14 thu

Elasticity

It is the property of the material to regain its original shape and dimensions, after the removal of external deforming force.

Perfectly elastic body – The body which complitely regains its original shape after the ~~normal~~

15 fri

removal of external deforming force.

Elastic limit – A certain limit below which if the deforming force is applied the body will regain its original dimensions but above the certain limit if deforming force is applied, body will get permanently deformed. This limit is called Elastic limit.

16 sat

Stress – When external forces or deforming forces are applied on the body under equilibrium there is a relative displacement of its particles and results in internal forces of reaction which tends to oppose or balance the deforming forces within elastic limit, then body is said to be under stress.

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Notes

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = N/m^2$$

Stress

↓ Tangential stress (shearing)
(when the stress is applied along the surface)
↓
results in the change of shape of the body.

↓ Normal stress
(when stress is applied perpendicular to the surface) ↓
results in change in length or volume.

<u>Strain</u> – When body is under stress, it undergoes a change in length or volume or shape. This change in the dimension of the body to the original dimension or measured per unit dimension is called as strain.

Linear strain $= \frac{l}{L}$ – change in length / – Original length.

Volume strain $= \frac{v}{V}$ – change in volume / – Original volume.

shearing strain = Angular deformation in the body 'θ'

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Notes

21 thu

Hooke's law — Provided the ~~station~~ strain is small, the stress is directly proportional to strain, within the elastic limit. or if the strain is small, ratio of stress to strain is constant within its elastic limit.

$E = \frac{\text{stress}}{\text{strain}}$ → Modulous of Elasticity or Coefficient of elasticity. [E].

22 fri

1) Young's Modulous $Y = \frac{\text{Linear stress}}{\text{Linear strain}} = \frac{F/A}{l/L}$

2) Bulk modulous $K = \frac{\text{Pressure}}{\text{Volume strain}} = \frac{F/A}{-v/V}$.

$K = \frac{-F/A}{v/V} = -\frac{PV}{v}$. Negative sign - if ~~the~~ pressure increases, the volume decreases.

23 sat

Incompressibility of the material of the body.

Bulk modulous is observed only in case of liquids and gases.

3) Modulous of Rigidity $\eta = \frac{\text{Tangential stress}}{\text{shear strain}} = \frac{F/A}{\theta}$.

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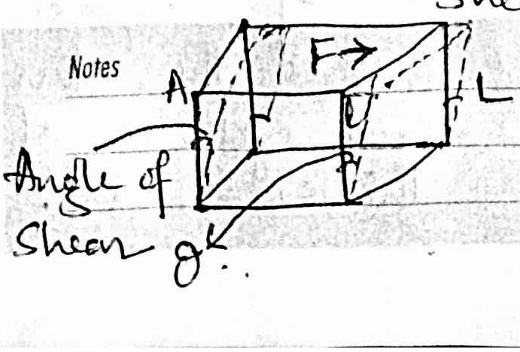
Notes



Poisson's ratio $\sigma = \dfrac{\text{Lateral strain}}{\text{Linear strain}}$

$= \dfrac{\text{Secondary strain}}{\text{primary strain}} = \dfrac{\beta}{\alpha}$

Lateral strain – is the strain at right to the applied force.

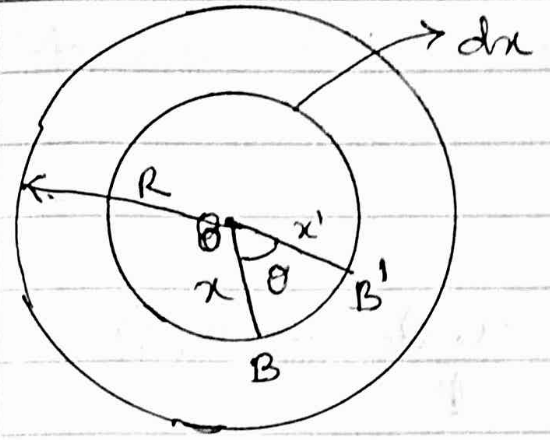
Linear strain – is the strain in the direction of applied force.

