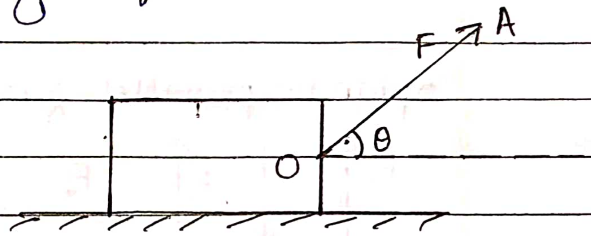
UNIT - 1

Force: An external agency which changes or tends to change the state of rest or of uniform motion of a body upon which it acts is known as force.

Force is a vector quantity since it has both magnitude and direction.

Force is represented by four quantities

1. Magnitude 'OA'
2. Direction '$\overrightarrow{OA}$'
3. Point of application 'O'
4. Angle of inclination 'θ'



One Newton Force: It is a force required to produce an acceleration of $1ms^{-2}$ in a body of mass 1kg

$F = ma$ ∴ Force = Mass × Acceleration

$$1N = 1kg \times 1ms^{-2}$$

$$N = kg\,ms^{-2}$$

Weight 'W' of a body of mass 'm' is a force and should be measured in Newtons.

$W = mg$ ∴ Weight of body = Mass × Acceleration due to gravity.

$$1\,kg\text{-}wt = 1kg \times 9.81ms^{-2}$$

$$1\,kg\text{-}wt = 9.81\,kg\,ms^{-2}$$

$$1\,kg\text{-}wt = 9.81N$$

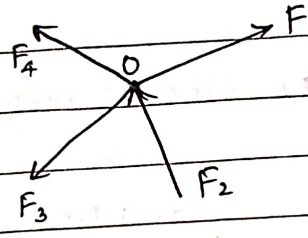
One kg-wt is the force required to move a mass of one kg with an acceleration equal to gravitational acceleration. (i.e., $g = 9.81ms^{-2}$)

Unit of force: N, kN, MN, GN

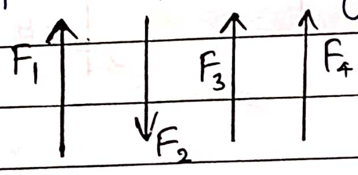
Classification of force system

1. Coplanar forces: Forces acting in same plane.

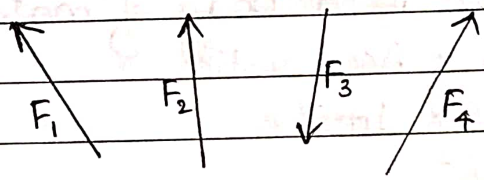
Coplanar concurrent forces: Forces are acting in same plane and meeting at a point.



Coplanar parallel forces: Forces are acting in the same plane and are parallel to each other.

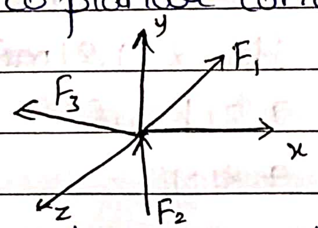


Coplanar non concurrent forces: Forces are acting in the same plane and forces neither meet at a point non parallel to each other

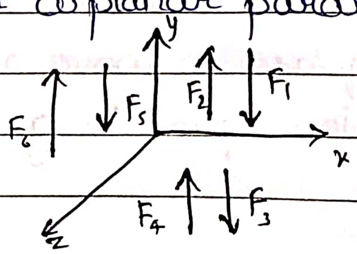


2. Non coplanar forces: Forces are acting in different planes.

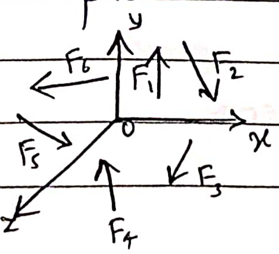
Non coplanar concurrent forces: Forces are acting in different planes and meeting at a point.



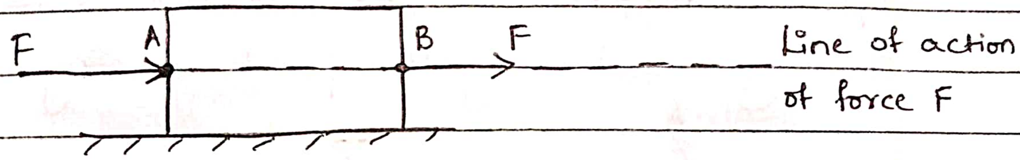
Non coplanar parallel forces: Forces are acting in different planes and are parallel to each other.



Non coplanar Non concurrent forces: Forces are acting in different planes and forces neither meet at a point nor parallel to each other.

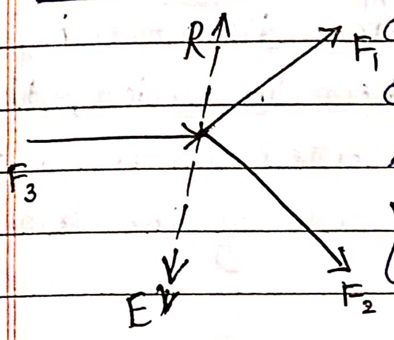


Principle of transmissibility of force :



It states that, "The point of application of a force may be transmitted (or replaced) to any point along its line of action without changing the conditions of equilibrium." [The effect is same].

Resultant (R): If number of forces are acting on a particle, it is possible to find out a single force which will have the same effect as produced by all the forces. This single force is called the resultant force or Resultant (R).

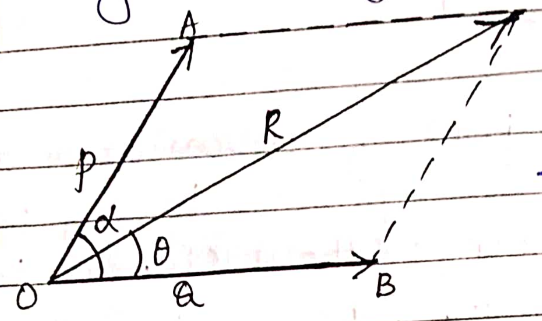


Equilibrant (E): The equilibrant of a system of forces is one which will balance the given system of forces. It is equal in magnitude and opposite in direction to that of the resultant.

