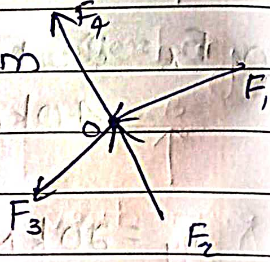
## Equilibrium of coplanar force system

If a rigid body is acted upon by a system of forces and remains at rest, the system is said to be in static equilibrium. Equilibrium of a rigid body is the state of ~~rest~~. balance

## Equilibrium of coplanar concurrent force system

The conditions of static equilibrium to be satisfied are:

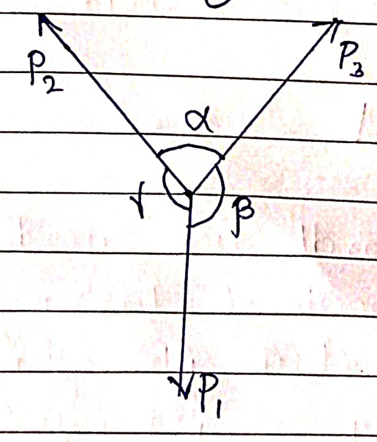


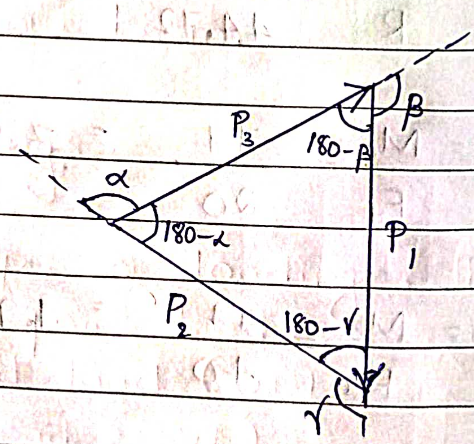

1. The algebraic sum of the horizontal components of forces acting on the body is zero. i.e., $\Sigma H = 0$

2. The algebraic sum of the vertical components of forces acting on the body is zero. i.e., $\Sigma V = 0$

## Lami's Theorem:

It states that, "If three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the two forces."




Force polygon

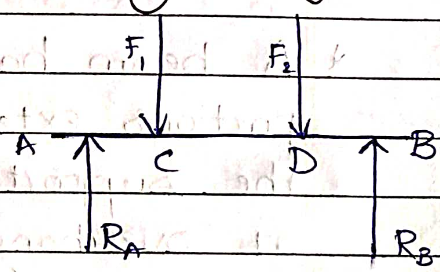
From Sine rule,

$$\frac{P_1}{\sin(180-\alpha)} = \frac{P_2}{\sin(180-\beta)} = \frac{P_3}{\sin(180-\gamma)}$$

$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$

## Equilibrium of coplanar parallel force system

The conditions of static equilibrium to be satisfied are :




1. The algebraic sum of the vertical components of forces acting on the body is zero i.e., $\Sigma V = 0$
2. The algebraic sum of the moments of all the forces about any point is zero i.e., $\Sigma M = 0$.

## BEAM

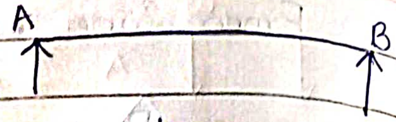
A beam is any structural member which carries forces or loads at right angles to the longitudinal axis of the member.

### Types of beams:

There are six types of beams. They are:

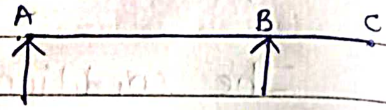
* Simply supported beam
* Cantilever beam
* Overhanging beam
* Fixed beam
* Propped Cantilever
* Continuous beam

* A beam supported or resting freely on the walls or columns at its both ends is known as simply supported beam.




* A beam fixed at one end and free at the other end is known as cantilever beam.

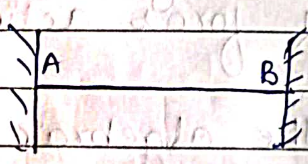

* A beam having its end portions extended beyond the support is known as overhanging beam.



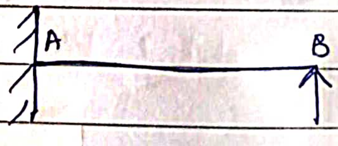

Single overhanging beam


Double overhanging beam

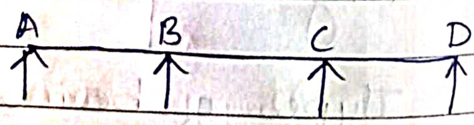
* A beam whose ends are rigidly fixed or built in walls is known as fixed beam.




* A beam which is fixed at one end and other end is freely supported on walls or columns is known as propped cantilever.




* A beam supported on more than two supports is known as continuous beam.

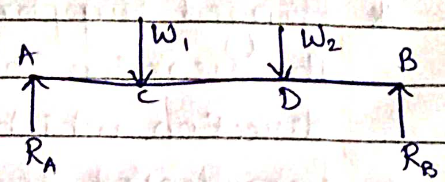



## Loadings

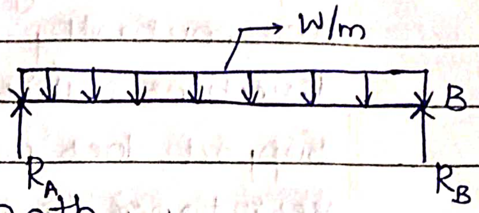
There are three types of loadings. They are

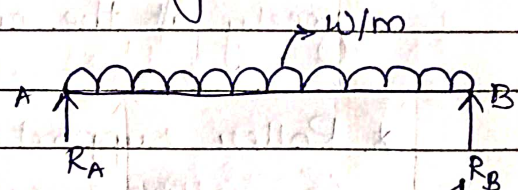
* Concentrate load or point load
* Uniformly distributed load (UDL)
* Uniformly varying load (UVL)

* A Load acting at a point on a beam is known as Concentrated load or point load

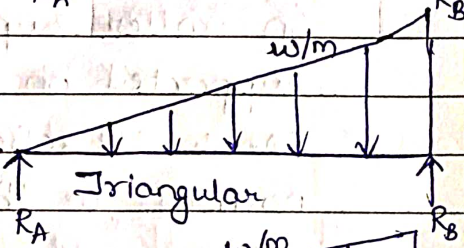


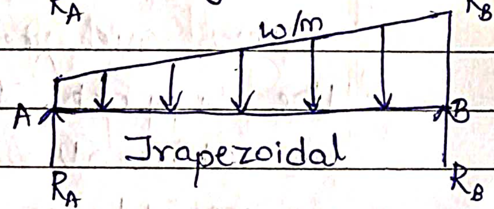

* A load which is spread over a beam in such a manner that each unit length is loaded to the same intensity




* A load is spread over a beam in such a manner that its intensity increases (or varies) linearly on each unit length is known as uniformly varying load.

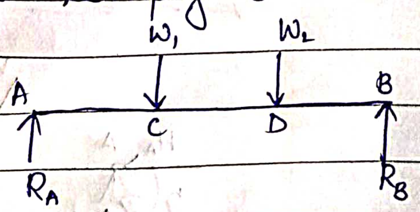


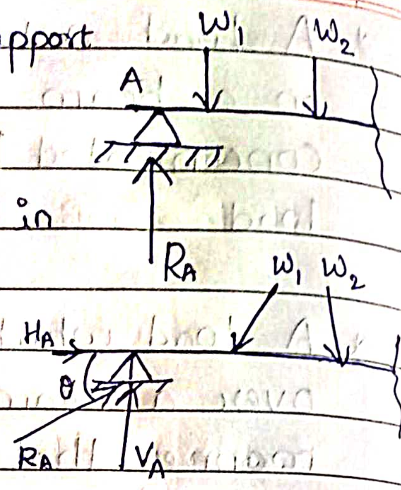

## Supports

There are four types of supports. They are:
* Simple support
* Hinged support (pinned support)
* Roller support
* Fixed support.

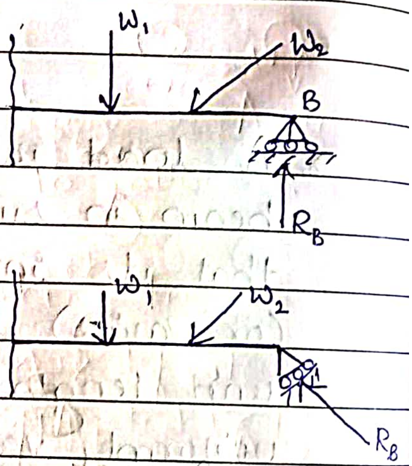
* The end of the beam rests simply on a rigid support. There is no resistance to the force in the direction of the support. Hence the reaction is always normal to the support. There is no moment resistance to the support (Simple support).



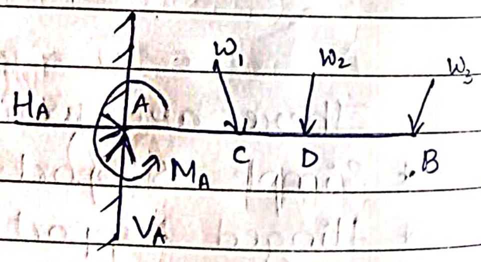
* **Hinged Support:** In hinged support the reaction can be in any direction which is usually represented by its components in two mutually perpendicular directions. This type of support does not provide any resistance to the moment, in other words it ~~provides~~ permits rotation freely at the end. It is free to rotate.



