PROBABILISTIC SPARSE VARIATIONAL MODEL BASED ON GAUSSIAN PROCESSES FOR ENERGY PARAMETER FORECASTING

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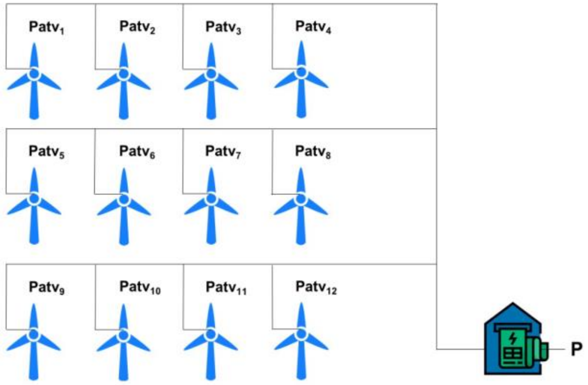
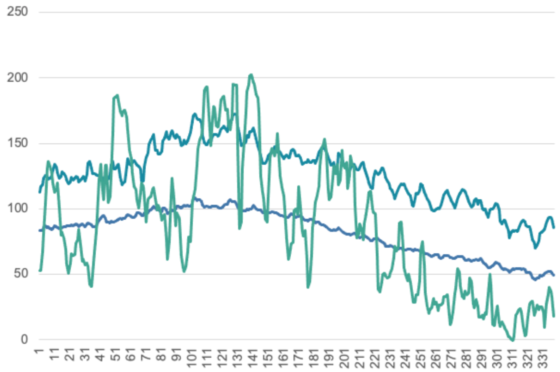
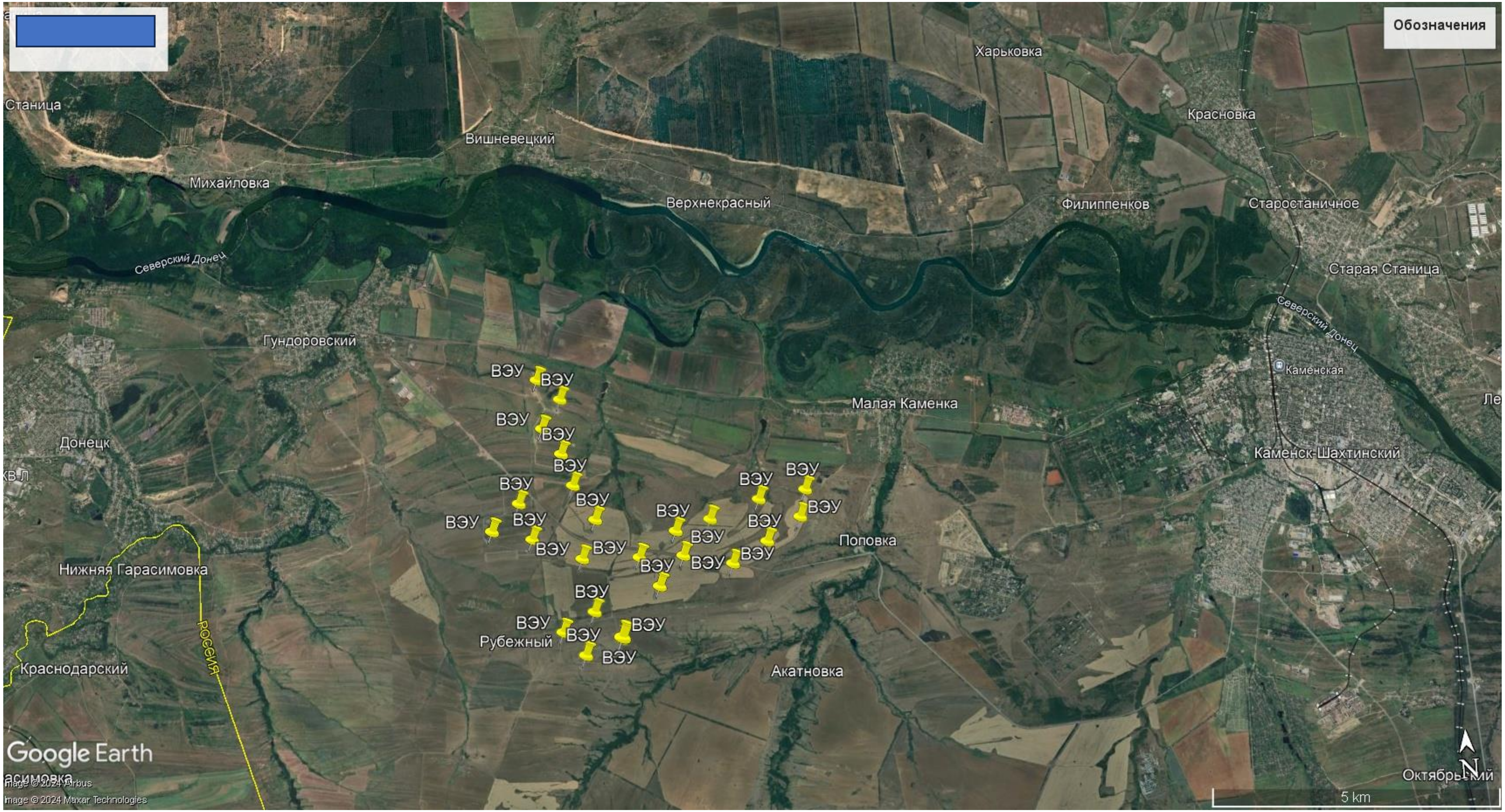
- Wind Energy, Power Plants and relevance of ML/AI models
- Data collection from a single wind turbine and power generator
- Data preprocessing
- Selection and development of SVGP model for power prediction
- SVGP model prediction results
- Analysis of the first results
- Further work planning

Relevance and problem statement

- More than **25 new wind farms** have been built in Russia in the last 5 years
- There is instability of power generation
- Dispatch schedule in power companies. GTP is a group of supply points.
- Forecast of electricity generation is necessary with an **accuracy of 90-95%.** It is important for the end consumer.
- Relevance: the need to develop ML/AI models for **Wind Power Forecasting** and **Cost of electricity**.
- Short-term models - 10-30 minutes; Long-term models (market) - 24-72 hours.
- **Deterministic and Probabilistic** power forecast models
- **Probabilistic models (PWPF):** Delta method, Bayesian estimation, Mean-Variance Estimation (MVE), Bootstrap technique; Kernel Density Estimation (KDE)
- Open data on meteorological parameters and power generation from competitions (hackathons) are available: GEFCom2014, **Baidu KDD Cup 2022**, Zendoo UK 2022, IEEE Power and Energy Society Working Group on Energy Forecasting and Analytics 2024 In many countries (China, Denmark, Germany, UK, USA, Germany, Denmark, Germany, UK, USA) there are active works on the development of new ML/AI models to evaluate the performance of WPPs
- **Problem statement:** to develop an ML model for one wind turbine and group with automated control system as part of a wind power plant





