Beginning Activities for Section 7.3

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Sets Associated with a Relation)

1. $R[b] = \{a, b, e\}$ $\quad$ $R[c] = \{c, d\}$ $\quad$ $R[d] = \{c, d\}$ $\quad$ $R[e] = \{a, b, e\}$.

2. By inspection of the directed graph, we see that the relation R is reflexive, symmetric, and transitive. Therefore, R is an equivalence relation on A.

3. It can be seen that $R[a] = R[b] = R[e]$ and $R[c] = R[d]$.

4. We also see that $R[a] \cap R[c] = \emptyset$. Because of the set equalities in (3), we also know that $R[a] \cap R[d] = \emptyset$, $R[b] \cap R[c] = \emptyset$, $R[b] \cap R[d] = \emptyset$, $R[e] \cap R[c] = \emptyset$, and $R[e] \cap R[d] = \emptyset$.

5. The directed graph has loops at vertices b, d, and e, an arrow from a to b, an arrow from b to c, an arrow from a to d, an arrow from c to d, and an arrow from d to c.

   By inspection, we see that the relation S is not reflexive, not symmetric, and not transitive. Therefore, S is not an equivalence relation on A.

6. $S\,[b] = \{a, b\}$ $\quad$ $S\,[c] = \{b, c, d\}$ $\quad$ $S\,[d] = \{a, c, d\}$ $\quad$ $S\,[e] = \{e\}$

7. None of the sets are equal.

8. $S\,[a]$ and each of the other sets are disjoint. $S\,[e]$ and each of the other sets are disjoint.

Beginning Activity 2 (Congruence Modulo 3)

1. $C\,[0] = \{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots\}$ $\quad$ $C\,[2] = \{\ldots, -7, -4, -1, 2, 5, 8, 11, \ldots\}$

   $C\,[1] = \{\ldots, -8, -5, -2, 1, 4, 7, 10, \ldots\}$ $\quad$ $C\,[3] = \{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots\}$

2. **(a)** The intersection of any two of the sets, $C\,[0]$, $C\,[1]$, $C\,[2]$, is the empty set.

   (b) Since $734 = 244 \cdot 3 + 2$, we know that $734 \equiv 2 \pmod 3$, and $734 \in C\,[2]$.

   (c) Since $79 = 26 \cdot 3 + 1$, we know that $79 \equiv 1 \pmod 3$, and $79 \in C\,[1]$.
   Since $-79 = (-27) \cdot 3 + 2$, we know that $-79 \equiv 2 \pmod 3$, and $-79 \in C\,[2]$.

   (d) Since each integer is congruent to 0, 1, or 2 modulo 3, we know that each integer is in precisely one of the three sets $C\,[0]$, $C\,[1]$, or $C\,[2]$. Hence, we may conclude that $C\,[0] \cup C\,[1] \cup C\,[2] = \mathbb{Z}$.

   (e) $C\,[3] = C\,[0]$.

   (f) $C\,[4] = C\,[1]$.
