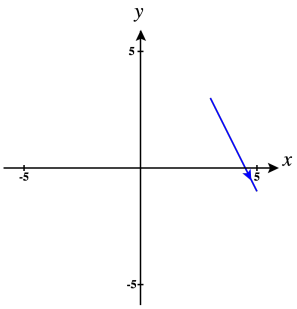
Homework 3: Parametrized Curves

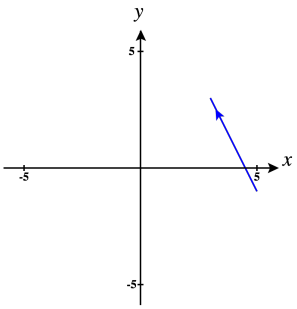
Online Problems

Problem 1 *Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases. Which is the graph of the path* $\vec{x}(t) = (4 - t, 2t + 1)$*, for* $-1 \leq t \leq 1$*?*

Multiple Choice:



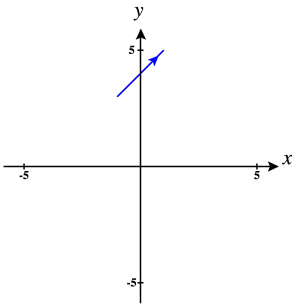
(a)



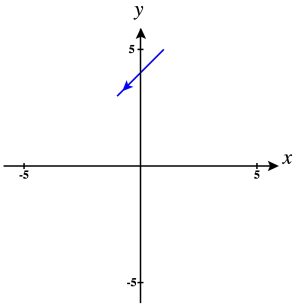
(b)

✓

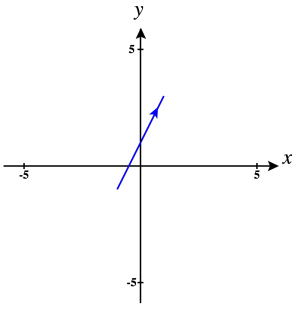
Learning outcomes:
Author(s):



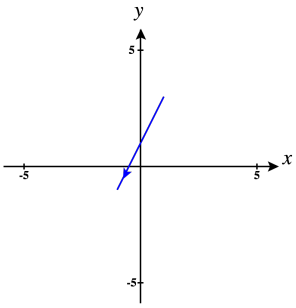
(c)



(d)



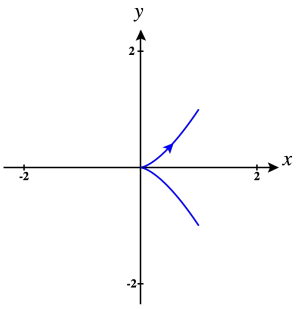
(e)



(f)

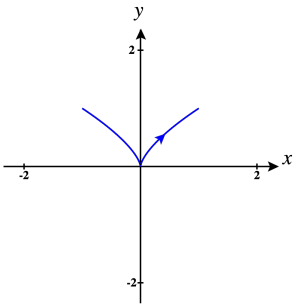
Problem 2 *Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases. Which is the graph of the path* $\vec{x}(t) = (t^2, t^3)$*, for* $-1 \leq t \leq 1$*?*

Multiple Choice:

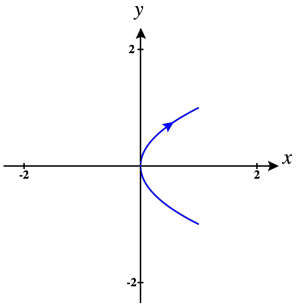


(a)

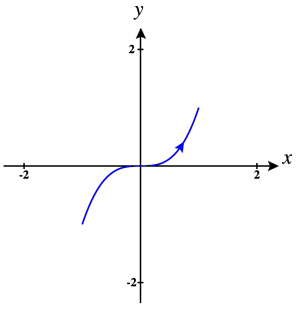
✓



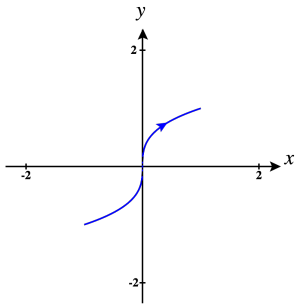
(b)



(c)

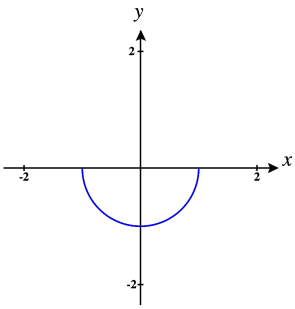


(d)



(e)

Problem 3 *Consider the curve below.*



Which of the following are parametrizations for the curve? Select all that apply.

Select All Correct Answers:

(a) $\vec{x}(t) = (\cos t, \sin t)$, *for* $0 \leq t \leq 2\pi$

(b) $\vec{x}(t) = (\sin t, \cos t)$, *for* $0 \leq t \leq \pi$

(c) $\vec{x}(t) = (\cos t, -\sin t)$, *for* $0 \leq t \leq \pi$ ✓

(d) $\vec{x}(t) = (-\cos t, \sin t)$, *for* $0 \leq t \leq \pi$

(e) $\vec{x}(t) = (\sin t, \cos t)$, *for* $\pi/2 \leq t \leq \pi/2$ ✓

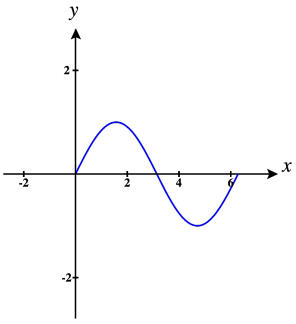
(f) $\vec{x}(t) = (\cos t, \sin t)$, *for* $\pi \leq t \leq 2\pi$ ✓

(g) $\vec{x}(t) = (-\sqrt{1-t^2}, t)$ *for* $-1 \leq t \leq 1$

(h) $\vec{x}(t) = (\sqrt{1-t^2}, t)$ *for* $0 \leq t \leq 1$

(i) $\vec{x}(t) = (t, -\sqrt{1-t^2})$ *for* $-1 \leq t \leq 1$ ✓

Problem 4 *Consider the curve below.*



Which of the following are parametrizations for the curve? Select all that apply.

