Practice Problems

Online Problems

Problem 1 *Complete the statement of the Fundamental Theorem of Line Integrals.*

Let $f : X \to \mathbb{R}$ *be* C^1*, where* $X \subset \mathbb{R}^n$ *is* $\boxed{open}$ *and* $\boxed{connected}$*. Then if* C *is any piecewise* C^1 *curve from* $\mathbf{A}$ *to* $\mathbf{B}$*, then*

$$\int_C \nabla f \cdot d\sim = \boxed{f(B) - f(A)}$$

Problem 2 *Complete the theorem statement.*

Let $F : X \to \mathbb{R}^n$ *be a* C^1 *vector field, where* $X \subset \mathbb{R}^n$ *is* $\boxed{open}$ *and* $\boxed{connected}$*. If* $\mathbf{F}$ *is conservative, then* $D\mathbf{F}$ *is symmetric.*

Problem 3 *Complete the definition.*

A continuous vector field is called $\boxed{pathindependent}$ *if* $\int_C \mathbf{F} \cdot d\mathbf{s} = \int_D \mathbf{F} \cdot d\mathbf{s}$ *for any two simple, piecewise* C^1*, oriented curves* C *and* D *with the same start and end points.*

Written Problems

Problem 4 *Sketch both of the following sets (in two different pictures) and determine whether they are open, closed, neither, or both and whether they are disconnected, connected, and/or simply connected. State all which apply and justify your answer using appropriate definitions and theorems.*

Learning outcomes:
Author(s):

(a) $A = \{x^2 + y^2 < 1\} \cup \{x = 0\} \subset \mathbb{R}^2$

(b) $B = \{x^2 + y^2 = 1\} \cup \{x = 0\} \subset \mathbb{R}^2$