

## Homework 3: Parametrized Curves

### Online Problems

**Problem 1** Two ants are running on the top of a table. Their paths are described by

$$\vec{x}(t) = (t^2 + 1, 2t - 1)$$

and

$$\vec{y}(t) = (\sqrt{t+3}, t),$$

with coordinates in inches, for  $t \geq 0$  in seconds.

At what time do the ants collide?

$$t = \boxed{1}$$

Where do the ants collide?

$$(x, y) = \boxed{(2, 1)}$$

**Problem 2** Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.

PICTURE

Which is the graph of the path  $\vec{x}(t) = (4 - t, 2t + 1)$ , for  $-1 \leq t \leq 1$ ?

**Multiple Choice:**

- (a) (a)
- (b) (b)
- (c) (c)
- (d) (d)
- (e) (e)

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Learning outcomes:  
Author(s):

Homework 3: Parametrized Curves

**Problem 3** Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.

PICTURE

Which is the graph of the path  $\vec{x}(t) = (3 \sin(t), 2 \cos(t))$ , for  $0 \leq t \leq \pi$ ?

**Multiple Choice:**

- (a) (a)
  - (b) (b)
  - (c) (c)
  - (d) (d)
  - (e) (e)
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**Problem 4** Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.

PICTURE

Which is the graph of the path  $\vec{x}(t) = (t^2, t^3)$ , for  $-1 \leq t \leq 1$ ?

**Multiple Choice:**

- (a) (a)
  - (b) (b)
  - (c) (c)
  - (d) (d)
  - (e) (e)
- 

**Problem 5** Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.

PICTURE

Which is the graph of the path  $\vec{x}(t) = (e^t, t)$ , for  $-1 \leq t \leq 1$ ?

**Multiple Choice:**

- (a) (a)

- (b) (b)
- (c) (c)
- (d) (d)
- (e) (e)

**Problem 6** Consider the curve below.

PICTURE

Which of the following are parametrizations for the curve? Select all that apply.

**Select All Correct Answers:**

- (a)  $\vec{x}(t) = (\cos t, \sin t)$ , for  $0 \leq t \leq 2\pi$
- (b)  $\vec{x}(t) = (\sin t, \cos t)$ , for  $0 \leq t \leq \pi$
- (c)  $\vec{x}(t) = (\cos t, -\sin t)$ , for  $0 \leq t \leq \pi$  ✓
- (d)  $\vec{x}(t) = (-\cos t, \sin t)$ , for  $0 \leq t \leq \pi$
- (e)  $\vec{x}(t) = (\sin t, \cos t)$ , for  $\pi/2 \leq t \leq \pi/2$  ✓
- (f)  $\vec{x}(t) = (\cos t, \sin t)$ , for  $\pi \leq t \leq 2\pi$  ✓
- (g)  $\vec{x}(t) = (-\sqrt{1-t^2}, t)$  for  $-1 \leq t \leq 1$
- (h)  $\vec{x}(t) = (\sqrt{1-t^2}, t)$  for  $0 \leq t \leq 1$
- (i)  $\vec{x}(t) = (t, -\sqrt{1-t^2})$  for  $-1 \leq t \leq 1$  ✓

**Problem 7** Consider the curve below.

PICTURE

Which of the following are parametrizations for the curve? Select all that apply.

**Select All Correct Answers:**

- (a)  $\vec{x}(t) = (t, \sin t)$  for  $0 \leq t \leq 2\pi$  ✓
- (b)  $\vec{x}(t) = (\arcsin t, t)$  for  $-1 \leq t \leq 1$
- (c)  $\vec{x}(t) = (2t^2, \sin(2t^2))$  for  $-1 \leq t \leq 1$  ✓

Homework 3: Parametrized Curves

(d)  $\vec{x}(t) = \left( t, \cos\left(\frac{\pi/2}{t} - 1\right) \right)$  for  $0 \leq t \leq 2$

(e)  $\vec{x}(t) = \left( \frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t}\right) \right)$  for  $0 \leq t \leq 2$

(f)  $\vec{x}(t) = \left( \frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t} - 1\right) \right)$  for  $0 \leq t \leq 2$

(g)  $\vec{x}(t) = \left( \frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t} - 1\right) \right)$  for  $0 \leq t \leq 4$  ✓

**Problem 8** Consider the path  $\vec{x}(t) = (3 \cos(t), -2 \sin(t))$ , for  $t \in \mathbb{R}$ .  
Compute the velocity of  $\vec{x}$ .

$$\vec{v}(t) = \boxed{(-3 \sin(t), -2 \cos(t))}$$

Compute the speed of  $\vec{x}$ .

$$\|\vec{x}'(t)\| = \boxed{\sqrt{13}}$$

**Problem 9** Consider the path  $\vec{x}(t) = (\cos(t^4), \sin(t^4), \frac{1}{2}t^4)$ , for  $t \geq 0$ .  
Compute the velocity.

