Practice Problems

Online Problems

Problem 1 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (x, y, z)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Problem 2 *Compute the 2-dimensional curl of the vector field* $\mathbf{F}(x, y) = (xy^2, y\sin(x^2))$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 2xy(\cos(x^2) - 1))}$$

Problem 3 *Compute the 2-dimensional curl of the vector field* $\mathbf{F}(x, y) = (e^{xy}, \cos(y))$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, -xe^{xy})}$$

Problem 4 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (y, z, x)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(-1, -1, -1)}$$

Learning outcomes:
Author(s):

Problem 5 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (yz, xz, xy)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Problem 6 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (\sin(x), \cos(y), xy)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(x, -y, 0)}$$

Problem 7 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (0, x^3, \sin(x+y)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(\cos(x+y), -\cos(x+y), 3x^2)}$$

Problem 8 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (e^{xy}, \cos(z), e^y)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(e^y + \sin(z), 0, -xe^{xy})}$$

Problem 9 *Compute the 2-dimensional curl of the vector field* $\mathbf{F}(x, y) = (y, x)$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Problem 10 *Compute the 2-dimensional curl of the vector field* $\mathbf{F}(x, y) = (y, -x)$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, -2)}$$

Problem 11 *Compute the 2-dimensional curl of the vector field* $\mathbf{F}(x,y) = (x^2y + 3xy, xy^4 - 2x^2y)$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{F}(x,y,z) = \boxed{(0, 0, -x^2 - 3x - 4xy + y^4)}$$

Problem 12 *Compute the curl of the vector field* $\mathbf{F}(x,y,z) = (yz, 2xz, 3xy)$.

$$\nabla \times \mathbf{F} = \boxed{(x, -2y, z)}$$

Find the curl of $\mathbf{F}$ *at the point* $(x,y,z) = (0,0,0)$.

$$\nabla \times \mathbf{F} = \boxed{(0,0,0)}$$

Is $\mathbf{F}$ *irrotational?*

Multiple Choice:

(a) *Yes.*

(b) *No.* ✓

(c) *Not enough information.*

Problem 13 *Compute the curl of the vector field* $\mathbf{F}(x,y,z) = (x^2, y^3, z^4)$.

$$\nabla \times \mathbf{F} = \boxed{(0,0,0)}.$$

Find the curl of $\mathbf{F}$ *at the point* $(x,y,z) = (1,2,3)$.

$$(\nabla \times \mathbf{F})(1,2,3) = \boxed{(0,0,0)}.$$

Is $\mathbf{F}$ *irrotational?*

Multiple Choice:

(a) *Yes.* ✓

(b) *No.*

(c) *Not enough information.*

Problem 14 *Compute the two-dimensional curl of the vector field* $\mathbf{F}(x, y) = (-xy, xy)$.

$$\nabla \times \mathbf{F} = \left(0, 0, \boxed{x + y}\right)$$

Describe the local rotation of $\mathbf{F}$ *at the point* $(1, 1)$.

Multiple Choice:

(a) *Counterclockwise.* ✓

(b) *Clockwise.*

(c) *No rotation.*

Describe the local rotation of $\mathbf{F}$ *at the point* $(-1, 1)$.

Multiple Choice:

(a) *Counterclockwise.*

(b) *Clockwise.*

(c) *No rotation.* ✓

Describe the local rotation of $\mathbf{F}$ *at the point* $(-1, -1)$.

Multiple Choice:

(a) *Counterclockwise.*

(b) *Clockwise.* ✓

(c) *No rotation.*

Problem 15 *Find the 2-dimensional curl of the vector field* $\mathbf{F}(x, y) = (-y, x)$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 2)}$$

Is this vector field irrotational?

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Describe the direction of the local rotation of this vector field.

Multiple Choice:

(a) *Clockwise*

(b) *Counterclockwise* ✓

(c) *No rotation*

Plot this vector field. INCLUDE GRAPHING STUFF

Find the 2-dimensional curl of the vector field $\mathbf{G}(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{-y}{x^2+y^2}\right)$. *Your answer should be given as a vector.*

$$\nabla \times \mathbf{G} = \boxed{(0,0,0)}$$

Is this vector field irrotational?

Multiple Choice:

(a) *Yes* ✓

(b) *No*

Describe the direction of the local rotation of this vector field.

Multiple Choice:

(a) *Clockwise*

(b) *Counterclockwise*

(c) *No rotation* ✓

Plot this vector field. INCLUDE GRAPHING STUFF

Although the vector field **F** *and* **G** *have the same flow lines, we see that one is irrotational and the other is not. Why does this happen?*

Multiple Choice:

(a) *Math is broken.*

(b) *Curl describes local rotation, not global rotation.* ✓

Problem 16 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (-2y\cos(3x), 3x\sin(-2y), 0)$.

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 2\cos(3x) - 3\sin(2y)}$$

Find the curl of $\mathbf{F}$ *at the point* $(x, y, z) = (\pi, \pi, \pi)$.

$$(\nabla \times \mathbf{F})(\pi, \pi, \pi) = \boxed{(0, 0, -2}$$

Is $\mathbf{F}$ *a conservative vector field?*

Multiple Choice:

(a) *Yes.*

(b) *No.* ✓

(c) *Not enough information.*

Justify your answer.

Free Response:

Problem 17 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (ze^{xz} + z\sin(x), xe^{xy}, -\cos(x))$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, x^2e^{xz}, xye^{xy} + e^{xy})}$$

Is this vector field conservative?

