Practice Problems: Review

Online Problems

Problem 1 *Compute the following:*

$$(1, 2, 3) + (8, 3, 6) = \boxed{(9, 5, 9)}$$

$$4(1, -2, 4) = \boxed{(4, -8, 16)}$$

$$-12((5, 2, 6) - (8, 2, 4)) = \boxed{(36, 0, 24)}$$

Problem 2 *Let h be a constant. Compute the following:*

$$(7, 2, -1) + (2h, 0, h) = \boxed{(7 + 2h, 2, h - 1)}$$

$$h(1, 8, 2) = \boxed{(h, 8h, 2h)}$$

Problem 3 *For each of the following, determine whether the quantity exists or does not exist.*

$(1, 8, 3, 7) + (-1, 7, 2, 7)$

Multiple Choice:

(a) *Exists.* ✓

(b) *Does not exist.*

$(2, 8, 3) + (1, 7)$

Multiple Choice:

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(a) *Exists.*

(b) *Does not exist.* ✓

$(2,7,3)+1$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$(2,8,3)(1,7,3)$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$2(7,2,3,7,2)$

Multiple Choice:

(a) *Exists.* ✓

(b) *Does not exist.*

Problem 4 *For points $P_1=(2,-3,7,1)$ and $P_2=(-1,7,2,1)$, compute the displacement vector $\vec{P_1P_2}$.*

$$\vec{P_1P_2} = \boxed{(-3,10,-5,0)}$$

Problem 5 *Write the vector $2\mathbf{i}-5\mathbf{j}+2\mathbf{k}$ in $\mathbb{R}^3$ in standard vector notation.*

$$2\mathbf{i}-5\mathbf{j}+2\mathbf{k} = \boxed{(2,-5,2)}$$

Problem 6 *Compute the dot product.*

$$(1, 8, 3) \cdot (-2, 6, 0) = \boxed{46}$$

Problem 7 *Compute the dot product.*

$$(1, -5, 0, 2) \cdot (2, -1, 4, 1) = \boxed{9}$$

Problem 8 *Compute the dot product.*

$$(1, 8, 3) \cdot (-3, 0, 1) = \boxed{0}$$

What can you conclude about the vectors?

Multiple Choice:

(a) *They're perpendicular.* ✓

(b) *They aren't perpendicular.*

Problem 9 *Compute the dot product.*

$$(1, 2, 3) \cdot (-3, -2, -1) = \boxed{-10}$$

What can you conclude about the vectors?

Multiple Choice:

(a) *They're perpendicular.*

(b) *They aren't perpendicular.* ✓

Problem 10 *Compute the dot product.*

$$(1, 8, 3, 6) \cdot (3, -3, -1, 4) = \boxed{-0}$$

What can you conclude about the vectors?

Multiple Choice:

(a) *They're perpendicular.* ✓

(b) *They aren't perpendicular.*

Problem 11 *For each expression, determine whether it exists or does not exist.*

$(2, 8, 3) \cdot (1, 8, 2, 4)$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$(2, 8, 3, 1, 8) \cdot (1, 8, 2, 4, 2)$

Multiple Choice:

(a) *Exists.* ✓

(b) *Does not exist.*

Problem 12 *Compute the angle between the vectors* $(2, 8)$ *and* $(-8, 2)$ *in degrees.*

$$\theta = \boxed{90}^\circ$$

Problem 13 *Compute the angle between the vectors* $(2, 1, 3, 1, 1)$ *and* $(-3, 1, 1, 1, -2)$ *in degrees.*

$$\theta = \boxed{104.48}^\circ$$

(Give your answer as a positive number to two decimal places.

