Beginning Activities for Section 8.3

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Integer Solutions for Linear Equations)

1. An integer solution for the equation is $x = 7$ since $6 \cdot 7 = 42$.

2. An integer solution for the equation is $x = -3$ since $7(-3) = -21$.

3. The equation has no solution that is an integer. There is no integer such that $4x = 9$.

4. The equation has no solution that is an integer. There is no integer such that $-3x = 20$.

5. **Theorem 8.18**. *Let $a, b \in \mathbb{Z}$ with $a \neq 0$.*

   - *If a does not divide b, then the equation $ax = b$ has no solution that is an integer.*
   - *If a divides b, then the equation $ax = b$ has exactly one solution that is an integer.*

   Proof. For the first statement, we will prove the contrapositive, which is "If the equation $ax = b$ has a solution that is an integer, then a divides b."

   So we assume the equation $ax = b$ has a solution that is an integer. This means there exists an integer q such that $aq = b$, and this means that a divides b. This completes the proof of the contrapositive.

   For the second statement, if a divides b, then there exists an integer q such that $aq = b$, and this proves that there exists at least one solution of the equation $ax = b$ that is an integer. So we assume that x_1 and x_2 are two solutions of the equation $ax = b$. Then, $ax_1 = b$ and $ax_2 = b$ and hence $ax_1 = ax_2$. Since $a \neq 0$,this implies that $x_1 = x_2$. This proves there is only one solution of the equation. ■

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Beginning Activity 2 (Exploring Linear Equations in Two Variables)

1. There are no integers x and y such that $2x + 6y = 25$. If there were such integers, the left side of the equation would be an even integer and the right side would be an odd integer. This is impossible.

2. There are no integers x and y such that $6x - 9y = 100$. If there were such integers, the left side of the equation would be an integer that is a multiple of 3 and the right side would be an integer that is not a multiple of 3. This is impossible.

3. (**a**) Examples are: $x = 12$, $y = -5$ and $x = 17$, $y = -8$.

   (**b**) Examples are: $x = -3$, $y = 4$ and $x = -8$, $y = 7$.

   (**c**) One possibility is:
$$x = 2 + 5k$$
$$y = 1 - 3k$$
   where k is an integer.

4. (**a**) Examples are: $x = 10$, $y = -4$ and $x = 13$, $y = -6$.

   (**b**) Examples are: $x = 1$, $y = 2$ and $x = -2$, $y = 4$.

   (**c**) One possibility is:
$$x = 4 + 3k$$
$$y = 0 - 3k$$
   where k is an integer.