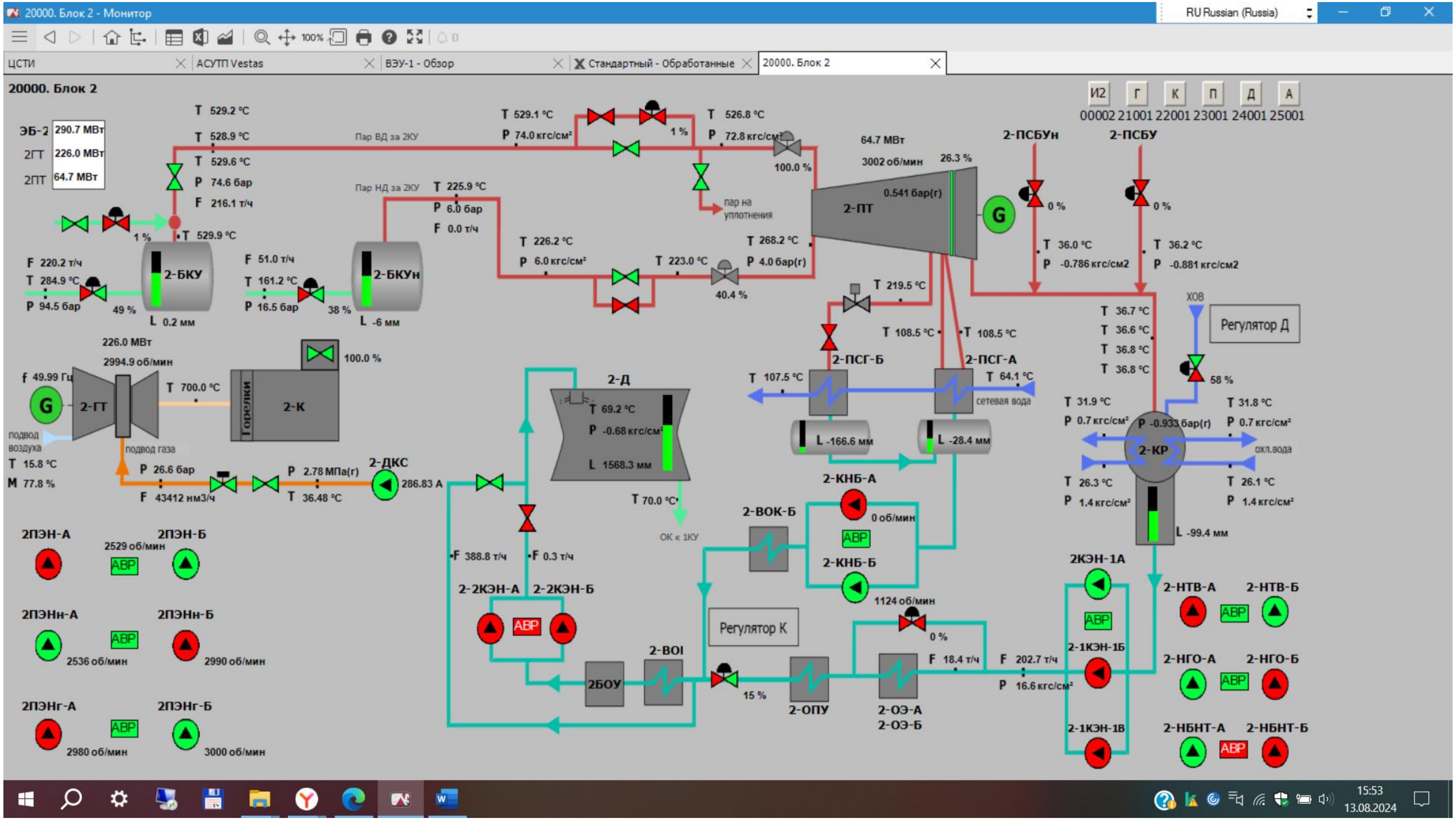


S = 6x6 km

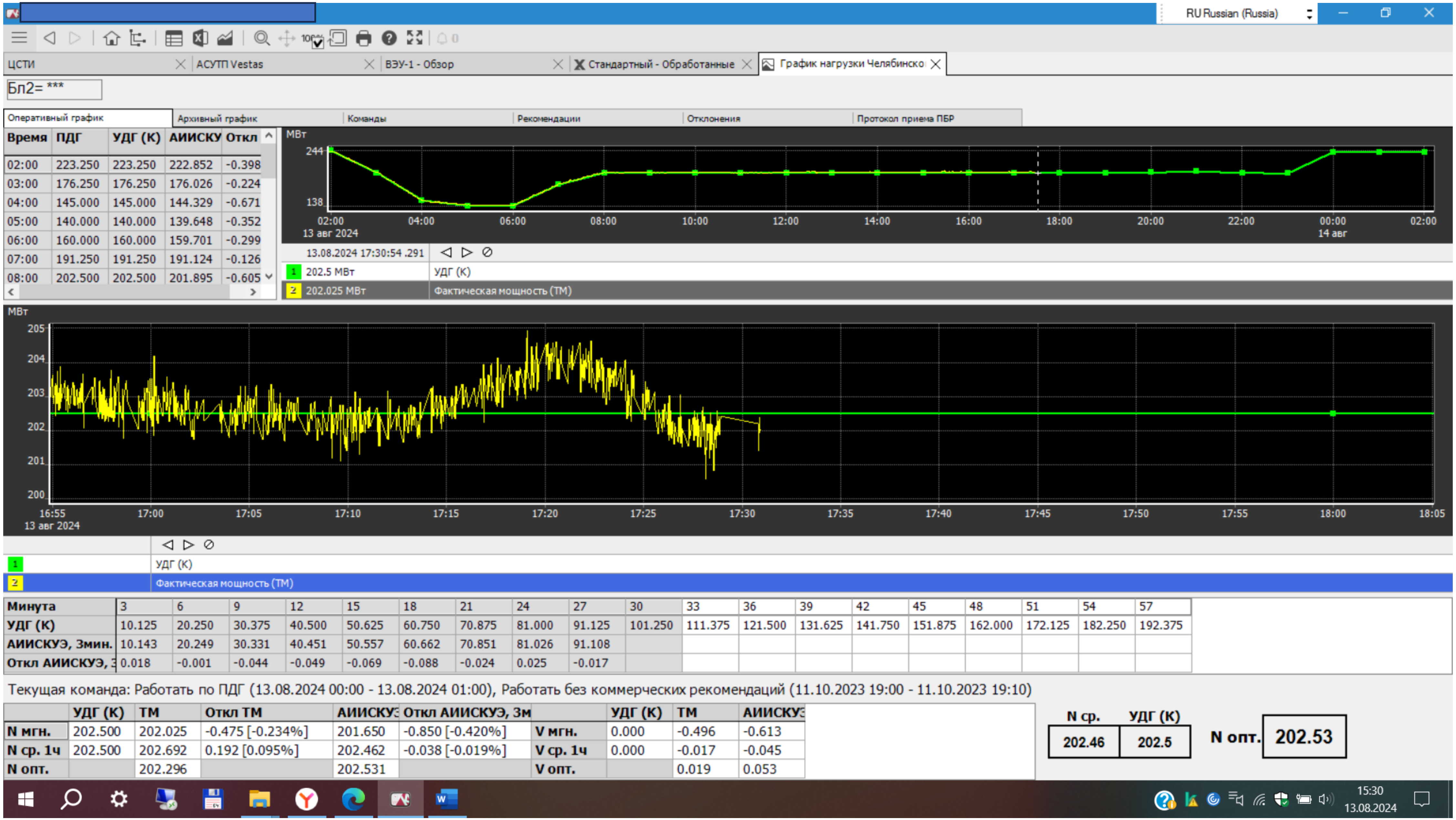
elevation - 125 meters above sea level

Date: 18.07.2024

Power Plant. Unit 2, Active load Pmax=202 MW.



Power Plant. Generation and prediction of Power.



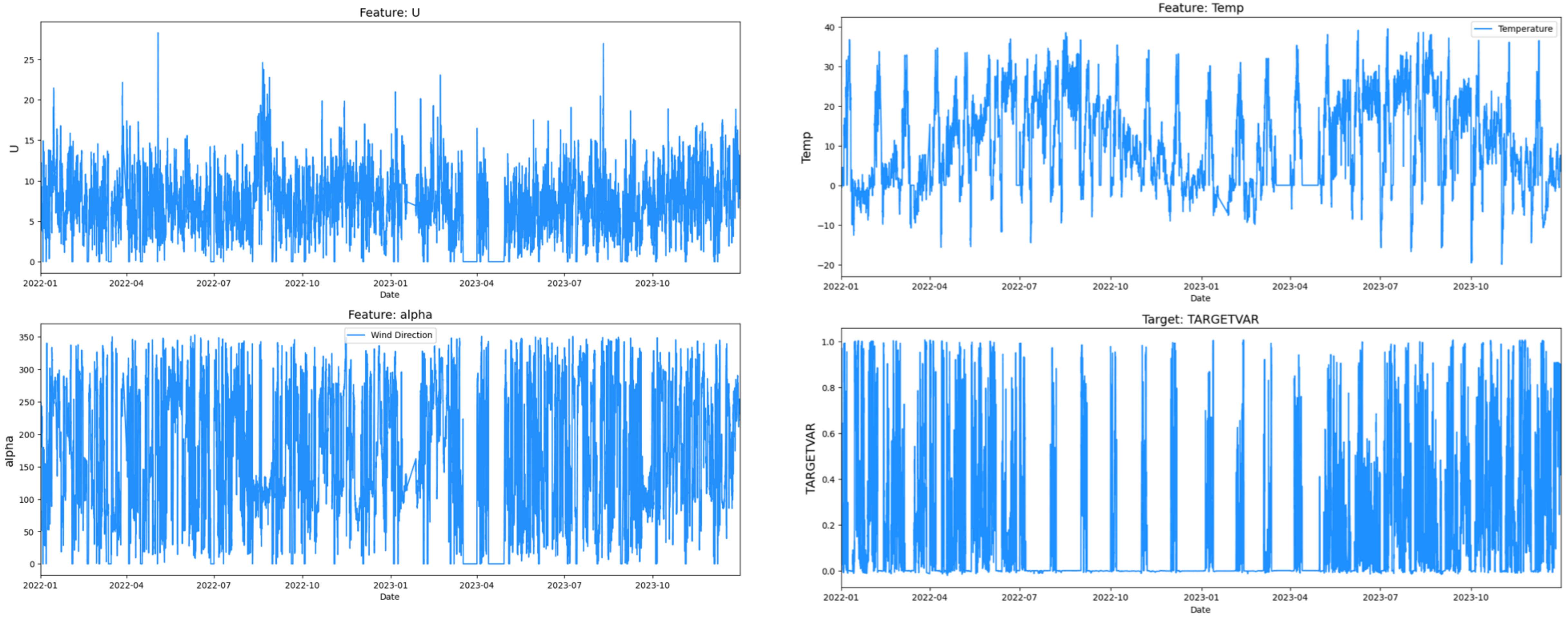
Data for wind turbine in 2022-2023

Тип отчета	Обработанные значения параметров с шагом по времени				
Время построения	15.05.2024 18:04:33				
Начало интервала	01.01.2022 00:00:00				
Конец интервала	01.01.2024 00:00:00				
Шаг времени	1h				
Функция	TimeAverage				
Свойства НСИ	Параметры				
SID	S_VESTAS.1010	S_VESTAS.1037	S_VESTAS.1035	S_VESTAS.1034	S_VESTAS.1057
Локальное имя	Ветрогенератор-1.Мощность со стороны сети	Ветрогенератор-1.Горизонтальная скорость ветра	Ветрогенератор-1.Горизонтальное направление ветра, абсолютное	Ветрогенератор-1.Температура окружающей среды	Ветрогенератор-1.Частота вращения ротора
Время	Значение	Значение	Значение	Значение	Значение
01.01.22 00:00	2636,934	9,513	189,367	0,492	12,531
01.01.22 01:00	1659,846	7,927	197,306	0,298	10,763
01.01.22 02:00	925,304	6,449	202,722	0,956	8,866
01.01.22 03:00	904,426	6,101	219,972	2,419	8,78
01.01.22 04:00	871,857	6,295	227,513	1,616	8,82
01.01.22 05:00	476,953	5,525	240,115	2,524	7,257
01.01.22 06:00	782,63	6,078	228,631	1,661	8,438
01.01.22 07:00	1112,761	6,659	219,148	1,182	9,443
01.01.22 08:00	1245,755	6,942	217,857	0,922	9,835
01.01.22 09:00	798,125	6,341	226,53	-0,428	8,454
01.01.22 10:00	623,247	5,913	239,8	-0,56	7,864
01.01.22 11:00	704,541	6,12	249,647	-0,696	8,161
01.01.22 12:00	746,209	6,191	238,679	-0,521	8,336
01.01.22 13:00	1566,007	7,82	238,397	-0,542	10,493
01.01.22 14:00	1169,655	7,154	230,738	0,335	9,538
01.01.22 15:00	476,908	5,329	215,842	0,651	7,296
01.01.22 16:00	731,012	5,871	211,415	0,525	8,244
01.01.22 17:00	1401,936	7,4	201,817	0,788	10,089
01.01.22 18:00	1774,197	7,746	196,696	1,879	10,958
01.01.22 19:00	2617,186	9,192	188,687	3,139	12,644
01.01.22 20:00	3426,749	10,376	180,433	3,51	13,515
01.01.22 21:00	3993,077	11,489	189,986	3,655	13,466

Total 160 attributes for wind turbines, we unload 4 attributes for the dataset. Pmax=4 200 kW.

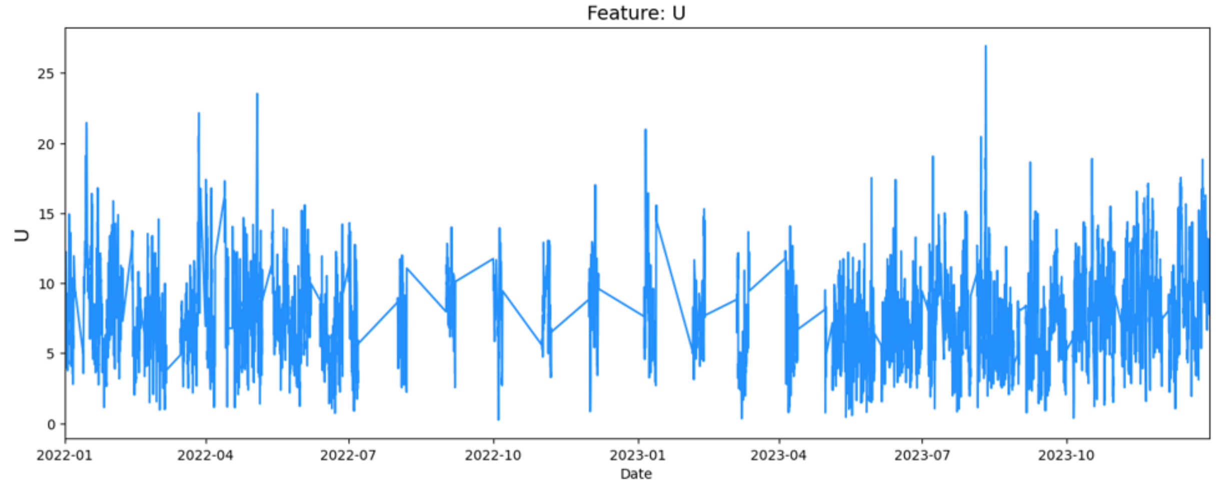
Data record multiplicity of 1 hour. We have a total of 17250 records. We use 4 attributes (U, alpha, Temp) and power.

