Problem Set - 19 Jan 2024

PROBLEM 1 (2017 AMC 10B #16)

How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

$\textbf{(A)}\ 469 \qquad \textbf{(B)}\ 471 \qquad \textbf{(C)}\ 475 \qquad \textbf{(D)}\ 478 \qquad \textbf{(E)}\ 481$

PROBLEM 2 (2010 AMC 12B #18)

A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

$\textbf{(A)}\ \dfrac{1}{6} \qquad \textbf{(B)}\ \dfrac{1}{5} \qquad \textbf{(C)}\ \dfrac{1}{4} \qquad \textbf{(D)}\ \dfrac{1}{3} \qquad \textbf{(E)}\ \dfrac{1}{2}$

PROBLEM 3 (2015 AIME II #6)

Steve says to Jon, "I am thinking of a polynomial whose roots are all positive integers. The polynomial has the form $P(x) = 2x^3-2ax^2+(a^2-81)x-c$ for some positive integers a and c. Can you tell me the values of a and c?"

After some calculations, Jon says, "There is more than one such polynomial."

Steve says, "You're right. Here is the value of a." He writes down a positive integer and asks, "Can you tell me the value of c?"

Jon says, "There are still two possible values of c."

Find the sum of the two possible values of c.

PROBLEM 4 (2015 AIME I #8)

For positive integer n, let $s(n)$ denote the sum of the digits of n. Find the smallest positive integer satisfying $s(n) = s(n+864) = 20$.

PROBLEM 5 (2013 USAJMO #5)

Quadrilateral $XABY$ is inscribed in the semicircle ω with diameter XY. Segments AY and BX meet at P. Point Z is the foot of the perpendicular from P to line XY. Point C lies on ω such that line XC is perpendicular to line AZ. Let Q be the intersection of segments AY and XC. Prove that

$$\frac{BY}{XP} + \frac{CY}{XQ} = \frac{AY}{AX}.$$