X. If y=(sin'x) know that, (16) $(1-x^2)y_{n+2}-x(2n+1)y_{n+1}-n^2y_n=0$ 7. Consider. (y= (sintx)2) => y = a sin-1x - 1 VI-x2 $\sqrt{1-x^2} = 2(\sin^2 x)^2$ $(1-x^2) \cdot y_1^2 = 4(\sin^2 x)^2$ [y=(sin-x)2) $(j-x^2)\cdot y_1^2 = 4y$. Differentiating both eides wirtx, we get. $(1-x^2)\cdot 2y_1y_2 - 2xy_1^2 = 4y_1$ ÷ by 24! $(1-x^2)y_2 - xy_1 - 2 = 0$ Differentiating each term n times, By Leibnitz's theorem, we get 76 (1-x²) yn+2 + n(1 yn+1 (-2x) +ng yn(-2) -xyn+1 (yn(1)=0 $\Rightarrow (1-x^2)^{\frac{1}{2}}n+2-2nx^{\frac{1}{2}}n+1-\frac{n(n-1)}{2}(2),y_n-xy_{n+1}-ny_{n-2},$ $= \frac{(2n+1)}{2^{2}} \frac{1}{2^{2}} \frac{1}{2^{2}} = \frac{(2n+1)}{2^{2}} \frac{2^{2}}{2^{2}} \frac{1}{2^{2}} = \frac{(2n+1)}{2^{2}} \frac{2^{2}}{2^{2}} = \frac{(2n+1)}{2^{2}} = \frac{(2n+1)}{2^{2}} \frac{2^{2}}{2^{2}} = \frac{(2n+1)}{2^{2}} = \frac{(2n+1)}{2$ $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ * If $y = [log(x+\sqrt{1+x^2})]^2$ show that $(1+\chi^2)$ Yn+2+ $(2n+1)\chi$ Yn+1 $-n^2$ Yn=0.

-> ble have $y = \left[\log \left(\chi + \sqrt{1 + \chi^2}\right)\right]^2$ $y_1 = 2. \log (x + \sqrt{1 + x^2}). \frac{1}{(x + \sqrt{1 + x^2})} \frac{\partial x}{(x + \sqrt{1 + x^2})}$ $y_1 = 2 \log (x + \sqrt{1 + x^2}) \cdot \left[\frac{1}{x + \sqrt{1 + x^2}} \right] \left[\frac{x + \sqrt{1 + x^2}}{\sqrt{1 + x^2}} \right]$ y = 2 log (x+ (1+x2). VI+x2 S.B.S $(1+x^2)y_1^2 - 4 [log(x+\sqrt{1+x^2})]^2$ Since $[y=[log(x+\sqrt{1+x^2})]^2$ $(1+x^2)y_1^2 = 4$ Since $[y=[log(x+\sqrt{1+x^2})]^2$ Diff Both Rides $x^2 + x^2$, we get VI+x2. 9, = 2. log(x+VI+x2). C1+x2).24142 +2x412=441 dividing throughout by dy, weget diff each team n times, by using Leibnitz's th we have, $+ n \cdot 2xynt1 + \frac{n(n-1)}{2} \cdot 2 \cdot ynt xynt1 + \frac{n \cdot 1 \cdot y}{n-1} = 0$ $(1+x^2) ynt2 + \frac{n \cdot 2xynt1}{n \cdot 1} + \frac{1}{n^2} \cdot \frac{1}{n \cdot 1} \cdot \frac{1}{n \cdot 1}$ $\Rightarrow (1+\chi^2)^{\frac{1}{2}} + (2n+1)^{\frac{1}{2}} + (2n+1)^{$ (1+x2) yn+2 + (2n+1) xyn+1 +n2yn=0

