Homework 10: Directional Derivatives

Graded Problems

Problem 1 *Consider the function*

$$f(x,y) = \begin{cases} \dfrac{2x^2y}{x^4+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0). \end{cases}$$

(a) *For the vector* $\mathbf{u} = (a,0)$ *in* $\mathbb{R}^2$, *find* $D_{\mathbf{u}}f(0,0)$.

(b) *For any vector of the form* $\mathbf{u} = (a,b)$ *with* $b \neq 0$ *in* $\mathbb{R}^2$, *find* $D_{\mathbf{u}}f(0,0)$.

(c) *Is* f *continuous at* $(0,0)$*?*

Problem 2 *Consider the function* $f(x,y,z) = 9x^2 + 4y^2 + z^2$. *At the point* $(2,-1,1)$, *find the direction in which* f *is changing most rapidly.*

Professional Problem

Problem 3 *You must email me the revised draft of your project by Wednesday, 11/21. Your revised draft must be sent as a pdf, and it must include your pin number, but not your names.*

Completion Packet

Problem 4 *For each function, compute the directional derivative* $D_{\mathbf{u}}f(\mathbf{a})$.

(a) $f(x,y) = x - 2xy$, $\mathbf{u} = (-1,1)$, $\mathbf{a} = (2,1)$.

Learning outcomes:
Author(s):

(b) $f(x,y) = \dfrac{1}{x-y}$, $\mathbf{u} = (1,1)$, $\mathbf{a} = (1,2)$.

(c) $f(x,y,z) = e^{xyz} + \dfrac{1}{x-y+2z}$, $\mathbf{u} = (2,1,0)$, $\mathbf{a} = (1,1,1)$.

Problem 5 *The alien Pmytmu is building a temple to honor their god, Suluclac. The temple is in the shape of a hemisphere with height 10 Azzips. Pmytmu is standing on the temple, at a vertical height of 5 Azzips above the ground. They are directly East of the center of the temple.*

(a) *Determine the slope of Pmytmu's path if they move east on the surface of the temple.*

(b) *Determine the slope of Pmytmu's path if they move west on the surface of the temple.*

(c) *Determine the slope of Pmytmu's path if they move southwest on the surface of the temple.*

Problem 6 *Through a series of unfortunate events, the unlucky alien Pmytmu has left their planet, and they are now floating around in space. It's very cold in space, and Pmytmu is afraid they might not survive due to the low temperatures. Fortunately, Pmytmu has some old textbooks with them. Due to conservation of momentum, when Pmytmu throws a textbook in one direction, Pmytmu will float in the exact opposite direction. The temperature in space is given by the function*

$$T(x,y,z) = x^2 + yz - e^{xy},$$

and Pmytmu is currently at the point $(1,1,1)$*. Pmytmu would like to move in the direction in which temperature is increasing the fastest. In which direction should Pmytmu throw their first textbook?*

Problem 7 *Find an equation for the tangent plane to each given surface at the given point.*

(a) $e^{xyz}\sin(x) = 1$, *at the point* $(\pi/2, -1, 0)$.

(b) $x^2 - z^2 + 2xy - 3yz = 3$ *at the point* $(1,1,0)$.

Problem 8 *Let* $\mathbf{x} : \mathbb{R} \to \mathbb{R}^2$ *be a smooth curve which is a level set of the graph of a function* $f : \mathbb{R}^2 \to \mathbb{R}$ *at height* $z = c$*. For any point* $\mathbf{a} = \mathbf{x}(t_0)$ *in* $\mathbb{R}^2$*, what is the directional derivative of* f *at* $\mathbf{a}$ *in the direction of* $\mathbf{x}'(t_0)$*?*