

Sage and Linear Algebra Worksheet

FCLA Section ILT

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1 Injective Linear Transformations

Two carefully-crafted linear transformations: T is injective, S is not.

```
A = matrix(QQ, [[1, 2, 2], [3, 7, 6], [1, 2, 1], [2, 5, 7]])
T = linear_transformation(QQ^3, QQ^4, A, side='right')
```

```
T.is_injective()
```

```
T.kernel()
```

```
B = matrix(QQ, [[0, 1, -2], [-1, 1, 3], [-2, 5, 0], [0, 2,
-4]])
S = linear_transformation(QQ^3, QQ^4, B, side='right')
```

```
S.is_injective()
```

```
K = S.kernel()
K
```

We create two different inputs, which differ by a random vector from the kernel (which we hope is not simply the zero vector, a distinct possibility). We always get the same output from S, predictably. If we try this with T then the kernel vector is *always* the zero vector and the demonstration is very uninteresting.

```
z = K.random_element()
u = random_vector(QQ, 3)
w = u + z
u, w, S(u), S(w), S(u) == S(w)
```

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