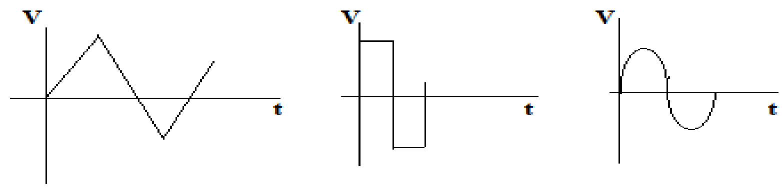
CHAPTER 2 : AC CIRCUITS

INTRODUCTION

All alternating waveform changes its magnitude and direction periodically. Figure shows various AC waveforms.



Many times alternating voltages and currents are represented by a sinusoidal waveforms. A sinusoidal voltage can be represented as

$$v = V_m \sin\theta$$
$$= V_m \sin\omega t$$
$$= V_m \sin 2\pi f t \text{ (wkt } f = 1/T)$$
$$\mathbf{v = V_m \sin\frac{2\pi}{T} t}$$

TERMS RELATED WITH ALTERNATING QUANTITIES

1. **Waveform**: A waveform is a graph in which the instantaneous value of any quantity is plotted against time.
2. **Cycle**: One complete set of positive and negative values of an alternating quantity is termed as cycle
3. **Frequency**: The number of cycles per second of an alternating quantity is known as frequency. It is denoted by 'f' and ins expressed in hertz (Hz) or cycles/sec (C/s)
4. **Time Period**: The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by 'T' and is expressed in seconds. ;T = $\frac{1}{f}$ sec
5. **Amplitude:** The max positive or negative values of an alternating quantity is called amplitude.
6. **Phase :** The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.
7. **Phase difference:** This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach the max and zero values at the same time. Their max value may be different in magnitude.

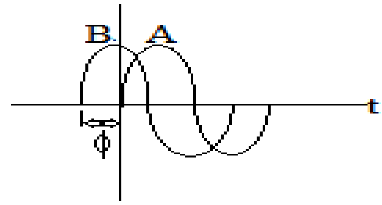
   A leading alternating quantity is one which reaches its max or zero value earlier as compared to the other quantity.

   A lagging alternating quantity is one which attains its max or zero value later as compared to the other quantity.

   A plus (+) sign which used in connection with the phase difference denotes leading, whereas minus (-) denotes lagging.

$$V_A = V_m \sin\omega t$$
$$V_B = V_m \sin(\omega t + \Phi)$$

Here quantity B leads A by a phase angle Φ



	Half wave rectified sine wave	Full wave rectified sine wave
Average Value	$V_{av} = \frac{Vm}{\pi}$	$V_{av} = \frac{2Vm}{\pi}$
RMS value	$I_{rms} = I_{eff} = \frac{Im}{2}$	$I_{rms} = I_{eff} = \frac{Im}{\sqrt{2}}$

Form factor : It is defined as the ratio of rms value of a wave and the average value of the wave.

$$\text{Form factor} = \frac{\text{rms value of the wave}}{\text{average value of the wave}}$$

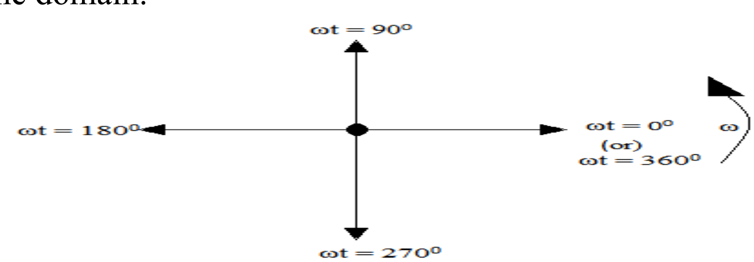
Peak factor or crest factor : It is the ratio of the peak value of the wave to its rms value.

$$\text{Peak factor} = \frac{\text{rms value of the wave}}{\text{average value of the wave}}$$

	Form factor	Peak factor
Sinusoidal wave	1.11	$\sqrt{2} = 1.414$
Triangular wave	$\frac{2}{\sqrt{3}}$	$\sqrt{3} = 1.732$

PHASOR DIAGRAMS:

- A geographical or pictorial representation of phases is known as phasor diagram.
- Phasor diagram is a pictorial representation of all the phase voltages and their respective currents in a network.
- Phasor is a **rotating vector**, which rotates in the **anticlockwise direction** with angular frequency ‘ω’ in the time domain.



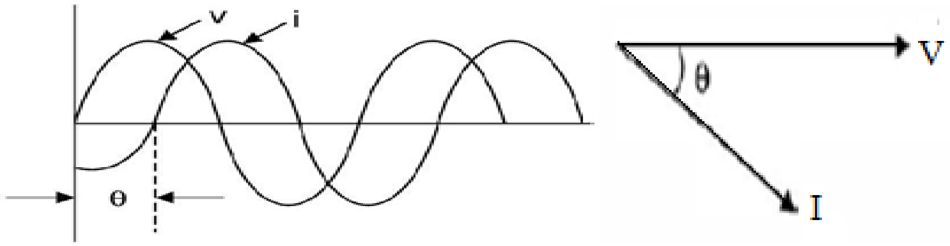
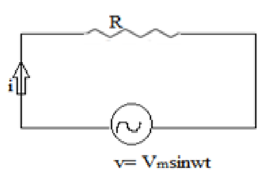
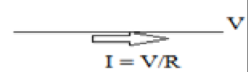
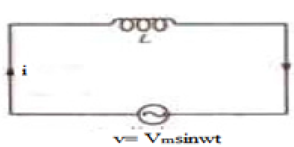
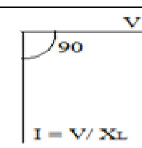
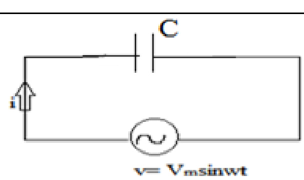
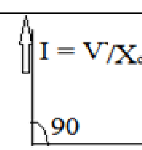


Fig: 2(a) VI sinusoidal wave 2(b) Phasor diagram of 2(a)

At each point on the time waveform, the angle of current lag is θ because the angle between the phasors V_m and I_m is at all time θ. Therefore, either the time waveform of the rotating phasors or the phasor diagram, can be used to describe the system. Since both the diagrams, the time diagram and the phasor diagram convey the same information, the phasor diagram being much more simpler, it is used for an explanation in circuit theory analysis. Since electrical data is given in terms of rms value, we draw phasor diagram with phasor values as rms rather than peak value used so far.

SINGLE PHASE SERIES CIRCUITS:

Single Phase series Circuits for pure passive elements

	Circuit	Voltage relation	Current relation	Phasor relation and power factor(PF)	Phasor diagram
Resistor		$v = V_m \sin\omega t$	$i = I_m \sin\omega t$	• Current is **in phase** with voltage • Power factor(PF) is unity	
Inductor **($X_L = 2\pi fL$)**		$v = V_m \sin\omega t$	$i = I_{max}\,(\omega t - \frac{\pi}{2})$	• Current **lags** voltage by an angle 90^0 • Power factor(PF) is zero (lagging)	
Capacitor **($X_c = \frac{1}{2\pi fC}$)**		$v = V_m \sin\omega t$	$I = I_m \sin(\omega t + \frac{\pi}{2})$	• Current **leads** voltage by an angle 90^0 • Power factor(PF) is zero (leading)	

1) <u>AC through series R-L circuit:</u>

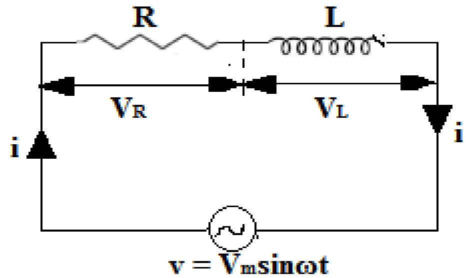


Fig 3(a): series RL circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in series with a pure inductance(L) in henry as shown in fig 3(a).

The series combination is connected across AC supply given by v = $V_m$sinωt

Circuit draws current I, then the two voltage drops are given by

a) Drop across pure resistance $V_R = IR$

b) Drop across pure inductance $V_L = IX_L$

Where, $X_L = 2\pi fL$ and I, V_R , V_L are the rms values.

KVL can be applied to get , $V = \overline{V_R} + \overline{V_L}$ ……………………..(1)

$$V = \overline{IR} + \overline{IX_L}$$

Let us draw the phasor diagram, Current I is taken as reference as it is common in both the elements.

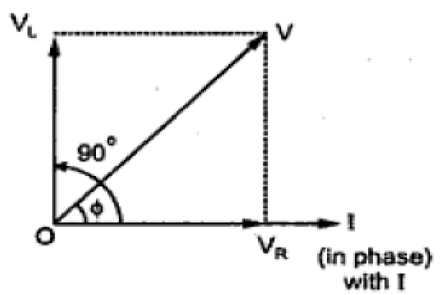


Fig 3(b) : Phasor diagram

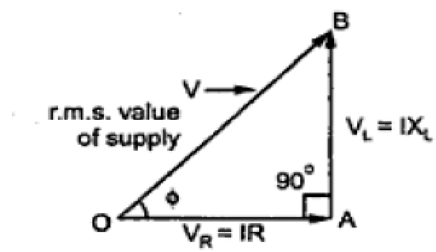


Fig 3(c) : Voltage triangle

❖ <u>**Impedance:**</u> Impedance is defined as the opposition in the circuit for the flow of alternating current. It is denoted by Z and its unit is ohms(Ω).

For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle Φ. From the voltage triangle, we can write,

$$\mathbf{tan\,\Phi = \frac{V_L}{V_R} = \frac{X_L}{R}\,;\quad cos\,\Phi = \frac{V_R}{V} = \frac{R}{Z}\,;\quad sin\,\Phi = \frac{V_L}{V} = \frac{X_L}{Z}}$$

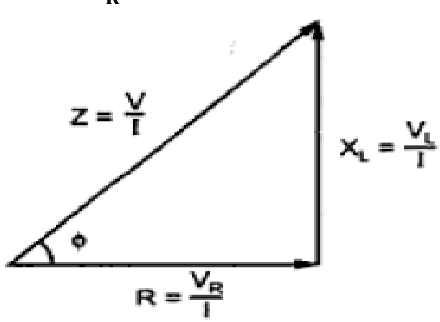


Fig 3(d) : Impedance triangle

If all the sides of voltage triangle are divided by current I, we get the triangle called impedance triangle as shown in fig 3(d).

Sides of this triangle are resistance R, inductive reactance X_L and an impedance Z.

From the impedance triangle, resistance R is given by **$R = Z\cos\Phi$**

Inductive reactance X_L is given by **$X_L = Z\sin\Phi$**

In rectangular form, impedance is given by **$Z = R + jX_L\ \Omega$**

In polar form, impedance is given by **$Z = |Z|\angle\Phi$**

Where, $|Z| = \sqrt{R^2 + X_L^2}$; $\Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

❖ **<u>Power and Power triangle</u>:**

- Average power $P_{av} = \frac{V_m I_m}{2}\cos\Phi = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} * \cos\Phi$

P = VI watts where V and I are rms values

If we multiply eq^n (1) by I, we get

$$\overline{V}I = \overline{IV_R} + \overline{IV_L}$$
$$\overline{V}I = \overline{V\cos\Phi I} + \overline{V\sin\Phi I}$$

From the above equation, power triangle can be obtained as shown in fig 3(e).

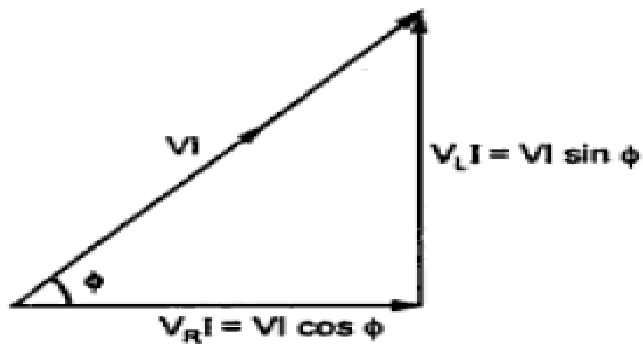


Fig 3(e) : Power triangle

So, the three sides of the triangle are (i) VI (ii) VIcosΦ (iii) VIsinΦ

❖ **<u>Apparent power (S):</u>** It is defined as the product of rms values of voltage(v) and current(I). It is denoted by S. Its unit is volt-amp(VA) or kilo volt-amp(KVA) **S = V*I VA**

❖ **<u>Real or True power (P):</u>** It is defined as the product of applied voltage and active component of current. It is the real component of apparent power. It is represented by 'P' and its unit is watts(W) or kilo watts(KW). **P = VIcos Φ watts**

❖ **<u>Reactive power (Q):</u>** It is defined as the product of applied voltage and reactive component of current. It is also defined as the imaginary component of apparent power. It is represented by 'Q' and its unit is volt-ampere reactive(VAR) or (KVAR). **Q = VIsin Φ VAR**

❖ **<u>Power factor (cos Φ):</u>** It is defined as a factor by which apparent power must be multiplied in order to obtain the true power.

