Beginning Activities for Section 5.2

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Working with Two Specific Sets)

In this beginning activity, we have our first example of proving one set is a subset of another set. Proving subset relationships is a frequent type of proof when studying sets, and one of the usual methods of doing so is the **choose-an-element method**. To prove a set A is a subset of a set B, he idea is to choose an arbitrary element of A and then use the definitions and properties of the two sets to prove that this element must also be an element of the set B.

2. $S = \{\ldots, -18, -12, -6, 0, 6, 12, 18, \ldots\}$ $\quad T = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$

It appears that S is a subset of T.

3. $S = \{x \in \mathbb{Z} \mid x \text{ is a multiple of } 6\}$ $\quad T = \{x \in \mathbb{Z} \mid x \text{ is even}\}$

4. An integer x is a multiple of 6 provided there exists an integer m such that $x = 6m$.

An integer y is an even integer provided there exists an integer k such that $y = 2k$.

5. Following is the completed know-show table.

Step	**Know**	**Reason**
P	S is the set of all integers that are multiples of 6. T is the set of all even integers.	Hypothesis
$P1$	Let $x \in S$.	Choose an arbitrary element of S.
$P2$	$(\exists m \in \mathbb{Z})\,(x = 6m)$	Definition of "multiple"
$P3$	$x = (2 \cdot 3)m$	$6 = 2 \cdot 3$
$P4$	$x = 2(3m)$	Associative Law
$P5$	x is even.	Definition of an even integer since $3m$ is an integer.
$Q2$	x is an element of T.	x is even
$Q1$	$(\forall x \in \mathbb{Z})\,[(x \in S) \to (x \in T)]$	Step $P1$ and Step $Q2$
Q	$S \subseteq T$.	Definition of "subset"

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Beginning Activity 2 (Working with Venn Diagrams)

In this beginning activity, we see how Venn diagrams can be used to help formulate conjectures about two sets. The conjectures are often in the form that one set is a subset of the other set or that the two sets are equal.

1. The region in the Venn diagram on the left in Figure 1 corresponding to B^c is the region inside the rectangle that is outside the circle for B. This appears to be contained in the shaded region for A^c. Thus, it would appear that $B^c \subseteq A^c$.

2. In the general Venn diagram shown above, both $A - B$ and $A \cap B^c$ are represented by region 1. This suggests that $A - B$ equals $A \cap B^c$.

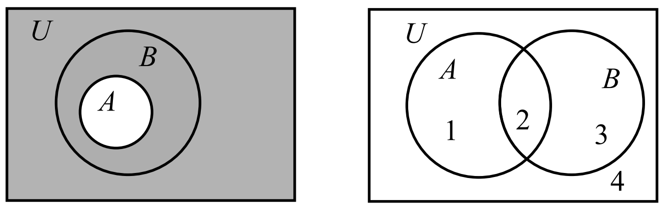

Figure 1: Venn Diagrams for Beginning Activity 2