Practical

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In [20]:
```
n(e^(pi)+12)
```

Out[20]:
```
35.1406926327793
```

In [24]:
```
g(x)= (2*x^2 - 6*x +5 )/(x-3)
solve(g == 0,x)
```

Out[24]:
```
[x == (-1/2*I + 3/2), x == (1/2*I + 3/2)]
```

In [25]:
```
find_root (g == -2, 0, 2)
```

```
---------------------------------------------------------------------------
RuntimeError                                Traceback (most recent call last)
<ipython-input-25-1ad598d8f389> in <module>()
----> 1 find_root (g == -Integer(2), Integer(0), Integer(2))

/Applications/SageMath-9.1.app/Contents/Resources/sage/local/lib/python2.7/site-packages/sage/misc/lazy_import.pyx in sage.misc.lazy_import.LazyImport.__call__ (build/cythonized/sage/misc/lazy_import.c:3686)()
    351             True
    352         """
--> 353         return self.get_object()(*args, **kwds)
    354
    355     def __repr__(self):

/Applications/SageMath-9.1.app/Contents/Resources/sage/local/lib/python2.7/site-packages/sage/numerical/optimize.pyc in find_root(f, a, b, xtol, rtol, maxiter, full_output)
    104     """
    105     try:
--> 106         return f.find_root(a=a,b=b,xtol=xtol,rtol=rtol,maxiter=maxiter,full_output=full_output)
    107     except AttributeError:
    108         pass

/Applications/SageMath-9.1.app/Contents/Resources/sage/local/lib/python2.7/site-packages/sage/symbolic/expression.pyx in sage.symbolic.expression.Expression.find_root (build/cythonized/sage/symbolic/expression.cpp:61766)()
  11853         elif self.number_of_arguments() == 1:
  11854             f = self._fast_float_(self.default_variable())
> 11855             return find_root(f, a=a, b=b, xtol=xtol,
  11856                              rtol=rtol,maxiter=maxiter,
  11857                              full_output=full_output)

/Applications/SageMath-9.1.app/Contents/Resources/sage/local/lib/python2.7/site-packages/sage/numerical/optimize.pyc in find_root(f, a, b, xtol, rtol, maxiter, full_output)
    123                 else:
    124                     return s
--> 125             raise RuntimeError("f appears to have no zero on the interval")
    126         # If we found such an s, then we just instead find
    127         # a root between left and s or s and right.

RuntimeError: f appears to have no zero on the interval
```

