Beginning Activities for Section 8.2

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Exploring Examples where a divides $b \cdot c$)

1. In all of the examples where $a \mid (bc)$ but a does not divide b and a does not divide c, we should see that $\gcd(a, b) \neq 1$ and $\gcd(a, c) \neq 1$.

2. In all of the examples where $\gcd(a, b) = 1$ and $a \mid (bc)$, we should see that a divides c.

3. Conjecture: Let $a, b, c \in \mathbb{Z}$. If $\gcd(a, b) = 1$ and $a \mid (bc)$, then a divides c.

We will prove this result in this section. (Theorem 8.12). If a, b, and c are non-zero integers and a divides bc, it is tempting to conclude that a divides b or a divides c. The examples in part 1 of this beginning activity show that this cannot be done. However, if we add the condition that $\gcd(a, b) = 1$, then we can conclude that a divides c.

Beginning Activity 2 (Prime Factorizations)

2.

$$\begin{aligned} 40 &= 2 \cdot 20 \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \end{aligned} \qquad \begin{aligned} 40 &= 5 \cdot 8 \\ &= 5 \cdot 2 \cdot 2 \cdot 2 \end{aligned}$$

3. In Part (2), the same prime number factors were obtained but in a different order. Since multiplication of natural numbers is commutative and associative, we can say that we have the same factorization.

4.

$$\begin{aligned} 150 &= 3 \cdot 50 \\ &= 3 \cdot 2 \cdot 5 \cdot 5 \end{aligned} \qquad \begin{aligned} 150 &= 5 \cdot 30 \\ &= 5 \cdot 2 \cdot 3 \cdot 5 \end{aligned}$$
