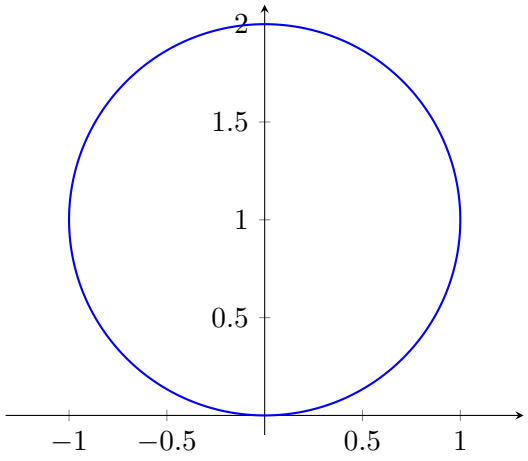

Figure 1: Exercise 3.2.

3 Math101 exercises

3.1 Let $f(x) = 3x^2 + 2x + 1$. Determine $f(-1)$ and $f(2)$.

3.2 The circle given by the equation $x^2 + (y-1)^2 = 1$ is sketched in Figure 1.

- Does there exist a function $f\colon [-1,1] \to [0,2]$ such that the graph of f is the circle in Figure 1?
- Determine a function $f_+\colon [-1,1] \to [1,2]$ such that the graph of f_+ is the upper semi circle in Figure 1.
- Determine a function $f_-\colon [-1,1] \to [0,1]$ such that the graph of f_- is the lower semi circle in Figure 1.

3.3 Let $f(x) = 3x - 2$ and $g(x) = \frac{1}{3}x + \frac{2}{3}$. Determine the function $f \circ g$.

3.4 Find the domain of the functions:

$$f(x) = \frac{1}{x+1}, \qquad g(x) = \frac{1}{1-x^2}, \qquad h(x) = \sqrt{2x-3}.$$

3.5 Let f, g be given by $f(x) = \sqrt{x}$ and $g(x) = 1/(1+x)$ on the domain $(0, \infty)$. Calculate $(f \circ g)(1)$ and $(g \circ f)(1)$. Is $f \circ g = g \circ f$?

3.6 Determine the intersection between $f(x) = 3x + 1$ and $g(x) = -x + 2$.

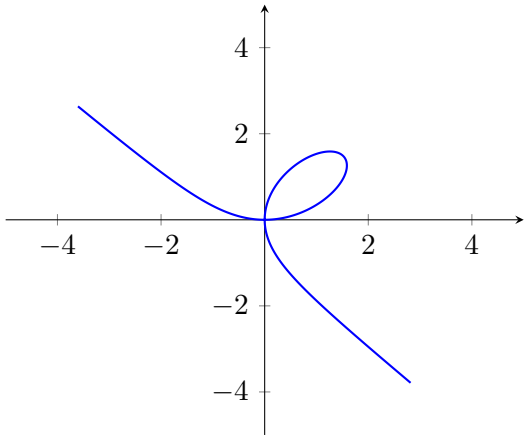

Figure 2: Exercise 3.13.

3.7 Let $f(x) = 1$ and $g(x) = 2x + 3$. Determine $f \circ g$ and $g \circ f$.

3.8 What is the largest possible domain of definition for the functions:

$$f(x) = \frac{1}{(1+x^2)^{\frac{1}{2}}}, \quad g(x) = \frac{2}{x^2 - 4x + 3}, \quad h(x) = \sqrt{-x^2 + 2x}.$$

3.9 Determine functions f and g such that $(f \circ g)(x) = e^{2x^2-1}$.

3.10 Determine all intersection points between $f(x) = x^2 + 4x + 4$ and $g(x) = 2x + 3$.

3.11 Determine functions f, g and h such that $(f \circ g \circ h)(x) = \sin^2(3x)$. (Hint: $\sin^2(x) = (\sin(x))^2$.)

3.12 Let $f(x) = 3(\frac{1}{x-2})^2$, $g(x) = \frac{1}{x}$ and $h(x) = \sqrt{x} + 2$ be functions defined on the domain $]2, \infty[$. Determine

$$f(g(x)), \qquad f(h(x)), \qquad h(g(x)), \qquad h(f(x)), \qquad g(f(h(x))).$$

3.13 Is the curve in Figure 2 the graph of a function?

3.14 Sketch the graph of a function which

3.14(a) has domain $[-1, 1]$,

3.14(b) intersects the points $(-1, 0)$ and $(1, 1)$,

3.14(c) intersects the y-axis at -1,