Practice Problems

Online Problems

Problem 1 *Find the Cartesian coordinates of each point, which is given in polar coordinates.* $(r, \theta) = (2, \pi/6)$

$$(x, y) = \boxed{(\sqrt{3}, 1)}$$

$(r, \theta) = (\sqrt{2}, 3\pi/4)$

$$(x, y) = \boxed{(-1, 1)}$$

$(r, \theta) = (1, 0)$

$$(x, y) = \boxed{(1, 0)}$$

$(r, \theta) = (3, \pi)$

$$(x, y) = \boxed{(-3, 0)}$$

Problem 2 *Find the polar coordinates of each point, which is given in Cartesian coordinates. Your answers should satify* $0 \leq r$ *and* $0 \leq \theta < 2\pi$. $(x, y) = (\sqrt{3}, 0)$

$$(r, \theta) = \boxed{(\sqrt{3}, 0)}$$

$(x, y) = (0, 2)$

$$(r, \theta) = \boxed{(2, \pi/2)}$$

$(x, y) = (-1, -1)$

$$(r, \theta) = \boxed{(\sqrt{2}, 5\pi/4)}$$

$(x, y) = (1, \sqrt{3})$

$$(r, \theta) = \boxed{(2, \pi/3)}$$

Learning outcomes:
Author(s):

Problem 3 *Find the Cartesian coordinates of the point $(\pi/2, \pi, 2)$, given in cylindrical coordinates.*

$$(x, y, z) = \boxed{(0, -\pi/2, 2)}$$

Problem 4 *Find cylindrical coordinates for the point $(0, -1, 3)$, written in Cartesian coordinates. Your answer should satisfy $0 \leq r$ and $0 \leq \theta < 2\pi$.*

$$(r, \theta, z) = \boxed{(1, \pi, 3)}$$

Problem 5 *Consider the surface described in Cartesian coordinates by*

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in cylindrical coordinates, of the form $0 = f(r, \theta, z)$.

$$0 = \boxed{r^2 - 2z^2}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

Multiple Choice:

(a) *Plane*

(b) *Cylinder*

(c) *Sphere*

(d) *Cone* ✓

(e) *Other*

Problem 6 *Consider the following region in $\mathbb{R}^3$.*

IMAGE

This region is the set of points $(r, \theta z)$, in cylindrical coordinates, satisfying the inequalities

$$\boxed{1} \leq r \leq \boxed{2}$$
$$\boxed{\pi/2} \leq \theta \leq \boxed{\pi}$$
$$\boxed{-1} \leq z \leq \boxed{1}$$

Problem 7 *For each of the following equations in cylindrical coordinates, select the type of shape they define.*

FIGURE OUT CORRECT ANSWERS

$r = \cos\theta$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$z = r\cos\theta$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$z = -r$

Multiple Choice:

(a) *plane* ✓

(b) *cylinder*

(c) *sphere*

(d) *other*

Problem 8 *Find the Cartesian coordinates of each point, which is given in cylindrical coordinates.* $(r, \theta, z) = (1, 1, 1)$

$$(x, y, z) = \boxed{(\cos(1), \sin(1), 1)}$$

$(r, \theta, z) = (\pi, \pi, \pi)$

$$(x, y, z) = \boxed{(-\pi, 0, \pi)}$$

$(r, \theta, z) = (2, 4\pi/3, -2)$

$$(x, y, z) = \boxed{(-\sqrt{3}, -1, -2)}$$

Problem 9 *Find the cylindrical coordinates of each point, which is given in Cartesian coordinates. Your answers should satisfy* $r \geq 0$ *and* $0 \leq \theta < 2\pi$.
$(x, y, z) = (1, 1, 1)$

$$(r, \theta, z) = \boxed{(\sqrt{2}, \pi/4, 1)}$$

$(x, y, z) = (\pi, \pi, \pi)$

$$(r, \theta, z) = \boxed{(\sqrt{2}\pi, \pi/4, \pi)}$$

$(x, y, z) = (2, 2\sqrt{3}, -2)$

$$(r, \theta, z) = \boxed{(4, \pi/6, -2)}$$

Problem 10 *Find the Cartesian coordinates of the point* $(2, \pi, \pi/2)$, *given in spherical coordinates.*

$$(x, y, z) = \boxed{(-2, 0, 0)}$$

Problem 11 *Find spherical coordinates for the point* $\left(-\sqrt{2}, sqrt2, 2\sqrt{3}\right)$, *written in Cartesian coordinates. Your answer should satisfy* $0 \leq \rho$, $0 \leq \theta \leq 2\pi$, *and* $0 \leq \phi \leq \phi$.

$$(\rho, \theta, \phi) = \boxed{(4, 3\pi/4, \pi/6)}$$

Problem 12 *Consider the surface described in Cartesian coordinates by*

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in spherical coordinates, of the form $0 = f(\rho, \theta, \phi)$.

$$0 = \boxed{\rho^2 \sin^2 \phi - 2\cos^2 \phi}$$

FIGURE OUT HOW TO HANDLE THIS!!! What type of shape is this?

Multiple Choice:

(a) *Plane*

(b) *Cylinder*

(c) *Sphere*

(d) *Cone* ✓

(e) *Other*

Problem 13 *Consider the following region in* $\mathbb{R}^3$.