$$\vec{v}(t) = \boxed{(-4t^3 \sin(t^4), 4t^3 \cos(t^4), 2t^3)}$$

Compute the speed.

$$\|\vec{x}'(t)\| = \boxed{\sqrt{20t^3}}$$

**Problem 10** Consider the curve  $\vec{x}(t) = (4t + 2, 1 - 3t)$  for  $t \in \mathbb{R}$ .  
Compute the velocity.

$$\vec{v}(t) = \boxed{(4, -3)}$$

Compute the speed.

$$\|\vec{x}'(t)\| = \boxed{4}$$

Homework 3: Parametrized Curves

**Problem 11** Consider the curve  $\vec{x}(t) = (2 \cos t, 5 \sin t, t^2)$  for  $t \in \mathbb{R}$ .

Find the velocity.

$$\vec{v}(t) = \boxed{(-2 \sin t, 5 \cos t, 2t)}$$

Find the velocity when  $t = \pi$ .

$$\vec{v}(\pi) = \boxed{(0, -5, 2\pi)}$$

Find a parametrization for the tangent line to  $\vec{x}$  at the point where  $t = \pi$ , so that  $L(0) = \vec{x}(\pi)$ .

$$L(t) = \boxed{(-2, 5, \pi^2) + t(0, -5, 2\pi)}$$

**Problem 12** Consider the curve  $\vec{x}(t) = (t, t^2, t^3)$  for  $t \in \mathbb{R}$ .

Find the velocity.

$$\vec{v}(t) = \boxed{(1, 2t, 3t^2)}$$

Find the velocity when  $t = 2$ .

$$\vec{v}(\pi) = \boxed{(1, 4, 12)}$$

Find a parametrization for the tangent line to  $\vec{x}$  at the point where  $t = 2$ , so that  $L(0) = \vec{x}(2)$ .

$$L(t) = \boxed{(2, 4, 8) + t(1, 4, 12)}$$

**Problem 13** Consider the curve  $\vec{x}(t) = (t, te^t, e^{t^2})$  for  $t \in \mathbb{R}$ .

Find the velocity.

$$\vec{v}(t) = \boxed{(1, e^t + te^t, 2te^{t^2})}$$

Find the velocity when  $t = 0$ .

$$\vec{v}(\pi) = \boxed{(1, 1, 0)}$$

Find a parametrization for the tangent line to  $\vec{x}$  at the point where  $t = 0$ , so that  $L(0) = \vec{x}(0)$ .

$$L(t) = \boxed{(0, 0, 1) + t(1, 1, 0)}$$

## Written Problems

**Problem 14** (a) Graph the surface  $z^2 = x^2 + y^2$  and the curve  $\vec{x}(t) = (t \cos(t), t \sin(t), t)$  for  $-5 \leq t \leq 5$ .

(b) Verify algebraically that the curve lies on the surface.

**Problem 15** (a) Graph the surface  $1 = x^2 + y^2 + z^2$  and the curve  $\vec{x}(t) = (\cos(8t) \sin(t), \sin(8t) \sin(t), \cos(t))$  for  $0 \leq t \leq \pi$ .

(b) Verify algebraically that the curve lies on the surface.

**Problem 16** Prove the following product rule for cross products.

Let  $\vec{x}$  and  $\vec{y}$  be paths in  $\mathbb{R}^3$ , then

$$(\vec{x} \times \vec{y})'(t) = \vec{x}'(t) \times \vec{y}(t) + \vec{x} \times \vec{y}'(t),$$

for  $t$  such that  $x'(t)$  and  $y'(t)$  exist.

**Problem 17** Consider the path  $\vec{x}(t) = (3t - 3^3, 3t^2)$ , for  $t \in \mathbb{R}$ .

(a) Graph  $\vec{x}$ .

(b) Find the point  $P$  where  $\vec{x}$  intersects itself.

(c) There are two tangent vectors  $\vec{x}$  at  $P$ , one for each time the path passes through this point. Find the angle between these two vectors.

**Problem 18** Consider the unit circle  $x^2 + y^2 = 1$  in  $\mathbb{R}^2$ , and consider all lines  $l_t$  passing through the point  $(1, 0)$ , indexed by their slopes  $t$ . For the line of slope  $t$ , let  $\vec{x}(t)$  be the point (other than  $(0, 1)$ ) where the line  $l_t$  intersects the unit circle.

(a) Find  $\vec{x}(0)$ ,  $\vec{x}(1)$ , and  $\vec{x}(-1)$ .

(b) Find an equation for  $l_t$ .

(c) Use your equation for  $l_t$  and the equation for the unit circle to find  $\vec{x}(t)$  in terms of only  $t$ .

*Homework 3: Parametrized Curves*

- (d) Consider the path  $\vec{x}(t)$ , for  $t \in \mathbb{R}$ , given by your answer to (c). What curve does this path parametrize? Are there any “missing” points?
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