Beginning Activities for Section 5.5

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (The Union and Intersection of a Family of Sets)

1. $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$ $\quad$ $A \cap B \cap C = \{3, 4, 5\}$
$B \cup C \cup D = \{2, 3, 4, 5, 6, 7, 8\}$ $\quad$ $B \cap C \cap D = \{4, 5, 6\}$

2. **(a)** $(A \cup B \cup C) \cup D = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(b) $A \cup (B \cup C \cup D) = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(c) $A \cup (B \cup C) \cup D = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(d) $(A \cup B) \cup (C \cup D) = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(e) $(A \cap B \cap C) \cap D = \{4, 5\}$
(f) $A \cap (B \cap C \cap D) = \{4, 5\}$
(g) $A \cap (B \cap C) \cap D = \{4, 5\}$
(h) $(A \cap B) \cap (C \cap D) = \{4, 5\}$

3. It appears that the placement of the parentheses in the union or intersection of these four sets does not make a difference. This means that we can define $A \cup B \cup C \cup D$ and $A \cap B \cap C \cap D$ as $(A \cup B \cup C) \cup D$ and $(A \cap B \cap C) \cap D$, respectively.

4. Since $\mathcal{A} = \{A, B, C, D\}$,

- $\bigcup\limits_{X \in \mathcal{A}} X$ consists of all the elements that are in one of the four sets A, B, C, and D. This is the same set as $A \cup B \cup C \cup D$.
- $\bigcap\limits_{X \in \mathcal{A}} X$ consists of all the elements that are in all of the four sets A, B, C, and D. This is the same set as $A \cap B \cap C \cap D$.

So,

$$\bigcup_{X \in \mathcal{A}} X = \{1, 2, 3, 4, 5, 6, 7, 8\} \qquad \text{and} \qquad \bigcap_{X \in \mathcal{A}} X = \{4, 5\}.$$

5. $\bigcup\limits_{X \in \mathcal{B}} X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $\quad$ and $\quad$ $\bigcap_{X \in \mathcal{B}} X = \emptyset$

6. Recall the the universal set is $\mathbb{N}$. Therefore,

$$\left(\bigcup_{X \in \mathcal{A}} X \right)^c = \{9, 10, 11, 12, 13, \ldots\}.$$

In addition,

$A^c = \{6, 7, 8, 9, \ldots\}$ $\quad$ $C^c = \{8, 9, 10, 11, \ldots\}$
$B^c = \{7, 8, 9, 10, \ldots\}$ $\quad$ $D^c = \{9, 10, 11, 12, \ldots\}$.

Therefore,

$$\bigcap_{X \in \mathcal{A}} X^c = \{9, 10, 11, 12, \ldots\}.$$

Beginning Activities for Section 5.5

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 2 (An Indexed Family of Sets)

1. $\bigcup_{j=1}^{4} C_j = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $\bigcap_{j=1}^{4} C_j = \{4, 5\}$

2. (**a**) $\bigcup_{j=1}^{6} C_j = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $\bigcap_{j=1}^{6} C_j = \emptyset$

(**b**) $\bigcup_{j=1}^{8} C_j = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and $\bigcap_{j=1}^{8} C_j = \emptyset$

(**c**) $\bigcup_{j=4}^{8} C_j = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and $\bigcap_{j=1}^{8} C_j = \{8\}$

(**d**) $(\bigcap_{j=1}^{4} C_j)^c = \{1, 2, 3, 6, 7, 8, 9, 10, \ldots\}$ and

$\bigcup_{j=1}^{4} C_j^c = \{1, 2, 3, 6, 7, 8, 9, 10, \ldots\}$
