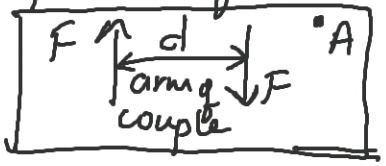
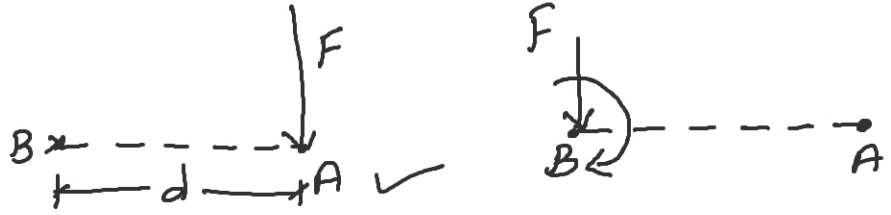
Characteristics of couple:

1. The algebraic sum of the components of two forces is zero i.e., The Resultant of a couple is zero
2. The moment of a couple is constant and is equal to the product of one of the forces and perpendicular distance between them ($M = F \times d$)
3. The couple can be balanced by equal and opposite couples only.
4. Two or more couples can be reduced to a single couple of moment equal to the algebraic sum of the given couples.
5. The moment of a couple is constant, irrespective of the point such as A, but same for all points in the plane of the couple.



Equivalent force-couple system:

A force acting at any point A on a rigid body may be replaced by another force of the same magnitude and direction at any other point B together with a couple whose moment is equal to the moment of F about the point B. [i.e, $M = F \times d$]

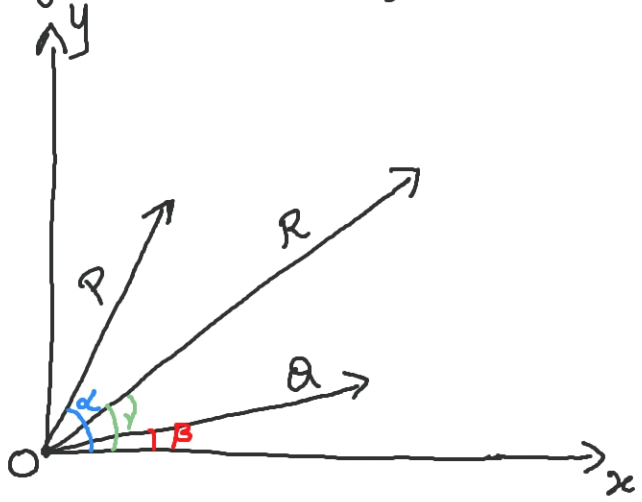


$M_B = F \times d.$

Varignon's theorem (or Principle of moments)

The moment of a force (i,e Resultant) at any point is equal to the algebraic sum of the moments of its components about that point.

Proof:

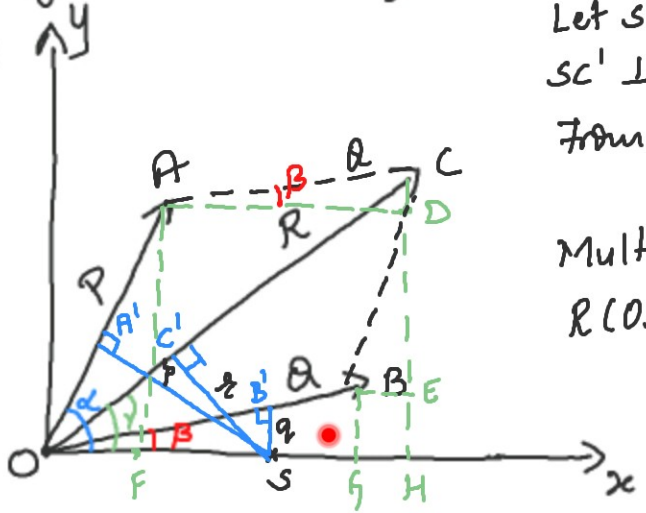


R is the resultant of P and Q.
Let α, β and γ be the inclination of P, Q & R w.r.t x-axis.

Varignon's theorem (or Principle of moments)

The moment of a force (i.e Resultant) at any point is equal to the algebraic sum of the moments of its components about that point.