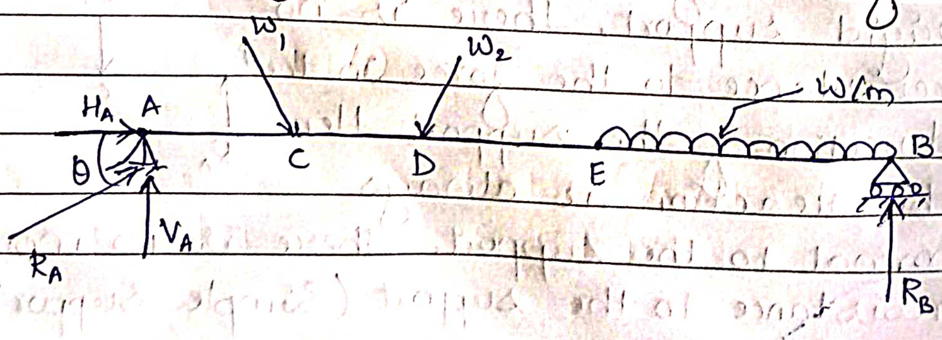

* **Roller support:**
In roller support, beam end is supported on rollers. In such cases, reaction is always normal to the support, since, rollers are free to roll along the support. The ends are free to rotate also. Hence, there is no resistance to moment.




* **Fixed support:** At fixed support, the end of the beam is neither permitted to move in any direction nor allowed to rotate. In this support, the reaction ($H_A$ and $V_A$) and moment ($M_A$) exists.




**Equilibrium of coplanar non concurrent force system**



The conditions of static equilibrium to be satisfied are:

1. The algebraic sum of the horizontal components of forces acting on the body is zero i.e., $\Sigma H = 0$
2. The algebraic sum of the vertical components of forces acting on the body is zero i.e., $\Sigma V = 0$
3. The algebraic sum of the moments of all the forces about any point is zero i.e., $\Sigma M = 0$.

Note 1. Hinged support will take horizontal and vertical reactions. Only moment $M_A = 0$.

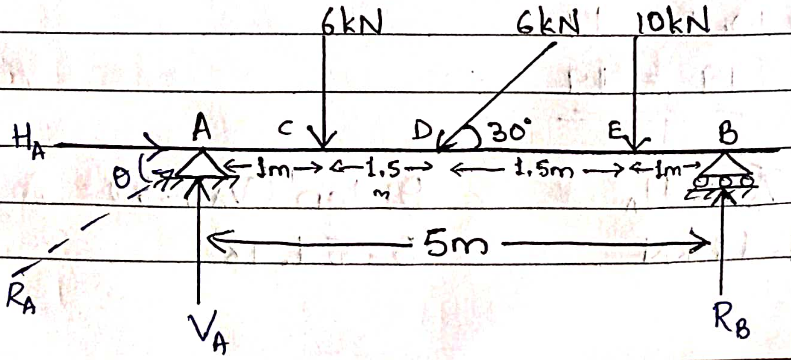
Note 2. Roller support will take vertical reactions perpendicular to its base. Therefore, horizontal reaction $H_B = 0$ and moment $M_B = 0$.

Note 3: To determine the support reactions at A and B, the following sign conventions have to be followed

| Taking moments about right hand support. | | Taking moments about left hand support | |
|---|---|---|---|
| +ve | -ve | -ve | +ve |

## Problems

1. Determine the support reactions at A and B.



Let $V_A$ and $R_B$ be the vertical reactions at A and B respectively.

Let $H_A$ be the horizontal reactions at the hinged support.

$\Sigma H = 0$

$H_A - 6\cos 30^\circ = 0$

$H_A = 5.19 \text{ kN}$

Taking moments ~~about~~ of all forces about B

+ve -ve

$\Sigma M_B = 0$

$V_A \times 5 - 6\times 4 - 6\sin 30^\circ \times 2.5 - 10\times 1 = 0$

$$V_A = \frac{24 + 7.5 + 10}{5}$$

$V_A = 8.3 \text{kN}$

Taking moments of all forces about A

+ve +ve

$\Sigma M_A = 0$

$R_B \times 5 - 10\times 4 - 6\sin 30^\circ \times 2.5 - 6\times 1 = 0$

$$R_B = \frac{40 + 7.5 + 6}{5}$$

$R_B = 10.7 \text{ kN}$

$\Sigma V = 0$

$V_A + R_B - 6 - 6\sin 30^\circ - 10 = 0$

$V_A + R_B = 6 + 3 + 10$

$8.3 + 10.7 = 19$

$\underline{\underline{19 = 19}}$

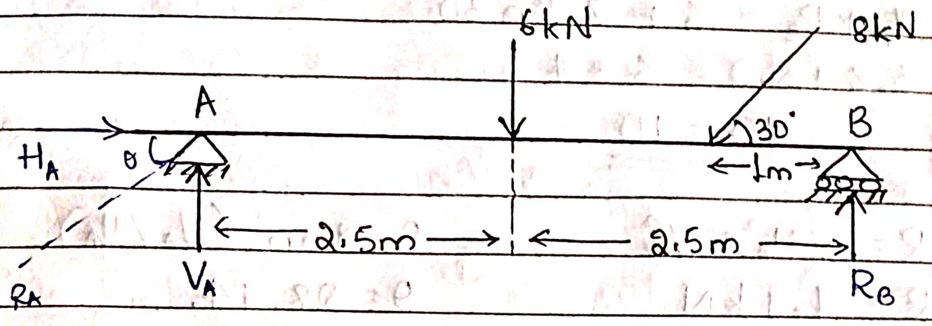
$R_A = \sqrt{V_A^2 + H_A^2}$

$R_A = 9.79 \text{ kN}$

$\theta = \tan^{-1}(V_A / H_A)$

$\theta = 57.98^\circ$

2. Determine the support reactions at A and B.




Let $V_A$ and $R_B$ be the vertical reactions at A and B respectively.

Let $H_A$ be the horizontal reactions at A.

$\Sigma H = 0$

$H_A - 8\cos 30° = 0$

$H_A = 6.93 kN$

Taking moments of all forces about B. +ve −ve

$\Sigma M_B = 0$

$V_A \times 5 - 6 \times 2.5 - 8 \sin 30° \times 1 = 0$

$V_A = \dfrac{15 + 4}{5}$

$V_A = 3.8 kN$

Taking moments of all forces about A. −ve +ve

$\Sigma M_A = 0$

$R_B \times 5 - 8 \sin 30° \times 4 - 6 \times 2.5 = 0$

$R_B = \dfrac{15 + 16}{5}$

$R_B = 6.2 kN$

$\Sigma V = 0$

$V_A + R_B = 6 + 8 \sin 30°$

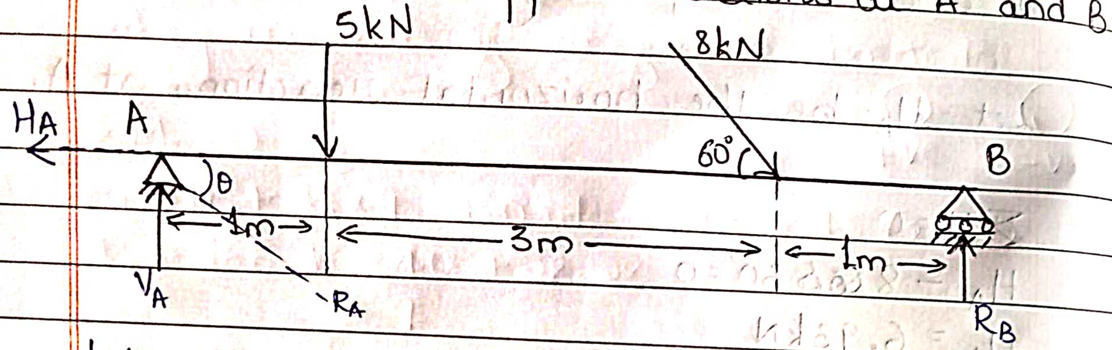
$3.8 + 6.2 = 6 + 4$

$\underline{\underline{10 = 10}}$

$R = \sqrt{H_A^2 + V_A^2}$ $\theta = \tan^{-1}(V_A / H_A)$

$R = 7.9\ kN$ $\theta = 28.74°$

3. Determine the support reactions at A and B




Let $V_A$ and $R_B$ be the vertical reactions at A and B respectively.

Let $H_A$ be the horizontal reaction at A

$\Sigma H = 0$

$8 \cos 60° - H_A = 0$

$H_A = 4\ kN$

Taking moments of all forces about B (+ve, −ve)

$\Sigma M_B = 0$

$V_A \times 5 - 5 \times 4 - 8 \sin 60 \times 1 = 0$

$V_A = \frac{20 + 6.93}{5}$

$V_A = 5.38\ kN$

Taking moments of all forces about A (−ve, +ve)

$\Sigma M_A = 0$

$R_B \times 5 - 8 \sin 60^\circ \times 4 - 5 \times 1 = 0$

$R_B = \dfrac{5 + 27.71}{5}$

$R_B = 6.54\,kN.$

$\Sigma V = 0$

$V_A + R_B = 5 + 8 \sin 60^\circ$

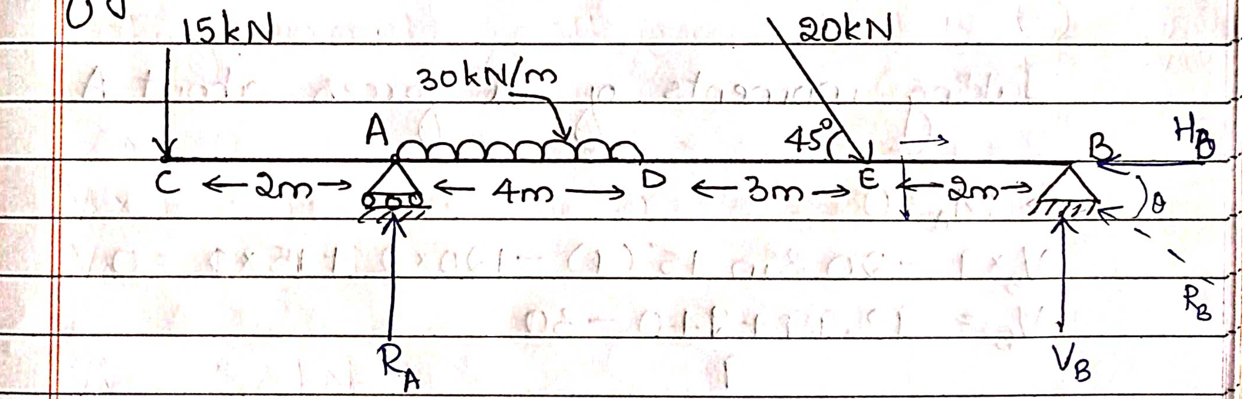
$5.38 + 6.54 = 5 + 6.92$

$11.92 = 11.92$

$R = \sqrt{H_A^2 + V_A^2}$ $\theta = \tan^{-1}(V_A / H_A)$

$R = 6.7\,kN$ $\theta = 53.37^\circ$

4. An overhanging beam is on rollers at A and is hinged at B and is loaded as shown in figure. Determine reactions at A and B.