In [37]:
```
g(x)= (2*x^2 - 6*x +5 )/(x-3)
```

In [38]:
```
g(0)
```

Out[38]:
```
-5/3
```

In [39]:
```
solve(1/g == 0, x)
```

Out[39]:
```
[x == 3]
```

In [41]:
```
limit(g, x=+infinity)
```

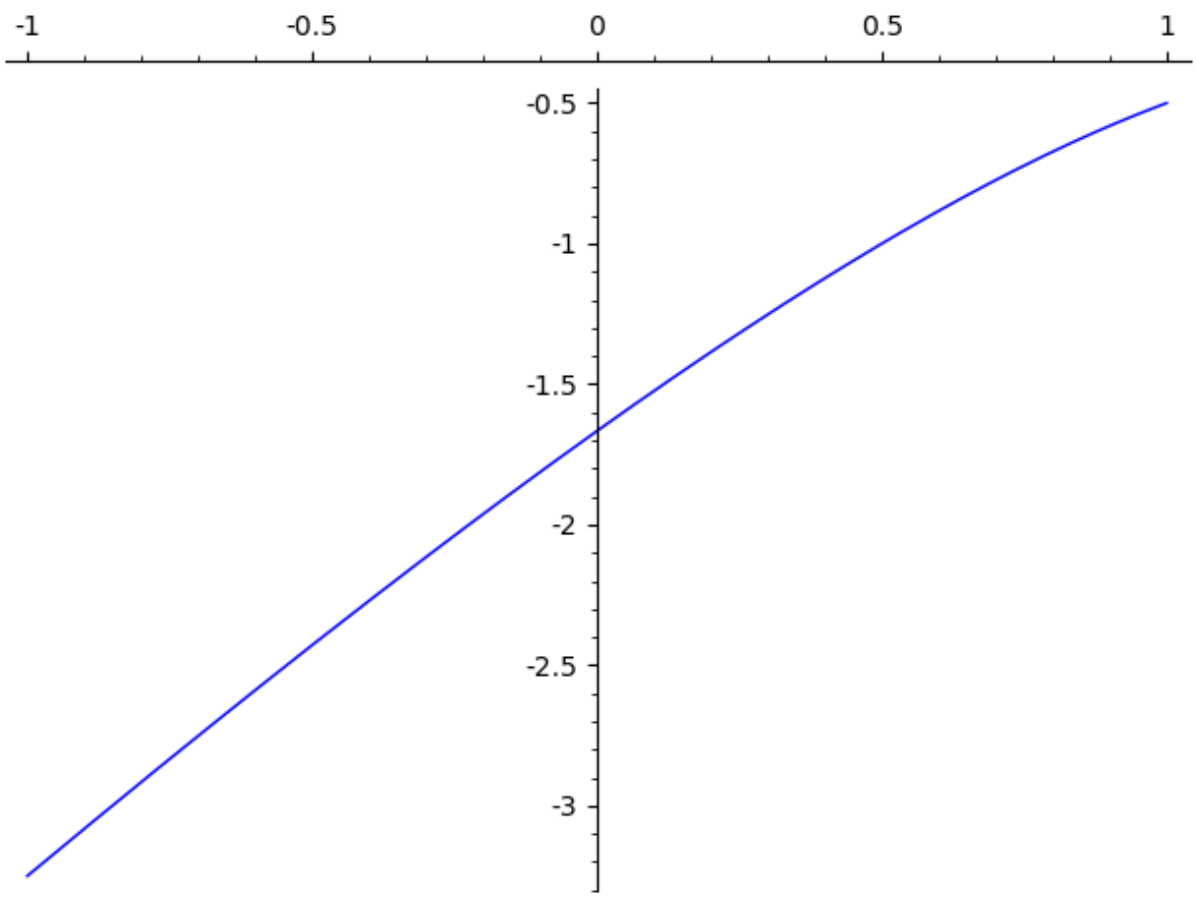
Out[41]:
```
x |--> +Infinity
```

In [42]:
```
limit(g, x=-infinity)
```

Out[42]:
```
x |--> -Infinity
```

In [46]:
```
plot(g(x))
```

Out[46]:



In [47]:
```
solve( diff(g, x) == 0, x)
```

Out[47]:
```
[x == -1/2*sqrt(10) + 3, x == 1/2*sqrt(10) + 3]
```

In [48]:
```
g2 = diff(g, x, 2)
g2(0)
```

Out[48]:
```
-10/27
```

In [51]:
```
f(x) = sin(x)*e^(-x^2)
```

In [77]:
```
def f(x): return sin(x)*e^(-x^2)
a, b = 1, 5
n= 100
```

In [78]:
```
print(list(range(n)))
print([i/2 + 1 for i in range(n)])
```

```
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 2
0, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 3
8, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 5
6, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 7
4, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 9
2, 93, 94, 95, 96, 97, 98, 99]
[1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 1
9/2, 10, 21/2, 11, 23/2, 12, 25/2, 13, 27/2, 14, 29/2, 15, 31/2, 16, 33/
2, 17, 35/2, 18, 37/2, 19, 39/2, 20, 41/2, 21, 43/2, 22, 45/2, 23, 47/2,
24, 49/2, 25, 51/2, 26, 53/2, 27, 55/2, 28, 57/2, 29, 59/2, 30, 61/2, 31,
63/2, 32, 65/2, 33, 67/2, 34, 69/2, 35, 71/2, 36, 73/2, 37, 75/2, 38, 77/
2, 39, 79/2, 40, 81/2, 41, 83/2, 42, 85/2, 43, 87/2, 44, 89/2, 45, 91/2,
46, 93/2, 47, 95/2, 48, 97/2, 49, 99/2, 50, 101/2]
```

In [79]:
```
x_n = [a + (b-a)/n*i for i in range(n+1)]
print(x_n)
```

```
[1, 26/25, 27/25, 28/25, 29/25, 6/5, 31/25, 32/25, 33/25, 34/25, 7/5, 36/
25, 37/25, 38/25, 39/25, 8/5, 41/25, 42/25, 43/25, 44/25, 9/5, 46/25, 47/
25, 48/25, 49/25, 2, 51/25, 52/25, 53/25, 54/25, 11/5, 56/25, 57/25, 58/2
5, 59/25, 12/5, 61/25, 62/25, 63/25, 64/25, 13/5, 66/25, 67/25, 68/25, 6
9/25, 14/5, 71/25, 72/25, 73/25, 74/25, 3, 76/25, 77/25, 78/25, 79/25, 1
6/5, 81/25, 82/25, 83/25, 84/25, 17/5, 86/25, 87/25, 88/25, 89/25, 18/5,
91/25, 92/25, 93/25, 94/25, 19/5, 96/25, 97/25, 98/25, 99/25, 4, 101/25,
102/25, 103/25, 104/25, 21/5, 106/25, 107/25, 108/25, 109/25, 22/5, 111/2
5, 112/25, 113/25, 114/25, 23/5, 116/25, 117/25, 118/25, 119/25, 24/5, 12
1/25, 122/25, 123/25, 124/25, 5]
```