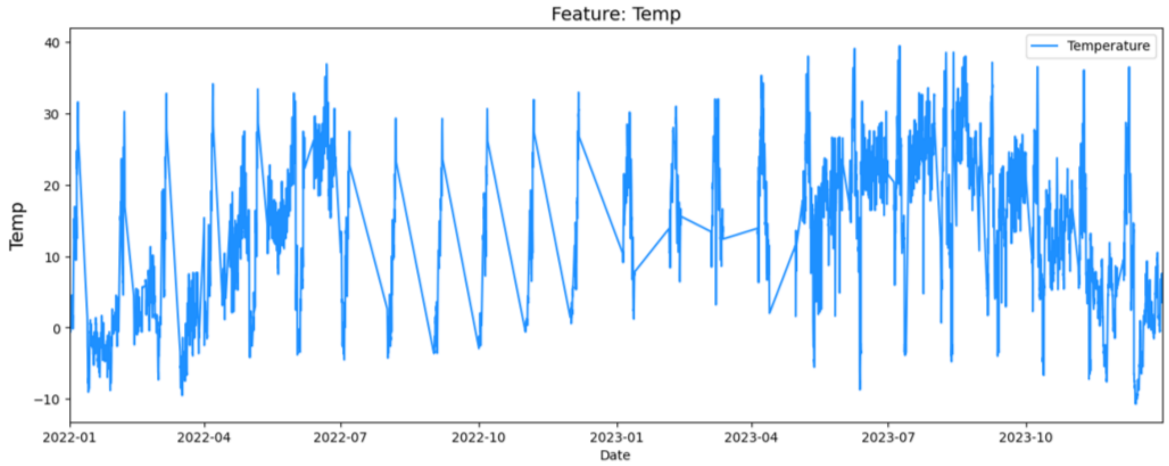
Python code: processing historical data (wind speed, wind direction, air temperature, wind turbine power) for the years 2022-2023 with 1 hour record multiplicity for wind turbine-S

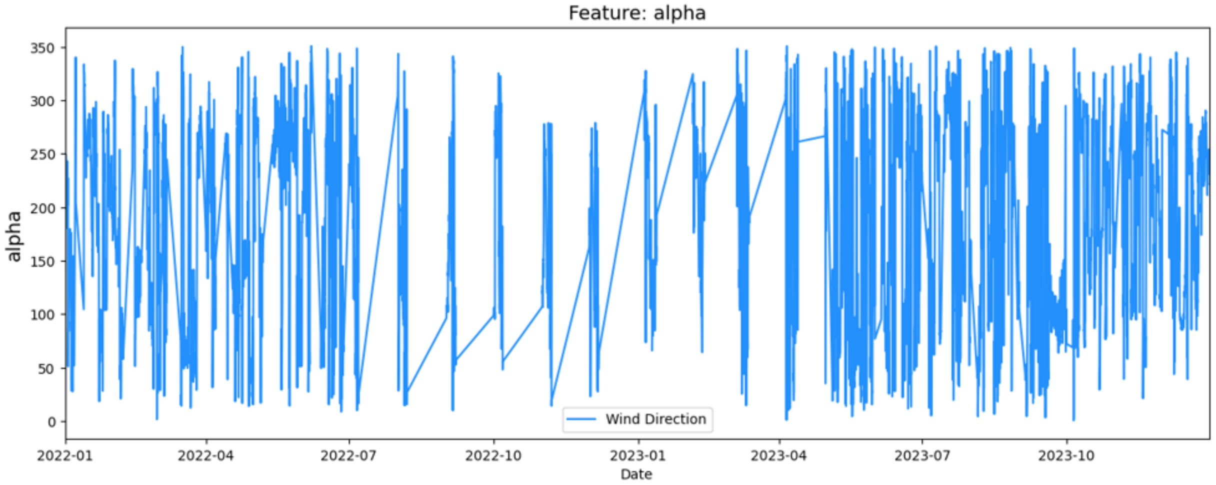


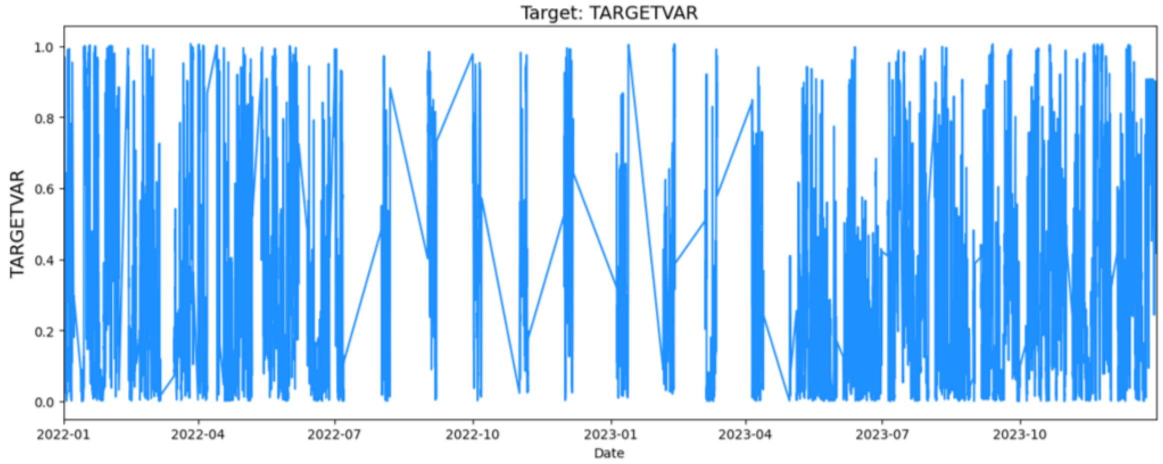
We processed negative values and peak power values. There is an annual trend for the temperature value.

Python code: processing historical data for 2022-2023 for wind turbine with consideration of removing rows with zeros by power



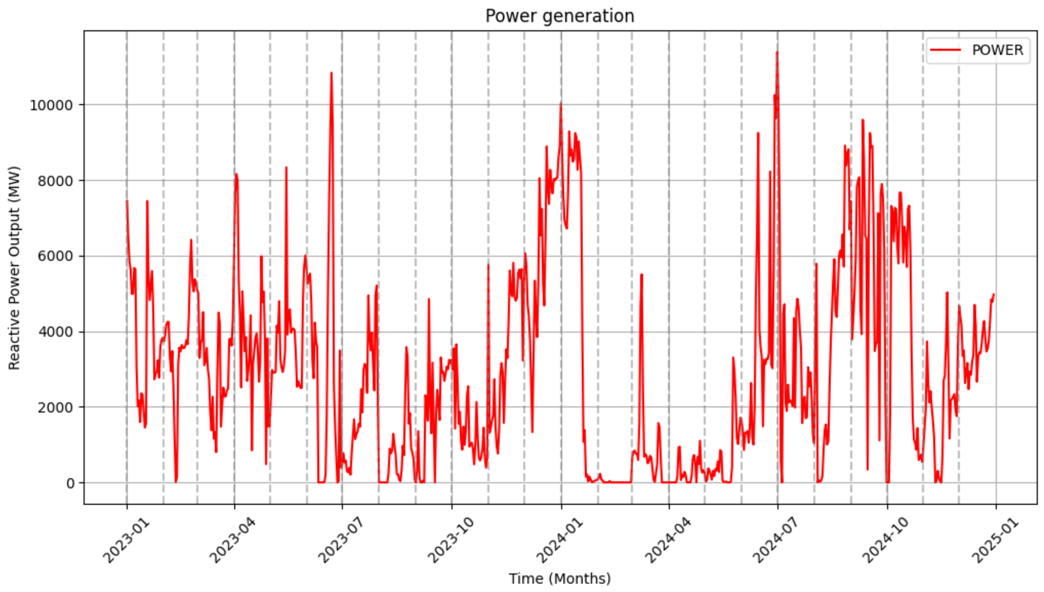






Data with zero values are not used in training the model

Data for power generator for 2023-2024 years



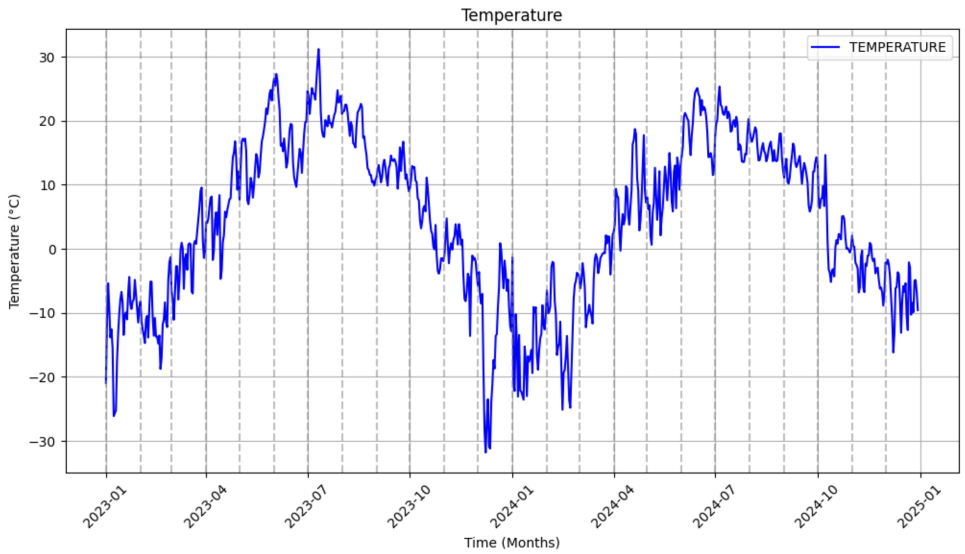


Table 2: Feature schema of the constructed dataset

Feature	**Description**
GEN1_ACTIVE_OUT	Active power output (Generator 1)
GEN1_ACTIVE_IN	Active power input (Generator 1)
GEN1_REACTIVE_OUT	Reactive power output (Generator 1)
GEN1_REACTIVE_IN	Reactive power input (Generator 1)
TEMPERATURE	Ambient temperature (°C)
HUMIDITY	Relative humidity (%)
PRECIPITATION	Precipitation (mm)
PRESSURE	Atmospheric pressure (hPa)
CLOUD_COVER	Cloud cover (%)
YEAR	Year of measurement
MONTH	Month of measurement
DAY	Day of measurement
HOUR	Hour of measurement

The data spanned from January 1, 2023, at 00:00:00 to December 31, 2024, at 00:00:00, recorded at different time intervals. Additionally, meteorological data were sourced from "Open-Meteo source" for the same period

