Beginning Activities for Section 8.1

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (The Greatest Common Divisor)

1. A nonzero integer m divides an integer n provided that there is an integer q such that $n = m \cdot q$.

2. The nonzero integer m does not divide the integer n means that for all $q \in \mathbb{Z}$, $n \neq m \cdot q$.

3. $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

4. $\{1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84\}$

5. $\{1, 2, 3, 4, 6, 12\}$

6. $\gcd(48, 84) = 12$

7.

a	b	Common Divisors of a and b	$\gcd(a, b)$
8	-12	1, 2, 4	4
0	5	1, 5	5
8	27	1	1
28	42	1, 2, 7, 14	14

8. One possible conjecture is: Any common divisor of a and b divides $\gcd(a, b)$.

Beginning Activity 2 (The GCD and the Division Algorithm)

The Division Algorithm. Let a and b be integers with $b > 0$. Then, there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < b$.

1.

a	b	$\gcd(a, b)$	Remainder r	$\gcd(b, r)$
44	12	4	8	4
75	21	3	12	3
50	33	1	17	1

2. Let a and b be integers with $b > 0$. If q and r are integers such that $a = bq + r$ and $0 \leq r < b$, then $\gcd(a, b) = \gcd(b, r)$.

We will prove a somewhat more general result in this section (Lemma 8.1). This will be the crucial result for the Euclidean Algorithm, which provides a method to find the greatest common divisor of two integers.