Composition of forces: The process of finding the single force (i.e., the resultant force) which would produce the same effect as that of the given system of forces is called the composition of forces.

1. Law of parallelogram of forces
2. Law of triangle of forces.
3. Law of polygon of forces.

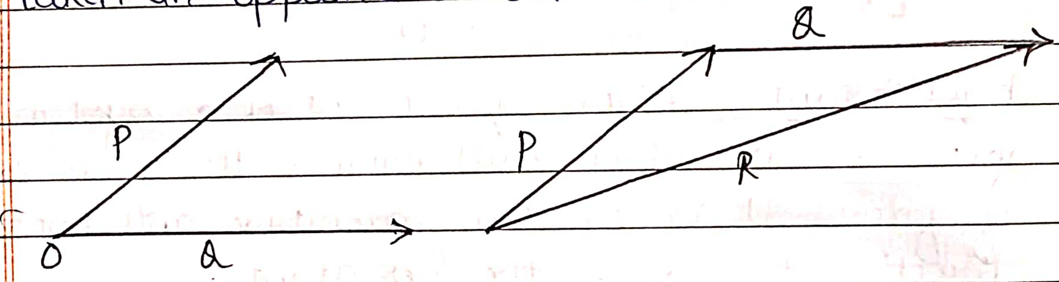
Law of parallelogram of forces: It states that "If two forces, which act at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from one of its angular points, their Resultant (R) is represented by the diagonal of the parallelogram passing through that angular point in magnitude & direction



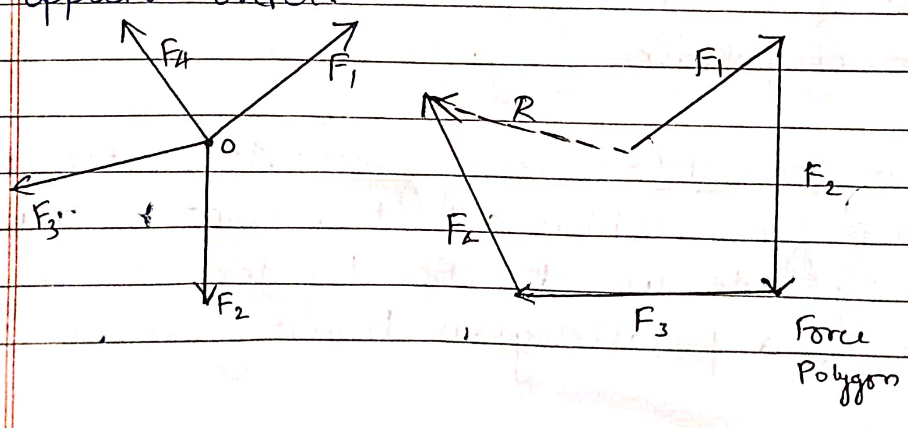
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\theta = \frac{P\sin\alpha}{Q + P\cos\alpha}$$

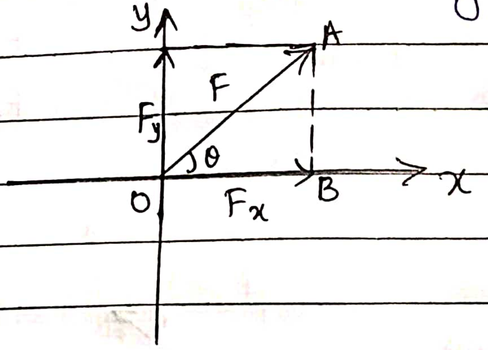
Law of triangle of forces: It states that, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle taken in order, their resultant (R) may be represented in magnitude and direction by the third side of the triangle taken in opposite order."



Law of polygon of forces: It states that "If number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order, their resultant (R) may be represented in magnitude and direction by the closing side of the polygon taken in opposite order."



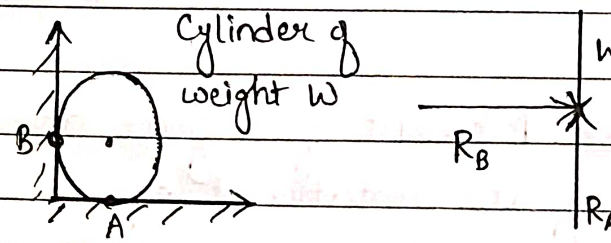
Resolution of a force: The splitting up of the force into number of its components without changing its effect on the body is called resolution of a force. A force is generally resolved along two mutually perpendicular directions (i.e., rectangular components).



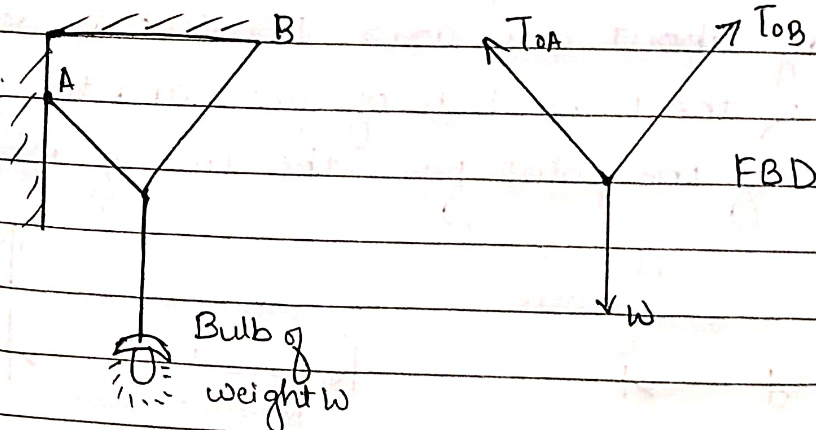
$$\frac{F_x}{F} = \cos\theta \Rightarrow F_x = F\cos\theta$$

$$\frac{F_y}{F} = \sin\theta \Rightarrow F_y = F\sin\theta$$

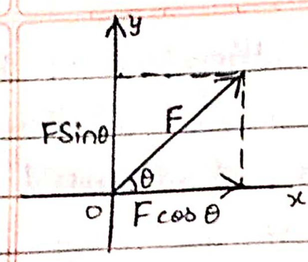
Free body diagram (FBD): A diagram of the body in which the body under consideration is isolated (or freed) from all the contact surfaces and all the forces acting on it (including reactions at the contact surfaces) are shown (or drawn) is called as free body diagram.



