“Matrix MoE is the best Fast Feed-Forward”

Vyacheslav Chaunin, Nickolay Mikhailovskiy

Higher IT School, Tomsk State University, NTR Labs

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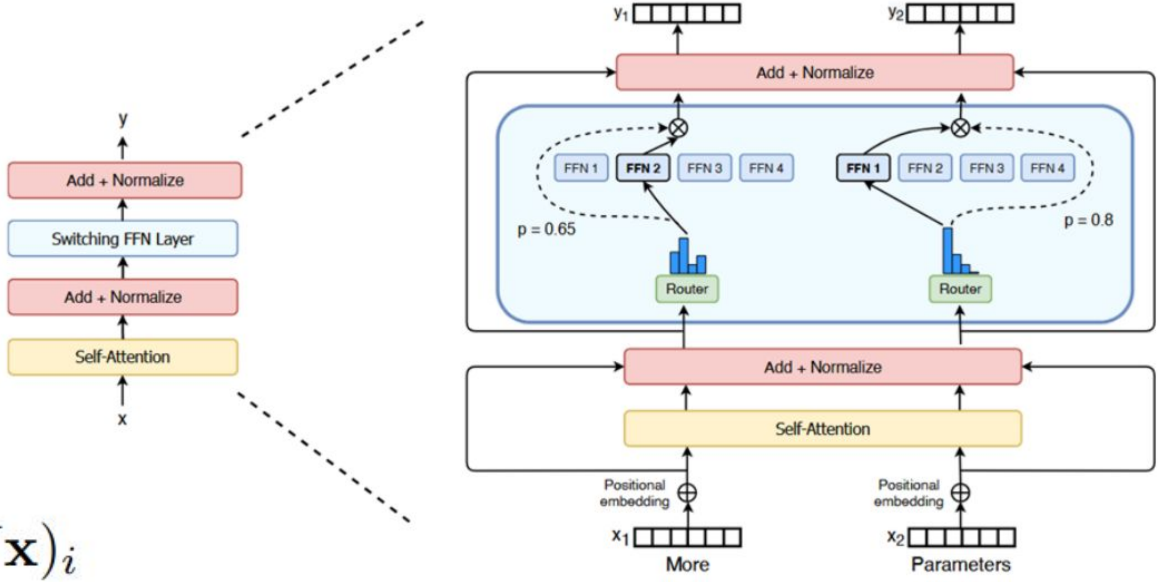
Mixture of Experts

- Smaller sub-models
- Routing mechanism

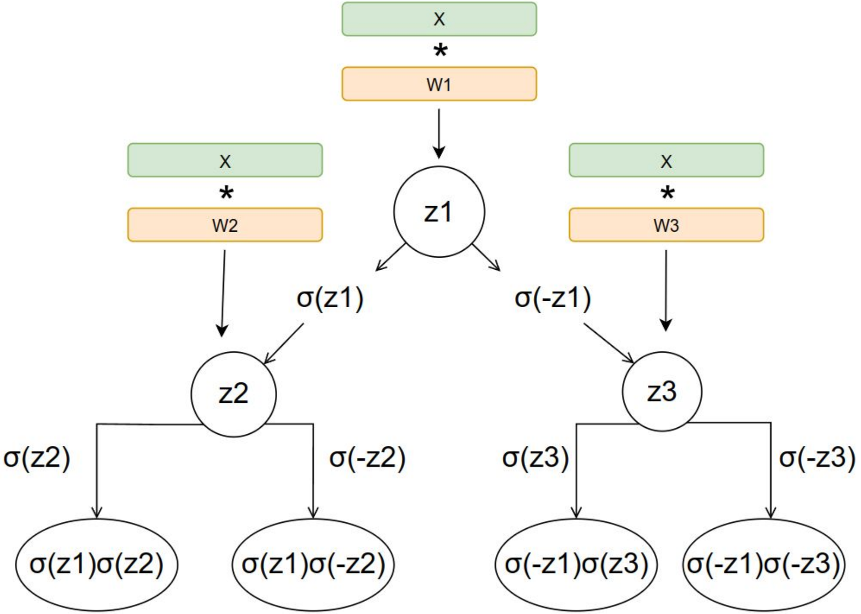


$$R(i|\mathbf{x}) = \text{Softmax}(W\mathbf{x})_i$$

$$\mathbf{y} = \sum_{i \in \text{top-}k(R)} R(i|\mathbf{x}) f_i(\mathbf{x})$$

Fast Feed-Forward

- Binary tree of routers
- $2^d - 1$ router parameter vectors
- 2^d experts



$$P(2i|\mathbf{x}, i) = \sigma(z_i) = \sigma(\mathbf{w}_i^\top \mathbf{x})$$

$$P(2i+1|\mathbf{x}, i) = \sigma(-z_i) = 1 - \sigma(\mathbf{w}_i^\top \mathbf{x})$$

