
Problem Set - 19 Jan 2024

PROBLEM 1 (2016 AMC 10A #8)

Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling, Fox is excited about his good fortune until he discovers that all his money is gone after crossing the bridge three times. How many coins did Fox have at the beginning?

- (A) 20 (B) 30 (C) 35 (D) 40 (E) 45

PROBLEM 2 (2015 AMC 10B #11)

Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?

- (A) $\frac{8}{99}$ (B) $\frac{2}{5}$ (C) $\frac{9}{20}$ (D) $\frac{1}{2}$ (E) $\frac{9}{16}$

PROBLEM 3 (2018 UNCO MATH CONTEST II #4)

How many positive integer factors of 36,000,000 are not perfect squares?

PROBLEM 4 (2011 USAJMO #1)

Find, with proof, all positive integers n for which $2^n + 12^n + 2011^n$ is a perfect square.

PROBLEM 5 (2013 USAJMO #5)

Quadrilateral $XABY$ is inscribed in the semicircle ω with diameter XY . Segments AY and BX meet at P . Point Z is the foot of the perpendicular from P to line XY . Point C lies on ω such that line XC is perpendicular to line AZ . Let Q be the intersection of segments AY and XC . Prove that

$$\frac{BY}{XP} + \frac{CY}{XQ} = \frac{AY}{AX}.$$