Problem Set - 19 Jan 2024

PROBLEM 1 (2018 AMC 10B #4)

A three-dimensional rectangular box with dimensions X, Y, and Z has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is X + Y + Z?

(A) 18

(B) 22

(C) 24

(D) 30

(E) 36

PROBLEM 2 (2016 AMC 12B #13)

Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

(A) 3.5

(B) 4

(C) 4.5

(D) 5

(E) 5.5

PROBLEM 3 (2020 AMC 12A #14)

Regular octagon ABCDEFGH has area n. Let m be the area of quadrilateral ACEG. What is $\frac{m}{n}$?

(A) $\frac{\sqrt{2}}{4}$

(B) $\frac{\sqrt{2}}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{3\sqrt{2}}{5}$ **(E)** $\frac{2\sqrt{2}}{3}$

PROBLEM 4 (2019 AMC 10A #20)

The numbers $1, 2, \dots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

 $(A) \frac{1}{21}$

(B) $\frac{1}{14}$ **(C)** $\frac{5}{63}$ **(D)** $\frac{2}{21}$ **(E)** $\frac{1}{7}$

PROBLEM 5 (2021 AIME | #15)

Let S be the set of positive integers k such that the two parabolas

$$y = x^2 - k$$
 and $x = 2(y - 20)^2 - k$

intersect in four distinct points, and these four points lie on a circle with radius at most 21. Find the sum of the least element of S and the greatest element of S.