FFF Training

The output of a layer is a weighted average across all experts

$$\mathbf{y} = \sum R(i|\mathbf{x}) f_i(\mathbf{x})$$

$$R(i|\mathbf{x}) = \prod_{j \in \mathrm{Path}(i)} P(j|\mathbf{x}, \lfloor \frac{j}{2} \rfloor)$$

The ***Path(i)*** contains indices of all router nodes that are parents of leaf ***i***

FFF Reformulation

$$\log R(i|\mathbf{x}) = \sum_{j \in \mathrm{Path}(i)} a((-1)^j z_{\lfloor j/2 \rfloor})$$

$$a(x) = \log \sigma(x) = -\mathrm{Softplus}(-x)$$

FFF Reformulation

$$\mathbf{z} = (z_1, \ldots, z_{2^d-1}) = W\mathbf{x}$$

$$R(i|\mathbf{x}) = \mathrm{Softmax}(Ta(S\mathbf{z}))_i$$

$$T \in \mathbb{R}^{2^d \times 2(2^d-1)} \qquad T_{ij} = 1 \text{ if } j \in \mathrm{Path}(i) \text{ and } 0 \text{ otherwise}$$

$$S \in \mathbb{R}^{2(2^d-1) \times 2^d-1} \qquad S_{ij} = \begin{cases} 1 & \text{if } 2i-1=j \\ -1 & \text{if } 2i=j \\ 0 & \text{otherwise} \end{cases}$$

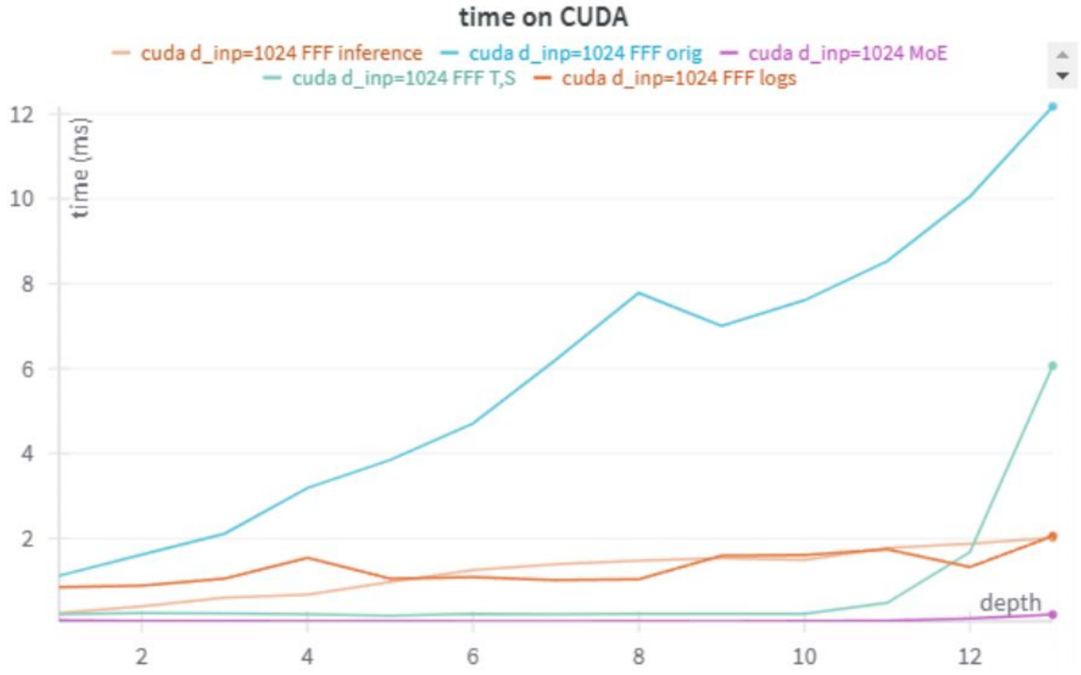
Example of FFF layer with d=2

$$T = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

Benchmarks

1024 input dimensions

Method	GPU		CPU	
	Up to 8	Up to 13	Up to 8	Up to 13
MoE	**45.63**	**57.84**	**7.27**	**4.55**
FFF T,S	11.89	8.93	3.55	0.88
FFF Inference	4.22	4.52	0.90	1.09
FFF Logs	2.58	3.22	1.27	1.07
FFF Orig	1.00	1.00	1.00	1.00



