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# Problem Set - 19 Jan 2024

## PROBLEM 1 (2019 AMC 10B #2)

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Consider the statement, "If  $n$  is not prime, then  $n - 2$  is prime." Which of the following values of  $n$  is a counterexample to this statement?

- (A) 11      (B) 15      (C) 19      (D) 21      (E) 27

## PROBLEM 2 (2019 AMC 8 #13)

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A *palindrome* is a number that has the same value when read from left to right or from right to left. (For example, 12321 is a palindrome.) Let  $N$  be the least three-digit integer which is not a palindrome but which is the sum of three distinct two-digit palindromes. What is the sum of the digits of  $N$ ?

- (A) 2      (B) 3      (C) 4      (D) 5      (E) 6

## PROBLEM 3 (2014 AMC 10A #5)

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On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz?

- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5

## PROBLEM 4 (2017 USAJMO #1)

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Prove that there are infinitely many distinct pairs  $(a, b)$  of relatively prime positive integers  $a > 1$  and  $b > 1$  such that  $a^b + b^a$  is divisible by  $a + b$ .

## PROBLEM 5 (2014 IMO #3)

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Convex quadrilateral  $ABCD$  has  $\angle ABC = \angle CDA = 90^\circ$ . Point  $H$  is the foot of the perpendicular from  $A$  to  $BD$ . Points  $S$  and  $T$  lie on sides  $AB$  and  $AD$ , respectively, such that  $H$  lies inside  $\triangle SCT$  and

$$\angle CHS - \angle CSB = 90^\circ, \quad \angle THC - \angle DTC = 90^\circ.$$

Prove that line  $BD$  is tangent to the circumcircle of  $\triangle TSH$ .