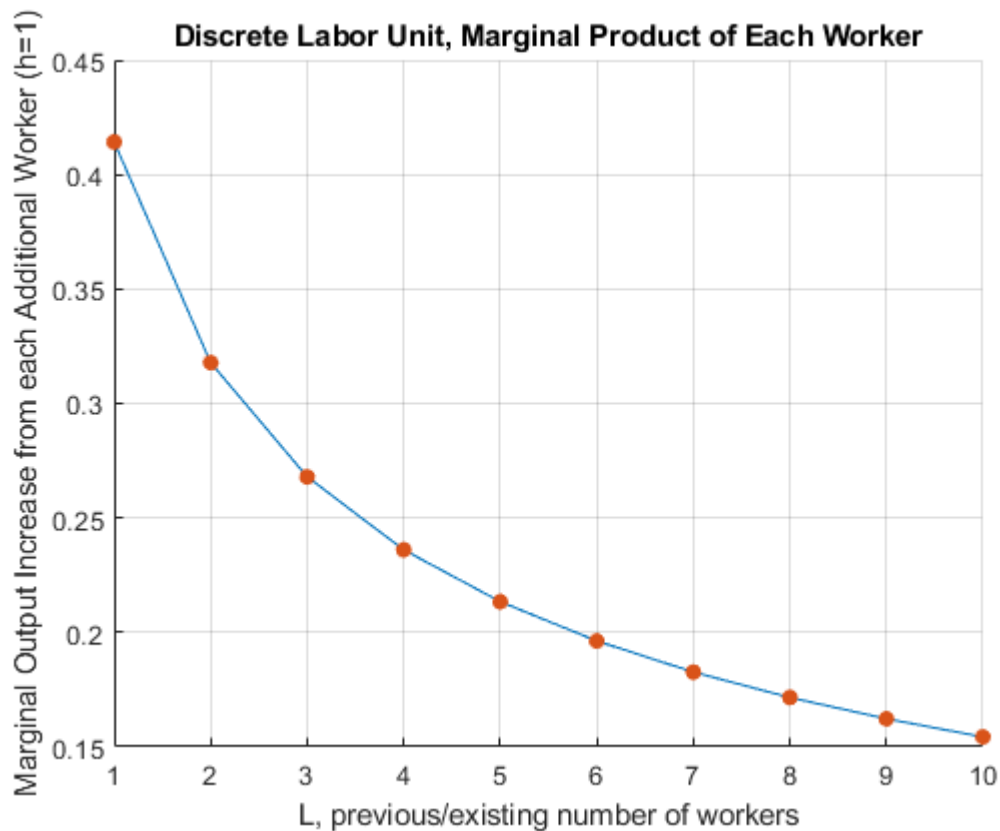
% derivatie
outputIncrease = (fx0plush - fx0)./h;

% Show Results in table
T = table(L', x0plush', fx0plush', outputIncrease');
T.Properties.VariableNames = {'L', 'x0plush', 'fx0plush', 'outputIncrease'};
disp(T);
```

L	x0plush	fx0plush	outputIncrease
1	2	1.4142	0.41421
2	3	1.7321	0.31784
3	4	2	0.26795
4	5	2.2361	0.23607
5	6	2.4495	0.21342
6	7	2.6458	0.19626
7	8	2.8284	0.18268
8	9	3	0.17157
9	10	3.1623	0.16228
10	11	3.3166	0.15435

```
% Graph
close all;
```

```
figure();
hold on;
plot(L, outputIncrease);
scatter(L, outputIncrease, 'filled');
grid on;
ylabel('Marginal Output Increase from each Additional Worker (h=1)')
xlabel('L, previous/existing number of workers')
title('Discrete Labor Unit, Marginal Product of Each Worker')
```



## Using Derivative to approximate Increase in Output from More Workers

We know the MPL formula, so we can evaluate MPL at the vector of L

```
% fixed capital level
K = 1;
% current labor level
L = [1,2,3,4,5,6,7,8,9,10];

% Cobb Douglas Production Parameters
alpha = 0.5;

% Output at x0
fprimeX0 = (1-alpha)*(K^alpha)*(L.^(-alpha));

T = table(L', outputIncrease', fprimeX0');
T.Properties.VariableNames = {'L', 'outputIncrease', 'fprimeX0'};
```