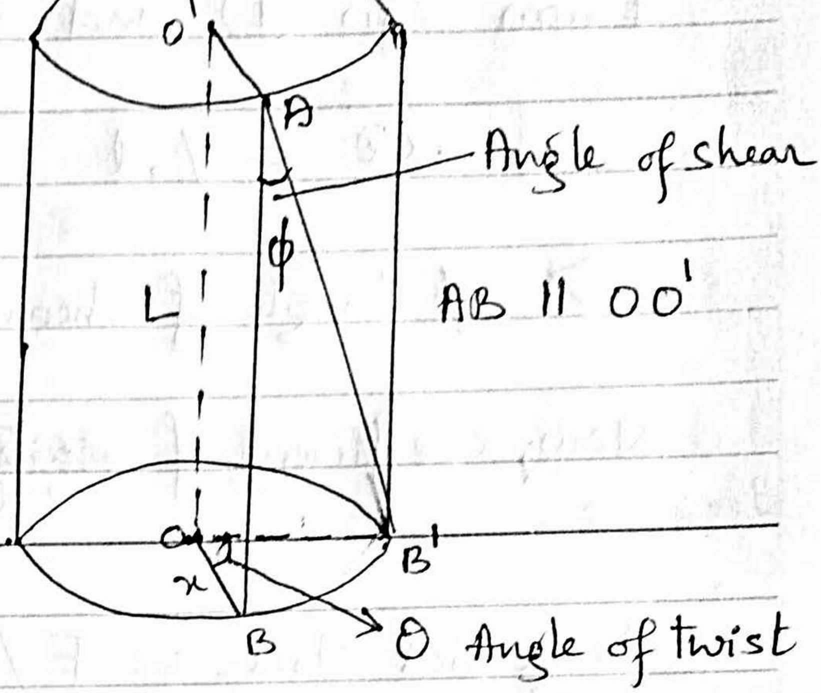
Expression for couple for a unit twist or torsion of a cylinder or Torsional rigidity of the material of wire

Consider a solid cylinder (wire) of length l and radius R to be fixed at its upper end.

Let the couple (chuck nuts) be applied at its lower end such that it is twisted by 'θ'. Now, naturally a resisting couple comes into action trying to oppose the twisting couple applied. and being the cylinder in equilibrium position.

In order to calculate the value of this couple, let us imagine the cylinder to consist of large number of co-axial cylinder, one inside the other and consider one such cylinder of radius 'x' and thickness 'dx'.





If we will twist AB from the bottom by angle θ it will acquire the position AB' with φ as angle of shear.

Now, twist at lower ~~[illegible]~~ end of rod/wire

$$\tan\theta = \frac{BB'}{OB}$$

Since θ is small, then $\tan\theta = \theta$ $\therefore \theta = \frac{BB'}{OB}$

$$BB' = OB.\theta \longrightarrow (1).$$
$$= x.\theta$$

At upper end.

$$\tan\phi = \frac{BB'}{AB} \qquad \phi = \frac{BB'}{AB}$$

$$BB' = \phi.AB \longrightarrow (2)$$
$$= \phi.l$$

Notes

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From eqn (1) and (2)

$$x.\theta = \phi.l$$

$$\Rightarrow \phi \text{ (angle of shear)} = \frac{x.\theta}{l} \longrightarrow (3).$$

Now, Coefficient of ~~rigidity~~ rigidity = $\frac{\text{Shearing / tangential Stress}}{\text{Shear}}$

Tangential stress = F / area of face of shell.

= F / circumference x thickness

$$= F / 2\pi x . dx$$

$$\therefore \eta = F / 2\pi x . dx \cdot \frac{1}{\phi} \quad \left[\eta = \frac{\text{Tangential stress}}{\text{shear}}\right]$$

$$\eta = \frac{F}{2\pi x . dx} \cdot \frac{l}{x.\theta} \quad [\because \text{from eqn } (3)]$$

$$F = \frac{2\pi\eta\theta}{l} . x^2 dx .$$

Moment of Force (couple) = Force × perpendicular distance from line of action

∴ Moment of force (About the axis) $= F \times x$.

Couple $= \frac{2\pi\eta\theta}{l} . x^3 dx \rightarrow (4)$ This is for only one layer of coaxial cylinder.

