

Practice Problems

Online Problems

Problem 1 Compute the following:

$$(1, 2, 3) + (8, 3, 6) = \boxed{(9, 5, 9)}$$

$$4(1, -2, 4) = \boxed{(4, -8, 16)}$$

$$-12((5, 2, 6) - (8, 2, 4)) = \boxed{(36, 0, 24)}$$

Problem 2 Let h be a constant. Compute the following:

$$(7, 2, -1) + (2h, 0, h) = \boxed{(7 + 2h, 2, h - 1)}$$

$$h(1, 8, 2) = \boxed{(h, 8h, 2h)}$$

Problem 3 For each of the following, determine whether the quantity exists or does not exist.

$$(1, 8, 3, 7) + (-1, 7, 2, 7)$$

Multiple Choice:

- (a) Exists. ✓
- (b) Does not exist.

$$(2, 8, 3) + (1, 7)$$

Multiple Choice:

Learning outcomes:
Author(s):

(a) *Exists.*

(b) *Does not exist.* ✓

$$(2, 7, 3) + 1$$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$$(2, 8, 3)(1, 7, 3)$$

Multiple Choice:

(a) *Exists.*

(b) *Does not exist.* ✓

$$2(7, 2, 3, 7, 2)$$

Multiple Choice:

(a) *Exists.* ✓

(b) *Does not exist.*

Problem 4 For points $P_1 = (2, -3, 7, 1)$ and $P_2 = (-1, 7, 2, 1)$, compute the displacement vector $P_1\vec{P}_2$.

$$P_1\vec{P}_2 = \boxed{(-3, 10, -5, 0)}$$

Problem 5 Write the vector $2\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ in \mathbb{R}^3 in standard vector notation.

$$2\mathbf{i} - 5\mathbf{j} + 2\mathbf{k} = \boxed{(2, -5, 2)}$$

Problem 6 Compute the dot product.

$$(1, 8, 3) \cdot (-2, 6, 0) = \boxed{46}$$

Problem 7 Compute the dot product.

$$(1, -5, 0, 2) \cdot (2, -1, 4, 1) = \boxed{9}$$

Problem 8 Compute the dot product.

$$(1, 8, 3) \cdot (-3, 0, 1) = \boxed{0}$$

What can you conclude about the vectors?

Multiple Choice:

- (a) They're perpendicular. ✓
- (b) They aren't perpendicular.

Problem 9 Compute the dot product.

$$(1, 2, 3) \cdot (-3, -2, -1) = \boxed{-10}$$

What can you conclude about the vectors?

Multiple Choice:

- (a) They're perpendicular.
- (b) They aren't perpendicular. ✓

Problem 10 Compute the dot product.

$$(1, 8, 3, 6) \cdot (3, -3, -1, 4) = \boxed{-0}$$

What can you conclude about the vectors?

Multiple Choice:

- (a) They're perpendicular. ✓
 (b) They aren't perpendicular.

Problem 11 For each expression, determine whether it exists or does not exist.

$$(2, 8, 3) \cdot (1, 8, 2, 4)$$

Multiple Choice:

- (a) Exists.
 (b) Does not exist. ✓
 $(2, 8, 3, 1, 8) \cdot (1, 8, 2, 4, 2)$

Multiple Choice:

- (a) Exists. ✓
 (b) Does not exist.

Problem 12 Compute the angle between the vectors $(2, 8)$ and $(-8, 2)$ in degrees.

$$\theta = \boxed{90}^\circ$$

Problem 13 Compute the angle between the vectors $(2, 1, 3, 1, 1)$ and $(-3, 1, 1, 1, -2)$ in degrees.

$$\theta = \boxed{104.48}^\circ$$

(Give your answer as a positive number to two decimal places.)

