Homework 7: Partial Derivatives

Completion Packet

Problem 1 *Compute the partial derivatives of the function $f(x, y) = ye^{x^2}$.*

Problem 2 *Compute the partial derivatives of the function $f(x, y) = \ln((x + y)^2)$.*

Problem 3 *Compute the partial derivatives of the function $f(x, y, z) = \sin(x^2 yz)$.*

Problem 4 *Consider the function $f(x, y) = 2x^2 + xy - y^2 + 3y + 1$.*

(a) *Explain how you can tell that this function is differentiable at the point $(2, 1)$.*

(b) *Find an equation for the tangent plane to the graph of f at the point $(2, 1)$.*

Problem 5 *Consider the function $f(x, y) = e^{x^2 + y^2}$.*

(a) *Explain how you can tell that this function is differentiable at the point $(1, 2)$.*

(b) *Find an equation for the tangent plane to the graph of f at the point $(1, 2)$.*

Problem 6 *Suppose we have a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(0,0) = 5$, $f(0.1,0) = 5.2$, and $f(0,0.1) = 4.7$.*

(a) *Estimate $f_x(0,0)$ and $f_y(0,0)$.*

(b) *Give an approximate equation for the tangent plane to the graph of f at the point $(0,0)$.*

(c) *Using your approximation from (b), estimate the value of $f(-0.2, 0.2)$.*

Problem 7 *Consider the function*

$$f(x,y) = \begin{cases} \dfrac{x^3 + 2x^2 y - xy^2 + 3y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

(a) *Compute the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ for $(x,y) \neq (0,0)$.*

(b) *Compute the partial derivatives $f_x(0,0)$ and $f_y(0,0)$.*

Problem 8 *Consider the function $f(x,y) = e^{x+y}$.*

(a) *Compute the partial derivatives $f_x(0,0)$ and $f_y(0,0)$.*

(b) *Graph the function.*

(c) *Based on your graph, is f differentiable at $(0,0)$?*

Problem 9 *Consider the function $f(x,y) = (xy)^{2/3}$.*

(a) *Compute the partial derivatives $f_x(1,1)$, $f_y(1,1)$, $f_x(0,0)$, and $f_y(0,0)$.*

(b) *Graph the function.*

(c) *Based on your graph, is f differentiable at $(1,1)$? Is f differentiable at $(0,0)$?*

Graded Problems

Problem 10 *Consider the function* $f(x, y) = x^2 + y^2$.

(a) *Find an equation for the tangent plane to the graph of* $f(x, y)$ *at the point* (a, b).

(b) *At which point(s)* (a, b) *is the tangent plane parallel to the plane* $x + 2y - z = 0$?

Problem 11 *Find the partial derivatives of the function*

$$f(x_1, ..., x_n) = x_1 x_2^2 x_3^3 x_4^4 \cdots x_n^n.$$