In [80]:
```
fx_n = [f(i).n() for i in x_n]
print(fx_n)
```

```
[0.309559875653112, 0.292400401754106, 0.274717752721139, 0.256750901375578, 0.238720338942892, 0.220825931642348, 0.203245419642334, 0.186133532622540, 0.169621681519360, 0.153818174091154, 0.138808892863245, 0.124658367813984, 0.111411172747751, 0.0990935734783959, 0.0877153574509864, 0.0772717779370975, 0.0677455510887231, 0.0591088505493526, 0.0513252516188460, 0.0443515847874897, 0.0381396664581107, 0.0326378825654309, 0.0277926083259767, 0.0235494543058786, 0.0198543352242381, 0.0166543633121944, 0.0138985725644791, 0.0115384838348930, 0.00952852345952665, 0.00782630999098311, 0.00639282476552042, 0.00519248249134697, 0.00419311793872497, 0.00336590423390758, 0.00268521731161033, 0.00212845986255690, 0.001675856713589390, 0.00131023208134385, 0.00101677760201941, 0.000782818553101594, 0.000597584229403280, 0.000451987126062069, 0.000338414374544427, 0.000250533824712573, 0.000183116258772145, 0.000131874463702694, 0.0000933192712603720, 0.0000646321889208313, 0.0000435538786141176, 0.0000282874786077099, 0.0000174155925473789, 9.82967365038414e-6, 4.67049717140445e-6, 1.27842738347440e-6, -8.47765387601282e-7, -2.08470700718842e-6, -2.71191822500611e-6, -2.93398508126704e-6, -2.89827048456174e-6, -2.70893125161009e-6, -2.43790373410213e-6, -2.13342635834542e-6, -1.82658048806161e-6, -1.53625282251561e-6, -1.27285335454771e-6, -1.04106262039280e-6, -8.41830164572916e-7, -6.73802208555987e-7, -5.34319720550729e-7, -4.20097646690000e-7, -3.27671181188172e-7, -2.53674849232000e-7, -1.95004123916758e-7, -1.48896633274408e-7, -1.12960142501552e-7, -8.51669010374490e-8, -6.38281780557870e-8, -4.75584958214714e-8, -3.52358937792586e-8, -2.59622589635493e-8, -1.90261312643275e-8, -1.38692682909774e-8, -1.00575025594716e-8, -7.25594140126904e-9, -5.20826924576003e-9, -3.71975392075288e-9, -2.64348975635816e-9, -1.86939886681811e-9, -1.31553545210939e-9, -9.21280633440756e-10, -6.42066662352696e-10, -4.45322501524353e-10, -3.07383606815132e-10, -2.11154934730116e-10, -1.44356694641148e-10, -9.82169051542238e-11, -6.65036367662569e-11, -4.48134412282119e-11, -3.00515225676058e-11, -2.00543689565934e-11, -1.33174864972730e-11]
```