Problem 14 Suppose you have vectors \vec{v} and \vec{w} such that $\|\vec{v}\| = 6$ and $\|\vec{w}\| = 2$, and the angle between \vec{v} and \vec{w} is $\frac{\pi}{4}$ radians. Compute the dot product of \vec{v} and \vec{w} .

$$\vec{v} \cdot \vec{w} = \boxed{6\sqrt{2}}$$

Problem 15 Compute the projection of $\vec{v} = (1, 7, 3)$ onto $\vec{w} = (-3, -2, 1)$.

$$\text{proj}_{\vec{w}}(\vec{v}) = \boxed{(7/2, 7/3, -7/6)}$$

Problem 16 Compute the projection of the vector $\vec{v} = (1, 2, 3)$ onto the vector $\vec{w} = (-3, 2, -1)$.

$$\text{proj}_{\vec{w}}(\vec{v}) = \boxed{(0, 0, 0)}$$

Problem 16.1 Why does your answer make sense?

Multiple Choice:

- (a) The vectors are parallel.
- (b) The vectors are perpendicular. ✓
- (c) They are the same length.

Problem 17 Compute the cross product.

$$(1, 7, 3) \times (2, 8, -1) = \boxed{(-31, 7, -6)}$$

Problem 18 Compute the cross product.

$$(-1, 7, 2) \times (2, 8, 3) = \boxed{(5, 7, -22)}$$

Problem 19 For each of the following, determine whether the expression exists or does not exist.

$$(-1, 7, 3) \times (1, 7, 2)$$

Multiple Choice:

- (a) *Exists.* ✓
- (b) *Does not exist.*

$$(-1, 7, 3, 8) \times (1, 7, 2, 0)$$

Multiple Choice:

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$(2, 0, 0) \times (1, 7, 2, 0)$$

Multiple Choice:

- (a) *Exists.*
- (b) *Does not exist.* ✓

$$(0, 0, 0) \times (0, 0, 0)$$

Multiple Choice:

- (a) *Exists.* ✓
- (b) *Does not exist.*

Problem 20 Compute the area of the parallelogram determined by $(1, 6)$ and $(1, 0)$.

$$\text{Area} = \boxed{6}$$

Problem 21 Compute the volume of the parallelepiped determined by $(1, 6, 2)$, $(-1, 2, 0)$, and $(0, 3, 1)$.

$$\text{Volume} = \boxed{2}$$

Problem 22 Suppose \vec{v} and \vec{w} are unit vectors in the xy -plane, and we know that they are perpendicular. What is $\vec{v} \times \vec{w}$?

Multiple Choice:

- (a) $(0, 0, 1)$
- (b) $(0, 0, -1)$
- (c) Not enough information. ✓

Problem 23 Find a parametrization $\vec{x}(t)$ of the line parallel to the vector $(1, 6, 3)$ and through the point $(1, 3, 2)$, such that $\vec{x}(0) = (1, 3, 2)$. $\vec{x}(t) =$
 $(t + 1, 6t + 3, 3t + 2)$

Problem 24 Find a parametrization $\vec{x}(s, t)$ of the plane containing vectors $(1, 6, 2)$ and $(1, 3, 2)$, and passing through the point $(1, 0, 0)$, such that $\vec{x}(0, 0) = (1, 0, 0)$ and $\vec{x}(1, 0) = (1, 6, 2)$.

$$\vec{x}(s, t) = (s + t + 1, 6s + 3t, 2s + 2t)$$

Problem 25 Give an equation which describes the plane perpendicular to the vector $(1, 7, 3)$ and through the point $(-2, 4, 1)$.

$$0 = (x + 2) + 7(y - 4) + 3(z - 1)$$

Written Problems

Problem 26 For any vector \vec{v} in \mathbb{R}^n , prove that $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$.

Practice Problems

Problem 27 Prove that vectors \vec{v} and \vec{w} in \mathbb{R}^n are perpendicular if and only if $\text{proj}_{\vec{w}}(\vec{v})$ is the zero vector.

Problem 28 For any vector \vec{v} in \mathbb{R}^3 , prove that $\vec{v} \times \vec{v}$ is the zero vector.