* If cas | = log (=) " chow that 22/n+2 + (2n+1) xyn+1 +2nyn=0 Sol" (oneider, cos-'[4] = log[x]" 古 = 608 [log | 光]] y= b. 608 [n log/2] -0 => y=-bein[nlog(x)] ono(x) oh. xy = -n·bsin[nlog(光)] diff both eides wit x, we get 242+41 = -nb.cos[n.log(2)].n.(2).h x2y2+xy1=-n2y differentiating each terms n'times, by Leibnitz's this we have, $2^{2}y_{n+2} + n(-2xy_{n+1} + n(-2-2y_{n+1} + n(-1) + n+n^{2})y_{n=0}$ $\Rightarrow \chi^{2}y_{n+2} + (2n+1)xy_{n+1} + [n(n-1) + n+n^{2}]y_{n=0}$ 2 ynt2 + (2n+1) xyn+1 + 2n2 yn=0

```
* I y'm+ y'm- or,
    (x2-1) yn+2 + (2n+1) xyn+1 + (n2-m2)yn=0
Sol": Let y'm z >> y'lm 1.
     => y'lm +y'lm= 2x
          z = \chi \pm \sqrt{\chi^2 - 1}
   taking the positive sign we have.
            y'lm = x+ \x2-1 => y= (x+\x2-1)m
         duit B.S wirt 'x', we go
              y_1 = m(x + \sqrt{x^2 - 1})^{m-1} \left(1 + \frac{\partial x}{\partial \sqrt{x^2 - 1}}\right)
               y_1 = m \left( \chi + \sqrt{\chi^2 - 1} \right) m \left[ \frac{\chi + \sqrt{\chi^2 - 1}}{\sqrt{\chi^2 - 1}} \right]
      Vx2-1.21= m(x+ Vx2-1)m L
        \sqrt{\chi^2 - 1 \cdot y_1} = my
[Since y = (\chi + \sqrt{\chi^2 - 1})^m)
  S.B.S (\chi^2-1)\chi^2 = m^2y^2.
               (x^{2}), 2y_{1}y_{2} + 2xy_{1}^{2} - m^{2}2yy_{1} = 0
              (x^{2}-1)y^{2} + xy_{1} - m^{2}y = 0
  Diff each team n times by wring Leibnitz's thm, 2yn-myn=0 (22-1)yn+2+ng 2xyn+1+ng 2yn+xyn+1.
     6,924, we got.
  (\chi^{2}-1) y_{n+2} + 2n\chi y_{n+1} + n(n-1)y_{2} + ny_{n+1} + ny_{n} - m^{2}y_{n}
    (\chi^{2}-1)y_{n+2} + (2n+1)\chi y_{n+1} + (n^{2}-n+n-m^{2})y_{n} = 0
       or (\chi^2 - 1)y_{n+2} + (\partial n + 1) \chi y_{n+1} + (n^2 - m^2), y_n = 0
```

```
\pm y = a \cdot (os(log x) + bsin(log x))

      \chi^{2}y_{n+2} + (2n+1)\chi y_{n+1} + (n^{2}+1)y_{n} = 0
           y = a cos(logx) + bsin(logx).
         y= - a sin (logx): + b. cos(logx): 1/x
          xy = -a sin ( log x) + b. cos (logi)
  diff xyz+y1 = - a. cos(logx): 1/2 - b sin(logx): 1/2
         \chi^2 y_2 + \chi y_1 = -\left[ a \cos(\log x) + b \sin(\log x) \right]
       each term n times by Leibnitz thm, weget
   22/n+2+n(1)x9n+1+n(229n+x4n+1+nGyn+4n=0
   \chi^2 y_{n+2} + 2n \chi y_{n+1} + n (n-1) y_n + \chi y_{n+1} + n y_n + y_n = 0
       \chi^{2}y_{n+2} + (2n+1)\chi y_{n+1} + (n^{2}-n+n+1)y_{n} = 0
     or \chi^2 y_{n+2} + (2n+1)\chi y_{n+1} + (n^2+1)y_{n=0}
* If y= emsinty show that,
     (1-\chi^2)y_{n+2} - (2n+1)\chi y_{n+1} - (n^2+m^2)y_n = 0
Sol" consider y= emsinty
Till mainty
Sol" coneaer y = emsinty m

Tily wirt v. y = emsinty Till Esince y = emsinty
           J1-x2.y= my.
 S. B.S (1-x2) y 2- m2y2
       (1-x2) 24142 - 2x412 - 2m21
         - by 291
```

```
(1-x^2)y_2 - xy_1 - m^2y = 0
           diff each term on times by using Leibnitz's thing
   (1-x2)yn+2+n(,(-2x)yn+,+n(2(-2)yn-xyn+1-n(,yn-m2)=
         => (1-x^2)y_{n+2}-2nxy_{n+1}-n(n-1)y_n-xy_{n+1}-ny_n-m^2y_{n}
         => (1-x2)yn+2-(2n+1)xyn+1-(n2-n+n+m2)yn=0
           \Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0
* If x = Sint and y = Cospt whom that
      (1-\chi^2) ynt 2 - (2n+1)\chi ynt 1 - (n^2-p^2) yn=0
                2=Sint > t=Sin-x.
                       y = Cospt => y = cos[p.sin x] - 0
      diff Both sides wort x we have
                      y_1 = -\sin\left(p \cdot \sin^{-1}x\right) \cdot \frac{p}{\sqrt{1-r^2}}
                          VI-x2, y, = -psin [psintx]
                S.B.S. (1-x2)y12=p2Sin2[psintx]
                 (1-x^2)y_1^2 = p^2[1-\cos^2(p\sin^2x)]
                                    (1-\chi^2)y_1^2 = \rho^2 - \rho^2 y_1^2 (from O)
               diff Bis wirt x, we get
                                (1-x^2) \cdot 2y_1y_2 - 2xy_1^2 = -2p^2y_1^2
             - by 241 (1-x2) 42 - x41 + p2y=0
       did reach term in times, by using heibnitz's thing
                   (1-x2) Yn+2+n((-2x)yn+1+n(2(-2)yn-xym+1-h(yn+p3)+c
                   \Rightarrow (1-x^2)^{\frac{1}{2}} + (2n+1)x^{\frac{1}{2}} + (n^2-n+n-p^2)^{\frac{1}{2}} + (2n+1)x^{\frac{1}{2}} + (2n+1)x^{\frac{1}{2}
                                                  (1-x2) yn+2 - (2n+1) x yn+1 - (n2-p2) yn=0
```

