A ***subpath*** of path $p = \langle v_0, v_1, \ldots, v_k \rangle$ is a contiguous subsequence of its vertices. That is, for any $0 \le i \le j \le k$, the subsequence of vertices $\langle v_i, v_{i+1}, \ldots, v_j \rangle$ is a subpath of $p$.

In a directed graph, a path $\langle v_0, v_1, \ldots, v_k \rangle$ forms a ***cycle*** if $v_0 = v_k$ and the path contains at least one edge. The cycle is ***simple*** if, in addition, $v_1, v_2, \ldots, v_k$ are distinct. A self-loop is a cycle of length 1. Two paths $\langle v_0, v_1, v_2, \ldots, v_{k-1}, v_0 \rangle$ and $\langle v'_0, v'_1, v'_2, \ldots, v'_{k-1}, v'_0 \rangle$ form the same cycle if there exists an integer $j$ such that $v'_i = v_{(i+j) \bmod k}$ for $i = 0, 1, \ldots, k-1$. In Figure 5.2(a), the path $\langle 1, 2, 4, 1 \rangle$ forms the same cycle as the paths $\langle 2, 4, 1, 2 \rangle$ and $\langle 4, 1, 2, 4 \rangle$. This cycle is simple, but the cycle $\langle 1, 2, 4, 5, 4, 1 \rangle$ is not. The cycle $\langle 2, 2 \rangle$ formed by the edge $(2, 2)$ is a self-loop. A directed graph with no self-loops is ***simple.*** In an undirected graph, a path $\langle v_0, v_1, \ldots, v_k \rangle$ forms a ***cycle*** if $v_0 = v_k$ and $v_1, v_2, \ldots, v_k$ are distinct. For example, in Figure 5.2(b), the path $\langle 1, 2, 5, 1 \rangle$ is a cycle. A graph with no cycles is ***acyclic.***

An undirected graph is ***connected*** if every pair of vertices is connected by a path. The ***connected components*** of a graph are the equivalence classes of vertices under the "is reachable from" relation. The graph in Figure 5.2(b) has three connected components: $\{1, 2, 5\}$, $\{3, 6\}$, and $\{4\}$. Every vertex in $\{1, 2, 5\}$ is reachable from every other vertex in $\{1, 2, 5\}$. An undirected graph is connected if it has exactly one connected component, that is, if every vertex is reachable from every other vertex.

A directed graph is ***strongly connected*** if every two vertices are reachable from each other. The ***strongly connected components*** of a graph are the equivalence classes of vertices under the "are mutually reachable" relation. A directed graph is strongly connected if it has only one strongly connected component. The graph in Figure 5.2(a) has three strongly connected components: $\{1, 2, 4, 5\}$, $\{3\}$, and $\{6\}$. All pairs of vertices in $\{1, 2, 4, 5\}$ are mutually reachable. The vertices $\{3, 6\}$ do not form a strongly connected component, since vertex 6 cannot be reached from vertex 3.

Two graphs $G = (V, E)$ and $G' = (V', E')$ are ***isomorphic*** if there exists a bijection $f : V \to V'$ such that $(u, v) \in E$ if and only if $(f(u), f(v)) \in E'$. In other words, we can relabel the vertices of $G$ to be vertices of $G'$, maintaining the corresponding edges in $G$ and $G'$. Figure 5.3(a) shows a pair of isomorphic graphs $G$ and $G'$ with respective vertex sets $V = \{1, 2, 3, 4, 5, 6\}$ and $V' = \{u, v, w, x, y, z\}$. The mapping from $V$ to $V'$ given by $f(1) = u$, $f(2) = v, f(3) = w, f(4) = x, f(5) = y, f(6) = z$ is the required bijective function. The graphs in Figure 5.3(b) are not isomorphic. Although both graphs have 5 vertices and 7 edges, the top graph has a vertex of degree 4 and the bottom graph does not.

We say that a graph $G' = (V', E')$ is a ***subgraph*** of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. Given a set $V' \subseteq V$, the subgraph of $G$ ***induced*** by $V'$ is the graph $G' = (V', E')$, where

$$E' = \{(u, v) \in E : u, v \in V'\} .$$