FBD of cylinder to find R_A and R_B

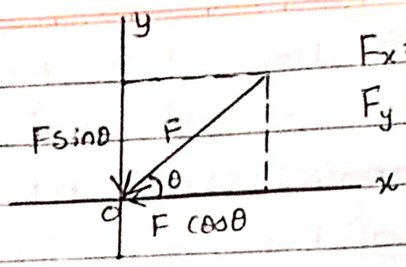


FBD to find T_{OA} and T_{OB}



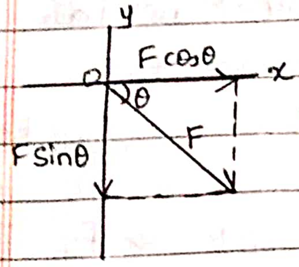
$F_x = F\cos\theta$

$F_y = F\sin\theta$



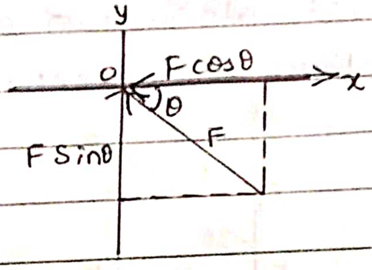
$F_x = -F\cos\theta$

$F_y = -F\sin\theta$



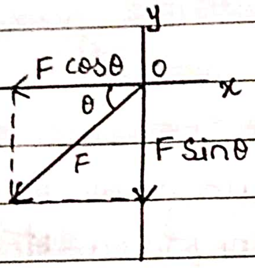
$F_x = F\cos\theta$

$F_y = -F\sin\theta$



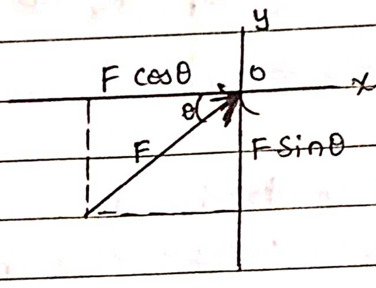
$F_x = -F\cos\theta$

$F_y = F\sin\theta$



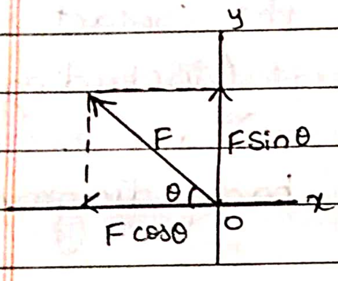
$F_x = -F\cos\theta$

$F_y = -F\sin\theta$



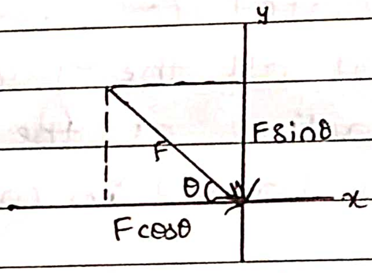
$F_x = F\cos\theta$

$F_y = F\sin\theta$



$F_x = -F\cos\theta$

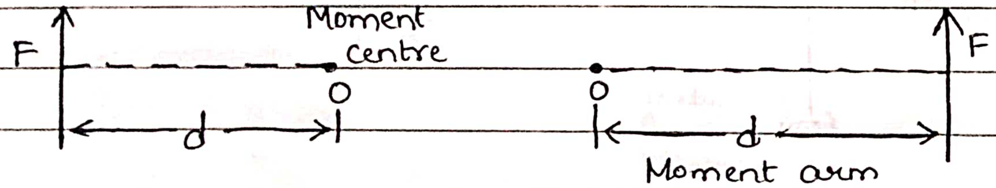
$F_y = F\sin\theta$



$F_x = F\cos\theta$

$F_y = -F\sin\theta$

Moment of a force: Moment of a force about a point is the measure of its rotational effect. The moment of a force is the capacity of a force to rotate the rigid body about any axis. Moment is defined as the product of the magnitude of force and the perpendicular distance of the point from the line of action of force

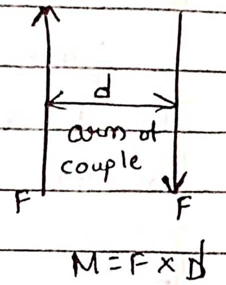


Clockwise moment ↻ +ve   Anticlockwise moment ↺ −ve

$M = F \times d$ Units: N-mm, N-m, kN-mm, kN-m.

Couple: The two parallel forces, equal in magnitude and opposite in direction and seperated by a definite distance are said to form a couple.

Characteristics of couple:



1. The algebraic sum of components of the two forces is zero, i.e., the resultant of couple of is Zero.

2. The moment of a couple is constant and is equal to the product of one of the forces and perpendicular distance between them ($M = F \times d$)

3. The couple can be balanced by equal and opposite couples only.

4. Two or more couples can be reduced to a single couple of moment equal to the algebraic sum of the given couples.

5. The moment of a couple is constant, irrespective of the point such as A, but same for all points in the plane of the couple.