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Problem 18 *Compute the curl of the vector field* $\mathbf{F}(x, y) = (2x - y, -x + 4y)$.

$$\nabla \times \mathbf{F} = (0, 0, \boxed{0})$$

Is $\mathbf{F}$ *conservative?*

Multiple Choice:

(a) *Yes.* ✓

(b) *No.*

Problem 18.1 *Find a potential function f for $\mathbf{F}$, so that $\nabla f = \mathbf{F}$.*

$$f(x, y) = \boxed{x^2 - xy + 2y^2}$$

Problem 19 *Compute the curl of the vector field $\mathbf{F}(x, y) = (2y, 3x)$.*

$$\nabla \times \mathbf{F} = (0, 0, \boxed{1})$$

Is $\mathbf{F}$ conservative?

Multiple Choice:

(a) *Yes.*

(b) *No.* ✓

Problem 20 *Compute the curl of the vector field $\mathbf{F}(x, y) = (2x, 3y)$.*

$$\nabla \times \mathbf{F} = (0, 0, \boxed{0})$$

Is $\mathbf{F}$ conservative?

Multiple Choice:

(a) *Yes.* ✓

(b) *No.*

Problem 20.1 *Find a potential function f for $\mathbf{F}$, so that $\nabla f = \mathbf{F}$.*

$$f(x, y) = \boxed{x^2 + \frac{3}{2}y^2}$$

Problem 21 *Compute the curl of the vector field* $\mathbf{F}(x,y) = (-4x + y\cos(x), \sin(x))$.

$$\nabla \times \mathbf{F} = (0, 0, \boxed{0})$$

Is $\mathbf{F}$ *conservative?*

Multiple Choice:

(a) *Yes.* ✓

(b) *No.*

Problem 21.1 *Find a potential function* f *for* $\mathbf{F}$, *so that* $\nabla f = \mathbf{F}$.

$$f(x,y) = \boxed{-2x^2 + y\sin(x)}$$

Problem 22 *Compute the curl of the vector field* $\mathbf{F}(x,y,z) = (\sin(x), y^2, e^z)$.

$$\nabla \times \mathbf{F} = \boxed{(0,0,0)}$$

Is $\mathbf{F}$ *conservative?*

Multiple Choice:

(a) *Yes.* ✓

(b) *No.*

Problem 22.1 *Find a potential function* f *for* $\mathbf{F}$, *so that* $\nabla f = \mathbf{F}$.

$$f(x,y,z) = \boxed{-\cos(x) + \frac{1}{2}y^2 + e^z}$$

Problem 23 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (x, y, z)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Is this vector field conservative?

Multiple Choice:

(a) *Yes* ✓

(b) *No*

Problem 23.1 *Find a potential function* f *for* $\mathbf{F}$, *so that* $\nabla f = \mathbf{F}$.

$$f(x, y, z) = \boxed{\frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2}$$

Problem 24 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (z, 4yz, x^2+3y)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(3 - 4y, 1 - 2x, 0)}$$

Is this vector field conservative?

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Problem 25 *Compute the curl of the vector field* $\mathbf{F}(x, y, z) = (ye^{xy}+z\cos(x), xe^{xy}, \sin(x))$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Is this vector field conservative?

Multiple Choice:

(a) *Yes* ✓

(b) *No*

Problem 25.1 *Find a potential function f for $\mathbf{F}$, so that $\nabla f = \mathbf{F}$.*

$$f(x, y, z) = \boxed{e^{xy} + \sin(x)z}$$

Written Problems

Problem 26 *Prove that, for C^1 vector fields $\mathbf{F}$ and $\mathbf{G}$ in $\mathbf{R}^3$,*

$$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}.$$

(This shows that the curl is an additive operator, *so the curl of a sum is the sum of the curls.)*

Problem 27 *Let $\mathbf{F}$ and $\mathbf{G}$ be vector fields in $\mathbb{R}^3$, and let a and b be real numbers. Prove that*

$$\nabla \times (a\mathbf{F} + b\mathbf{G}) = a(\nabla \times \mathbf{F}) + b(\nabla \times \mathbf{G}).$$

Problem 28 *Let $\mathbf{F}$ be a vector field in $\mathbb{R}^3$, and let $f : \mathbb{R}^3 \to \mathbb{R}$ be a scalar valued function. Prove that*

$$\nabla \times (f\mathbf{F}) = f(\nabla \times \mathbf{F}) + (\nabla f) \times \mathbf{F}.$$

Problem 29 *Let $\mathbf{F}(x, y, z) = \dfrac{c\mathbf{r}}{\|\mathbf{r}\|^3}$, where c is constant and $\mathbf{r} = (x, y, z)$. Prove that the curl of $\mathbf{F}$, $\nabla \times \mathbf{F}$, is zero.*

Problem 30 *Let* $\mathbf{F}(x, y, z) = (x, y, z)$*, and let* $\mathbf{a} \in \mathbb{R}^3$ *be a vector with constant entries. Prove that*

$$\nabla \times (\mathbf{a} \times \mathbf{F}) = 2\mathbf{a}.$$

Problem 31 (a) *Compute the curl of the vector field* $\mathbf{F} = (x, y, z)$*. Explain why your answer makes sense geometrically.*

(b) *Suppose we have a* C^1 *vector field* $\mathbf{F}(x, y, z) = (f(x), g(y), h(z))$*. Compute the curl of* $\mathbf{F}$*, and explain why your answer makes sense geometrically.*

Problem 32 (a) *Consider the function* $f(x, y, z) = e^x \sin(y) + z^2 \cos(y)$*. Compute* ∇f*, and verify that* $\nabla \times (\nabla f) = \mathbf{0}$*.*

(b) *Prove that for any* C^2 *function* $f : \mathbb{R}^3 \to \mathbb{R}$*, the curl of the gradient of* f *is zero. That is, prove that*

$$\nabla \times (\nabla f) = \mathbf{0}.$$
