7 Math101 exercises

7.1 Is $F(x) = 2x^3 - x^2 + x - 7$ an antiderivative of $f(x) = 6x^2 - 2x + 1$?

7.2 Is $F(x) = (x-1)e^x$ an antiderivative of $f(x) = 2xe^x$?

7.3 Determine an antiderivative F to the functions f given by $f(x) = 3x-7$ such that $F(1) = 7$.

7.4 Calculate the following indefinite integrals:

$$\int_0^1 x^2\,dx, \qquad \int_{-1}^1 x^3 + x\,dx, \qquad \int_1^2 \frac{2}{x}\,dx.$$

7.5 Calculate the following indefinite integrals:

$$\int x - 1\,dx, \qquad \int x^2 + e^x\,dx, \qquad \int 2\sin(x)\,dx.$$

7.6 Calculate the following definite integrals:

$$\int_0^{2\pi} \cos(x)\,dx, \qquad \int_{-1}^2 e^x\,dx, \qquad \int_\pi^{2\pi} \sin(x)\,dx.$$

7.7 Calculate the following indefinite integrals:

$$\int x^{-2} - e^{3x}\,dx, \qquad \int e^x - \frac{2}{x}\,dx$$

7.8 Calculate the following definite integrals:

$$\int_0^1 e^{2x}\,dx, \qquad \int_{-3}^1 x^2 - 7x + 1\,dx, \qquad \int_{-1}^0 \sin(x) + x\,dx.$$

7.9 Show that $F(x) = \frac{5}{7}x^{\frac{14}{5}}$ is an antiderivative of $f(x) = 2x^{\frac{9}{5}}$.

7.10 Calculate the following indefinite integrals:

$$\int 3x^2 + 2x\,dx, \qquad \int 3(e^{6x} - \cos x)\,dx, \qquad \int \ln(x) - \frac{1}{\sqrt{x}}\,dx.$$

7.11 Calculate the following indefinite integrals:

$$\int \frac{2}{x} + 3\sqrt{x} + 4x\,dx, \qquad \int \frac{5}{4}x^{\frac{3}{8}} - \frac{1}{x^{-2}}\,dx, \qquad \int x^{\frac{5}{4}} - \sqrt[4]{x^5}\,dx$$

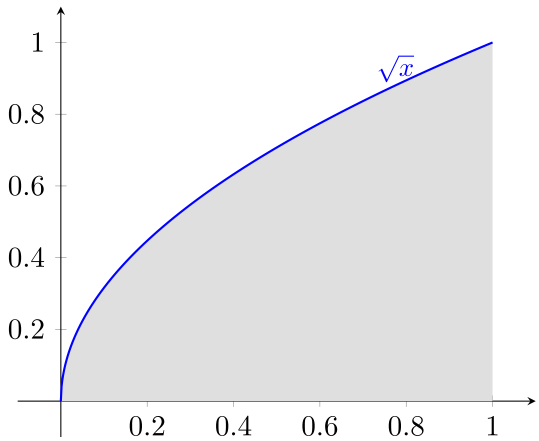

Figure 7: Exercise 7.12

7.12 How large is the grey area in Figure 7 relative to the area of the square with corners $(0,0)$, $(1,0)$, $(0,1)$ og $(1,1)$.

7.13 Show that

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$$

(Hint: Calculate on the right hand side.)

7.14 Sow that

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$$

(Hint: Calculate on the right hand side.)