If $y = (\sin^2 x)^2$ Show that $(1-x^2)d^2y - x dy - 2 - 0$ differentiale the above equation n times with x, Alio VI-x2 4, = 25in 7. (1-x2)y12 = 4 [Sin-x]2. $(1-x^2)y_1^2 = 4y$. diff. B.S wirt h' $(1-\chi^2)2y_1y_2-2\chi y_1^2=4y_1$ - py 24 [) $(1-x^2)y_2 - xy_1 - 2=0$ diff each team n times by using Leibnitz's the weget (1-x2) yn+2 + ng (-2x) yn+1 +n(2(-2) yn- 24yntl - 1/4 yn=0 \Rightarrow $(1-x^2)$ $y_{n+2}-2n x y_{n+1}$ n(n-1) yn - xyn+1 - nym2 (1-x2) yn+2 - (2n+1) x yn+1 putting x=oin 0 &0 we get 4(0)=0,111 (y210)=0.

Again putting x=0in(3) yn+10) - n19n(0)=0 => Yn+2(0) = n2yn(0) put n=1,3,5,... wegt 95(0)=12 410)=0 [:410)=0 95(0)=32 43(0)=0 [:410]=0 y=(0) = +2 y=(0) =0 [: 4=10] · yn(0) = 0 when n'is odd putting n= 2, 4, 6, ... weg 9410) = 22 42(0) = 2.22 [::4,10]=2) $y_6(0) = 4^{\frac{1}{2}} y_4(0) = 2 \cdot 2^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} \left(\frac{1}{2} y_4 \log^2 2 \cdot 2^{\frac{1}{2}} \right)$ (y8(0) = 6246(0) = 2-22, 42-62 $y_n(0) = (n-2)^2 y_{n-2}(0)$ $y_n(0) = 2 - 2^2 \cdot 4^2 \cdot 6^2 \cdot \cdot \cdot (n-2)^2$ Thus when n'is even $y_{n(0)} = 2.2.4^{2}.6^{2}...(n-2)^{2}$ million Dans a rol (b. 13

If It= tan (logy), show that, $(1+x^2)y_{n+2} + [a(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$ \Rightarrow . $\chi = \tan(\log y)$. $tan^{-1}x = log y$ $y = e^{tan^{-1}x}$ $diff y = e^{tan^{-1}x}$ $(1+x^{2})y_{1} = e^{tan^{-1}x}$ A by Eq. (1+x2) y1 = y. diff (1+x2) y2 + 2xy1 = y1 Diff each team to times by using Leibnitz's then weg (1+x²) yn+2 + ng 2xyn+1+ng 2,yn+ (2x-1)yn+1+ng 2,yn = 0 => (1+x²)yn+2+2nxyn+1+n(n-1)yn+(2x-1)ym+2nyn=D -> (1+x2)yn+2 + [2 (n+1) x - 1] yn+1 + (n2-n+2n)yn=D -> (1+x2)yn+2+ [2(n+1)x-1)yn+1+n(n+1)yn=0

Find the nth derivative of x2 log x x 3. cos 1. 3. x2. ex. coxx. 4. x3. Sinat If y = Sin (m. sin x). Rhow that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-m^2)y_n = 0$ $J = e^{m \cos^{-1} y} \text{ Show Hot.}$ $(n^2 + m^2) y_{n-1} = 0$ $(1 - x^2) y_{n+2} = (2n+1) x \cdot y_{n+1} = (n^2 + m^2) y_{n-1} = 0$ $(1-2^2)$ ynt 2 Comment of the state of the sta Louis The Hall Hall Annie Landin Andrew And Thomas I