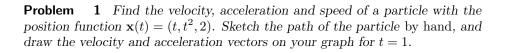
Homework 5: Moving Frames and Acceleration

SOME PROBLEMS COPIED FROM HANDOUTS - NOT SURE OF THEIR SOURCE

Completion Packet



Problem 2 Find the velocity, acceleration and speed of a particle with the position function $\mathbf{x}(t) = (t, 2\cos t, \sin t)$. Sketch the path of the particle by hand, and draw the velocity and acceleration vectors on your graph for t = 0.

Problem 3 Find the moving frame for the path $\vec{x}(t) = (2t, 6\sin(3t), -6\cos(3t))$.

Problem 4 Find the the acceleration of the path $\vec{x}(t) = (t, t^2)$, and express it as a linear combination of the unit tangent vector and the unit normal vector.

Problem 5 Find the the acceleration of the path $\vec{x}(t) = (e^t, t)$, and express it as a linear combination of the unit tangent vector and the unit normal vector.

Problem 6 Suppose we have a path $\vec{x}(t)$ and a point \vec{c} in \mathbb{R}^3 such that $||\vec{x}(t) - \vec{c}|| = R$ for some constant R and for all t.

(a) Geometrically, what can you say about this path?

Learning outcomes: Author(s):

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- (b) Prove that $\vec{v}(t)$ is perpendicular to $\vec{x}(t) \vec{c}$.
- (c) If $\vec{x}(t)$ has constant speed, show that $\kappa(t)$ is nonzero for all t.

Problem 7 Prove that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

Problem 8 Find the tangential and normal components of the acceleration vector for $\mathbf{x}(t) = (1 + t, t^2 - 2t)$.

Problem 9 Find the tangential and normal components of the acceleration vector for $\mathbf{x}(t) = (t, t^2, 3t)$.

Problem 10 Compute the following limits. Show all work. ("Wolfram Alpha" is not work.)

- (a) $\lim_{x \to 3} \frac{x^2 9}{x^2 + 2x 3}$
- (b) $\lim_{x \to \pi^-} \ln(\sin x)$
- (c) $\lim_{v \to 4^+} \frac{4-v}{|4-v|}$
- (d) $\lim_{x \to \infty} (\sqrt{x^2 + 4x + 1} x)$
- (e) $\lim_{x \to \infty} e^{x-x^2}$
- (f) $\lim_{x \to 0} \frac{\sin 2x}{x}$

Graded Problems

Problem 11 Find the moving frame for the path $\vec{x}(t) = (e^t \cos(t), e^t \sin(t), 5)$.

Problem 12 Prove that

$$\frac{|(\vec{v}\times\vec{a})\cdot\vec{a}'|}{\|\vec{v}\times\vec{a}\|^2} = \frac{\left\|d\vec{B}/dt\right\|}{|ds/dt|}.$$

Professional Problem

Problem 13 (a) Beginning with the formula $\mathbf{a} = s''\mathbf{T} + (s')^2\kappa\mathbf{N}$, cross both sides with the velocity vector $\mathbf{v} = s'\mathbf{T}$ and derive the formula

$$\kappa = \frac{||\mathbf{v} \times \mathbf{a}||}{(s')^3}.$$

Make sure to write your solution in your own words!

(b) Let y = f(x). The graph of y = f(x) is therefore a plane curve. Assuming $f \in C^2$ (that is, it is twice differentiable and its second derivatives are continuous), use the formula from (a) to prove the following well-known formula for the curvature of such a curve:

$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}.$$