Beginning Activities for Section 4.3

Mathematical Reasoning: Writing and Proof, Version 3

Beginning Activity 1 (Recursively Defined Sequences)

Notice that this beginning activity consists mainly of exploration and conjecture.

1. Based on the following, it appears that as n gets larger, b_n gets closer to zero.

$$b_4 = 2 \qquad b_5 = 1 \qquad b_6 = \frac{1}{2} \qquad b_7 = \frac{1}{4}$$
$$b_8 = \frac{1}{8} \qquad b_9 = \frac{1}{16} \qquad b_{10} = \frac{1}{32}$$

2. Based on the following, it appears that as n gets larger, T_n gets closer to 32.

$$T_3 = 28 \qquad T_4 = 30 \qquad T_5 = 31 \qquad T_6 = 31$$
$$T_7 = 31.75 \qquad T_8 = 31.875 \qquad T_9 = 31.9375 \qquad T_{10} = 31.96875$$

3. Based on the following, a conjecture could be: $a_n = r^{n-1} \cdot a$.

$$a_2 = r \cdot a \qquad a_3 = r^2 \cdot a \qquad a_4 = r^3 \cdot a$$
$$a_5 = r^4 \cdot a \qquad a_6 = r^5 \cdot a$$

4. Based on the following, a conjecture could be: $S_n = a + ra + r^2 + \cdots r^{n-1}a$.

$$S_2 = a + ra \qquad S_3 = a + ra + r^2a \qquad S_4 = a + ra + r^2a + r^3a$$
$$S_5 = a + ra + r^2a + r^3ar^4a \qquad S_6 = a + ra + r^2a + r^3ar^4a + r^5a$$

5.

n	1	2	3	4	5	6	7
a_n	1	2	6	24	120	720	5040
$n!$	1	2	6	24	120	720	5040

6. In order to calculate a_{100}, we first need to calculate $a_0, a_1, a_2, \ldots, a_{99}$.

7. For each n, in order to calculate a_n, we first need to calculate $a_0, a_1, a_2, \ldots, a_{n-1}$.

8. It seems that for each nonnegative integer n, $a_n = n!$.

BY NC SA

Beginning Activities for Section 4.3

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Beginning Activity 2 (The Fibonacci Numbers)

We will study the Fibonacci sequence in this section.

$f_5 = 5$	$f_9 = 34$	$f_{13} = 233$	$f_{17} = 1597$
$f_6 = 8$	$f_{10} = 55$	$f_{14} = 377$	$f_{18} = 2584$
$f_7 = 13$	$f_{11} = 89$	$f_{15} = 610$	$f_{19} = 4181$
$f_8 = 21$	$f_{12} = 144$	$f_{16} = 987$	$f_{20} = 6765$

2. The Fibonacci numbers f_3, f_6, f_9, f_{12}, f_{15}, and f_{18} are even.
The Fibonacci numbers f_4, f_8, f_{12}, f_{16}, and f_{20} are multiples of three.

3. In each case, the sum of the first $(n-1)$ Fibonacci numbers is equal to $f_{n+1} - 1$.

4. One other observation is that it appears that for each $n \in \mathbb{N}$, f_{5n} is a multiple of 5.