Power factor $= \frac{\text{True power}}{\text{Apparent power}} = \frac{VI\cos\Phi}{VI} = \cos\Phi$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. **It is always less than 1.**

It is also defined as the ratio of resistance to impedance. **$\cos\Phi = \frac{R}{Z}$**

NOTE: *If current lags voltage, power factor is said to be lagging, if current leads voltage, power factor is said to be leading.*

So ,for pure inductance, the power factor is $\cos(90°)$ *ie.,zero power factor(ZPF) lagging, while for pure capacitance, the power factor is* $\sin(90°)$ *ie.,zero power factor(ZPF) leading. For a purely resistive circuit volatge and current are in phase ie.,* $\Phi = 0°$*. Therefore, power factor is* $\cos(0°) = 1$*. Such circuit is called unity power factor(UPF) circuit.*

$\boldsymbol{Power\ factor\ =\ \cos\Phi}$ *, where* Φ *is the angle between voltage and current.*

2) **AC through series R-C circuit:**

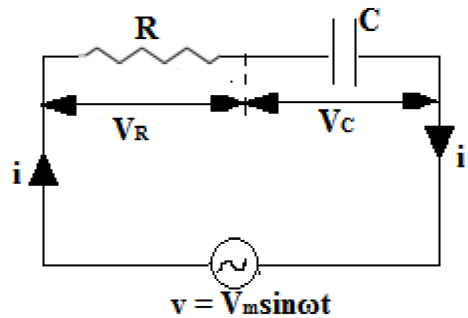


Fig 4(a): series RC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in series with a pure capacitance(C) in farad as shown in fig 4(a).

The series combination is connected across AC supply given by $v = V_m \sin\omega t$

Circuit draws current I, then the two voltage drops are given by

a) Drop across pure resistance $V_R = IR$

b) Drop across pure capacitance $V_C = IX_C$

Where, $X_C = \frac{1}{2\pi fC}$ and I, V_R , V_C are the rms values.

KVL can be applied to get , $V = \overline{V_R} + \overline{V_C}$ ……………………..(1)

$V = \overline{IR} + \overline{IX_C}$

Let us draw the phasor diagram, Current I is taken as reference as it is common in both the elements.

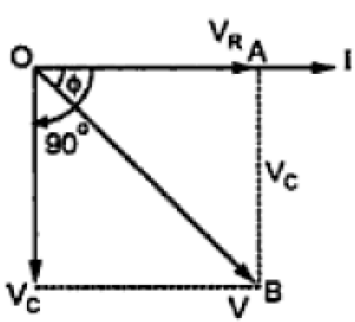


Fig 4(b) : Phasor diagram

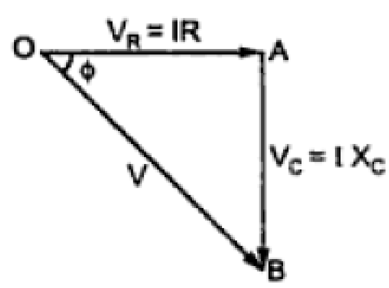
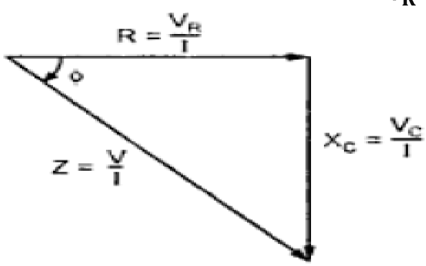


Fig 4(c) : Voltage triangle

❖ **Impedance:** Impedance is defined as the opposition in the circuit for the flow of alternating current. It is denoted by Z and its unit is ohms(Ω).

For the R-C series circuit, it can be observed from the phasor diagram that the current leads the applied voltage by an angle Φ. From the voltage triangle, we can write,

$$\tan\Phi = \frac{V_C}{V_R} = \frac{X_C}{R}\ ; \quad \cos\Phi = \frac{V_R}{V} = \frac{R}{Z}\ ; \qquad \sin\Phi = \frac{V_C}{V} = \frac{X_C}{Z}$$



If all the sides of voltage triangle are divided by current I, we get the triangle called impedance triangle as shown in fig 4(d).

Sides of this triangle are resistance R, inductive reactance X_C and an impedance Z.

Fig 4(d) : Impedance triangle

From the impedance triangle, resistance R is given by **$R = Z\cos\Phi$**

Inductive reactance X_L is given by **$X_L = Z\sin\Phi$**

In rectangular form, impedance is given by **$Z = R - jX_C\ \Omega$**

In polar form, impedance is given by **$Z = |Z|\angle -\Phi\ \Omega$**

$$\text{Where, } |Z| = \sqrt{R^2 + X_C^{\,2}}\ ;\ \Phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

❖ **Power and Power triangle**:

Average power $P_{av} = \frac{V_m I_m}{2}\cos\Phi = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} * \cos\Phi$

P = VI watts where V and I are rms values

If we multiply eq^n (1) by I, we get

$$\overline{VI} = \overline{IV_R} + \overline{IV_L}$$

$$\overline{VI} = \overline{V\cos\Phi I} + \overline{V\sin\Phi I}$$

From the above equation, power triangle can be obtained as shown in fig 4(e).

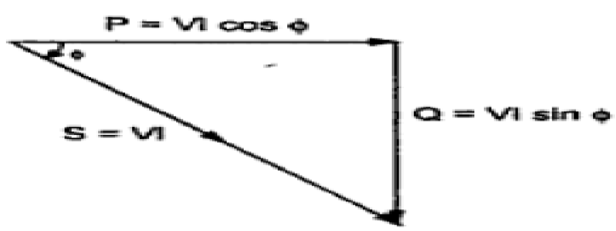


Fig 4(e) : Power triangle

Thus, the various powers are,

a. **Apparent power, S = VI VA**

b. **True or avg power, P = VIcos Φ watts**

c. **Reactive power, Q = VIsin Φ VAR**

Note: *Remember that, X_L term appears to be positive in Z*

ie., $Z = R + jX_L\ \Omega = |Z|\angle\Phi$, where Φ is positive for Z.

X_C term appears to be negative in Z

ie., $Z = R - jX_C\ \Omega = |Z|\angle -\Phi$, where Φ is negative for Z.

3) **AC through series R-L-C circuit:**

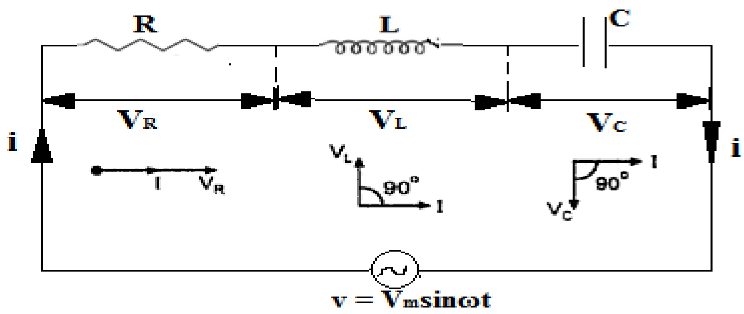


Fig 5(a) : Series RLC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in series with a pure inductance(L) in henry and a pure capacitance(C) in farad as shown in fig 5(a).

The series combination is connected across AC supply given by $v = V_m \sin\omega t$

Circuit draws current I, then the two voltage drops are given by

a) Drop across pure resistance $V_R = IR$

b) Drop across pure inductance $V_L = IX_L$; Where, $X_L = 2\pi fL$

c) Drop across pure capacitance $V_C = IX_C$ Where, $X_C = \frac{1}{2\pi fC}$

The values of I, V_R , V_L, V_C are the rms values.

The characteristics of three drops are

(a) V_R is in phase with current I.

(b) V_L leads current by 90°

(c) V_C lags current by 90°

According to kirchoff's law, we can write,

$$\bar{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

Let us draw the phasor diagram, Current I is taken as reference as it is common to all the elements.

The phasor diagram depends on the condition ofmagnitudes of V_L and V_C which ultimately depends on the valurs of X_L and X_C.

Let us consider the different cases,

(i) $X_L > X_C$: When $X_L > X_C$, obviously IX_L ie.,V_L is greater than IX_C ie., V_C. So the resultant of V_L and $V_C$will be directed towards V_L leading current I. Current I will lag the resultant of V_L and V_C ie., $(V_L – V_C)$.

The circuit is said to be inductive in nature. The phasor sum of V_R and $(V_L – V_C)$ gives the resultant supply voltage V. This is shown in fig 5(b).

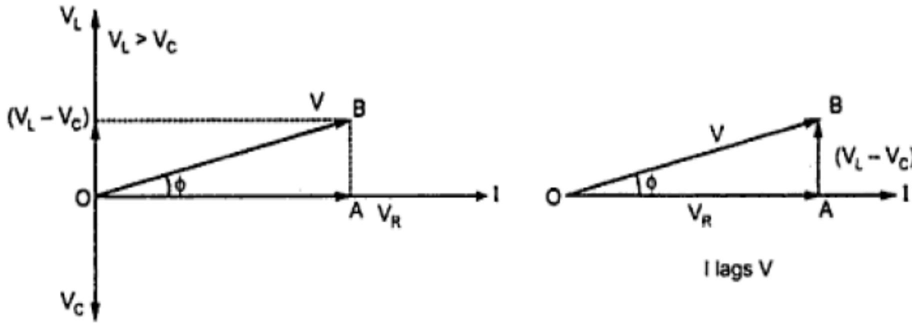


Fig 5(b) : Phasor diagram and voltage triangle for ($X_L > X_C$)

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where, $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$; $(\mathbf{X_L - X_C})$ **is positive**.

(ii) **$X_L < X_C$** : When $X_L < X_C$, obviously IX_C ie.,V_C is greater than IX_L ie., V_L. So the resultant of V_L and V_C will be directed towards V_C lagging current I. Current I will leads the resultant of V_L and V_C ie., $(V_C - V_L)$.

circuit is said to be capacitive in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in fig 5(c).

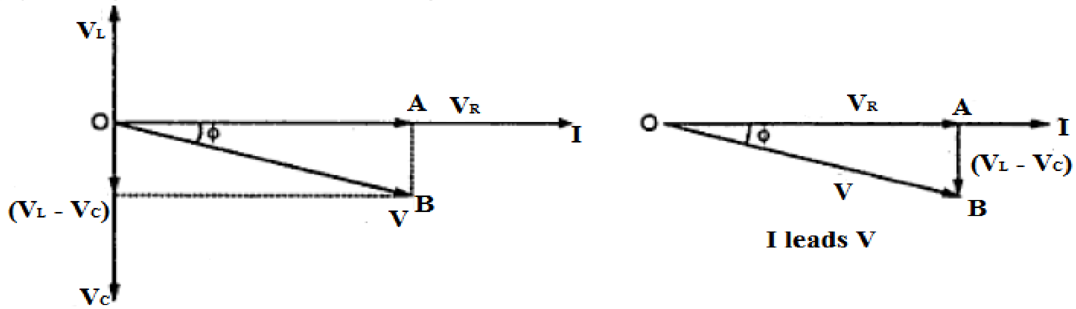


Fig 5(c) : Phasor diagram and voltage triangle for ($X_L < X_C$)

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_C - V_L)^2} = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$

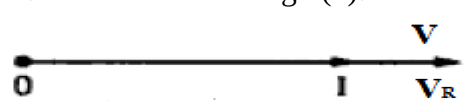
$$V = I\sqrt{(R)^2 + (X_C - X_L)^2}$$

$$V = IZ$$

Where, $Z = \sqrt{(R)^2 + (X_C - X_L)^2}$; $(\mathbf{X_C - X_L})$ **is positive**

(iii) **$X_L = X_C$:** When $X_L = X_C$, obviously IX_C ie.,V_C is equal to IX_L ie., V_L ie., $(V_L - V_C) = 0$. Current I be in phase with the voltage V_R.

circuit is said to be resistive in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage V ie., V_R itself. This is shown in fig 5(d).



