Problem Set - 19 Jan 2024

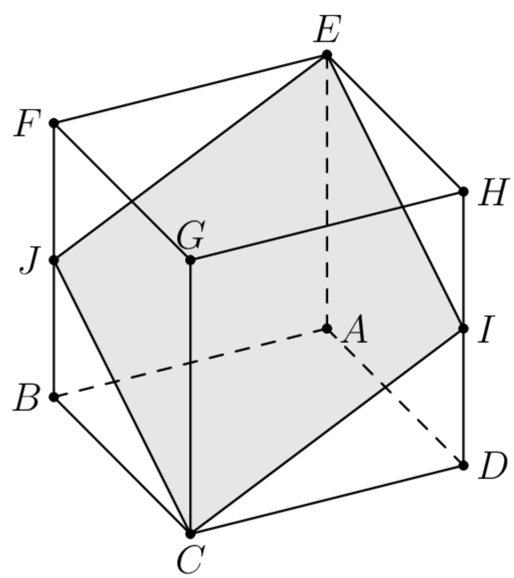
PROBLEM 1 (2018 AMC 8 #10)

The harmonic mean of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?

(A) $\frac{3}{7}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{12}{7}$ **(D)** $\frac{7}{4}$ **(E)** $\frac{7}{3}$

PROBLEM 2 (2018 AMC 8 #24)

In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of segments $\overline{FB}$ and $\overline{HD}$, respectively. Let R be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is R^2?



(A) $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{25}{16}$ **(E)** $\frac{9}{4}$

PROBLEM 3 (2012 AMC 12B #13)

Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers, each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas will have at least one point in common?

(A) $\frac{1}{2}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{31}{36}$ **(E)** 1

PROBLEM 4 (2010 AIME II #3)

Let K be the product of all factors $(b - a)$ (not necessarily distinct) where a and b are integers satisfying $1 \leq a < b \leq 20$. Find the greatest positive integer n such that 2^n divides K.

PROBLEM 5 (2022 AIME II #10)

Find the remainder when

$$\binom{\binom{3}{2}}{2} + \binom{\binom{4}{2}}{2} + \cdots + \binom{\binom{40}{2}}{2}$$

is divided by 1000.