# Density of states.

DOS is defined as the no of electronic states per unit energy range. $\left(\frac{dn}{dE}\right)$

The free $\bar{e}$ theory describes that the outer $\bar{e}$ of all the atoms in metal move freely about the entire metal solid, but very rarely do they actually leave.

Since $\bar{e}$ are trapped within the metal, we can imagine that the metal is a finite potential well i.e., $\bar{e}$ have a P.E of zero inside the well but P.E jumps to some high value at edges of metal.

Recall that a particle trapped in a potential well has quantised energies given by following eqn.

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

$n \rightarrow$ principal quantum no $n = 1, 2, 3 \ldots$
$L \rightarrow$ width.

Similarly $\bar{e}$ trapped in metal solids have quantised energy levels but how many states are there?

By Pauli's exclusion principle, all $\bar{e}$ must be given by a unique set of quantum no and only two $\bar{e}$ can be found in a solid metals, we can presume that there must be very large no of possible energy states.

W.K.T $E_n = \frac{n^2 h^2}{8mL^2} \rightarrow (1)$

Differentiate eqn (1) we get.

$$dE_n = (2n)\frac{h^2}{8mL^2}\, dn.$$

$$\frac{dn}{dE_n} = \frac{8mL^2}{h^2}\left(\frac{1}{2n}\right) \rightarrow (2).$$

$$g(E) = 2 \times \frac{dn}{dE_n} = 2 \times \frac{8mL^2}{h^2} \cdot \frac{1}{2n}$$

$$g(E) = \frac{8mL^2}{h^2} \cdot \frac{1}{n}. \rightarrow (3)$$

$\left(\frac{dn}{dE_n}\right)$ denotes energy levels per unit energy range corresponding to 2-spin states.

Upon Simplification (3) $\boxed{g(E) = \frac{4\pi}{h^3}(2m)^{3/2} \cdot E^{1/2}}$

Considering the cubic metal piece with cube edge l.

## Carrier concentration (Free Electron Density) in Metals:—

The concentration of the carrier $n_c$ ie number of electrons per unit volume in the given energy levels E and E+dE is obtained by integrating the product of the density of states and the probability of occupancy f(E).

Therefore, $$n_c = \int g(E) f(E)\, dE \longrightarrow (1)$$

Substituting the value of DOS g(E) and f(E)

$$= \frac{1}{1+e^{(E-E_F)/KT}} \longrightarrow (2)$$

$$\therefore n_c = \int \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} \cdot \frac{1}{1+e^{(E-E_F)/KT}} \cdot dE. \longrightarrow (3)$$

Eqn (3) gives the carrier concentration in the given energy level E+dE.