Homework 5: Moving Frames and Acceleration

SOME PROBLEMS COPIED FROM HANDOUTS - NOT SURE OF THEIR SOURCE

Completion Packet

Problem 1 *Find the velocity, acceleration and speed of a particle with the position function* $\mathbf{x}(t) = (t, t^2, 2)$. *Sketch the path of the particle* by hand, *and draw the velocity and acceleration vectors on your graph for* $t = 1$.

Problem 2 *Find the velocity, acceleration and speed of a particle with the position function* $\mathbf{x}(t) = (t, 2\cos t, \sin t)$. *Sketch the path of the particle* by hand, *and draw the velocity and acceleration vectors on your graph for* $t = 0$.

Problem 3 *Find the moving frame for the path* $\vec{x}(t) = (2t, 6\sin(3t), -6\cos(3t))$.

Problem 4 *Find the the acceleration of the path* $\vec{x}(t) = (t, t^2)$, *and express it as a linear combination of the unit tangent vector and the unit normal vector.*

Problem 5 *Find the the acceleration of the path* $\vec{x}(t) = (e^t, t)$, *and express it as a linear combination of the unit tangent vector and the unit normal vector.*

Problem 6 *Suppose we have a path* $\vec{x}(t)$ *and a point* $\vec{c}$ *in* $\mathbb{R}^3$ *such that* $\|\vec{x}(t) - \vec{c}\| = R$ *for some constant* R *and for all* t.

(a) *Geometrically, what can you say about this path?*

Learning outcomes:
Author(s):

(b) *Prove that $\vec{v}(t)$ is perpendicular to $\vec{x}(t) - \vec{c}$.*

(c) *If $\vec{x}(t)$ has constant speed, show that $\kappa(t)$ is nonzero for all t.*

Problem 7 *Prove that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.*

Problem 8 *Find the tangential and normal components of the acceleration vector for $\mathbf{x}(t) = (1 + t, t^2 - 2t)$.*

Problem 9 *Find the tangential and normal components of the acceleration vector for $\mathbf{x}(t) = (t, t^2, 3t)$.*

Problem 10 *Compute the following limits. Show all work. ("Wolfram Alpha" is not work.)*

(a) $\displaystyle\lim_{x\to 3} \frac{x^2 - 9}{x^2 + 2x - 3}$

(b) $\displaystyle\lim_{x\to \pi^-} \ln(\sin x)$

(c) $\displaystyle\lim_{v\to 4^+} \frac{4 - v}{|4 - v|}$

(d) $\displaystyle\lim_{x\to \infty} (\sqrt{x^2 + 4x + 1} - x)$

(e) $\displaystyle\lim_{x\to \infty} e^{x - x^2}$

(f) $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{x}$

Graded Problems

Problem 11 *Find the moving frame for the path $\vec{x}(t) = (e^t \cos(t), e^t \sin(t), 5)$.*

Problem 12 *Prove that*

$$\frac{|(\vec{v} \times \vec{a}) \cdot \vec{a}'|}{\|\vec{v} \times \vec{a}\|^2} = \frac{\left\|d\vec{B}/dt\right\|}{|ds/dt|}.$$

Professional Problem

Problem 13 (a) *Beginning with the formula* $\mathbf{a} = s''\mathbf{T} + (s')^2\kappa\mathbf{N}$, *cross both sides with the velocity vector* $\mathbf{v} = s'\mathbf{T}$ *and derive the formula*

$$\kappa = \frac{||\mathbf{v} \times \mathbf{a}||}{(s')^3}.$$

Make sure to write your solution in your own words!

(b) *Let* $y = f(x)$. *The graph of* $y = f(x)$ *is therefore a plane curve. Assuming* $f \in C^2$ *(that is, it is twice differentiable and its second derivatives are continuous), use the formula from (a) to prove the following well-known formula for the curvature of such a curve:*

$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}.$$