Proof:



Let S be the point on x-axis. Draw SA', SB' and SC' ⊥re to the lines of action of forces P, Q & R proms.

From fig, CH = DH + CD

$$R\sin\gamma = P\sin\alpha + Q\sin\beta \rightarrow (1)$$

Multiply (1) by distance OS.

$$R(OS)\sin\gamma = P(OS)\sin\alpha + Q(OS)\sin\beta$$

$$R(SC') = P(SA') + Q(SB')$$

$$\boxed{R.r = P.p + Q.q.}$$

SC' is the moment arm of R = r
SA' is the moment arm of P = p
SB' is the moment arm of Q = q.

R is the resultant of P and Q.
Let α, β and γ be the inclination of P, Q & R w.r.t x-axis.

(continuation of Varignon's theorem...)

$$R.r = P.p + Q.q$$

∴ Moment of R about S = Algebraic sum of the moments of P and Q about S.

∴ Moment of the resultant force about a point is equal to the algebraic sum of the moments of its components about that point.

Equilibrium of coplanar force system.

If a rigid body is acted upon by a system of forces and remains at rest, the system is said to be in static equilibrium. Equilibrium of a rigid body is a state of balance.

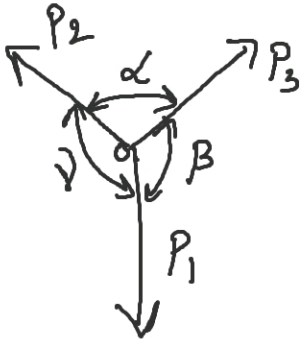
$$\Sigma F_H = 0 \quad \& \quad \Sigma F_V = 0 \quad \Sigma M = 0$$

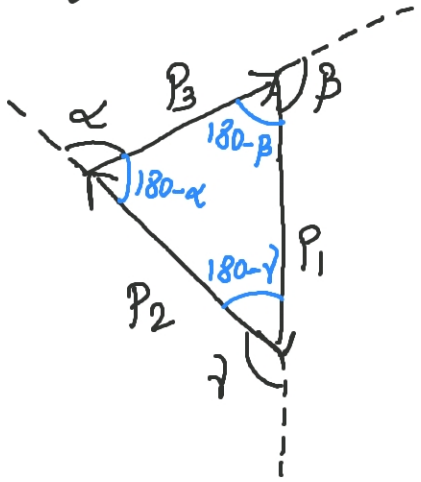
Equilibrium of coplanar concurrent force system:

The conditions of static equilibrium to be satisfied are:

1. The algebraic sum of the horizontal components of forces acting on the body is zero i.e $\Sigma F_H = 0$ (or) $\Sigma H = 0$
2. The algebraic sum of the vertical components of the forces acting on the body is zero i.e $\Sigma F_V = 0$ (or) $\Sigma V = 0$.

Lami's theorem: It states that "If three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between other two forces."





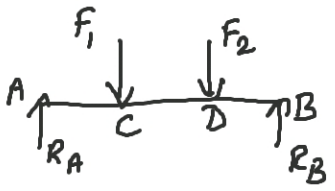
Force polygon

From sine rule,

$$\frac{P_1}{\sin(180-\alpha)} = \frac{P_2}{\sin(180-\beta)} = \frac{P_3}{\sin(180-\gamma)}$$

$$\therefore \boxed{\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}}$$

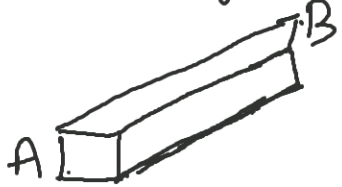
Equilibrium of coplanar parallel force system.



The conditions of static equilibrium to be satisfied are:

1. The algebraic sum of the vertical components of forces acting on the body is zero. i.e., $\Sigma F_V = 0$ or $\Sigma V = 0$
2. The algebraic sum of the moments of all the forces about any point is zero. i.e., $\Sigma M = 0$.

BEAM: A beam is any structural member which carries forces or loads at right angle to the longitudinal axis of the member.