V and I are in phase

$V = IZ$, where $Z = R$

1. **AC through parallel R-L circuit**

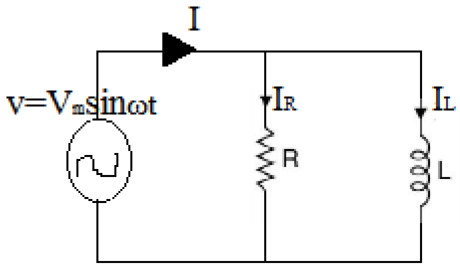


Fig :6(a)

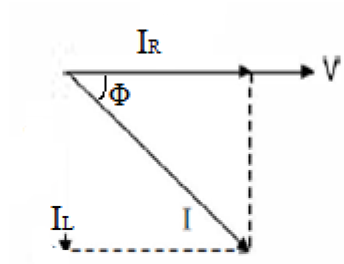


Fig: 6(b)

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure inductance(L) in henry as shown in fig 6(a).

We have taken here V as the reference quantity as it is the voltage which is common to both the elements R and L and not the current. Now current through R will be in phase with the voltage across it whereas current through inductor will lag by 90° the voltage across it. Hence, the phasor diagram in Fig. 6(b) follows and current I supplied by the source equals the phasor sum of the current I_1 and I_2.

Now, the current is given by , $I_R = \frac{V}{R}$; $I_L = \frac{V}{X_L} \angle -90°$; $I = I_R + I_L = \frac{V}{R} + \frac{V}{j\omega L}$

Here, $\frac{1}{R}$ is the reciprocal of resistance and is termed as conductance(G), whereas $\frac{1}{\omega L}$ is the reciprocal of the inductive reactance and is known as inductive susceptance(B_L).

From the above equation, we have

$$\frac{I}{V} = \frac{1}{R} + \frac{1}{j\omega L} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Now dimensionally $\frac{I}{V}$ is reciprocal of impedance and is known as admittance(Y). The above equation can be rewritten as **$Y = G - jB_L$**

Where, Impedance $Y = \frac{1}{Z}$; Conducatnce $G = \frac{1}{R}$; Susceptance $B_L = \frac{1}{\omega L}$

It is to be noted that whereas for an inductive circuit $Z = R + jX_L$, the admittance is $Y = G - jB_L$

The units of admittance is mho or siemens and denoted by ℧ or S respectively.

The power consumed by the circuit is again P = VI cosΦ where I is the total current and Φ is the angle between V and I. $\Phi = \tan^{-1}\left(\frac{-B_L}{G}\right)$; Power factor , $\cos\Phi = \frac{G}{Y} = \frac{I_R}{I}$ (lag)

From eq^n (1), we have $\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L}$

If $Z_1 = R$ and $Z_2 = j\omega L$, we have

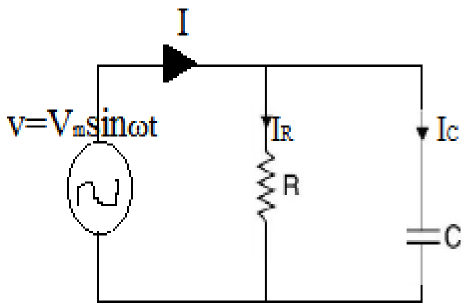
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}\ ;\ \ Z = \frac{Z_1 * Z_2}{Z_1 + Z_2} \qquad \ldots\ldots\ldots(2)$$

i.e. if there are two impedances Z_1, Z_2 connected in parallel, their equivalent impedance equals the ratio of the product of the two impedance to the sum of the two impedances. Also from eq^n (2)

$$\mathbf{Y = Y_1 + Y_2}$$

i.e. if there are two admittances Y_1 and Y_2 connected in parallel, the equivalent admittance is the sum of the two admittances.

2. <u>AC through parallel R-C circuit</u>



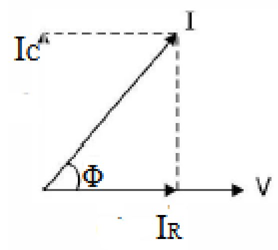


Fig 7(a) **Fig 7(b)**

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure capacitance(c) in farad as shown in fig 7(a).

We have taken here V as the reference quantity as it is the voltage which is common to both the elements R and C and not the current. Now current through R will be in phase with the voltage across it whereas current through capacitor will lead by 90° the voltage across it. Hence, the phasor diagram in Fig. 7(b) follows and current I supplied by the source equals the phasor sum of the current I_1 and I_2.

Now, the current is given by, $I_R = \frac{V}{R}$; $I_C = \frac{V}{X_C} \angle 90°$; $I = I_R + I_C = \frac{V}{R} + jV\omega C$

Here, $\frac{1}{R}$ is the reciprocal of resistance and is termed as conductance(G), whereas ωC is the reciprocal of the capacitive reactance and is known as capacitive susceptance(B_C).

From the above equation, we have

$$\frac{I}{V} = \frac{1}{R} + j\omega C \qquad \ldots\ldots\ldots\ldots(3)$$

Now dimensionally $\frac{I}{V}$ is reciprocal of impedance and is known as admittance(Y). The above equation can be rewritten as **$Y = G + jB_C$**

Where, Impedance $Y = \frac{1}{Z}$; Conductance $G = \frac{1}{R}$; Susceptance $B_C = \omega C$

It is to be noted that whereas for an inductive circuit $Z = R - jX_C$, the admittance is $Y = G + jB_C$

The units of admittance is mho or simens and denoted by ℧ or S respectively.

The power consumed by the circuit is again P = VI cos Φ where I is the total current and Φ is the angle between V and I. $\Phi = \tan^{-1}\left(\frac{B_C}{G}\right)$; Power factor , $\cos\Phi = \frac{G}{Y} = \frac{I_R}{I}$ (lead)

3. <u>AC through parallel R-L-C circuit</u>

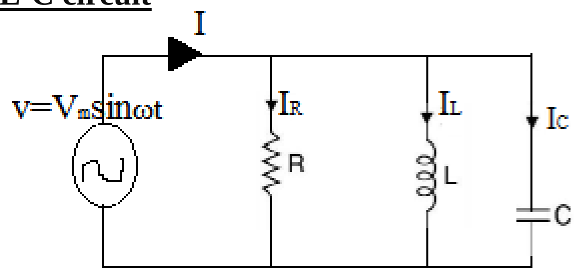


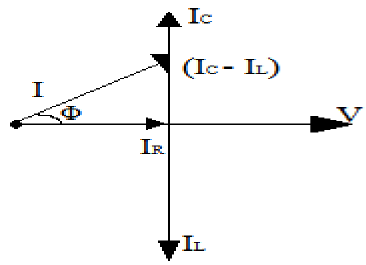
Fig 8(a) : parallel RLC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure inductance(L) in henry and a pure capacitance(C) in farad as shown in fig 8(a).

The parallel combination is connected across AC supply given by $v = V_m \sin\omega t$

The current is given by , $I_R = \frac{V}{R}$; $I_L = \frac{V}{X_L} \angle -90°$; $I_C = \frac{V}{X_C} \angle 90°$

I. $B_C > B_L$ ($I_C > I_L$)

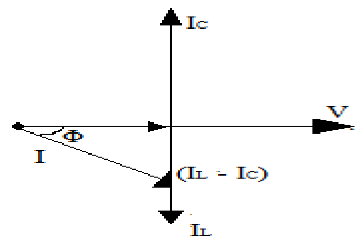


Current, $I = \sqrt{(I_R)^2 + (I_C - I_L)^2}$

Admittance angle, $\Phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right) = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$

Power factor, $\cos\Phi = \frac{I_R}{I}$ (PF lead)

II. **$B_C < B_L$ ($I_C < I_L$)**

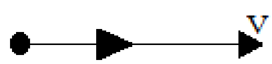


Current, $I = \sqrt{(I_R)^2 + (I_L - I_C)^2}$

Admittance angle, $\Phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right) = \tan^{-1}\left(\frac{B_L - B_C}{G}\right)$

Power factor, $\cos\Phi = \frac{I_R}{I}$ (ZPF lag)

III. $B_L = B_C$ ($I_L = I_C$)



Current, $I = I_R$; $\Phi = 0°$; $\cos\Phi = 1$ (UPF)

Circuit	Voltage and current relation	Impedance(Z) or Admittance(Y)	Phase angle Φ	Phasor relation	Power factor(PF) = cosΦ
Series RL circuit	$V = \sqrt{V_R^2 + V_L^2}$	$Z = (R + jX_L)$	$\tan^{-1}\left(\frac{X_L}{R}\right)$	I lags V by an angle Φ	$\frac{R}{z} = \frac{V_R}{V}$
Series RC circuit	$V = \sqrt{V_R^2 + V_c^2}$	$Z = (R - jX_C)$	$\tan^{-1}\left(\frac{-X_C}{R}\right)$	I leads V by an angle Φ	$\frac{R}{z} = \frac{V_R}{V}$
Series RLC circuit	$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$	$Z = R + j(X_L - X_C)$	$\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$	$X_L > X_C$; I lags V $X_L < X_C$; I leads V $X_L = X_C$;I in phase with V	$\frac{R}{z} = \frac{V_R}{V}$
Parallel RL circuit	$I = \sqrt{I_R^2 + I_L^2}$	$Y = (G - jB_L)$	$\tan^{-1}\left(\frac{-B_L}{G}\right)$	I lags V by an angle Φ	$\frac{G}{Y} = \frac{I_R}{I}$
Parallel RC circuit	$I = \sqrt{I_R^2 + I_c^2}$	$Y = (G + jB_C)$	$\tan^{-1}\left(\frac{B_C}{G}\right)$	I leads V by an angle Φ	$\frac{G}{Y} = \frac{I_R}{I}$
Parallel RLC circuit	$I = \sqrt{(I_R)^2 + (I_L - I_C)^2}$	$Y = G + j(B_C - B_L)$	$\tan^{-1}\left(\frac{B_C - B_L}{G}\right)$	$B_L > B_C$; I lags V $B_L < B_C$; I leads V $B_L = B_C$;I in phase with V	$\frac{G}{Y} = \frac{I_R}{I}$

R = Resistance ; X = Reactance; Z = Impedance

$G = \frac{1}{R}$ = Conductance; $B = \frac{1}{X}$ = susceptance; $Y = \frac{1}{Z}$ = Admittance

Concept of Time constant:

A. Time Constant of an RC Circuit

Let us take a simple RC circuit, as shown below.

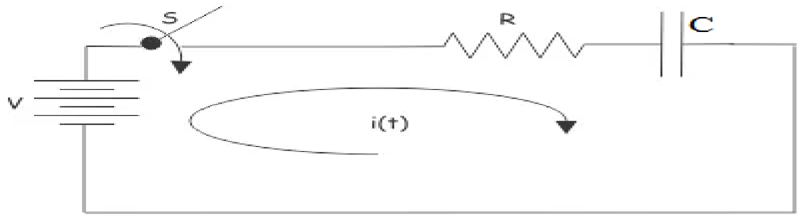


Fig 9: Series RC circuit

Let us assume the capacitor is initially unchanged and the switch S is closed at time t = 0. After closing the switch, electric current i(t) starts flowing through the circuit. Applying Kirchhoff Voltage Law in that single mesh circuit, we get,

$$V = Ri(t) + \frac{1}{C}\int i(t)dt$$

Differentiating both sides with respect to time t, we get,

$$R\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

$$R\frac{di(t)}{dt} = \frac{-i(t)}{C}$$

$$-RC\,\frac{di(t)}{i(t)} = dt$$

$$\frac{di(t)}{i(t)} = \frac{-1}{RC}dt$$

Integrating on both sides, we get,

$$\int \frac{di(t)}{i(t)} = = \frac{-1}{RC}dt$$

$$\log_e i(t) + \log_e i(k) = \frac{-t}{RC}$$

$$\log_e(ki(t)) = \frac{-t}{RC}$$

$$k*i(t) = e^{\frac{-t}{RC}} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Now, at t = 0, the capacitor behaves as a short circuit, so just after closing the switch. The current through the circuit will be $I_o = \frac{V}{R}$

Now, substituting the above eq^n in eq^n(1), we get

$$k*\frac{V}{R} = e^0 \;;\; k = \frac{R}{V}$$

Now, putting the value of k in eq^n (1), we get

$$i(t) = \frac{V}{R}e^{\frac{-t}{RC}}$$

Now, if we put t = RC in the final expression of circuit current i(t), we get,

$$I_{t=RC} = = \frac{V}{R}e^{-1} = \mathbf{0.367}\,\frac{\mathbf{V}}{\mathbf{R}} = \mathbf{0.364I_o = 36.7\%\ I_o}$$

From the above mathematical expression, it is clear that RC is the time in second during which the current in a charging capacitor diminishes to 36.7 percent from its initial value. Initial value means current at the time of switching on the unchanged capacitor.

