Beginning Activities for Section 3.4

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (A Logical Equivalency)

1.

P	Q	R	$P \vee Q$	$(P \vee Q) \to R$	$P \to R$	$Q \to R$	$(P \to R) \wedge (Q \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

2. If we prove both $P \to R$ and $Q \to R$, then we have proven $(P \to R) \wedge (Q \to R)$. Since the statements $(P \vee Q) \to R$ and $(P \to R) \wedge (Q \to R)$ are logically equivalent, this means we have also proven $(P \vee Q) \to R$.

3. The contrapositive is: For all integers x and y, if x is even or y is even, then xy is even.

4. If x is an even integer, then there exists an integer k such that $x = 2k$. So if y is an integer, then

$$xy = (2k)y = 2(ky).$$

Since ky is an integer, this proves that if x is even, then xy is even.

If y is an even integer, then there exists an integer m such that $y = 2m$. So if x is an integer, then

$$xy = x(2m) = 2(xm).$$

Since xm is an integer, this proves that if y is even, then xy is even.

5. The proposition in part (3) is of the form $(P \vee Q) \to R$, which is logically equivalent to $(P \to R) \wedge (Q \to R)$. In part (4), we proved $P \to R$ and $Q \to R$ and so we proved $(P \to R) \wedge (Q \to R)$. Because of the logical equivalency, we have proved the proposition in part (3).

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Beginning Activity 2 (Using Cases in a Proof)

1. Let n be an even integer. We will show that $n^2 + n$ is an even integer. By the definition of an even integer, there exists an integer m such that

$$n = 2m.$$

Substituting this into the expression $n^2 + n$ yields

$$\begin{aligned} n^2 + n &= (2m)^2 + 2m \\ &= 4m^2 + 2m \\ &= 2(2m^2 + m). \end{aligned}$$

By the closure properties of the integers, $2m^2 + m$ is an integer, and hence $n^2 + n$ is even. So this proves that when n is an even integer, $n^2 + n$ is an even integer. ■

2. Let n be an odd integer. We will show that $n^2 + n$ is an even integer. By the definition of an odd integer, there exists an integer m such that

$$n = 2m + 1.$$

Substituting this into the expression $n^2 + n$ yields

$$\begin{aligned} n^2 + n &= (2m + 1)^2 + (2m + 1) \\ &= 4m^2 + 6m + 2 \\ &= 2(2m^2 + 3m + 1). \end{aligned}$$

By the closure properties of the integers, $2m^2 + 3m + 1$ is an integer, and hence $n^2 + n$ is even. So this proves that when n is an odd integer, $n^2 + n$ is an even integer. ■

3. The proofs of Propositions 2 and 3 constitute a proof of Proposition 1 since the integer n must be even or must be odd.
