

# Practice Problems

## Online Problems

**Problem 1** Find the Cartesian coordinates of each point, which is given in polar coordinates.

$$(r, \theta) = (2, \pi/6)$$

$$(x, y) = \boxed{(\sqrt{3}, 1)}$$

$$(r, \theta) = (\sqrt{2}, 3\pi/4)$$

$$(x, y) = \boxed{(-1, 1)}$$

$$(r, \theta) = (1, 0)$$

$$(x, y) = \boxed{(1, 0)}$$

$$(r, \theta) = (3, \pi)$$

$$(x, y) = \boxed{(-3, 0)}$$

**Problem 2** Find the polar coordinates of each point, which is given in Cartesian coordinates. Your answers should satisfy  $0 \leq r$  and  $0 \leq \theta < 2\pi$ .

$$(x, y) = (\sqrt{3}, 0)$$

$$(r, \theta) = \boxed{(\sqrt{3}, 0)}$$

$$(x, y) = (0, 2)$$

$$(r, \theta) = \boxed{(2, \pi/2)}$$

$$(x, y) = (-1, -1)$$

$$(r, \theta) = \boxed{(\sqrt{2}, 5\pi/4)}$$

$$(x, y) = (1, \sqrt{3})$$

$$(r, \theta) = \boxed{(2, \pi/3)}$$

Learning outcomes:

Author(s):

**Problem 3** Find the Cartesian coordinates of the point  $(\pi/2, \pi, 2)$ , given in cylindrical coordinates.

$$(x, y, z) = \boxed{(0, -\pi/2, 2)}$$

**Problem 4** Find cylindrical coordinates for the point  $(0, -1, 3)$ , written in Cartesian coordinates. Your answer should satisfy  $0 \leq r$  and  $0 \leq \theta < 2\pi$ .

$$(r, \theta, z) = \boxed{(1, \pi, 3)}$$

**Problem 5** Consider the surface described in Cartesian coordinates by

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in cylindrical coordinates, of the form  $0 = f(r, \theta, z)$ .

$$0 = \boxed{r^2 - 2z^2}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

**Multiple Choice:**

- (a) Plane
- (b) Cylinder
- (c) Sphere
- (d) Cone ✓
- (e) Other

**Problem 6** Consider the following region in  $\mathbb{R}^3$ .

IMAGE

This region is the set of points  $(r, \theta, z)$ , in cylindrical coordinates, satisfying the inequalities

$$\begin{aligned} \boxed{1} &\leq r \leq \boxed{2} \\ \boxed{\pi/2} &\leq \theta \leq \boxed{\pi} \\ \boxed{-1} &\leq z \leq \boxed{1} \end{aligned}$$

**Problem 7** For each of the following equations in cylindrical coordinates, select the type of shape they define.

FIGURE OUT CORRECT ANSWERS

$$r = \cos \theta$$

**Multiple Choice:**

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

$$z = r \cos \theta$$

**Multiple Choice:**

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

$$z = -r$$

**Multiple Choice:**

- (a) plane ✓
- (b) cylinder
- (c) sphere
- (d) other

**Problem 8** Find the Cartesian coordinates of each point, given in cylindrical coordinates.

$$(r, \theta, z) = (1, 1, 1)$$

$$(x, y, z) = \boxed{(\cos(1), \sin(1), 1)}$$

$$(r, \theta, z) = (\pi, \pi, \pi)$$

$$(x, y, z) = \boxed{(-\pi, 0, \pi)}$$

$$(r, \theta, z) = (2, 4\pi/3, -2)$$

$$(x, y, z) = \boxed{(-\sqrt{3}, -1, -2)}$$

**Problem 9** Find the cylindrical coordinates of each point, given in Cartesian coordinates. Your answers should satisfy  $r \geq 0$  and  $0 \leq \theta < 2\pi$ .

$$(x, y, z) = (1, 1, 1)$$

$$(r, \theta, z) = \boxed{(\sqrt{2}, \pi/4, 1)}$$

$$(x, y, z) = (\pi, \pi, \pi)$$

$$(r, \theta, z) = \boxed{(\sqrt{2}\pi, \pi/4, \pi)}$$

$$(x, y, z) = (2, 2\sqrt{3}, -2)$$

$$(r, \theta, z) = \boxed{(4, \pi/6, -2)}$$

**Problem 10** Find the Cartesian coordinates of the point  $(2, \pi, \pi/2)$ , given in spherical coordinates.

$$(x, y, z) = \boxed{(-2, 0, 0)}$$

**Problem 11** Find spherical coordinates for the point  $(-\sqrt{2}, \sqrt{2}, 2\sqrt{3})$ , written in Cartesian coordinates. Your answer should satisfy  $0 \leq \rho$ ,  $0 \leq \theta \leq 2\pi$ , and  $0 \leq \phi \leq \pi$ .

$$(\rho, \theta, \phi) = \boxed{(4, 3\pi/4, \pi/6)}$$

**Problem 12** Consider the surface described in Cartesian coordinates by

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in spherical coordinates, of the form  $0 = f(\rho, \theta, \phi)$ .

$$0 = \boxed{\rho^2 \sin^2 \phi - 2 \cos^2 \phi}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

**Multiple Choice:**

- (a) Plane
- (b) Cylinder
- (c) Sphere
- (d) Cone ✓
- (e) Other

**Problem 13** Consider the following region in  $\mathbb{R}^3$ .

IMAGE

This region is the set of points  $(\rho, \theta, \phi)$ , in spherical coordinates, satisfying the inequalities

$$\begin{aligned} 0 &\leq \rho \leq 2 \\ 0 &\leq \theta \leq \pi/2 \\ 0 &\leq \phi \leq \pi \end{aligned}$$