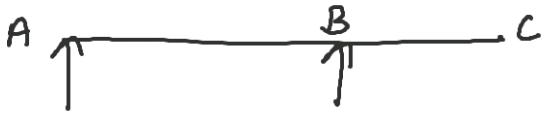


Types of beams:

① **simply supported beam**: A beam supported or resting freely on the walls or columns at its both ends is known as simply supported beam.

② **cantilever beam**: A beam fixed at one end and free at the other end is known as cantilever beam

③ Overhanging beam: A beam having its end portions extended beyond the support is known as overhanging beam.



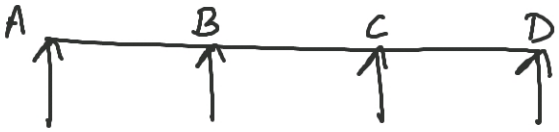
single overhanging beam.

Double overhanging beam.

④ Fixed beam: A beam whose ends are rigidly fixed or built in walls in known as fixed beam.

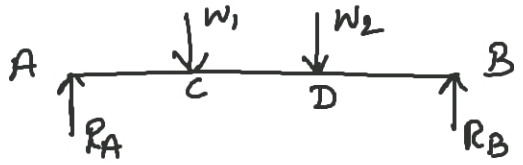
⑤ Propped Cantilever: A beam which is fixed at one end and other end is freely supported on walls or columns is known as Propped Cantilever.

⑥ Continuous beam: A beam supported on more than two supports is known as continuous beam.

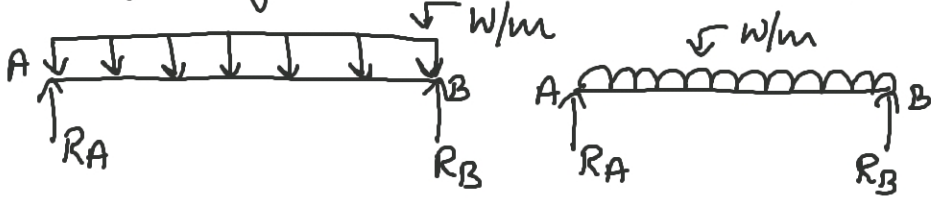


Types of loadings:

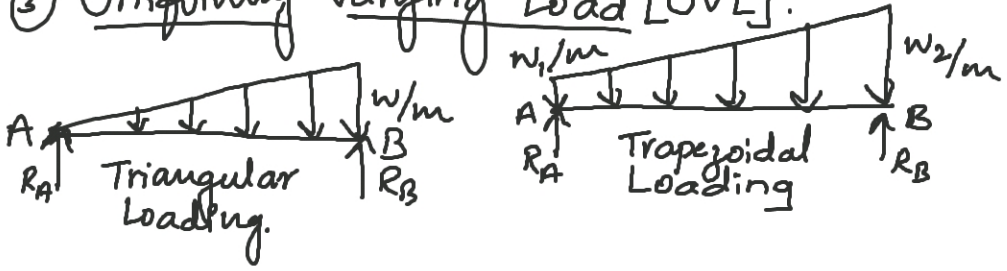
① Concentrated load or point load: A load acting at a point on a beam is known as concentrated load or point load.



② Uniformly distributed load [UDL]: A load which is spread over a beam in such a manner that each unit length is loaded to the same intensity is known as UDL.

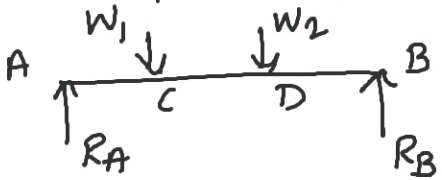


③ Uniformly varying Load [UVL]: A load is spread over a beam in such a manner that its intensity increases (or varies) linearly on each unit length is known as UVL.

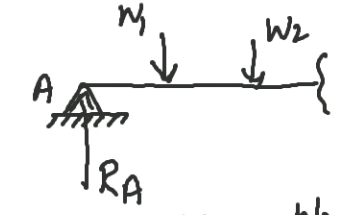


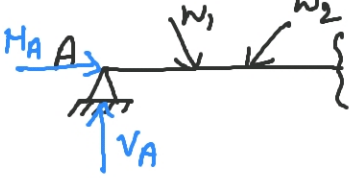
Type of supports :

① **Simple support :** The end of the beam rests simply on a rigid support. There is no resistance to the force in the direction of the support. Hence the reaction is always normal to the support. There is no moment resistance at the support.

