CHAPTER 1: DC CIRCUITS

BASIC CONCEPTS AND DEFINITIONS

1. **CHARGE**: Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). Charge, positive or negative, is denoted by the letter q or Q.

   All matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. The charge 'e' on an electron is negative and equal in magnitude to 1.602x10-19 C, while a proton carries a positive charge of the same magnitude as the electron and the neutron has no charge. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

2. **CURRENT** : Current can be defined as the motion of charge through a conducting material, measured in Ampere (A). Electric current, is denoted by the letter i or I.

   The unit of current is the ampere abbreviated as (A) and corresponds to the quantity of total charge that passes through an arbitrary cross section of a conducting material per unit second.

$$I=\frac{Q}{T} \text{ (or) } Q=IT$$

   Where Q is the symbol of charge measured in Coulombs (C), I is the current in amperes (A) and t is the time in second (s).

   The current can also be defined as the rate of charge passing through a point in an electric circuit**.**

   Mathematically, $i=\frac{dq}{dt}$

   The charge transferred between time t_1 and t_2 is obtained as $q=\int_{t1}^{t2} i dt$

   A constant current (also known as a direct current or DC) is denoted by symbol I whereas a time-varying current (also known as alternating current or AC) is represented by the symbol i or i(t). Figure 1.1 shows direct current and alternating current.

   Current is always measured through a circuit element as shown in Fig. 1.1

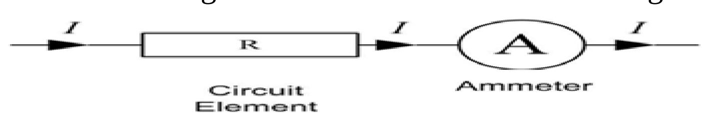


Fig. 1.1 Current through Resistor (R)

Two types of currents:

1) A direct current (DC) is a current that remains constant with time.
2) An alternating current (AC) is a current that varies with time.

Example 1

Determine the current in a circuit if a charge of 80 coulombs passes a given point in 20 seconds.

Solution:

$$I =\frac{Q}{T} = \frac{80}{20} = 4A$$

Example 2

How much charge is represented by 4,600 electrons?

Solution:

Each electron has - 1.602×10^{-19} C. Hence 4,600 electrons will have:

Q= -1.602x10-19x4600 = **-7.369×10^{-16} C**

Example 3

The total charge entering a terminal is given by =5tsin4πtmC . Calculate the current at =0.5sec

Solution:

$$i= \frac{dq}{dt} = \frac{d}{dt}(5tsin4\pi t)= (5\sin4\pi t +20\pi t\cos4\pi t) \text{ mA}$$

At =0.5sec

i= 31.42 mA.

Example 4

Determine the total charge entering a terminal between t_1 =1 and t_2 =2 if the current passing the terminal is = $(3t^2 - t)$.

Solution:

$$q= \int_{t=1}^{t=2}(3t2 - t)\,dt = (8\text{-}2)\text{-}(1\text{-}\tfrac{1}{2}) = 5.5C$$

3. **VOLTAGE (or) POTENTIAL DIFFERENCE**

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig.1.3. This emf is also known as voltage or potential difference. The voltage V_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b.

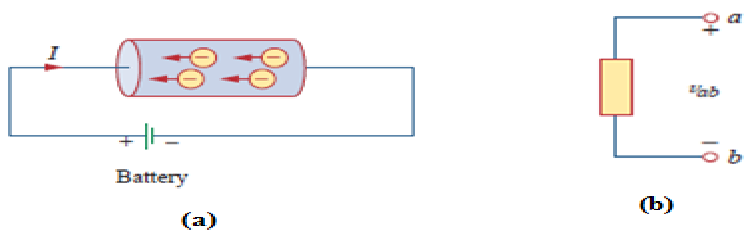


Fig. 1.3(a) Electric Current in a conductor, (b)Polarity of voltage Vab

Voltage (or potential difference) is the energy required to move charge from one point to the other, measured in volts (V). Voltage is denoted by the letter v or V.