For the complete / whole cylinder, integrate couple between $x=0$ to $x=R$.

∴ The twisting couple for the entire cylinder is

$$= \int_0^R \frac{2\pi\eta\theta}{l} . x^3 dx \quad \text{or} \quad \frac{2\pi\eta\theta}{l}\left[\frac{x^4}{4}\right]_0^R$$

$$= \frac{2\pi\eta\theta}{l} . \frac{R^4}{\cancel{4}\,2} \Rightarrow \frac{\pi\eta R^4}{2l} . \theta$$

$$\therefore \boxed{\text{Twisting couple} = \frac{\pi\eta R^4}{2l} . \theta}$$

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Notes

In the above expression, If $\theta = 1$,

Twisting couple $= \dfrac{\pi \eta R^4}{2l}$

$\therefore$ $\boxed{C = \dfrac{\pi \eta R^4}{2l}}$ → Torsional rigidity of the material of wire.

Since, twisting couple is analogous to restoring couple, So twisting couple can also be called as restoring couple per unit twist.

Torsional Pendulum –

Any rigid body, capable of executing angular simple harmonic motion in horizontal

plane about the vertical axis passing through centre of the gravity of the body is called as torsional pendulum.

Consider a torsional pendulum in any forms, here as a rectangular bar, suspended by a wire. One end of the wire is fixed to a rigid and frictionless support. And other end is attached to centre of of mass of solid, here 'O' is the Centre of bar.

When the bar is twisted from A to B, then the wire is also twisted by same angle and restoring Torque is produced in wire, which tends to bring the wire in its equilibrium position. So, the restoring torque or twisting torque acting on The wire is $\Rightarrow C.\theta$.

$$= \frac{\pi \eta R^4}{2l} . \theta$$

$\therefore C = \frac{\pi \eta R^4}{2l} \rightarrow$ Torsional rigidity of the material of wire. $\longrightarrow$ (1)

Now, If 'I' moment of Inertia of the rectangular bar, about the axis of the wire, passing

through its centre,

$\frac{d^2\theta}{dt^2}$ = Angular acceleration,

$I.\frac{d^2\theta}{dt^2}$ = Couple acting on rectangular bar $\longrightarrow$ (2).

Eqn (1) and (2) implies couple acting on the bar and wire.

$$\therefore I \frac{d^2\theta}{dt^2} = - C.\theta$$

$\Rightarrow$ Negative sign indicates that restoring couple is oppositely directed to angular displacement θ

$$\frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta.$$

If $\frac{C}{I} = \omega^2$ then $\frac{d^2\theta}{dt^2} = -\omega^2\theta.$

$$\boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$$ → Differential eqn of the angular Simple Harmonic Motion of torsional pendulum.

Its solution is given by

$$\boxed{\theta = \theta_m \sin(\omega t + \phi)}$$

Frequency $n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$

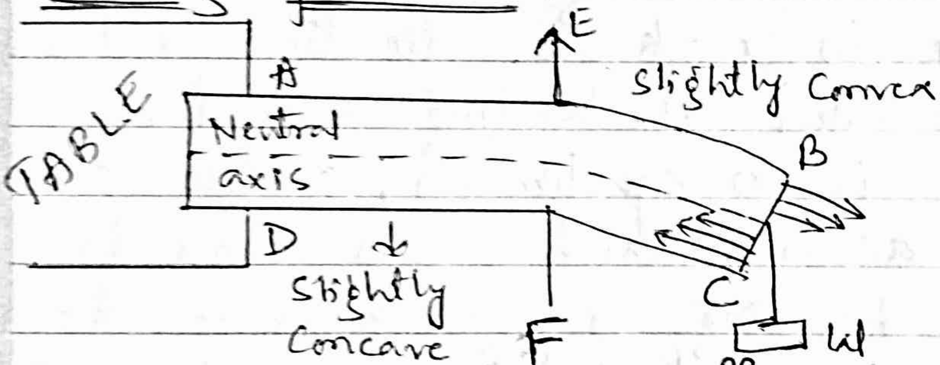
Time period $T = \frac{1}{n} = \frac{1}{2\pi}\sqrt{\frac{I}{C}}$

Beam A rod or bar of a circular or rectangular cross-section with its length very much greater than its thickness (so that no shearing stresses over any section of it) is called a beam.

