# **Practice Problems**

## Online Problems

**Problem 1** Find the Cartesian coordinates of each point, which is given in polar coordinates.

$$(r,\theta)=(2,\pi/6)$$

$$(x,y) = \boxed{(\sqrt{3},1)}$$

$$(r,\theta) = (\sqrt{2}, 3\pi/4)$$

$$(x,y) = \boxed{(-1,1)}$$

$$(r,\theta) = (1,0)$$

$$(x,y) = \boxed{(1,0)}$$

$$(r,\theta) = (3,\pi)$$

$$(x,y) = \boxed{(-3,0)}$$

**Problem 2** Find the polar coordinates of each point, which is given in Cartesian coordinates. Your answers should satisfy  $0 \le r$  and  $0 \le \theta < 2\pi$ .

$$(x,y) = (\sqrt{3},0)$$

$$(r,\theta) = \boxed{(\sqrt{3},0)}$$

$$(x,y) = (0,2)$$

$$(r,\theta) = \boxed{(2,\pi/2)}$$

$$(x,y) = (-1,-1)$$

$$(r,\theta) = \sqrt{(\sqrt{2}, 5\pi/4)}$$

$$(x,y) = (1,\sqrt{3})$$

$$(r,\theta) = \boxed{(2,\pi/3)}$$

**Problem 3** Find the Cartesian coordinates of the point  $(\pi/2, \pi, 2)$ , given in cylindrical coordinates.

$$(x,y,z) = \boxed{(0,-\pi/2,2)}$$

**Problem 4** Find cylindrical coordinates for the point (0,-1,3), written in Cartesian coordinates. Your answer should satisfy  $0 \le r$  and  $0 \le \theta < 2\pi$ .

$$(r, \theta, z) = \boxed{(1, \pi, 3)}$$

Problem 5 Consider the surface described in Cartesian coordinates by

$$2z^2 = x^2 + y^2$$
.

Describe this surface with an equation in cylindrical coordinates, of the form  $0 = f(r, \theta, z)$ .

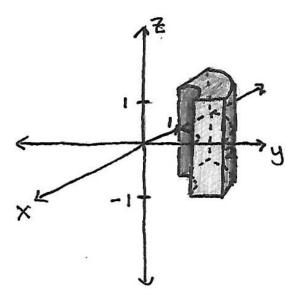
$$0 = \boxed{r^2 - 2z^2}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

#### Multiple Choice:

- (a) Plane
- (b) Cylinder
- (c) Sphere
- (d) Cone ✓
- (e) Other

**Problem 6** Consider the following region in  $\mathbb{R}^3$ .



This region is the set of points  $(r, \theta, z)$ , in cylindrical coordinates, satisfying the inequalities

$$\boxed{1} \le r \le \boxed{2}$$

$$\boxed{1} \leq r \leq \boxed{2}$$
 
$$\boxed{\pi/2} \leq \theta \leq \boxed{\pi}$$
 
$$\boxed{-1} \leq z \leq \boxed{1}$$

$$-1 \le z \le \boxed{1}$$

Problem 7 For each of the following equations in cylindrical coordinates, select the type of shape they define.

FIGURE OUT CORRECT ANSWERS

 $r = \cos \theta$ 

## Multiple Choice:

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

 $z=r\cos\theta$ 

Multiple Choice:

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

$$z = -r$$

Multiple Choice:

- (a) plane ✓
- (b) cylinder
- (c) sphere
- (d) other

Problem 8 Find the Cartesian coordinates of each point, given in cylindrical coordinates.

$$(r,\theta,z) = (1,1,1)$$

$$(x,y,z) = \boxed{(\cos(1),\sin(1),1)}$$

$$(r,\theta,z)=(\pi,\pi,\pi)$$

$$(x,y,z) = \boxed{(-\pi,0,\pi)}$$

$$(r, \theta, z) = (2, 4\pi/3, -2)$$

$$(x, y, z) = (-\sqrt{3}, -1, -2)$$

Problem 9 Find the cylindrical coordinates of each point, given in Cartesian coordinates. Your answers should satisfy  $r \ge 0$  and  $0 \le \theta < 2\pi$ .

