

Practice Problems

Online Problems

Problem 1 Complete the statement of the Fundamental Theorem of Line Integrals.

Let $f : X \rightarrow \mathbb{R}$ be C^1 , where $X \subset \mathbb{R}^n$ is open and connected. Then if C is any piecewise C^1 curve from \mathbf{A} to \mathbf{B} , then

$$\int_C \nabla f \cdot d\mathbf{r} = \boxed{f(B) - f(A)}$$

Problem 2 Complete the theorem statement.

Let $F : X \rightarrow \mathbb{R}^n$ be a C^1 vector field, where $X \subset \mathbb{R}^n$ is open and connected. If \mathbf{F} is conservative, then $D\mathbf{F}$ is symmetric.

Problem 3 Complete the definition.

A continuous vector field is called pathindependent if $\int_C \mathbf{F} \cdot d\mathbf{s} = \int_D \mathbf{F} \cdot d\mathbf{s}$ for any two simple, piecewise C^1 , oriented curves C and D with the same start and end points.

Written Problems

Problem 4 Sketch both of the following sets (in two different pictures) and determine whether they are open, closed, neither, or both and whether they are disconnected, connected, and/or simply connected. State all which apply and justify your answer using appropriate definitions and theorems.

Learning outcomes:
Author(s):

Practice Problems

(a) $A = \{x^2 + y^2 < 1\} \cup \{x = 0\} \subset \mathbb{R}^2$

(b) $B = \{x^2 + y^2 = 1\} \cup \{x = 0\} \subset \mathbb{R}^2$