Mathematical model of power prediction

Ordinary deterministic model

$$\hat{y} = g(x; w), \qquad (1)$$

forecast value

g – math model prediction, x – NWP, w - parameters

$$g(x; w) = \phi(x)^T w, \qquad (2)$$

Analysis of uncertainties

$$\epsilon = y - f. \qquad (3)$$

f - power curve of wind turbine

observed value, mathematical model and error

$$y = g(x; w) + \epsilon. \qquad (4)$$

Bayesian modeling

$$p(f|x) = \int p(f|w)p(w|X, Y), \qquad (5)$$

$$p(y|x) = \int p(y|w)p(w|X, Y), \qquad (6)$$

Uncertainty for the class model

Training dataset $X = \{x_1, \ldots, x_N\}$, $Y = \{y_1, \ldots, y_N\}$,

$$f_i = g_i(x_i; w_i). \qquad (7)$$

$$f \sim g(x; \mathcal{W}), \qquad (8)$$

$$y \sim \mathcal{N}(f, \sigma^2_{obs}), \qquad (9)$$

σ^2_{obs} Dispersion of random noise

Gaussian distribution prediction model

$$\xi = l(y) = \ln\left(\frac{y}{1-y}\right), \qquad (10)$$

logit-normal transformation $y \in (0, 1)$

$$p(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{y(1-y)} \exp\left[-\frac{1}{2}\left(\frac{\ln(\frac{y}{1-y}) - \mu}{\sigma}\right)^2\right], \qquad (11)$$

Applying the transformation to the GP probability

$$\begin{cases} f \sim \mathcal{GP} \\ l(y) = f + \epsilon; \epsilon \sim \mathcal{N}(0, \sigma^2_{obs} I) \end{cases}, \qquad (12)$$

$$p(\xi^*|x^*) \sim \mathcal{N}(\xi^*|\mu_f(x^*), \sigma^2_f(x^*) + \sigma^2_{obs}). \qquad (13)$$

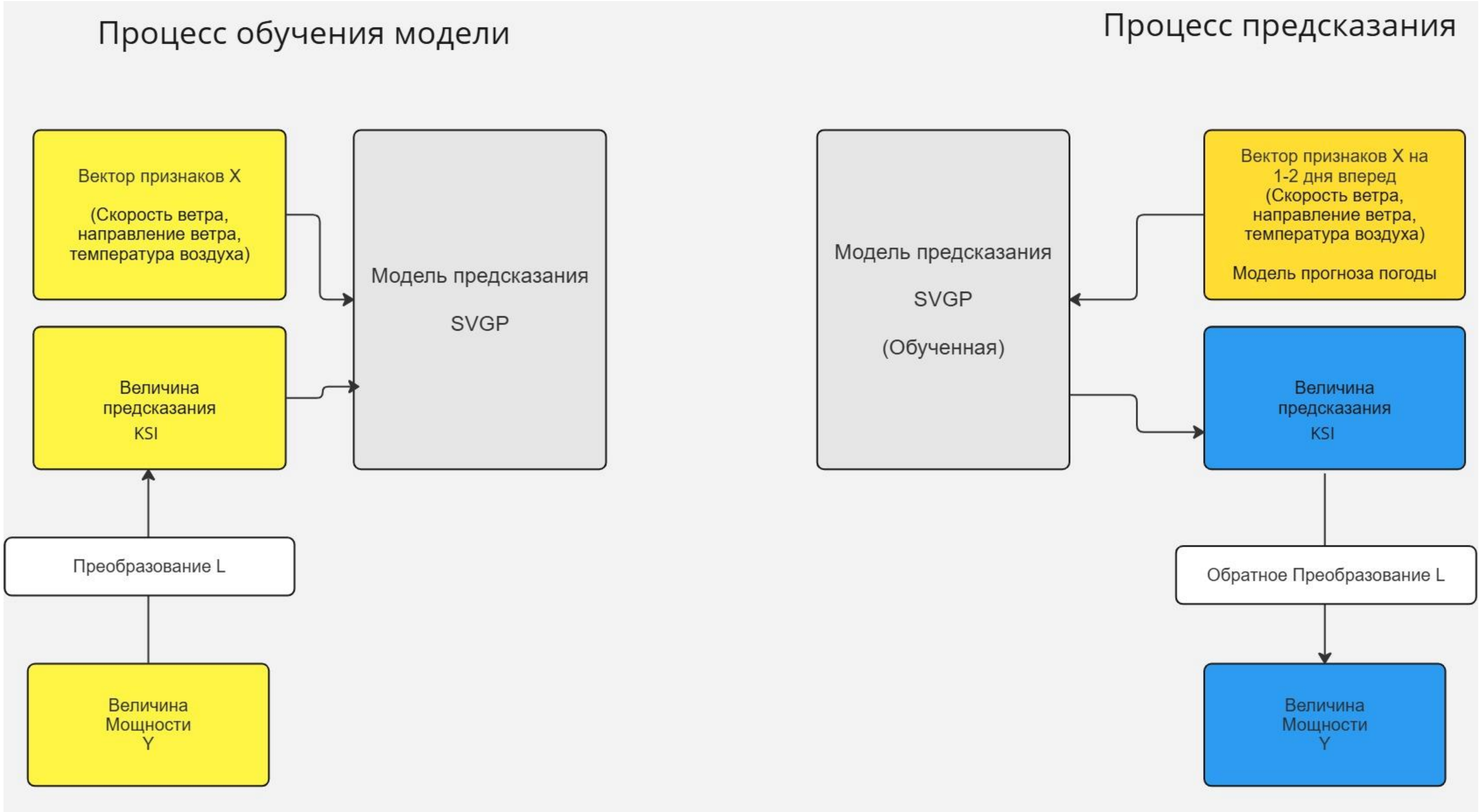
Density Probability forecast

$\mu_f(\cdot)$ Mean value

$$\left[q_{\beta/2}(x^*), q_{1-\beta/2}(x^*)\right], \qquad (14)$$

$q\alpha(x*)$ - α-th quantile of $p(y*|x*)$.

Probabilistic power prediction model based on Gaussian process



SVGP - regression model considering Gaussian process of function behavior

PWPF VIA SPARSE VARIATIONAL GAUSSIAN PROCESS (SVGP)

$$m(x) = \mathbb{E}(g(x)), \qquad (15)$$

- Mean function

$$k(x, x') = \mathbb{E}[(g(x) - m(x))(g(x') - m(x'))]. \qquad (16)$$

- Covariance function

$$p(f|X) = \mathcal{N}(f|\mu, K), \qquad (17)$$

$$p(\xi|f) = \mathcal{N}(\xi|f, \sigma^2_{obs} I). \qquad (18)$$

Under the assumption of Gaussian noise

A Gaussian process contains a set of random variables, any finite number of which has a joint Gaussian distribution

Neural network model for predicting the mean function

$$h_k = T^{(k)}(h_{k-1}) = W_k h_{k-1} + b_k, \qquad (19)$$

$$\sigma(h_k)_i = \max(h_{k,i}, 0). \qquad (20)$$

- ReLu activation function

$$m(x) = (T^{(L)} \circ \sigma \circ T^{(L-1)} \circ \cdots \circ \sigma \circ T^{(1)})(x), \qquad (21)$$

finite function is a composition of affine functions and activation functions

$$k(x, x') = c\exp[-\frac{1}{2}\sum_{d=1}^{D} b_d(x_d - x'_d)^2], \qquad (22)$$

quadratic exponential kernel with hyperparameters θ = {c, b}

Approximated Sparse GP

$\mathcal{O}(N^3)$ to $\mathcal{O}(M^2 N)$.

M – inducing points (M ≪ N)

Z = [z_1, . . . , z_M] – model point set

$$p(u|Z) = \mathcal{N}(u|\mu_M, K_{MM}), \qquad (23)$$

K_{MM} - Covariance matrix

$u = g(Z)$. - Vector

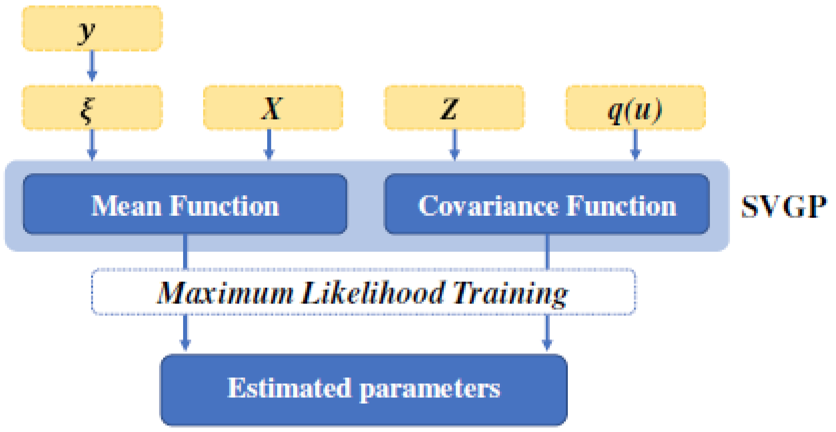
$$\begin{pmatrix} u \\ f \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_M \\ \mu \end{pmatrix}, \begin{pmatrix} K_{MM} & K_{MN} \\ K_{NM} & K \end{pmatrix}\right), \qquad (24)$$

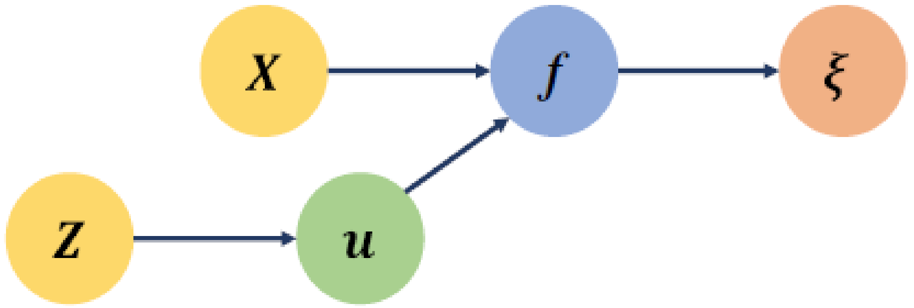
$$p(f|u, X, Z) = \mathcal{N}(f|\mu^*, \Sigma^*), \qquad (25)$$

$$\mu^* = \mu + K_{NM} K_{MM}^{-1}(u - \mu_M), \qquad (26)$$

$$\Sigma^* = K - K_{NM} K_{MM}^{-1} K_{MN}. \qquad (27)$$

Основные компоненты модели SVGP





Probabilistic graph of the model

TABLE I: Parameters and hyperparameters of the proposed model.

	Description
Parameters	weights and biases in mean function $\{W_k, b_k\}$, noise σ_{obs}, kernel hyperparameters θ, inducing points Z, mean and variance of variational distribution m and S
Hyperparameters	number of inducing points, layers and units in the mean function

$$\mathrm{CRPS}(F, y) = \int_{-\infty}^{\infty} (F(p_w) - \mathbb{1}(p_w - y))^2 dp_w,$$

- continuous ranked probability score

H. Wen, J. Ma, J. Gu, L. Yuan, and Z. Jin, “Sparse variational gaussian process based day-ahead probabilistic wind power forecasting,” IEEE Transactions on Sustainable Energy, vol. 13, no. 2, pp. 957–970, 2022.
H. Wen, P. Pinson, J. Ma, J. Gu, and Z. Jin, “Continuous and distribution free probabilistic wind power forecasting: A conditional normalizing flow approach,” IEEE Transactions on Sustainable Energy, vol. 13, no. 4, pp. 2250–2263, 2022.