Problem 14 *Suppose you have vectors* $\vec{v}$ *and* $\vec{w}$ *such that* $\|\vec{v}\| = 6$ *and* $\|\vec{w}\| = 2$*, and the angle between* $\vec{v}$ *and* $\vec{w}$ *is* $\frac{\pi}{4}$ *radians. Compute the dot product of* $\vec{v}$ *and* $\vec{w}$.

$$\vec{v} \cdot \vec{w} = \boxed{6\sqrt{2}}$$

Problem 15 *Compute the projection of $\vec{v} = (1, 7, 3)$ onto $\vec{w} = (-3, -2, 1)$.*

$$proj_{\vec{w}}(\vec{v}) = \boxed{(7/2, 7/3, -7/6)}$$

Problem 16 *Compute the projection of the vector $\vec{v} = (1, 2, 3)$ onto the vector $\vec{w} = (-3, 2, -1)$.*

$$\text{proj}_{\vec{w}}(\vec{v}) = \boxed{(0, 0, 0)}$$

Problem 16.1 *Why does your answer make sense?*

Multiple Choice:

(a) *The vectors are parallel.*

(b) *The vectors are perpendicular.* ✓

(c) *They are the same length.*

Problem 17 *Compute the cross product.*

$$(1, 7, 3) \times (2, 8, -1) = \boxed{(-31, 7, -6)}$$

Problem 18 *Compute the cross product.*

$$(-1, 7, 2) \times (2, 8, 3) = \boxed{(5, 7, -22)}$$

Problem 19 *For each of the following, determine whether the expression exists or does not exist.*

$(-1, 7, 3) \times (1, 7, 2)$

Multiple Choice:

(a) *Exists.* ✓

(b) *Does not exist.*

$(-1, 7, 3, 8) \times (1, 7, 2, 0)$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$(2, 0, 0) \times (1, 7, 2, 0)$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$(0, 0, 0) \times (0, 0, 0)$

Multiple Choice:

(a) *Exists.* ✓

(b) *Does not exist.*

Problem 20 *Compute the area of the parallelogram determined by* $(1, 6)$ *and* $(1, 0)$.

$$\textit{Area} = \boxed{6}$$

Problem 21 *Compute the volume of the parallelepiped determined by* $(1, 6, 2)$, $(-1, 2, 0)$, *and* $(0, 3, 1)$.

$$\textit{Volume} = \boxed{2}$$

Problem 22 *Suppose $\vec{v}$ and $\vec{w}$ are unit vectors in the xy-plane, and we know that they are perpendicular. What is $\vec{v} \times \vec{w}$?*

Multiple Choice:

(a) $(0, 0, 1)$

(b) $(0, 0, -1)$

(c) *Not enough information.* ✓

Problem 23 *Find a parametrization $\vec{x}(t)$ of the line parallel to the vector $(1, 6, 3)$ and through the point $(1, 3, 2)$, such that $\vec{x}(0) = (1, 3, 2)$. $\vec{x}(t) =$* $\boxed{(t + 1, 6t + 3, 3t + 2)}$

Problem 24 *Find a parametrization $\vec{x}(s, t)$ of the plane containing vectors $(1, 6, 2)$ and $(1, 3, 2)$, and passing through the point $(1, 0, 0)$, such that $\vec{x}(0, 0) = (1, 0, 0)$ and $\vec{x}(1, 0) = (1, 6, 2)$.*

$$\vec{x}(s, t) = \boxed{(s + t + 1, 6s + 3t, 2s + 2t)}$$

Problem 25 *Give an equation which describes the plane perpendicular to the vector $(1, 7, 3)$ and through the point $(-2, 4, 1)$.*

$$0 = \boxed{(x + 2) + 7(y - 4) + 3(z - 1)}$$

Written Problems

Problem 26 *For any vector $\vec{v}$ in $\mathbb{R}^n$, prove that $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$.*

Problem 27 *Prove that vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$ are perpendicular if and only if $\mathrm{proj}_{\vec{w}}(\vec{v})$ is the zero vector.*

Problem 28 *For any vector $\vec{v}$ in $\mathbb{R}^3$, prove that $\vec{v} \times \vec{v}$ is the zero vector.*