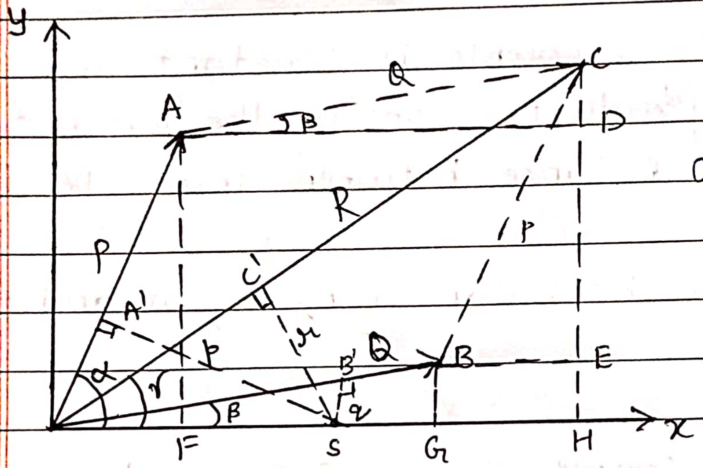
Equivalent force-couple system:

A force acting at any point A on a rigid body may be replaced by another force of the same magnitude and direction at any other point B together with a couple whose moment is equal to the moment of F about the point B. [i.e., $M = F \times d$]

Varignon's Theorem (or Principle of moments.)

The moment of a force (i.e., Resultant) at any point is equal to the algebraic sum of moments of its components about that point.

Proof:



R is the resultant of P and Q. Let α, β and γ be the inclination of P, Q and R wrt x-axis.

Let S be the point on x-axis. Draw SA', SB' and SC' perpendicular to the lines of action of forces P, Q and R from S. Draw AF, BG and CH perpendicular to x-axis and draw AD and BE parallel to x-axis.

From the figure, $CH = DH + CD$

$$R \sin \gamma = P \sin \alpha + Q \sin \beta$$

Multiplying the above equation by the distance OS

$$R(OS) \sin \gamma = P(OS) \sin \alpha + Q(OS) \sin \beta$$

$$R(SC') = P(SA') + Q(SB')$$

[But SC' is the moment arm of R = r (say)
SA' is the moment arm of P = p (say)
SB' is the moment arm of Q = q (say)]

$$Rr = Pp = Qq$$

$\therefore$ Moment of R about S = Algebraic sum of the moments of P and Q about S.

Therefore, Moment of the Resultant force about a point is equal to the algebraic sum of the moments of its components about that point.

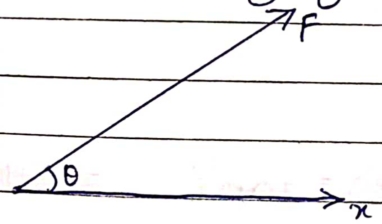
Sign conventions to be used while solving problems

1. Vertical forces → upward forces positive ↑ +ve
   → downward forces negative ↓ −ve

2. Horizontal forces → forward forces positive ⟶ +ve
   → Reverse forces negative −ve ⟵

3. Moments → clockwise moments positive (+ve)
   → Anticlockwise moments negative (−ve)

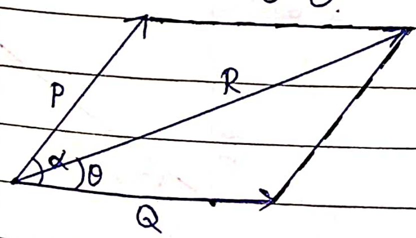
Resolution of forces



F is a force inclined at an angle θ

Horizontal component, $F_x = F\cos\theta$
Vertical component, $F_y = F\sin\theta$

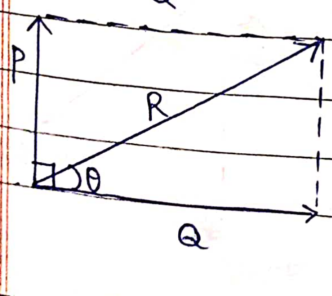
Composition of forces



Resultant, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\tan\theta = \dfrac{P\sin\alpha}{Q + P\cos\alpha}$

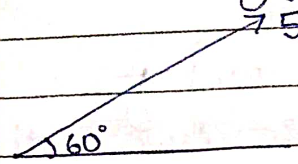
If $\alpha = 90°$



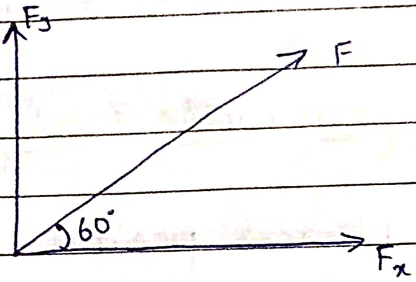
Resultant, $R = \sqrt{P^2 + Q^2}$

$\tan\theta = \dfrac{P}{Q}$

1. Determine the resultant of the forces 50kN as shown in figure. Find the magnitude of force also.



Solution



Horizontal component

$\theta = 60°$

$F_x = F\cos\theta$

$= 50 \times \frac{1}{2} \times 10^3$

$= 25\,kN$

Vertical component $F_y = F\sin 60$

$= 50 \times \frac{\sqrt{3}}{2} \times 10^3$

$= 43.3\,kN$

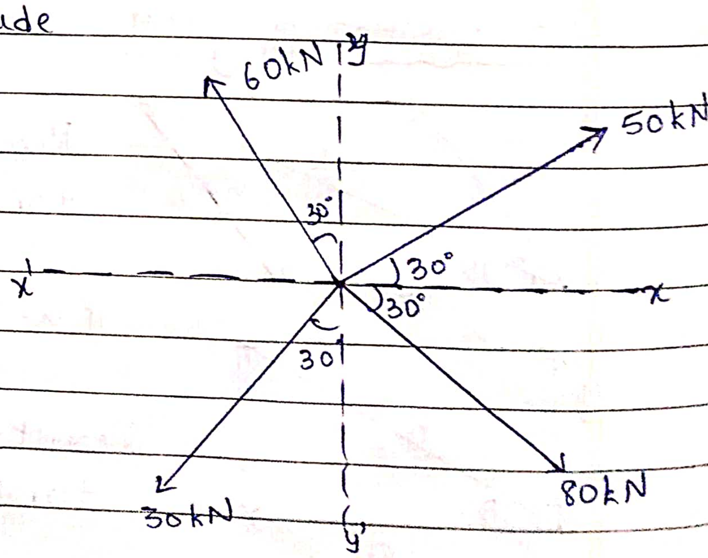
Resultant, $F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(25)^2 + (43.3)^2} \times 10^3$

