Active unsupervised texture segmentation on a diffusion based feature space

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Active Unsupervised Texture Segmentation on a Diffusion Based Feature Space

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Abstract

In this paper, we propose a novel and efficient approach for active unsurpervised texture segmentation. First, we show how we can extract a small set of good features for texture segmentation based on the structure tensor and nonlinear diffusion. Then, we propose a variational framework that incorporates these features in a level set based unsupervised segmentation that adaptively takes into account their estimated statistical information inside and outside the region to segment. The approach has been tested on various textured images, and its performance is favorably compared to recent studies.

1 Introduction

In recent time it has become very popular to use prior knowledge in the field of image segmentation. There are many techniques proposed in the literature, some of them including priors on the shape of objects [12, 7, 22], others, in the field of texture segmentation, use learned models of textures to retrieve them during the segmentation process [18, 4, 14]. With no doubt the usage of appropriate prior knowledge is very important for the ability to deal with difficult image scenes. However, there is very few work so far on how to obtain this prior knowledge automatically. So far, segmentation algorithms with the ability to handle difficult image scenes need a supervised initialization step, where they are told the right segmentations of a training set of images. It would be preferable to extract prior knowledge automatically from simpler image scenes, in order to use it in succeeding scenes that are more difficult to deal with. This requires a powerful image segmentation method that does not depend on prior knowledge, but can nevertheless handle the whole set of possible objects as long as the scene is not spoiled by clutter.

Basically such a segmentation process splits into two parts: the acquisition of suitable features that are powerful enough to discriminate regions that a human observer would describe as different, and a model for the partitioning and the statistics of these features.

Recently, mostly Gabor filters are used to extract texture features for the segmentation [9, 18, 25, 24]. Unfortunately, Gabor filters have the decisive drawback that they induce a lot of redundancy and thus lots of feature channels. An interesting work that helps to overcome this problem is that of Bigün et al. [5]. They used the structure tensor in order to discriminate textures. This method yields only three feature channels for each scale. However, the Gaussian smoothing used for the structure tensor dislocates the edges in feature space leading to inaccurate segmentation results. In [6] a nonlinear structure tensor based on nonlinear matrix-valued diffusion has been proposed that tackles this problem. A very similar outcome is achieved by orientation estimation based on robust statistics [28]. We stick to the idea of the nonlinear structure tensor, yet we use the matrix-valued diffusion technique from [26, 27] that also takes us back to vector-valued diffusion. Vector-valued PDEs were used for smoothing texture features in [24], too. However, their extraction of features based on Gabor filters is completely different.

Once the features are extracted, a segmentation model that accounts for the information must be defined. Variational formulations have been shown to be very powerful in integrating different cues, e.g. boundary information [15] or region information [17]. Moreover, the use of level set functions to represent evolving curves gives many good properties to the segmentation process: the curve is represented implicitly, topological changes are *naturally* possible, it can be used in any dimension and efficient techniques for numerical implementation exist. Another key point of our method is the unsupervised aspect. Following the idea of [21], the minimization of the proposed energy gives the maximum a posteriori segmentation. This EM like approach adapts the region parameters actively to the evolving curve.

Paper organization. In the next section we derive our feature extraction method from the structure tensor and nonlinear diffusion. Section 3 then deals with the adaptive segmentation based on these features. In Section 4 we show some results and compare them to previous approaches. The paper is concluded by a brief summary.

2 Feature Extraction

Our feature extraction approach is based on the classical *structure tensor* [8, 5, 13, 29]

$$J_\rho = K_\rho * (\nabla I \nabla I^\top) = \begin{pmatrix} K_\rho * I_x^2 & K_\rho * I_x I_y \\ K_\rho * I_x I_y & K_\rho * I_y^2 \end{pmatrix} \quad (1)$$

where K_ρ is a Gaussian kernel with standard deviation ρ and subscripts denote partial derivatives. Obviously the structure tensor yields three feature channels for each scale. In order to keep things simple, we will only consider one scale in this paper. One could incorporate further scales by adding additional feature channels very easily. Comparing the number of features obtained by the structure tensor to that of Gabor filters reveals that the degree of freedom for the orientation known from Gabor filters is replaced by the smoothed versions of the image derivatives. The image derivatives hold the whole orientation information, so the components of the structure tensor are as powerful for the discrimination of textures as a whole set of Gabor filters of a fixed scale. Due to the mixed matrix component, texture discrimination is also fully rotation invariant.
The major problem of the classic structure tensor is the dislocation of edges due to the smoothing with Gaussian kernels. This leads to inaccurate segmentation results near region boundaries. The basic idea in [6] to address this problem is the replacement of the Gaussian smoothing by nonlinear diffusion. We stick to this idea but enhance the technique for its application to texture segmentation.
Nonlinear diffusion is based on the early work of Perona and Malik [19]. The main idea is to reduce the smoothing in the presence of edges. The resulting diffusion equation is

