Problem Set - 19 Jan 2024

PROBLEM 1 (2016 AMC 8 #19)

The sum of 25 consecutive even integers is $10,000$. What is the largest of these 25 consecutive integers?

$\textbf{(A)}\ 360 \qquad \textbf{(B)}\ 388 \qquad \textbf{(C)}\ 412 \qquad \textbf{(D)}\ 416 \qquad \textbf{(E)}\ 424$

PROBLEM 2 (2014 AMC 12A #13)

A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

$\textbf{(A)}\ 2100 \qquad \textbf{(B)}\ 2220 \qquad \textbf{(C)}\ 3000 \qquad \textbf{(D)}\ 3120 \qquad \textbf{(E)}\ 3125$

PROBLEM 3 (2016 AMC 10A #17)

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\tfrac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5)=1$ and that $P(N)$ approaches $\tfrac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \tfrac{321}{400}$?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 20$

PROBLEM 4 (2010 AMC 12A #20)

Arithmetic sequences (a_n) and (b_n) have integer terms with $a_1=b_1=1<a_2 \le b_2$ and $a_n b_n = 2010$ for some n. What is the largest possible value of n?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 288 \qquad \textbf{(E)}\ 2009$

PROBLEM 5 (2013 USAMO #1)

In triangle ABC, points P,Q,R lie on sides BC,CA,AB respectively. Let ω_A, ω_B, ω_C denote the circumcircles of triangles AQR, BRP, CPQ, respectively. Given the fact that segment AP intersects ω_A, ω_B, ω_C again at X,Y,Z respectively, prove that $YX/XZ=BP/PC$.