IMAGE

This region is the set of points (ρ, θ, ϕ), *in spherical coordinates, satisfying the inequalities*

$$\boxed{0} \leq \rho \leq \boxed{2}$$
$$\boxed{0} \leq \theta \leq \boxed{pi/2}$$
$$\boxed{0} \leq \phi \leq \boxed{pi}$$

Problem 14 *For each of the following equations in spherical coordinates, select the type of shape they define.*

FIGURE OUT CORRECT ANSWERS

$\rho = \cos\phi$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$\rho = \sin\theta$

Multiple Choice:

(a) *plane*

(b) *cylinder*

(c) *sphere*

(d) *other*

$\rho \cos\theta \sin\phi = 1$

Multiple Choice:

(a) *plane* ✓

(b) *cylinder*

(c) *sphere*

(d) *other*

Problem 15 *Find the spherical coordinates of each point, which is given in Cartesian coordinates. Your answers should satisfy* $0 \leq r$, $0 \leq \theta < 2\pi$, *and* $0 \leq \phi \leq \pi$. $(x, y, z) = (1, 1, 1)$

$$(\rho, \theta, \phi) = \boxed{(\sqrt{3}, \pi/4, \pi/4)}$$

$(x, y, z) = (1, -1, -1)$

$$(\rho, \theta, \phi) = \boxed{(\sqrt{3}, 3\pi/4, 3\pi/4)}$$

$(x, y, z) = (1, \sqrt{3}, 0)$

$$(\rho, \theta, \phi) = \boxed{(\sqrt{4}, \pi/6, \pi/2)}$$

Problem 16 *Find the Cartesian coordinates of each point, which is given in spherical coordinates.* $(\rho, \theta, \phi) = (\pi, \pi, \pi)$

$$(x, y, z) = \boxed{(0, 0, -\pi)}$$

$(\rho, \theta, \phi) = (3, \pi/2, \pi/4)$

$$(x, y, z) = \boxed{(0, 3\sqrt{2}/2, 3\sqrt{2}/2)}$$

$(\rho, \theta, \phi) = (2, 7\pi/6, 3\pi/4)$

$$(x, y, z) = \boxed{(-\sqrt{6}/2, -\sqrt{2}/2, -\sqrt{2})}$$

Written Problems

Problem 17 *For several values of the constant a, sketch the graph of the curve in $\mathbb{R}^2$ given by the polar equation*

$$r = a\sin\theta.$$

What do you notice about these curves?

Problem 18 *Which points in $\mathbb{R}^2$ have the same coordinates when written in Cartesian and polar coordinates? (That is, for what points do we have $x = r$ and $y = \theta$?)*

Problem 19 *Consider the surface described by $(r-3)^2 + z^2 = 1$ in cylindrical coordinates, with the restriction $r \geq 0$.*

(a) *Sketch the intersection of the surface with the half-plane $\theta = 0$.*

(b) *Sketch the intersection of the surface with the half-plane $\theta = \dfrac{\pi}{2}$.*

(c) *Sketch the intersection of the surface with the plane $z = 0$.*

(d) *Sketch the surface.*

Problem 20 *Sketch the region in $\mathbb{R}^3$ with cylindrical coordinates satisfying the inequality*

$$r \leq z \leq 4 - 2r$$

Problem 21 *Convert the following equation, given in cylindrical coordinates, into Cartesian coordinates.*

$$r = 0$$

Problem 22 *Sketch the region in* $\mathbb{R}^3$ *given by*

$$r \leq z \leq 3$$
$$0 \leq \theta \leq \pi/2$$

in cylindrical coordinates.

Problem 23 **Problem 23.1** *Sketch the region in* $\mathbb{R}^3$ *given by*

$$r^2 - 2 \leq z \leq 2 - r^2$$

in cylindrical coordinates.

Problem 24 *Which points in* $\mathbb{R}^3$ *have the same coordinates when written in Cartesian and cylindrical coordinates?*

Problem 25 (a) *Given a function* f*, consider the graphs of the equations* $r = f(\theta)$ *and* $r = 2f(\theta)$*, in polar coordinates. How are these graphs related?*

(b) *Given a function* f*, consider the graphs of the equations* $\rho = f(\theta, \phi)$ *and* $\rho = 2f(\theta, \phi)$*, in spherical coordinates. How are these graphs related?*

(c) *Given a function* f*, consider the graphs of the equations* $r = f(\theta)$ *and* $r = -f(\theta)$*, in polar coordinates. How are these graphs related?*

(d) *Given a function* f*, consider the graphs of the equations* $\rho = f(\theta, \phi)$ *and* $\rho = -f(\theta, \phi)$*, in spherical coordinates. How are these graphs related?*

Problem 26 *Sketch the surface in* $\mathbb{R}^3$ *given by the equation*

$$1 - \cos\phi$$

in spherical coordinates.

Problem 27 *Consider the surface in* $\mathbb{R}^3$ *given by the equation*

$$\rho \sin\phi \sin\theta = 1$$

in spherical coordinates.

Convert this equation to Cartesian coordinates and cylindrical coordinates, and sketch the surface.

Problem 28 *Consider the region in* $\mathbb{R}^3$ *consisting of points whose spherical coordinates satisfy*

$$1 \leq \rho \leq 3$$

Sketch this region.

Problem 29 *Consider the region in* $\mathbb{R}^3$ *consisting of points whose spherical coordinates satisfy*

$$0 \leq \phi \leq \pi/2$$
$$0 \leq \rho \leq 1$$

Sketch this region.

Problem 30 *Consider the region in* $\mathbb{R}^3$ *consisting of points whose spherical coordinates satisfy*

$$\cos\phi \leq \rho \leq 2$$

Sketch this region.

Problem 31 **Problem 31.1** *Which points in* $\mathbb{R}^3$ *have the same coordinates when written in Cartesian and spherical coordinates?*
