

Extra Problems

Online Problems

Problem 1 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (x, y, z)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Problem 2 Compute the 2-dimensional curl of the vector field $\mathbf{F}(x, y) = (xy^2, y \sin(x^2))$. Your answer should be given as a vector.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 2xy(\cos(x^2) - 1))}$$

Problem 3 Compute the 2-dimensional curl of the vector field $\mathbf{F}(x, y) = (e^{xy}, \cos(y))$. Your answer should be given as a vector.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, -xe^{xy})}$$

Problem 4 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (y, z, x)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(-1, -1, -1)}$$

Learning outcomes:
Author(s):

Problem 5 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (yz, xz, xy)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Problem 6 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (\sin(x), \cos(y), xy)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(x, -y, 0)}$$

Problem 7 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (0, x^3, \sin(x+y))$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(\cos(x+y), -\cos(x+y), 3x^2)}$$

Problem 8 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (e^{xy}, \cos(z), e^y)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(e^y + \sin(z), 0, -xe^{xy})}$$

Problem 9 Compute the 2-dimensional curl of the vector field $\mathbf{F}(x, y) = (y, x)$. Your answer should be given as a vector.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Problem 10 Compute the 2-dimensional curl of the vector field $\mathbf{F}(x, y) = (y, -x)$. Your answer should be given as a vector.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, -2)}$$

Problem 11 Compute the 2-dimensional curl of the vector field $\mathbf{F}(x, y) = (x^2y + 3xy, xy^4 - 2x^2y)$. Your answer should be given as a vector.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, -x^2 - 3x - 4xy + y^4)}$$

Problem 12 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (yz, 2xz, 3xy)$.

$$\nabla \times \mathbf{F} = \boxed{(x, -2y, z)}$$

Find the curl of \mathbf{F} at the point $(x, y, z) = (0, 0, 0)$.

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 0)}$$

Is \mathbf{F} irrotational?

Multiple Choice:

- (a) Yes.
- (b) No. ✓
- (c) Not enough information.

Problem 13 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (x^2, y^3, z^4)$.

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 0)}.$$

Find the curl of \mathbf{F} at the point $(x, y, z) = (1, 2, 3)$.

$$(\nabla \times \mathbf{F})(1, 2, 3) = \boxed{(0, 0, 0)}.$$

Is \mathbf{F} irrotational?

Multiple Choice:

- (a) Yes. ✓
- (b) No.
- (c) Not enough information.

Problem 14 Compute the two-dimensional curl of the vector field $\mathbf{F}(x, y) = (-xy, xy)$.

$$\nabla \times \mathbf{F} = \left(0, 0, \boxed{x + y}\right)$$

Describe the local rotation of \mathbf{F} at the point $(1, 1)$.

Multiple Choice:

- (a) Counterclockwise. ✓
- (b) Clockwise.
- (c) No rotation.

Describe the local rotation of \mathbf{F} at the point $(-1, 1)$.

Multiple Choice:

- (a) Counterclockwise.
- (b) Clockwise.
- (c) No rotation. ✓

Describe the local rotation of \mathbf{F} at the point $(-1, -1)$.

Multiple Choice:

- (a) Counterclockwise.
 - (b) Clockwise. ✓
 - (c) No rotation.
-

Problem 15 Find the 2-dimensional curl of the vector field $\mathbf{F}(x, y) = (-y, x)$. Your answer should be given as a vector.

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 2)}$$

Is this vector field irrotational?

Multiple Choice:

- (a) Yes
- (b) No ✓

Describe the direction of the local rotation of this vector field.

Multiple Choice:

- (a) Clockwise
- (b) Counterclockwise ✓
- (c) No rotation

Plot this vector field. *INCLUDE GRAPHING STUFF*

Find the 2-dimensional curl of the vector field $\mathbf{G}(x, y) = \left(\frac{-y}{x^2 + y^2}, \frac{-y}{x^2 + y^2} \right)$.

Your answer should be given as a vector.

$$\nabla \times \mathbf{G} = \boxed{(0, 0, 0)}$$

Is this vector field irrotational?

Multiple Choice:

- (a) Yes ✓
- (b) No

Describe the direction of the local rotation of this vector field.

Multiple Choice:

- (a) Clockwise
- (b) Counterclockwise
- (c) No rotation ✓

Plot this vector field. *INCLUDE GRAPHING STUFF*

Although the vector field \mathbf{F} and \mathbf{G} have the same flow lines, we see that one is irrotational and the other is not. Why does this happen?

Multiple Choice:

- (a) Math is broken.

- (b) Curl describes local rotation, not global rotation. ✓

Problem 16 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (-2y \cos(3x), 3x \sin(-2y), 0)$.

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 2 \cos(3x) - 3 \sin(2y))}$$

Find the curl of \mathbf{F} at the point $(x, y, z) = (\pi, \pi, \pi)$.

$$(\nabla \times \mathbf{F})(\pi, \pi, \pi) = \boxed{(0, 0, -2)}$$

Is \mathbf{F} a conservative vector field?

Multiple Choice:

- (a) Yes.
 (b) No. ✓
 (c) Not enough information.

Justify your answer.

Free Response:

Problem 17 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (ze^{xz} + z \sin(x), xe^{xy}, -\cos(x))$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, x^2 e^{xz}, xye^{xy} + e^{xy})}$$

Is this vector field conservative?