**Problem 14** For each of the following equations in spherical coordinates, select the type of shape they define.

FIGURE OUT CORRECT ANSWERS

$$\rho = \cos \phi$$

**Multiple Choice:**

- (a) plane
- (b) cylinder
- (c) sphere
- (d) other

$$\rho = \sin \theta$$

**Multiple Choice:**

- (a) plane

- (b) *cylinder*
- (c) *sphere*
- (d) *other*

$$\rho \cos \theta \sin \phi = 1$$

**Multiple Choice:**

- (a) *plane* ✓
- (b) *cylinder*
- (c) *sphere*
- (d) *other*

**Problem 15** Find the spherical coordinates of each point, given in Cartesian coordinates. Your answers should satisfy  $0 \leq r$ ,  $0 \leq \theta < 2\pi$ , and  $0 \leq \phi \leq \pi$ .

$$(x, y, z) = (1, 1, 1)$$

$$(\rho, \theta, \phi) = (\sqrt{3}, \pi/4, \pi/4)$$

$$(x, y, z) = (1, -1, -1)$$

$$(\rho, \theta, \phi) = (\sqrt{3}, 3\pi/4, 3\pi/4)$$

$$(x, y, z) = (1, \sqrt{3}, 0)$$

$$(\rho, \theta, \phi) = (\sqrt{4}, \pi/6, \pi/2)$$

**Problem 16** Find the Cartesian coordinates of each point, given in spherical coordinates.

$$(\rho, \theta, \phi) = (\pi, \pi, \pi)$$

$$(x, y, z) = (0, 0, -\pi)$$

$$(\rho, \theta, \phi) = (3, \pi/2, \pi/4)$$

$$(x, y, z) = (0, 3\sqrt{2}/2, 3\sqrt{2}/2)$$

$$(\rho, \theta, \phi) = (2, 7\pi/6, 3\pi/4)$$

$$(x, y, z) = (-\sqrt{6}/2, -\sqrt{2}/2, -\sqrt{2})$$

## Written Problems

**Problem 17** For several values of the constant  $a$ , sketch the graph of the curve in  $\mathbb{R}^2$  given by the polar equation

$$r = a \sin \theta.$$

What do you notice about these curves?

**Problem 18** Which points in  $\mathbb{R}^2$  have the same coordinates when written in Cartesian and polar coordinates? (That is, for what points do we have  $x = r$  and  $y = \theta$ ?)

**Problem 19** Consider the surface described by  $(r-3)^2 + z^2 = 1$  in cylindrical coordinates, with the restriction  $r \geq 0$ .

- (a) Sketch the intersection of the surface with the half-plane  $\theta = 0$ .
- (b) Sketch the intersection of the surface with the half-plane  $\theta = \frac{\pi}{2}$ .
- (c) Sketch the intersection of the surface with the plane  $z = 0$ .
- (d) Sketch the surface.

**Problem 20** Sketch the region in  $\mathbb{R}^3$  with cylindrical coordinates satisfying the inequality

$$r \leq z \leq 4 - 2r$$

**Problem 21** Convert the following equation, given in cylindrical coordinates, into Cartesian coordinates.

$$r = 0$$

**Problem 22** Sketch the region in  $\mathbb{R}^3$  given by

$$\begin{aligned} r &\leq z \leq 3 \\ 0 &\leq \theta \leq \pi/2 \end{aligned}$$

in cylindrical coordinates.

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**Problem 23** Sketch the region in  $\mathbb{R}^3$  given by

$$r^2 - 2 \leq z \leq 2 - r^2$$

in cylindrical coordinates.

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**Problem 24** Which points in  $\mathbb{R}^3$  have the same coordinates when written in Cartesian and cylindrical coordinates?

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**Problem 25** (a) Given a function  $f$ , consider the graphs of the equations  $r = f(\theta)$  and  $r = 2f(\theta)$ , in polar coordinates. How are these graphs related?

(b) Given a function  $f$ , consider the graphs of the equations  $\rho = f(\theta, \phi)$  and  $\rho = 2f(\theta, \phi)$ , in spherical coordinates. How are these graphs related?

(c) Given a function  $f$ , consider the graphs of the equations  $r = f(\theta)$  and  $r = -f(\theta)$ , in polar coordinates. How are these graphs related?

(d) Given a function  $f$ , consider the graphs of the equations  $\rho = f(\theta, \phi)$  and  $\rho = -f(\theta, \phi)$ , in spherical coordinates. How are these graphs related?

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**Problem 26** Sketch the surface in  $\mathbb{R}^3$  given by the equation

$$1 - \cos \phi$$

in spherical coordinates.

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**Problem 27** Consider the surface in  $\mathbb{R}^3$  given by the equation

$$\rho \sin \phi \sin \theta = 1$$

in spherical coordinates.



Convert this equation to Cartesian coordinates and cylindrical coordinates, and sketch the surface.

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**Problem 28** Consider the region in  $\mathbb{R}^3$  consisting of points whose spherical coordinates satisfy

$$1 \leq \rho \leq 3$$

Sketch this region.

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**Problem 29** Consider the region in  $\mathbb{R}^3$  consisting of points whose spherical coordinates satisfy

$$0 \leq \phi \leq \pi/2$$

$$0 \leq \rho \leq 1$$

Sketch this region.

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**Problem 30** Consider the region in  $\mathbb{R}^3$  consisting of points whose spherical coordinates satisfy

$$\cos \phi \leq \rho \leq 2$$

Sketch this region.

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**Problem 31** Which points in  $\mathbb{R}^3$  have the same coordinates when written in Cartesian and spherical coordinates?

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