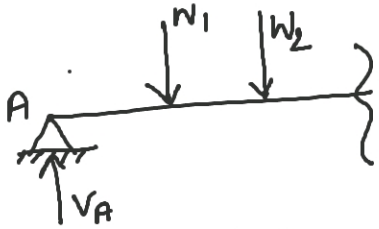


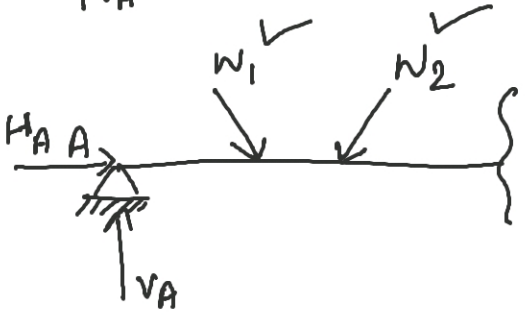
② **Hinged support (or) (Pinned support) :** In hinged support, the reaction can be in any direction which is usually represented by its components in two mutually ⊥re directions. This type of support does not provide any resistance to the moment, in other words it permits rotation freely at the end. It is free to rotate.

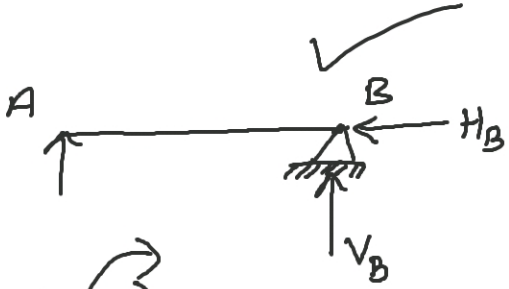


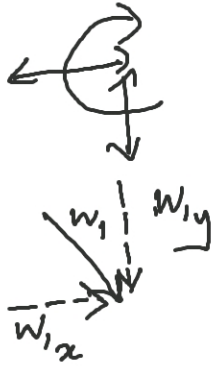


Hinged support (or) Pinned support.

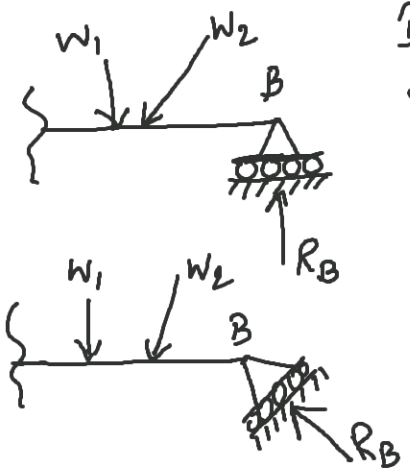




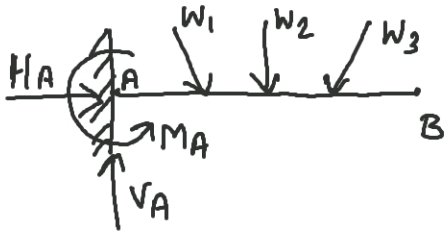




③ Roller support : In roller support, beam end is supported on rollers. In such cases, reaction is always normal to the support, since rollers are free to roll along the support. The ends are free to rotate also. Hence there is no resistance to moment.



④ Fixed support : At fixed support, the end of beam is neither permitted to move in any direction nor allowed to rotate. In this support, the reactions (H_A & V_A) and moment (M_A) exists.



Equilibrium of coplanar non-concurrent force system:

The conditions of static equilibrium to be satisfied are:

1. The algebraic sum of the horizontal components of forces acting on the body is zero i.e $\Sigma F_H = 0$ (or) $\Sigma H = 0$
2. The algebraic sum of the vertical components of forces acting on the body is zero i.e., $\Sigma F_V = 0$ (or) $\Sigma V = 0$
3. The algebraic sum of the moments of all the forces about any point is zero i.e., $\Sigma M = 0$

$\Sigma F_H = 0$, $\Sigma F_V = 0$, $\Sigma M = 0$