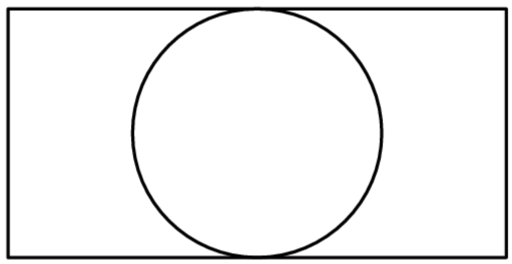
Problem Set - 19 Jan 2024

PROBLEM 1 (2012 AMC 12B #2)

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?



(A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

PROBLEM 2 (2019 AMC 10A #21)

A sphere with center O has radius 6. A triangle with sides of length $15, 15$, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

(A) $2\sqrt{3}$ **(B)** 4 **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{5}$ **(E)** 5

PROBLEM 3 (2018 AIME II #7)

Triangle ABC has side lengths $AB = 9$, $BC = 5\sqrt{3}$, and $AC = 12$. Points $A = P_0, P_1, P_2, ..., P_{2450} = B$ are on segment $\overline{AB}$ with P_k between P_{k-1} and P_{k+1} for $k = 1, 2, ..., 2449$, and points $A = Q_0, Q_1, Q_2, ..., Q_{2450} = C$ are on segment $\overline{AC}$ with Q_k between Q_{k-1} and Q_{k+1} for $k = 1, 2, ..., 2449$. Furthermore, each segment $\overline{P_kQ_k}$, $k = 1, 2, ..., 2449$, is parallel to $\overline{BC}$. The segments cut the triangle into 2450 regions, consisting of 2449 trapezoids and 1 triangle. Each of the 2450 regions has the same area. Find the number of segments $\overline{P_kQ_k}$, $k = 1, 2, ..., 2450$, that have rational length.

PROBLEM 4 (2011 AMC 12A #24)

Consider all quadrilaterals $ABCD$ such that $AB = 14$, $BC = 9$, $CD = 7$, and $DA = 12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

(A) $\sqrt{15}$ **(B)** $\sqrt{21}$ **(C)** $2\sqrt{6}$ **(D)** 5 **(E)** $2\sqrt{7}$

PROBLEM 5 (2010 USAMO #3)

The 2010 positive numbers $a_1, a_2, \ldots, a_{2010}$ satisfy the inequality $a_i a_j \le i + j$ for all distinct indices i, j. Determine, with proof, the largest possible value of the product $a_1 a_2 \cdots a_{2010}$.