$$\partial_t u = \text{div}\left(g(|\nabla u|)\nabla u\right) \quad (2)$$

with $u(t=0)$ being the image I and g a decreasing function.
So far, this equation can only be used with scalar-valued data like a gray value image. Gerig et al. [10] provided a version for vector-valued data

$$\partial_t u_i = \text{div}\left(g\left(\sum_{k=1}^{N} |\nabla u_k|^2\right)\nabla u_i\right) \qquad \forall i \quad (3)$$

where u_i is an evolving vector channel and N the number of channels. Note that all channels are coupled by a joint diffusivity, so an edge in one channel also inhibits smoothing in the others.
If we regard the components of a matrix as components of a vector, what is reasonable, since the Frobenius norm of a matrix equals the Euclidean norm of the resulting vector, it is possible to diffuse a matrix, such as the structure tensor, with the above-mentioned scheme. This yields a nonlinear structure tensor that is a bit different from that in [6]. The matrix-valued diffusion in fact equals a scheme proposed in [26]. Note that according to [30] the coupling of the channels preserves the semipositive definiteness.
A rather critical issue is the good choice of the diffusivity function g. For the application to texture features *total variation (TV) flow* [23, 2, 16] seems to suit very well, since it removes oscillations and leads to piecewise constant results. This is very useful, as the structure tensor contains first derivatives, which have very local responses. Task of the smoothing process is mainly to close the areas between these local phenomena while preserving the important edges. Furthermore, TV flow has the nice property of not causing any additional parameters. However, when the gradient gets close to zero, TV flow leads to numerical problems. This case is circumvented by adding a small positive constant ϵ to the gradient magnitude.

$$g(|\nabla u|) = \frac{1}{|\nabla u| + \epsilon} \quad (4)$$

We deviate a little from the actual structure tensor by adding the image gray value, which is certainly a very important feature, to the feature vector. So finally our features are computed by applying Eq.3 with initial conditions $u_1 = I$, $u_2 = I_x^2$, $u_3 = I_y^2$, $u_4 = I_x I_y$ and the diffusivity function $g(s) = 1/s$. For implementation we apply the AOS scheme [31], that allows efficient computation of TV flow also for small ϵ in the area of 0.001, causing less blurring effects than larger ϵ. In this case AOS is around 4 orders of magnitude faster than a simple explicit scheme.

3 Adaptive segmentation based on these features

Variational formulation. We restrict our study to the segmentation into two textured regions. Basically, this includes all images with one object in front of a background. Following [18], the image segmentation can be found by maximizing the *a posteriori* partitioning probability $p(\mathcal{P}(\Omega)|I)$ where $\mathcal{P}(\Omega) = \{\Omega_1, \Omega_2\}$ is a partitioning of the image domain Ω. Instead of the original image I, we use the vector-valued image $u = (u_1, ..., u_4)$ obtained by smoothing $(I, I_x^2, I_y^2, I_x I_y)$ with Eq.3. Two hypotheses are necessary: all the partitions are equally probable and the pixels within each region are independent. Let $p_1(u(x))$ and $p_2(u(x))$ be the probability density functions for the value $u(x)$ to be in Ω_1 and Ω_2 respectively. With $\partial\Omega$ being the boundary between Ω_1 and Ω_2, the segmentation is found by minimizing the energy

$$E(\Omega_1, \Omega_2) = -\int_{\Omega_1} \log p_1(u(x))\,dx - \int_{\Omega_2} \log p_2(u(x))\,dx. \quad (5)$$

Gaussian approximation for all channels. This formula still lacks the definition of the probability density function (pdf). First, a general Gaussian approximation is used for all four channels to model the statistics of each region. Since the image u is vector-valued, we have to deal with covariance matrices. Let $\{\mu_1, \Sigma_1\}$ and $\{\mu_2, \Sigma_2\}$ be the vector's means and covariance matrices of the Gaussian approximation in Ω_1 and Ω_2. The probability of $u(x)$ to be in Ω_i is:

$$p_i(u(x)) = \frac{1}{(2\pi)^2|\Sigma_i|^{1/2}} e^{-\frac{1}{2}(u(x)-\mu_i)^T \Sigma_i^{-1}(u(x)-\mu_i)} \quad (6)$$

