Problem Set - 19 Jan 2024

PROBLEM 1 (2017 AMC 8 #5)

What is the value of the expression $\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}{1+2+3+4+5+6+7+8}$?

$\textbf{(A)}\ 1020 \qquad \textbf{(B)}\ 1120 \qquad \textbf{(C)}\ 1220 \qquad \textbf{(D)}\ 2240 \qquad \textbf{(E)}\ 3360$

PROBLEM 2 (2013 AMC 8 #11)

Ted's grandfather used his treadmill on 3 days this week. He went 2 miles each day. On Monday he jogged at a speed of 5 miles per hour. He walked at the rate of 3 miles per hour on Wednesday and at 4 miles per hour on Friday. If Grandfather had always walked at 4 miles per hour, he would have spent less time on the treadmill. How many minutes less?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

PROBLEM 3 (2010 AMC 12A #8)

Triangle ABC has $AB=2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

$\textbf{(A)}\ 60^\circ \qquad \textbf{(B)}\ 75^\circ \qquad \textbf{(C)}\ 90^\circ \qquad \textbf{(D)}\ 105^\circ \qquad \textbf{(E)}\ 120^\circ$

PROBLEM 4 (2021 AMC 10A #20)

In how many ways can the sequence $1,2,3,4,5$ be rearranged so that no three consecutive terms are increasing and no three consecutive terms are decreasing?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 44$

PROBLEM 5 (2019 IMO #1)

Let $\mathbb{Z}$ be the set of integers. Determine all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that, for all integers a and b,

$$f(2a)+2f(b)=f(f(a+b)).$$