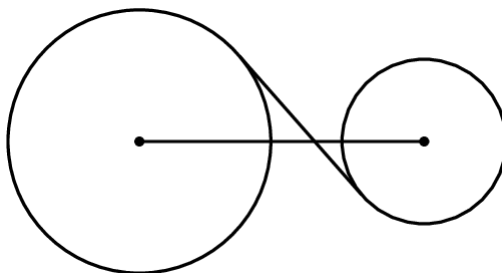


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# Problem Set - 19 Jan 2024

## PROBLEM 1 (2017 UNCO MATH CONTEST II #1)

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A circle has radius 24, a second circle has radius 15, and the centers of the two circles are 52 units apart. A line tangent to both circles crosses the line connecting the two centers at a point P between the two centers. How much farther is P from the center of the bigger circle than it is from the center of the smaller circle?

## PROBLEM 2 (2022 AMC 10B #5)

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What is the value of

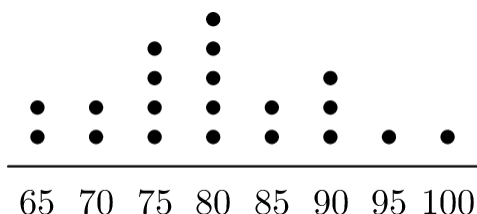
$$\frac{\left(1 + \frac{1}{3}\right) \left(1 + \frac{1}{5}\right) \left(1 + \frac{1}{7}\right)}{\sqrt{\left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{5^2}\right) \left(1 - \frac{1}{7^2}\right)}}?$$

- (A)  $\sqrt{3}$     (B) 2    (C)  $\sqrt{15}$     (D) 4    (E)  $\sqrt{105}$

**PROBLEM 3** (2022 AMC 8 #19)

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Mr. Ramos gave a test to his class of 20 students. The dot plot below shows the distribution of test scores.



Later Mr. Ramos discovered that there was a scoring error on one of the questions. He regraded the tests, awarding some of the students 5 extra points, which increased the median test score to 85.

What is the minimum number of students who received extra points?

(Note that the *median* test score equals the average of the 2 scores in the middle if the 20 test scores are arranged in increasing order.)

- (A) 2    (B) 3    (C) 4    (D) 5    (E) 6

**PROBLEM 4** (2017 UNCO MATH CONTEST II #4)

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Monkey Business

Harold writes an integer; its right-most digit is 4. When Curious George moves that digit to the far left, the new number is four times the integer that Harold wrote. What is the smallest possible positive integer that Harold could have written?

**PROBLEM 5** (2021 AMC 12A #18)

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Let  $f$  be a function defined on the set of positive rational numbers with the property that  $f(a \cdot b) = f(a) + f(b)$  for all positive rational numbers  $a$  and  $b$ . Suppose that  $f$  also has the property that  $f(p) = p$  for every prime number  $p$ . For which of the following numbers  $x$  is  $f(x) < 0$ ?

- (A)  $\frac{17}{32}$     (B)  $\frac{11}{16}$     (C)  $\frac{7}{9}$     (D)  $\frac{7}{6}$     (E)  $\frac{25}{11}$