Mathematically, $$\mathbf{V= \frac{dw}{dq}}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage ab or simply V is measured in volts (V).

1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

Fig. 1.3 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (-) signs are used to define reference direction or voltage polarity. The Vab can be interpreted in two ways: (1) point a is at a potential of Vab volts higher than point b, or (2) the potential at point a with respect to point b is Vab . It follows logically that in general $V_{ab} = -V_{ba}$

Voltage is always measured across a circuit element as shown in Fig. 1.4

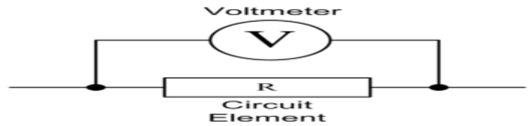


Fig. 1.4 Voltage across Resistor (R)

Example 5

An energy source forces a constant current of 2 A for 10s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

Total charge dq= i*dt = 2*10 = 20 C

The voltage drop is

V= $\frac{dw}{dq}$ = $\frac{2.3k}{20}$ = 115J

4. POWER

Power is the time rate of expending or absorbing energy, measured in watts (W). Power, is denoted by the letter p or P.

Mathematically, $$\mathbf{P = \frac{dw}{dt}}$$

Where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s).

From voltage and current equations, it follows that;

$$\mathbf{P = \frac{dw}{dt} = \frac{dw}{dq} * \frac{dq}{dt} = V*I}$$

Thus, I is the magnitude of current I and voltage are given, then power can be evaluated as the product of the two quantities and is measured in watts (W).

Sign of power:

- **Plus sign:** Power is absorbed by the element. (Resistor, Inductor)
- **Minus sign:** Power is supplied by the element. (Battery, Generator)

Passive sign convention:

- If the current enters through the positive polarity of the voltage, p = +vi
- If the current enters through the negative polarity of the voltage, p = – vi

5. ENERGY :

Energy is the capacity to do work, and is measured in joules (J). The energy absorbed or supplied by an element from time 0 to t is given by,

$$W= \int_0^t p\,dt = \int_0^t vi\,dt$$

The electric power utility companies measure energy in watt-hours (WH) or Kilo watt-hours (KWH) ; **1 WH = 3600 J**

Example 6
A source EMF of 5 V supplies a current of 3A for 10 minutes. How much energy is provided in this time?
Solution:

W=VIt = 5 × 3 × 10 × 60 = **9W**

Example 7
An electric heater consumes 1.8MJ when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.
Solution:

P=W /t = $(1.8\times10^6)/(30\times60)$ = 1000
Power rating of heater = 1kW
Thus
I=P /V =1000/250=4
Hence the current taken from the supply is 4A.

Example 8
Find the power delivered to an element at time t=ms if the current entering its positive terminals is i=5cos60πt A and the voltage is: (a)v =3i , (b)v =3didt.
Solution:

(a) The voltage is v=3i =15cos60 πt V ; hence, the power is: p=vi =75cos260πt W
At t=3ms ,

P=75cos260 πt $\times 3\times10^{-3}$ =53.48 W

(b) We find the voltage and the power as
V =3didt =3 −60π5sin60 πt = −900πsin60 πt V
P=VI =−4500πsin60πtcos60πt W
At t =3ms ,
P =−4500π sin0.18 π cos0.18 π = −6.396 W

OHM'S LAW

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

Ohm's law states that at constant temperature, the voltage (V) across a conducting material is directly proportional to the current (I) flowing through the material.

Mathematically,

$$V \propto I$$
$$\mathbf{V=RI}$$

Where the constant of proportionality R is called the resistance of the material. The V-I relation for resistor according to Ohm's law is depicted in Fig.1.6

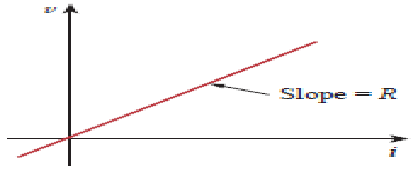


Fig. 1.6 V-I Characteristics for resistor

Limitations of Ohm's Law:

1. Ohm's law is not applicable to non-linear elements like diode, transistor etc.
2. Ohm's law is not applicable for non-metallic conductors like silicon carbide.

