Problem Set - 19 Jan 2024

PROBLEM 1 (2011 AMC 10A #9)

A rectangular region is bounded by the graphs of the equations $y = a, y = -b, x = -c$, and $x = d$, where a, b, c, and d are all positive numbers. Which of the following represents the area of this region?

$\textbf{(A)}\ ac + ad + bc + bd \qquad \textbf{(B)}\ ac - ad + bc - bd \qquad \textbf{(C)}\ ac + ad - bc - bd \qquad \textbf{(D)}\ -ac - ad + bc + bd \qquad \textbf{(E)}\ ac - ad - bc + bd$

PROBLEM 2 (2015 AMC 10A #12)

Points $(\sqrt{\pi}, a)$ and $(\sqrt{\pi}, b)$ are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a - b|$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{\pi}{2} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \sqrt{1 + \pi} \qquad \textbf{(E)}\ 1 + \sqrt{\pi}$

PROBLEM 3 (2013 AIME II #1)

Suppose that the measurement of time during the day is converted to the metric system so that each day has 10 metric hours, and each metric hour has 100 metric minutes. Digital clocks would then be produced that would read $9:99$ just before midnight, $0:00$ at midnight, $1:25$ at the former $3:00$ AM, and $7:50$ at the former $6:00$ PM. After the conversion, a person who wanted to wake up at the equivalent of the former $6:36$ AM would set his new digital alarm clock for $\mathrm{A:BC}$, where A, B, and C are digits. Find $100\mathrm{A} + 10\mathrm{B} + \mathrm{C}$.

PROBLEM 4 (2015 AMC 12B #19)

In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $CBWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

$\textbf{(A)}\ 12 + 9\sqrt{3} \qquad \textbf{(B)}\ 18 + 6\sqrt{3} \qquad \textbf{(C)}\ 12 + 12\sqrt{2} \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

PROBLEM 5 (2014 USAJMO #1)

Let a, b, c be real numbers greater than or equal to 1. Prove that

$$\min\left(\frac{10a^2 - 5a + 1}{b^2 - 5b + 10}, \frac{10b^2 - 5b + 1}{c^2 - 5c + 10}, \frac{10c^2 - 5c + 1}{a^2 - 5a + 10}\right) \leq abc$$