Multiple Choice:

- (a) Yes
 (b) No ✓

Problem 18 Compute the curl of the vector field $\mathbf{F}(x, y) = (2x - y, -x + 4y)$.

$$\nabla \times \mathbf{F} = (0, 0, \boxed{0})$$

Is \mathbf{F} conservative?

Multiple Choice:

(a) Yes. ✓

(b) No.

Problem 18.1 Find a potential function f for \mathbf{F} , so that $\nabla f = \mathbf{F}$.

$$f(x, y) = \boxed{x^2 - xy + 2y^2}$$

Problem 19 Compute the curl of the vector field $\mathbf{F}(x, y) = (2y, 3x)$.

$$\nabla \times \mathbf{F} = (0, 0, \boxed{1})$$

Is \mathbf{F} conservative?**Multiple Choice:**

(a) Yes.

(b) No. ✓

Problem 20 Compute the curl of the vector field $\mathbf{F}(x, y) = (2x, 3y)$.

$$\nabla \times \mathbf{F} = (0, 0, \boxed{0})$$

Is \mathbf{F} conservative?**Multiple Choice:**

(a) Yes. ✓

(b) No.

Problem 20.1 Find a potential function f for \mathbf{F} , so that $\nabla f = \mathbf{F}$.

$$f(x, y) = \boxed{x^2 + \frac{3}{2}y^2}$$

Problem 21 Compute the curl of the vector field $\mathbf{F}(x, y) = (-4x + y \cos(x), \sin(x))$.

$$\nabla \times \mathbf{F} = (0, 0, \boxed{0})$$

Is \mathbf{F} conservative?

Multiple Choice:

- (a) Yes. ✓
- (b) No.

Problem 21.1 Find a potential function f for \mathbf{F} , so that $\nabla f = \mathbf{F}$.

$$f(x, y) = \boxed{-2x^2 + y \sin(x)}$$

Problem 22 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (\sin(x), y^2, e^z)$.

$$\nabla \times \mathbf{F} = \boxed{(0, 0, 0)}$$

Is \mathbf{F} conservative?

Multiple Choice:

- (a) Yes. ✓
- (b) No.

Problem 22.1 Find a potential function f for \mathbf{F} , so that $\nabla f = \mathbf{F}$.

$$f(x, y, z) = \boxed{-\cos(x) + \frac{1}{2}y^2 + e^z}$$

Problem 23 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (x, y, z)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Is this vector field conservative?

Multiple Choice:

- (a) Yes ✓
- (b) No

Problem 23.1 Find a potential function f for \mathbf{F} , so that $\nabla f = \mathbf{F}$.

$$f(x, y, z) = \boxed{\frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2}$$

Problem 24 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (z, 4yz, x^2 + 3y)$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(3 - 4y, 1 - 2x, 0)}$$

Is this vector field conservative?

Multiple Choice:

- (a) Yes
- (b) No ✓

Problem 25 Compute the curl of the vector field $\mathbf{F}(x, y, z) = (ye^{xy} + z \cos(x), xe^{xy}, \sin(x))$.

$$\nabla \times \mathbf{F}(x, y, z) = \boxed{(0, 0, 0)}$$

Is this vector field conservative?

Multiple Choice:

- (a) Yes ✓

(b) No

Problem 25.1 Find a potential function f for \mathbf{F} , so that $\nabla f = \mathbf{F}$.

$$f(x, y, z) = \boxed{e^{xy} + \sin(x)z}$$

Written Problems

Problem 26 Prove that, for C^1 vector fields \mathbf{F} and \mathbf{G} in \mathbb{R}^3 ,

$$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}.$$

(This shows that the curl is an additive operator, so the curl of a sum is the sum of the curls.)

Problem 27 Let \mathbf{F} and \mathbf{G} be vector fields in \mathbb{R}^3 , and let a and b be real numbers. Prove that

$$\nabla \times (a\mathbf{F} + b\mathbf{G}) = a(\nabla \times \mathbf{F}) + b(\nabla \times \mathbf{G}).$$

Problem 28 Let \mathbf{F} be a vector field in \mathbb{R}^3 , and let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a scalar valued function. Prove that

$$\nabla \times (f\mathbf{F}) = f(\nabla \times \mathbf{F}) + (\nabla f) \times \mathbf{F}.$$

Problem 29 Let $\mathbf{F}(x, y, z) = \frac{c\mathbf{r}}{\|\mathbf{r}\|^3}$, where c is constant and $\mathbf{r} = (x, y, z)$. Prove that the curl of \mathbf{F} , $\nabla \times \mathbf{F}$, is zero.

Problem 30 Let $\mathbf{F}(x, y, z) = (x, y, z)$, and let $\mathbf{a} \in \mathbb{R}^3$ be a vector with constant entries. Prove that

$$\nabla \times (\mathbf{a} \times \mathbf{F}) = 2\mathbf{a}.$$

Problem 31 (a) Compute the curl of the vector field $\mathbf{F} = (x, y, z)$. Explain why your answer makes sense geometrically.

(b) Suppose we have a C^1 vector field $\mathbf{F}(x, y, z) = (f(x), g(y), h(z))$. Compute the curl of \mathbf{F} , and explain why your answer makes sense geometrically.

Problem 32 (a) Consider the function $f(x, y, z) = e^x \sin(y) + z^2 \cos(y)$. Compute ∇f , and verify that $\nabla \times (\nabla f) = \mathbf{0}$.

(b) Prove that for any C^2 function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$, the curl of the gradient of f is zero. That is, prove that

$$\nabla \times (\nabla f) = \mathbf{0}.$$