CIRCUIT ELEMENTS

An element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are 2 types of elements found in electrical circuits.

a) Active elements (Energy sources): The elements which are capable of generating or delivering the energy are called active elements.

E.g., Generators, Batteries

b) Passive element (Loads): The elements which are capable of receiving the energy are called passive elements.

E.g., Resistors, Capacitors and Inductors

➢ ACTIVE ELEMENTS (ENERGY SOURCES)

The energy sources which are having the capacity of generating the energy are called active elements. The most important active elements are voltage or current sources that generally deliver power/energy to the circuit connected to them.

There are two kinds of sources

- Independent sources
- Dependent sources

▪ INDEPENDENT SOURCES:

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

i. Ideal Independent Voltage Source:

An ideal independent voltage source is an active element that gives a constant voltage across its terminals irrespective of the current drawn through its terminals. In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. The symbol of idea independent voltage source and its V-I characteristics are shown in Fig. 1.7

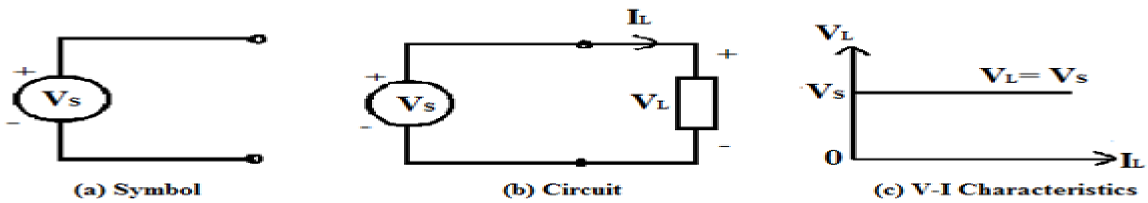


Fig. 1.7 Ideal Independent Voltage Source

ii. **<u>Practical Independent Voltage Source:</u>**

Practically, every voltage source has some series resistance across its terminals known as internal resistance, and is represented by Rse. For ideal voltage source $R_{se} = 0$. But in practical voltage source Rse is not zero but may have small value. Because of this Rse voltage across the terminals decreases with increase in current as shown in Fig. 1.8

Terminal voltage of practical voltage source is given by

$$V_L = V_S - IL\ R_{se}$$

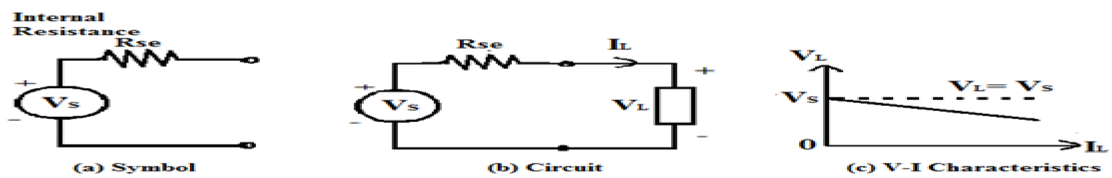


Fig. 1.8 Practical Independent Voltage Source

iii. **<u>Ideal Independent Current Source:</u>**

An ideal independent Current source is an active element that gives a constant current through its terminals irrespective of the voltage appearing across its terminals. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol of idea independent current source and its V-I characteristics are shown in Fig. 1.9

