Homework 5: Moving Frames and Acceleration

SOME PROBLEMS COPIED FROM HANDOUTS - NOT SURE OF THEIR SOURCE

Completion Packet

Problem 1 *Find the velocity, acceleration and speed of a particle with the position function* $\mathbf{x}(t) = (t, t^2, 2)$. *Sketch the path of the particle* by hand, *and draw the velocity and acceleration vectors on your graph for* $t = 1$.

Problem 2 *Find the velocity, acceleration and speed of a particle with the position function* $\mathbf{x}(t) = (t, 2\cos t, \sin t)$. *Sketch the path of the particle* by hand, *and draw the velocity and acceleration vectors on your graph for* $t = 0$.

Problem 3 *Find the moving frame for the path* $\vec{x}(t) = (2t, 6\sin(3t), -6\cos(3t))$.

Problem 4 *Find the the acceleration of the path* $\vec{x}(t) = (t, t^2)$, *and express it as a linear combination of the unit tangent vector and the unit normal vector.*

Problem 5 *Find the the acceleration of the path* $\vec{x}(t) = (e^t, t)$, *and express it as a linear combination of the unit tangent vector and the unit normal vector.*

Problem 6 *Suppose we have a path* $\vec{x}(t)$ *and a point* $\vec{c}$ *in* $\mathbb{R}^3$ *such that* $\|\vec{x}(t) - \vec{c}\| = R$ *for some constant* R *and for all* t.

(a) *Geometrically, what can you say about this path?*

Learning outcomes:
Author(s):

(b) *Prove that* $\vec{v}(t)$ *is perpendicular to* $\vec{x}(t) - \vec{c}$.

(c) *If* $\vec{x}(t)$ *has constant speed, show that* $\kappa(t)$ *is nonzero for all* t.

Problem 7 *Prove that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.*

Problem 8 *Find the tangential and normal components of the acceleration vector for* $\mathbf{x}(t) = (1 + t, t^2 - 2t)$.

Problem 9 *Find the tangential and normal components of the acceleration vector for* $\mathbf{x}(t) = (t, t^2, 3t)$.

Problem 10 *Compute the following limits. Show all work. ("Wolfram Alpha" is not work.)*

(a) $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

(b) $\lim_{x \to \pi^-} \ln(\sin x)$

(c) $\lim_{v \to 4^+} \dfrac{4 - v}{|4 - v|}$

(d) $\lim_{x \to \infty} (\sqrt{x^2 + 4x + 1} - x)$

(e) $\lim_{x \to \infty} e^{x - x^2}$

(f) $\lim_{x \to 0} \dfrac{\sin 2x}{x}$

Graded Problems

Problem 11 *Find the moving frame for the path* $\vec{x}(t) = (e^t \cos(t), e^t \sin(t), 5)$.

Problem 12 *Prove that*

$$\frac{|(\vec{v}\times\vec{a})\cdot\vec{a}'|}{\|\vec{v}\times\vec{a}\|^2}=\frac{\left\|d\vec{B}/dt\right\|}{|ds/dt|}.$$

Professional Problem

Problem 13 (a) *Beginning with the formula* $\mathbf{a}=s''\mathbf{T}+(s')^2\kappa\mathbf{N}$, *cross both sides with the velocity vector* $\mathbf{v}=s'\mathbf{T}$ *and derive the formula*

$$\kappa=\frac{||\mathbf{v}\times\mathbf{a}||}{(s')^3}.$$

Make sure to write your solution in your own words!

(b) *Let* $y=f(x)$. *The graph of* $y=f(x)$ *is therefore a plane curve. Assuming* $f\in C^2$ *(that is, it is twice differentiable and its second derivatives are continuous), use the formula from (a) to prove the following well-known formula for the curvature of such a curve:*

$$\kappa(x)=\frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}.$$