Let $R_A$ and $V_B$ be the vertical reactions at A and B respectively

Let $H_B$ be the horizontal reaction at B

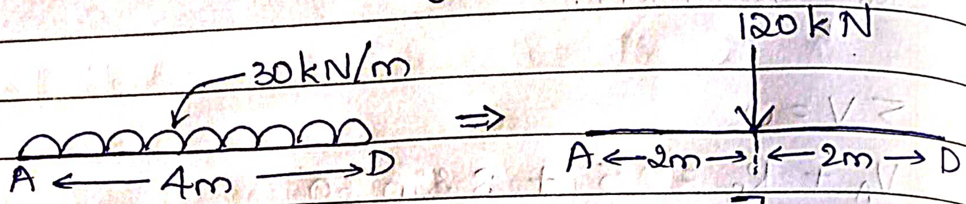
$\Sigma H = 0$

$20 \cos 45^\circ - H_B = 0$

$H_B = 14.14\,kN$

[The uniformly distributed weight of 30 kN/m acting over 4m is assumed as 120 kN (30 × 4) acting at the centre of 4m.

i.e.,




∴ Equilibrium is not disturbed]

Taking moments of all forces about B

+ve −ve

$\sum M_B = 0$

$-15 \times 11 + R_A \times 9 - 120 \times 7 - 20 \sin 45^\circ (2) = 0$

$R_A = \dfrac{165 + 840 + 28.28}{9}$

$R_A = 114.81\,kN$

Taking moments of all forces about A

$\sum M_A = 0$

$V_B \times 9 - 20 \sin 45^\circ (7) - 120 \times 2 + 15 \times 2 = 0$

$V_B = \dfrac{98.99 + 240 - 30}{9}$

$V_B = 34.33\,kN.$

$\sum V = 0$

$V_B + R_A = 15 + 120 + 20 \sin 45^\circ$

$114.81 + 34.33 = 15 + 120 + 14.14$

$\underline{149.14 = 149.14}$

$R_B = \sqrt{V_B^2 + H_B^2}$

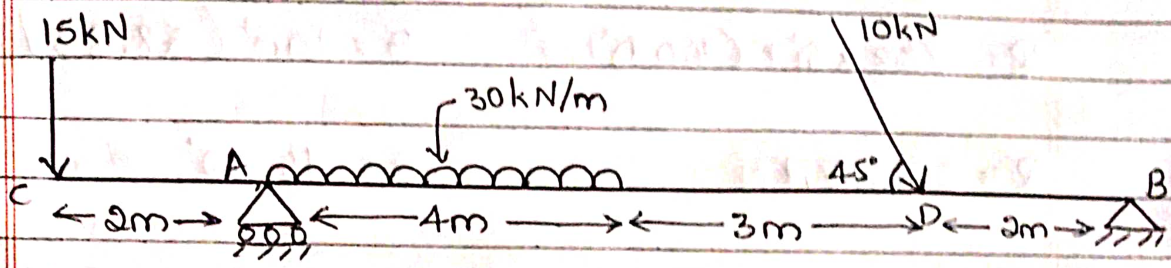
$R_B = 37.13$

~~$R_B = 115.68\,kN$~~

$\theta = \tan^{-1}(V_B / H_B)$

$\underline{\underline{\theta = 67.61^\circ}}$

5. Find the reactions at A and B.




Let $R_A$ and $V_B$ be the vertical reactions at A and B respectively

Let $H_B$ be the horizontal reaction at B

$\Sigma H = 0$

$10 \cos 45° - H_B = 0$

$H_B = 7.07 kN$

Taking moments of all forces about B (clockwise +ve, anticlockwise −ve)

$\Sigma M_B = 0$

$-15 \times 11 + R_A \times 9 - 120 \times 7 - 10 \sin 45° (2) = 0$

$R_A = \dfrac{165 + 840 + 14.14}{9}$

$R_A = 113.24 kN$

Taking moments of all forces about A (clockwise −ve, anticlockwise +ve)

$\Sigma M_A = 0$

$V_B \times 9 - 10 \sin 45° (7) - 120 \times 2 + 15 \times 2 = 0$

$V_B = \dfrac{49.495 + 240 - 30}{9}$

$V_B = 28.83 kN$

$\Sigma V = 0$

$V_B + R_A = 15 + 120 + 10 \sin 45°$

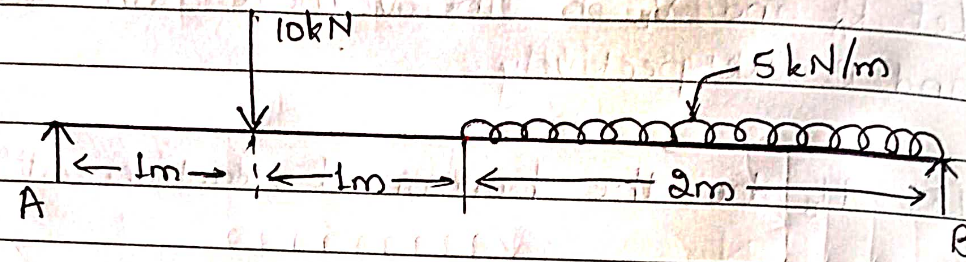
$142.07 = 142.07$

$R_B = \sqrt{H_B^2 + V_B^2}$ $\theta = \tan^{-1}(V_B / H_B)$

$R = \sqrt{(28.83)^2 + (7.07)^2}$ $\theta = \tan^{-1}(28.83 / 7.07)$

$R = 29.68\ kN$ $\theta = 76.22^\circ$

6. Find the reactions at A and B.




Let $R_A$ and $R_B$ be the support reactions at A and B respectively.

Taking moments of all forces about B +ve -ve

$\Sigma M_B = 0$
$R_A \times 4 - 10(3) - 10(1) = 0$
$R_A = 10\ kN$

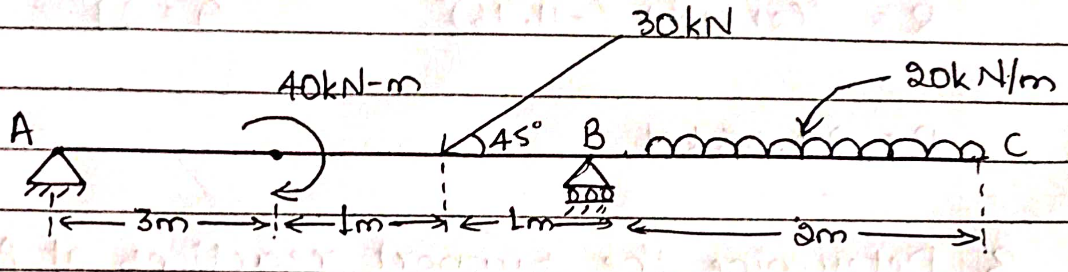
Taking moments of all forces about A -ve +ve

$\Sigma M_A = 0$
$R_B \times 4 - 10 \times 3 - 10(1) = 0$
$R_B = 10kN$

$\Sigma V = 0$
$R_A - 10 - 5(2) + R_B = 0$
$R_A + R_B = 20kN$

Check: $R_A + R_B = 10 + 10 = 20kN.$

7. Determine Reactions developed at supports in the beam shown.




Let $V_A$ and $R_B$ be the vertical reactions at A and B respectively.

Let $H_A$ be the horizontal reaction at A.

$\sum H = 0$

$H_A - 30\cos 45° = 0$

$H_A = 21.21\ kN$

Taking moments of all forces about B ↻ +ve ↺ −ve

$\sum M_B = 0$

$V_A \times 5 + 40 - 30\sin 45 \times 1 + 20(2)(1) = 0$

$V_A = -11.76\ kN$

∴ Assumed direction is wrong ∴ 9.798 is acting in downward direction.

