Homework 3: Parametrized Curves

Online Problems

Problem 1 *Two ants are running on the top of a table. Their paths are described by*

$$\vec{x}(t) = (t^2 + 1, 2t - 1)$$

and

$$\vec{y}(t) = (\sqrt{t+3}, t),$$

with coordinates in inches, for $t \geq 0$ *in seconds.*

At what time do the ants collide?

$$t = \boxed{1}$$

Where do the ants collide?

$$(x, y) = \boxed{(2, 1)}$$

Problem 2 *Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.*

PICTURE

Which is the graph of the path $\vec{x}(t) = (4 - t, 2t + 1)$, *for* $-1 \leq t \leq 1$?

Multiple Choice:

(a) *(a)*

(b) *(b)*

(c) *(c)*

(d) *(d)*

(e) *(e)*

Learning outcomes:
Author(s):

Problem 3 *Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.*

PICTURE

Which is the graph of the path $\vec{x}(t) = (3\sin(t), 2\cos(t))$, for $0 \leq t \leq \pi$?

Multiple Choice:

(a) *(a)*

(b) *(b)*

(c) *(c)*

(d) *(d)*

(e) *(e)*

Problem 4 *Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.*

PICTURE

Which is the graph of the path $\vec{x}(t) = (t^2, t^3)$, for $-1 \leq t \leq 1$?

Multiple Choice:

(a) *(a)*

(b) *(b)*

(c) *(c)*

(d) *(d)*

(e) *(e)*

Problem 5 *Several parametrized curves are graphed below, and the arrow indicates the direction in which the parameter increases.*

PICTURE

Which is the graph of the path $\vec{x}(t) = (e^t, t)$, for $-1 \leq t \leq 1$?

Multiple Choice:

(a) *(a)*

(b) *(b)*

(c) *(c)*

(d) *(d)*

(e) *(e)*

Problem 6 *Consider the curve below.*

PICTURE

Which of the following are parametrizations for the curve? Select all that apply.

Select All Correct Answers:

(a) $\vec{x}(t) = (\cos t, \sin t)$, *for* $0 \leq t \leq 2\pi$

(b) $\vec{x}(t) = (\sin t, \cos t)$, *for* $0 \leq t \leq \pi$

(c) $\vec{x}(t) = (\cos t, -\sin t)$, *for* $0 \leq t \leq \pi$ ✓

(d) $\vec{x}(t) = (-\cos t, \sin t)$, *for* $0 \leq t \leq \pi$

(e) $\vec{x}(t) = (\sin t, \cos t)$, *for* $\pi/2 \leq t \leq \pi/2$ ✓

(f) $\vec{x}(t) = (\cos t, \sin t)$, *for* $\pi \leq t \leq 2\pi$ ✓

(g) $\vec{x}(t) = (-\sqrt{1-t^2}, t)$ *for* $-1 \leq t \leq 1$

(h) $\vec{x}(t) = (\sqrt{1-t^2}, t)$ *for* $0 \leq t \leq 1$

(i) $\vec{x}(t) = (t, -\sqrt{1-t^2})$ *for* $-1 \leq t \leq 1$ ✓

Problem 7 *Consider the curve below.*

PICTURE

Which of the following are parametrizations for the curve? Select all that apply.

Select All Correct Answers:

(a) $\vec{x}(t) = (t, \sin t)$ *for* $0 \leq t \leq 2\pi$ ✓

(b) $\vec{x}(t) = (\arcsin t, t)$ *for* $-1 \leq t \leq 1$

(c) $\vec{x}(t) = (2t^2, \sin(2t^2))$ *for* $-1 \leq t \leq 1$ ✓

(d) $\vec{x}(t) = \left(t, \cos\left(\frac{\pi/2}{(}t-1)\right)\right)$ *for* $0 \le t \le 2$

(e) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{t}\right)\right)$ *for* $0 \le t \le 2$

(f) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{(}t-1)\right)\right)$ *for* $0 \le t \le 2$

(g) $\vec{x}(t) = \left(\frac{\pi}{2}t, \cos\left(\frac{\pi/2}{(}t-1)\right)\right)$ *for* $0 \le t \le 4$ ✓

Problem 8 *Consider the path* $\vec{x}(t) = (3\cos(t), -2\sin(t))$, *for* $t \in \mathbb{R}$.

Compute the velocity of $\vec{x}$.

$$\vec{v}(t) = \boxed{(-3\sin(t), -2\cos(t))}$$

Compute the speed of $\vec{x}$.

$$\|\vec{x}'(t)\| = \boxed{\sqrt{13}}$$

Problem 9 *Consider the path* $\vec{x}(t) = (\cos(t^4), \sin(t^4), \frac{1}{2}t^4)$, *for* $t \ge 0$.

