

## Practice Problems

### Online Problems

**Problem 1** Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  given by

$$f(x, y) = x^2 + 4y^2 - 2.$$

What is the domain of  $f$ ?

**Multiple Choice:**

- (a)  $\mathbb{R}$
- (b)  $\mathbb{R} \setminus \{0\}$
- (c)  $[0, \infty)$
- (d)  $(0, \infty)$
- (e)  $\mathbb{R}^2$  ✓
- (f)  $\mathbb{R}^2 \setminus \{(0, 0)\}$

What is the range of  $f$ ?

$$\text{Range } f = \boxed{[-2, \infty)}$$

Is  $f$  onto?

**Multiple Choice:**

- (a) yes
- (b) no ✓

**Problem 1.1** We would like to restrict the codomain of the function  $f$  so that it becomes onto. We'll describe our new codomain as the set of numbers  $a$  in  $\mathbb{R}$  such that some condition holds. Which condition gives us the largest possible codomain such that  $f$  is onto?

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Learning outcomes:  
Author(s):

**Multiple Choice:**

- (a)  $a \in \mathbb{R}$
  - (b)  $a \geq 0$
  - (c)  $a > 0$
  - (d)  $a \neq 0$
  - (e)  $a = 0$
  - (f)  $a \geq 2$
  - (g)  $a > 2$
  - (h)  $a \neq 2$
  - (i)  $a = 2$
  - (j)  $a \geq -2$  ✓
  - (k)  $a > -2$
  - (l)  $a \neq -2$
  - (m)  $a = -2$
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Is  $f$  one-to-one?

**Multiple Choice:**

- (a) yes
- (b) no ✓

**Problem 1.2** We would like to restrict the domain of the function  $f$ , so that it becomes one-to-one. We'll describe our new domain as the set of points  $(x, y)$  in  $\mathbb{R}^2$  such that some condition(s) hold. Which condition(s) give us the largest possible domain such that  $f$  is one-to-one?

**Select All Correct Answers:**

- (a)  $x \neq 0$
- (b)  $x \geq 0$  ✓
- (c)  $x > 0$

- (d)  $y \neq 0$   
 (e)  $y \geq 0$  ✓  
 (f)  $y > 0$
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**Problem 2** Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the function defined by

$$f(\vec{x}) = 3\vec{x} + \mathbf{i} - 2\mathbf{j}.$$

Find the component functions of  $f$  in terms of  $x$ ,  $y$ , and  $z$ .

$$f_1(x, y, z) = \boxed{3x + 1}$$

$$f_2(x, y, z) = \boxed{3y - 2}$$

$$f_3(x, y, z) = \boxed{3z}$$


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**Problem 3** Consider the linear function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by  $f(\vec{x}) = A\vec{x}$ , where

$$A = \begin{pmatrix} 1 & 5 & 2 \\ -2 & 0 & 1 \end{pmatrix},$$

and  $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$

- (a) Determine the component functions of  $f$  in terms of  $x_1$ ,  $x_2$ , and  $x_3$ .

$$f_1(x_1, x_2, x_3) = \boxed{x_1 + 5x_2 + 2x_3}$$

$$f_2(x_1, x_2, x_3) = \boxed{-2x_1 + x_3}$$

- (b) Is  $f$  one-to-one?

**Multiple Choice:**

- (i) Yes  
 (ii) No ✓  
 (c) Is  $f$  onto?

**Multiple Choice:**

- (i) Yes ✓
  - (ii) No
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**Problem 4** Consider the function

$$f(x, y) = xy.$$

What is the shape of the level curve at height 0 of  $f$ ?

**Multiple Choice:**

- (a) Empty
- (b) A single line
- (c) Two intersecting lines ✓
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola

What is the shape of the level curve at height 1 of  $f$ ?

**Multiple Choice:**

- (a) Empty
- (b) A single line
- (c) Two intersecting lines
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola ✓

What is the shape of the level curve at height  $-1$  of  $f$ ?