For minimizing the energy, we introduce the level set function $\phi : \Omega \to \Re$ with $\phi(x) = \mathcal{D}(x, \partial\Omega)$ if $x \in \Omega_1$, and $\phi(x) = -\mathcal{D}(x, \partial\Omega)$ if $x \in \Omega_2$. ($\mathcal{D}(x, \partial\Omega)$ stands for the Euclidean distance between x and $\partial\Omega$). Furthermore, let $H_\epsilon(z)$ and $\delta_\epsilon(z)$ be regularized versions of the Heaviside and Dirac functions. Adding a regularization constraint on the length of $\partial\Omega$, the energy (5) can be minimized with respect to the whole set of parameters $\{\partial\Omega, \mu_1, \mu_2, \Sigma_1, \Sigma_2\}$ using the following evolution equation (see [21] for details):

$$\phi_t(x) = \delta_\epsilon \phi(x)) \left(\nu \, \text{div}(\nabla\phi/|\nabla\phi|) + \log \frac{p_1(u(x))}{p_2(u(x))} \right) \quad (7)$$

while the Gaussian parameters are updated at each iteration following:

$$\begin{cases} \mu_i(\phi) = \int_\Omega u(x)\chi_i dx / \int_\Omega \chi_i dx \\ \Sigma_i(\phi) = \int_\Omega (\mu_i - u(x))(\mu_i - u(x))^T \chi_i dx / \int_\Omega \chi_i dx \end{cases} \quad (8)$$

where $\chi_1(z) = H_\epsilon(z)$ and $\chi_2(z) = 1 - H_\epsilon(z)$.

Considering full covariance matrices leads to lots of unknown parameters. This can result in multiple local minima and suboptimal solutions like shown in Fig. 1a. If we further analyze the channels depicted in Fig. 2, we can see that the information in each channel is not much correlated. Thus, the hypothesis of the channels not to be correlated is reasonable, and the pdf $p_i(u(x))$ can be estimated using the joint density probability of each component:

$$p_i(u(x)) = \Pi_{k=1}^4 p_{k,i}(u_k(x)) \quad (9)$$

what is equivalent to consider a diagonal covariance matrix. With this new approximation the energy has only 8 instead of 14 unknown statistical parameters for each region and we obtain the favorable result depicted in Fig. 1b.

Non-parametric approximation for $\mathbf{u_1}$. Seemingly, the Gaussian approximation does not suit well to the first channel, which corresponds to the smoothed version of the gray value image I (see the first feature in Fig. 2). So for this channel, an estimation of the pdf based on the histogram should be preferred. Since we make the assumption that different channels are not correlated, the pdf in each channel can be estimated using different approaches. As in [11], we propose to use a continuous version of the Parzen density for the first channel. The probability of $u_1(x)$ to be in Ω_i is given by:

Figure 1: (a) LEFT: Result with full Σ. (b) RIGHT: Result with diagonal Σ.

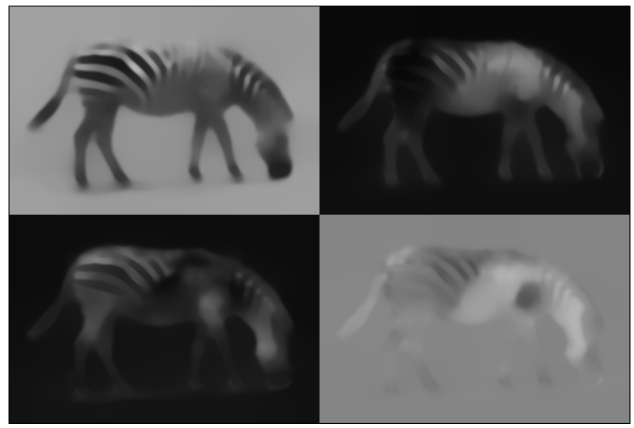

Figure 2: Feature channels $(u_1, .., u_4)$ obtained by smoothing $(I, I_x^2, I_y^2, I_x I_y)$ from left to right and top to bottom.

$$p_{1,i}(u_1(x)) = \frac{1}{|\Omega_i|} \int_{\Omega_i} g_\sigma(u_1(x) - u_1(\hat{x})) \, d\hat{x} \quad (10)$$

where the Gaussian kernel is $g_\sigma(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$. This new approximation gives us a new functional:

$$F(\Omega_i) = \int_{\Omega_i} \log \left(\frac{1}{|\Omega_i|} \int_{\Omega_i} g_\sigma(u_1(x) - u_1(\hat{x})) \, d\hat{x} \right) dx \quad (11)$$

