

## Homework 7: Partial Derivatives

### Completion Packet

**Problem 1** Compute the partial derivatives of the function  $f(x, y) = ye^{x^2}$ .

**Problem 2** Compute the partial derivatives of the function  $f(x, y) = \ln((x + y)^2)$ .

**Problem 3** Compute the partial derivatives of the function  $f(x, y, z) = \sin(x^2yz)$ .

**Problem 4** Consider the function  $f(x, y) = 2x^2 + xy - y^2 + 3y + 1$ .

- (a) Explain how you can tell that this function is differentiable at the point  $(2, 1)$ .
- (b) Find an equation for the tangent plane to the graph of  $f$  at the point  $(2, 1)$ .

**Problem 5** Consider the function  $f(x, y) = e^{x^2+y^2}$ .

- (a) Explain how you can tell that this function is differentiable at the point  $(1, 2)$ .
- (b) Find an equation for the tangent plane to the graph of  $f$  at the point  $(1, 2)$ .

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Learning outcomes:  
Author(s):

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**Problem 6** Suppose we have a differentiable function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  such that  $f(0,0) = 5$ ,  $f(0.1,0) = 5.2$ , and  $f(0,0.1) = 4.7$ .

- (a) Estimate  $f_x(0,0)$  and  $f_y(0,0)$ .
  - (b) Give an approximate equation for the tangent plane to the graph of  $f$  at the point  $(0,0)$ .
  - (c) Using your approximation from (b), estimate the value of  $f(-0.2,0.2)$ .
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**Problem 7** Consider the function

$$f(x,y) = \begin{cases} \frac{x^3 + 2x^2y - xy^2 + 3y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

- (a) Compute the partial derivatives  $f_x(x,y)$  and  $f_y(x,y)$  for  $(x,y) \neq (0,0)$ .
  - (b) Compute the partial derivatives  $f_x(0,0)$  and  $f_y(0,0)$ .
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**Problem 8** Consider the function  $f(x,y) = e^{x+y}$ .

- (a) Compute the partial derivatives  $f_x(0,0)$  and  $f_y(0,0)$ .
  - (b) Graph the function.
  - (c) Based on your graph, is  $f$  differentiable at  $(0,0)$ ?
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**Problem 9** Consider the function  $f(x,y) = (xy)^{2/3}$ .

- (a) Compute the partial derivatives  $f_x(1,1)$ ,  $f_y(1,1)$ ,  $f_x(0,0)$ , and  $f_y(0,0)$ .
  - (b) Graph the function.
  - (c) Based on your graph, is  $f$  differentiable at  $(1,1)$ ? Is  $f$  differentiable at  $(0,0)$ ?
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## Graded Problems

**Problem 10** Consider the function  $f(x, y) = x^2 + y^2$ .

- (a) Find an equation for the tangent plane to the graph of  $f(x, y)$  at the point  $(a, b)$ .
  - (b) At which point(s)  $(a, b)$  is the tangent plane parallel to the plane  $x + 2y - z = 0$ ?
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**Problem 11** Find the partial derivatives of the function

$$f(x_1, \dots, x_n) = x_1 x_2^2 x_3^3 x_4^4 \cdots x_n^n.$$

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