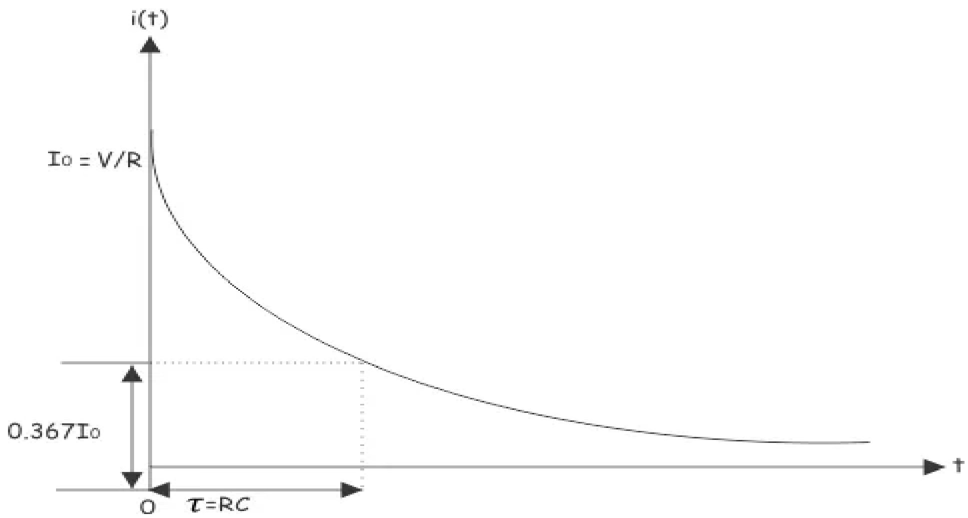
This term is quite significant in analyzing the behavior of capacitive as well as inductive circuits. This term is known as the **time constant**.

So **time constant** is the duration in seconds during which the current through a capacities circuit becomes 36.7 percent of its initial value. This is numerically equal to the product of resistance and capacitance value of the circuit. The time **constant** is normally denoted by τ (tau).

So, **τ = RC**

In a complex RC circuit, the **time constant** will be the equivalent resistance and capacitance of the circuit.

Let us discuss the significance of the time constant in more detail. To do this, let us first plot current i(t).



At t = 0, the current through the capacitor circuit is $\mathbf{I_o = \frac{V}{R}}$

At t = RC, the current through the capacitor is $\mathbf{0.364I_o = 0.367\,\frac{V}{R}}$

B. <u>Time Constant of an RC Circuit</u>

Let us take a simple RC circuit, as shown below

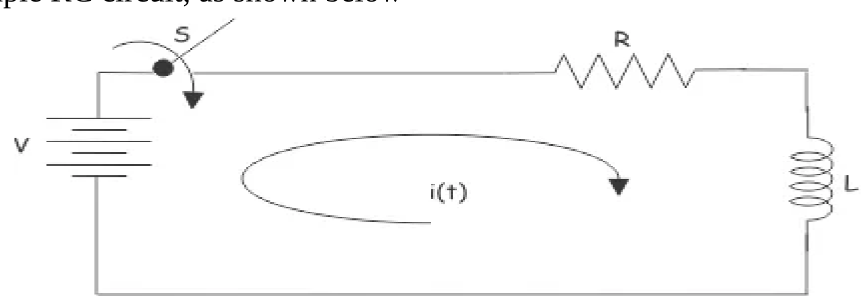


Fig 10 : Series RL circuit

Applying Kirchhoff Voltage Law in the above circuit. We get,

$$V = Ri(t) + L\frac{di(t)}{dt}$$

The equation can also be solved Laplace Transformation technique. For that, we have to take Laplace Transformation of the equation at both sides,

$$\frac{V}{s} = L[sI(s) - i(0)] + RI(s)$$

$$\frac{V}{s} = LsI(s) + RI(s)$$

$$I(s) = \frac{V}{s(sL+R)}$$

Now, $$I(s) = \frac{V_o}{R}\frac{1}{s(sL+R)}$$

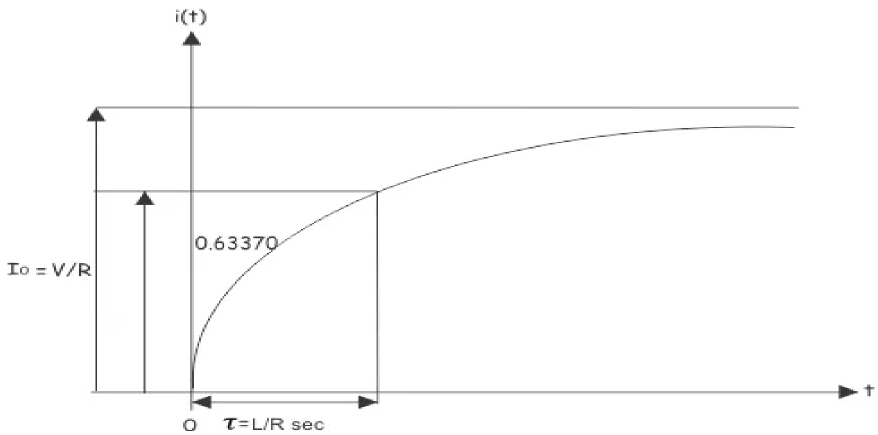
Taking inverse laplace transform of the above equation, we get

$$i(t) = \frac{V_o}{R}(1\text{- } e^{\frac{-Rt}{L}})$$

Now, if we put $t = \frac{L}{R}$, we get, $$i(\tfrac{L}{R}) = \frac{V_o}{R}(1\text{- } e^{-1}) = 0.633\frac{V_o}{R}$$

At the RL circuit, at time t = L/R sec, the current becomes 63.3% of its final steady-state value. The L/R is known as the time constant of an LR circuit ie., $\boldsymbol{\tau = \frac{L}{R}}$

Let us plot the current of the inductor circuit.



<u>Resonance:</u> Resonance in electric circuit is because of the presence of both energy storing elements ie., inductor and capacitor.

Under resonance, for an electric circuit, if the applied voltage and resultant current, both are in phase (Φ = 0°) , then the system is under resonace.

Z = Resistive (ie., imaginary part = 0)

(OR) The total impedance of any electrical circuit is only resistive (imaginary part = 0) then circuit is under resonance.

(OR) If any electrical circuit is operating at UPF(ie., cos Φ = 0), then the circuit is under resonance.

Resonance

Series Resonance

Parallel Resonance

- R-L-C Parallel circuit
- R-L & C parallel circuit
- R-L & R-C parallel circuit

- **Series Resonance**
  - **R-L-C series circuit:**

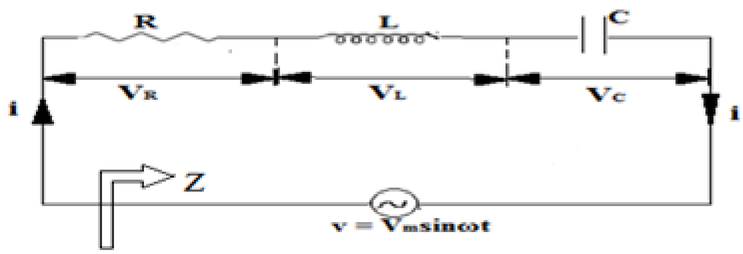


Fig 11: series RLC circuit for resonance

The impedance of the circuit at any frequency ω is given as

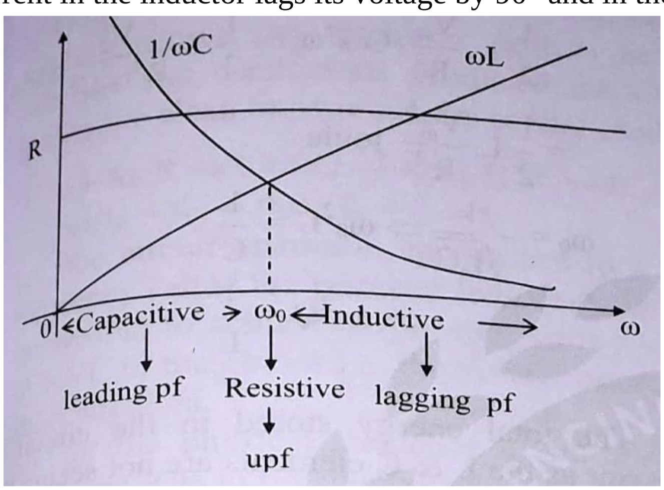
$$Z = Z_R + Z_L + Z_C$$

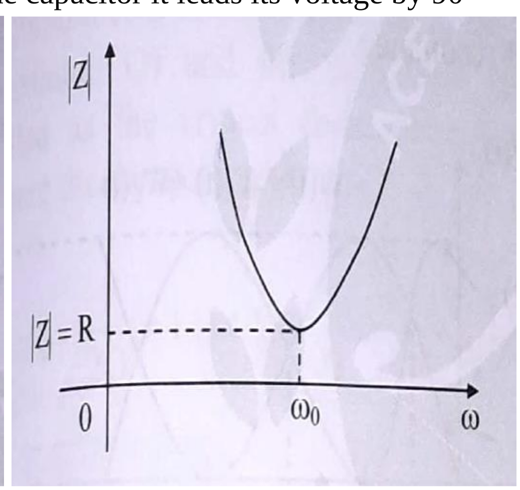
$$Z = R + j(\omega L - \frac{1}{\omega C})$$

Since resistance is independent of frequency the circuit will have minimum impedance at some frequency.

When $Z = R$; Or when, $\omega_o L - \frac{1}{\omega_o C} = 0$; $\boldsymbol{\omega_o = \frac{1}{\sqrt{LC}}}$; $\boldsymbol{f_o = \frac{1}{2\pi\sqrt{LC}}}$

Here f_o is the frequency of resonance i.e. if a circuit has fixed values of R, L and C, resonance will take place if the supply frequency $f_o = \frac{1}{2\pi\sqrt{LC}}$, when the impedance of the circuit is purely resistive i.e. the power factor of the circuit is unity, the supply voltage and current are in phase. However, it is to be noted that the phase relations between the voltage and current in the individual elements R, L and C are not same. The current in the inductor lags its voltage by 90° and in the capacitor it leads its voltage by 90° •





Since R is independent of frequency it is shown by a horizontal line Z = R. Also $X_L = \omega L$ the inductive reactance is a straight line passing through the origin and inductive reactance is taken as +ve, where as $X_C = \frac{1}{\omega C}$ the capacitive reactance as a function of ω is a rectangular hyperbola and the reactance is taken as -ve. The net impedance is shown a positive quantity. The resonance frequency is f_0 where $(\omega L - \frac{1}{\omega C})$ is zero and at this frequency the impedance curve has minimum value equal to R.

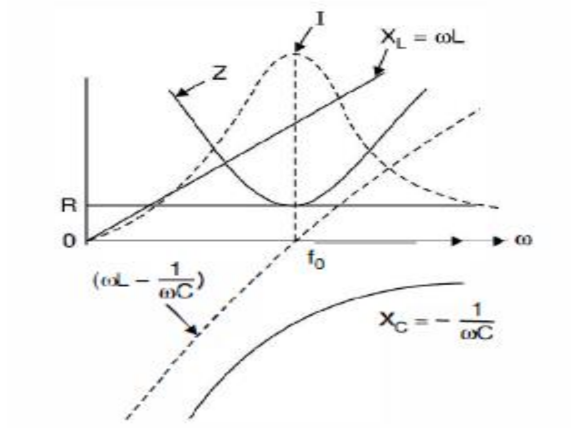


Fig 12: Z Vs ω for series RLC circuit

The variation of current is also shown in Fig 12 as a function of frequency and is maximum at f_0 whereas on either side the current decreases. It is to be noted that at $\omega = 0$, the current in the RLC series circuit is zero as the capacitor reactance is infinite and, therefore, the graph starts from origin whereas it is again zero at $\omega = \infty$ and hence the graph should not be passing through zero rather it should have some finite value as indicated in the diagram.

Again, it can be seen that the series circuit is capacitive for all frequency $\omega < \omega_0$, inductive for all frequency $\omega > \omega_0$ and at $\omega = \omega_0$ the circuit is resistive.