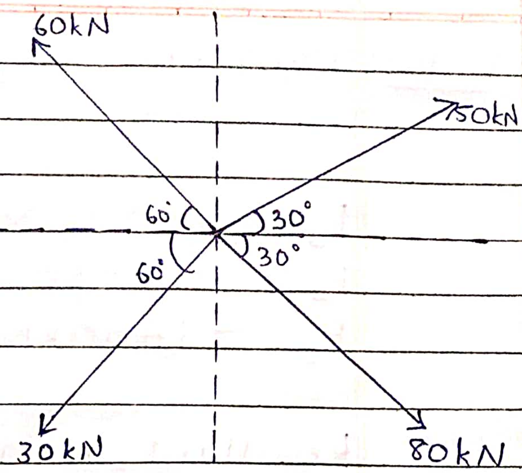
$= 49.99\,kN$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{43.3}{25}\right) = \tan^{-1}(\ ) = 59.99°$

2. Determine magnitude and direction of resultant of the concurrent non coplanar forces shown in figure.



Solution Resolving forces horizontally



2.

$F_x = 50\cos 30^\circ + 80\cos 30^\circ - 30\cos 60^\circ - 60\cos 60^\circ$

$F_x = 50(\sqrt{3}/2) + 80(\sqrt{3}/2) - 30(1/2) - 60(1/2)$

$F_x = 43.3 + 69.28 - 15 - 30$

$F_x = 67.58\ kN$

$F_x = F\cos\theta \qquad F_y = F\sin\theta$

Resolving forces vertically.

$F_y = 50\sin 30^\circ - 80\sin 30^\circ - 30\sin 60^\circ + 60\sin 60^\circ$

$F_y = 50(1/2) - 80(1/2) - 30(\sqrt{3}/2) + 60(\sqrt{3}/2)$

$F_y = 25 - 40 - 25.98 + 51.96$

$F_y = 10.98\ kN.$

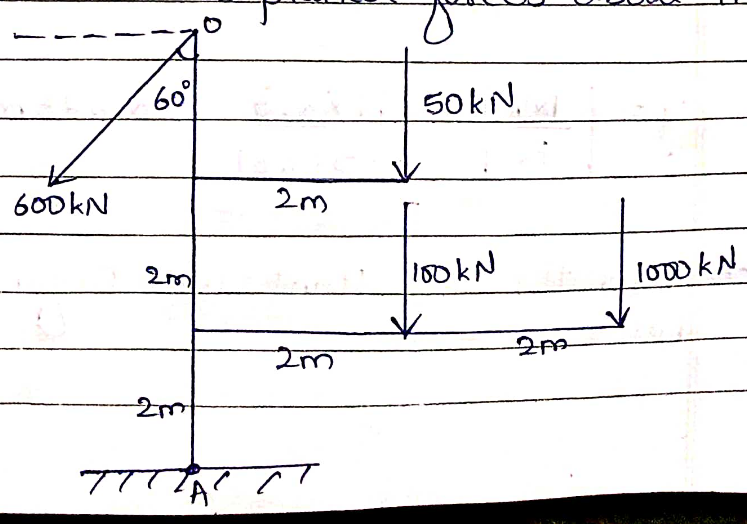
Resultant, $R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(67.58)^2 + (10.98)^2} \times 10^3$

$= 68.46\ kN$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{10.98}{67.58}\right) = \tan^{-1}(0.162) = 9.2^\circ$

3. Determine the magnitude and resultant of following non concurrent non co planar forces about the point A.



Solution: Horizontal component, $F_x = 600 \cos 30^\circ$

$F_x = -519.61 \text{ kN}$

$F_y = -600 \sin 30^\circ - 50 - 100 - 1000$

$F_y = -300 - 50 - 100 - 1000$

$F_y = -1450 \text{ kN}$

Resultant, $R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(519.61)^2 + (-1450)^2} \times 10^3$

$= 1540.3 \text{ kN}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-1450}{-519.61}\right) = \tan^{-1}(2.7905) = 70.28^\circ$

Taking moments of all forces about A.

$M_A = -600 \cos 30^\circ \times 6 + 600 \sin 30^\circ \times 0 + 50 \times 2 + 100 \times 2$
$+ 1000 \times 4$

