Extetion of Chebyshev's Inequality

1 Argument

When specific two progressions,$\{a\}$ and $\{b\}$, both of which have n elements, meet the following conditions

$$\forall k \quad s.t. \quad 1 \leqq k \leqq n, \qquad \prod_{i=1}^{k} a_i \geq \prod_{i=1}^{k} b_i$$

then this formula will be held.

$$\forall k \quad s.t. \quad 1 \leqq k \leqq n, \qquad \sum_{i=1}^{k} a_i - b_i \geq 0$$

1.1 A case where the number of the elements is "2"

The conditions are as follows.

$$\begin{aligned} a_1 &\geq a_2 \\ b_1 &\geq b_2 \\ a_1 \times a_2 &= b_1 \times b_2 \end{aligned}$$

The expression to be proved is as follows.

$$a_1 + a_2 - b_1 - b_2 \geq 0$$

This is proved as so

$$\begin{aligned} a_1 + a_2 - b_1 - b_2 &= a_1 + \frac{b_1 \times b_2}{a_1} - b_1 - b_2 \\ &= \frac{1}{a_1}(a_1 - b_1)(a_1 - b_2) \\ &\geq 0 \end{aligned}$$