Sets and indices

i, j, k, α : $indices\ for\ nodes, components, subassemblies, final\ assemblies\ and\ plants$

n : $indices\ for\ disruption\ scenarios, n = 1,2, .., R$

t^n : $duration\ of\ disruption\ in\ disruption\ scenarion\ n, (time\ to\ recover - TTR)$

$\Phi^{(n)}$: $set\ of\ nodes(facilities)\ disrupted\ in\ scenarion\ n$

$Nodes$: $set\ of\ all\ nodes\ in\ the\ supply\ chain$

Ω_{jk} : $set\ of\ all\ nodes\ (facilities) that\ are\ in\ the\ upstream\ of\ node\ j\ and\ of\ part\ type\ k$

$N^+(i)$: $set\ of\ subassembly\ or\ final\ assemblies\ that\ require\ component\ or\ subassembly\ i$

$N^-(i)$:
$set\ of\ subassemblies\ or\ components\ that\ require\ to\ produce\ subassembly\ or\ final\ assembly\ i$

D : $all\ final\ assembly\ and\ subassemblies\ in\ BOM\ (except\ the\ last\ tier)$

U : $all\ nodes\ except\ the\ final\ (assembly) nodes$

Υ : *set of final products /assemblies*

A_α : $set\ of\ all\ nodes\ (components, suabassemblies\ and\ assemblies) produced\ in\ plant\ \alpha$

Input parameters

GHG_i: $green\ house\ gas\ emmission\ index\ of\ node\ (plant) i\ for\ each\ product\ produced$

GHG_{ij} : $green\ house\ gas\ emmission\ of\ transporting\ a\ unit\ of\ product\ from\ plant\ i\ to\ j.$

SI_i : $societal\ impact\ (employability, following\ legislations) index\ of\ plant\ i$

f_j : $profit\ margin\ of\ product\ j$

$c_\alpha t^{(n)}$: $capacity\ of\ plant\ \alpha\ during\ the\ all\ disruption\ duration\ t^{(n)} in\ scenario\ n$

Decision variables

l_j : $lost\ production\ amount\ of\ product\ j$

y_{ij} : *amount allocated from upstream node i to downstream node j during disruption duration*
u_j : total production quantity of node *j* during disruption scenario $t^{(n)}$.

Mathematical model:

$$Minimize \left\{\sum_{j\in\Upsilon} f_j l_j\,;\ \left\{\sum_{i\epsilon Nodes} GHG_i u_i + \sum_{i\epsilon Nodes, j\epsilon N^+(i)} GHG_{ij} y_{ij}\right\}\right\} \quad (1)$$
$$Maximize \sum_{i\epsilon Nodes} SI_i u_i$$

$Subject\ to:$

$$u_j \le \sum_{i\in\Omega_{jk}} \left(y_{ij}/r_{kj}\right) \qquad \forall k \in N^-(j), \forall j \in D \qquad (2)$$

$$\sum_{j\in N^+(i)} y_{ij} \le u_i + s_i \qquad \forall i \in U \qquad (3)$$

$$u_j = 0 \qquad \forall j \in \Phi^{(n)} \qquad (4)$$

$$l_j + \sum_{k\in V_j} u_k \ge d_j t^{(n)} \qquad \forall j \in \Upsilon \qquad (5)$$

$$\sum_{k\in A_\alpha} u_k \le c_\alpha t^{(n)} \qquad \forall \alpha \in \mathrm{A} \qquad (6)$$

$$y_{ij}, u_j, l_j \ge 0 \qquad (7)$$

Multi-objective function (1) minimizes the lost sales due to disruption scenario $t^{(n)}$ while trying to best resource allocation plans that minimize GHG index and maximize the societal impact lexicographically.

Constraints (2) are Bill of Material (BOM) inequalities for subassemblies and final products and ensure that for each final product and subassembly j, production is limited with the most scarce parent subassembly/component. See the below figure for representing the relationship.

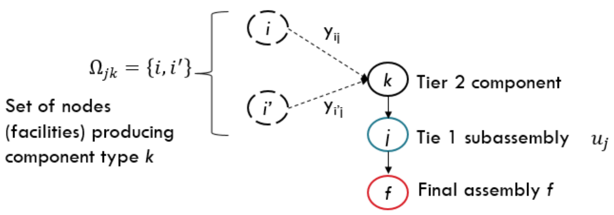

Figure 1: A representative BOM structure

Constraints (3) ensure the BOM inequality for subassemblies and components that total outflow of that subassembly/component to other subassemblies and final products is limited with the production and inventory amount.

Constraints (4) limits the production amount for each facilities in the disruption scenario *n*.

Constraints (5) guarantee that the lost sales (amount of final products can not be delivered) and the production amount for each final product is at least equal to the demand of that final product during the disruption duration considered.

Constraints (6) ensures that the total production amount in each facility is limited with the capacity of the facility during disruption duration.