


Simple cycles of length two in an undirected graph

 cs.stackexchange.com/questions/115158/simple-cycles-of-length-two-in-an-undirected-graph

Pedagogical question.

Background

A cycle in a graph can be defined as a sequence of vertices v_1, \dots, v_n with $v_1 = v_n$ such that, for each $i \in \{1, \dots, n-1\}$, the graph has an edge (v_i, v_{i+1}) . (One can define it differently.)

The point

Every definition of *simple* cycle I have seen is: a cycle with no repeated vertices, except the first and last.

But this definition implies that even in undirected graphs, we can have simple cycles of length two, e.g. $u \rightarrow v \rightarrow u$. However, we probably do not want to consider this a simple cycle because it re-uses the edge (u, v) and this would break several algorithms and proofs.

Of course for cycles of length 3 or more, requiring no repeated vertices implies there are no repeated edges.

The question

Which of the following is right? Justify your answer.

- (1) The standard definitions are incorrect, they ought to explicitly exclude simple cycles of length 2 in undirected graphs.
- (2) The standard definitions are correct, the cycle of length 2 **is simple**.
- (3) This question is wrong in claiming this is the standard definition of simple cycle. There are other better definitions that are standard (references please!).

asked Sep 27 '19 at 13:31

I'll go with (3)

From Bundy and Murty's *Graph Theory with Applications*

(http://www.maths.lse.ac.uk/Personal/jozef/LTCC/Graph_Theory_Bondy_Murty.pdf), sections 1.6 "Paths and Connections" and 1.7 "Cycles":

A *walk* is a finite non-null sequence of alternating vertices and edges (repetition of both allowed).

If the edges of a walk are distinct then it is also a *trail*. If in addition the vertices are distinct then the trail is a *path*.

A walk is *closed* if it has positive length (defined as its number of edges) and its origin and terminus are the same (vertex). **A closed trail whose origin and internal vertices are distinct is a cycle.**

SUMMARY: So in this system of definitions, a *cycle* cannot repeat edges or vertices except the origin/terminus, which excludes walks of length two.

However, it is true that there are other contradictory definitions in the literature which is why it is best to always restate the definitions in say a paper or course.

PS: For the sake of completion, I believe that in this system of definitions (although I did look for it in this particular reference), a *circuit* is a closed trail (repetition of vertices allowed, but not of edges).

edited Sep 28 '19 at 10:06

answered Sep 28 '19 at 9:52

It turns out that the very popular textbook *Introduction to Algorithms* by Cormen, Leiserson, Rivest, and Stein, sneakily avoids this issue by giving a different definition of cycle in undirected vs directed graphs.

In Section B.4, it defines cycle and simple cycle as I do (they also require at least one edge), *in directed graphs*. In an undirected graph, they define cycle the same way *except* they require that all of the edges in the cycle be distinct; and they say the cycle is simple if in addition the vertices are distinct, other than $v_1 = v_k$.

answered Sep 30 '19 at 23:22

Arguably two of the most definite textbooks on graph theory are Bundy & Murty (BM) (roughly 15000 citations at the time of writing) and Diestel (at least hundreds of citations).

In BM (Section 1.6), the definition of a cycle is obtained via walks and trails, i.e., a *cycle* is a non-empty *trail* in which the first and last vertex are repeated. In your example, *uvuvuvu* is not a cycle because in a trail all edges are distinct. So in particular, there's no need to explicitly forbid length-2 cycles as the definition of a trail takes care of this.

In Diestel (Section 1.3), the author begins by defining a *path* $P=(V,E)$ with $V=\{x_0, x_1, \dots, x_k\}$ and $E=\{x_0x_1, x_1x_2, \dots, x_{k-1}x_k\}$. Then, if $P=x_0, \dots, x_{k-1}$ is a path and $k \geq 3$, the graph $C:=P+x_{k-1}x_0$ is a

cycle.

In addition, the current Wikipedia entry on a cycle refers to Bender & Williamson which seems to use the same definition as BM.

In summary, both definitions should capture precisely the set of objects.

answered Sep 30 '19 at 16:25