Problem Set - 19 Jan 2024

PROBLEM 1 (2019 AMC 10B #6)

There is a positive integer n such that $(n+1)! + (n+2)! = n! \cdot 440$. What is the sum of the digits of n?

(A) 3 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 12

PROBLEM 2 (2016 AMC 10B #13)

At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

(A) 25 **(B)** 40 **(C)** 64 **(D)** 100 **(E)** 160

PROBLEM 3 (2013 AMC 12B #17)

Let a, b, and c be real numbers such that

$$a+b+c = 2, \text{ and}$$

$$a^2+b^2+c^2 = 12$$

What is the difference between the maximum and minimum possible values of c?

(A) 2 (B) $\frac{10}{3}$ (C) 4 (D) $\frac{16}{3}$ (E) $\frac{20}{3}$

PROBLEM 4 (2022 AMC 10B #22)

Let S be the set of circles in the coordinate plane that are tangent to each of the three circles with equations $x^2+y^2=4$, $x^2+y^2=64$, and $(x-5)^2+y^2=3$. What is the sum of the areas of all circles in S?

(A) 48π **(B)** 68π **(C)** 96π **(D)** 102π **(E)** 136π

PROBLEM 5 (2018 AIME II #15)

Find the number of functions f from $\{0,1,2,3,4,5,6\}$ to the integers such that $f(0)=0$, $f(6)=12$, and

$$|x-y| \leq |f(x)-f(y)| \leq 3|x-y|$$

for all x and y in $\{0,1,2,3,4,5,6\}$.