Taking moments of all forces about A ↻ −ve ↺ +ve

$\sum M_A = 0$

$R_B \times 5 - 30\sin 45°(4) - 40 - 40(6) = 0$

$R_B = 72.97\ kN$

$\sum V = 0$

$-V_A - 30\sin 45° + R_B - 20(2) = 0$

$R_B - V_A = 61.21\ kN$

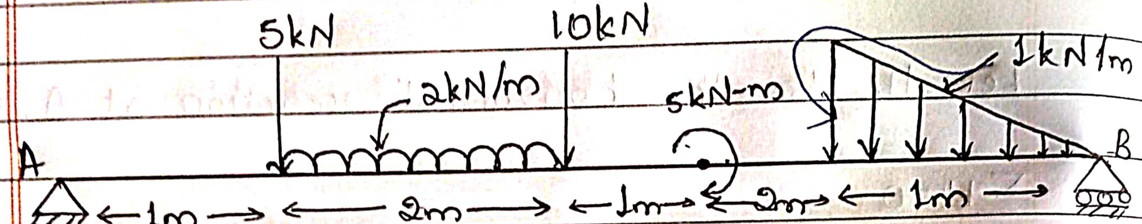
Check : $R_B - V_A = 72.97 - 11.76 = 61.21 kN$

$R = \sqrt{H_A^2 + V_A^2}$ $\qquad$ $\theta = \tan^{-1}(V_A / H_A)$

$R = \sqrt{(21.21)^2 + (-11.76)^2}$ $\qquad$ $\theta = 29^\circ$

$R = 24.25\ kN$

8. Determine the support reactions at A and B for the simply supported beam.




There are no horizontal reaction ∴ $H_A = 0$

Taking moments of all forces about B $\quad$ +ve $\quad$ -ve

$\Sigma M_B = 0$

$V_A \times 7 - 5 \times 6 - 2(2)(5) - 10(4) + 5 - \frac{1}{2}(1)(1)\left(\frac{2}{3}\right) = 0$

$V_A = 12.19\ kN$

Taking moments of all forces about A $\quad$ -ve $\quad$ +ve

$\Sigma M_A = 0$

$R_B \times 7 - \frac{1}{2}(1)(1)(6.333) - 5 - 10(3) - 4(2) - 5(1) = 0$

$R_B = 7.31\ kN$

$\Sigma V = 0$

$V_A - 5 - 2(2) - 10 - \frac{1}{2}(1)(1) + R_B = 0$

$V_A + R_B = 19.5\ kN$

Check : $V_A + R_B = 12.19 + 7.31 = 19.5\ kN$

9 Determine the reaction developed in simply supported beam as shown in fig.

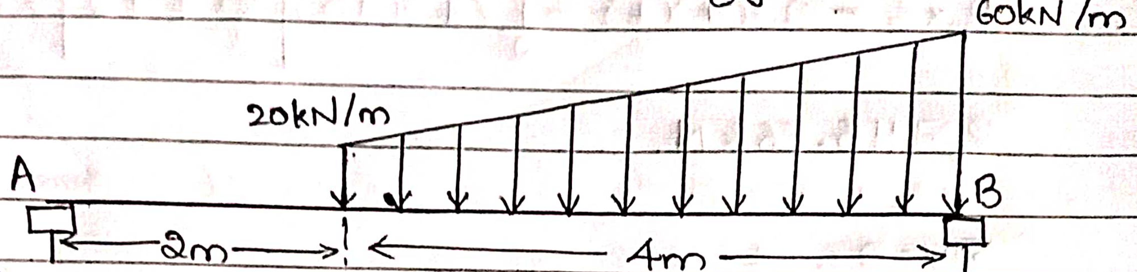



Fig A

Solution.

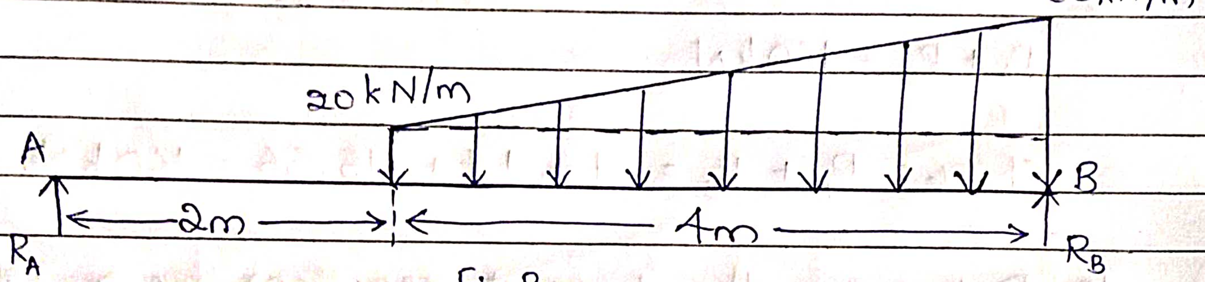



Fig. B

Fig A shows uniformly loaded load (Trapezoidal) 20kN/m to 60kN/m from C to B in the given problem.

In the solution, as shown in fig B, this uniformly varying load will be having two components for analysis purpose

A UDL of 20kN/m is superimposed on triangular load. Hence, UDL will be having intensity of 20kN/m and triangular load will be having intensity 0 to 40kN/m as shown in Fig B

Let $R_A$ and $R_B$ be the reactions at the supports A and B.

Taking moments of all forces about B (+ve) (−ve)

$\Sigma M_B = 0$

$$R_A \times 6 - 20(4)(2) - \frac{1}{2}(4)(40)\left(\frac{4}{3}\right) = 0$$

$R_A = 44.44\,kN$

Taking moments of all forces about A (↺ −ve, ↻ +ve)

$\sum M_A = 0$

$R_B \times 6 - 20(4)(4) - \frac{1}{2}(4)(40)\left[\frac{2 \times 4}{3} + 2\right] = 0$

$R_B = 115.56\ kN.$

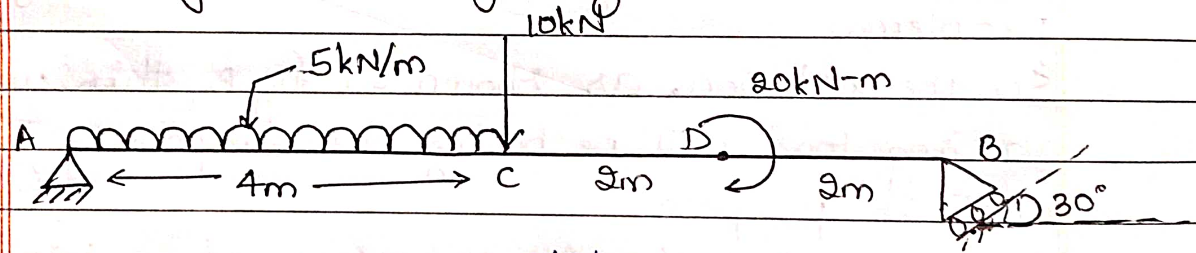
$\sum V = 0$

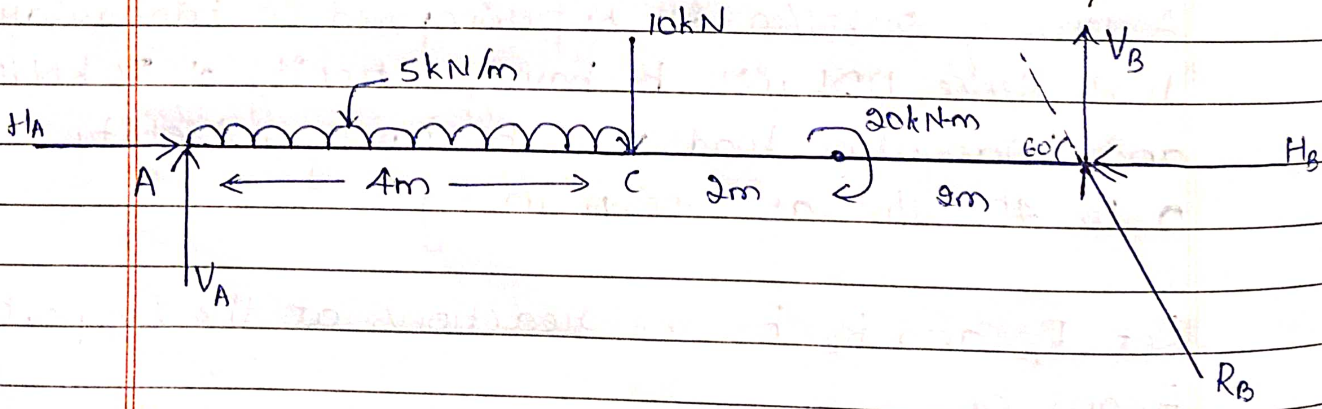
$R_A - 20(4) - \frac{1}{2}(40)(4) + R_B = 0$

$R_A + R_B = 160\ kN$

Check: $R_A + R_B = 44.44 + 115.56 = 160\ kN.$

10. Determine the support reactions for a simply supported beam shown in fig. wherein A is having hinged support and roller support is at B inclining at an angle of 30°.




Let $V_A$ and $H_A$ be the support reactions at A.

