Practice Problems

Online Problems

Problem 1 *Find the Cartesian coordinates of the point* $(\pi/2, \pi, 2)$*, given in cylindrical coordinates.*

$$(x, y, z) = \boxed{(0, -\pi/2, 2)}$$

Problem 2 *Find cylindrical coordinates for the point* $(0, -1, 3)$*, written in Cartesian coordinates. Your answer should satisfy* $0 \leq r$ *and* $0 \leq \theta < 2\pi$*.*

$$(r, \theta, z) = \boxed{(1, \pi, 3)}$$

Problem 3 *Consider the surface described in Cartesian coordinates by*

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in cylindrical coordinates, of the form $0 = f(r, \theta, z)$*.*

$$0 = \boxed{r^2 - 2z^2}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

Multiple Choice:

(a) *Plane*

(b) *Cylinder*

(c) *Sphere*

Learning outcomes:
Author(s):

(d) *Cone* ✓

(e) *Other*

Problem 4 *Consider the following region in* $\mathbb{R}^3$.

IMAGE

This region is the set of points $(r, \theta z)$*, in cylindrical coordinates, satisfying the inequalities*

$$\boxed{1} \leq r \leq \boxed{2}$$
$$\boxed{\pi/2} \leq \theta \leq \boxed{\pi}$$
$$\boxed{-1} \leq z \leq \boxed{1}$$

Problem 5 *For each of the following equations in cylindrical coordinates, select the type of shape they define.*

FIGURE OUT CORRECT ANSWERS

$r = \cos\theta$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$z = r\cos\theta$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$z = -r$

Multiple Choice:

(a) *plane* ✓

(b) *cylinder*

(c) *sphere*

(d) *other*

Problem 6 *Find the Cartesian coordinates of the point* $(2, \pi, \pi/2)$*, given in spherical coordinates.*

$$(x, y, z) = \boxed{(-2, 0, 0)}$$

Problem 7 *Find spherical coordinates for the point* $\left(-\sqrt{2}, sqrt2, 2\sqrt{3}\right)$*, written in Cartesian coordinates. Your answer should satisfy* $0 \leq \rho$*,* $0 \leq \theta \leq 2\pi$*, and* $0 \leq \phi \leq \phi$*.*

$$(\rho, \theta, \phi) = \boxed{(4, 3\pi/4, \pi/6)}$$

Problem 8 *Consider the surface described in Cartesian coordinates by*

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in spherical coordinates, of the form $0 = f(\rho, \theta, \phi)$*.*

$$0 = \boxed{\rho^2 \sin^2 \phi - 2\cos^2 \phi}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

Multiple Choice:

(a) *Plane*

(b) *Cylinder*

(c) *Sphere*

(d) *Cone* ✓

(e) *Other*

Problem 9 *Consider the following region in* $\mathbb{R}^3$.

IMAGE

This region is the set of points (ρ, θ, ϕ)*, in spherical coordinates, satisfying the inequalities*

$$\boxed{0} \leq \rho \leq \boxed{2}$$
$$\boxed{0} \leq \theta \leq \boxed{pi/2}$$
$$\boxed{0} \leq \phi \leq \boxed{pi}$$

Problem 10 *For each of the following equations in spherical coordinates, select the type of shape they define.*

FIGURE OUT CORRECT ANSWERS

$\rho = \cos\phi$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$\rho = \sin\theta$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$\rho\cos\theta\sin\phi = 1$

Multiple Choice:

(a) *plane* ✓

(b) *cylinder*

(c) *sphere*

(d) *other*

Written Problems

Problem 11 *Consider the surface described by* $(r-3)^2+z^2=1$ *in cylindrical coordinates, with the restriction* $r \geq 0$.

(a) *Sketch the intersection of the surface with the half-plane* $\theta = 0$.

(b) *Sketch the intersection of the surface with the half-plane* $\theta = \dfrac{\pi}{2}$.

(c) *Sketch the intersection of the surface with the plane* $z = 0$.

(d) *Sketch the surface.*

Problem 12 *Sketch the region in* $\mathbb{R}^3$ *with cylindrical coordinates satisfying the inequality*

$$r \leq z \leq 4 - 2r$$

Problem 13 (a) *Given a function* f, *consider the graphs of the equations* $r = f(\theta)$ *and* $r = 2f(\theta)$, *in polar coordinates. How are these graphs related?*

(b) *Given a function* f, *consider the graphs of the equations* $\rho = f(\theta, \phi)$ *and* $\rho = 2f(\theta, \phi)$, *in spherical coordinates. How are these graphs related?*

(c) *Given a function* f, *consider the graphs of the equations* $r = f(\theta)$ *and* $r = -f(\theta)$, *in polar coordinates. How are these graphs related?*

(d) *Given a function* f, *consider the graphs of the equations* $\rho = f(\theta, \phi)$ *and* $\rho = -f(\theta, \phi)$, *in spherical coordinates. How are these graphs related?*
