# Problem Set - 19 Jan 2024

## PROBLEM 1 (2018 AMC 10B #1)

Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 180 \qquad \textbf{(D)}\ 200 \qquad \textbf{(E)}\ 360$

## PROBLEM 2 (2014 AIME II #7)

Let $f(x)=(x^2+3x+2)^{\cos(\pi x)}$. Find the sum of all positive integers $n$ for which

$$\left|\sum_{k=1}^n\log_{10}f(k)\right|=1.$$

## PROBLEM 3 (2010 AMC 12B #22)

Let $ABCD$ be a cyclic quadrilateral. The side lengths of $ABCD$ are distinct integers less than 15 such that $BC\cdot CD=AB\cdot DA$. What is the largest possible value of $BD$?

$\textbf{(A)}\ \sqrt{\dfrac{325}{2}} \qquad \textbf{(B)}\ \sqrt{185} \qquad \textbf{(C)}\ \sqrt{\dfrac{389}{2}} \qquad \textbf{(D)}\ \sqrt{\dfrac{425}{2}} \qquad \textbf{(E)}\ \sqrt{\dfrac{533}{2}}$

## PROBLEM 4 (2011 AMC 12B #22)

Let $T_1$ be a triangle with side lengths 2011, 2012, and 2013. For $n \ge 1$, if $T_n = \Delta ABC$ and $D, E$, and $F$ are the points of tangency of the incircle of $\Delta ABC$ to the sides $AB$, $BC$, and $AC$, respectively, then $T_{n+1}$ is a triangle with side lengths $AD, BE$, and $CF$, if it exists. What is the perimeter of the last triangle in the sequence $(T_n)$?

$\textbf{(A)}\ \frac{1509}{8} \qquad \textbf{(B)}\ \frac{1509}{32} \qquad \textbf{(C)}\ \frac{1509}{64} \qquad \textbf{(D)}\ \frac{1509}{128} \qquad \textbf{(E)}\ \frac{1509}{256}$

## PROBLEM 5 (2013 AIME I #13)

Triangle $AB_0C_0$ has side lengths $AB_0 = 12$, $B_0C_0 = 17$, and $C_0A = 25$. For each positive integer $n$, points $B_n$ and $C_n$ are located on $\overline{AB_{n-1}}$ and $\overline{AC_{n-1}}$, respectively, creating three similar triangles $\triangle AB_nC_n \sim \triangle B_{n-1}C_nC_{n-1} \sim \triangle AB_{n-1}C_{n-1}$. The area of the union of all triangles $B_{n-1}C_nB_n$ for $n \geq 1$ can be expressed as $\tfrac{p}{q}$, where $p$ and $q$ are relatively prime positive integers. Find $q$.