Beginning Activities for Section 4.1

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Exploring Statements of the Form $(\forall n \in \mathbb{N})\,(P(n))$)

Notice the experimentation in all parts of this beginning activity. This is a good practice to use to explore whether a proposition appears to be true. In this section, we will study a method of proof that can be used to prove the two propositions in this beginning activity.

The following table shows that $P(1)$ through $P(7)$ are true.

n	$(5^n - 1)$	Does 4 divide $(5^n - 1)$?
1	4	yes
2	24	yes
3	124	yes
4	624	yes
5	3124	yes
6	15,624	yes
7	78,124	yes

The following table shows that $Q(1)$ through $Q(7)$ are true.

n	$1^2 + 2^2 + \cdots + n^2$	$\dfrac{n(n+1)(2n+1)}{6}$
1	1	1
2	5	5
3	14	14
4	30	30
5	55	55
6	91	91
7	140	140

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Beginning Activity 2 (A Property of the Natural Numbers)

The concept of an inductive set will provide the logical foundation for proofs by induction, which will be studied in this section. Again, notice that after the definition was given, we explored examples of sets that are inductive and examples that are not inductive. We also wrote what it means to say that a subset of the integers is not an inductive set.

1. A set T that is a subset of $\mathbb{Z}$ is not an inductive set provided that there exists an integer k such that $k \in T$ and $k + 1 \notin T$

2. (**a**) The set $A = \{1, 2, 3, \ldots, 20\}$ is not inductive since $20 \in A$, but $21 \notin A$.

(**b**) The set of natural numbers, $\mathbb{N}$, is inductive since the natural numbers are closed with respect to addition.

(**c**) The set $B = \{n \in \mathbb{N} | \, n \geq 5\}$ is inductive. If $k \in \mathbb{Z}$ and $k \geq 5$, then $(k + 1) \geq 5$.

(**d**) The set $S = \{n \in \mathbb{N} | \, n \geq -3\}$ is inductive. If $k \in \mathbb{Z}$ and $k \geq -3$, then $(k + 1) \geq -3$.

(**e**) The set $R = \{n \in \mathbb{Z} \, | n \leq 100\}$ is not inductive since $100 \in R$ but $101 \notin R$.

(**f**) The set of integers, $\mathbb{Z}$, is inductive since the set of integers is closed with respect to addition.

(**g**) The set of all odd integers is not an inductive set. For example, 1 is in the set of all odd integers but 2 is not in the set of all odd integers.

3. Now assume that $T \subseteq \mathbb{N}$ and assume that $1 \in T$ and that T is inductive.

(**a**) Since $1 \in T$ and T is inductive, $2 \in T$.

(**b**) Since $2 \in T$ and T is inductive, $3 \in T$.

(**c**) Since $3 \in T$ and T is inductive, $4 \in T$.

(**d**) Since $4 \in T$ and T is inductive, $5 \in T$. This in turn guarantees that $6 \in T$, which guarantees that $7 \in T$. We can continue this process to argue that any succeeding natural number is in T. So, $100 \in T$.

(**e**) The argument in Part (d) suggests that $T = \mathbb{N}$.
