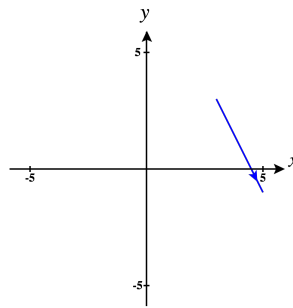


Homework 3: Parametrized Curves

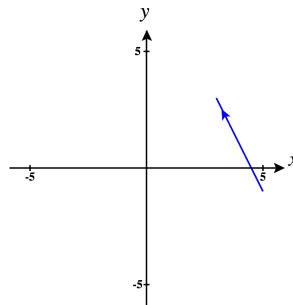
Online Problems

Problem 1 Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases. Which is the graph of the path $\vec{x}(t) = (4 - t, 2t + 1)$, for $-1 \leq t \leq 1$?

Multiple Choice:



(a)

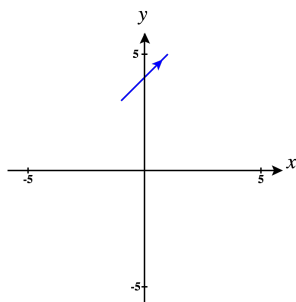


(b)

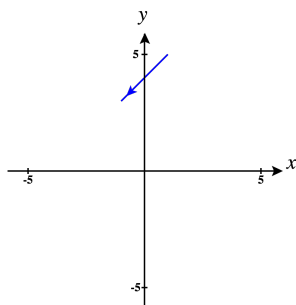
✓

Learning outcomes:
Author(s):

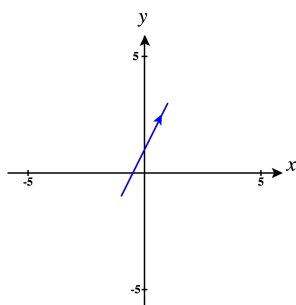
Homework 3: Parametrized Curves



(c)

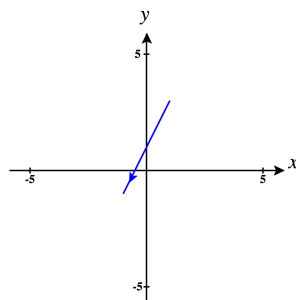


(d)



(e)

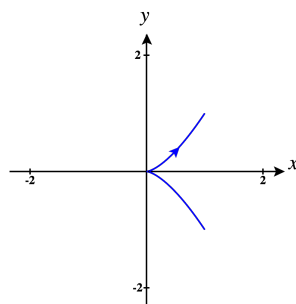
Homework 3: Parametrized Curves



(f)

Problem 2 Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases. Which is the graph of the path $\vec{x}(t) = (t^2, t^3)$, for $-1 \leq t \leq 1$?

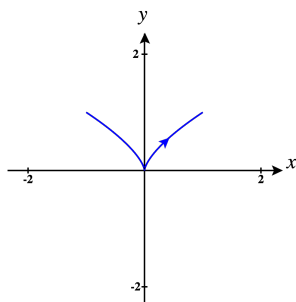
Multiple Choice:



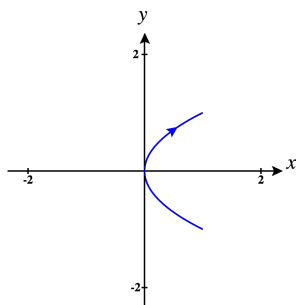
(a)

✓

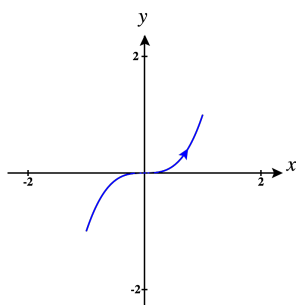
Homework 3: Parametrized Curves



(b)

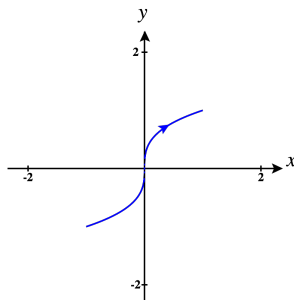


(c)



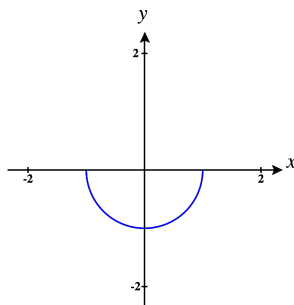
(d)

Homework 3: Parametrized Curves



(e)

Problem 3 Consider the curve below.



Which of the following are parametrizations for the curve? Select all that apply.

Select All Correct Answers:

- (a) $\vec{x}(t) = (\cos t, \sin t)$, for $0 \leq t \leq 2\pi$
- (b) $\vec{x}(t) = (\sin t, \cos t)$, for $0 \leq t \leq \pi$
- (c) $\vec{x}(t) = (\cos t, -\sin t)$, for $0 \leq t \leq \pi$ ✓
- (d) $\vec{x}(t) = (-\cos t, \sin t)$, for $0 \leq t \leq \pi$
- (e) $\vec{x}(t) = (\sin t, \cos t)$, for $\pi/2 \leq t \leq 3\pi/2$ ✓
- (f) $\vec{x}(t) = (\cos t, \sin t)$, for $\pi \leq t \leq 2\pi$ ✓

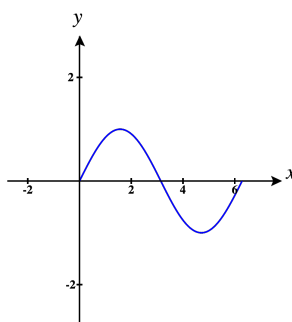
Homework 3: Parametrized Curves

(g) $\vec{x}(t) = (-\sqrt{1-t^2}, t)$ for $-1 \leq t \leq 1$

(h) $\vec{x}(t) = (\sqrt{1-t^2}, t)$ for $0 \leq t \leq 1$

(i) $\vec{x}(t) = (t, -\sqrt{1-t^2})$ for $-1 \leq t \leq 1$ ✓

Problem 4 Consider the curve below.



