

# Practice Problems

## Online Problems

**Problem 1** Compute the following:

$$(1, 2, 3) + (8, 3, 6) = \boxed{(9, 5, 9)}$$

$$4(1, -2, 4) = \boxed{(4, -8, 16)}$$

$$-12((5, 2, 6) - (8, 2, 4)) = \boxed{(36, 0, 24)}$$

**Problem 2** Let  $h$  be a constant. Compute the following:

$$(7, 2, -1) + (2h, 0, h) = \boxed{(7 + 2h, 2, h - 1)}$$

$$h(1, 8, 2) = \boxed{(h, 8h, 2h)}$$

**Problem 3** For each of the following, determine whether the quantity exists or does not exist.

$$(1, 8, 3, 7) + (-1, 7, 2, 7)$$

**Multiple Choice:**

- (a) *Exists.* ✓
- (b) *Does not exist.*

$$(2, 8, 3) + (1, 7)$$

**Multiple Choice:**

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Learning outcomes:  
Author(s):

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$(2, 7, 3) + 1$$

**Multiple Choice:**

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$(2, 8, 3)(1, 7, 3)$$

**Multiple Choice:**

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$2(7, 2, 3, 7, 2)$$

**Multiple Choice:**

- (a) *Exists.* ✓
- (b) *Does not exist.*

**Problem 4** For points  $P_1 = (2, -3, 7, 1)$  and  $P_2 = (-1, 7, 2, 1)$ , compute the displacement vector  $P_1\vec{P}_2$ .

$$P_1\vec{P}_2 = \boxed{(-3, 10, -5, 0)}$$

**Problem 5** Write the vector  $2\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$  in  $\mathbb{R}^3$  in standard vector notation.

$$2\mathbf{i} - 5\mathbf{j} + 2\mathbf{k} = \boxed{(2, -5, 2)}$$

**Problem 6** Compute the dot product.

$$(1, 8, 3) \cdot (-2, 6, 0) = \boxed{46}$$

**Problem 7** Compute the dot product.

$$(1, -5, 0, 2) \cdot (2, -1, 4, 1) = \boxed{9}$$

**Problem 8** Compute the dot product.

$$(1, 8, 3) \cdot (-3, 0, 1) = \boxed{0}$$

What can you conclude about the vectors?

**Multiple Choice:**

- (a) They're perpendicular. ✓
- (b) They aren't perpendicular.

**Problem 9** Compute the dot product.

$$(1, 2, 3) \cdot (-3, -2, -1) = \boxed{-10}$$

What can you conclude about the vectors?

**Multiple Choice:**

- (a) They're perpendicular.
- (b) They aren't perpendicular. ✓

**Problem 10** Compute the dot product.

$$(1, 8, 3, 6) \cdot (3, -3, -1, 4) = \boxed{-0}$$

What can you conclude about the vectors?

**Multiple Choice:**

- (a) They're perpendicular. ✓  
 (b) They aren't perpendicular.

**Problem 11** For each expression, determine whether it exists or does not exist.

$$(2, 8, 3) \cdot (1, 8, 2, 4)$$

**Multiple Choice:**

- (a) Exists.  
 (b) Does not exist. ✓  
 $(2, 8, 3, 1, 8) \cdot (1, 8, 2, 4, 2)$

**Multiple Choice:**

- (a) Exists. ✓  
 (b) Does not exist.

**Problem 12** Compute the angle between the vectors  $(2, 8)$  and  $(-8, 2)$  in degrees.

$$\theta = \boxed{90}^\circ$$

**Problem 13** Compute the angle between the vectors  $(2, 1, 3, 1, 1)$  and  $(-3, 1, 1, 1, -2)$  in degrees.

$$\theta = \boxed{104.48}^\circ$$

(Give your answer as a positive number to two decimal places.)

