Problem Set - 19 Jan 2024

PROBLEM 1 (2015 AMC 10B #12)

For how many integers x is the point $(x, -x)$ inside or on the circle of radius 10 centered at $(5, 5)$?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

PROBLEM 2 (2010 AMC 10B #13)

What is the sum of all the solutions of $x = |2x - |60 - 2x||$?

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 92 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 124$

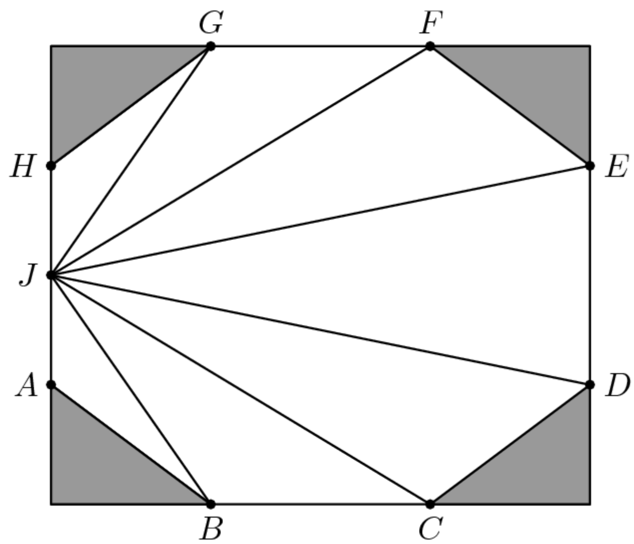
PROBLEM 3 (2020 AMC 10B #16)

Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where n is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

$\textbf{(A)}$ Bela will always win. $\qquad \textbf{(B)}$ Jenn will always win.
$\textbf{(C)}$ Bela will win if and only if n is odd.
$\textbf{(D)}$ Jenn will win if and only if n is odd. $\qquad \textbf{(E)}$ Jenn will win if and only if $n > 8$.

PROBLEM 4 (2018 AIME II #9)

Octagon $ABCDEFGH$ with side lengths $AB = CD = EF = GH = 10$ and $BC = DE = FG = HA = 11$ is formed by removing 6-8-10 triangles from the corners of a 23×27 rectangle with side $\overline{AH}$ on a short side of the rectangle, as shown. Let J be the midpoint of $\overline{AH}$, and partition the octagon into 7 triangles by drawing segments $\overline{JB}$, $\overline{JC}$, $\overline{JD}$, $\overline{JE}$, $\overline{JF}$, and $\overline{JG}$. Find the area of the convex polygon whose vertices are the centroids of these 7 triangles.



PROBLEM 5 (2014 USAMO #5)

Let ABC be a triangle with orthocenter H and let P be the second intersection of the circumcircle of triangle AHC with the internal bisector of the angle $\angle BAC$. Let X be the circumcenter of triangle APB and Y the orthocenter of triangle APC. Prove that the length of segment XY is equal to the circumradius of triangle ABC.