

Practice Problems

Online Problems

Problem 1 Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by

$$f(x, y) = x^2 + 4y^2 - 2.$$

What is the domain of f ?

Multiple Choice:

- (a) \mathbb{R}
- (b) $\mathbb{R} \setminus \{0\}$
- (c) $[0, \infty)$
- (d) $(0, \infty)$
- (e) \mathbb{R}^2 ✓
- (f) $\mathbb{R}^2 \setminus \{(0, 0)\}$

What is the range of f ?

$$\text{Range } f = \boxed{[-2, \infty)}$$

Is f onto?

Multiple Choice:

- (a) yes
- (b) no ✓

Problem 1.1 We would like to restrict the codomain of the function f so that it becomes onto. We'll describe our new codomain as the set of numbers a in \mathbb{R} such that some condition holds. Which condition gives us the largest possible codomain such that f is onto?

Learning outcomes:
Author(s):

Multiple Choice:

- (a) $a \in \mathbb{R}$
 - (b) $a \geq 0$
 - (c) $a > 0$
 - (d) $a \neq 0$
 - (e) $a = 0$
 - (f) $a \geq 2$
 - (g) $a > 2$
 - (h) $a \neq 2$
 - (i) $a = 2$
 - (j) $a \geq -2$ ✓
 - (k) $a > -2$
 - (l) $a \neq -2$
 - (m) $a = -2$
-

Is f one-to-one?

Multiple Choice:

- (a) yes
- (b) no ✓

Problem 1.2 We would like to restrict the domain of the function f , so that it becomes one-to-one. We'll describe our new domain as the set of points (x, y) in \mathbb{R}^2 such that some condition(s) hold. Which condition(s) give us the largest possible domain such that f is one-to-one?

Select All Correct Answers:

- (a) $x \neq 0$
- (b) $x \geq 0$ ✓
- (c) $x > 0$

- (d) $y \neq 0$
 (e) $y \geq 0$ ✓
 (f) $y > 0$
-
-

Problem 2 Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the function defined by

$$f(\vec{x}) = 3\vec{x} + \mathbf{i} - 2\mathbf{j}.$$

Find the component functions of f in terms of x , y , and z .

$$f_1(x, y, z) = \boxed{3x + 1}$$

$$f_2(x, y, z) = \boxed{3y - 2}$$

$$f_3(x, y, z) = \boxed{3z}$$

Problem 3 Consider the linear function $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ given by $f(\vec{x}) = A\vec{x}$, where

$$A = \begin{pmatrix} 1 & 5 & 2 \\ -2 & 0 & 1 \end{pmatrix},$$

and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$

- (a) Determine the component functions of f in terms of x_1 , x_2 , and x_3 .

$$f_1(x_1, x_2, x_3) = \boxed{x_1 + 5x_2 + 2x_3}$$

$$f_2(x_1, x_2, x_3) = \boxed{-2x_1 + x_3}$$

- (b) Is f one-to-one?

Multiple Choice:

- (i) Yes
 (ii) No ✓
 (c) Is f onto?

Multiple Choice:

- (i) Yes ✓
 - (ii) No
-

Problem 4 Consider the function

$$f(x, y) = xy.$$

What is the shape of the level curve at height 0 of f ?

Multiple Choice:

- (a) Empty
- (b) A single line
- (c) Two intersecting lines ✓
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola

What is the shape of the level curve at height 1 of f ?

Multiple Choice:

- (a) Empty
- (b) A single line
- (c) Two intersecting lines
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola ✓

What is the shape of the level curve at height -1 of f ?

Multiple Choice:

- (a) Empty
- (b) A single line
- (c) Two intersecting lines
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola ✓

What is the shape of the level curve at height 2 of f ?

Multiple Choice:

- (a) Empty
- (b) A single line
- (c) Two intersecting lines
- (d) Two parallel lines
- (e) Circle
- (f) Ellipse
- (g) Parabola
- (h) Hyperbola ✓

Which of the following is the graph of f ?

GRAPHS

Problem 5 Consider the function

$$f(x, y) = |x|.$$

What is the shape of the level curve at height 0 of f ?

Multiple Choice:

- (a) *Empty*
- (b) *A single line* ✓
- (c) *Two intersecting lines*
- (d) *Two parallel lines*
- (e) *Circle*
- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

What is the shape of the level curve at height 1 of f ?

Multiple Choice:

- (a) *Empty*
- (b) *A single line*
- (c) *Two intersecting lines*
- (d) *Two parallel lines* ✓
- (e) *Circle*
- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

What is the shape of the level curve at height -1 of f ?

Multiple Choice:

- (a) *Empty* ✓
- (b) *A single line*
- (c) *Two intersecting lines*
- (d) *Two parallel lines*
- (e) *Circle*

- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

What is the shape of the level curve at height 2 of f ?

Multiple Choice:

- (a) *Empty*
- (b) *A single line*
- (c) *Two intersecting lines*
- (d) *Two parallel lines* ✓
- (e) *Circle*
- (f) *Ellipse*
- (g) *Parabola*
- (h) *Hyperbola*

Which of the following is the graph of f ?

GRAPHS

Problem 6 Which of the following is the graph of the ellipsoid

$$\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 1?$$

Is there a function $f(x, y)$ such that the graph of f is the ellipsoid above?

Multiple Choice:

- (a) *Yes*
- (b) *No* ✓

Problem 6.1 Why is this impossible?

Multiple Choice:

- (a) *It wouldn't be one-to-one.*

- (b) *It wouldn't be onto.*
- (c) *There would be multiple inputs with the same output.*
- (d) *A single input would need to have two outputs. ✓*

Problem 7 Classify the quadric surface defined by the equation

$$x^2 + 4y^2 + z^2 + 8y = 0.$$

Multiple Choice:

- (a) *Ellipsoid ✓*
- (b) *Elliptic Paraboloid*
- (c) *Hyperbolic Paraboloid*
- (d) *Elliptic Cone*
- (e) *Hyperboloid of One Sheet*
- (f) *Hyperboloid of Two Sheets*

It is centered at $(0, -1, 0)$.

Problem 7.1 Which of the following is the graph of the quadric surface given above?

GRAPHS

Problem 8 Classify the quadric surface defined by the equation

$$2x^2 + 2y^2 - 8y - z + 4 = 0$$

Multiple Choice:

- (a) *Ellipsoid*
- (b) *Elliptic Paraboloid ✓*

- (c) *Hyperbolic Paraboloid*
- (d) *Elliptic Cone*
- (e) *Hyperboloid of One Sheet*
- (f) *Hyperboloid of Two Sheets*

It is centered at $\boxed{(0, 2, -4)}$.

Problem 8.1 Which of the following is the graph of the quadric surface given above?

GRAPHS

Written Problems

Problem 9 Consider the function

$$f(x, y, z) = \frac{4}{\sqrt{9 - x^2 - y^2 - z^2}}.$$

- (a) What is the domain of f ? Describe this domain as a region in \mathbb{R}^3 .
- (b) What is the range of f ?

Problem 10 Consider the function

$$f(x) = x^2 + y^2 - 4.$$

- (a) Draw at least five level curves of f .
- (b) Use these level curves to sketch the graph of f .

Problem 11 Draw the graph of the surface in \mathbb{R}^3 determined by the equation

$$x = y^2/4 - z^2/9.$$

Use level curves and/or sections to justify why your drawing is correct.
