Problem Set - 19 Jan 2024

PROBLEM 1 (2018 AMC 8 #11)

Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.

$$\begin{matrix} X & X & X \\ X & X & X \end{matrix}$$

If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column?
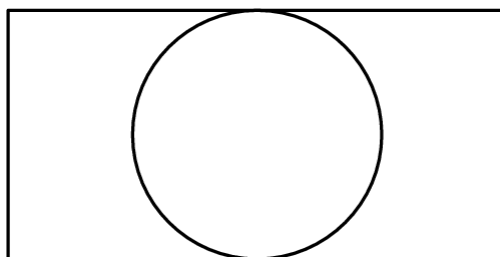
(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{7}{15}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

PROBLEM 2 (2011 UNCO MATH CONTEST II #1)

The largest integer n so that 3^n evenly divides $9! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9$ is $n = 4$. Determine the largest integer n so that 3^n evenly divides $85! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots 84 \cdot 85$.

PROBLEM 3 (2012 AMC 12B #2)

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?

(A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

PROBLEM 4 (2016 AMC 10A #16)

A triangle with vertices $A(0,2)$, $B(-3,2)$, and $C(-3,0)$ is reflected about the x-axis, then the image $\triangle A'B'C'$ is rotated counterclockwise about the origin by $90°$ to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

(A) counterclockwise rotation about the origin by $90°$.

(B) clockwise rotation about the origin by $90°$.

(C) reflection about the x-axis

(D) reflection about the line $y = x$

(E) reflection about the y-axis.

PROBLEM 5 (2019 AIME II #8)

The polynomial $f(z) = az^{2018} + bz^{2017} + cz^{2016}$ has real coefficients not exceeding 2019, and $f\left(\frac{1+\sqrt{3}i}{2}\right) = 2015 + 2019\sqrt{3}i$. Find the remainder when $f(1)$ is divided by 1000.