Practice Problems

Online Problems

Problem 1 *Complete the statement of the Fundamental Theorem of Line Integrals.*

Let $f : X \to \mathbb{R}$ be C^1, where $X \subset \mathbb{R}^n$ is $\boxed{\textit{open}}$ *and* $\boxed{\textit{connected}}$*. Then if C is any piecewise C^1 curve from $\mathbf{A}$ to $\mathbf{B}$, then*

$$\int_C \nabla f \cdot d\sim = \boxed{f(B) - f(A)}$$

Problem 2 *Complete the theorem statement.*

Let $F : X \to \mathbb{R}^n$ be a C^1 vector field, where $X \subset \mathbb{R}^n$ is $\boxed{\textit{open}}$ *and* $\boxed{\textit{connected}}$*. If $\mathbf{F}$ is conservative, then $D\mathbf{F}$ is symmetric.*

Problem 3 *Complete the definition.*

A continuous vector field is called $\boxed{\textit{pathindependent}}$ *if $\int_C \mathbf{F} \cdot d\mathbf{s} = \int_D \mathbf{F} \cdot d\mathbf{s}$ for any two simple, piecewise C^1, oriented curves C and D with the same start and end points.*

Written Problems

Problem 4 *Sketch both of the following sets (in two different pictures) and determine whether they are open, closed, neither, or both and whether they are disconnected, connected, and/or simply connected. State all which apply and justify your answer using appropriate definitions and theorems.*

Learning outcomes:
Author(s):

(a) $A = \{x^2 + y^2 < 1\} \cup \{x = 0\} \subset \mathbb{R}^2$

(b) $B = \{x^2 + y^2 = 1\} \cup \{x = 0\} \subset \mathbb{R}^2$