There are various applications of a series resonant circuit where the frequency is fixed and either L or C is varied to obtain the condition of resonance. A typical example is that of tuning a radio receiver to a particular desired station that is operating at a fixed frequency. Here a circuit or L and C is adjusted to resonance at the operating frequency of the desired station. The capacitor C (parallel plate) is variable in most portable radio receivers and the inductance of the coil is usually varied in tuning of an automobile radio receiver.

Under resonance, $I_o = \frac{V}{R}$

Therefore, $V_R = I_oR = \frac{V}{R} * R = V$ is the supply voltage

$V_L = I_o\omega L = \frac{V}{R}\omega L = V\frac{\omega_o L}{R}$

where Q is known as the quality factor of the series network. Usually Q >> 1, hence it is also known as voltage gain, as the voltage across the inductor is much greater than the supply voltage.

Also, $V_C = \frac{I_O}{\omega_O C} = V\frac{1}{\omega_O CR} = VQ;$ $\quad V_L = V_c >> V$

Therefore, extreme care must be taken when working on series circuits that may become resonant when connected to power line sources.

Quality factor (Q):

In a practical circuit R is essentially resistance of the coil since practical capacitors have very low loss in comparison to practical inductor. Hence Q is a measure of the energy storage property (LI^2) in relation to the energy dissipation property (I^2R) of a coil or a circuit.

The Q is, therefore, defined as

$$\mathbf{Q = 2\pi \frac{maximum\ energy\ stored}{energy\ dissipated\ per\ cycle}}$$

In electric circuit energy is stored in the form of electromagnetic field in the inductance where as in electrostatic form of energy across a capacitance. It can be proved that at any instant at a certain frequency the sum of energy stored by the inductor and the capacitor is constant. At the extreme situation when the current through the inductor is maximum, the voltage across the capacitor is zero hence the total energy is

$$\frac{1}{2}L(\sqrt{2}\,I)^2 = LI^2$$

where $\sqrt{2}I$ is the instantaneous maximum value of the current. At this since V_C is zero, therefore maximum energy stored is LI^2 . The power consumed per cycle is the energy per sec divided

by f_0 under resonance condition. Therefore $P_R = \frac{I^2R}{f_o}$

Hence $Q = \frac{2\pi LI^2}{I^2R}$; $\mathbf{Q = \frac{\omega_o L}{R}}$ (Q for series RL circuit)

NOTE:

Q can also be looked as a ratio of

$$Q = \frac{Time\ rate\ of\ change\ of\ energy\ stored}{Time\ rate\ of\ change\ of\ energy\ dissipated} = \frac{Reactive\ power\ absored\ by\ inductor}{Active\ power\ consumed\ by\ resistor}$$

The great advantage of this definition of Q is that it is also applicable to more complicated lumped circuits, to distributed circuits such as transmission lines and to non-electrical circuits.

Q is also a measure of the frequency selectivity of the circuit. A circuit with high Q will have a very sharp current response curve as compared to one which has a low value of Q. To understand this let us consider Fig. 13 . Here we find that the current response is maximum at f_0 and on either side of f_0, the current decreases sharply.

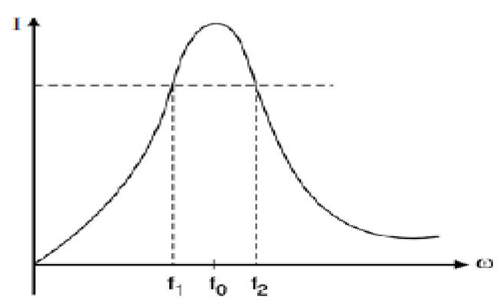


Fig 13 : Frequency selectivity

Frequencies f_1 and f_2 are known as half-power frequencies as at these frequencies the power dissipated by the circuit is half of that dissipated at f_o.

The band width of a resonant circuit is defined as the frequency range between the 70.7% current points. **Bandwidth, BW =f_2 - f_1**

Resonant frequency , $\omega_o = \sqrt{\omega_1\omega_2}$

This means the resonance frequency is the geometric mean of the half-power frequencies.

$\frac{\omega_2 - \omega_1}{\omega_o} = \omega_o CR = \frac{1}{Q}$ (where, Q = $\frac{1}{\omega o CR}$ is the Q factor value for RC series circuit)

$$f_2 - f_1 = \frac{f_o}{Q}$$

Bandwidth is thus given by the ratio of the frequency of resonance to the quality factor and selectivity is defined as the ratio of resonant frequency to the bandwidth $f_0/(f_2 - f_1)$. This, therefore, shows that the larger the value of Q the smaller is $(f_2 - f_1)$ and hence sharper is the current response.

$$\textbf{Selectivity} \propto \textbf{Q-factor} \propto \frac{1}{\text{bandwidth}};\ Q = \frac{f_o}{f_2 - f_1};\ Q = \frac{\omega_o}{\omega_2 - \omega_1}$$

NOTE

Q factor for

- *RL series circuit:* $Q = \frac{\omega_o L}{R}$
- *RC series circuit:* $Q = \frac{1}{\omega o CR}$
- *RLC series circuit:* $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Parallel Resonance

Parallel R-L-C circuit

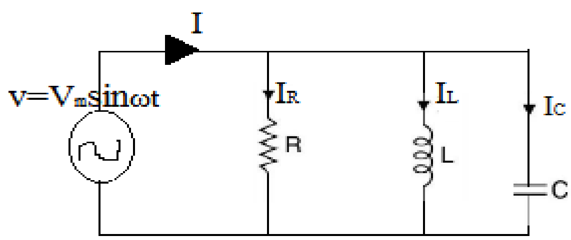


Fig 14 : Parallel RLC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure inductance(L) in henry and a pure capacitance(C) in farad as shown in fig 14.

Wkt, $Y = G + j(B_C - B_L)$

Under resonance, $(B_C - B_L) = 0$; $B_C = B_L$

$$B_C = \frac{1}{X_C} = \omega_o C\ ; \qquad B_L = \frac{1}{X_L} = \frac{1}{\omega_o L}$$

$$\omega_o C = \frac{1}{\omega_o L}; \qquad \omega_o^2 = \frac{1}{LC}; \qquad \omega_o = \frac{1}{\sqrt{LC}} \text{ rad/sec}; \qquad f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Therefore, the expression for resonant frequency is $\mathbf{f_o = \frac{1}{2\pi\sqrt{LC}}}$ **Hz**

❖ R-L & C parallel circuit

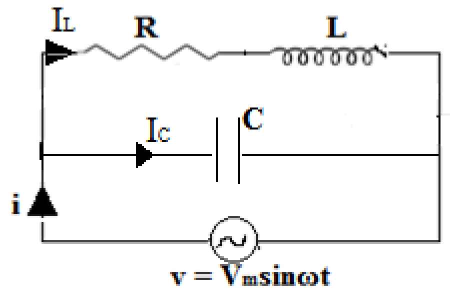


Fig 15: R-L & C parallel circuit

Consider the circuit in which one branch consists of resistor R in series with the inductor L. So it is series R-L circuit with impedance Z_{RL}. The other branch is pure capacitive with capacitor C. Both the branches are connected in parallel across a variable frequency constant voltage source.

The current drawn by inductive branch is I_L, while drawn by capacitive branch is I_C.

$$I_L = \frac{V}{Z_{RL}}, \text{ where } Z_L = R + jX_L$$

$$I_C = \frac{V}{Z_C}, \text{ where } Z_C = \frac{1}{2\pi fC}$$

$$Y = \frac{1}{Z_{RL}} + \frac{1}{Z_C} = \frac{1}{R+jX_L} + \frac{1}{-jX_C}$$

$$Y = \frac{1}{R+jX_L} + \frac{1}{-jX_C} = \frac{R-jX_L}{R^2+{X_L}^2} + \frac{j}{X_C}$$

$$Y = \frac{R}{R^2+{X_L}^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+{X_L}^2}\right)$$

At resonance, imaginary part is equal to zero.

$$\frac{1}{X_C} - \frac{X_L}{R^2+{X_L}^2} = 0; \quad \frac{1}{X_C} = \frac{X_L}{R^2+{X_L}^2}$$

$$R^2 + {X_L}^2 = X_L X_C \quad \left(X_L X_C = \omega_o L * \frac{1}{\omega_o C} = \frac{L}{C}\right)$$

$$R^2 + {X_L}^2 = \frac{L}{C}; \quad {X_L}^2 = \frac{L}{C} - R^2$$

$$X_L = \sqrt{\frac{L}{C} - R^2}$$

$$\omega_o = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}; \quad \boldsymbol{\omega_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} \textbf{ rad/sec}$$

$$\boldsymbol{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} \textbf{ Hz}$$

❖ R-L and R-C parallel circuit:

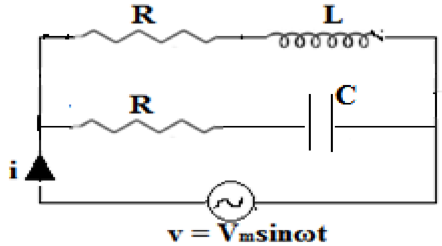


Fig 16: R-L and R-C parallel circuit

Consider the circuit in which one branch consists of resistor R in series with the inductor L, So it is series R-L circuit with impedance Z_{RL}. The other branch is a resistor R in series with a pure capacitive with capacitor C, So it is series R-C circuit with impedance Z_{RC}. Both the branches are connected in parallel across a variable frequency constant voltage source v.

$$Y = \frac{1}{Z_{RL}} + \frac{1}{Z_{RC}} = \frac{1}{R+jX_L} + \frac{1}{R-jX_C}$$

$$Y = \frac{1}{R_L+jX_L} + \frac{1}{R_C-jX_C} = \frac{R_L-jX_L}{{R_L}^2+{X_L}^2} + \frac{R_C+jX_C}{{R_C}^2+{X_C}^2}$$

$$Y = \frac{R_L}{{R_L}^2+{X_L}^2} + \frac{R_C}{{R_C}^2+{X_C}^2} + j\left(\frac{X_C}{{R_C}^2+{X_C}^2} - \frac{X_L}{{R_L}^2+{X_L}^2}\right)$$

At resonance, imaginary part is equal to zero.

$$\frac{X_C}{{R_C}^2+{X_C}^2} - \frac{X_L}{{R_L}^2+{X_L}^2} = 0$$

$$\frac{X_C}{{R_C}^2+{X_C}^2} = \frac{X_L}{{R_L}^2+{X_L}^2}$$

$$X_C\,({R_L}^2 + {X_L}^2) = X_L({R_C}^2 + {X_C}^2)$$

$$X_C{R_L}^2 + X_C{X_L}^2 = X_L{R_C}^2 + X_L{X_C}^2$$

$$\frac{{R_L}^2}{\omega_o C} + \frac{L}{C}\,\omega_o L = {R_C}^2\,\omega_o L + \frac{L}{C}\,X_C$$

Multiplying by $\omega_o C$, we get,

$$R_L^2 + \frac{L}{C}\,\omega_o^2\,LC = R_C^2\,\omega_o^2\,LC + \frac{L}{C}$$

$$\omega_o^2\,LC\,(R_C^2 - \frac{L}{C}) = R_L^2 - \frac{L}{C}$$

$$\omega_o^2 = \frac{1}{LC}\,\frac{{R_L}^2 - \frac{L}{C}}{{R_C}^2 - \frac{L}{C}}; \qquad \boldsymbol{\omega_o = \sqrt{\frac{1}{LC}}\sqrt{\frac{{R_L}^2 - L/C}{{R_C}^2 - L/C}}}\ \textbf{rad/sec}$$

$$\boldsymbol{f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{{R_L}^2 - L/C}{{R_C}^2 - L/C}}}\ \textbf{Hz}$$

NOTE:

Dynamic impedance at resonance (Z_D): *The impedance offered by the parallel circuit at resistance is denoted as dynamic resistance (Z_D). This is maximum at resistance.*

$$Z_D = \frac{L}{RC} = \textit{dynamic resistance}$$

Quality factor of parallel circuit: Parallel circuit is used to magnify the current and hence known as current resonance circuit.

