

Homework 7: Partial Derivatives

Completion Packet

Problem 1 Compute the partial derivatives of the function $f(x, y) = ye^{x^2}$.

Problem 2 Compute the partial derivatives of the function $f(x, y) = \ln((x + y)^2)$.

Problem 3 Compute the partial derivatives of the function $f(x, y, z) = \sin(x^2yz)$.

Problem 4 Consider the function $f(x, y) = 2x^2 + xy - y^2 + 3y + 1$.

- (a) Explain how you can tell that this function is differentiable at the point $(2, 1)$.
 - (b) Find an equation for the tangent plane to the graph of f at the point $(2, 1)$.
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Problem 5 Consider the function $f(x, y) = e^{x^2+y^2}$.

- (a) Explain how you can tell that this function is differentiable at the point $(1, 2)$.
 - (b) Find an equation for the tangent plane to the graph of f at the point $(1, 2)$.
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Learning outcomes:
Author(s):

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Problem 6 Suppose we have a differentiable function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $f(0,0) = 5$, $f(0.1,0) = 5.2$, and $f(0,0.1) = 4.7$.

- (a) Estimate $f_x(0,0)$ and $f_y(0,0)$.
 - (b) Give an approximate equation for the tangent plane to the graph of f at the point $(0,0)$.
 - (c) Using your approximation from (b), estimate the value of $f(-0.2,0.2)$.
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Problem 7 Consider the function

$$f(x,y) = \begin{cases} \frac{x^3 + 2x^2y - xy^2 + 3y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

- (a) Compute the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ for $(x,y) \neq (0,0)$.
 - (b) Compute the partial derivatives $f_x(0,0)$ and $f_y(0,0)$.
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Problem 8 Consider the function $f(x,y) = e^{x+y}$.

- (a) Compute the partial derivatives $f_x(0,0)$ and $f_y(0,0)$.
 - (b) Graph the function.
 - (c) Based on your graph, is f differentiable at $(0,0)$?
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Problem 9 Consider the function $f(x,y) = (xy)^{2/3}$.

- (a) Compute the partial derivatives $f_x(1,1)$, $f_y(1,1)$, $f_x(0,0)$, and $f_y(0,0)$.
 - (b) Graph the function.
 - (c) Based on your graph, is f differentiable at $(1,1)$? Is f differentiable at $(0,0)$?
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Graded Problems

Problem 10 Consider the function $f(x, y) = x^2 + y^2$.

- (a) Find an equation for the tangent plane to the graph of $f(x, y)$ at the point (a, b) .
 - (b) At which point(s) (a, b) is the tangent plane parallel to the plane $x + 2y - z = 0$?
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Problem 11 Find the partial derivatives of the function

$$f(x_1, \dots, x_n) = x_1 x_2^2 x_3^3 x_4^4 \cdots x_n^n.$$