Compute the velocity.

$$\vec{v}(t) = \boxed{(-4t^3\sin(t^4), 4t^3\cos(t^2), 2t^3)}$$

Compute the speed.

$$\|\vec{x}'(t)\| = \boxed{\sqrt{20t^3}}$$

Problem 10 *Consider the curve* $\vec{x}(t) = (4t+2, 1-3t)$ *for* $t \in \mathbb{R}$.

Compute the velocity.

$$\vec{v}(t) = \boxed{(4, -3)}$$

Compute the speed.

$$\|\vec{x}'(t)\| = \boxed{4}$$

Problem 11 *Consider the curve* $\vec{x}(t) = (2\cos t, 5\sin t, t^2)$ *for* $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(-2\sin t, 5\cos t, 2t)}$$

Find the velocity when $t = \pi$.

$$\vec{v}(\pi) = \boxed{(0, -5, 2\pi)}$$

Find a parametrization for the tangent line to $\vec{x}$ *at the point where* $t = \pi$, *so that* $L(0) = \vec{x}(\pi)$.

$$L(t) = \boxed{(-2, 5, \pi^2) + t(0, -5, 2\pi)}$$

Problem 12 *Consider the curve* $\vec{x}(t) = (t, t^2, t^3)$ *for* $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(1, 2t, 3t^2)}$$

Find the velocity when $t = 2$.

$$\vec{v}(\pi) = \boxed{(1, 4, 12)}$$

Find a parametrization for the tangent line to $\vec{x}$ *at the point where* $t = 2$, *so that* $L(0) = \vec{x}(2)$.

$$L(t) = \boxed{(2, 4, 8) + t(1, 4, 12)}$$

Problem 13 *Consider the curve* $\vec{x}(t) = (t, te^t, e^{t^2})$ *for* $t \in \mathbb{R}$.

Find the velocity.

$$\vec{v}(t) = \boxed{(1, e^t + te^t, 2te^{t^2})}$$

Find the velocity when $t = 0$.

$$\vec{v}(\pi) = \boxed{(1, 1, 0)}$$

Find a parametrization for the tangent line to $\vec{x}$ *at the point where* $t = 0$, *so that* $L(0) = \vec{x}(0)$.

$$L(t) = \boxed{(0, 0, 1) + t(1, 1, 0)}$$

Written Problems

Problem 14 (a) *Graph the surface* $z^2 = x^2 + y^2$ *and the curve* $\vec{x}(t) = (t\cos(t), t\sin(t), t)$ *for* $-5 \le t \le 5$.

(b) *Verify algebraically that the curve lies on the surface.*

Problem 15 (a) *Graph the surface* $1 = x^2 + y^2 + z^2$ *and the curve* $\vec{x}(t) = (\cos(8t)\sin(t), \sin(8t)\sin(t), \cos(t))$ *for* $0 \le t \le \pi$.

(b) *Verify algebraically that the curve lies on the surface.*

Problem 16 *Prove the following product rule for cross products.*

Let $\vec{x}$ *and* $\vec{y}$ *be paths in* $\mathbb{R}^3$, *then*

$$(\vec{x} \times \vec{y})'(t) = \vec{x}'(t) \times \vec{y}(t) + \vec{x} \times \vec{y}'(t),$$

for t *such that* $x'(t)$ *and* $y'(t)$ *exist.*

Problem 17 *Consider the path* $\vec{x}(t) = (3t - 3^3, 3t^2)$, *for* $t \in \mathbb{R}$.

(a) *Graph* $\vec{x}$.

(b) *Find the point* P *where* $\vec{x}$ *intersects itself.*

(c) *There are two tangent vectors* $\vec{x}$ *at* P, *one for each time the path passes through this point. Find the angle between these two vectors.*

Problem 18 *Consider the unit circle* $x^2 + y^2 = 1$ *in* $\mathbb{R}^2$, *and consider all lines* l_t *passing through the point* $(1, 0)$, *indexed by their slopes* t. *For the line of slope* t, *let* $\vec{x}(t)$ *be the point (other than* $(0, 1)$*) where the line* l_t *intersects the unit circle.*

(a) *Find* $\vec{x}(0)$, $\vec{x}(1)$, *and* $\vec{x}(-1)$.

(b) *Find an equation for* l_t.

(c) *Use your equation for* l_t *and the equation for the unit circle to find* $\vec{x}(t)$ *in terms of only* t.

(d) *Consider the path* $\vec{x}(t)$, *for* $t \in \mathbb{R}$, *given by your answer to (c). What curve does this path parametrize? Are there any "missing" points?*