$$(x, y, z) = (1, 1, 1)$$

$$(r,\theta,z) = \boxed{(\sqrt{2},\pi/4,1)}$$

$$(x, y, z) = (\pi, \pi, \pi)$$

$$(r, \theta, z) = \boxed{\left(\sqrt{2}, \pi/4, 1\right)}$$
  
 $(r, \theta, z) = \boxed{\left(\sqrt{2}\pi, \pi/4, \pi\right)}$ 

$$(x, y, z) = (2, 2\sqrt{3}, -2)$$
 
$$(r, \theta, z) = \boxed{(4, \pi/6, -2)}$$

**Problem 10** Find the Cartesian coordinates of the point  $(2, \pi, \pi/2)$ , given in spherical coordinates.

$$(x, y, z) = (-2, 0, 0)$$

**Problem 11** Find spherical coordinates for the point  $\left(-\sqrt{2},\sqrt{2},2\sqrt{3}\right)$ , written in Cartesian coordinates. Your answer should satisfy  $0 \le \rho$ ,  $0 \le \theta \le 2\pi$ , and  $0 \le \phi \le \phi$ .

$$(\rho,\theta,\phi) = \boxed{(4,3\pi/4,\pi/6)}$$

**Problem 12** Consider the surface described in Cartesian coordinates by

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in spherical coordinates, of the form  $0 = f(\rho, \theta, \phi)$ .

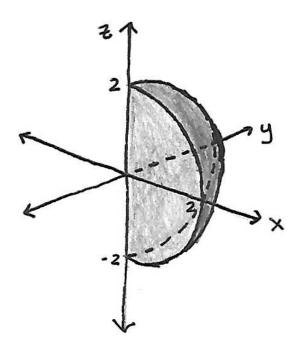
$$0 = \rho^2 \sin^2 \phi - 2 \cos^2 \phi$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

#### Multiple Choice:

- (a) Plane
- (b) Cylinder
- (c) Sphere
- (d) Cone ✓
- (e) Other

**Problem 13** Consider the following region in  $\mathbb{R}^3$ .



This region is the set of points  $(\rho, \theta, \phi)$ , in spherical coordinates, satisfying the in equalities

$$0 \le \rho \le 2$$

$$\boxed{0 \le \rho \le \boxed{2}}$$

$$\boxed{0 \le \theta \le \boxed{pi/2}}$$

$$\boxed{0} \le \phi \le \boxed{pi}$$

Problem 14 For each of the following equations in spherical coordinates, select the type of shape they define.

FIGURE OUT CORRECT ANSWERS

 $\rho = \cos \phi$ 

## Multiple Choice:

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

$$\rho = \sin \theta$$

 ${\it Multiple~Choice:}$ 

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

 $\rho\cos\theta\sin\phi = 1$ 

#### Multiple Choice:

- (a) plane ✓
- (b) cylinder
- (c) sphere
- (d) other

**Problem 15** Find the spherical coordinates of each point, given in Cartesian coordinates. Your answers should satisfy  $0 \le r$ ,  $0 \le \theta < 2\pi$ , and  $0 \le \phi \le \pi$ .

