Problem Set - 19 Jan 2024

PROBLEM 1 (2021 AMC 12A #9)

Which of the following is equivalent to

$$(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})?$$

(A) $3^{127}+2^{127}$ **(B)** $3^{127}+2^{127}+2\cdot 3^{63}+3\cdot 2^{63}$ **(C)** $3^{128}-2^{128}$ **(D)** $3^{128}+2^{128}$ **(E)** 5^{127}

PROBLEM 2 (2018 AMC 10B #18)

Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

(A) 60 **(B)** 72 **(C)** 92 **(D)** 96 **(E)** 120

PROBLEM 3 (2014 AMC 12B #15)

When $p=\sum_{k=1}^{6} k\ln k$, the number e^p is an integer. What is the largest power of 2 that is a factor of e^p?

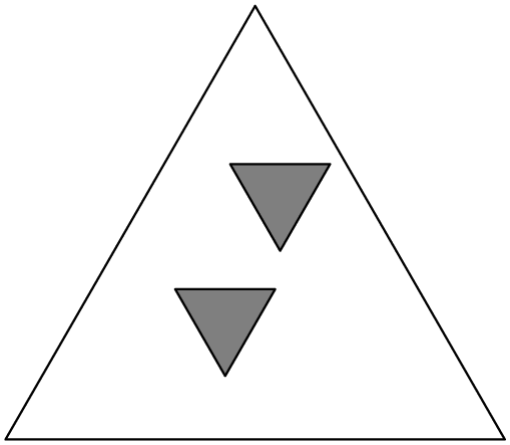
(A) 2^{12} **(B)** 2^{14} **(C)** 2^{16} **(D)** 2^{18} **(E)** 2^{20}

PROBLEM 4 (2013 AIME I #3)

Let $ABCD$ be a square, and let E and F be points on $\overline{AB}$ and $\overline{BC}$, respectively. The line through E parallel to $\overline{BC}$ and the line through F parallel to $\overline{AB}$ divide $ABCD$ into two squares and two nonsquare rectangles. The sum of the areas of the two squares is $\frac{9}{10}$ of the area of square $ABCD$. Find $\frac{AE}{EB}+\frac{EB}{AE}$.

PROBLEM 5 (2021 USAJMO #3)

An equilateral triangle Δ of side length $L > 0$ is given. Suppose that n equilateral triangles with side length 1 and with non-overlapping interiors are drawn inside Δ, such that each unit equilateral triangle has sides parallel to Δ, but with opposite orientation. (An example with $n = 2$ is drawn below.)



Prove that

$$n \leq \frac{2}{3}L^2.$$
