Math101 Cheat Sheet

1 Fractions

Fractions are numbers on the form

$$\frac{a}{b},$$

where a, b are numbers with $b \neq 0$. a is called the *numerator* and b is called the *denominator*.

1.1 Rules

We have that

$$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}, \quad \frac{a}{b}\frac{c}{d} = \frac{ac}{bd}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc},$$

$$a\frac{b}{c} = \frac{ab}{c}, \quad \frac{\frac{a}{b}}{c} = \frac{a}{bc}, \quad \frac{a}{\frac{b}{c}} = \frac{ac}{b}.$$

1.2 Reducing Fractions

Common factors can be reduced:

$$\frac{a}{b} = \frac{ac}{bc}$$

2 Powers

Powers are numbers on the form

$$x^a.$$

x is the *base* and a is the *exponent*.

2.1 Rules

We have that

$$x^a x^b = x^{a+b}, \quad \frac{x^a}{x^b} = x^{a-b}, \quad (xy)^a = x^a y^a,$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}, \quad (x^a)^b = x^{ab}, \quad x^{-a} = \frac{1}{x^a}.$$

3 Roots

If $x \geq 0$ and $n \in \mathbb{Z}_+$ then there exists a number $\sqrt[n]{x} > 0$ such that

$$(\sqrt[n]{x})^n = x.$$

Note that $\sqrt[n]{x} = x^{\frac{1}{n}}$.

3.1 Rules

We ave that

$$\sqrt[n]{x} = x^{\frac{1}{n}}, \quad \sqrt[n]{x^m} = x^{\frac{m}{n}} = (\sqrt[n]{x})^m,$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}, \quad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}.$$

4 Squared Sum

We have that

$$(a+b)^2 = a^2 + b^2 + 2ab$$
$$(a-b)^2 = a^2 + b^2 - 2ab$$
$$(a+b)(a-b) = a^2 - b^2.$$

5 Equations

Equations can be reduced with the following rules:

1. You can add/subtract with the same number on both sides of an equation.
2. You can multiply/divide with the same number (except 0) on both sides of an equation.

5.1 Second Order Equations

Second order equations are on the form

$$ax^2 + bx + c = 0, \qquad (1)$$

The solutions to (1) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

5.2 Factorization

If $ax^2 + bx + c = 0$ has roots r_1 and r_2 then.

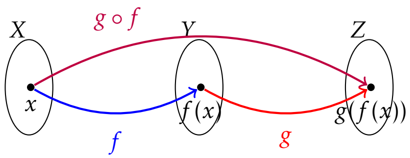
$$ax^2 + bx + c = a(x - r_1)(x - r_2).$$

6 Funktioner

A function $f\colon X \to Y$ associates to all $x \in X$ *exactly one* element $f(x) \in Y$.

6.1 Function Composition

If $f\colon X \to Y$ and $g\colon Y \to Z$ then the composition $g \circ f\colon X \to Z$ is defined by $(g \circ f)(x) = g(f(x))$. f is called the *inner function*, g is called the *outer function*

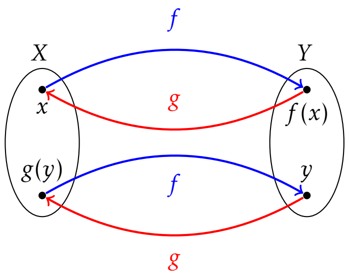


6.2 Inverse Functions

Two functions $f\colon X \to Y$ and $g\colon Y \to X$ are *inverse functions* if

$$f(g(y)) = y, \quad \text{and} \quad g(f(x)) = x$$

for all x in X and y in Y.



6.3 Polynomials

A first order polynomial is on the form:

$$f(x) = ax + b.$$

A second order polynomial is on the form:

$$f(x) = ax^2 + bx + c.$$

6.4 Logarithms and Exponential Functions

The *logarithm with base* a, $\log_a\colon]0,\infty[\to \mathbb{R}$ is inverse to the exponential function $f_a(x) = a^x$ ($a > 0$, $a \neq 1$). We have that

$$log_a(a^x) = x \quad \text{and} \quad a^{\log_a(y)} = y$$

and that

$$\ln x = \log_e x, \qquad \log x = \log_{10} x$$

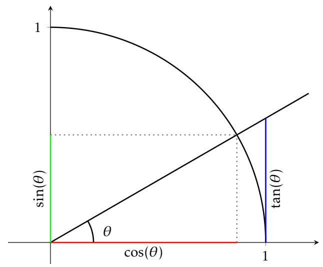
6.5 Rules

We have that

$$\log_a(xy) = \log_a(x) + \log_a(y),$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y),$$
$$\log_a(x^r) = r\log_a(x).$$

7 Trigonometric Functions

The trigonometric functions are defined by using the unit circle:



We have that

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

and that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$.

8 Derivatives

The derivative of f is denoted $f' = \frac{d}{dx}f = \frac{df}{dx}$.

8.1 Rules

We have that

$f(x)$	$f'(x)$
c	0
x	1
x^n	nx^{n-1}
e^x	e^x
e^{cx}	ce^{cx}
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$1 + \tan^2(x)$

8.2 General Rules

We have that

$$(cf)'(x) = cf'(x)$$
$$(f \pm g)'(x) = f'(x) \pm g'(x)$$
$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$
$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x).$$

The last formula is also called the *chain rule*.

9 Indefinite Integrals

A function f has *antiderivative* F if

$$F'(x) = f(x).$$

The indefinite integral of f is defined as

$$\int f(x)\,dx = F(x) + k,$$

where F is an antiderivative of f and $k \in \mathbb{R}$.

9.1 General Rules

$$\int cf(x)\,dx = c\int f(x)\,dx$$
$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$$
$$\int f(x)g(x)\,dx = f(x)G(x) - \int f'(x)G(x)\,dx$$
$$\int f(g(x))g'(x)\,dx = F(g(x)) + k.$$

The 3rd rule is called *integration by parts* and the last is called *integration by substitution*.

9.2 Rules

We have that

$f(x)$	$\int f(x)\,dx$
c	$cx + k$
x	$\frac{1}{2}x^2 + k$
x^n	$\frac{1}{n+1}x^{n+1} + k$
e^x	$e^x + k$
e^{cx}	$\frac{1}{c}e^{cx} + k$
$\frac{1}{x}$	$\ln(\lvert x\rvert) + k$
$\ln x$	$x\ln(x) - x + k$
$\cos x$	$\sin x + k$
$\sin x$	$-\cos x + k$
$\tan x$	$-\ln(\lvert\cos(x)\rvert) + k$

9.3 Integration by substitution

Given an integral on the form $\int f(g(x))g'(x)\,dx$ you can use the method:

1. Let $u = g(x)$.
2. Calculate $\frac{du}{dx}$ and solve for dx.
3. Substitute $g(x)$ and dx.
4. Calculate the integral wrt. u.
5. Substitute back.

10 Definite Integrals

The definite integral of f in the interval $[a,b]$ is defined as

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a),$$

where F is an antiderivative to f.

10.1 General Rules

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$
$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$
$$\int_a^b f(x)g(x)\,dx = [f(x)G(x)]_a^b - \int_a^b f'(x)G(x)\,dx$$
$$\int_a^b f(g(x))g'(x)\,dx = [F(x)]_{g(a)}^{g(b)}.$$

10.2 Integration by substitution

Given an integral on the form $\int_a^b f(g(x))g'(x)\,dx$ ou can use the method:

1. Let $u = g(x)$.
2. Calculate $\frac{du}{dx}$ and solve for dx.
3. Substitute $g(x)$, dx and limits.
4. Calculate the integral wrt. u.