Problem Set - 19 Jan 2024

PROBLEM 1 (2019 AMC 10B #13)

What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17,$ and x is equal to the mean of those five numbers?

$\textbf{(A) }-5 \qquad \textbf{(B) }0 \qquad \textbf{(C) }5 \qquad \textbf{(D) }\frac{15}{4} \qquad \textbf{(E) }\frac{35}{4}$

PROBLEM 2 (2010 AMC 12B #11)

A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?

$\textbf{(A)}\ \dfrac{1}{10} \qquad \textbf{(B)}\ \dfrac{1}{9} \qquad \textbf{(C)}\ \dfrac{1}{7} \qquad \textbf{(D)}\ \dfrac{1}{6} \qquad \textbf{(E)}\ \dfrac{1}{5}$

PROBLEM 3 (2013 AIME I #2)

Find the number of five-digit positive integers, n, that satisfy the following conditions:

(a) the number n is divisible by $5,$

(b) the first and last digits of n are equal, and

(c) the sum of the digits of n is divisible by $5.$

PROBLEM 4 (2010 AIME II #9)

Let $ABCDEF$ be a regular hexagon. Let G, H, I, J, K, and L be the midpoints of sides AB, BC, CD, DE, EF, and AF, respectively. The segments $\overline{AH}$, $\overline{BI}$, $\overline{CJ}$, $\overline{DK}$, $\overline{EL}$, and $\overline{FG}$ bound a smaller regular hexagon. Let the ratio of the area of the smaller hexagon to the area of $ABCDEF$ be expressed as a fraction $\frac {m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

PROBLEM 5 (2015 USAMO #5)

Let a,b,c,d,e be distinct positive integers such that $a^4 + b^4 = c^4 + d^4 = e^5$. Show that $ac + bd$ is a composite number.