4 Math101 answers

4.1 The answers are:

$$27, \qquad \frac{1}{2}, \qquad -1, \qquad 8, \qquad 1.$$

4.2 The answers are:

$$7, \qquad 2, \qquad -2, \qquad 3, \qquad 0.$$

4.3 The answers are:

$$\sqrt{2}, \qquad \frac{3\sqrt{3}}{2}, \qquad \frac{2}{\sqrt{3}}.$$

4.4 The answers are:

$$3, \qquad 1, \qquad 3$$

4.5 The answers are:

$$-\frac{\sqrt{2}}{2}, \qquad -\frac{\sqrt{3}}{2}, \qquad -1, \qquad -\frac{1}{2}.$$

4.6 The answers are:

$$\frac{3}{2}\ln(2), \qquad 2, \qquad \frac{1}{2}.$$

4.7 The answers are:

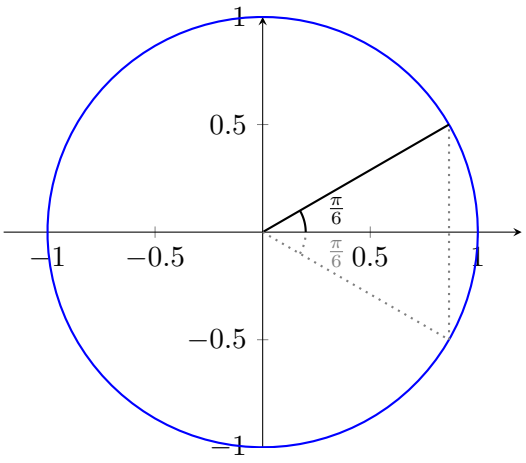
$$1, \qquad 3e, \qquad \frac{1}{7}, \qquad \frac{7}{9}, \qquad \frac{1}{9}.$$

4.8 The answers are:

$$\frac{1}{2}, \qquad 0, \qquad -1, \qquad 0.$$

4.9 The answers are:

$$x = \ln(3), \qquad x = e^4, \qquad x = 18, \qquad x = 3.$$



Figur 2: Exercise 4.11

4.10 he answers could be:

$$x = \frac{\pi}{4}, x = \frac{3\pi}{4}, \qquad x = \frac{\pi}{6}, x = -\frac{\pi}{6}, \qquad x = \frac{2\pi}{3}, x = \frac{4\pi}{3}.$$

Note that there are infinitely many correct answers.

4.11 The answers could be:

4.11(a) The triangle in Figure 2 is equilateral since all angles are $\frac{\pi}{3}$. Since two sides of the triangle has length 1 it follows that the last side must be of length 1. Hence it follows that $\sin(\frac{\pi}{6})$, which is half of the dotted line, must be $\frac{1}{2}$.

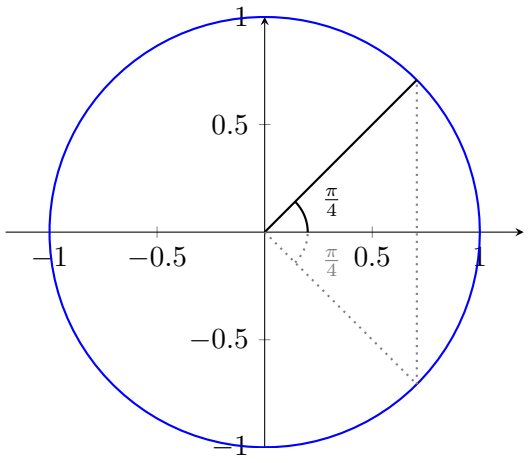
4.11(b) The Pythagorean trigonometric identity gives that $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6} = 1$ and solving for $\cos(\frac{\pi}{6})$ gives that $\cos(\frac{\pi}{6}) = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$.

4.11(c) Using the hint we obtain that

$$\sin(\frac{\pi}{3}) = \sin(2\frac{\pi}{6}) = 2\sin(\frac{\pi}{6})\cos(\frac{\pi}{6}) = 2\frac{1}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}.$$

4.11(d) It follows that

$$\sin^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{3} = 1 \quad \Leftrightarrow \quad \cos^2 \frac{\pi}{3} = 1 - \frac{3}{4} \quad \Leftrightarrow \quad \cos \frac{\pi}{3} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$



Figur 3: Exercise 4.12

4.12 The answers could be:

4.12(a) The triangle in Figure 3 is a right triangle where each leg has length 1. Using the Pythagorean theorem it follows that the hypotenuse must have length $\sqrt{1+1} = \sqrt{2}$. singe $\sin \frac{\pi}{4}$ is half the length of the hypotenuse we have that $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

4.12(b) It follows that

$$\cos \frac{\pi}{4} = \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2}.$$