$$(x, y, z) = (1, 1, 1)$$

$$(\rho, \theta, \phi) = \boxed{(\sqrt{3}, \pi/4, \pi/4)}$$

$$(x, y, z) = (1, -1, -1)$$

$$(\rho, \theta, \phi) = \boxed{(\sqrt{3}, 3\pi/4, 3\pi/4)}$$

$$(x, y, z) = (1, \sqrt{3}, 0)$$

$$(\rho,\theta,\phi) = \boxed{(\sqrt{4},\pi/6,\pi/2)}$$

**Problem 16** Find the Cartesian coordinates of each point, given in spherical coordinates.

$$(\rho, \theta, \phi) = (\pi, \pi, \pi)$$

$$(x,y,z) = \boxed{(0,0,-\pi)}$$

$$(\rho, \theta, \phi) = (3, \pi/2, \pi/4)$$

$$(x, y, z) = (0, 3\sqrt{2}/2, 3\sqrt{2}/2)$$

$$(\rho, \theta, \phi) = (2, 7\pi/6, 3\pi/4)$$

$$(x, y, z) = \sqrt{(-\sqrt{6}/2, -\sqrt{2}/2, -\sqrt{2})}$$

## Written Problems

**Problem 17** For several values of the constant a, sketch the graph of the curve in  $\mathbb{R}^2$  given by the polar equation

$$r = a \sin \theta$$
.

What do you notice about these curves?

**Problem 18** Which points in  $\mathbb{R}^2$  have the same coordinates when written in Cartesian and polar coordinates? (That is, for what points do we have x = r and  $y = \theta$ ?)

**Problem 19** Consider the surface described by  $(r-3)^2 + z^2 = 1$  in cylindrical coordinates, with the restriction  $r \ge 0$ .

- (a) Sketch the intersection of the surface with the half-plane  $\theta = 0$ .
- (b) Sketch the intersection of the surface with the half-plane  $\theta = \frac{\pi}{2}$ .
- (c) Sketch the intersection of the surface with the plane z = 0.
- (d) Sketch the surface.

**Problem 20** Sketch the region in  $\mathbb{R}^3$  with cylindrical coordinates satisfying the inequality

$$r \le z \le 4 - 2r$$

**Problem 21** Convert the following equation, given in cylindrical coordinates, into Cartesian coordinates.

$$r = 0$$

**Problem 22** Sketch the region in  $\mathbb{R}^3$  given by

$$r \le z \le 3$$
$$0 \le \theta \le \pi/2$$

in cylindrical coordinates.

**Problem 23** Sketch the region in  $\mathbb{R}^3$  given by

$$r^2 - 2 \le z \le 2 - r^2$$

in cylindrical coordinates.

**Problem 24** Which points in  $\mathbb{R}^3$  have the same coordinates when written in Cartesian and cylindrical coordinates?

- **Problem 25** (a) Given a function f, consider the graphs of the equations  $r = f(\theta)$  and  $r = 2f(\theta)$ , in polar coordinates. How are these graphs related?
  - (b) Given a function f, consider the graphs of the equations  $\rho = f(\theta, \phi)$  and  $\rho = 2f(\theta, \phi)$ , in spherical coordinates. How are these graphs related?
  - (c) Given a function f, consider the graphs of the equations  $r = f(\theta)$  and  $r = -f(\theta)$ , in polar coordinates. How are these graphs related?
  - (d) Given a function f, consider the graphs of the equations  $\rho = f(\theta, \phi)$  and  $\rho = -f(\theta, \phi)$ , in spherical coordinates. How are these graphs related?

**Problem 26** Sketch the surface in  $\mathbb{R}^3$  given by the equation

$$1-\cos\phi$$

in spherical coordinates.

**Problem 27** Consider the surface in  $\mathbb{R}^3$  given by the equation

$$\rho \sin \phi \sin \theta = 1$$

in spherical coordinates.

Convert this equation to Cartesian coordinates and cylindrical coordinates, and sketch the surface.

**Problem 28** Consider the region in  $\mathbb{R}^3$  consisting of points whose spherical coordinates satisfy

$$1 \le \rho \le 3$$

Sketch this region.

**Problem 29** Consider the region in  $\mathbb{R}^3$  consisting of points whose spherical coordinates satisfy

$$0 \le \phi \le \pi/2$$

$$0 \le \rho \le 1$$

Sketch this region.

**Problem 30** Consider the region in  $\mathbb{R}^3$  consisting of points whose spherical coordinates satisfy

$$\cos \phi \le \rho \le 2$$

Sketch this region.

**Problem 31** Which points in  $\mathbb{R}^3$  have the same coordinates when written in Cartesian and spherical coordinates?