Select All Correct Answers:

(a) $\vec{x}(t) = (t, \sin t)$ *for* $0 \leq t \leq 2\pi$ ✓

(b) $\vec{x}(t) = (\arcsin t, t)$ *for* $-1 \leq t \leq 1$

(c) $\vec{x}(t) = (2t^2, \sin(2t^2))$ *for* $-1 \leq t \leq 1$ ✓

(d) $\vec{x}(t) = \left(t, \cos\left(\frac{\pi/2}{(}t - 1)\right)\right)$ *for* $0 \leq t \leq 2$

(e) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t}\right)\right)$ *for* $0 \leq t \leq 2$

(f) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{(}t - 1)\right)\right)$ *for* $0 \leq t \leq 2$

(g) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{(}t - 1)\right)\right)$ *for* $0 \leq t \leq 4$ ✓

Problem 5 *Consider the path* $\vec{x}(t) = (3\cos(t), -2\sin(t))$, *for* $t \in \mathbb{R}$.

Compute the velocity of $\vec{x}$.

$$\vec{v}(t) = \boxed{(-3\sin(t), -2\cos(t))}$$

Compute the speed of $\vec{x}$.

$$\|\vec{x}'(t)\| = \boxed{\sqrt{13}}$$

Compute the acceleration of $\vec{x}$.

$$\vec{a}(t) = \boxed{(-3\cos(t), 2\cos(t))}$$

Problem 6 *Two ants are running on the top of a table. Their paths are described by*

$$\vec{x}(t) = (t^2 + 1, 2t - 1)$$

and

$$\vec{y}(t) = (\sqrt{t+3}, t),$$

with coordinates in inches, for $t \geq 0$ *in seconds.*

At what time do the ants collide?

$$t = \boxed{1}$$

Where do the ants collide?

$$(x, y) = \boxed{(2, 1)}$$

Problem 7 *Consider the curve* $\vec{x}(t) = (4t + 2, 1 - 3t)$ *for* $t \in \mathbb{R}$.

Compute the velocity.

$$\vec{v}(t) = \boxed{(4, -3)}$$

Compute the speed.

$$\|\vec{x}'(t)\| = \boxed{4}$$

Compute the acceleration.

$$\vec{a}(t) = \boxed{(0, 0)}$$

Problem 8 *Consider the curve* $\vec{x}(t) = (2\cos t, 5\sin t, t^2)$ *for* $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(-2\sin t, 5\cos t, 2t)}$$

Find the velocity when $t = \pi$.

$$\vec{v}(\pi) = \boxed{(0, -5, 2\pi)}$$

Find a parametrization for the tangent line to $\vec{x}$ *at the point where* $t = \pi$, *so that* $L(0) = \vec{x}(\pi)$.

$$L(t) = \boxed{(-2, 5, \pi^2) + t(0, -5, 2\pi)}$$

Problem 9 *Consider the curve* $\vec{x}(t) = (t, t^2, t^3)$ *for* $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(1, 2t, 3t^2)}$$

Find the velocity when $t = 2$.

$$\vec{v}(\pi) = \boxed{(1, 4, 12)}$$

Find a parametrization for the tangent line to $\vec{x}$ *at the point where* $t = 2$, *so that* $L(0) = \vec{x}(2)$.

$$L(t) = \boxed{(2, 4, 8) + t(1, 4, 12)}$$

Problem 10 *Consider the curve* $\vec{x}(t) = (t, te^t, e^{t^2})$ *for* $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(1, e^t + te^t, 2te^{t^2})}$$

Find the velocity when $t = 0$.

$$\vec{v}(\pi) = \boxed{(1, 1, 0)}$$

Find a parametrization for the tangent line to $\vec{x}$ *at the point where* $t = 0$, *so that* $L(0) = \vec{x}(0)$.

$$L(t) = \boxed{(0, 0, 1) + t(1, 1, 0)}$$

Written Problems

Problem 11 (a) *Graph the surface* $z^2 = x^2 + y^2$ *and the curve* $\vec{x}(t) = (t\cos(t), t\sin(t), t)$ *for* $-5 \leq t \leq 5$.

(b) *Verify algebraically that the curve lies on the surface.*

Problem 12 *Prove the following product rule for cross products.*

Let $\vec{x}$ *and* $\vec{y}$ *be paths in* $\mathbb{R}^3$*, then*

$$(\vec{x} \times \vec{y})'(t) = \vec{x}'(t) \times \vec{y}(t) + \vec{x} \times \vec{y}'(t),$$

for t *such that* $x'(t)$ *and* $y'(t)$ *exist.*

Professional Problem

Problem 13 (a) *Let* $\vec{x}(t)$ *be a curve lying on a sphere in* $\mathbb{R}^n$ *of radius* C*. Prove that* $\vec{x}(t)$ *and* $\vec{x}'(t)$ *are perpendicular.*

(b) *For the curve* $\vec{x}(t) = (\cos(t), \sin^2(t), \cos(t)\sin(t))$*, verify computationally that* $\vec{x}(t)$ *lies on the sphere, and that* $\vec{x}'(t)$ *is perpendicular to* $\vec{x}(t)$*.*
