Homework 1: Coordinates

Graded Problems

Problem 1 *Consider the surface in $\mathbb{R}^3$ described by the equation $(r-2)^2 + z^2 = 1$ in cylindrical coordinates.*

(a) *Sketch the intersection of this surface with the xy-plane.*

(b) *Given that θ does not appear in the equation, what can you say about the surface?*

(c) *Sketch the surface.*

Problem 2 (a) *Suppose we have a curve in $\mathbb{R}^2$ defined by the polar equation $r = f(\theta)$, for some function f. What is the relationship between this curve and the curve defined by $r = f(\theta + \pi/4)$ in polar coordinates?*

(b) *Suppose we have a surface in $\mathbb{R}^3$ defined by the cylindrical equation $r = g(\theta, z)$, for some function g. What is the relationship between this surface and the surface defined by $r = g(\theta + \pi/4, z)$ in cylindrical coordinates?*

(c) *Suppose we have a surface in $\mathbb{R}^3$ defined by the spherical equation $r = h(\theta, \phi)$, for some function h. What is the relationship between this surface and the surface defined by $r = h(\theta + \pi/4, \phi)$ in spherical coordinates?*

(d) *Again, suppose we have a surface in $\mathbb{R}^3$ defined by the spherical equation $r = h(\theta, \phi)$, for some function h. What is the relationship between this surface and the surface defined by $r = h(\theta, \phi + \pi)$ in spherical coordinates? Hint: it's not a rotation.*
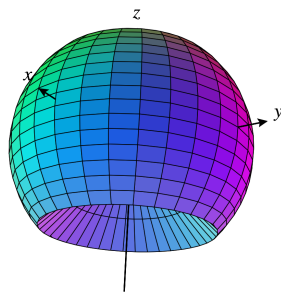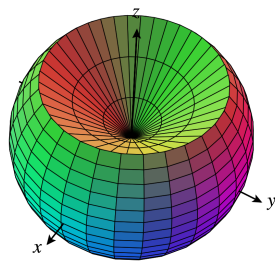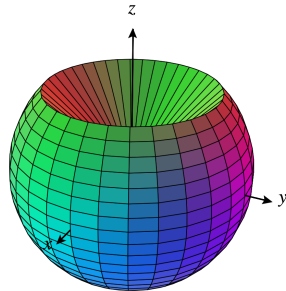
Professional Problem

Problem 3 *Consider the solid below, which is obtained by taking the portion of a solid sphere of radius 2 which is outside of the (double) cone $z^2 = x^2 + y^2$.*

(a) *Is this region easier to describe in spherical coordinates or in cylindrical coordinates? Justify your answer.*

(b) *Describe this region in either spherical coordinates or cylindrical coordinates (based on your answer to (a)).*

Completion Packet

Problem   4 *Consider the point $P = \left(\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$, given in crylindrical coordinates. What are the Cartesian coordinates of P?*

Problem   5 *Consider the point $P = \left(\dfrac{pi}{2}, \dfrac{pi}{2}, \dfrac{pi}{2}\right)$, given in spherical coordinates. What are the spherical coordinates of P?*

Problem   6 *Consider the point $P = (2, 1, 3)$, given in Cartesian coordinates. Find all possible ways to write P in cylindrical coordinates.*

Problem   7 *Consider the point $P = (2, 1, 2)$, given in Cartesian coordinates. Find all possible ways to write P in spherical coordinates.*

Problem   8 *Sketch the graph of the surface in $\mathbb{R}^3$ defined by the equation $\rho = \cos(\phi)$ in spherical coordinates, for $0 \leq \phi \leq \pi$.*

Problem   9 *Consider the surface in $\mathbb{R}^3$ defined by the equation $2x^2 + 2y^2 + z^2 = 1$ in Cartesian coordinates.*

Convert the equation to cylindrical coordinates.

Convert the equation to spherical coordinates.

Sketch the surface.

Problem   10 *Sketch the region defined by the inequalities $\pi/4 \leq \theta \leq \pi$, $0 \leq r \leq 2$, and $0 \leq z \leq r$ in cylindrical coordinates.*

Problem 11 *Sketch the region defined by the inequalities $0 \leq \phi \leq \pi$, $0 \leq \theta \leq \pi$, and $0 \leq \rho \leq \theta$.*

Problem 12 *Consider the region defined by the inequalities $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$, and $0 \leq z \leq 2$ in cylindrical coordinates.*

(a) *Sketch this region.*

(b) *Describe this region in spherical coordinates.*