Problem Set - 19 Jan 2024

PROBLEM 1 (2014 AMC 10B #7)

Suppose $A > B > 0$ and A is $x\%$ greater than B. What is x?

$\textbf{(A)}\ 100\left(\frac{A-B}{B}\right) \qquad \textbf{(B)}\ 100\left(\frac{A+B}{B}\right) \qquad \textbf{(C)}\ 100\left(\frac{A+B}{A}\right) \qquad \textbf{(D)}\ 100\left(\frac{A-B}{A}\right) \qquad \textbf{(E)}\ 100\left(\frac{A}{B}\right)$

PROBLEM 2 (2020 AIME I #7)

A club consisting of 11 men and 12 women needs to choose a committee from among its members so that the number of women on the committee is one more than the number of men on the committee. The committee could have as few as 1 member or as many as 23 members. Let N be the number of such committees that can be formed. Find the sum of the prime numbers that divide N.

PROBLEM 3 (2014 AIME I #7)

Let w and z be complex numbers such that $|w| = 1$ and $|z| = 10$. Let $\theta = \arg \left(\tfrac{w-z}{z}\right)$. The maximum possible value of $\tan^2 \theta$ can be written as $\tfrac{p}{q}$, where p and q are relatively prime positive integers. Find $p+q$. (Note that $\arg(w)$, for $w \neq 0$, denotes the measure of the angle that the ray from 0 to w makes with the positive real axis in the complex plane)

PROBLEM 4 (2019 AIME I #11)

In $\triangle ABC$, the sides have integer lengths and $AB=AC$. Circle ω has its center at the incenter of $\triangle ABC$. An *excircle* of $\triangle ABC$ is a circle in the exterior of $\triangle ABC$ that is tangent to one side of the triangle and tangent to the extensions of the other two sides. Suppose that the excircle tangent to $\overline{BC}$ is internally tangent to ω, and the other two excircles are both externally tangent to ω. Find the minimum possible value of the perimeter of $\triangle ABC$.

PROBLEM 5 (2014 IMO #2)

Let $n \ge 2$ be an integer. Consider an $n \times n$ chessboard consisting of n^2 unit squares. A configuration of n rooks on this board is $\textit{peaceful}$ if every row and every column contains exactly one rook. Find the greatest positive integer k such that, for each peaceful configuration of n rooks, there is a $k \times k$ square which does not contain a rook on any of its k^2 squares.