Taking moments of all forces about B

$\sum M_B = 0$

$V_A \times 8 - 5(4)(6) - 10(4) + 20 = 0$

$V_A = 17.5\ kN$

Taking the moments of all forces about A

$\Sigma M_A = 0$

$V_B \times 8 - 20 - 10(4) - 5(4)(2) = 0$

$V_B = 12.5\,kN$

$R_B \sin 60^\circ = 12.5$

$R_B = 14.43\,kN$

$\Sigma H = 0$

$H_A - R_B \cos 60^\circ = 0$

$H_A = 7.215\,kN$

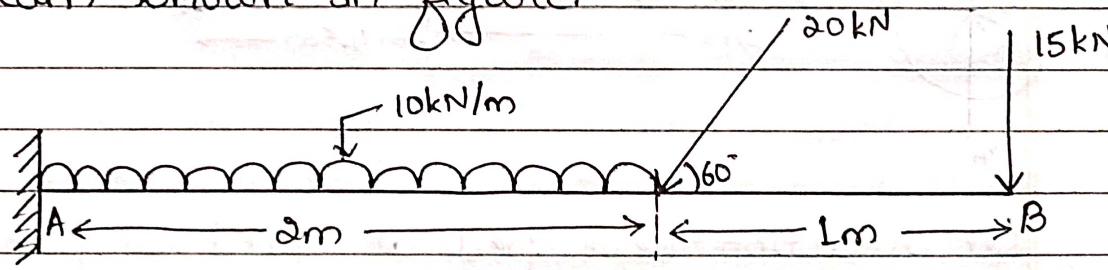
$\Sigma V = 0$

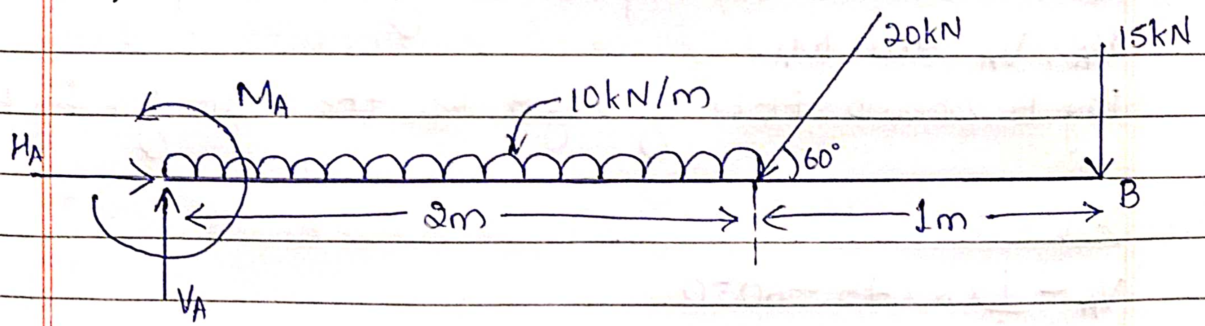
$V_A - 5(4) - 10 + V_B = 0$

$V_A + V_B = 30\,kN$

Check: $V_A + V_B = 17.5 + 12.5 = 30\,kN$

11. Determine the reactions for the cantilever beam shown in figure.




Let the reaction developed at fixed support A be $V_A$, $H_A$ and $M_A$ as shown in fig.

$\Sigma V = 0$

$V_A - 10(2) - 20 \sin 60^\circ - 15 = 0$

$V_A = 52.32\,kN$

$\Sigma H = 0$

$H_A - 20\cos 60^\circ = 0$

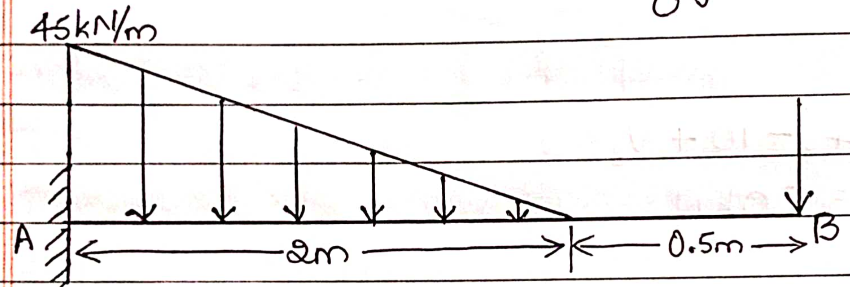
$H_A = 10 kN$

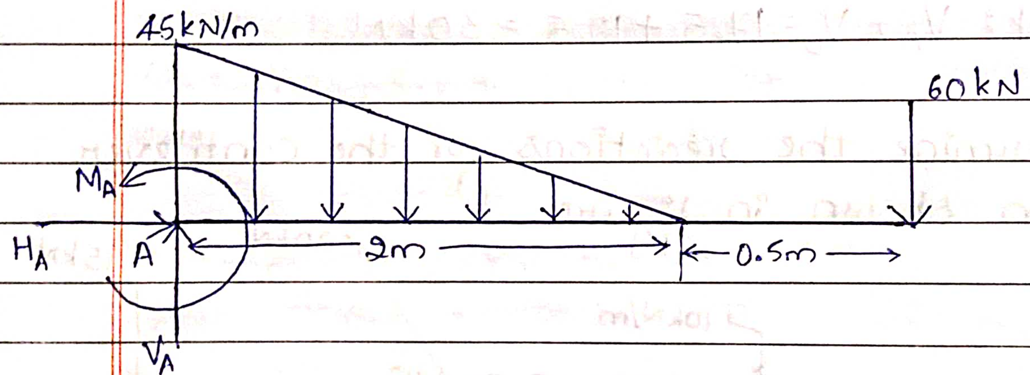
$\Sigma M = 0$

$-M_A + 10(2)(1) + 20\sin 60^\circ (2) + 15(3) = 0$

$M_A = 99.64 kN\text{-}m$

12. Determine the reactions developed in the cantilever beam shown in figure:




Let reaction developed at fixed support A be $H_A$, $V_A$ and $M_A$.
Fig B shows FBD of Fig A i.e., the soln for fig A.

$\Sigma V = 0$

$V_A - \frac{1}{2} \times 2 \times 45 - 60 = 0$

$V_A = 115 kN$

$\Sigma H = 0$

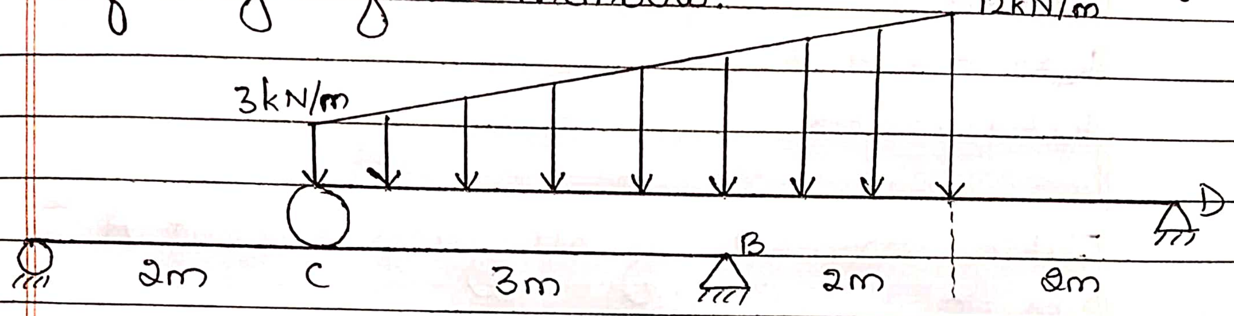
$H_A = 0$

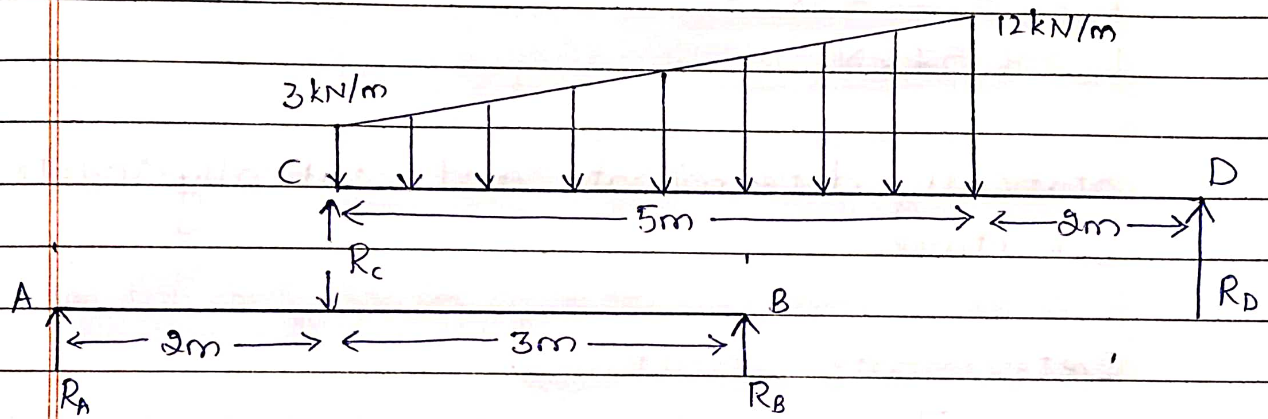
Taking moments of all forces about A

$$-M_A + \frac{1}{2}(2)(45)\frac{1}{3}(2) + 60(2.5) = 0$$

$$M_A = 180\,kN\text{-}m.$$

13. Determine the support reactions at A, B and D of compound beam as shown in fig. A. Neglect self weight of the members.




FBD of beams CD and AB are as shown in fig. The load may be divided into a UDL of 3kN/m and a triangular load of intensity 0 at C and 9kN/m at 2m from D.

Consider the beam CD,

Taking moments of all forces about C.

$$\Sigma M_c = 0$$

$$R_D \times 7 - 3 \times 5 \times 2.5 - \tfrac{1}{2}(5)(9)(2/3)(5) = 0$$

$$R_D = 16.07\,kN$$

Taking moments of all forces about D.

$\Sigma M_D = 0$

$R_c \times 7 - 3 \times 5 \times 4.5 - \frac{1}{2}(5)(9)\left[\frac{5}{3}+2\right] = 0$

$R_c = 21.43$ kN.

Consider the beam AB

Taking moments of all forces about B

$\Sigma M_B = 0$

$R_A \times 5 - 21.43 \times 3 = 0$

$R_A = 12.86$ kN

Taking moments of all forces about A

$\Sigma M_A = 0$

$R_B \times 5 - 21.43 \times 2 = 0$

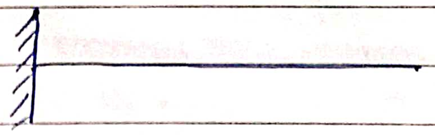
$R_B = 8.57$ kN.

Statically determinate and statically indeterminate structures.

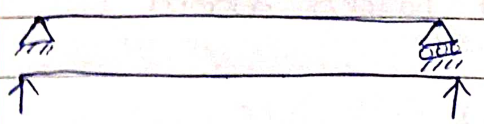
Determinate structures:

The structures which can be analyzed by using 3 static equilibrium conditions i.e., $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$ are called determinate structures.