Probabilistic prediction model based on Gaussian process

Variational inference

According to the probability graph of the model

$$p(X, Z, \boldsymbol{\xi}, \boldsymbol{u}, \boldsymbol{f}) = p(X, Z)p(\boldsymbol{f}|X, \boldsymbol{u})p(\boldsymbol{u}|Z)p(\boldsymbol{\xi}|\boldsymbol{f}), \quad (28)$$

$$p(\boldsymbol{\xi}, \boldsymbol{u}, \boldsymbol{f}|X, Z) = p(\boldsymbol{f}|X, \boldsymbol{u})p(\boldsymbol{u}|Z)p(\boldsymbol{\xi}|\boldsymbol{f}). \quad (29)$$

$$p(\boldsymbol{\xi}, \boldsymbol{u}|X, Z) = \int p(\boldsymbol{f}|X, \boldsymbol{u})p(\boldsymbol{u}|Z)p(\boldsymbol{\xi}|\boldsymbol{f})d\boldsymbol{f}. \quad (30)$$

$$\begin{aligned} \log p(\boldsymbol{\xi}, \boldsymbol{u}|X, Z) &= \log \int p(\boldsymbol{f}|X, \boldsymbol{u})p(\boldsymbol{u}|Z)p(\boldsymbol{\xi}|\boldsymbol{f})d\boldsymbol{f} \\ &\geq \mathbb{E}_{p(\boldsymbol{f}|X,\boldsymbol{u})} \log p(\boldsymbol{\xi}|\boldsymbol{f}) + \log p(\boldsymbol{u}|Z) \overset{def}{=} \mathcal{L}_1. \end{aligned} \quad (31)$$

$$\begin{aligned} \mathcal{L}_1 = \sum_{i=1}^{N} \log \mathcal{N}(\xi_i | \boldsymbol{k}_i^T \boldsymbol{K}_{MM}^{-1} \boldsymbol{u}, \sigma_{obs}^2) - \frac{1}{2\sigma_{obs}^2} Tr(\boldsymbol{\Sigma}^*) \\ + \log p(\boldsymbol{u}|Z), \end{aligned} \quad (32)$$

Lower bound

Variational distribution

Multivariate normal variation distribution of model points to approximate the original GP distribution

$$q(\boldsymbol{u}) = \mathcal{N}(\boldsymbol{m}, \boldsymbol{S}), \quad (33)$$

$$\mathbb{E}_{q(\boldsymbol{u})}[\log p(\boldsymbol{\xi}, \boldsymbol{u}|X, Z)] \overset{def}{=} \mathcal{L}_2. \quad (34)$$

$$\begin{aligned} \mathcal{L}_2 = \sum_{i=1}^{N} \log \mathcal{N}(\xi_i | \boldsymbol{k}_i^T \boldsymbol{K}_{MM}^{-1} \boldsymbol{m}, \sigma_{obs}^2) \\ - \frac{1}{2\sigma_{obs}^2} (\sum_{i=1}^{N} \boldsymbol{k}_i^T \boldsymbol{K}_{MM}^{-1} \boldsymbol{S} \boldsymbol{K}_{MM}^{-1} \boldsymbol{k}_i + Tr(\boldsymbol{\Sigma}^*)). \end{aligned} \quad (35)$$

$$\mu_{\boldsymbol{f}}(\boldsymbol{x}_i) = \boldsymbol{k}_i^T \boldsymbol{K}_{MM}^{-1} \boldsymbol{m}, \quad (36)$$

$$\sigma_{\boldsymbol{f}}^2(\boldsymbol{x}_i) = \boldsymbol{k}_i^T \boldsymbol{K}_{MM}^{-1} \boldsymbol{S} \boldsymbol{K}_{MM}^{-1} \boldsymbol{k}_i + \boldsymbol{\Sigma}_{i,i}^*. \quad (37)$$

Mean value (36) and variance (37)

Probabilistic prediction model based on Gaussian process

Training procedure

Here we introduce the training procedure that involves a loss function based on the maximum likelihood

Finding the maximum of the loss function

$$
\begin{aligned}
\mathcal{L} &= \mathbb{E}_{p_{data}(\xi,\boldsymbol{x})}[\log p(\xi|\boldsymbol{x})] \\
&= \sum_{i=1}^{N} \log \mathcal{N}(\xi_i|\mu_{\boldsymbol{f}}(\boldsymbol{x}_i), \sigma^2_{\boldsymbol{f}}(\boldsymbol{x}_i) + \sigma^2_{obs}) \\
&= \sum_{i=1}^{N} [-\frac{1}{2}\log(\sigma^2_{\boldsymbol{f}}(\boldsymbol{x}_i) + \sigma^2_{obs}) - \frac{(\xi_i - \mu_{\boldsymbol{f}}(\boldsymbol{x}_i))^2}{2(\sigma^2_{\boldsymbol{f}}(\boldsymbol{x}_i) + \sigma^2_{obs})}].
\end{aligned} \tag{38}
$$

$$
\mathcal{L} = \mathbb{E}_{p_{data}(\xi,\boldsymbol{x})}[\log p(\xi|\boldsymbol{x})] - KL(p(\boldsymbol{u})||q(\boldsymbol{u})), \tag{39}
$$

Algorithm 1 Gradient descent algorithm for SVGP

Input: Training data: $\mathcal{D} = \{\boldsymbol{x}_i, \xi_i\}_{i=1}^{N}$, γ, training epochs E, learning rate λ
Output: estimated parameters $\boldsymbol{m}, \boldsymbol{S}, \sigma_{obs}, Z$, $\{W_k, \boldsymbol{b}_k\}$, and kernel hyperparameters θ

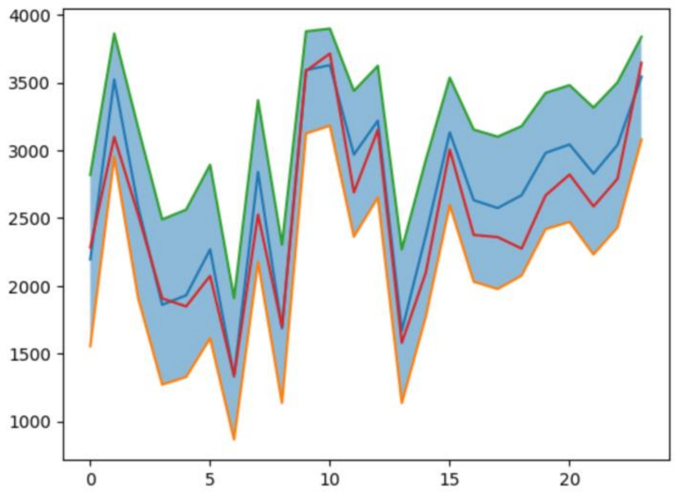
```
for epoch = 0 to E do
    while D ≠ ∅ do
        Choose a random mini-batch D_mb ⊂ D
        Calculate gradient of −L(D_mb): Δm, ΔS, Δθ, Δσ_obs, ΔZ, ΔW_k, Δb_k
        Parameter update: m = m − λΔm, S = S − λΔS,
            θ = θ − λΔθ, σ_obs = σ_obs − λΔσ_obs, Z = Z − λΔZ,
            W_k = W_k − λΔW_k, b_k = b_k − λΔb_k
        D = D − D_mb
    end while
end for
```

MLPMean NN: input layer [1,512], 2 hidden layers [512,512], [512,512]; 1 output layer [512,1], RELu. Adam optimizer. The learning rates Lr = 1e-3. num_epoch=200. M= 700, 1000, 1200.

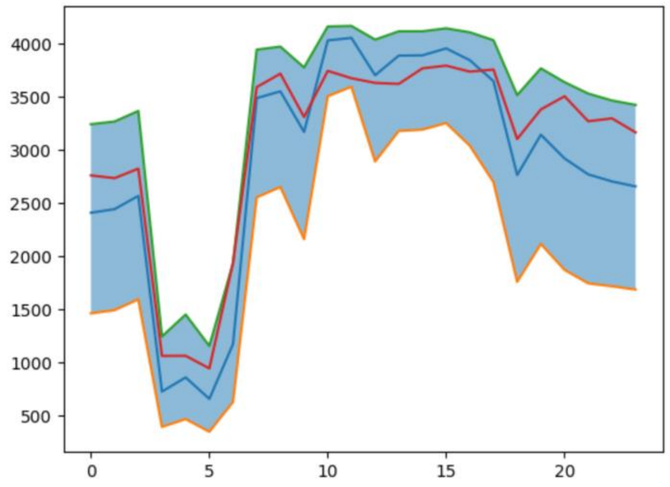
Software: PyTorch, gpytorch, scikit-learn, tqdm, properscoring, numpy, pandas, matplotlib, CUDA.

The first 98% of the data is used for training. The next 2% of the data is used for testing.

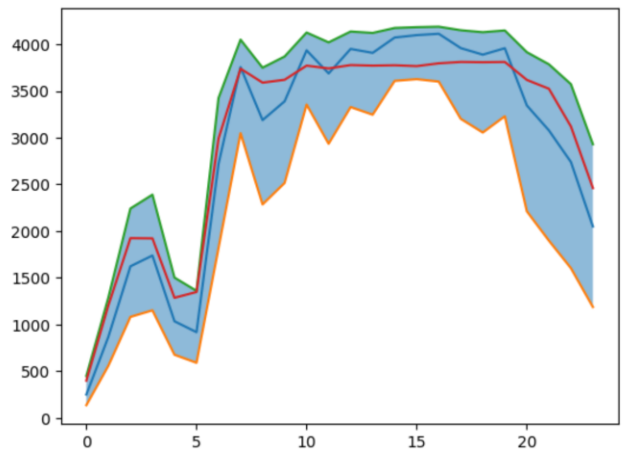
Results of power prediction for different wind turbines, 1 and 2 days



