OPTIMIZING STOCK INVESTMENT PORTFOLIO USING DYNAMIC GRAPH NEURAL NETWORKS

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Representation of the asset market

The asset market is represented as a collection of financial time series X. A **time series** can be defined as a sequence of vectors (or scalars) that depend on time:

$$X = \{x(t_0), x(t_1), x(t_2), \ldots, x(t_i), \ldots\}, \tag{1}$$

here, t_i is the i-th time index, where $i = 0, 1, \ldots, N$.

Preprocessing time series to stationarity

A time series is considered stationary if its statistical properties remain constant over time. Specifically, stationarity implies that:

1. The expectation remains constant:

$$E[X_t] = \mu, \quad \forall t \tag{2}$$

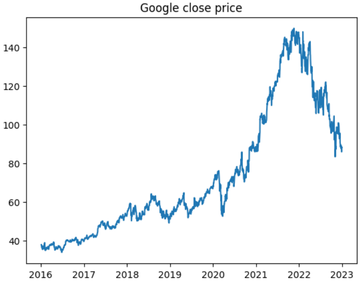
2. The variance does not change over time:

$$Var(X_t) = \sigma^2, \quad \forall t \tag{3}$$

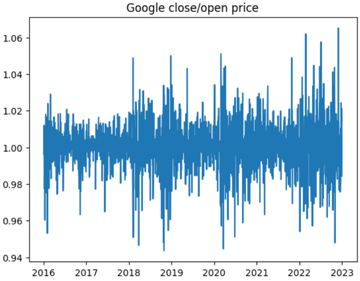
3. The covariance between observations depends only on the time lag h, not on the absolute time:

$$Cov(X_t, X_{t+h}) = f(h) \tag{4}$$

Preprocessing time series to stationarity



(a) Non-stationary series



(b) Stationary series

Рис. 1: a) The original (non-stationary) closing price series of Google (GOOGL) stocks for 2016–2023, b) The transformed (stationary) series.

Normalization time series

We applied and compared several normalization techniques:

- **StandardScaler normalization**, which standardizes data to have zero mean and unit variance:

$$x^* = \frac{x - \mu}{\sigma}$$

- **MinMax normalization**, which scales the data to the $[0, 1]$ range:

$$x^* = \frac{x - \min(x)}{\max(x) - \min(x)}$$

- **Combined normalization**, where StandardScaler is applied first, followed by MinMax:

$$x^{**} = \frac{x^* - \min(x^*)}{\max(x^*) - \min(x^*)}$$

after time series transformation

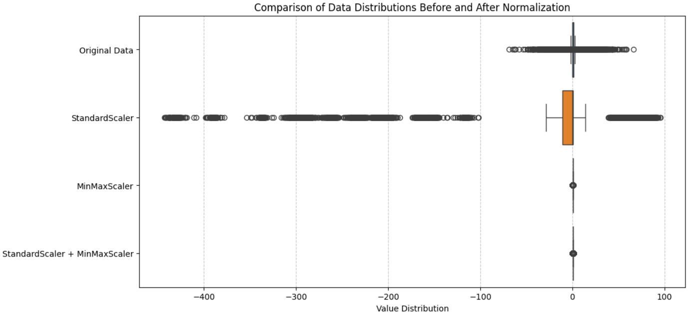


Рис. 2: Comparison of normalization methods applied to financial time series. Top-left: boxplot summary across methods. Top-right: StandardScaler. Bottom-left: MinMaxScaler. Bottom-right: Combined StandardScaler + MinMax.

Correlation Structure After Pre

figures/f_correlation_matrix.png

Definition of Temporal Financial Graph

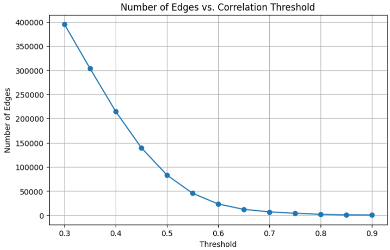
A temporal graph is a graph whose structure and attributes evolve over time. Formally, we define a temporal graph at time t as:

$$G_t = (V_t, E_t, W_t, X_t) \tag{5}$$

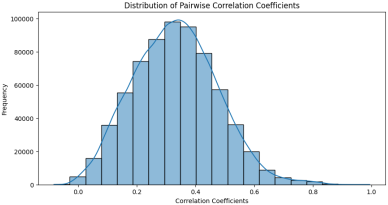
where:

- V_t is the set of vertices at time t, where each vertex represents one of n assets.
- E_t is the set of edges at time t, representing relationships (e.g., correlation or covariance) between assets.
- W_t is the weight matrix associated with the edges, encoding the strength of relationships at time t.
- Each node $V_{i,t}$ has a feature vector $X_{i,t}$, representing the time series of asset i up to time t.