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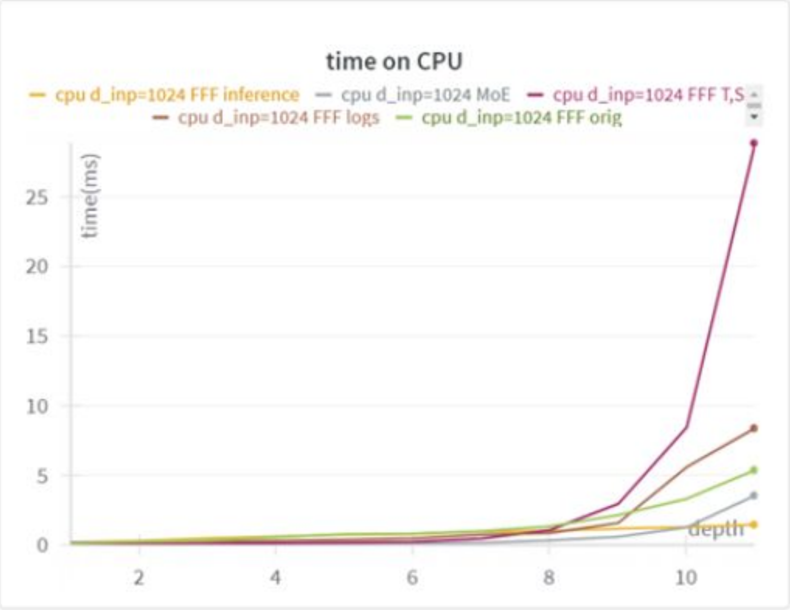
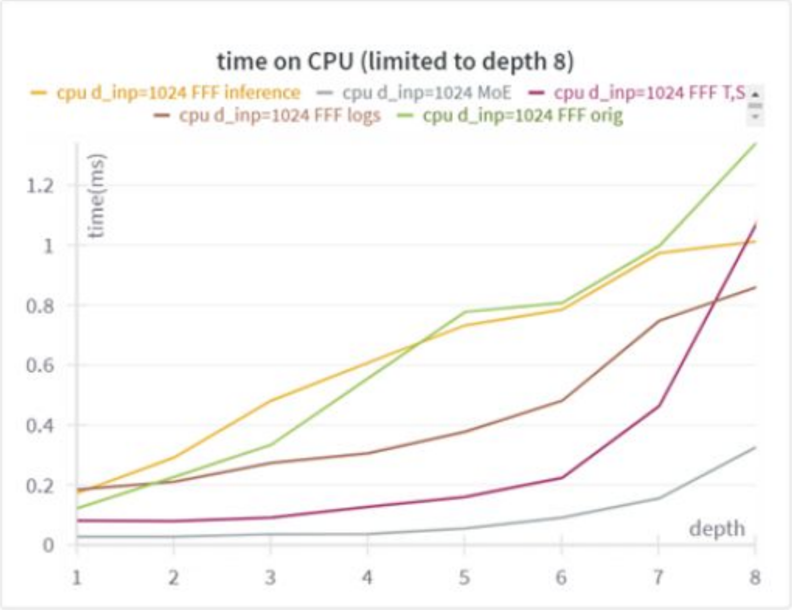