Q factor for

- RL parallel circuit: $Q = \frac{R}{\omega_o L}$
- RC parallel circuit: $Q = \omega_o CR$
- RLC parallel circuit: $Q = R\sqrt{\frac{C}{L}}$

	Q factor	
	Series	Parallel
R-L	$\frac{\omega_o L}{R} = \frac{X_L}{R}$	$\frac{R}{\omega_o L} = \frac{R}{X_L}$
R-C	$\frac{1}{\omega_o CR}$	$\omega_o CR = \frac{R}{X_C}$
R-L-C	$Q_S = \frac{1}{R}\sqrt{\frac{L}{C}}$	$Q_P = R\sqrt{\frac{C}{L}}$

Resonant frequency:

Series RLC	$f_o = \frac{1}{2\pi\sqrt{LC}}$ Hz	$\omega_o = \frac{1}{\sqrt{LC}}$ rad/sec
Parallel RLC	$f_o = \frac{1}{2\pi\sqrt{LC}}$ Hz	$\omega_o = \frac{1}{\sqrt{LC}}$ rad/sec
R-L & C in parallel	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz	$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ rad/sec
R-L & R-C in parallel	$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$ Hz	$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$ rad/sec

NOTE:

Series RLC resonant circuit – acceptor circuit

Parallel RLC resonant circuit – rejector circuit

Concept of power factor improvement:

Significance of power factor: Apparent power drawn by the circuit has two components

(i) Active power or true power(P)

(ii) Reactive power(Q)

True power should be as large as possible because it does useful work in the circuit. This is possible if the reactive power component is small.

The greater the power factor(PF) of the circuit, greater will be the abilty to utilize apparent power. ∴ PF of of the circuit should be nearly equal to 1.

$$P = \sqrt{3}V_L I_L \cos\Phi \rightarrow I_L = \frac{P}{\sqrt{3}V_L \cos\Phi}$$

From the above eq^n for fixed power and voltage, the load current is inversly proportional to the PF, ie.,smaller the PF, higher is the load current and vice versa.

Power factor improvement: there are three main ways to improve power factor

1. **Capacitor bank**: Improving PF means reducing the phase difference between voltage and current. Since the majority of loads are of inductive nature, they require some amount of reactive power for them to function.

   A capacitor or bank of capacitors installed parallel to the load provides this reactive power. They act as a source of local reactive power, thus less reactive power flows through the line.

   Capacitor banks reduce the phase difference between the voltage and current.

2. **Synchronous condenser:** Synchronous condensers are 3-Φ synchronous motor with no load attached to its shaft.

   The synchronous motor has the characteristics of operating under any power factor ie., leading or lagging or UPF depending upon the excitation. For inductive loads, a synchronous condenser is connected towards load side and is overexcited.

   Synchronous condensers make it behave like a capacitor. It draws the lagging current from the supply or supplies the reactive power.

3. **Phase advancers:** This is an AC exciter used to improve the PF of an induction motor. They are mounted on the shaft of the motor and are connected to the rotor circuit of the motor. It improves te PF by providing excitating ampere turns to produce the required flux at the given slip frequency. Further, if ampere turns increases, it can be made to operate at leading power factor.

Measurement of power in three phase (3-Φ) circuits:

Following are the methods used for measurement of active power in 3-Φ circuits

1. Three wattmeter method – both balanced and unbalanced loads.
2. Two wattmeter method – both balanced and unbalanced loads.
3. One wattmeter method – balanced loads only.

Two wattmeter method:

Figure (a) shows the two wattmeter method for the measurement of power. This is the most commonly used method for the measurement of power as this can be used for both balanced and unbalanced loads and requires two watt meters.

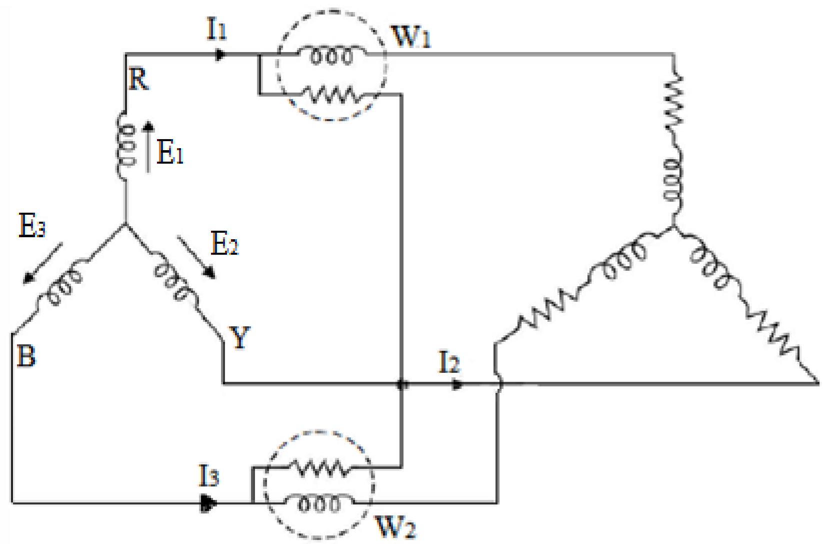


Fig (a) : Two wattmeter method for measuring 3-Φ power

The voltage across the pressure coil of W_1 is $(E_1 – E_2)$ and the current I_1 whereas voltage across the pressure coil of W_2, the voltage is $(E_3 – E_2)$ and the current I_3. Hence, instantaneous power recorded by the two watt meters.

$$= (E_1 – E_2)I_1 + (E_3 – E_2)I_3$$
$$= E_1I_1 + E_3I_3 – E_2 (I_1 + I_3)$$
$$= E_1I_1 + E_3I_3 + E_2I_2$$

As $I_1 + I_3 = -I_2$

Hence the two wattmeters connected in this fashion i.e. current coil in phase 1 and 3 and potential coils between phase 1 and 2 & 3 and 2, measures instantaneous 3-phase power.

However, if the load is balanced, the following analysis can be used to calculate the power factor of the balanced load.

Fig (b) shows the phasor diagram for the connections of Fig(a) for balanced loads. Phasors have been drawn taking rms values into considerations.

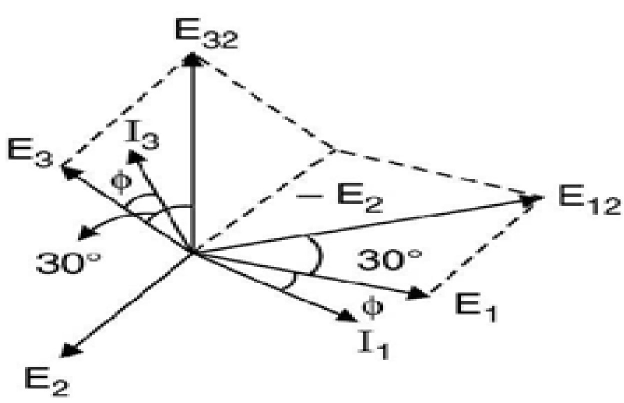


Fig (b) : Phasor diagram for fig (a)

Power $W = W_1 + W_2 = E_{12} I_1 \cos(30° + \Phi) + E_{23} I_3 \cos(30° - \Phi)$

Since it is a balanced circuit,

$$|E_{12}| = |E_{23}| = \sqrt{3} * |E_{ph}|$$

$$|I_1| = |I_3| = |I|$$

Therefore, $W = W_1 + W_2 = \sqrt{3}\, E_{ph}\, I \cos(30° + \Phi) + \sqrt{3}\, E_{ph}\, I \cos(30° - \Phi)$

$$= \sqrt{3}\, E_{ph}\, I\, [(\cos(30° + \Phi) + \cos(30° - \Phi)] = 2 * \sqrt{3}\, E_{ph}\, I * \cos 30° * \cos\Phi$$

$$\mathbf{W = W_1 + W_2 = 3\, E_{ph}\, I \cos\Phi}$$

Which is nothing but the three phase power in a balanced system.

It can be seen that if the p.f. of the load i s 0.5 ie., . $\Phi = 60°$ the reading of W_1 becomes zero and if the p.f. is less than 0.5, the p.f. angle is greater than 60° and with the usual connection of the wattmeter W_1 will read negative and hence the connection of either the current coil or the pressure coil should be reversed to obtain its reading and in such a situation this reading is to be reckoned negative and the total power consumed by the load is the algebraic sum of the two wattmeter readings.

Now, to find out $W_1 – W_2$

$$= \sqrt{3}\, E_{ph}\, I\, [(\cos(30° + \Phi) - \cos(30° - \Phi)]$$

$$= \sqrt{3}\, E_{ph}\, I\, [-2\sin\Phi \sin 30°]$$

(or) $$W_2 – W_1 = \sqrt{3}\, E_{ph}\, I\, [\, 2\sin\Phi \sin 30°] = \sqrt{3}\, E_{ph}\, I * 2 * \frac{1}{2} * \sin\Phi = \sqrt{3}\, E_{ph}\, I \sin\Phi$$

$$\mathbf{W_2 – W_1 = \sqrt{3}\, E_{ph}\, I \sin\Phi}$$

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{\sqrt{3}\, E_{ph}\, I \sin\Phi}{3\, E_{ph}\, I \cos\Phi} = \frac{\tan\Phi}{\sqrt{3}}$$

$$\mathbf{\tan\Phi = \sqrt{3} * \frac{W_2 - W_1}{W_2 + W_1}}$$

Hence, Power factor $\cos\Phi$ of the circuit can be calculated.

For reactive power , $Q = 3\, E_{ph}\, I \sin\Phi$

Advantages of 3Φ system:

In the three phase system, alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase differnce of 120° between each other. Such a three phase system has following advantages over single phase system:

1. The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2. For transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating, so transmission becomes very much economical.
3. It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.

4. In a single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t time. This fluctuating power causesconsiderable vibrations in single phase motor. Hence the performance of single phase motor is poor. While instantaneous power in symmetrical three phase system is constant.
5. Three phase systems give steady output.
6. Single phase supply can be obtained from three phase but three phase canont be obtained from single phase.
7. Power factor of a single phase motors is poor than three phase motors of same ratings.
8. For converting machines like rectifiers, the DC output voltage becomes smoother if number of phases are increased.

But it is found that optimum number of phases required to get all the above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard polyphase system throught the world.

NOTE:

- $W_1 = \sqrt{3}\, E_{ph}\, I\, (\cos(30° + \Phi) = E_L\, I_L\, (\cos(30° + \Phi)$

  $W_2 = \sqrt{3}\, E_{ph}\, I\, (\cos(30° - \Phi) = E_L\, I_L\, (\cos(30° - \Phi)$

  1. $W_1 = W_2$, $\cos\Phi = 1$
  2. W_1 - +ve, W_2 - +ve ($W_1 \neq W_2$) , $0.5 < \cos\Phi < 1$
  3. W_1 - -ve, W_2 - +ve ($W_1 \neq W_2$) , $0 < \cos\Phi < 0.5$
  4. $W_1 = 0$, $\cos\Phi = 0.5$
  5. $W_2 = -W_1$, $\cos\Phi = 0$

- *For star connection,* $E_L = \sqrt{3}\, E_{ph}$, $I_L = I_{ph}$
- *For delta connection,* $E_L = E_{ph}$, $I_L = \sqrt{3}\, I_{ph}$

Problems on RL, RC and RLC circuits:

1. A 50Hz, alternating voltage of 150V (rms) is applied independently to (i) Resistance of 10Ω (ii) Inducatance of 0.2H (iii) Capacitance of 50μF.Find the expression for instantaneous current in each case. Draw the phasor diagram in each case.

