Beginning Activities for Section 9.2

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Introduction to Infinite Sets)

1. The contrapositive of "If A is a finite set, then A is not equivalent to any of its proper subsets" is

   If A is equivalent to one of its proper subsets, then A is an infinite set.

   The contrapositive of "For each set A, if A is a finite set, then for each proper subset of B of A, $A \not\approx B$" is

   For each set A, if there exists a proper subset B of A such that $A \approx B$, then A is an infinite set.

   This means that if we can show that there exists a set B such that $B \subset A$ and $A \approx B$, then we can conclude that A is an infinite set.

2. **(a)** Since D^+ is a proper subset of $\mathbb{N}$ and $\mathbb{N} \approx D^+$, we conclude that $\mathbb{N}$ is an infinite set.

   (b) We can also conclude that D^+ is an infinite set since if D^+ is finite, then there exists a $k \in \mathbb{N}$ such that $D^+ \approx \mathbb{N}_k$. Since we proved in part (a) that $\mathbb{N} \approx D^+$, we can then use part (c) of Theorem 9.1, to conclude that $\approx$, $\mathbb{N} \approx \mathbb{N}_k$ and this is a contradiction since $\mathbb{N}$ is infinite and $\mathbb{N}_k$ is finite.

3. **(a)** If $0 < b < 1$, then $(0, b)$ is a proper subset of $(0, 1)$ and $(0, 1) \approx (0, b)$. Hence, $(0, 1)$ is an infinite set.

   (b) If $b > 1$, then $(0, 1)$ is a proper subset of $(0, b)$ and $(0, 1) \approx (0, b)$. Hence, $(0, b)$ is an infinite set.

Beginning Activities for Section 9.2

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 2 (A Function from $\mathbb{N}$ to $\mathbb{Z}$)

1. If the pattern continues,

$f(8) = 4$	$f(9) = -4$
$f(10) = 5$	$f(11) = -5$
$f(12) = 6$	$f(13) = -6$

2. If the pattern continues, it would appear that f is a bijection.

3. It appears that if n is even, then $f(n) = \dfrac{n}{2}$.

4. It appears that if n is odd, then $f(n) = \dfrac{1-n}{2}$.

5. Define $f : \mathbb{N} \to \mathbb{Z}$ where

$$f(n) = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even} \\[2ex] \dfrac{1-n}{2} & \text{if } n \text{ is odd} \end{cases}$$

6. (**a**) The results are consistent with the pattern.

(**b**) $f(1000) = 500$ and $f(1001) = -500$.

(**c**) $f(2000) = 1000$.

We will prove that this function is a bijection in Theorem 9.13, and hence $\mathbb{N} \approx \mathbb{Z}$.