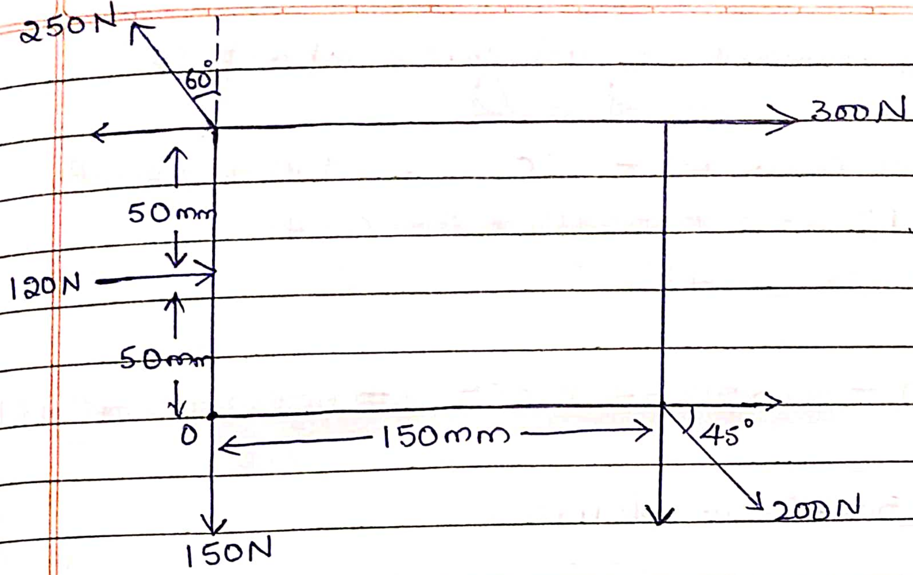
$M_A = 1182.31 \text{ kN-m}$

$r = \frac{M_A}{R} = \frac{1182.3}{1540.3} = 0.7676 \text{m}$

$x = \left|\frac{M_A}{F_y}\right| = \frac{1182.3}{1450} = 0.815 \text{m}$

$y = \left|\frac{M_A}{F_x}\right| = \frac{1182.3}{519.61} = 2.275 \text{m}$

4. Find the resultant wrt O for the figure shown below.



Solution Horizontal component of forces at 'O'

$$F_x = -250\sin 60^\circ + 300 + 120 + 200\cos 45^\circ$$
$$= -216.506 + 300 + 120 + 141.421$$
$$= 344.92\,N$$

Vertical component of forces at 'O'

$$F_y = 250\cos 60^\circ - 150 - 200\sin 45^\circ$$
$$= 125 - 150 - 141.421$$
$$= -166.42\,N$$

Resultant, $R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(344.92)^2 + (-166.42)^2}$$

$$= \sqrt{146665.41}$$

$$= 382.96\,N$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-166.42}{344.92}\right) = \tan^{-1}(-0.482) = -25.75^\circ$$

Taking moments of all forces about 'O'

$$M_o = 250\cos 60^\circ \times 0 - 250\sin 60^\circ \times 100 + 300\times 100 + 120\times 50 + 150\times 0 + 200\cos 45^\circ \times 0 + 200\sin 45^\circ \times 150$$

$$M_o = 0 - 21650.64 + 30000 + 6000 + 0 + 0 + 21213.2$$

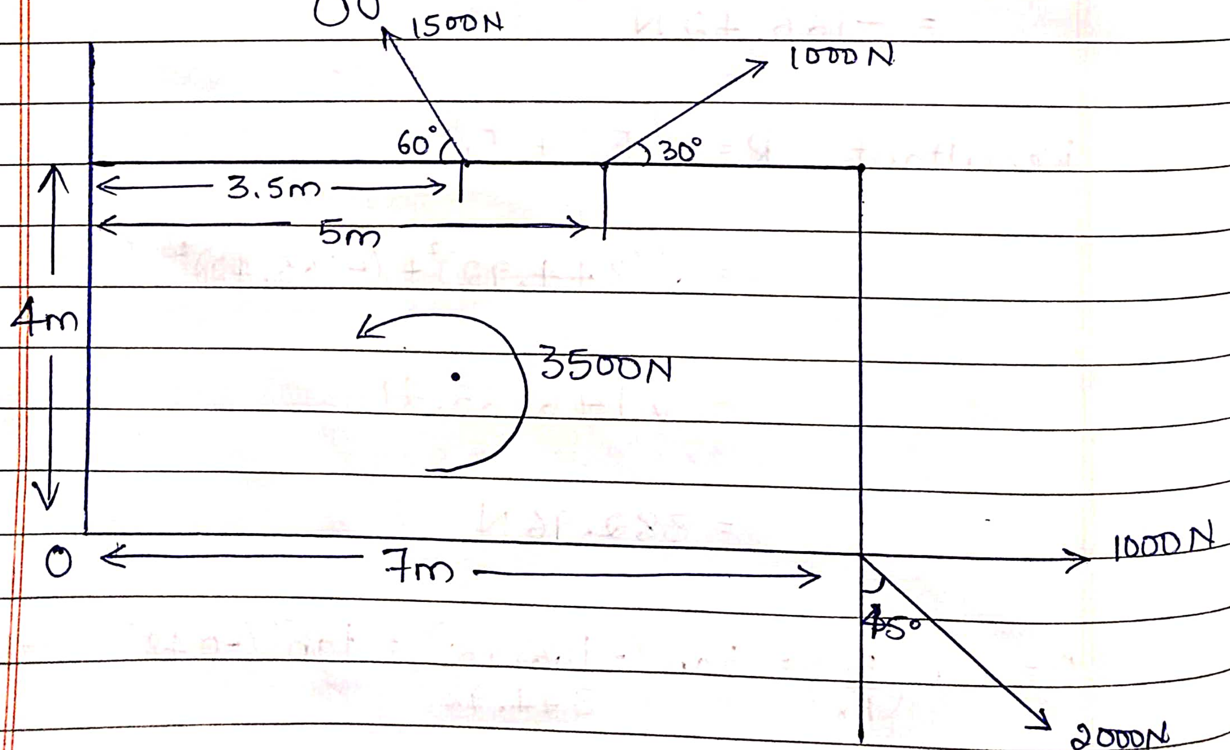
$$M_o = 35562.56 \text{ N-mm}$$

$$r = \frac{M_o}{R} = \frac{35562.56}{382.96} = 92.86 \text{ mm}$$

$$x = \frac{M_o}{F_y} = \frac{35562.56}{+166.42} = 213.69 \text{ mm}$$

$$y = \frac{M_o}{F_x} = \frac{35562.56}{344.92} = 103.1 \text{ mm}$$

5. Determine the resultant of coplanar forces shown in fig. 1 wrt point 'O'.



chandra's

Solution: Horizontal component

$F_x = -1500\cos 60^\circ + 1000\cos 30^\circ + 1000 + 2000\sin 45^\circ$
$= -750 + 866.02 + 1000 + 1414.21$
$= 2530.22\,N$