Solution: (i) Given : R = 10 Ω

$V = V_m \sin\omega t$

$V_m = \sqrt{2}\, V_{rms} = \sqrt{2} * 150 = 212.13V$

$I_m = \frac{V_m}{R} = \frac{212.13}{10} = 21.213A$

In a pure resistive circuit, current is in phase with voltage,ie., $\Phi = 0°$

$i = I_m \sin\omega t = I_m \sin(2\pi f t)$

$\mathbf{i = 21.213\sin(100\pi t)\ A}$

(ii) **Given ; L = 0.2H**

Inductive reactance $X_L = 2\pi fL = 2\pi*50*0.2 = 62.83\ \Omega$

$I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.37A$

In a pure inductive circuit, current lags voltage by an angle 90° ie., $\Phi = 90° = \frac{\pi}{2}$

$i = I_m \sin(\omega t - \Phi)$

$\mathbf{i = 3.37\sin(2\pi f t - \frac{\pi}{2})\ A}$

(iii) **Given C = 50μF**

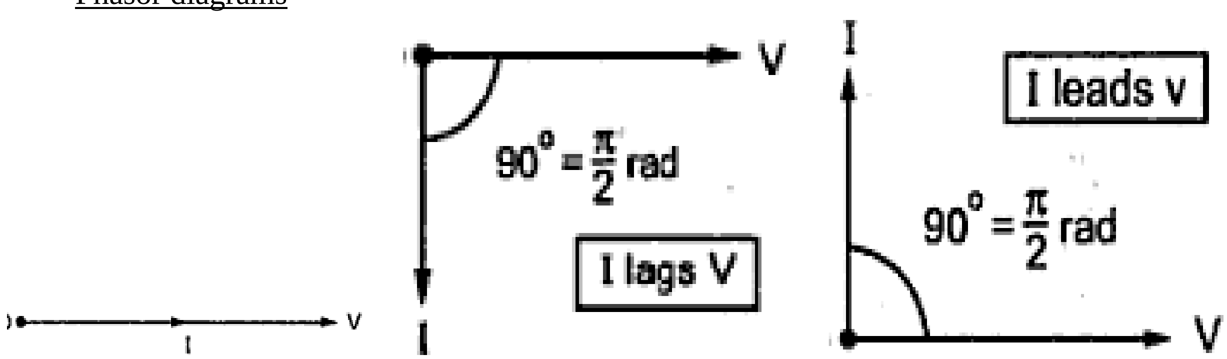
Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi*50*50*10^{-6}} = 63.66\ \Omega$

$I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33A$

In a pure capacitive circuit, current leads voltage by an angle 90° ie., $\Phi = 90° = \frac{\pi}{2}$

$i = I_m \sin(\omega t + \Phi)$ $\mathbf{i = 3.33\sin(2\pi f t + \frac{\pi}{2})\ A}$

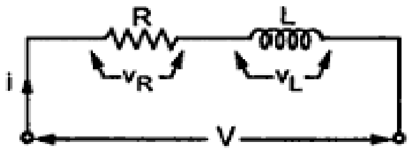
<u>Phasor diagrams</u>



(i) Pure resistive circuit (ii) Pure inductive circuit (iii) Pure capacitive circuit

2. An alternating current, $i = 1.414\sin(2\pi*50*t)$A is passed through a series circuit consisting of a resistance of 100Ω and inductance of 0.31831H. Find the expression for instantaneous values of volatge across (i) resistance (ii)inducatabce (iii) combination

Sol^n :



Given : $i = 1.414\sin(2\pi*50*t)$A , f = 50Hz , $\omega = 2\pi f = 2*\pi*50$, R = 100Ω , L = 0.31831H ,
$X_L = 2\pi fL = 2*\pi*50*0.31831 = 100\Omega$

(i) The voltage across resistance is,

$V_R = iR = 1.414\sin(2\pi*50*t)*100$

$\mathbf{V_R = 141.4\sin(2\pi*50*t)\ V}$

(ii) The voltage across inductor leads current by an angle 90°,

$V_L = iX_L = 1.414\sin(2\pi*50t + \Phi) * 100$

$\mathbf{V_L = 141.4\sin(2\pi*50t + 90°)V}$

(iii) From the expression of V_R, we can write,

$V_{R(rms)} = \frac{V_R}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100V$, $\Phi = 0°$

$V_{R(rms)} = 100\angle 0° = 100 + j0$ V

$V_{L(rms)} = \frac{V_L}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100V$, $\Phi = 90°$

$V_{L(rms)} = 100\angle 90° = 0 + j100$ V

$V = 100 + j0 + 0 + j100$

$= 100 + j100 = 141.42\angle 45°$

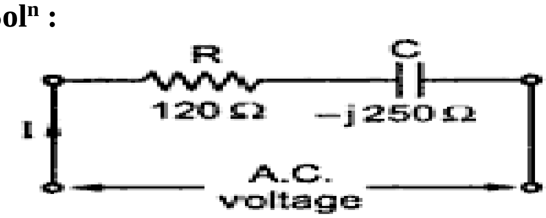
$V_m = \sqrt{2} * 141.42 = 200V$

Hence the expression for instantaneous value of resultant voltage is,

$\mathbf{V = 200\sin(2\pi*50t + 45°)V}$

3. A resistance of 120Ω and a capacitive reactance of 250 Ω are connected in series across a AC voltage source. If the current of 0.9A is flowing in a circuit, then find (i) Power factor (ii) Supply voltage (iii) Voltage across resistance and capacitance (iv) Active and reactive power

Sol^n :



Given : R = 120 Ω , X_C = 250 Ω , I = $0.9\angle 0°$A

$Z = R - jX_C = 120 - j250\ \Omega = 277.308\angle -64.358°$

(i) Power factor = $\cos\Phi = \cos(-64.358°)$; **PF = 0.4327 leading**

(ii) Supply voltage, $V = I*Z = (0.9\angle 0°)*(277.308\angle -64.358°)$

$$V = 249.5772\angle -64.358° \text{ V}$$

(iii) $V_R = IR = 0.9*120 = 108V$

$V_C = IX_C = 0.9*250 = 225V$

(iv) $P = VI\cos\Phi = 249.5772*0.9*0.4327$

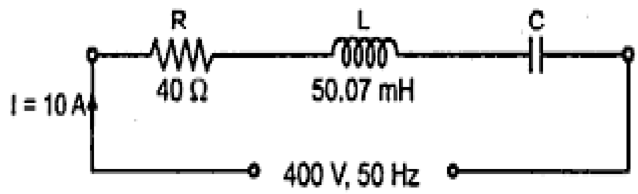
$P = 97.1928W$

$Q = VI\sin\Phi = 249.5772*0.9*\sin(-64.358°)$

$Q = -202.49VAR$

4. A series circuit having pure resistance of 40Ω, pure inductance of 50.07mH and a capacitor is connected across a 400V, 50Hz, AC supply. This R, L , C combination draws acurrent of 10A. Calculate (i) Power factor of the circuit (ii) Capacitor value

Sol^n:



Given: R = 40 Ω , L = 50.07mH , I = 10A

$X_L = 2\pi fL = 2\pi * 50*50.07*10^{-3} = 15.73\ \Omega$

$Z = R + j(X_L - X_C)$

$|Z| = \sqrt{(R)^2 + (X_L - X_C)^2} = \sqrt{(40)^2 + (15.73 - X_C)^2}$

$|Z| = \frac{|V|}{|I|} = \frac{400}{10} = 40\ \Omega$

$40 = \sqrt{(40)^2 + (15.73 - X_C)^2} = 1600 = 40^2 + (15.73 - X_C)^2$

$(15.73 - X_C)^2 = 0$; $X_C = 15.73\ \Omega$

$X_C = \frac{1}{2\pi fC}$; $C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi*50*15.73} = 2.02358*10^{-4}$

$C = 202.358\mu F$

$Z = 40 + j(15.73 - 15.73) = 40 + j0\ \Omega = 40\angle 0°$

$PF = \cos\Phi = \cos(0°)$; **PF = 1 (unity power factor)**

5. A voltage $v = 200\sin100\pi t$ is applied to a load having R = 200Ω in series with L = 638mH. Estimate
   (i) Expression for current in $i = I_m\sin(\omega t \pm \Phi)$ form **(Ans: $i = 0.7071\sin(100\pi t - 45.06°)$)**
   (ii) Power consumed by the load **(Ans: P = 50W)**
   (iii) Reactive power of the load **(Ans: Q = 50VAR)**
   (iv) Voltage across R and L **(Ans: $V_R = 100V$; $V_L = 100.21V$)**

6. The impedance of a circuit placed across a 120V, 50 Hz source is (10 + j20). Find current and the power.
(Ans: i = 2.4 – j4.8A , P = 288W)

7. Calculate the resistance, inductance or capacitance in series for each of the following impedances. Assume the frequency to be 60Hz. (i) (12 + j30)Ω (ii) (-j60) Ω (iii) $20\angle 60°$
(Ans: (i) R = 10 Ω , L = 79.58mH; (ii) R = 0 Ω, C = 44.209µF; (iii) R = 10 Ω, L = 45.94mH)

8. The waveform of volatge and current of a circuit are given by e = 120sin(314t)V and i = 10sin(314t + $\frac{\pi}{6}$). Calculate the values of resistance, capacitance which are connected in series to form the circuit and also the power consumed by the circuit. **(Ans: R = 10.393Ω, C = 530.45μF, P = 519.52W)**

9. A 15V source is applied to a capacitive circuit that has an impedance of (10 - j25)Ω . Determine the value of current and active and reactive power in the circuit. **(Ans: i = 0.55∟68.2°, P = 3W , Q = -7.7VAR)**

10. Two impedances Z_1 and Z_2 are connected in parallel across a applied voltage of (100+j200)volts. The total power supplied to the circuit is 5KW. The first branch takes aleading current of 16A and has a resistance of 5Ω while the second branch takes a lagging current at 0.8 power factor. Calculate (i) current in second branch (ii) Total current (iii) Circuit constants
(Ans: I_2 = 20.79∟26.55° , I = 22.48∟69.70° , R_1 = 5 Ω, R_2 = 8.60 Ω, X_1 = 13.04 Ω, X_2 = 6.4531 Ω)

11. A parallel circuit having resistance of 25 Ω, inducatance of 64mH and a capacitance of 80μF are connected across 110V, 50Hz single phase AC supply. Calculate the current in individual elements, the total current from the supply and the overall power factor of the circuit. Draw the phasor diagram showing $\overline{V}$, $\overline{I_R}$, $\overline{I_L}$, $\overline{I_C}$ and $\overline{I}$.
(Ans: I_R = 4.4∟0°, I_L = 5.47∟-90°, I_C = 2.76∟90°, I = 5.1676∟-31.62°, PF = 0.851 lagging)

12. The impedance of a parallel RC circuit is (10 - j30)Ω at 1 MHz. Determine the values of the components.
(Ans: R = 100 Ω, C = $4.7*10^{-9}$F)

13. Determine the line current and the total impedance and admittance of the circuit having R and L in series and that in parallel with C where R = 25 Ω , X_L = 50 Ω , X_C = 40 Ω across volatge source of 100V.
(Ans: Z = 83.12∟-48.4° , Y = $1.2*10^{-2}$∟48.4°, I = 1.2∟48.4° A)

14. A 4700 Ω resistor and a 2μF capacitor are connected in parallel across a 240 V 60 Hz source. Determine the circuit impedance and the line current. **(Ans: Z = 1276∟-74.3° Ω, I = 0.188∟74.3°**)

15. A room heater of 2KW, 125V rating is to be operated at 230V , 50Hz AC supply. Calculate the value of inductance, that must be connected in series with the heater, so that the heater will not get damaged due too over heating. **(Ans: L = 0.0384H)**

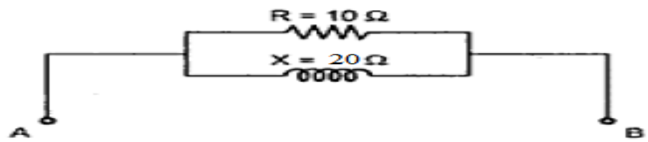
16. A choke coil and a pure resistance are connected in series across 230V, 50Hz AC supply. If the voltage drop across the coil is 190V and across resistance is 80V while the current drawn by the circuit is 5A. Calculate (i) internal resistance of coil (ii) inductance of coil (iii) Resistance R (iv) Power factor of circuit (v) Power consumed by the circuit **(Ans: r = 13Ω, L = 0.1136H, R = 16Ω, PF = 0.6304lag, P = 724.96W)**

17. Two coils A and B are connected in series across 200V, 50Hz AC supply. The power input to the circuit is 2.2KW and 1.5KVAR. If the resistance of coil A is 4Ω and the reactance id 8Ω. Calculate the resiatance and the reactance of the coil B. Also calculate the active power consumed by coil A and coil B, total impedance of the circuit. **(Ans: R_B = 8.4 Ω, X_B = 0.5Ω, P_A = 707.56W, P_B = 1.4858KW, Z_T = 15.037Ω)**

18. A sereis R-C is connected across 200V, 50Hz AC supply draws a current of 20A. When the frequency of the supply is increased to 100Hz, the current increases to 23.40821a. Calculate the value of resistance and capaciatnce of the circuit. **(Ans: R = 8Ω, L = 0.5305mH)**

19. Two Impedance Z_1 and Z_2 havind same numerical value are connected in series. If Z_1 is having power factor of 0.866 lagging and Z_2 is having power factor of 0.6 leading. Calculate the PF of the series combination. **(PF = cosΦ = 0.9796 leading)**

20. A circuit is shown in the figure Draw its equivalent admittance circuit. Also calculate admittance, conductance and susceptance of the circuit.



