Problem Set - 19 Jan 2024

PROBLEM 1 (2010 AMC 8 #11)

The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

(A) 48 **(B)** 64 **(C)** 80 **(D)** 96 **(E)** 112

PROBLEM 2 (2020 AMC 8 #20)

A scientist walking through a forest recorded as integers the heights of 5 trees standing in a row. She observed that each tree was either twice as tall or half as tall as the one to its right. Unfortunately some of her data was lost when rain fell on her notebook. Her notes are shown below, with blanks indicating the missing numbers. Based on her observations, the scientist was able to reconstruct the lost data. What was the average height of the trees, in meters?

Tree 1	___ meters
Tree 2	11 meters
Tree 3	___ meters
Tree 4	___ meters
Tree 5	___ meters
Average height	___ .2 meters

(A) 22.2 **(B)** 24.2 **(C)** 33.2 **(D)** 35.2 **(E)** 37.2

PROBLEM 3 (2019 AMC 10A #6)

For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

- a square
- a rectangle that is not a square
- a rhombus that is not a square
- a parallelogram that is not a rectangle or a rhombus
- an isosceles trapezoid that is not a parallelogram

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

PROBLEM 4 (2016 AMC 12B #25)

The sequence (a_n) is defined recursively by $a_0 = 1$, $a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \geq 2$. What is the smallest positive integer k such that the product $a_1 a_2 \cdots a_k$ is an integer?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

PROBLEM 5 (2014 USAMO #5)

Let ABC be a triangle with orthocenter H and let P be the second intersection of the circumcircle of triangle AHC with the internal bisector of the angle $\angle BAC$. Let X be the circumcenter of triangle APB and Y the orthocenter of triangle APC. Prove that the length of segment XY is equal to the circumradius of triangle ABC.
