Practice Problems

Online Problems

Problem 1 *Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by*

$$f(x, y) = x^2 + 4y^2 - 2.$$

What is the domain of f?

Multiple Choice:

(a) $\mathbb{R}$

(b) $\mathbb{R} \setminus \{0\}$

(c) $[0, \infty)$

(d) $(0, \infty)$

(e) $\mathbb{R}^2$ ✓

(f) $\mathbb{R}^2 \setminus \{(0, 0)\}$

What is the range of f?

$$\text{Range } f = \boxed{[-2, \infty)}$$

Is f onto?

Multiple Choice:

(a) *yes*

(b) *no* ✓

Problem 1.1 *We would like to restrict the codomain of the function f so that it becomes onto. We'll describe our new codomain as the set of numbers a in $\mathbb{R}$ such that some condition holds. Which condition gives us the largest possible codomain such that f is onto?*

Learning outcomes:
Author(s):

Multiple Choice:

(a) $a \in \mathbb{R}$

(b) $a \geq 0$

(c) $a > 0$

(d) $a \neq 0$

(e) $a = 0$

(f) $a \geq 2$

(g) $a > 2$

(h) $a \neq 2$

(i) $a = 2$

(j) $a \geq -2$ ✓

(k) $a > -2$

(l) $a \neq -2$

(m) $a = -2$

Is f one-to-one?

Multiple Choice:

(a) *yes*

(b) *no* ✓

Problem 1.2 *We would like to restrict the domain of the function f, so that it becomes one-to-one. We'll describe our new domain as the set of points (x, y) in $\mathbb{R}^2$ such that some condition(s) hold. Which condition(s) give us the largest possible domain such that f is one-to-one?*

Select All Correct Answers:

(a) $x \neq 0$

(b) $x \geq 0$ ✓

(c) $x > 0$

(d) $y \neq 0$

(e) $y \geq 0$ ✓

(f) $y > 0$

Problem 2 *Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by*

$$f(\vec{x}) = 3\vec{x} + \boldsymbol{i} - 2\boldsymbol{j}.$$

Find the component functions of f in terms of x, y, and z.

$$f_1(x, y, z) = \boxed{3x + 1}$$
$$f_2(x, y, z) = \boxed{3y - 2}$$
$$f_3(x, y, z) = \boxed{3z}$$

Problem 3 *Consider the linear function $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by $f(\vec{x}) = A\vec{x}$, where*

$$A = \begin{pmatrix} 1 & 5 & 2 \\ -2 & 0 & 1 \end{pmatrix},$$

and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

(a) *Determine the component functions of f in terms of x_1, x_2, and x_3.*

$$f_1(x_1, x_2, x_3) = \boxed{x_1 + 5x_2 + 2x_3}$$
$$f_2(x_1, x_2, x_3) = \boxed{-2x_1 + x_3}$$

(b) *Is f one-to-one?*

Multiple Choice:

(i) *Yes*

(ii) *No* ✓

(c) *Is f onto?*

Multiple Choice:

(i) *Yes* ✓
(ii) *No*

Problem 4 *Consider the function*

$$f(x, y) = xy.$$

What is the shape of the level curve at height 0 *of* f*?*

Multiple Choice:

(a) *Empty*
(b) *A single line*
(c) *Two intersecting lines* ✓
(d) *Two parallel lines*
(e) *Circle*
(f) *Ellipse*
(g) *Parabola*
(h) *Hyperbola*

What is the shape of the level curve at height 1 *of* f*?*

Multiple Choice:

(a) *Empty*
(b) *A single line*
(c) *Two intersecting lines*
(d) *Two parallel lines*
(e) *Circle*
(f) *Ellipse*
(g) *Parabola*
(h) *Hyperbola* ✓

What is the shape of the level curve at height -1 of f?

Multiple Choice:

(a) *Empty*

(b) *A single line*

(c) *Two intersecting lines*

(d) *Two parallel lines*

(e) *Circle*

(f) *Ellipse*

(g) *Parabola*

(h) *Hyperbola* ✓

What is the shape of the level curve at height 2 of f?

