4 Math101 exercises

4.1 Calculate the following:

$$3^3, \qquad 2^{-1}, \qquad \left(\frac{1}{-1}\right)^3, \qquad \left(\frac{1}{2}\right)^{-3}, \qquad 123^0.$$

4.2 Calculate the following:

$$\log_2(128), \quad \log_{10}(100), \quad \log_5\left(\frac{1}{25}\right), \quad \ln(e^3), \quad \log_{123}(1).$$

4.3 Calculate the following:

$$\sin(\frac{\pi}{4}) + \cos(\frac{\pi}{4}), \qquad \tan(\frac{\pi}{3}) + \cos(\frac{\pi}{6}), \qquad \frac{\sin(\frac{\pi}{6}) + \cos(\frac{\pi}{3})}{\sin(\frac{2\pi}{3})}.$$

4.4 Calculate the following:

$$\log_{10}(4) + \log_{10}(250), \quad \log_{10}(25) - \log_{10}(5) + \log_{10}(2), \quad \log_3(54) + \log_3\left(\frac{1}{2}\right)$$

4.5 Calculate the following:

$$\cos(-\frac{5\pi}{4}), \qquad \sin(\frac{5\pi}{3}), \qquad \tan(-\frac{5\pi}{4}), \qquad \cos(\frac{8\pi}{3}).$$

4.6 Reduce the following expressions:

$$\ln(\sqrt{2}) + \ln(2), \quad \log_{10}(5^{3/2}) + \frac{1}{2}\log_{10}(5) + \log_{10}(4), \quad \frac{1}{4}\log_5(4^2 + 3^2).$$

4.7 Calculate the following:

$$3^{\log_3(1)}, \qquad e^{1+\ln(3)}, \qquad 10^{-\log_{10}(7)}, \qquad 7^{1-\log_7(9)}, \qquad 4^{-\log_2(3)}.$$

4.8 Calculate the following:

$$\cos(\frac{13\pi}{3}), \qquad \tan(\frac{12\pi}{6}), \qquad \sin(-\frac{10\pi}{4}), \qquad \tan(\frac{15\pi}{5}).$$

4.9 Solve the equations:

$$e^x = 3, \quad \ln(x) = 4, \quad \ln(2x - 4) = \ln(8) + \ln(4), \quad 3\log_{10}(x) = \log_{10}(27).$$

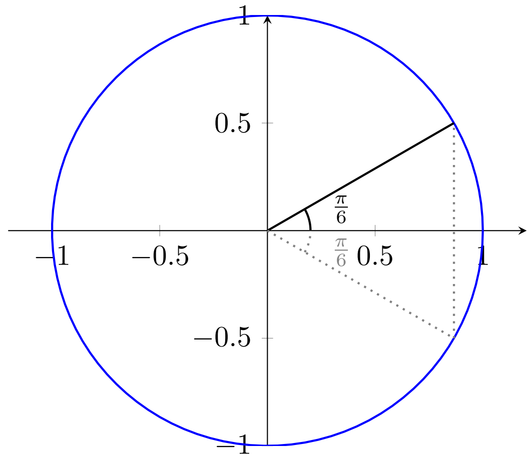

Figure 3: Exercise 4.11

4.10 Determine two different solutions to the equations:

$$\sin(x) = \frac{\sqrt{2}}{2}, \quad \cos(x - \pi) = -\frac{\sqrt{3}}{2}, \quad 2\cos^2(x) + 5\cos(x) + 2 = 0.$$

4.11 In this exercise we determine exact values of sine and cosine for the angles $\frac{\pi}{6}$ and $\frac{\pi}{6}$.

4.11(a) Show that $\sin(\frac{\pi}{6}) = \frac{1}{2}$ by considering the triangle in Figure 3. (Hint: What kind of triangle is it?)

4.11(b) Use the Pythagorean trigonometric identity ($\cos^2(x) + \sin^2(x) = 1$) to show that $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.

4.11(c) Show that $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$. (Hint: Use that $\sin(\frac{\pi}{3}) = \sin(\frac{\pi}{6} + \frac{\pi}{6}) = 2\sin(\frac{\pi}{6})\cos(\frac{\pi}{6})$)

4.11(d) Use the Pythagorean trigonometric identity to show that $\cos(\frac{\pi}{3}) = \frac{1}{2}$.

4.12 In this exercise we determine exact values of sine and cosine for the angle $\frac{\pi}{4}$.

4.12(a) Show that $\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ by considering the triangle in Figure 4.(Hint: Pythagorean theorem)

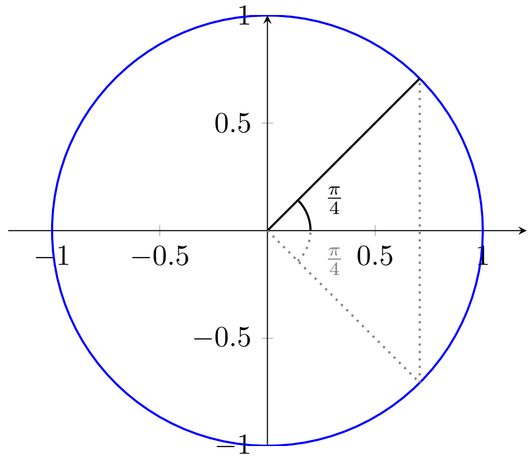

Figure 4: Exercise 4.12

4.12(b) Use the Pythagorean trigonometric identity to show that $\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.