(Ans: G = 0.1mho, B = 0.05mho, Y = 0.1118mho)

21. A heater operates at 100V, 50Hz AC supply and takes a current of 8A and consumes 1.2KW power. A choke coil is having ratio of reactance to resistance as 10, is connected in series with the heater. The series combination is connected across 230V, 50Hz AC supply. Calculate (i) Resistance of choke coil (ii) Reactanc eof choke coil (iii) Power consumed by the choke coil (iv) Total power consumed.

(**Ans: r = 2.576Ω, X_L = 25.576 Ω, P_{coil} = 164.87W, P_{total} = 964.864W)**

22. Two impedances (8 + j6)Ω and (3 – j4)Ω are connected in parallel. If the total current drawn by the combination is 25A, find the current and power taken by each impedance.

(Ans: I_1 = 11.1803∟-63.4349° A, I_2 = 22.3607∟26.5642° A, P_1 = 1000W, P_2 = 1500W)

23. Two impedances Z_1 = (12 + j15)Ω and Z_2 = (8 – j4)Ω are connected in parallel across a potential difference of (230 + j0)V. Calculate (i) total current drawn (ii) total power and branch power consumed (iii) overall power factor of the circuit.

(Ans: I_T= 30.55∟4.03°A, P_T = 7.0091KW, P_1 = 7.719KW, P_2 = 5.289KW, cosΦ = 0.9975leading)

24. A parallel circuit consists of two branches. Branch (i) consists of R of 100Ω connected in series with inducatnce of 1H and branch (ii) consists of R of 50Ω in series with capacitance of 79.5µF. This parallel circuit is connected across single phase 200V, 50Hz AC supply. Calculate (i) Branch currents (ii) total current of the circuit (iii) total power factor of the circuit (iv) total power drawn by the circuit.**(Ans: I_1= 0.6066∟-72.343°A,I_2 = 3.1223∟38.687°A, I_T = 2.9592∟27.6563°A, PF = 08857leading ,P = 524W)**

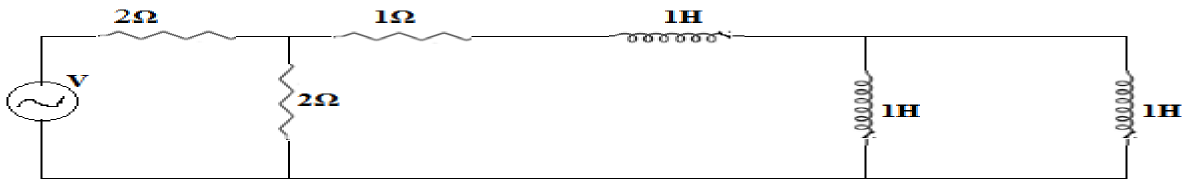
25. Two impedances Z_1 = 40∟30°Ω and Z_2 = 30∟60°Ω are connected in series across a 230V, 50Hz AC supply. Calculate (i) current (ii) Power factor (iii) Power consumed by the circuit

(Ans: I = 3.399∟-42.807°A, cosϕ = .7336 lagging, P = 573.543watts)

Problems on time constant:

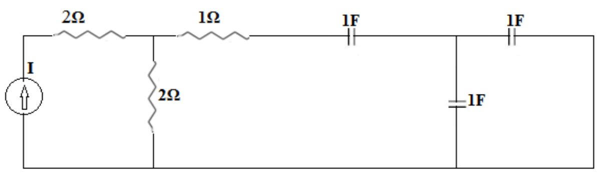
1) Obtain the value of time contant for the given circuit.

   a.

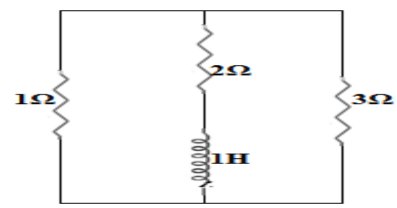


(Ans: $\tau = \frac{3}{4}$ sec)

   b.



c.



(Ans: τ = 2sec) **(Ans: $\tau = \frac{4}{11}$sec)**

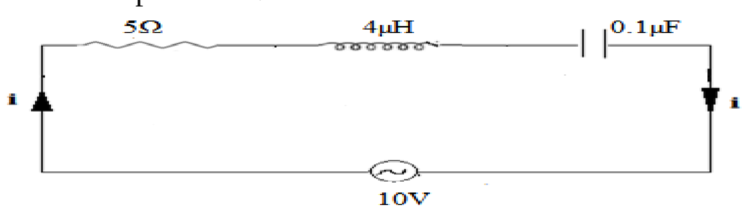
Problems on Resonance:

1) A series RLC circuit is connected to 230V AC supply. The current drawn by the circuit ar resonance is 25A. The voltage drop across the capacitor is 4000V. At series resonace, calculate the value of resistance, inductance if the value of capacitance is 5μF, also calculate the resonant frequency.

(Ans: R = 9.2Ω, L = 0.128H, f_o = 198.943Hz)

2) An inductive coil of resistance 10Ω and an inductance of 0.1H is connected in parallel with 150μF capacitor to a variable frequency, 200V supply. Find the resonant frequency at which the total current taken from the supply is in phase with the supply voltage. Also find the value of current in the circuit and draw the phasor diagram for the same. **(f_o = 37.8865Hz, I = 3A)**

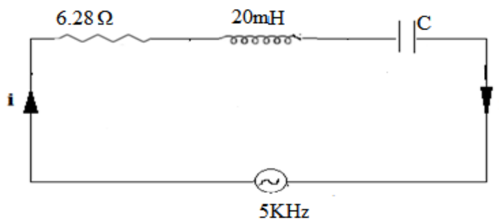
3) For the Fig(3) shown determine the maximum current, the frequency at which it occurs and the resulting voltage across the inductance and capacitance.



Fig(3)

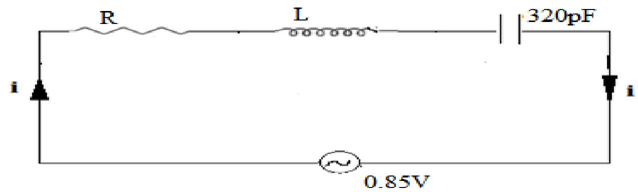
(Ans:I = 2A, f_o= 251.64KHz, $V_L = V_C$ = 12.63V)

4) In a series RLC circuit shown in fig(4), determine (i) The necessary value of capacitor, (ii) The supply voltage to produce a voltage of 5V across the capacitor if f_o = 5KHz (iii) If the capacitance is made $\frac{1}{2}$ at (i), determine f_o , the Q value of new circuit.



(**Ans: C = 0.0507 μF, V = 0.05V, f_o = 7071Hz, Q = 141.49**)

5) For the circuit shown, determine the value of inductance for resonance if Q = 50 and f_o = 175KHz. Also find the circuit current, the voltage across the capacitor and the bandwidth of the circuit.

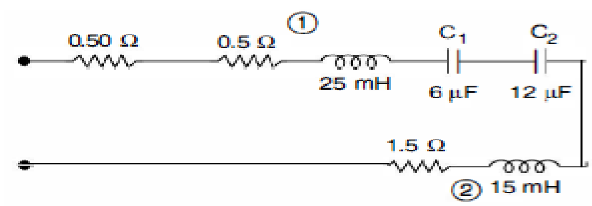


(Ans: L=2.58mH, I = 14.96mA, V_C = 42.5V, BW = 3.5KHz)

6) For a series RLC circuit having R = 10Ω, L = 100mH, C = 0.01μF, determine the impedance magnitude at resonance, 1KHz below resonance and at 1KHz above resosnace.

(Ans: Z_1 = 1.42KΩ, Z_2 = 1.15KΩ)

7) For the circuit shown in Fig R_1 = 0.5 Ω, R_2 = 0.5Ω, R_3 = 1.5 Ω, C_1 = 6 μF and C_2 = 12 μF, L_1 = 25 mH and L_2 = 15mH. Determine (i) the frequency of resonance (ii) Q of the circuit (iii) Q of coil 1 and coil 2 individually.



(Ans: ω_o = 2.5*10^3 rad/sec, Q = 40, Q_1 = 62.5, Q_2 = 25)

8) A coil having a 5Ω resistor is connected in series with a 50 μF capacitor. The circuit resonates at 100 Hz (a) Determine the inductance of the coil. (b) If the circuit is connected across a 200 V 100 Hz a.c. source, determine the power delivered to the coil (c) the voltage across the capacitor and the coil (d) the bandwidth of the circuit.

(Ans: L = 50.66mH, P = 8KW, V_C = 1273.24V,V_L = 1256V, Q = 6.3, BW = 16Hz)

9) A coil having a resistance of 50Ω and inductances 10mH is connected in series with a capacitor and is supplied at constant voltage and variable frequency source. The maximum current is 1A at 750Hz. Determine the bandwidth and half power frequencies. **(Ans: BW = 795.8Hz, f_2 = 1246.9Hz, f_1 = 451Hz)**

10) Two branches of a parallel circuit have element R_L = 6Ω, L = 1mH and R_C = 4 Ω and C = 20 μF. Determine the frequency at resonance. **(Ans: ω_o = 4537 rad/sec)**

Problems on two wattmter method:

1. A 500 volt 3-phase motor has an output of 37.3 KW and operates at a p.f. of 0.85 with an efficiency of 90%. Calculate the reading on each of the two wattmeters connected to measure the input.

(Ans: W_1 = 13.31KW , W_2 = 28.13KW)

2. Two wattmetrs reading 4KW and 2KW are connected for measuring the active power of $3-\Phi$ circuit. Calculate the power factor of the circuit. **(Ans: cosΦ = 0.866lag)**

3. E_L = 400V , I_L = 10A, cosΦ = 0.5lag. Calculate the two wattmter readings. **(Ans: W_1 =0 , W_2 = 3.464KW)**

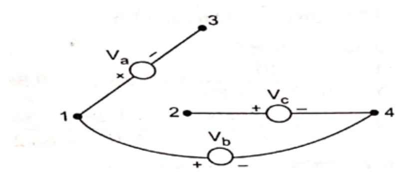
4. A balanced star connected lod of (8+j6)Ω is connected to a 3 phase 230V supply. Find the line current, power factor, reactive voltamperes and total voltamperes.

(Ans: $I_L = 13.279\angle{-36.86°}$, P = 4.231KW, Q = 3.173KVAR, S = 5289.95)

5. Three 100Ω resistors are connected (i) star (ii) delta across a 415V 50Hz 3-phase supply.Calculate the line current and power consumed in each case.

(Ans: (i) <u>star</u>: I_L = 2.396A, P = 1.72KW; (ii) <u>Delta</u>: I_L = 4.15A, P = 2.98KW)

6. The current of average value 18.019A is flowing in a circuit in which a voltage of peak value of 141.42V is applied, V lags I by $\frac{\pi}{6}$ radians. Determine (i) Z = R + jX (ii) Power (**Ans: $Z = 7.85\angle 30°$ Ω, P = 2.206KW)**
7. Three volatges are connected as shown in the figure below, If V_a = 17.32 + j10 V, $V_b = 30\angle 80°$ V, $V_c = 15\angle{-100°}$ V. Find (i) V_{12} , (ii) V_{23} , (iii) V_{34}



(Ans: V_{12} = 7.804+j44.312V, V_{23} = 9.52-j34.31V, V_{34} = -12.12+j19.54V)

8. How is current of 10A shared by three impedance Z_1 = (2-j5)Ω, $Z_2 = 6.708\angle 26.56°$ Ω , Z_3 = (3+j4)Ω all connected in parallel? **(Ans: $I_1 = 5.8\angle 77.76°$A, $I_2 = 4.65\angle{-17°}$ A, $I_3 = 6.25\angle{-43.57°}$ A)**

9. A 3-phase star connected supply with phase voltage of 230V is supplying a balanced delta (Δ) load. The load draws a power of 15KW at 0.8 PF lagging. Find the line current and the current in each phase of the load. What is the load impedance per phase? **(Ans: I_L = 27.17A, I_{ph} = 15.68A, $Z_{ph} = 14.66\angle 0.8°$ Ω)**

10. Power is measured in a 3 phase balanced load using two wattmetrs. The line voltage is 400V. The load and its power factor is so adjusted that the line current is always 10A. Find the readings of the wattmeter when the PF is (i) unity (Ans: **W_1 = 3.46KW, W_2 = 3.46KW**
    (ii) 0.866 **W_1 = 2 KW, W_2 = 4KW**
    (iii) 0.5 **W_1 = 0 watts, W_2 = 3.464KW**
    (iv) Zero **W_1 = -2000 watts, W_2 = 2000 watts**)