In [81]:
```
print("left endpoints: ", fx_n[:-1])
print("Right endpoints: ", fx_n[1:])
```

```
('left endpoints: ', [0.309559875653112, 0.292400401754106, 0.27471775272
1139, 0.256750901375578, 0.238720338942892, 0.220825931642348, 0.20324541
9642334, 0.186133532622540, 0.169621681519360, 0.153818174091154, 0.13880
8892863245, 0.124658367813984, 0.111411172747751, 0.0990935734783959, 0.0
877153574509864, 0.0772717779370975, 0.0677455510887231, 0.05910885054935
26, 0.0513252516188460, 0.0443515847874897, 0.0381396664581107, 0.0326378
825654309, 0.0277926083259767, 0.0235494543058786, 0.0198543352242381, 0.
0166543633121944, 0.0138985725644791, 0.0115384838348930, 0.0095285234595
2665, 0.00782630999098311, 0.00639282476552042, 0.00519248249134697, 0.00
419311793872497, 0.00336590423390758, 0.00268521731161033, 0.002128459862
55690, 0.00167585671389390, 0.00131023208134385, 0.00101677760201941, 0.0
0078281855310159, 0.00059758422940328 0, 0.00045198712606206 9, 0.00033841
4374544427, 0.00025053382471257 3, 0.00018311625877214 5, 0.000131874463702
694, 0.00009331927126037 20, 0.00006646321889208313, 0.00004355387861411 76,
0.00002828747860770 99, 0.00001741559254737 89, 9.82967365038414e-6, 4.6704
9717140445e-6, 1.27842738347440e-6, -8.47765387601282e-7, -2.084707007188
42e-6, -2.71191822500611e-6, -2.93398508126704e-6, -2.89827048456174e-6,
-2.70893125161009e-6, -2.43790373410213e-6, -2.13342635834542e-6, -1.8265
8048806161e-6, -1.53625282251561e-6, -1.27285335454771e-6, -1.04106262039
280e-6, -8.41830164572916e-7, -6.73802208555987e-7, -5.34319720550729e-7,
-4.20097646690000e-7, -3.27671181188172e-7, -2.53674849232000e-7, -1.9500
4123916758e-7, -1.48896633274408e-7, -1.12960142501552e-7, -8.51669010374
490e-8, -6.38281780557870e-8, -4.75584958214714e-8, -3.52358937792586e-8,
-2.59622589635493e-8, -1.90261312643275e-8, -1.38692682909774e-8, -1.0057
5025594716e-8, -7.25594140126904e-9, -5.20826924576003e-9, -3.71975392075
288e-9, -2.64348975635816e-9, -1.86939886681811e-9, -1.31553545210939e-9,
-9.21280633440756e-10, -6.42066662352696e-10, -4.45322501524353e-10, -3.0
7383606815132e-10, -2.11154934730116e-10, -1.44356694641148e-10, -9.82169
051542238e-11, -6.65036367662569e-11, -4.48134412282119e-11, -3.005152256
76058e-11, -2.00543689565934e-11])
('Right endpoints: ', [0.292400401754106, 0.274717752721139, 0.2567509013
75578, 0.238720338942892, 0.220825931642348, 0.203245419642334, 0.1861335
32622540, 0.169621681519360, 0.153818174091154, 0.138808892863245, 0.1246
58367813984, 0.111411172747751, 0.0990935734783959, 0.0877153574509864,
0.0772717779370975, 0.0677455510887231, 0.0591088505493526, 0.05132525161
88460, 0.0443515847874897, 0.0381396664581107, 0.0326378825654309, 0.0277
926083259767, 0.0235494543058786, 0.0198543352242381, 0.0166543633121944,
0.0138985725644791, 0.0115384838348930, 0.00952852345952665, 0.0078263099
9098311, 0.00639282476552042, 0.00519248249134697, 0.00419311793872497,
0.00336590423390758, 0.00268521731161033, 0.00212845986255690, 0.00167585
671389390, 0.00131023208134385, 0.00101677760201941, 0.00078281855310159
4, 0.00059758422940328 0, 0.00045198712606206 9, 0.00033841437454442 7, 0.00
025053382471257 3, 0.00018311625877214 5, 0.00013187446370269 4, 0.000093319
2712603720, 0.00006646321889208313, 0.00004355387861411 76, 0.0000282874786
077099, 0.00001741559254737 89, 9.82967365038414e-6, 4.67049717140445e-6,
1.27842738347440e-6, -8.47765387601282e-7, -2.08470700718842e-6, -2.71191
822500611e-6, -2.93398508126704e-6, -2.89827048456174e-6, -2.708931251610
09e-6, -2.43790373410213e-6, -2.13342635834542e-6, -1.82658048806161e-6,
-1.53625282251561e-6, -1.27285335454771e-6, -1.04106262039280e-6, -8.4183
0164572916e-7, -6.73802208555987e-7, -5.34319720550729e-7, -4.20097646690
000e-7, -3.27671181188172e-7, -2.53674849232000e-7, -1.95004123916758e-7,
-1.48896633274408e-7, -1.12960142501552e-7, -8.51669010374490e-8, -6.3828
1780557870e-8, -4.75584958214714e-8, -3.52358937792586e-8, -2.59622589635
493e-8, -1.90261312643275e-8, -1.38692682909774e-8, -1.00575025594716e-8,
```

```
-7.25594140126904e-9, -5.20826924576003e-9, -3.71975392075288e-9, -2.6434
8975635816e-9, -1.86939886681811e-9, -1.31553545210939e-9, -9.21280633440
756e-10, -6.42066662352696e-10, -4.45322501524353e-10, -3.07383606815132e
-10, -2.11154934730116e-10, -1.44356694641148e-10, -9.82169051542238e-11,
-6.65036367662569e-11, -4.48134412282119e-11, -3.00515225676058e-11, -2.0
0543689565934e-11, -1.33174864972730e-11])
```

In [82]:
```
L = (b-a)/n * sum(fx_n[:-1])
R = (b-a)/n * sum(fx_n[1:])
print("Left Approximation: ", L)
print("Right Approximation: ", R)
```

```
('Left Approximation: ', 0.135985460464793)
('Right Approximation: ', 0.123603065438135)
```