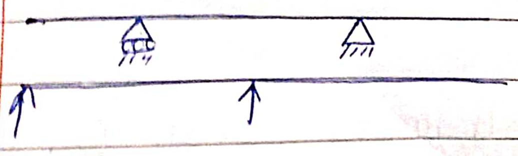
Examples are:



Cantilever beam



Simply supported beam.

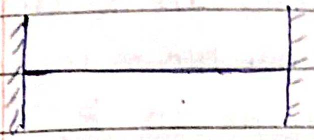


Overhanging beam

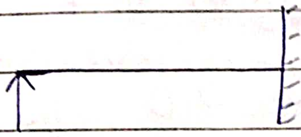
Indeterminate structures

The structures which can't be analyzed by using 3 equilibrium conditions $\Sigma H=0$, $\Sigma V=0$ and $\Sigma M=0$ are called as indeterminate structures.

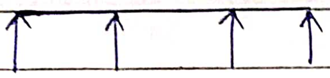
Examples are:



Fixed beam (6 unknowns)



Propped cantilever (4 unknowns)



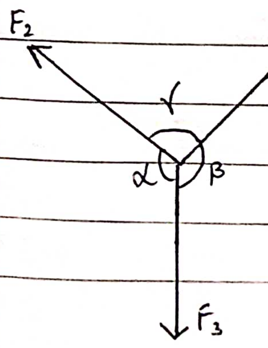
Continuous beam (>3 unknowns)

## Lami's Theorem

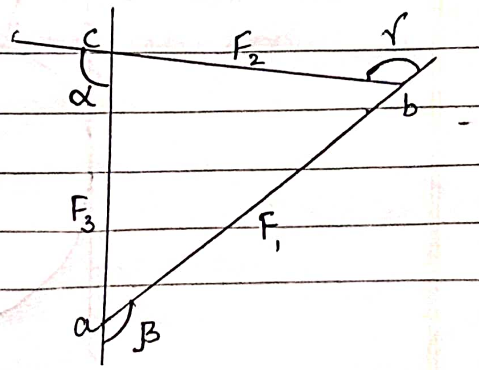
If a body is in equilibrium, it may be analysed using equations of equilibrium. However, if the body is in equilibrium under the action of only three forces. Lami's theorem can be used conveniently.

Lami's theorem states that, if a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the other two forces. Thus for the system of forces shown in fig A.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$




(A)



Proof: Draw the three forces $F_1$, $F_2$ and $F_3$ one after the other in direction and magnitude starting from point a. Since the body is in equilibrium the resultant should be zero, which means the last point of force diagram should coincide with a. Thus, it results in a triangle of forces abc as shown in fig (b). Now the external angles at a, b and c are equal to $\beta$, $\gamma$ and $\alpha$, since ab, bc and ca are parallel to $F_1$, $F_2$ and $F_3$ respectively. In the triangle of forces abc,

$$ab = F_1$$
$$bc = F_2$$
$$ca = F_3$$

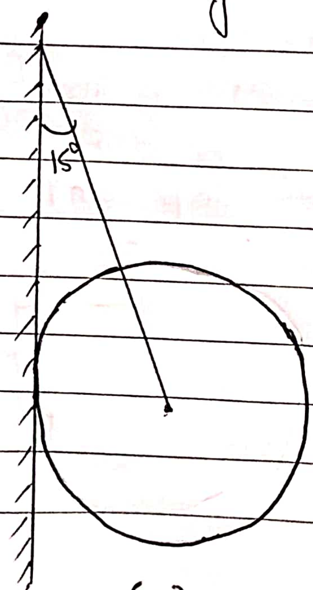
Applying sine rule for the triangle abc,

$$\frac{ab}{\sin(180^\circ - \alpha)} = \frac{bc}{\sin(180^\circ - \beta)} = \frac{ca}{\sin(180^\circ - \gamma)}$$

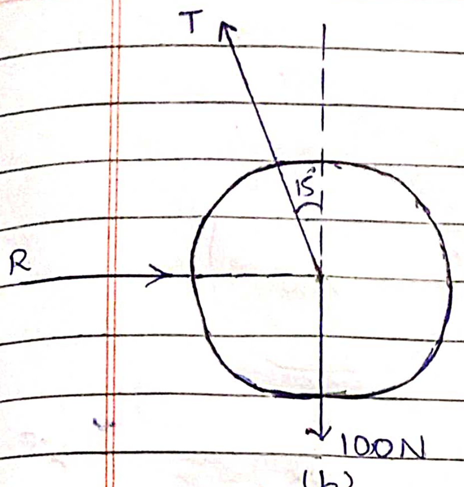
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

## Problems

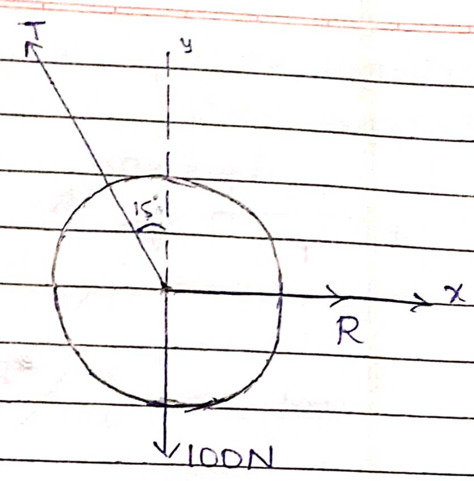
1. A sphere weighing 100N is tied to a smooth wall by a string as shown in fig (a). Find the tension T in the string and the reaction R from the wall.




(a)



(b)



(c)

Free body diagram of sphere is shown in fig b and c shows all the forces acting away from the centre of the ball, which is permissible as per the law of transmissibility of forces. Appling Lami's theorem to the system of forces, we get,

$$\frac{T}{\sin 90^\circ} = \frac{R}{\sin(180-15)} = \frac{100}{\sin(90+15)}$$

$R = 26.8N$

$T = 103.53N$

or

$\Sigma F_y = 0$

$T\cos 15^\circ - 100 = 0$

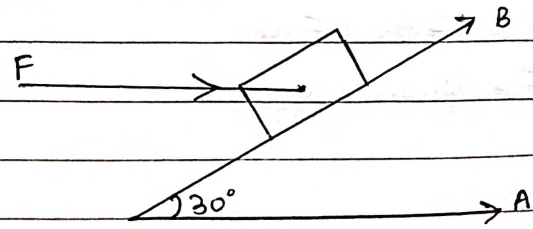
$T = 103.53N$

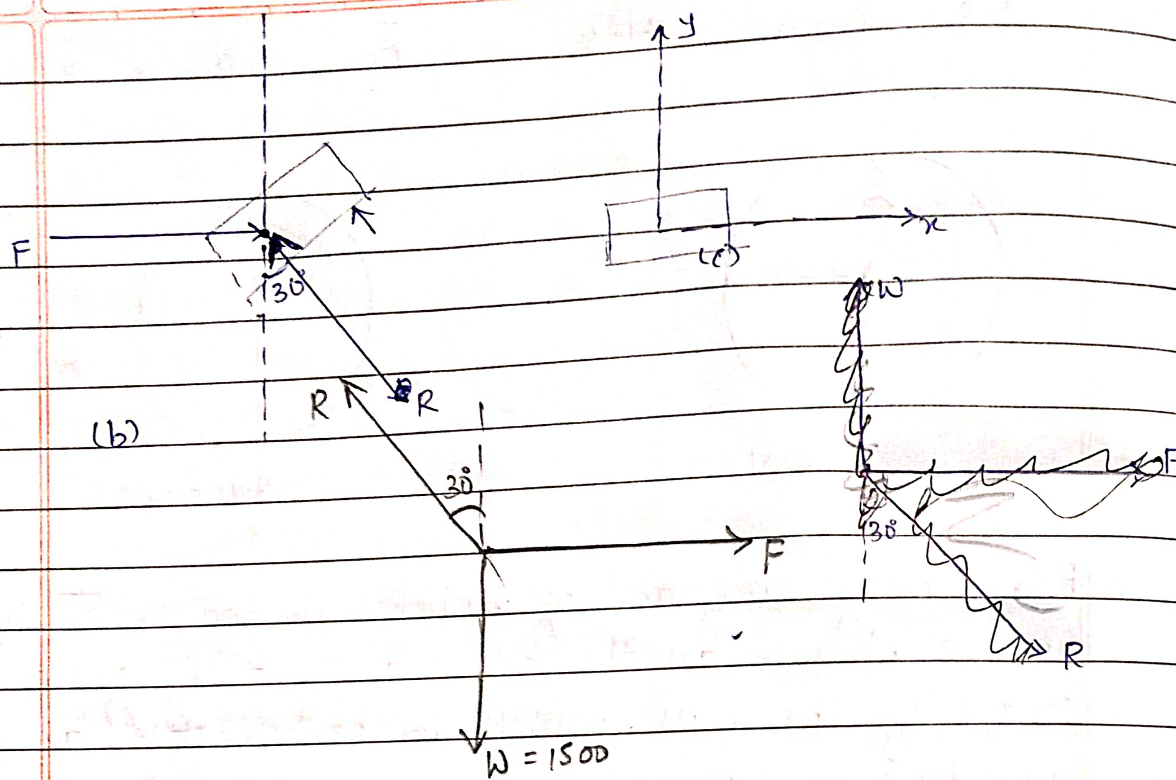
$\Sigma F_x = 0$

$R - T\sin 15^\circ = 0$

$R = 26.8N$

2. Determine the horizontal force to be applied to the block weighing 1500N to hold it in position on a smooth inclined plane AB which makes an angle 30° with the horizontal as shown in fig A.






