Problem Set - 19 Jan 2024

PROBLEM 1 (2011 AMC 12B #9)

Two real numbers are selected independently and at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{2}{3}$

PROBLEM 2 (2022 AMC 12B #16)

Suppose x and y are positive real numbers such that

$$x^y = 2^{64} \text{ and } (\log_2 x)^{\log_2 y} = 2^7.$$

What is the greatest possible value of $\log_2 y$?

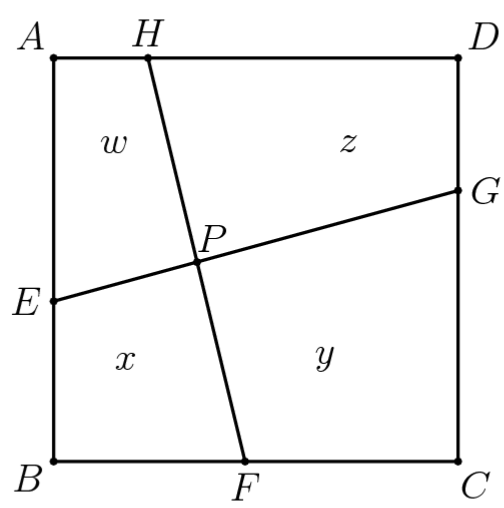
$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 3+\sqrt{2} \qquad \textbf{(D)}\ 4+\sqrt{3} \qquad \textbf{(E)}\ 7$

PROBLEM 3 (2011 UNCO MATH CONTEST II #6)

What is the remainder when $1! + 2! + 3! + \cdots + 2011!$ is divided by 18?

PROBLEM 4 (2014 AIME I #13)

On square $ABCD$, points E, F, G, and H lie on sides $\overline{AB}, \overline{BC}, \overline{CD}$, and $\overline{DA}$, respectively, so that $\overline{EG} \perp \overline{FH}$ and $EG = FH = 34$. Segments $\overline{EG}$ and $\overline{FH}$ intersect at a point P, and the areas of the quadrilaterals $AEPH, BFPE, CGPF$, and $DHPG$ are in the ratio $269 : 275 : 405 : 411$. Find the area of square $ABCD$.



PROBLEM 5 (2011 USAMO #4)

This problem is from both the 2011 USAJMO and the 2011 USAMO, so both problems redirect here.

Consider the assertion that for each positive integer $n \geq 2$, the remainder upon dividing 2^{2^n} by $2^n - 1$ is a power of 4. Either prove the assertion or find (with proof) a counter-example.