**Multiple Choice:**

- (a) Empty
- (b) A single line
- (c) Two intersecting lines
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola ✓

What is the shape of the level curve at height  $2$  of  $f$ ?

**Multiple Choice:**

- (a) Empty
- (b) A single line
- (c) Two intersecting lines
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola ✓

Which of the following is the graph of  $f$ ?

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**Problem 5** Consider the function

$$f(x, y) = |x|.$$

What is the shape of the level curve at height  $0$  of  $f$ ?

**Multiple Choice:**

- (a) *Empty*
- (b) *A single line* ✓
- (c) *Two intersecting lines*
- (d) *Two parallel lines*
- (e) *Circle*
- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

What is the shape of the level curve at height 1 of  $f$ ?

**Multiple Choice:**

- (a) *Empty*
- (b) *A single line*
- (c) *Two intersecting lines*
- (d) *Two parallel lines* ✓
- (e) *Circle*
- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

What is the shape of the level curve at height  $-1$  of  $f$ ?

**Multiple Choice:**

- (a) *Empty* ✓
- (b) *A single line*
- (c) *Two intersecting lines*
- (d) *Two parallel lines*
- (e) *Circle*

- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

What is the shape of the level curve at height 2 of  $f$ ?

**Multiple Choice:**

- (a) *Empty*
- (b) *A single line*
- (c) *Two intersecting lines*
- (d) *Two parallel lines* ✓
- (e) *Circle*
- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

Which of the following is the graph of  $f$ ?

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**Problem 6** Which of the following is the graph of the ellipsoid

$$\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 1?$$

Is there a function  $f(x, y)$  such that the graph of  $f$  is the ellipsoid above?

**Multiple Choice:**

- (a) *Yes*
- (b) *No* ✓

**Problem 6.1** Why is this impossible?

**Multiple Choice:**

- (a) *It wouldn't be one-to-one.*

- (b) *It wouldn't be onto.*
- (c) *There would be multiple inputs with the same output.*
- (d) *A single input would need to have two outputs. ✓*

**Problem 7** Classify the quadric surface defined by the equation

$$x^2 + 4y^2 + z^2 + 8y = 0.$$

**Multiple Choice:**

- (a) *Ellipsoid ✓*
- (b) *Elliptic Paraboloid*
- (c) *Hyperbolic Paraboloid*
- (d) *Elliptic Cone*
- (e) *Hyperboloid of One Sheet*
- (f) *Hyperboloid of Two Sheets*

It is centered at  $(0, -1, 0)$ .

**Problem 7.1** Which of the following is the graph of the quadric surface given above?

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**Problem 8** Classify the quadric surface defined by the equation

$$2x^2 + 2y^2 - 8y - z + 4 = 0$$

**Multiple Choice:**

- (a) *Ellipsoid*
- (b) *Elliptic Paraboloid ✓*



- (c) *Hyperbolic Paraboloid*
- (d) *Elliptic Cone*
- (e) *Hyperboloid of One Sheet*
- (f) *Hyperboloid of Two Sheets*

It is centered at  $\boxed{(0, 2, -4)}$ .

**Problem 8.1** Which of the following is the graph of the quadric surface given above?

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## Written Problems

**Problem 9** Consider the function

$$f(x, y, z) = \frac{4}{\sqrt{9 - x^2 - y^2 - z^2}}.$$

- (a) What is the domain of  $f$ ? Describe this domain as a region in  $\mathbb{R}^3$ .
- (b) What is the range of  $f$ ?

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**Problem 10** Consider the function

$$f(x) = x^2 + y^2 - 4.$$

- (a) Draw at least five level curves of  $f$ .
- (b) Use these level curves to sketch the graph of  $f$ .

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**Problem 11** Draw the graph of the surface in  $\mathbb{R}^3$  determined by the equation

$$x = y^2/4 - z^2/9.$$

Use level curves and/or sections to justify why your drawing is correct.

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