Calculus I **Answer Key**
Fall 2020
Lab 5

Show all your work, cite your sources, and type your answers for full credit.

Materials needed: none

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1. (0 points) (In Class Activity) draw a function and the derivative of that function on the same graph. Draw the derivative of the derivative function (this is called the second derivative) on that same graph as well. Label each I, II, III and provide the answers under a piece of paper taped on the white board

2. (5 points) Using what you've learned from Calculus lecture, find a formula for

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)h(x)}\right)$$

Answer: One method is to treat the bottom as one function while doing the quotient rule, then do the product rule, which gives us

$$\begin{aligned}\frac{d}{dx}\left(\frac{f(x)}{g(x)h(x)}\right) &= \frac{g(x)h(x)f'(x) - f(x)\frac{d}{dx}\left(g(x)h(x)\right)}{g(x)^2h(x)^2} \\ &= \frac{g(x)h(x)f'(x) - f(x)g'(x)h(x) - f(x)g(x)h'(x)}{g(x)^2h(x)^2}\end{aligned}$$

The above answer is correct. They might have also split the fraction as

$$= \frac{f'(x)}{g(x)(h(x)} - \frac{f(x)g'(x)}{g(x)^2h(x)} - \frac{f(x)h'(x)}{g(x)h(x)^2}$$

3. (5 points) Using what you've learned from Calculus lecture, find a formula for

$$\frac{d}{dx}\Big(f(x)g(x)h(x)\Big)$$

Answer: One method is to treat the first two as one function then do the product rule twice, which gives us

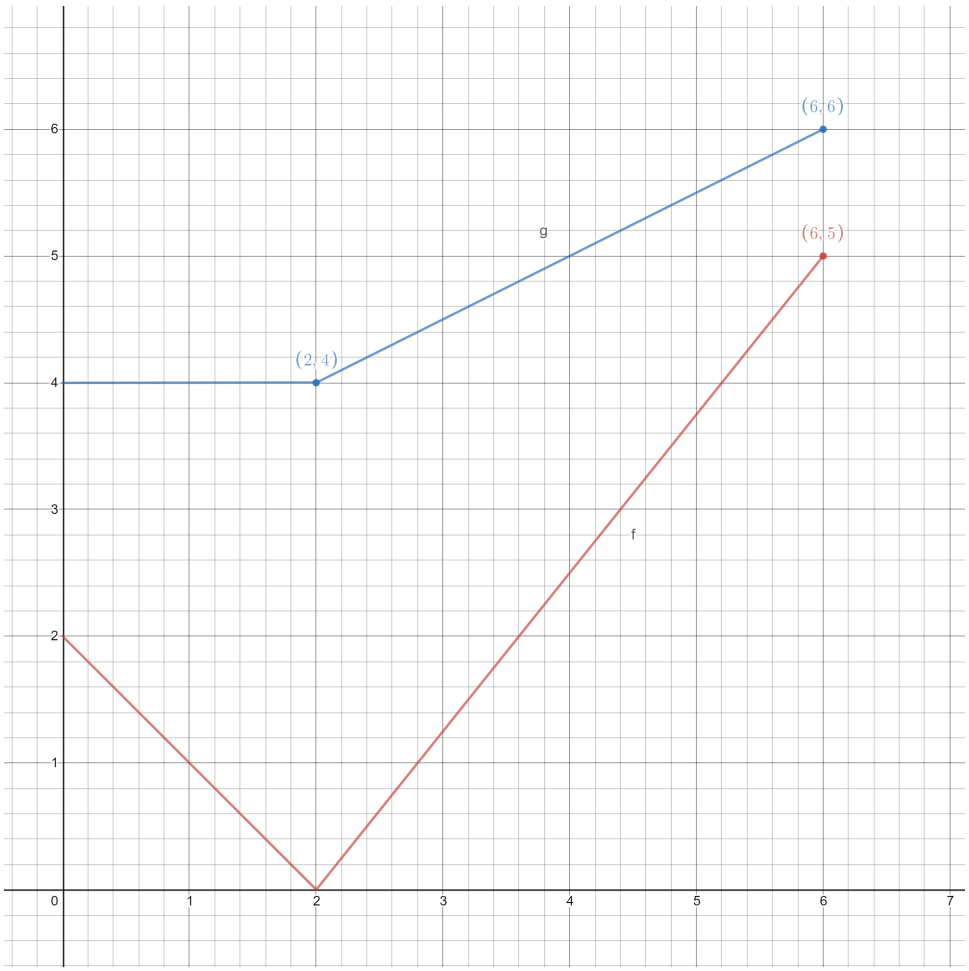
$$\begin{aligned}\frac{d}{dx}\left(f(x)g(x)h(x)\right) &= \frac{d}{dx}\left(f(x)g(x)\right)h(x) + f(x)g(x)h'(x) \\ &= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)\end{aligned}$$

4. (5 points) Find the three tangent lines to the curve $f(x) = (x+2)(x-3)(x+4)$ that go through the point $(4, -7)$.

Answer: This question was harder than I intended. Give them credit if they found the derivative of $f(x)$. From above, we know that

$$\begin{aligned} f'(x) &= (x+2)(x-3) + (x+2)(x+4) + (x-3)(x+4) \\ &= (x^2 - x - 6) + (x^2 + 6x + 8) + (x^2 + x - 12) \\ &= 3x^2 + 6x - 10 \end{aligned}$$

5. (10 points) Let f and g be defined by the graph below.



Let

$$r(x) = f(g(x)) \qquad s(x) = g(f(x)), \qquad t(x) = f(x)g(x), \qquad u(x) = \frac{f(x)}{g(x)}$$

Compute the following:

(a) $r'(1) = 0$
(b) $r'(4) = \frac{5}{8} = 0.625$
(c) $s'(1) = 0$
(d) $s'(4) = \frac{1}{4} = 0.25$
(e) $t'(1) = -4$
(f) $t'(4) = -7.5$
(g) $u'(1) = -\frac{1}{4} = -0.25$
(h) $u'(4) = \frac{1}{5} = 0.2$