Vertical component

$F_y = 1500\sin 60^\circ + 1000\sin 30^\circ - 2000\sin 45^\circ$
$= 1299.03 + 500 - 1414.21$
$= 384.82\,N$

Resultant, $R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(2530.22)^2 + (384.82)^2}$

$= 2559.31\,N$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{384.82}{2530.22}\right) = 8.65^\circ$

Taking moments of all forces about O

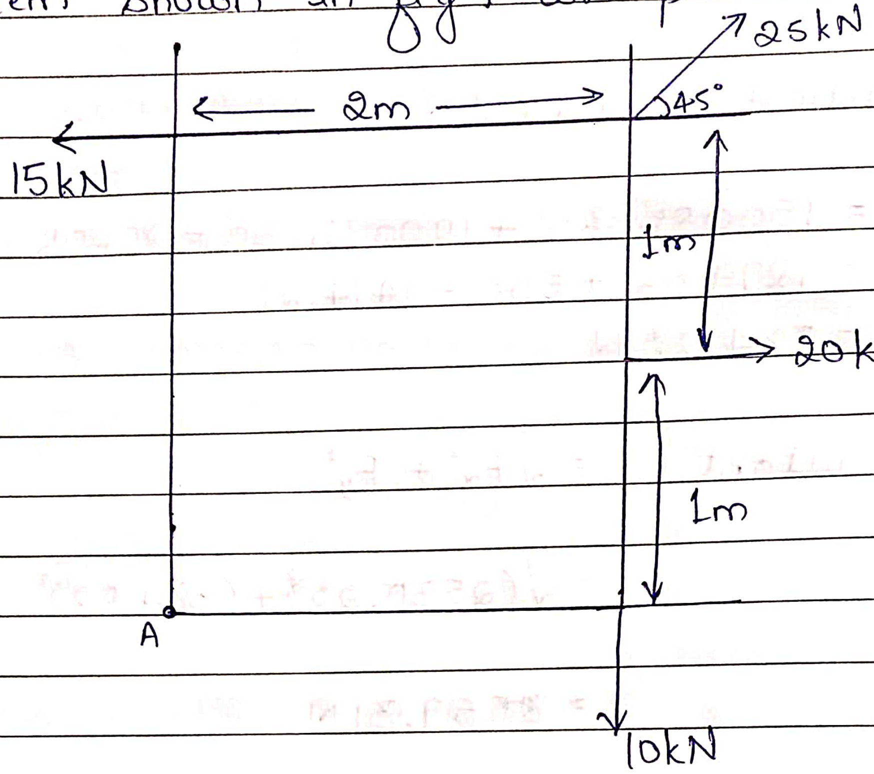
$M_o = -1500\cos 60^\circ(4) - 1500\sin 60^\circ(3.5)$
$+ 1000\cos 30^\circ(4) - 1000\sin 30^\circ(5)$
$+ 2000\cos 45^\circ(7) - 3500$
$= -3000 - 4546.63 + 3464.1 - 2500 + 9899.49$
$- 3500$
$= -183.04\,N\text{-}m$

$r = \left|\frac{M_o}{R}\right| = \left|\frac{-183.04}{2559.31}\right| = 0.0715\,m$

$x = \left|\frac{M_o}{F_y}\right| = \left|\frac{-183.04}{384.82}\right| = 0.4756\,m$

$$y = \left| \frac{M_A}{F_x} \right| = \left| \frac{-183.04}{2530.22} \right| = 0.0723 \text{ m}$$

6. Determine the resultant of coplanar force system shown in fig 1 wrt point A.



Solution Horizontal component

$$F_x = -15 + 20 + 25 \cos 45^\circ$$
$$= -15 + 20 + 17.67$$
$$= 22.67 \text{ kN}$$

$$F_y = 25 \sin 45^\circ - 10$$
$$= 17.68 - 10$$
$$= 7.68 \text{ kN}$$

$$R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{(22.67)^2 + (7.68)^2}$$
$$= 23.94 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$= \tan^{-1}\left(\frac{7.68}{22.67}\right)$$
$$= 18.72^\circ$$

Taking moments of all forces about A

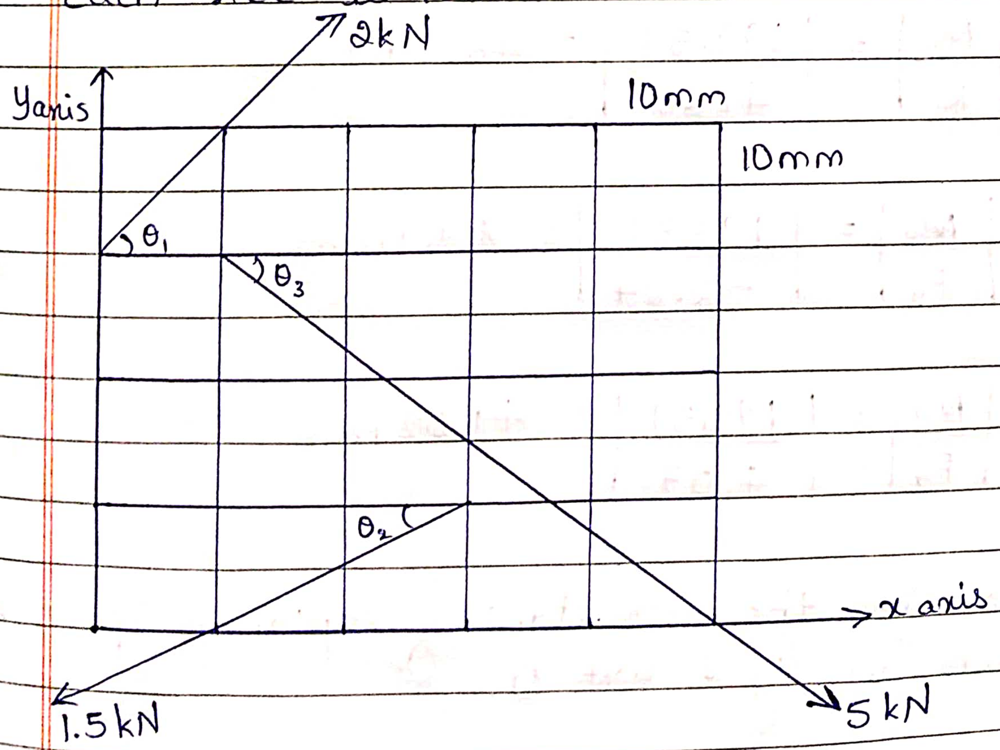
$M_A = -15(2) + 25\cos 45^\circ(2) - 25\sin 45^\circ(2)$
$+20(1) + 10(2)$
$= -30 + 35.36 - 35.36 + 20 + 20$
$= 10\ kN\text{-}m$

