Deep Plug-and-Play Gradient Method for Super-Resolution

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Abstract

- We extend pre-existing hybrid algorithms (combining model-based and learning-based approaches) to use gradient priors.
- We demonstrate the fixed-point convergence theorem in a general framework.
- We show that the gradient prior outperforms the image prior, in terms of the SSIM index, on heavily degraded gray-level images.

Introduction

We consider a degraded Low Resolution (LR) image v and we aim at recovering the clean High Resolution image u. The two images are related by the following equation

$$v = Au + \eta \qquad with\ A = SH \tag{1}$$

where S is a downsampling operator, H is a blur matrix and η a gaussian additive noise.
Two main approaches to solve the inverse problem (1):

- **model-based** uses a classical optimization method on an objective function to get the solution
$$\hat{u} = \arg\min_u \left\{ \frac{1}{2}\|Au - v\|_2^2 + \lambda\phi(Lu) \right\} \tag{2}$$
where ϕ is a fixed regularization function acting on some linear transform of the image;
- **learning-based** computes the solution
$$\hat{u} = N(v, \Theta) \tag{3}$$
using a trained neural network N, whose weights are Θ.

Hybrid approach

We introduce the auxiliary variable $t = Lu$ and we minimize

$$\mathcal{Q}_\rho(u,t) = \frac{1}{2}\|Au - v\|_2^2 + \lambda\phi(t) + \frac{\rho}{2}\|Lu - t\|_2^2 \tag{4}$$

where $\rho > 0$ is an adaptive penalty parameter.
By applying the Half Quadratic Splitting (HQS) method [2], we decouple the cost function (4) into two subproblems which can be solved by the following iterative scheme:

$$\begin{cases} t_{k+1} = \arg\min_t \frac{\rho_k}{2}\|Lu_k - t\|_2^2 + \lambda\phi(t) & (5) \\ u_{k+1} = \arg\min_u \frac{1}{2}\|Au - v\|_2^2 + \frac{\rho_k}{2}\|Lu - t_{k+1}\|_2^2 & (6) \end{cases}$$

In the proposed *hybrid* Plug-and-Play (PnP) approach:

- equation (5) corresponds to a gaussian denoising on the t, thus t_{k+1} is computed as $D_{\sigma_k}(Lu_k)$, where D_{σ_k} is a Convolutional Neural Network (CNN) Denoiser, trained on the noise level $\sigma_k = \sqrt{\frac{\lambda}{\rho_k}}$;
- the quadratic problem (6) is a standard Tikhonov problem.

PnP-HQS methods

Algorithm 1 Plug-and-Play HQS

Require: Starting point u_0
Require: A maximum number of iterations k_{max}
1: **for** $k = 0, 1, \ldots, k_{max}$ **do**
2: $t_{k+1} = \mathcal{D}_{\sigma_k}(Lu_k)$
3: $u_{k+1} = \arg\min_u \frac{1}{2}\|Au - v\|_2^2 + \frac{\rho_k}{2}\|Lu - t_{k+1}\|_2^2$
4: Choose a new penalty parameter $\rho_{k+1} \geq \rho_k$
5: Choose a new denoising parameter $\sigma_{k+1} \leq \sigma_k$
6: **end for**

Algorithm insights

- We perform $k_{max} = 30$ iterations.
- We have trained the CNNs on 25 different noise levels $\sigma_k \in [0, 50]$. The training can be parallelized on GPU.
- When $L = I$, the PnP-HQS method corresponds to the IRCNN algorithm in [3]. In particular:
  - $\mathcal{D}_{\sigma_k}$ is trained on 400 images from the BSD data set [1];
  - step 3 is solved with a Gradient Descent method.
- We propose to exploit $L = D$ as the discretization operator of the first order image derivative. In addition:
  - we train $\mathcal{D}_{\sigma_k}$ on the gradient of the 400 images from the BSD data set;
  - we solve step 3 with Conjugate Gradients method.

