Problem Set - 19 Jan 2024

PROBLEM 1 (2012 AMC 8 #10)

How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 12

PROBLEM 2 (2021 AMC 12A #17)

Trapezoid $ABCD$ has $\overline{AB} \parallel \overline{CD}, BC = CD = 43$, and $\overline{AD} \perp \overline{BD}$. Let O be the intersection of the diagonals $\overline{AC}$ and $\overline{BD}$, and let P be the midpoint of $\overline{BD}$. Given that $OP = 11$, the length of AD can be written in the form $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. What is $m + n$?

(A) 65 **(B)** 132 **(C)** 157 **(D)** 194 **(E)** 215

PROBLEM 3 (2018 AMC 12A #20)

Triangle ABC is an isosceles right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $AI > AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a + b + c$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

PROBLEM 4 (2013 AIME II #7)

A group of clerks is assigned the task of sorting 1775 files. Each clerk sorts at a constant rate of 30 files per hour. At the end of the first hour, some of the clerks are reassigned to another task; at the end of the second hour, the same number of the remaining clerks are also reassigned to another task, and a similar assignment occurs at the end of the third hour. The group finishes the sorting in 3 hours and 10 minutes. Find the number of files sorted during the first one and a half hours of sorting.

PROBLEM 5 (2011 AMC 10B #25)

Let T_1 be a triangle with side lengths $2011, 2012$, and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and D, E, and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$