Beam

Supported beam | Fixed beam (cantilever)

Bending of a Beam –



Consider a beam of rectangular cross section fixed at one end and loaded at other end.

ABCD is the beam fixed from AD end and free from BC end and 'W' load is suspended at free end.

a) Due to moment of load 'W' it will bend down.

b) AB portion of beam will increase in length due to tension or tensile stress acting outward.

c) CD portion of beam will get compressed due to compressing stress acting inward.

Neutral axis – It is the axis of the beam in which ~~[illegible]~~ neither get extended nor get compressed ie retain original length.

Bending Moment – ABCD is a beam.

a) Due to moment of 'W' the beam will get bend is downloaded direction the ~~equal~~ equal

and opposite force acts in upward direction along EF ↑. Both these forces constitute a couple called as bending couple which will bend beam in clockwise direction and its moment is called as bending moment.

b) Since beam is in equilibrium, another couple comes into action, which causes beam to resist the bending or will balance the beam.

This couple is called balancing couple. This arises due to the compressive stress acting on 'DC' portion and tensile stress acting on 'AB' portion of the bar or due to forces acting above and below neutral axis NN'

Expression for Bending Moment

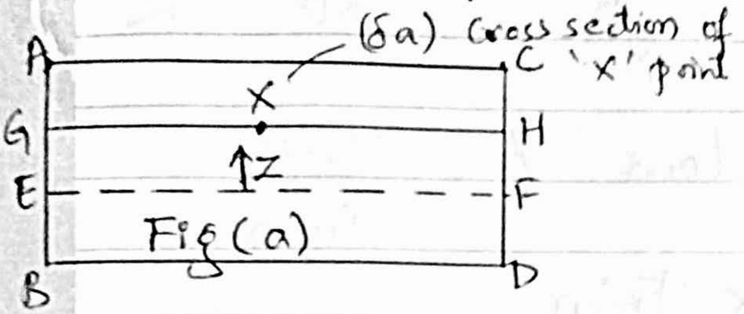
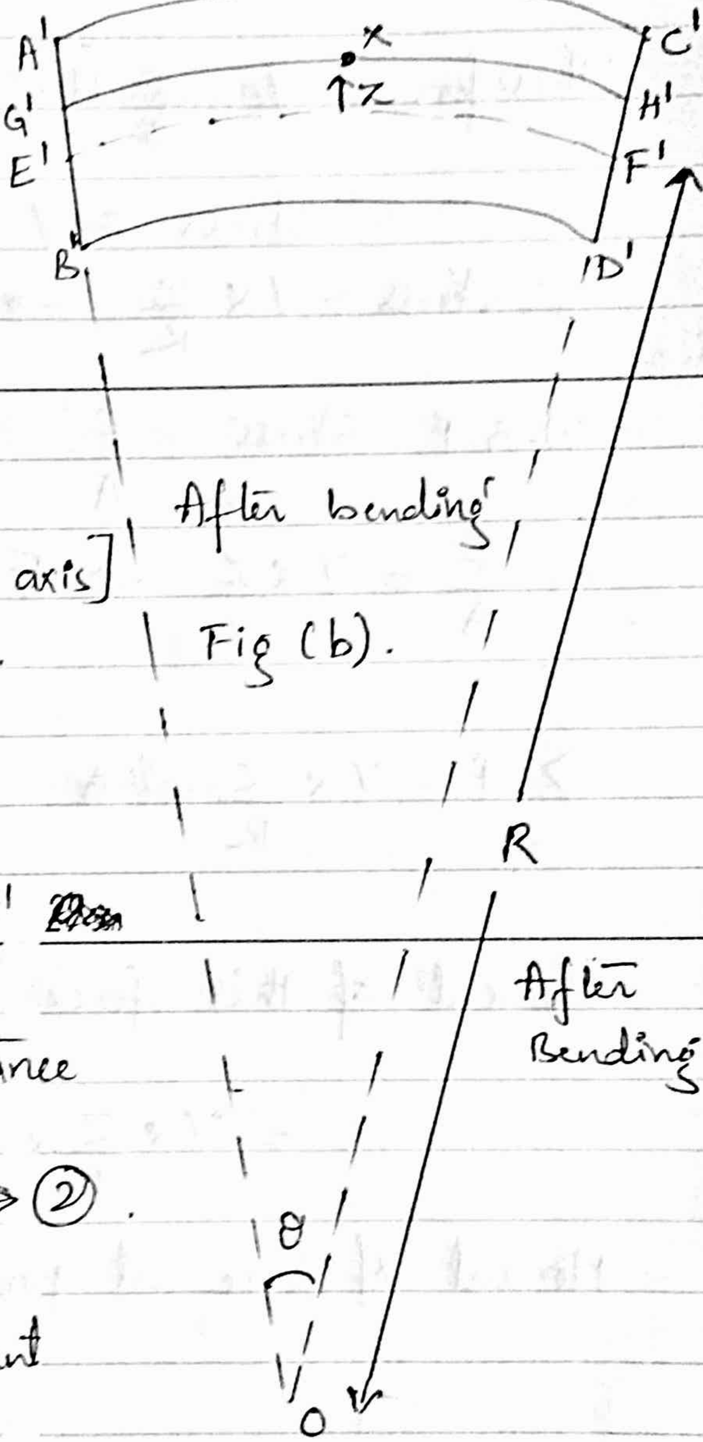


