Beginning Activities for Section 7.4

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Congruence Modulo 6)

1. $[3] = \{\ldots, -15, -9, -3, 3, 9, 15, \ldots\}$ and $[4] = \{\ldots, -14, -8, -2, 4, 10, 16, \ldots\}$.

2. In all cases, $s \equiv 1 \pmod 6$.

3. In all cases, $p \equiv 0 \pmod 6$.

4. In all cases, $q \equiv 3 \pmod 6$.

These results with congruence modulo 6 illustrate general properties of the equivalence classes for congruence modulo n. After a little more work at the start of this section, this will allow us to define an addition and multiplication of congruence classes modulo n on page 404.

Beginning Activity 2 (The Remainder When Dividing by 9)

n	$s(n)$	Remainder when n is divided by 9	Remainder when $s(n)$ is divided by 9
498	21	3	3
7319	20	2	2
4672	19	1	1
9845	26	8	8
51381	18	0	0
305877	30	3	3

In each case, n and $s(n)$ have the same remainder when divided by 9. We will prove this result in this section and it will help justify a so-called **divisibility test** for division by 9. That result is, 9 divides an integer n if and only if 9 divides the sum of its digits.

For example, 9 divides the integer 5832 since $5 + 8 + 3 + 2 = 18$ and 9 divides 18. Also, 9 does not divide the integer 7265 since $7 + 2 + 6 + 5 = 20$ and 9 does not divide 20.