Modeling of Temporal Financial Graph



Number of Edges vs. Correlation Threshold



Distribution of Pairwise Correlation Coefficients

- Most asset pairs exhibit moderate correlation (peak around 0.35).
- Increasing the correlation threshold drastically reduces the number of edges in the financial graph.
- This sparsity can be leveraged in graph-based models to focus

Architecture of Temporal Financial Graph

We model the graph sequence $\{G_1, G_2, \ldots, G_T\}$

1. **Graph Convolutional Layer (GCN):** For each node $v_i \in V_t$, we aggregate features from its neighbors using:

$$H_t^{(1)} = \mathrm{ReLU}\left(\hat{D}_t^{-1/2}\hat{A}_t\hat{D}_t^{-1/2}X_tW^{(gcn)}\right),$$

2. **Temporal Layer (GRU):** To capture temporal evolution of asset embeddings, we use a gated recurrent unit:

$$H_t^{(2)}, h_t = \mathrm{GRU}(H_t^{(1)}, h_{t-1}),$$

3. **Output Layer (Linear + Softmax):** The final layer maps the temporal embeddings to scalar scores and applies the softmax function to obtain valid portfolio weights:

$$\hat{w}_t = \mathrm{Softmax}(W^{(out)}H_t^{(2)} + b),$$

Formulation of the Optimization Problem

In each time step t, the goal is to determine optimal portfolio weights $\mathrm{w}_t \in \mathbb{R}^n$ for n assets that maximize cumulative profit and risk-adjusted performance:

Objective function:

$$\mathcal{L}_{\text{portfolio}} = -\sum_{t=1}^{T} \mathrm{w}_t^\top \mathrm{r}_t + \lambda \cdot \left(\frac{1}{\text{Sharpe} + \epsilon} \right) \tag{6}$$

where r_t is the asset return vector, λ is the regularization coefficient, and ϵ is a small constant to avoid division by zero.

Constraints:

$$\mathrm{w}_t^\top \Sigma_t \mathrm{w}_t \leq \sigma^2_{\max} \qquad \text{(Risk constraint)} \tag{7}$$

$$\sum_{i=1}^{n} w_{i,t} = 1 \qquad \text{(Budget constraint)} \tag{8}$$

Investment Portfolio

In financial terms, a **portfolio** is a collection of financial assets such as stocks, bonds, or other instruments.
Each asset i in the portfolio is assigned a weight $w_{i,t}$ at time t, representing the proportion of the total investment allocated to it.

Portfolio vector:

$$\mathrm{w}_t = [w_{1,t}, w_{2,t}, \ldots, w_{n,t}]^\top, \quad \sum_{i=1}^{n} w_{i,t} = 1$$

The total return of the portfolio at time t is the weighted sum of individual asset returns:

$$r_t^{(p)} = \mathrm{w}_{t-1}^\top \mathrm{r}_t$$

Portfolio Evaluation Metrics

To evaluate the performance of the portfolio, we use the following metrics:

1. Profit and Loss (PnL):

$$\text{PnL} = \sum_{t=1}^{T} \mathrm{w}_{t-1}^{\top} \mathrm{r}_t$$

2. Sharpe Ratio:

$$\text{Sharpe} = \frac{E[r^{(p)} - r_f]}{\sigma_p}$$

where r_f is the risk-free rate and σ_p is the portfolio volatility.

3. Maximum Drawdown (MDD):

$$\text{MDD} = \max_{t \in [1,T]} \left(\frac{\max_{s \le t} P_s - P_t}{\max_{s \le t} P_s} \right)$$

1. Data
2. Modeling of Temporal Financial Graph
3. The Optimization Problem
4. **Results**

Comparison of Portfolio Optimization Models

Metric	DGNN (Full)	DGNN (Lite)	Markowitz	WA
PnL	0.180	0.070	0.010	0.008
Sharpe	0.082	0.042	0.016	0.014
Risk	0.007	0.007	0.007	0.007

Таблица 1: Performance metrics for different portfolio optimization models.

Conclusion: The proposed DGNN model significantly outperforms traditional approaches in both profitability and risk-adjusted return, while maintaining the same level of risk.

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