Experiments with Activation function

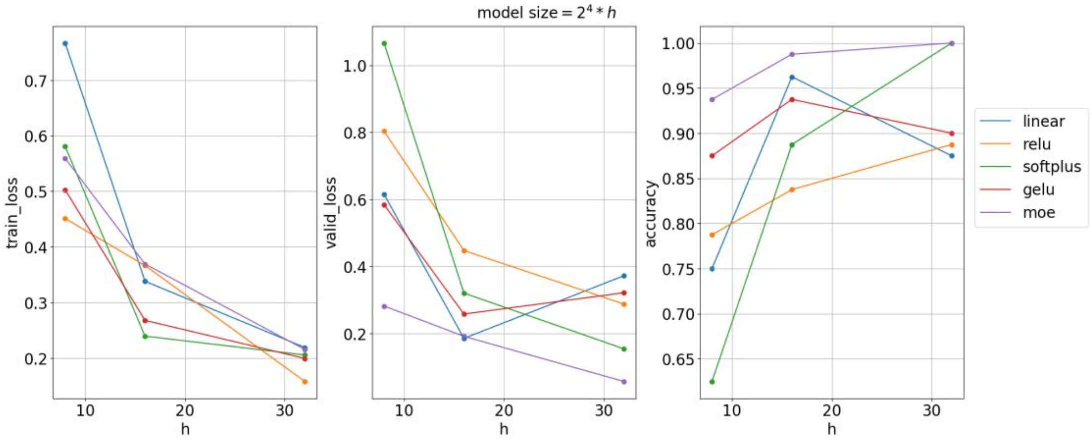
- Candidate functions:
  - Softplus
  - Linear
  - ReLU
  - GELU
- Setups:
  - Clusters of Gaussians
  - CIFAR-10
  - Cramming BERT

$$R(i|\mathbf{x}) = \mathrm{Softmax}(Ta(S\mathbf{z}))_i$$

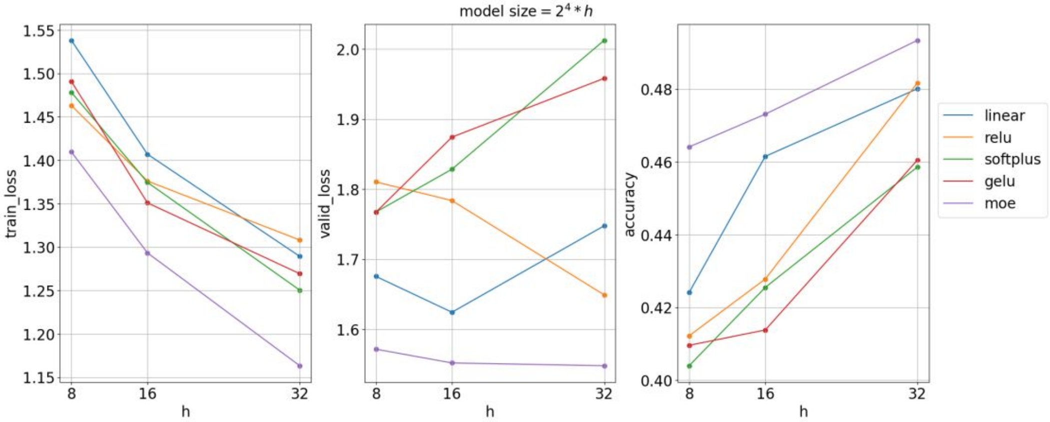
Synthetic and CIFAR-10 setup

- Single FFF/MoE routing layer
- Experts are 2-layer MLPs of varying width from [8, 16, 32]
- Batch size 16 (64)
- Adam optimizer with one-cycle schedule and max lr 8e-4
- Dropout 0.2
- 15 (20) epochs

Experiments



Synthetic dataset



CIFAR-10

Results

CIFAR-10 accuracy across layer depths

Activation	d=2.0	d=3.0	d=4.0	d=5.0
Linear	**0.470**	0.461	**0.455**	**0.447**
ReLU	0.464	**0.468**	0.441	0.416
Softplus	0.458	0.461	0.429	0.427
GELU	0.461	0.456	0.428	0.408
MOE	**0.472**	**0.483**	**0.477**	**0.470**

Cramming BERT results

Activation	STSB	SST2	RTE	MRPC	COLA	Average GLUE
Linear	**0.564**	0.859	0.484	0.706	0.129	0.57
Softplus	0.439	**0.860**	0.509	0.708	0.112	0.54
ReLU	0.560	0.849	**0.560**	0.716	0.117	**0.58**
GELU	0.506	0.855	0.545	**0.723**	**0.165**	**0.58**

Cramming BERT https://arxiv.org/abs/2212.14034

Conclusions

- FFF can be practically used only with large number of experts, i.e. in very fragmented models
- For training it’s more optimal to vectorize the process
- For many models using linear activation might improve the results

Thank you for attention!