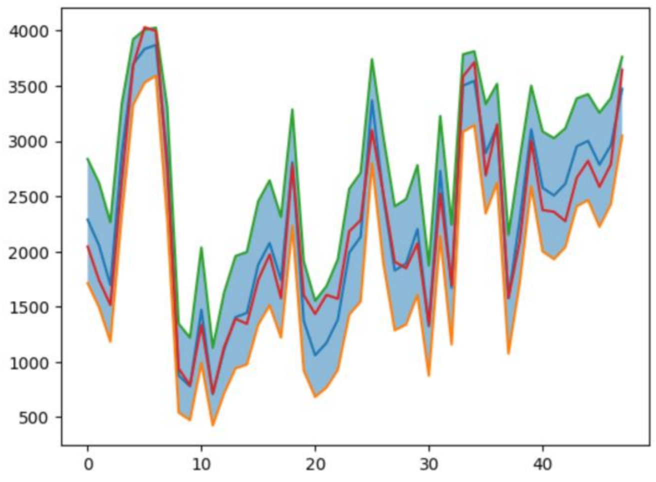
WT-K

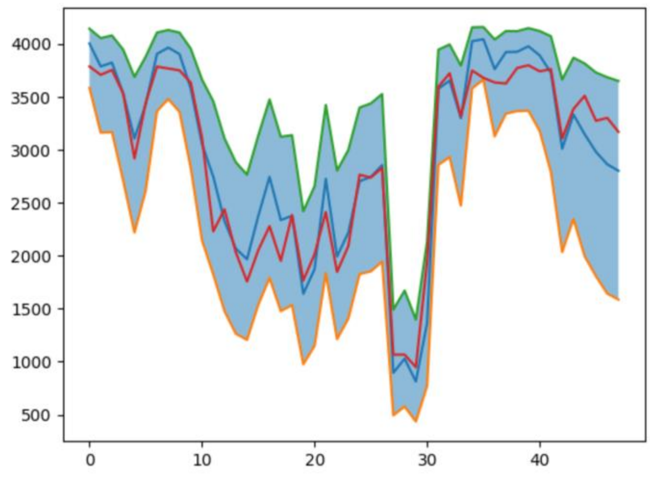


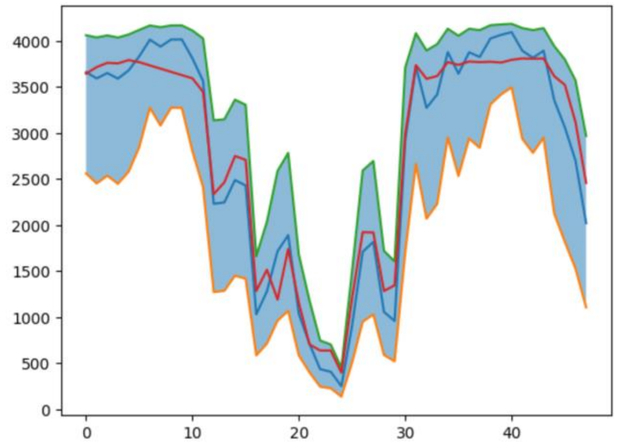
WT-S



WT-T







Red color – real power. Blue color – prediction value. Lower and upper bounds – green and gray.

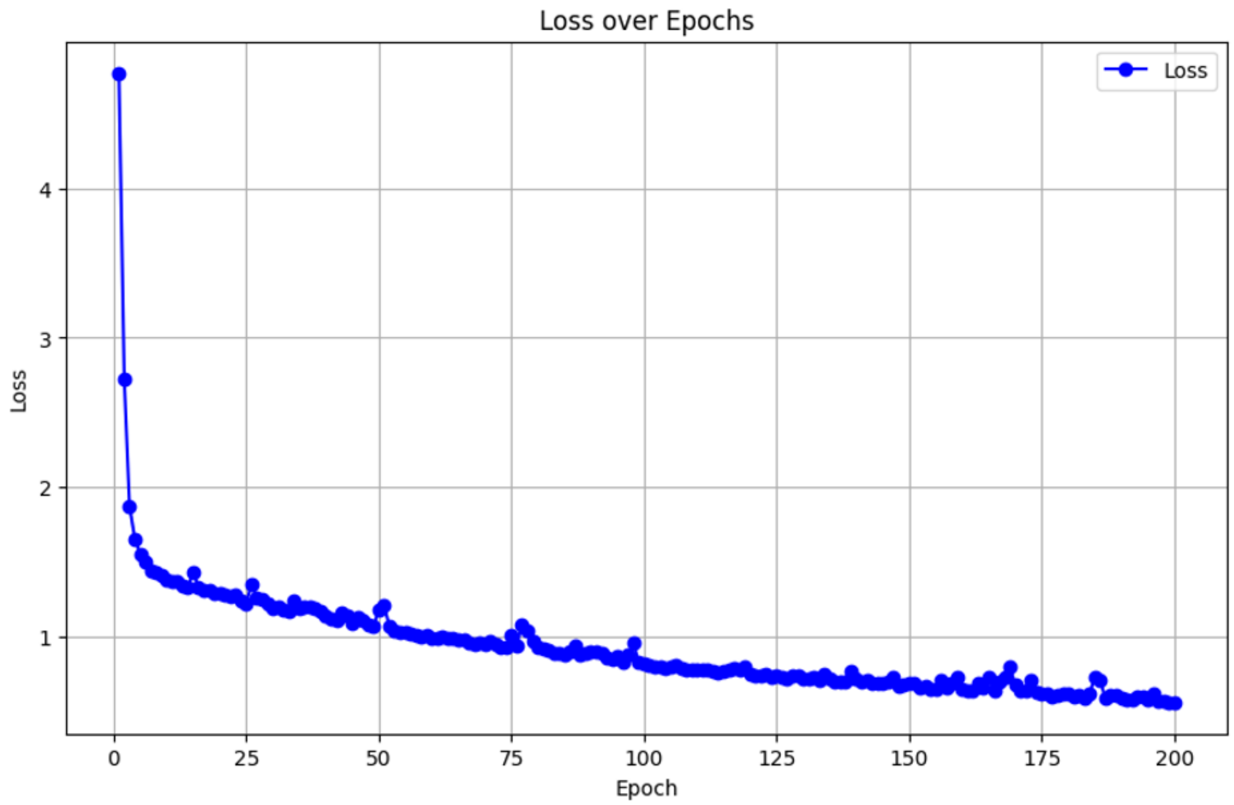


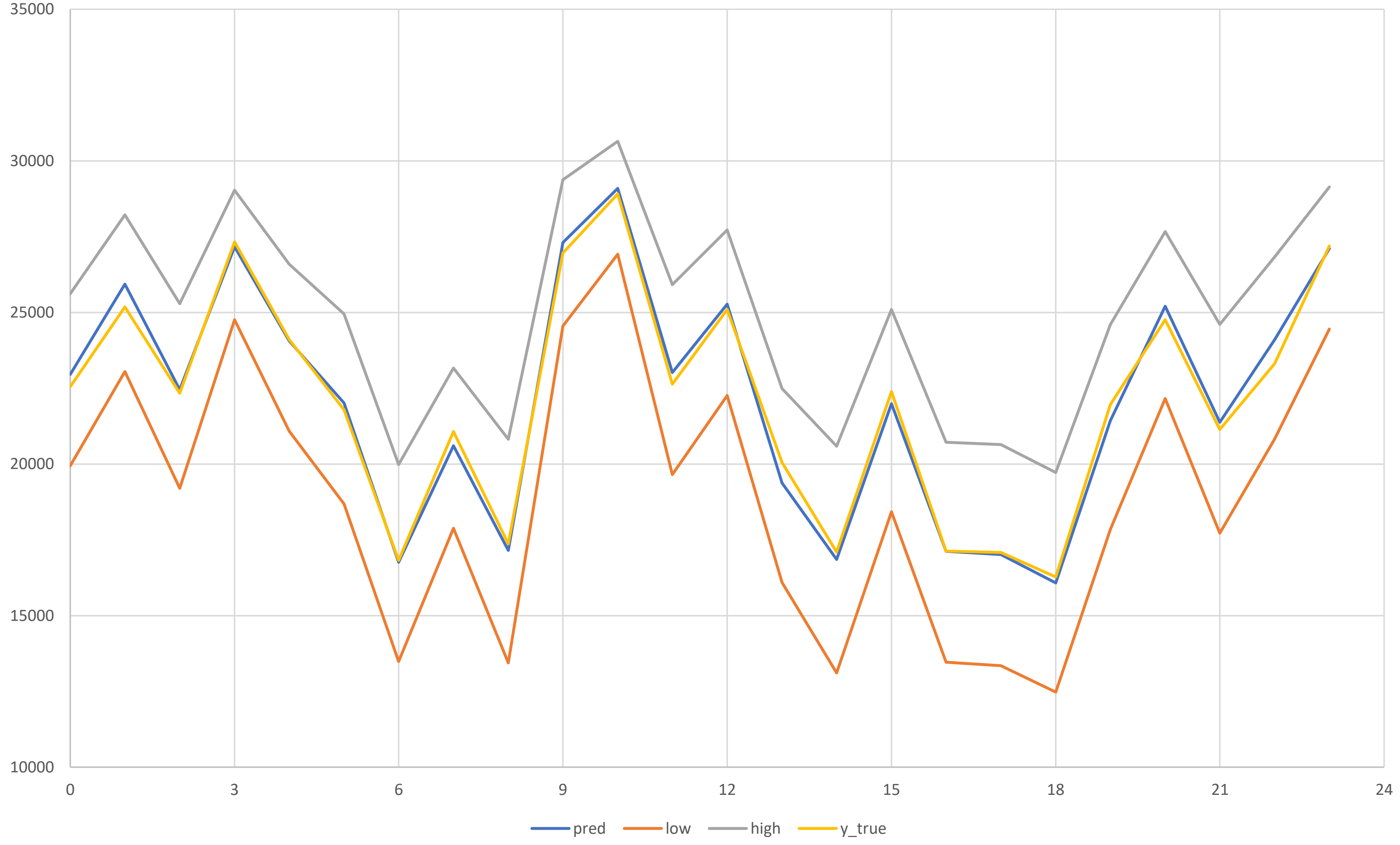
Table 5: Evaluation metric for a single wind turbine and for the 24-hour forecast

Metric	**Definition**	**Value**
CRPS	Continuous Ranked Probability Score	0.034
MAE	Dimensionless Mean Absolute Error	0.037
RMSE	Dimensionless Root Mean Square Error	0.041
R^2	Coefficient of Determination	0.899

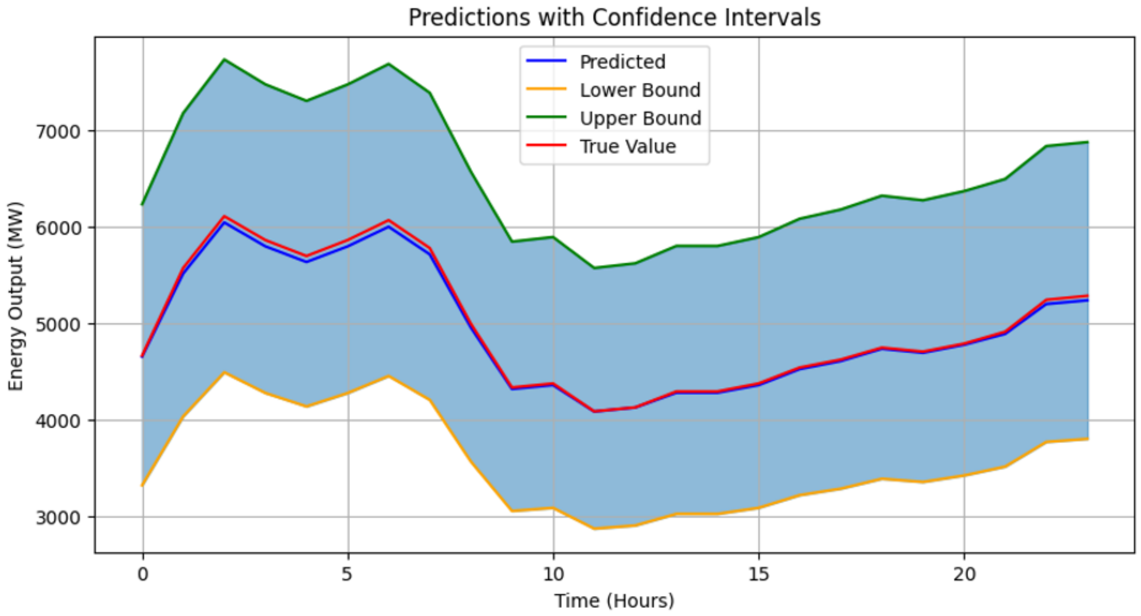
Table 6: Evaluation metric for a single wind turbine and for the 48-hour forecast

