Beginning Activities for Section 6.3

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Functions with Finite Domains)

Notice that for a function with a finite domain, we can specify the "rule" for the function simply by giving the output for each input. In this situation, each function has the set $A = \{1, 2, 3\}$ as its domain. So for the function f, for example, all we have to do is specify $f(1)$, $f(2)$, and $f(3)$. In this beginning activity, we actually did this in two ways. One was the arrow diagram for f and the other just listed $f(1) = a$, $f(2) = b$, and $f(3) = c$.

1. The function f satisfies the stated property. The functions g and h do not.
2. The property in (2) is the contrapositive of the property in (1). So the function f satisfies the stated property. The functions g and h do not.
3. $\text{range}(f) = \{a, b, c\}$, $\text{range}(g) = \{a, b\}$, $\text{range}(h) = \{s, t\}$.
4. For the function h, the range is equal to the codomain. For the functions f and g, the range is not equal to the codomain.
5. The function h satisfies the stated property. The functions f and g do not. Note: The property in (5) is asking if the codomain of the function is a subset of the range. Since the range is always a subset of the codomain, if a function satisfies this property, then its range is equal to its codomain. This is why the answer to (5) is the same as the answer to (4).

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Beginning Activity 2 (Statements Involving Functions)

1. (**a**) The contrapositive is: For all $x, y \in A$, if $f(x) = f(y)$, then $x = y$.

   (**b**) The negation is: There exist $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

2. The negation is: There exists a y in B such that for all x in A, $f(x) \neq y$.

3. (**a**) For all $a, b \in \mathbb{R}$, if $g(a) = g(b)$, then $a = b$.

   Proof. We let $a, b \in \mathbb{R}$, and we assume that $g(a) = g(b)$. This means that

   $$5a + 3 = 5b + 3.$$

   By subtracting 3 from both sides of this equation and then dividing both sides by 5, we obtain

   $$\begin{aligned} 5a &= 5b \\ a &= b \end{aligned}$$

   This proves that for all $a, b \in \mathbb{R}$, if $g(a) = g(b)$, then $a = b$.

   (**b**) For all $b \in \mathbb{R}$, there exists an $a \in \mathbb{R}$ such that $g(a) = b$.

   Proof. We let $b \in \mathbb{R}$. We need to find an $a \in \mathbb{R}$ such that $g(a) = b$. In order for this to happen, we need $5a + 3 = b$. Solving this equation for a gives $a = \dfrac{b-3}{5}$. We then see that $a \in \mathbb{R}$ and

   $$\begin{aligned} g(a) &= g\left(\frac{b-3}{5}\right) \\ &= 5\left(\frac{b-3}{5}\right) + 3 \\ &= (b-3) + 3 \\ &= b \end{aligned}$$

   This proves that for all $b \in \mathbb{R}$, there exists an $a \in \mathbb{R}$ such that $g(a) = b$.