Multiple Choice:

(a) *Empty*

(b) *A single line*

(c) *Two intersecting lines*

(d) *Two parallel lines*

(e) *Circle*

(f) *Ellipse*

(g) *Parabola*

(h) *Hyperbola* ✓

Which of the following is the graph of f?

Problem 5 *Consider the function*

$$f(x,y) = |x|.$$

What is the shape of the level curve at height 0 of f?

Multiple Choice:

(a) *Empty*

(b) *A single line* ✓

(c) *Two intersecting lines*

(d) *Two parallel lines*

(e) *Circle*

(f) *Ellipse*

(g) *Parabola*

(h) *Hyperbola*

What is the shape of the level curve at height 1 of f?

Multiple Choice:

(a) *Empty*

(b) *A single line*

(c) *Two intersecting lines*

(d) *Two parallel lines* ✓

(e) *Circle*

(f) *Ellipse*

(g) *Parabola*

(h) *Hyperbola*

What is the shape of the level curve at height -1 of f?

Multiple Choice:

(a) *Empty* ✓

(b) *A single line*

(c) *Two intersecting lines*

(d) *Two parallel lines*

(e) *Circle*

(f) *Ellipse*

(g) *Parabola*

(h) *Hyperbola*

What is the shape of the level curve at height 2 of f?

Multiple Choice:

(a) *Empty*

(b) *A single line*

(c) *Two intersecting lines*

(d) *Two parallel lines* ✓

(e) *Circle*

(f) *Ellipse*

(g) *Parabola*

(h) *Hyperbola*

Which of the following is the graph of f?

Problem 6 *Which of the following is the graph of the ellipsoid*

$$\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 1?$$

PICTURES

Is there a function $f(x, y)$ such that the graph of f is the ellipsoid above?

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Problem 6.1 *Why is this impossible?*

Multiple Choice:

(a) *It wouldn't be one-to-one.*

(b) *It wouldn't be onto.*

(c) *There would be multiple inputs with the same output.*

(d) *A single input would need to have two outputs.* ✓

Problem 7 *Classify the quadric surface defined by the equation*

$$x^2 + 4y^2 + z^2 + 8y = 0.$$

Multiple Choice:

(a) *Ellipsoid* ✓

(b) *Elliptic Paraboloid*

(c) *Hyperbolic Paraboloid*

(d) *Elliptic Cone*

(e) *Hyperboloid of One Sheet*

(f) *Hyperboloid of Two Sheets*

It is centered at $\boxed{(0, -1, 0)}$.

Problem 7.1 *Which of the following is the graph of the quadric surface given above?*

GRAPHS

Problem 8 *Classify the quadric surface defined by the equation*

$$2x^2 + 2y^2 - 8y - z + 4 = 0$$

Multiple Choice:

(a) *Ellipsoid*

(b) *Elliptic Paraboloid* ✓

(c) *Hyperbolic Paraboloid*

(d) *Elliptic Cone*

(e) *Hyperboloid of One Sheet*

(f) *Hyperboloid of Two Sheets*

It is centered at $\boxed{(0, 2, -4)}$.

Problem 8.1 *Which of the following is the graph of the quadric surface given above?*

GRAPHS

Written Problems

Problem 9 *Consider the function*

$$f(x, y, z) = \frac{4}{\sqrt{9 - x^2 - y^2 - z^2}}.$$

(a) *What is the domain of f? Describe this domain as a region in $\mathbb{R}^3$.*

(b) *What is the range of f?*

Problem 10 *Consider the function*

$$f(x) = x^2 + y^2 - 4.$$

(a) *Draw at least five level curves of f.*

(b) *Use these level curves to sketch the graph of f.*

Problem 11 *Draw the graph of the surface in $\mathbb{R}^3$ determined by the equation*

$$x = y^2/4 - z^2/9.$$

Use level curves and/or sections to justify why your drawing is correct.