$r = \left|\frac{M_A}{R}\right| = \frac{10}{23.94} = 0.418m$

$x = \frac{M_A}{F_y} = \frac{10}{7.68} = 1.3m$

$y = \frac{M_A}{F_x} = \frac{10}{22.67} = 0.44m$

7. Find the resultant of the system of coplanar forces acting on a Lamina wrt O as shown. Each side is 10mm.



Solution: $\theta_1 = \tan^{-1}\left(\frac{10}{10}\right) = 45^\circ$ $\theta_2 = \tan^{-1}\left(\frac{10}{20}\right) = 26.56^\circ$ $\theta_3 = \tan^{-1}\left(\frac{30}{40}\right) = 36.86^\circ$

$F_x = 2\cos 45^\circ + 5\cos(36.86^\circ) - 1.5\cos(26.56^\circ)$
$= 1.414 + 3.99 - 1.34$
$= 4.07\,kN$

$F_y = 2\sin 45^\circ - 5\sin(36.86^\circ) - 1.5\sin(26.56^\circ)$
$= 1.414 - 3 - 0.68$
$= -2.27\,kN$

$R = \sqrt{F_x^2 + F_y^2}$ $\qquad$ $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \sqrt{(4.07)^2 + (-2.27)^2}$ $\qquad$ $= \tan^{-1}\left(\frac{-2.27}{4.07}\right)$

$= 4.66\,kN$ $\qquad$ $= 29.15^\circ$

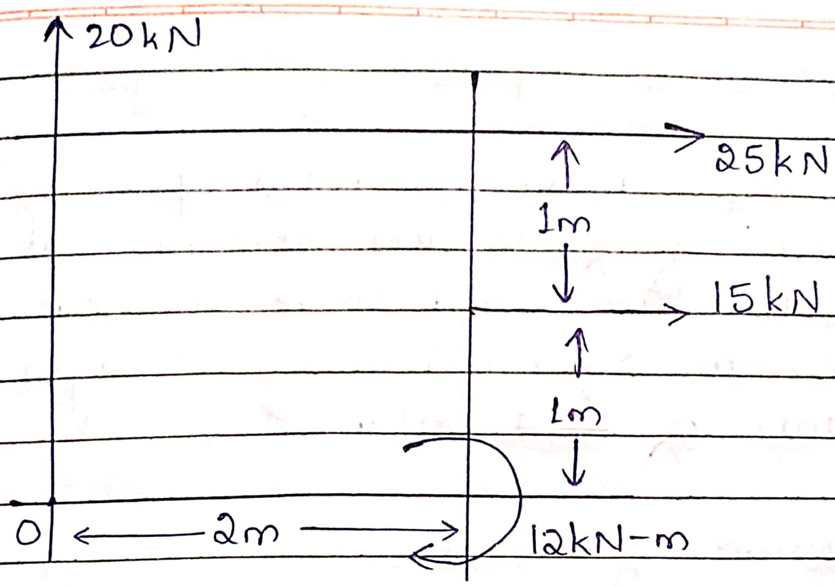
$M_o = 2\cos 45^\circ(30) + 5\cos 36.86^\circ(37.67) - 1.5\cos 26.56(10)$
$+ 1.5(\sin 26.56)(30)$ $\qquad$ $(5\cos 37.67(30) + 5\sin 37.67(10))$
$= 42.42 + 150.42 - 13.42 + 20.12$
$= 199.51\,kN\text{-}mm$

$r = \left|\frac{M_o}{R}\right| = \left|\frac{199.51}{4.66}\right| = 42.81\,mm$

$x = \left|\frac{M_o}{F_y}\right| = \left|\frac{199.51}{-2.27}\right| = 87.9\,mm$

$y = \left|\frac{M_o}{F_x}\right| = \left|\frac{199.51}{4.07}\right| = 49.02\,mm$

8. Determine the resultant of coplanar force system shown in fig 1 wrt O.



dution. $F_x = 25 + 15$

$= 40 kN$

$F_y = 20 kN$

$R = \sqrt{F_x^2 + F_y^2}$ $\qquad$ $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \sqrt{(40)^2 + (20)^2}$ $\qquad$ $= \tan^{-1}\left(\frac{20}{40}\right)$

$= 44.72 kN$ $\qquad$ $= 26.56^\circ$

$M_o = 25(2) + 15(1) + 12$

$= 50 + 15 + 12$

$= 77 kN\text{-}m$

$r = \frac{M_o}{R} = \frac{77}{44.72} = 1.72 m$

$x = \frac{M_o}{F_y} = \frac{77}{20} = 3.85 m$

$y = \frac{M_o}{F_x} = \frac{77}{40} = 1.925 m$