Networks Architecture

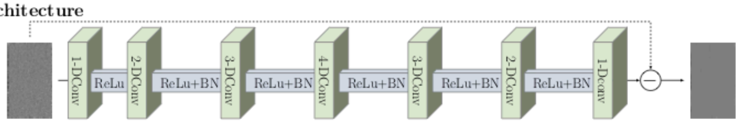

Figure 1:Denoisers architecture (*BN* represents the batch normalization and *m-DConv* denotes *m*-dilated convolution)

Fixed point convergence theorem for the PnP-HQS method

We were able to demonstrate that:
Under a monotone update rule of the penalty parameter ($\rho_{k+1} = \gamma\rho_k, \quad \gamma > 1$), if L is a full rank operator, for any set of bounded denoisers $\mathcal{D}_\sigma$ there exists a point (u^*, t^*) such that $\|u_k - u^*\|_2 \to 0$ and $\|t_k - t^*\|_2 \to 0$ as $k \to \infty$.

Results

We test the proposed approach on simulated LR images: the ground truth are blurred, with a gaussian kernel of size 9×9 and variance 1.6, and then down-sampled with a down-factor equals to 4. Moreover, we add gaussian noise components (with different noise-levels) in order to reproduce real LR acquisitions. The HR reconstructions are compared in terms of PSNR and SSIM with their ground truth.

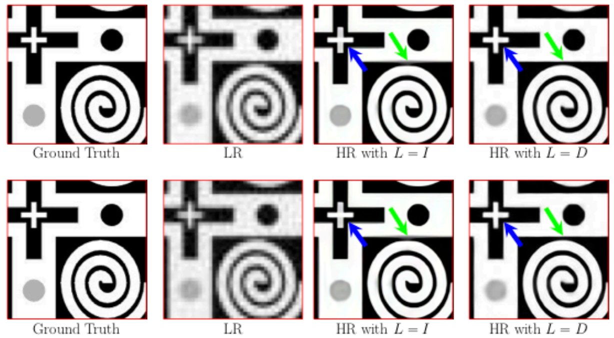

Figure 2:Geometric image with sharp edge objects. First row: $\sigma = 2.55$. Second row: $\sigma = 5$.

Noise level		$L = I$	$L = D$
$\sigma = 0$	PSNR	27.464	**28.783**
	SSIM	0.955	**0.969**
$\sigma = 2.55$	PSNR	27.005	**28.146**
	SSIM	0.945	**0.958**
$\sigma = 5$	PSNR	26.427	**27.003**
	SSIM	0.932	**0.937**

Table 1:Metrics on the geometric image, at different noise-levels

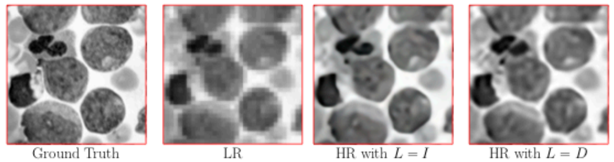

Figure 3:Cell image with both separated and overlapping low-contrasted objects. Here $\sigma = 2.55$.

Noise level		$L = I$	$L = D$
$\sigma = 0$	PSNR	**28.528**	28.208
	SSIM	0.864	**0.911**
$\sigma = 2.55$	PSNR	**28.180**	27.709
	SSIM	0.851	**0.901**
$\sigma = 5$	PSNR	**27.557**	26.667
	SSIM	0.830	**0.876**

Table 2:Metrics on the cell image, at different noise-levels

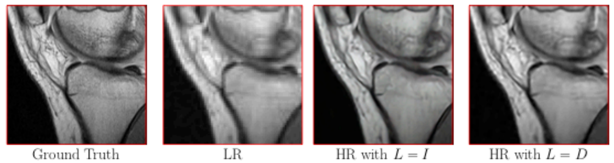

Figure 4:MRI knee image with fine details such as veins. Here $\sigma = 2.55$.

Noise level		$L = I$	$L = D$
$\sigma = 0$	PSNR	**30.616**	30.247
	SSIM	0.827	**0.909**
$\sigma = 2.55$	PSNR	**30.263**	29.872
	SSIM	0.811	**0.899**
$\sigma = 5$	PSNR	**29.711**	29.123
	SSIM	0.789	**0.880**

Table 3:Metrics on the knee image, at different noise-levels

Conclusion

Our method is the natural generalisation of the state-of-the-art [3]. We show, through several experiments, that the PnP-HQS framework inherits the stability and flexibility from the variational. Enforcing CNN denoisers to capture the gradient statistics ($L = D$) leads to better results in terms of SSIM index with respect to the case $L = I$. In general, the gain is more noticeable on images with sharp edges and low-contrasted objects.

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