Metric	**Definition**	**Value**
CRPS	Continuous Ranked Probability Score	0.033
MAE	Dimensionless Mean Absolute Error	0.037
RMSE	Dimensionless Root Mean Square Error	0.045
R^2	Coefficient of Determination	0.949

Power forecast P for GTP-1 (8 wind turbines), for 24 hours ahead



Results for prediction of power generator for 1 and 7 days



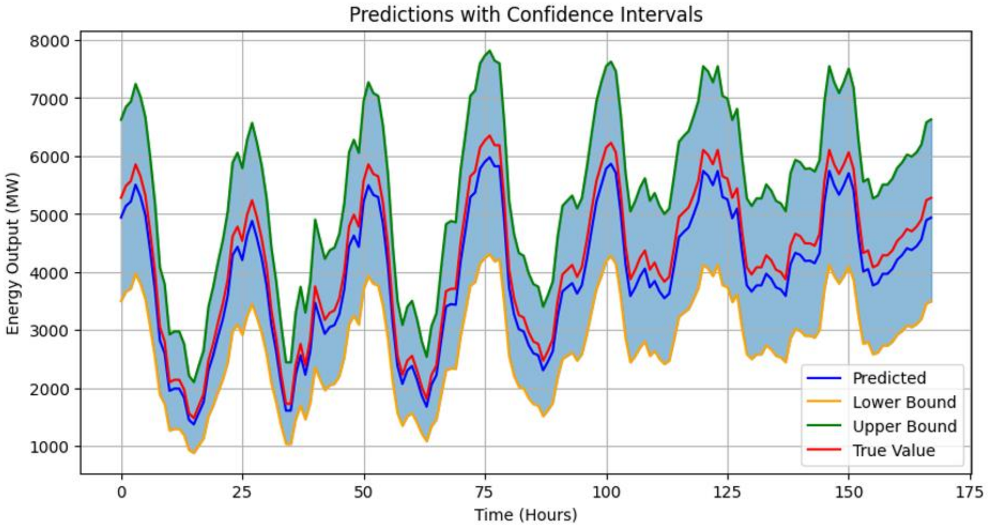


Table 3: Evaluation metrics for the 24-hour forecast

Metric	Definition	Value
CRPS	Continuous Ranked Probability Score	0.0234
MAE	Dimensionless Mean Absolute Error	0.00518
RMSE	Dimensionless Root Mean Square Error	0.00643
R^2	Coefficient of Determination	0.99625

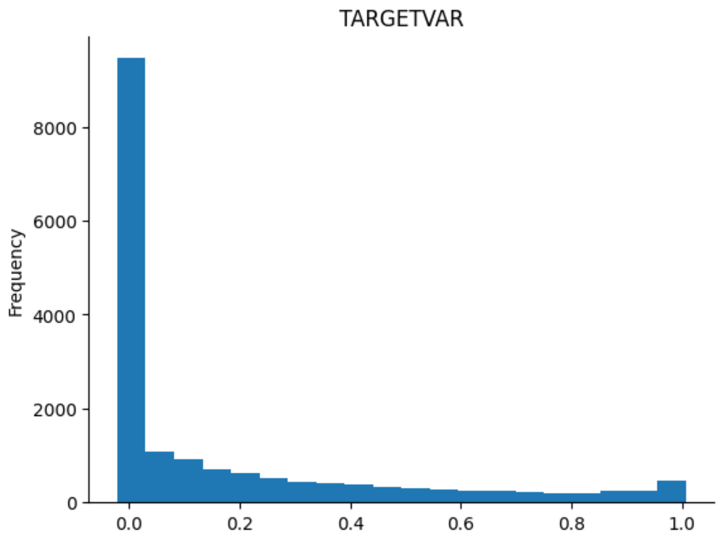
Table 4: Evaluation metrics for the 168-hour forecast

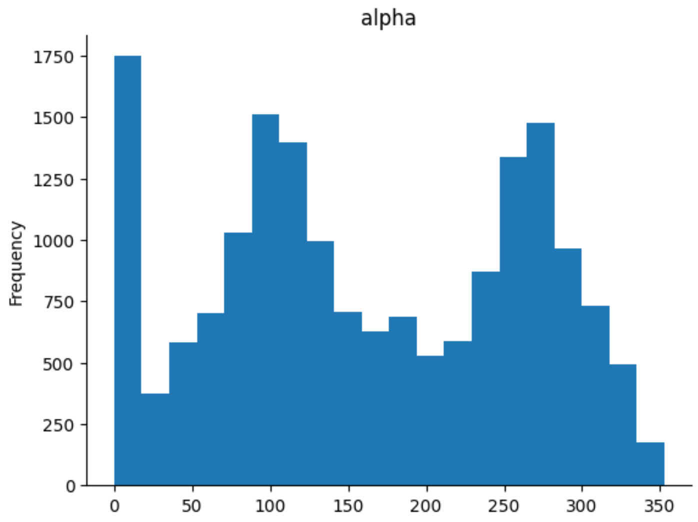
Metric	Definition	Value
CRPS	Continuous Ranked Probability Score	0.0229
MAE	Dimensionless Mean Absolute Error	0.0457
RMSE	Dimensionless Root Mean Square Error	0.0471
R^2	Coefficient of Determination	0.9421

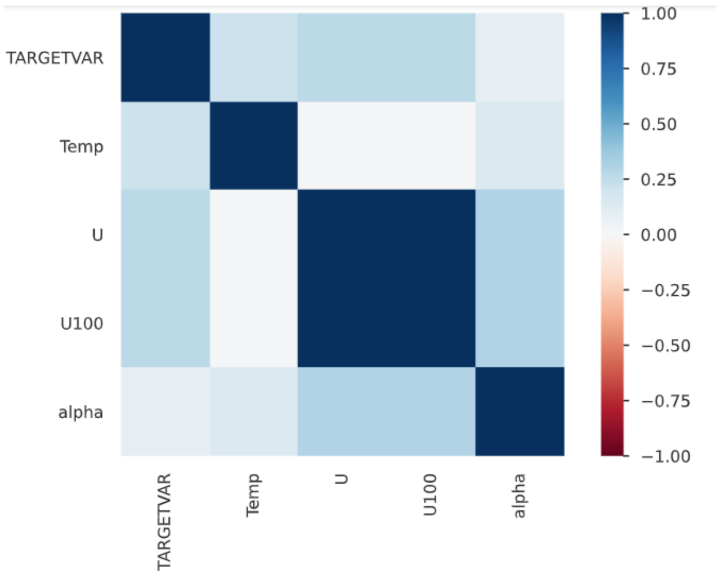
Conclusions

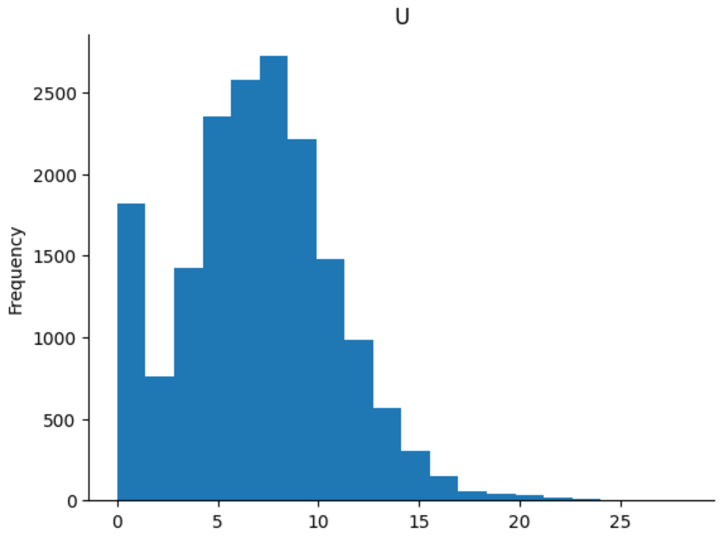
- SVGP model was developed
- SVGP was tested on different wind turbines and power generator
- For better predictions we can use more features
- Metrics: CRPS, RMSE, MAE, MSE. Error is less then 6 %.
- Hardware: Nvidia GeForce RTX 3060 and Nvidia GPU A100. Time learning ~ 20 mins.
- Software: Python, PyTorch, gptorch, scikit-learn, tqdm, properscoring, numpy, pandas, Jupyter Notebook
- Plans: RNN, CNN, GNN, k-NN, Gradient Boosting and Decision Tree, Hybrid models
- SVGP can be used for other problems

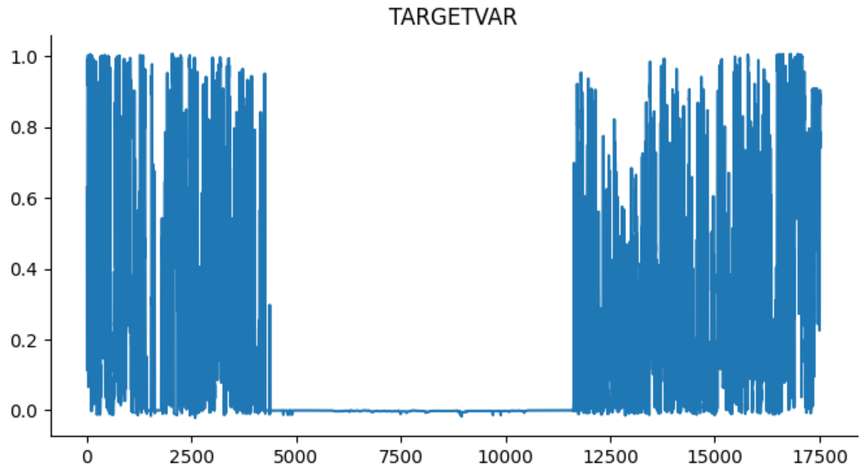
Python code: данные по ВЭУ-С с ВЭС в форме гистограмм и корреляционной таблицы











