Problem Set - 19 Jan 2024

PROBLEM 1 (2019 AMC 8 #1)

Ike and Mike go into a sandwich shop with a total of $\$30.00$ to spend. Sandwiches cost $\$4.50$ each and soft drinks cost $\$1.00$ each. Ike and Mike plan to buy as many sandwiches as they can, and use any remaining money to buy soft drinks. Counting both sandwiches and soft drinks, how many items will they buy?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

PROBLEM 2 (2020 AMC 10B #5)

How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row from left to right? (Tiles of the same color are indistinguishable.)

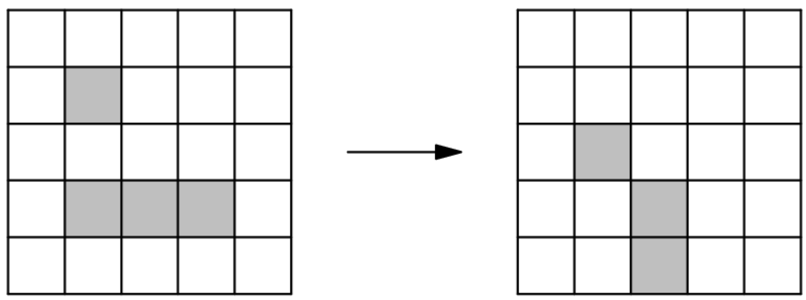
$\textbf{(A)}\ 210 \qquad \textbf{(B)}\ 420 \qquad \textbf{(C)}\ 630 \qquad \textbf{(D)}\ 840 \qquad \textbf{(E)}\ 1050$

PROBLEM 3 (2022 AMC 10B #19)

Each square in a 5×5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules:

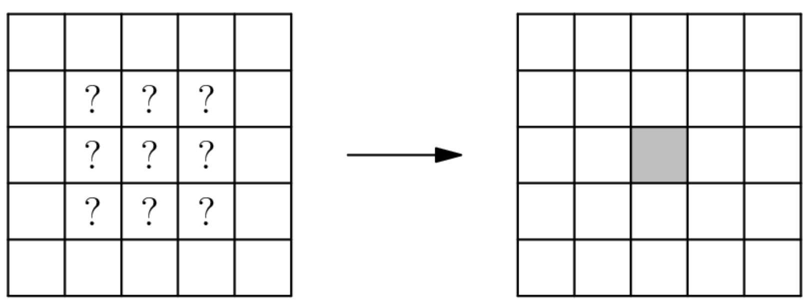
- Any filled square with two or three filled neighbors remains filled.
- Any empty square with exactly three filled neighbors becomes a filled square.
- All other squares remain empty or become empty.

A sample transformation is shown in the figure below.



Initial Transformed

Suppose the 5×5 grid has a border of empty squares surrounding a 3×3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)



Initial Transformed

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 30$

PROBLEM 4 (2014 AMC 10A #21)

Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

$\textbf{(A)}\ -20 \qquad \textbf{(B)}\ -18 \qquad \textbf{(C)}\ -15 \qquad \textbf{(D)}\ -12 \qquad \textbf{(E)}\ -8$

PROBLEM 5 (2019 AMC 12A #23)

Define binary operations $\diamondsuit$ and $\heartsuit$ by

$$a \diamondsuit b = a^{\log_7(b)} \qquad \text{and} \qquad a \heartsuit b = a^{\frac{1}{\log_7(b)}}$$

for all real numbers a and b for which these expressions are defined. The sequence (a_n) is defined recursively by $a_3 = 3 \heartsuit 2$ and

$$a_n = (n \heartsuit (n-1)) \diamondsuit a_{n-1}$$

for all integers $n \geq 4$. To the nearest integer, what is $\log_7(a_{2019})$?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$