

Tuesday, November 29, 2022 20:29

Diagram illustrating a neural network structure with inputs x , weights w_1, w_2 , biases b_1, b_2 , and hidden layer nodes z_1, z_2 . The output is y . The loss function is $J = \frac{1}{2}(\hat{y} - y)^2$.

$$\frac{\partial J}{\partial \hat{y}} = \hat{y} - y$$

$$\frac{\partial \hat{y}}{\partial z_2} = g(1 - g)$$

$$\textcircled{1} z_1 = w_1 x + b_1$$

$$a = \sigma(z_1)$$

$$\textcircled{2} z_2 = w_2 a + b_2$$

$$\hat{y} = \sigma(z_2)$$

$$\frac{\partial J}{\partial z_2} = \frac{\partial J}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z_2}$$

$$\frac{\partial z_2}{\partial w_2} = a^T$$

$$\frac{\partial z_2}{\partial b_2} = 1$$

$$\frac{\partial a}{\partial z_1} = a(1 - a)$$

$$\frac{\partial J}{\partial z_1} = \frac{\partial J}{\partial a} \times \frac{\partial a}{\partial z_1}$$

$$\frac{\partial z_1}{\partial w_1} = x^T$$

$$\frac{\partial z_1}{\partial b_1} = 1$$

$$\frac{\partial J}{\partial w_1} = \frac{\partial J}{\partial z_1} \times \frac{\partial z_1}{\partial w_1}$$

$$\frac{\partial J}{\partial b_1} = \frac{\partial J}{\partial z_1} \times \frac{\partial z_1}{\partial b_1}$$

$$\frac{\partial J}{\partial w_2} = \frac{\partial J}{\partial z_2} \times \frac{\partial z_2}{\partial w_2}$$

$$\frac{\partial J}{\partial b_2} = \frac{\partial J}{\partial z_2} \times \frac{\partial z_2}{\partial b_2}$$

$$\frac{\partial z_2}{\partial a} = w_2^T$$

$$\frac{\partial J}{\partial a} = \frac{\partial J}{\partial z_2} \times \frac{\partial z_2}{\partial a}$$

$$y = \sigma(x)$$

$$y' = g'(y)$$