Суммарные данные прогноза по ГТП-1 и ВЭУ1…ВЭУ4 К-ВЭС

	ГТП-1				ВЭУ-1				ВЭУ-2				ВЭУ-3				ВЭУ-4			
Time	pred	low	high	y_true	pred	low	high	y_true	pred	low	high	y_true	pred	low	high	y_true	pred	low	high	y_true
0	22953,48	19942,07	25622,19	22549,13	2308,837	1823,018	2773,082	2283,755	3612,313	3348,1	3804,268	3678,706	3729,103	3461,733	3907,816	3657,241	3898,383	3696,807	4023,074	3655,372
1	25930	23048,25	28215,69	25185,03	3398,419	3053,989	3657,713	3098,576	3470,47	3159,166	3703,304	3400,963	3688,407	3392,359	3885,981	3398,105	3971,367	3783,084	4077,376	4142,47
2	22451,53	19200,34	25285,94	22334,68	2599,885	2134,004	3018,857	2524,548	3306,027	2941,033	3587,25	3387,382	3606,822	3287,46	3827,095	3413,29	3553,143	3246,209	3774,259	3335,1
3	27166,01	24762,1	29032,05	27331,79	1892,524	1437,912	2367,659	1907,364	3620,194	3358,377	3810,021	3648,789	3900,519	3700,491	4024,25	3866,749	3972,845	3794,048	4075,477	3988,406
4	24063,16	21085,02	26590,62	24109,76	1873,05	1429,785	2337,768	1848,349	3486,707	3176,865	3716,984	3469,493	3804,367	3564,895	3959,633	3729,545	3880,989	3677,664	4009,272	3871,136
5	22016,25	18684,09	24952,94	21791	2115,377	1662,533	2566,796	2073,794	2792,473	2336,942	3184,998	2827,213	3313,744	2900,976	3621,509	3120,573	3609,377	3322,723	3813,268	3398,646
6	16761,49	13481,73	19982,83	16824,01	1314,289	953,0293	1739,141	1333,766	2749,585	2293,923	3146,352	2842,718	3062,337	2583,452	3441,034	3071,054	3160,756	2744,14	3489,04	3063,481
7	20606,37	17877,75	23165,72	21071,29	2504,426	2047,967	2924,373	2526,133	3622,81	3341,796	3822,21	3774,89	3950,979	3739,63	4068,708	4111,881	4015,997	3848,557	4105,62	4108,792
8	17151,99	13435,99	20818,83	17365,76	1614,255	1205,196	2066,356	1696,477	2211,998	1749,163	2664,172	2366,707	2434,106	1893,178	2933,075	2488,012	2720,438	2252,332	3129,509	2646,414
9	27305,51	24553,93	29377,66	26964,45	3706,27	3459,173	3878,607	3585,475	3189,469	2798,931	3498,433	3181,234	3467,705	3093,941	3734,182	3241,756	3563,426	3254,219	3784,452	3506,214
10	29092,14	26920,17	30645,05	28910,16	3746,87	3511,106	3908,643	3713,881	3256,82	2894,356	3541,551	3334,755	3837,517	3617,425	3979,527	3645,031	3781,004	3553,293	3934,526	3744,128
11	23016,27	19647,88	25920,62	22636,63	2820,331	2386,814	3193,889	2690,012	2410,473	1944,799	2846,891	2450,919	3080,193	2596,681	3459,475	2917,168	3300,012	2932,106	3583,606	3113,869
12	25274,96	22259,98	27718,89	25096,56	3269,442	2911,032	3550,43	3151,388	2239,325	1763,819	2700,901	2325,139	3144,502	2671,225	3509,154	2990,224	3427,672	3093,43	3677,992	3269,446
13	19376,52	16090,35	22488,79	20049,4	1555,525	1156,149	2002,116	1580,13	1120,507	793,8403	1521,551	1266,315	1809,514	1303,877	2351,991	1865,946	2298,25	1817,164	2759,218	2338,764
14	16855,53	13106,2	20590,23	17097,55	2019,167	1573,498	2472,249	2097,341	1968,781	1518,82	2431,173	2074,575	1970,197	1449,49	2507,435	2024,664	2486,473	2005,767	2928,621	2414,018
15	21991,19	18427,14	25093,46	22392,1	3028,7	2626,185	3361,151	3005,091	2559,619	2102,271	2975,341	2749,969	2896,546	2387,125	3315,839	3013,254	3127,189	2721,638	3452,061	3087,812
16	17122,08	13459,48	20721,29	17125,54	2405,616	1950,498	2833,181	2375,94	2249,146	1787,739	2696,521	2364,135	2317,156	1775,837	2830,795	2365,714	2301,089	1818,829	2762,853	2211,528
17	17010,1	13349,07	20639,76	17086,06	2359,012	1902,171	2792,246	2359,689	1603,488	1190,459	2061,651	1693,421	1918,725	1404,848	2455,446	1965,182	2454,006	1972,327	2900,172	2426,499
18	16078,35	12477,02	19720,12	16274,73	2337,17	1881,549	2771,244	2275,192	1876,686	1433,709	2340,719	1972,286	2112,125	1582,09	2640,619	2159,084	2301,918	1818,432	2764,721	2269,785
19	21430,59	17844,28	24600,66	21957,42	2632,139	2180,827	3036,399	2667,763	2628,81	2168,321	3040,729	2763,695	2905,067	2400,531	3319,999	2977,794	3099,515	2687,634	3431,306	3073,508
20	25205,03	22156,02	27664,38	24759,55	2999,504	2591,445	3338,46	2821,229	2526,714	2067,357	2947,093	2594,316	3181,128	2719,712	3533,941	3031,62	3469,019	3147,09	3707,909	3287,31
21	21376,24	17720,16	24603,89	21139,31	2698,508	2256,71	3089,3	2586,185	2335,392	1869,609	2778,844	2417,105	2935,376	2424,096	3351,011	2818,38	3163,75	2762,695	3481,982	3011,426
22	24100,57	20828,16	26831,69	23306,3	2966,535	2553,873	3311,729	2789,914	2533,529	2062,98	2962,614	2598,097	3185,688	2727,356	3536,097	2934,681	3441,429	3111,309	3687,924	3209,534
23	27103,23	24453,38	29137,84	27192,21	3694,634	3447,59	3868,366	3645,679	3199,207	2823,602	3497,803	3395,994	3731,922	3458,141	3913,048	3635,287	3640,177	3373,092	3830,447	3577,675

ГТП-1: ошибки MAE=0.009 RMSE=0.011

Данные прогноза по ВЭУ5…ВЭУ8 К-ВЭС

ВЭУ-5

pred	low	high	y_true
2259,922	1797,134	2707,596	2479,615
3335,489	2991,241	3601,319	3394,379
2340,936	1886,128	2773,938	2469,1
3565,38	3261,405	3783,481	3632,69
2925,457	2507,068	3278,454	3052,9
2510,534	2058,946	2925,817	2592,744
2315,176	1861,679	2749,26	2338,97
1978,226	1532,367	2435,379	2109,865
2392,142	1935,594	2822,165	2417,369
3644,508	3372,897	3836,537	3669,824
3890,587	3704,394	4010,41	3971,994
3438,325	3126,115	3675,01	3420,803
3608,956	3343,386	3801,994	3733,594
2739,131	2303,219	3121,762	2949,701
2032,202	1584,822	2485,828	2110,86
2288,039	1834,565	2724,499	2362,411
1449,918	1069,787	1883,832	1531,214
1799,368	1371,222	2254,647	1850,123
1631,261	1224,442	2078,815	1761,593
2272,809	1818,803	2711,114	2502,857
3073,856	2681,133	3395,505	3002,783
2560,57	2109,53	2970,989	2552,858
3453,92	3145,701	3686,734	3409,005
3538,965	3255,441	3749,17	3629,618

ВЭУ-6

pred	low	high	y_true
2437,144	1906,41	2927,069	2282,79
3123,093	2654,708	3487,607	2894,51
3033,622	2552,939	3417,045	2962,94
3668,835	3367,425	3871,764	3750,225
3116,64	2656,307	3477,061	3129,925
2719,768	2198,617	3168,858	2773,115
2088,492	1571,44	2606,944	2067,999
2057,841	1535,809	2585,139	1990,154
2125,939	1606,15	2642,568	2064,004
2679,158	2163,893	3128,589	2655,017
3025,193	2553,209	3404,072	2999,017
2183,405	1657,898	2698,655	2169,147
3263,801	2841,471	3583,332	3210,652
3162,835	2714,386	3510,311	3231,918
2514,69	1988,09	2992,091	2496,584
3091,676	2630,985	3455,383	3061,735
2527,39	2002,41	3001,99	2449,959
2700,454	2184,816	3147,7	2673,241
2815,788	2311,915	3240,972	2817,52
3350,584	2950,126	3646,795	3338,979
3667,347	3374,622	3866,514	3663,917
2923,232	2432,384	3326,714	3011,48
2506,556	1980,054	2984,822	2411,259
3131,44	2678,481	3485,522	3046,368

ВЭУ-7

pred	low	high	y_true
1613,199	1201,473	2068,679	1611,998
2459,259	1994,907	2890,037	2326,805
1573,675	1164,746	2030,146	1750,915
3058,052	2665,175	3381,256	3099,766
2324,989	1864,98	2764,146	2369,385
1629,444	1219,662	2080,782	1681,629
1092,572	782,2283	1472,978	1118,461
738,7202	512,4401	1036,825	760,872
1845,993	1412,753	2302,346	1837,852
3687,427	3387,251	3886,983	3778,883
3905,505	3703,294	4029,192	4035,54
3464,896	3149,118	3700,827	3542,311
3773,64	3551,429	3925,6	3841,458
3193,402	2807,552	3499,028	3241,355
1887,66	1454,077	2340,34	1899,06
2148,582	1700,093	2592,682	2191,317
1368,483	1002,638	1792,961	1388,77
1595,11	1194,015	2039,523	1581,742
1516,434	1125,126	1957,21	1549,539
2154,669	1707,539	2596,893	2299,294
2672,727	2233,89	3063,45	2595,313
2169,698	1721,893	2611,245	2136,253
3459,243	3150,831	3691,622	3376,967
3572,472	3289,567	3778,726	3632,109

ВЭУ-8

pred	low	high	y_true
3094,578	2707,392	3410,611	2899,653
2483,5	2018,795	2912,349	2529,222
2437,417	1987,824	2857,353	2491,407
3487,663	3177,27	3718,14	3437,798
2650,962	2207,458	3047,299	2639,028
3325,535	2983,694	3590,915	3323,29
978,2792	691,8347	1338,086	987,556
1737,375	1319,189	2187,467	1688,704
1807,119	1381,627	2258,635	1848,922
3367,548	3023,626	3629,88	3346,048
3648,64	3383,091	3837,128	3465,813
2318,634	1854,351	2762,268	2332,404
2547,627	2084,189	2969,488	2574,66
3497,359	3194,164	3722,813	3575,272
1976,358	1531,635	2432,496	1980,448
2850,838	2424,275	3216,502	2920,506
2503,281	2051,747	2919,156	2438,28
2579,934	2129,216	2988,376	2536,159
1486,967	1099,758	1925,82	1469,727
2386,994	1930,499	2817,428	2333,532
3614,738	3340,776	3811,505	3763,066
2589,712	2143,247	2993,805	2605,625
2553,668	2096,057	2970,147	2576,846
2594,41	2127,468	3014,758	2629,477