```
disp(T);
```

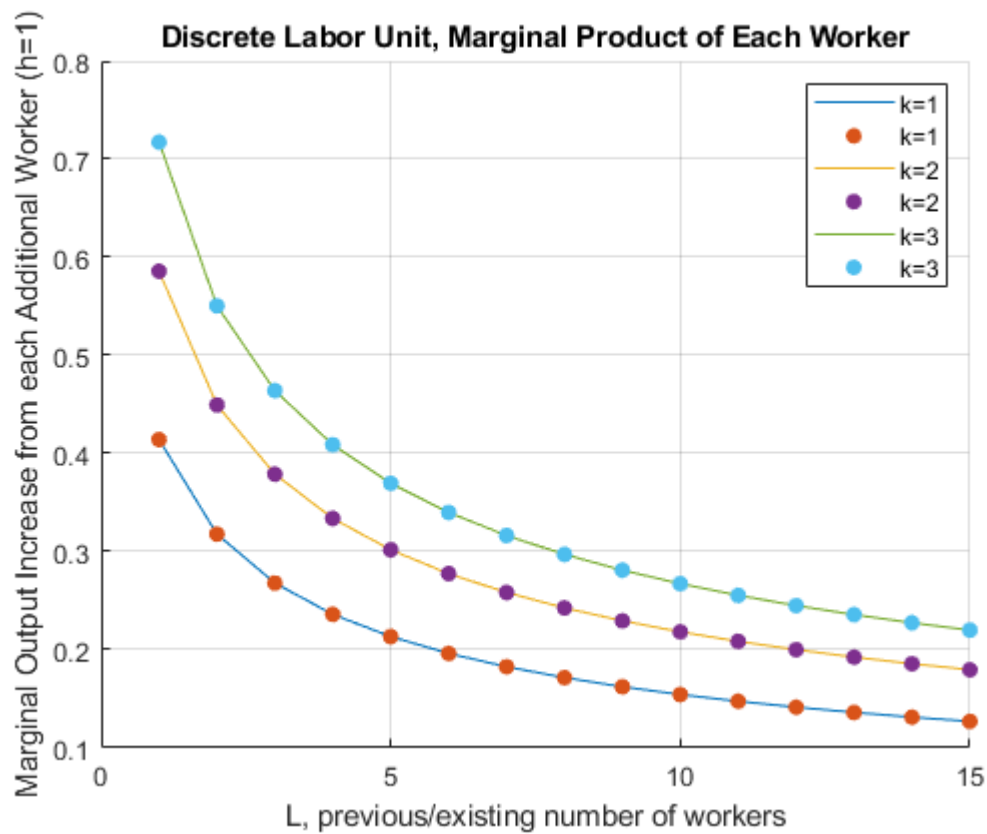
L	outputIncrease	fprimeX0
1	0.41421	0.5
2	0.31784	0.35355
3	0.26795	0.28868
4	0.23607	0.25
5	0.21342	0.22361
6	0.19626	0.20412
7	0.18268	0.18898
8	0.17157	0.17678
9	0.16228	0.16667
10	0.15435	0.15811

## Marginal Product of Additional Workers Different Capital (Discrete Workers)

Suppose we can not hire fractions of workers, but have to hire 1, 2, 3, etc.. What is the marginal product of each additional worker?

```
% fixed capital level
K1 = 1;
[fprimeX0K1, L] = MPKdiscrete(K1);
K2 = 2;
[fprimeX0K2, L] = MPKdiscrete(K2);
K3 = 3;
[fprimeX0K3, L] = MPKdiscrete(K3);

% Graph
close all;
figure();
hold on;
plot(L, fprimeX0K1);
scatter(L, fprimeX0K1, 'filled');
plot(L, fprimeX0K2);
scatter(L, fprimeX0K2, 'filled');
plot(L, fprimeX0K3);
scatter(L, fprimeX0K3, 'filled');
grid on;
ylabel('Marginal Output Increase from each Additional Worker (h=1)')
xlabel('L, previous/existing number of workers')
title('Discrete Labor Unit, Marginal Product of Each Worker')
legend(['k=', num2str(K1)], ['k=', num2str(K1)], ...
        ['k=', num2str(K2)], ['k=', num2str(K2)], ...
        ['k=', num2str(K3)], ['k=', num2str(K3)]]);
```



```
function [fprimeX0, L] = MPKdiscrete(K)
% current labor level
L = [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15];

% Cobb Douglas Production Parameters
alpha = 0.5;
beta = 1-alpha;

% Output at x0
fx0 = (K^alpha)*(L.^beta);

% a vector of h
h = 1;

% output at fx0plush
x0plush = L+h;
fx0plush = (K^alpha)*((x0plush).^beta);

% derivatie
fprimeX0 = (fx0plush - fx0)./h;

end
```