

## Practice Problems

### Online Problems

**Problem 1** Complete the statement of the Fundamental Theorem of Line Integrals.

Let  $f : X \rightarrow \mathbb{R}$  be  $C^1$ , where  $X \subset \mathbb{R}^n$  is open and connected. Then if  $C$  is any piecewise  $C^1$  curve from  $\mathbf{A}$  to  $\mathbf{B}$ , then

$$\int_C \nabla f \cdot d\mathbf{r} = \boxed{f(\mathbf{B}) - f(\mathbf{A})}$$

**Problem 2** Complete the theorem statement.

Let  $F : X \rightarrow \mathbb{R}^n$  be a  $C^1$  vector field, where  $X \subset \mathbb{R}^n$  is open and connected. If  $\mathbf{F}$  is conservative, then  $D\mathbf{F}$  is symmetric.

**Problem 3** Complete the definition.

A continuous vector field is called pathindependent if  $\int_C \mathbf{F} \cdot d\mathbf{s} = \int_D \mathbf{F} \cdot d\mathbf{s}$  for any two simple, piecewise  $C^1$ , oriented curves  $C$  and  $D$  with the same start and end points.

### Written Problems

**Problem 4** Sketch both of the following sets (in two different pictures) and determine whether they are open, closed, neither, or both and whether they are disconnected, connected, and/or simply connected. State all which apply and justify your answer using appropriate definitions and theorems.

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Learning outcomes:  
Author(s):

*Practice Problems*

(a)  $A = \{x^2 + y^2 < 1\} \cup \{x = 0\} \subset \mathbb{R}^2$

(b)  $B = \{x^2 + y^2 = 1\} \cup \{x = 0\} \subset \mathbb{R}^2$

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