The body is in equilibrium under the action of applied force F, self weight 1500N and reaction R from the plane. Applied force is horizontal and self weight is vertically downward. Reaction is normal to the plane AB, since the plane AB is smooth. Since, plane makes 30° to horizontal, normal to it makes 60° to horizontal i.e, 30° to vertical as shown in fig B.

$\Sigma F_y = 0$

$R\cos 30^\circ - 1500 = 0$

$R = 1732.05\,N$

$\Sigma F_x = 0$

$F - R\sin 30^\circ = 0$

$F = 866.02\,N$

(c)

By Lami's theorem,

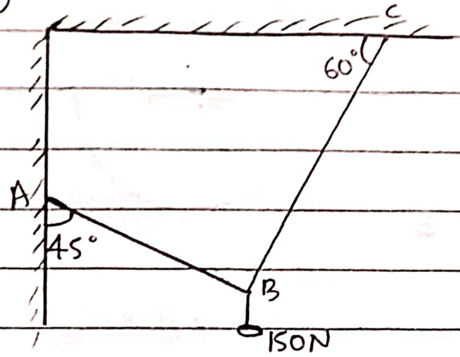
$$\frac{R}{\sin 90^\circ} = \frac{F}{\sin 150^\circ} = \frac{1500}{\sin 120^\circ}$$

$R = 1732.05\,N$

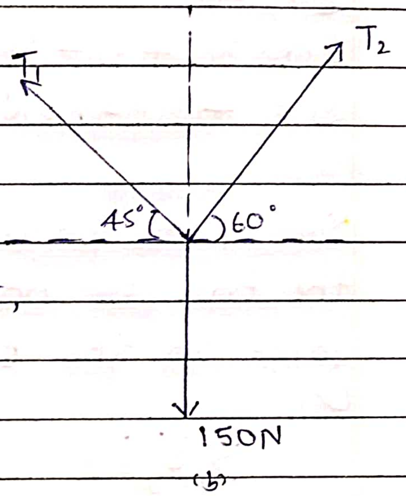
$F = 866.02\,N$

3. Determine the horizontal force F to be applied

3. Find the forces developed in the wires, supporting an electric fixture as shown in fig A.




Let the forces developed in the wires BA and BC be $T_1$ and $T_2$ as shown in fig B. Applying Lami's theorem to the system of forces, we get,

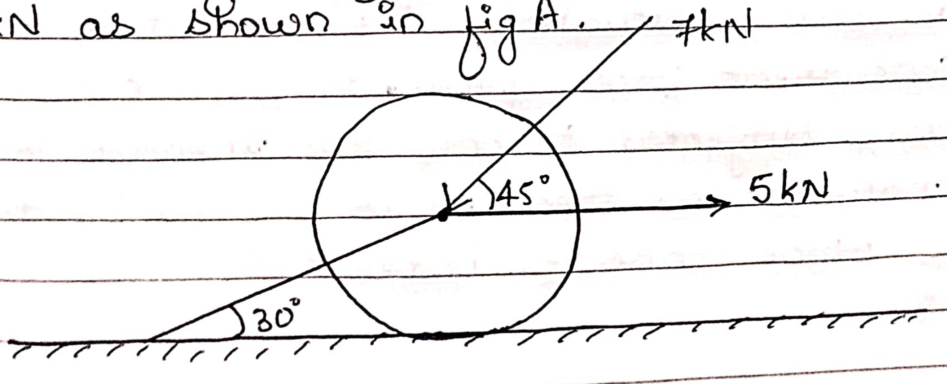


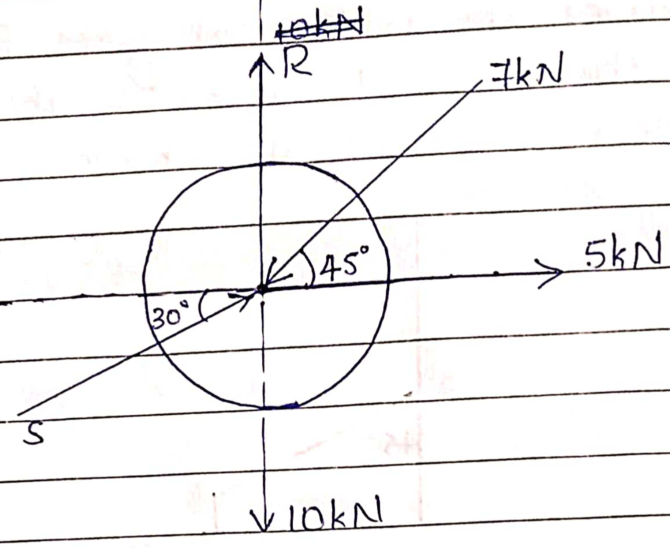

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{150}{\sin 75^\circ}$$

$T_1 = 77.64\,N$

$T_2 = 109.81\,N$

4. A roller weighing 10kN rests on a smooth horizontal floor and is connected to the floor by the bar AC as shown in fig A. Determine the force in the bar AC and reaction from the floor, if the roller is subjected to a horizontal force of 5kN and an inclined force 7kN as shown in fig A.






$\sum F_x = 0$

$S \cos 30^\circ + 5 - 7 \cos 45^\circ = 0$

$S = -0.058 \text{ kN}$

Since, the value is negative the reaction from the bar is not push, but it is pull (tensile force in the bar) of magnitude 0.058 kN.

$\sum F_y = 0$

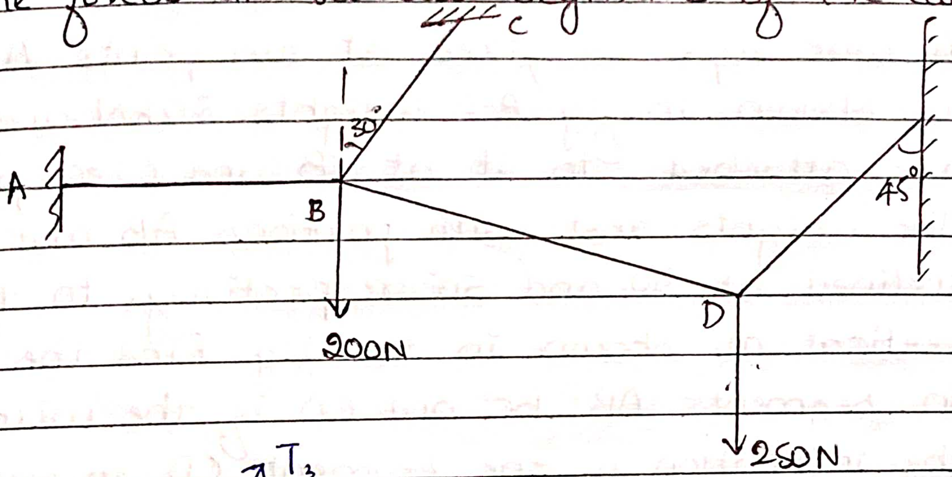
$R - 10 - 7 \sin 45^\circ + S \sin 30^\circ = 0$

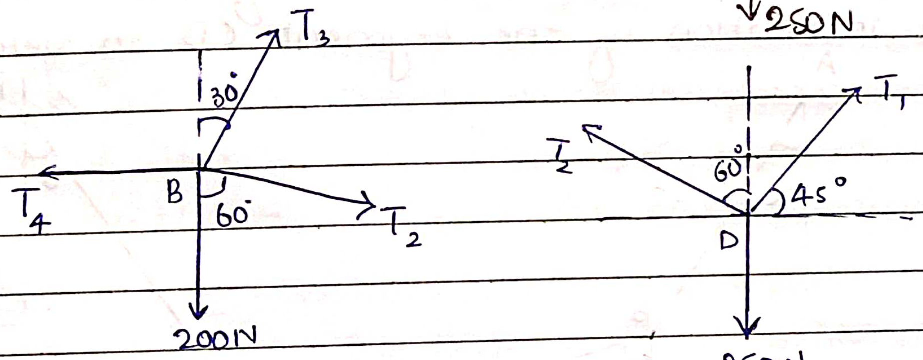
$R = 14.98 \text{ kN}$

## Equilibrium of connected bodies

When two or more bodies are in contact with one another, the system of forces appear as though it is a non-concurrent force system. However, when each body is considered seperately in many situations it turns out to be a set of concurrent force system. In such cases, first, the body subjected to only two unkoown forces is analysed and then it is followed by the analysis of other connected bodies.

1. A system of connected flexible cables shown in fig (a) is supporting two vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable.