Fig (a)



Before bending (Fig a)

$GH = EF$

$EF = E'F'$ [∵ EF is neutral axis]

$\therefore GH = E'F' = R\theta \longrightarrow (1)$.

After bending (Fig b)

R – Radius of curvature E'F'

G'H' – Filament at 'z' distance from E'F'

Now $G'H' = (R+z)\theta \longrightarrow (2)$.

$\therefore$ change in length of filament

$GH = G'H' - GH$.

$= (R+z)\theta - R\theta$

$= z\theta$.

Original length $= R\theta$.

30 mon

Now Strain = $\frac{\text{Change in length}}{\text{Original length}} = \frac{Z\theta}{R\theta} = \frac{Z}{R}$

Therefore, Young Modulous 'Y' = $\frac{\text{Stress}}{\text{Strain}}$

Stress = Y x Strain

$\therefore$ stress = $Y \times \frac{Z}{R}$ → This is the stress at point 'x' on G'H' filament.

31 tue

W.K.T Stress = $\frac{F}{A}$.

$\therefore \frac{F}{A} = Y \times \frac{Z}{R}$ ⇒ F on the 'x' point = $Y \times \frac{Z}{R} \times A$

$\Rightarrow F = Y \times \frac{Z}{R} \times \delta a$ δa → area of cross section of 'x' point.

1 wed

Moment of this force 'F' = F x perpendicular distance from line of action (E'F')

$= Y \times \frac{Z}{R} \times \delta a \times Z.$

Moment of force at point 'x' is $Y . \frac{\delta a}{R} . Z^2$

2 thu

Now the total moment of force acting on the complete filament / bar, ABCD $= \sum \frac{Y.\,\delta a.}{R} z^2$

or $\frac{Y}{R} \sum \delta a . z^2$ or $\frac{Y}{R} . I_g$

Where $I_g = \sum \delta a . z^2 \rightarrow$ Moment of Inertia or Radius of Gyration

3 fri

The quantity $\frac{Y I_g}{R} \rightarrow$ Restoring couple or bending moment of beam.

$\therefore \boxed{\text{Bending moment} = \frac{Y I_g}{R} = M.}$

If $R = 1$, Radius of curvature
then $\boxed{M = Y I_g} \rightarrow$ Flexural Rigidity of wire

4 sat

Measure of resistance of beam to bending.

If beam is Rectangular cross section
$I_g = \frac{bd^3}{12} \Rightarrow \boxed{M = \frac{Y bd^3}{12 R}}$

If beam is circular cross section
$I_g = \frac{\pi r^4}{4} \Rightarrow \boxed{M = \frac{Y \pi r^4}{4 R}}$.