Which of the following are parametrizations for the curve? Select all that apply.

Select All Correct Answers:

(a) $\vec{x}(t) = (t, \sin t)$ for $0 \leq t \leq 2\pi$ ✓

(b) $\vec{x}(t) = (\arcsin t, t)$ for $-1 \leq t \leq 1$

(c) $\vec{x}(t) = (2t^2, \sin(2t^2))$ for $-1 \leq t \leq 1$ ✓

(d) $\vec{x}(t) = \left(t, \cos\left(\frac{\pi/2}{t} - 1\right)\right)$ for $0 \leq t \leq 2$

(e) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t}\right)\right)$ for $0 \leq t \leq 2$

(f) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t} - 1\right)\right)$ for $0 \leq t \leq 2$

(g) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t} - 1\right)\right)$ for $0 \leq t \leq 4$ ✓

Homework 3: Parametrized Curves

Problem 5 Consider the path $\vec{x}(t) = (3 \cos(t), -2 \sin(t))$, for $t \in \mathbb{R}$.

Compute the velocity of \vec{x} .

$$\vec{v}(t) = \boxed{(-3 \sin(t), -2 \cos(t))}$$

Compute the speed of \vec{x} .

$$\|\vec{x}'(t)\| = \boxed{\sqrt{13}}$$

Compute the acceleration of \vec{x} .

$$\vec{a}(t) = \boxed{(-3 \cos(t), 2 \sin(t))}$$

Problem 6 Two ants are running on the top of a table. Their paths are described by

$$\vec{x}(t) = (t^2 + 1, 2t - 1)$$

and

$$\vec{y}(t) = (\sqrt{t+3}, t),$$

with coordinates in inches, for $t \geq 0$ in seconds.

At what time do the ants collide?

$$t = \boxed{1}$$

Where do the ants collide?

$$(x, y) = \boxed{(2, 1)}$$

Problem 7 Consider the curve $\vec{x}(t) = (4t + 2, 1 - 3t)$ for $t \in \mathbb{R}$.

Compute the velocity.

$$\vec{v}(t) = \boxed{(4, -3)}$$

Compute the speed.

$$\|\vec{x}'(t)\| = \boxed{4}$$

Compute the acceleration.

$$\vec{a}(t) = \boxed{(0, 0)}$$

Homework 3: Parametrized Curves

Problem 8 Consider the curve $\vec{x}(t) = (2 \cos t, 5 \sin t, t^2)$ for $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(-2 \sin t, 5 \cos t, 2t)}$$

Find the velocity when $t = \pi$.

$$\vec{v}(\pi) = \boxed{(0, -5, 2\pi)}$$

Find a parametrization for the tangent line to \vec{x} at the point where $t = \pi$, so that $L(0) = \vec{x}(\pi)$.

$$L(t) = \boxed{(-2, 5, \pi^2) + t(0, -5, 2\pi)}$$

Problem 9 Consider the curve $\vec{x}(t) = (t, t^2, t^3)$ for $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(1, 2t, 3t^2)}$$

Find the velocity when $t = 2$.

$$\vec{v}(2) = \boxed{(1, 4, 12)}$$

Find a parametrization for the tangent line to \vec{x} at the point where $t = 2$, so that $L(0) = \vec{x}(2)$.

$$L(t) = \boxed{(2, 4, 8) + t(1, 4, 12)}$$

Problem 10 Consider the curve $\vec{x}(t) = (t, te^t, e^{t^2})$ for $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(1, e^t + te^t, 2te^{t^2})}$$

Find the velocity when $t = 0$.

$$\vec{v}(0) = \boxed{(1, 1, 0)}$$

Find a parametrization for the tangent line to \vec{x} at the point where $t = 0$, so that $L(0) = \vec{x}(0)$.

$$L(t) = \boxed{(0, 0, 1) + t(1, 1, 0)}$$

Written Problems

Problem 11 (a) Graph the surface $z^2 = x^2 + y^2$ and the curve $\vec{x}(t) = (t \cos(t), t \sin(t), t)$ for $-5 \leq t \leq 5$.

(b) Verify algebraically that the curve lies on the surface.

Problem 12 Prove the following product rule for cross products.

Let \vec{x} and \vec{y} be paths in \mathbb{R}^3 , then

$$(\vec{x} \times \vec{y})'(t) = \vec{x}'(t) \times \vec{y}(t) + \vec{x} \times \vec{y}'(t),$$

for t such that $x'(t)$ and $y'(t)$ exist.

Professional Problem

Problem 13 (a) Let $\vec{x}(t)$ be a curve lying on a sphere in \mathbb{R}^n of radius C . Prove that $\vec{x}(t)$ and $\vec{x}'(t)$ are perpendicular.

(b) For the curve $\vec{x}(t) = (\cos(t), \sin^2(t), \cos(t) \sin(t))$, verify computationally that $\vec{x}(t)$ lies on the sphere, and that $\vec{x}'(t)$ is perpendicular to $\vec{x}(t)$.