**Problem 14** Suppose you have vectors  $\vec{v}$  and  $\vec{w}$  such that  $\|\vec{v}\| = 6$  and  $\|\vec{w}\| = 2$ , and the angle between  $\vec{v}$  and  $\vec{w}$  is  $\frac{\pi}{4}$  radians. Compute the dot product of  $\vec{v}$  and  $\vec{w}$ .

$$\vec{v} \cdot \vec{w} = \boxed{6\sqrt{2}}$$

**Problem 15** Compute the projection of  $\vec{v} = (1, 7, 3)$  onto  $\vec{w} = (-3, -2, 1)$ .

$$\text{proj}_{\vec{w}}(\vec{v}) = \boxed{(7/2, 7/3, -7/6)}$$

**Problem 16** Compute the projection of the vector  $\vec{v} = (1, 2, 3)$  onto the vector  $\vec{w} = (-3, 2, -1)$ .

$$\text{proj}_{\vec{w}}(\vec{v}) = \boxed{(0, 0, 0)}$$

**Problem 16.1** Why does your answer make sense?

**Multiple Choice:**

- (a) The vectors are parallel.
- (b) The vectors are perpendicular. ✓
- (c) They are the same length.

**Problem 17** Compute the cross product.

$$(1, 7, 3) \times (2, 8, -1) = \boxed{(-31, 7, -6)}$$

**Problem 18** Compute the cross product.

$$(-1, 7, 2) \times (2, 8, 3) = \boxed{(5, 7, -22)}$$

**Problem 19** For each of the following, determine whether the expression exists or does not exist.

$$(-1, 7, 3) \times (1, 7, 2)$$

**Multiple Choice:**

- (a) *Exists.* ✓
- (b) *Does not exist.*

$$(-1, 7, 3, 8) \times (1, 7, 2, 0)$$

**Multiple Choice:**

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$(2, 0, 0) \times (1, 7, 2, 0)$$

**Multiple Choice:**

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$(0, 0, 0) \times (0, 0, 0)$$

**Multiple Choice:**

- (a) *Exists.* ✓
- (b) *Does not exist.*

**Problem 20** Compute the area of the parallelogram determined by  $(1, 6)$  and  $(1, 0)$ .

$$\text{Area} = \boxed{6}$$

**Problem 21** Compute the volume of the parallelepiped determined by  $(1, 6, 2)$ ,  $(-1, 2, 0)$ , and  $(0, 3, 1)$ .

$$\text{Volume} = \boxed{2}$$

**Problem 22** Suppose  $\vec{v}$  and  $\vec{w}$  are unit vectors in the  $xy$ -plane, and we know that they are perpendicular. What is  $\vec{v} \times \vec{w}$ ?

**Multiple Choice:**

- (a)  $(0, 0, 1)$
- (b)  $(0, 0, -1)$
- (c) Not enough information. ✓

**Problem 23** Find a parametrization  $\vec{x}(t)$  of the line parallel to the vector  $(1, 6, 3)$  and through the point  $(1, 3, 2)$ , such that  $\vec{x}(0) = (1, 3, 2)$ .  $\vec{x}(t) =$   
 $(t + 1, 6t + 3, 3t + 2)$

**Problem 24** Find a parametrization  $\vec{x}(s, t)$  of the plane containing vectors  $(1, 6, 2)$  and  $(1, 3, 2)$ , and passing through the point  $(1, 0, 0)$ , such that  $\vec{x}(0, 0) = (1, 0, 0)$  and  $\vec{x}(1, 0) = (1, 6, 2)$ .

$$\vec{x}(s, t) = (s + t + 1, 6s + 3t, 2s + 2t)$$

**Problem 25** Give an equation which describes the plane perpendicular to the vector  $(1, 7, 3)$  and through the point  $(-2, 4, 1)$ .

$$0 = (x + 2) + 7(y - 4) + 3(z - 1)$$

## Written Problems

**Problem 26** For any vector  $\vec{v}$  in  $\mathbb{R}^n$ , prove that  $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$ .

*Practice Problems*

**Problem 27** Prove that vectors  $\vec{v}$  and  $\vec{w}$  in  $\mathbb{R}^n$  are perpendicular if and only if  $\text{proj}_{\vec{w}}(\vec{v})$  is the zero vector.

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**Problem 28** For any vector  $\vec{v}$  in  $\mathbb{R}^3$ , prove that  $\vec{v} \times \vec{v}$  is the zero vector.

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