(c) (b)

In fig B, applying Lami's theorem,

$$\frac{T_1}{\sin(30+90)} = \frac{T_2}{\sin(45+90)} = \frac{250}{\sin(60+45)}$$

$$T_1 = 224.14 N$$
$$T_2 = 183.01 N$$

In fig C, applying

$\Sigma F_y = 0$

$\cancel{T_3 \sin 30°}$

$-200 + T_3 \cos 30° - T_2 \cos 60° = 0$

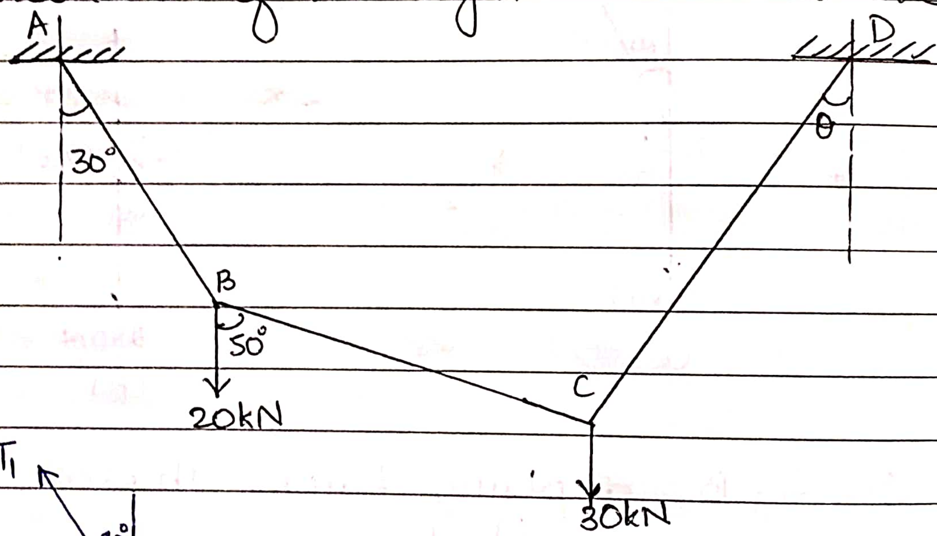
$T_3 = 336.6 N$

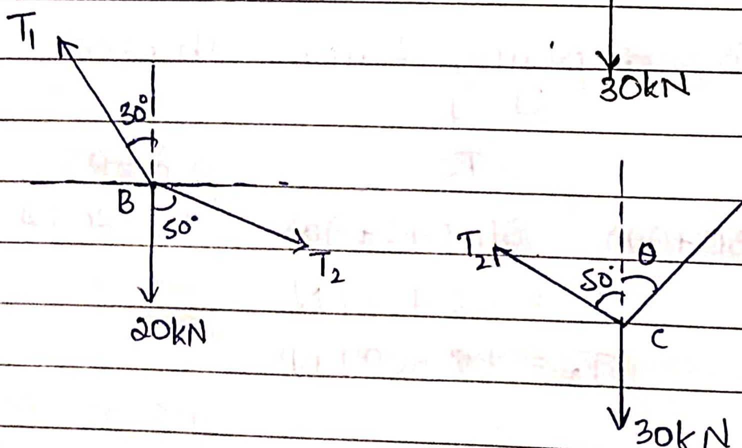
$\Sigma F_x = 0$

$-T_4 + T_2 \sin 60^\circ + T_3 \sin 30^\circ = 0$

$T_4 = 326.79\,N$

2. A wire rope is fixed at two points A and D as shown in fig A. Weights 20 kN and 30 kN are attached to it at B and C respectively. The weights rest with portions AB and BC inclined at 30° and 50° respectively, to the vertical as shown in the fig. Find the tension in segments AB, BC and CD of the wire. Determine the inclination of the segment CD to vertical.




At B,

$$\frac{20}{\sin(160)} = \frac{T_1}{\sin 50^\circ} = \frac{T_2}{\sin 150^\circ}$$

$T_1 = 44.8\,kN$

$T_2 = 29.24\,kN$

At C,

$\Sigma F_x = 0$

$T_3 \sin\theta - T_2 \sin 50^\circ - \cancel{T_1 \sin 30^\circ} = 0$

$T_3 \sin\theta = \cancel{44.8\,kN} = 22.4\,kN \rightarrow (1)$

$\Sigma F_y = 0$

$T_3 \cos\theta - 30 + T_2 \cos 50^\circ = 0$

$T_3 \cos\theta = 11.2\,kN \rightarrow (2)$

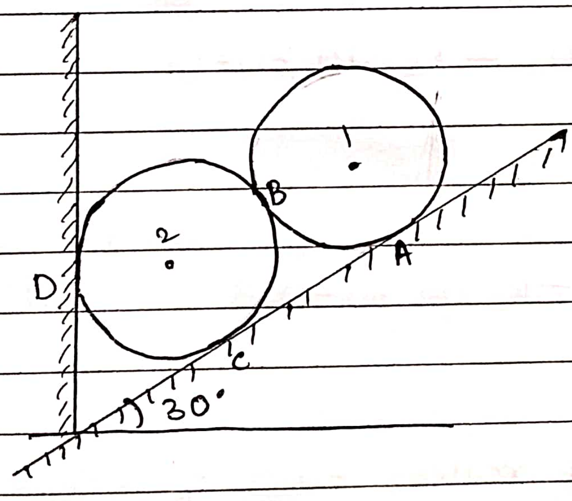
$$\frac{(1)}{(2)} = \frac{T_3 \sin\theta}{T_3 \cos\theta} = \tan\theta = 2$$

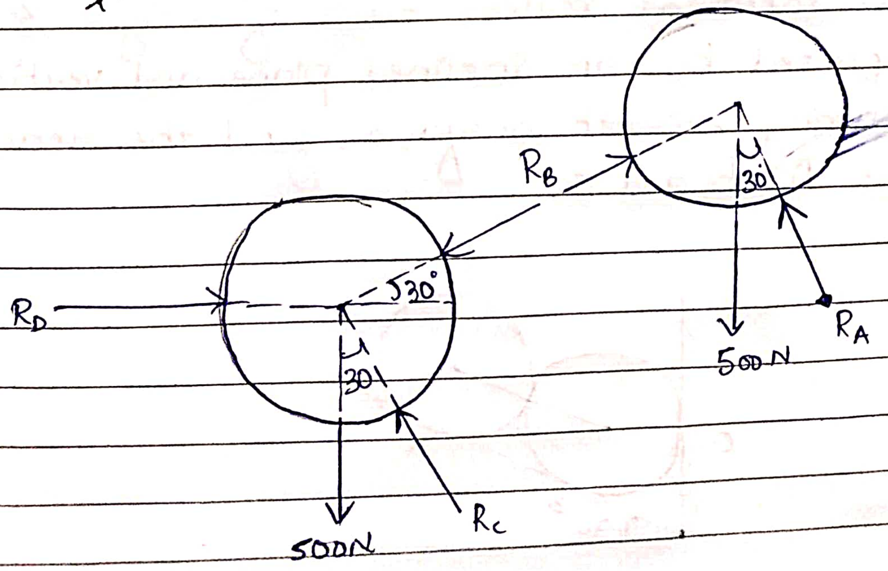
$\theta = 63.43^\circ$

$T_3 \cos\theta = 11.2$

$T_3 = 25.04\,kN$

3. Two identical rollers each weighing 500N are placed inclined as shown in fig A. Determine the reactions developed by each contacted points A, B, C and D.



Free body diagrams of two cylinders are as shown in fig. Consider the equilibrium of cylinder 1, since $R_A$ is at right angles to the plane, it makes 60° to the horizontal i.e, 30° to vertical. $R_B$ is parallel to plane since the cylinders are identical. Thus $R_A$ and $R_B$ are at right angles to each other.

$\Sigma$ Forces normal to plane $=0$

$R_A - 500 \cos 30° = 0$

$R_A = 433 N$

$\Sigma$ Forces parallel to plane $=0$

$R_B - 500 \sin 30° = 0$

$R_B = 250 N$.

$\Sigma F_y = 0$

$-500 + R_C \cos 30° - R_B \sin 30° = 0$

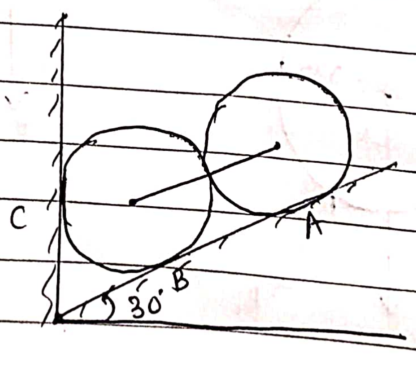
$R_C = 721.7 N$

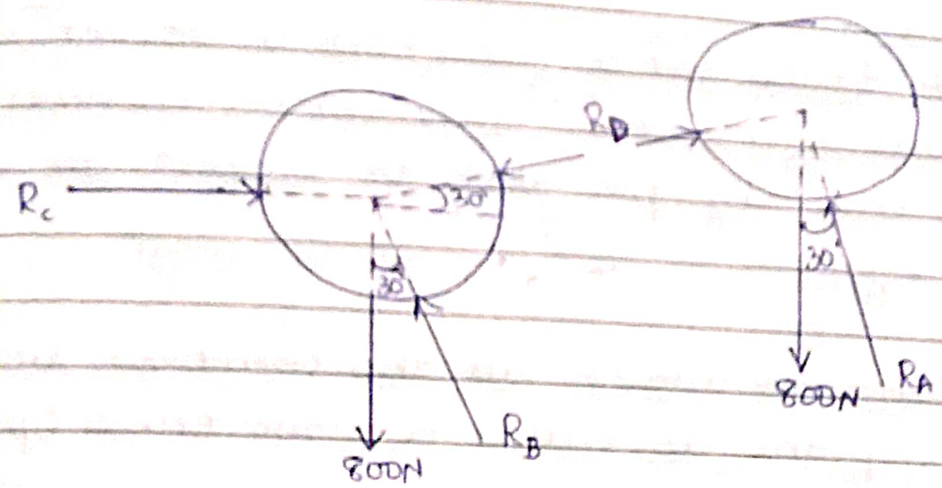
$\Sigma F_x = 0$

$R_D - R_C \sin 30° - R_B \cos 30° = 0$

$R_D = 577.4 N$

4. Two identical rollers each of weight 800N are supported by an inclined plane and vertical wall. Assuming smooth surfaces, find the reactions at point A, B and C.






$R_A - 800\cos 30^\circ = 0$

$R_A = 692.82N$

$R_D - 800 \sin 30^\circ = 0$

$R_D = 400N$

$\Sigma F_y = 0$

$-800 + R_B \cos 30^\circ - R_D \sin 30^\circ = 0$

$R_B = 1154.7N$

$\Sigma F_x = 0$

$R_c - R_B \sin 30^\circ - R_D \cos 30^\circ = 0$

$R_c = 923.76N$