The shape derivative tool introduced in [3] drives to:

$$\begin{aligned} < F'(\Omega_i), V >= - \int_{\Omega_i} \Big(& \log p_{1,i}(u(x)) \\ & + \tfrac{1}{|\Omega_i|} \int_{\Omega_i} \tfrac{g_\sigma(u_1(x) - u_1(\hat{x}))}{p_{1,i}(\hat{x})} \, d\hat{x} \Big) (V.N) da(x) \quad i = \{1, 2\} \end{aligned} \quad (12)$$

Refer to [20] for details on derivation. Using the level set representation, it yields a new evolution equation for ϕ:

$$\phi_t(x) = \delta_\epsilon(\phi)\Bigg(\underbrace{\nu\,\mathrm{div}(\nabla\phi/|\nabla\phi|) + \sum_{k=1}^{4} \log \frac{p_{k,1}(u(x))}{p_{k,2}(u(x))}}_{\text{usual term}} + \underbrace{\sum_{i=\{1,2\}} \frac{1}{|\Omega_i|} \int_\Omega \chi_i(\phi) \frac{g_\sigma(u_1(x)-u_1(\hat{x}))}{p_{1,i}(u(\hat{x}))}\, d\hat{x}}_{\text{additional term}} \Bigg) \tag{13}$$

while Gaussian parameters for the channels u_2, u_3 and u_4 are updated at each iteration according to (8). In this new evolution equation an additional term appears due to the first channel.

The Gaussian approximation is very robust for the texture channels $u_2..u_4$. When it is combined with the non-parametric approach for the gray value channel, the method can deal with a much larger range of images, and experimental results are improved a lot, as can be seen in Fig. 3.

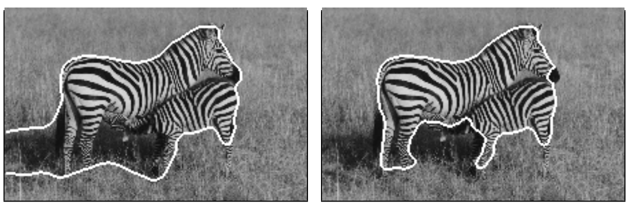

Figure 3: (a) LEFT: Result with Gaussian approximation for each channel. (b) RIGHT: Result with non-parametric approximation for the 1st channel.

Implementation remarks. To implement the evolution equation (13), we used an explicit scheme with centered finite difference to estimate the curvature. The level set is updated only in a narrow band around its zero crossing (12 pixels width). Therefore, the active area is relatively small, and it is possible to reinitialize the level set function to the distance function at each iteration in reasonable time. We used the reinitialization method described in [1] which limits the displacements of the zero level. For all the examples shown in the next section, the same initialization of the level set and the same parameters were used. Although our initialization with small circles yields the active region to cover almost the whole image at the beginning, the final solution is reached after only few iterations (mostly less than 30). Other initializations lead to the same result (see Fig. 8), so the method seems to be quite robust. Two parameters must be set, the regularization weight ν and the Parzen window size σ, we set them once and for all: $\nu = 0.5$ and $\sigma = 5$. We were able to get good results on a wide range of textured images with these parameters.

Since we use an explicit scheme and so a small time step, the second order terms in (13) are negligible and can be omitted. With this approximation the curve evolution takes around ten seconds on 250x200 images and standard hardware. Applying AOS, the same time is taken by the feature extraction part.

4 Results

We first tested the performance of our method with synthetic images composed of textures from the Brodatz texture database. Fig. 4a and 4c reveal our method to work quite fine with common textured images. Note the average gray value between the two textured regions not to be much different, and there is also no dominant orientation for the textures in Fig. 4c. Also note that the correct way of smoothing the structure tensor is very important to get such good results. Fig. 4b shows that the segmentation fails completely if the smoothing is omitted. Actually the smoothing is responsible for the fact that the three feature channels based on the nonlinear structure tensor can keep up with a set of Gabor features. However, Fig. 4d also shows the limitation that was already mentioned in Section 2: if two textures can only be distinguished due to their scale, there is no possibility for the structure tensor of only one scale to separate the regions. This problem could be fixed easily by adding the features of a structure tensor of different scale.

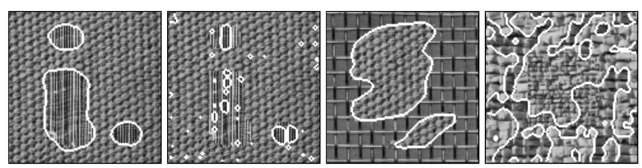

Figure 4: FROM LEFT TO RIGHT: (a) Result for a synthetic test image. (b) Segmentation fails if the feature channels are not smoothed. (c) Result for a synthetic test image with non-oriented texture. (d) Segmentation fails if texture only differs in scale.

