Homework 1: Coordinates

Graded Problems

Problem 1 Consider the surface in \mathbb{R}^3 described by the equation $(r-2)^2 + z^2 = 1$ in cylindrical coordinates.

- (a) Sketch the intersection of this surface with the xy-plane.
- (b) Given that θ does not appear in the equation, what can you say about the surface?
- (c) Sketch the surface.

Problem 2 (a) Suppose we have a curve in \mathbb{R}^2 defined by the polar equation $r = f(\theta)$, for some function f. What is the relationship between this curve and the curve defined by $r = f(\theta + \pi/4)$ in polar coordinates?

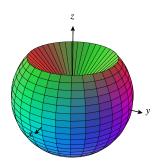
- (b) Suppose we have a surface in \mathbb{R}^3 defined by the cylindrical equation $r = g(\theta, z)$, for some function g. What is the relationship between this surface and the surface defined by $r = g(\theta + \pi/4, z)$ in cylindrical coordinates?
- (c) Suppose we have a surface in \mathbb{R}^3 defined by the spherical equation $r = h(\theta, \phi)$, for some function h. What is the relationship between this surface and the surface defined by $r = h(\theta + \pi/4, \phi)$ in spherical coordinates?
- (d) Again, suppose we have a surface in \mathbb{R}^3 defined by the spherical equation $r = h(\theta, \phi)$, for some function h. What is the relationship between this surface and the surface defined by $r = h(\theta, \phi + \pi)$ in spherical coordinates? Hint: it's not a rotation.

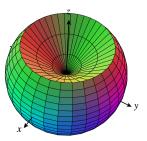
Professional Problem

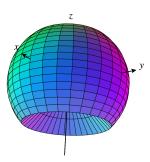
Problem 3 Consider the solid below, which is obtained by taking the portion of a solid sphere of radius 2 which is outside of the (double) cone $z^2 = x^2 + y^2$.

Learning outcomes: Author(s):

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(a) Is this region easier to describe in spherical coordinates or in cylindrical coordinates? Justify your answer.

(b) Describe this region in either spherical coordinates or cylindrical coordinates (based on your answer to (a)).

Completion Packet

Problem 4 Consider the point $P = \left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}\right)$, given in crylindrical coordinates. What are the Cartesian coordinates of P?

Problem 5 Consider the point $P=\left(\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2}\right)$, given in spherical coordinates. What are the spherical coordinates of P?

Problem 6 Consider the point P = (2, 1, 3), given in Cartesian coordinates. Find all possible ways to write P in cylindrical coordinates.

Problem 7 Consider the point P = (2, 1, 2), given in Cartesian coordinates. Find all possible ways to write P in spherical coordinates.

Problem 8 Sketch the graph of the surface in \mathbb{R}^3 defined by the equation $\rho = \cos(\phi)$ in spherical coordinates, for $0 \le \phi \le \pi$.

Problem 9 Consider the surface in \mathbb{R}^3 defined by the equation $2x^2+2y^2+z^2=1$ in Cartesian coordinates.

Convert the equation to cylindrical coordinates.

Convert the equation to spherical coordinates.

Sketch the surface.

Problem 10 Sketch the region defined by the inequalities $\pi/4 \le \theta \le \pi$, $0 \le r \le 2$, and $0 \le z \le r$ in cylindrical coordinates.

Problem 11 Sketch the region defined by the inequalities $0 \le \phi \le \pi$, $0 \le \theta \le \pi$, and $0 \le \rho \le \theta$.

Problem 12 Consider the region defined by the inequalities $0 \le r \le 1$, $0 \le \theta \le 2\pi$, and $0 \le z \le 2$ in cylindrical coordinates.

- (a) Sketch this region.
- (b) Describe this region in spherical coordinates.