Notes $r \rightarrow$ Radius of circular beam
$R \rightarrow$ Radius of curvature.

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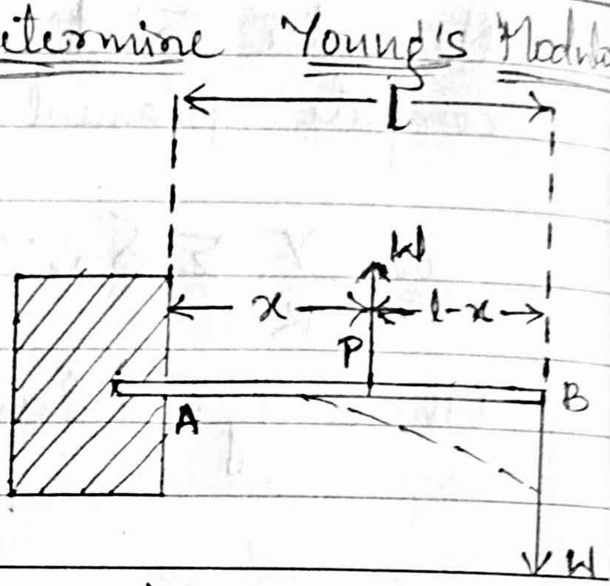
Cantilever Experiment to determine Young's Modulus



A cantilever is a beam fixed horizontally at one end and loaded at the other end.

Let AB be a beam of

length 'l' fixed at the end A and loaded by a weight W at B. The bent position is shown by the dotted line. Consider a section of the beam at P, distant x from the fixed end A. Since the beam is in equilibrium in the bent position, the external bending couple at P is balanced by the internal bending moment.

$$W(l-x) = \frac{YI}{R} \longrightarrow (1)$$

Where R is the radius of curvature of the neutral axis at P and I is the geometrical moment of inertia of the section of the beam at P.

$$\text{But} \quad R = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

∵ Differential Calculus.

9 thu

If the slope of the beam $\left(\frac{dy}{dx}\right)$ with the horizontal is small, then

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Hence, $W(l-x) = YI\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \frac{W}{YI}\left(lx - \frac{x^2}{2}\right) + C_1$$

10 fri

Where C_1 is the constant of integration.

At the end A, $x=0$, $dy/dx = 0$.

$\therefore C_1 = 0$.

Hence, $\frac{dy}{dx} = \frac{W}{YI}\left(lx - \frac{x^2}{2}\right)$.

Integrating again, $y = \frac{W}{YI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$

11 sat

Where C_2 is the constant of integration.

Again at the end A, $x=0$ and $y=0$.

$\therefore C_2 = 0$

$$\therefore y = \frac{W}{YI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$$

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Notes

13 mon

If y_0 is the maximum depression of the end B of the beam ie at $x = l$ then,

$$y_0 = \frac{W}{YI}\left(\frac{l\,l^2}{2} - \frac{l^3}{6}\right)$$

$$= \frac{W}{YI}\left(\frac{l^3}{2} - \frac{l^3}{6}\right)$$

14 tue

$$= \frac{W}{YI}\left(\frac{l^3}{3}\right)$$

$$\boxed{y_0 = \frac{Wl^3}{3YI}}$$

i) For a beam of rectangular cross-section.

$$I = \frac{bd^3}{12}$$

where b is the breadth and d the thickness of the beam

15 wed

$$\therefore y_0 = \frac{4Wl^3}{3bd^3Y}$$

If $W = Mg$ then $y_0 = \frac{4Mgl^3}{3bd^3Y}$

or $$\boxed{Y = \frac{4Mgl^3}{3bd^3y_0}}$$

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16 thu

ii) For a beam of circular cross-section

$$I = \frac{\pi r^4}{4}$$ where 'r' is the radius of the beam

$$y_0 = \frac{4WL^3}{3\pi r^4 Y}$$

17 fri

$$y_0 = \frac{4Mgl^3}{3\pi r^4 Y} \quad \text{or} \quad \boxed{Y = \frac{4Mgl^3}{3\pi r^4 y_0}}$$

18 sat

19 sun

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