More interesting than synthetic images are real-world images. Fig. 5 proves our method to be fully competitive to recent approaches published in [11] and [25] (these results as well as the results shown Fig. 6 compare our method to recent studies by using the images of the original articles). Considering the results from the method in [24], where they used smoothed features based on Gabor filters, our method uses less feature channels and compares favorably. Fig. 6b shows that we can even improve the good results obtained in [18] where a supervised scheme was used. Finally, Fig. 7 illustrates the capabilities of our approach on another set of synthetic and natural images.

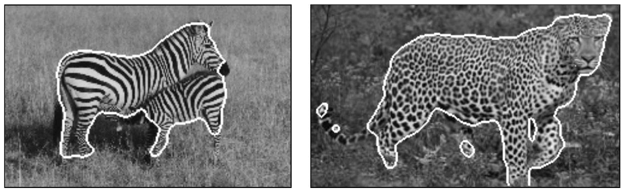

Figure 5: Our results on test images from: (a) LEFT: [25]. (b) RIGHT: [11].

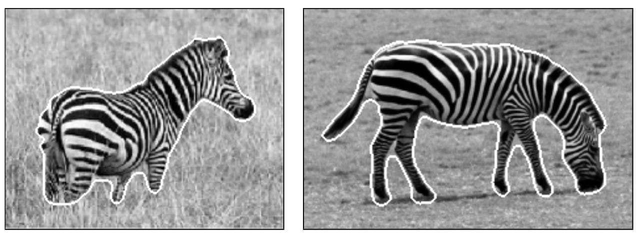

Figure 6: Our results on test images from: (a) LEFT: [24]. (b) RIGHT: [18].

So obviously, our features based on the nonlinear structure tensor are very powerful in discriminating different textured regions, and our dynamic modelling of the active regions is able to incorporate the features in a way that enables the algorithm to cope with real textured images. We also want to stress that our method is almost free of parameters. The parameters that appear for the nonlinear diffusion and the active contour are very robust and can be set to fixed values. All our results have been computed with the same parameters. This property is very important for an unsupervised approach, because we think an approach is not really unsupervised, if for each image someone has to figure out the right parameters first. We are also in the process to apply our method to many more images.

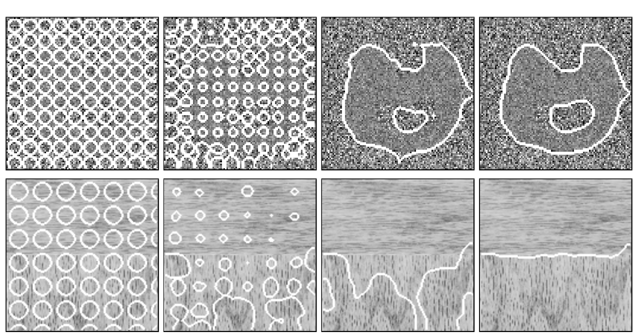

Figure 7: Contour evolution. TOP: Synthetic image with regions of same mean but different variance. BOTTOM: *Wood* image.

5 Conclusions

Two novel ideas were proposed and applied in order to cope with unsupervised texture segmentation. First, the idea of using the structure tensor for feature extraction was emphasized and a nonlinear version was shown to be competitive or even superior to a whole set of Gabor features. Second, we have proposed a new segmentation model based on a variational formulation and on the level representation. Starting from a general multi-dimensional Gaussian to approximate region information of the feature channels, we have simplified and specialized the model so as to get a robust segmentation process. The robustness of the approach allowed to fix all appearing parameters. The comparison of our method to some recent approaches was very convincing. It was possible to reproduce or even improve the former results. Of course, there still exist several examples where our method is not appropriate. One problem is caused by textures that only differ in their scale. Another limitation is given by the fact that we only consider two regions. Our future research will focus on resolving these problems.

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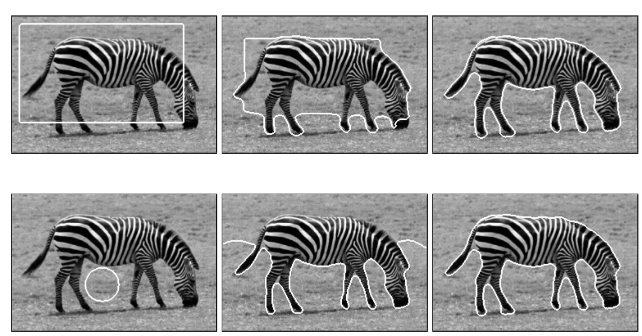

Figure 8: Curve evolution with two different level set initializations.

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