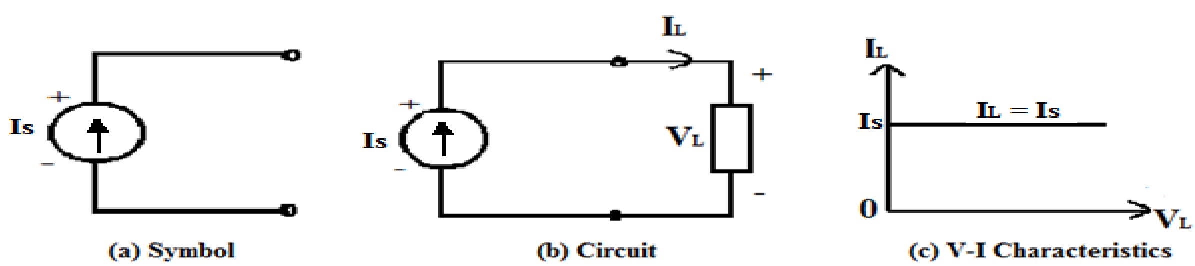


Fig. 1.9 Ideal Independent Current Source

iv. **<u>Practical Independent Current Source:</u>**

Practically, every current source has some parallel/shunt resistance across its terminals known as internal resistance, and is represented by Rsh. For ideal current source $R_{sh} = \infty$ (infinity). But in practical voltage source Rsh is not infinity but may have a large value. Because of this Rsh current through the terminals slightly decreases with increase in voltage across its terminals as shown in Fig. 1.10.

Terminal current of practical current source is given by $I_L = I_s - I_{sh}$

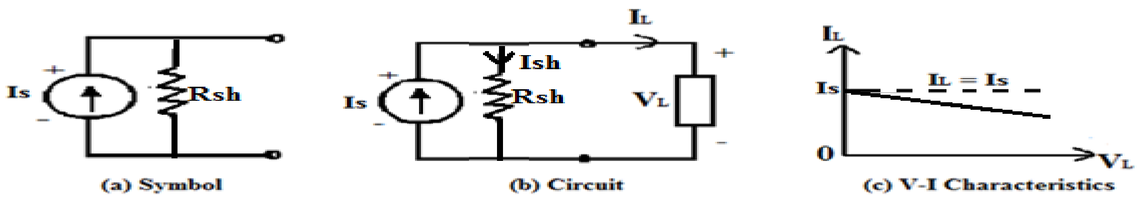


Fig. 1.10 Practical Independent Current Source

- **DEPENDENT (CONTROLLED) SOURCES**

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.11. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)

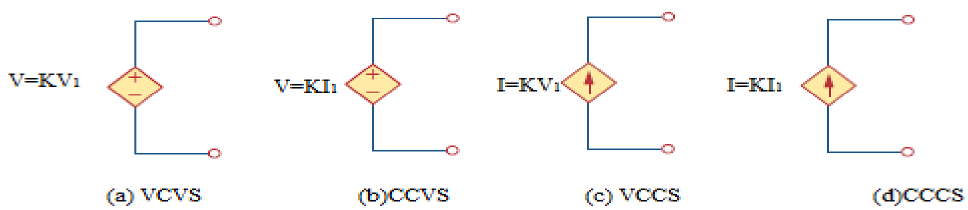


Fig. 1.11 Symbols for Dependent voltage source and Dependent current source

➢ **PASSIVE ELEMENTS (LOADS)**

Passive elements are those elements which are capable of receiving the energy. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, a passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors, Inductors fall in this category.

- **RESISTOR**

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist the flow of current, is known as resistance and is represented by the symbol R.The Resistance is measured in ohms (Ω). The circuit element used to model the current-resisting behavior of a material is called the resistor.

The resistance of a resistor depends on the material of which the conductor is made and geometrical shape of the conductor. The resistance of a conductor is proportional to the its length (l) and inversely proportional to its cross sectional area (A). Therefore the resistance of a conductor can be written as,

$$\mathbf{R} = \frac{\rho l}{A}$$

The proportionality ρ constant is called the specific resistance or resistivity of the conductor and its value depends on the material of which the conductor is made.

The inverse of the resistance is called the conductance and inverse of resistivity is called specific conductance or conductivity. The symbol used to represent the conductance is G and conductivity is $\sigma = 1/\rho$. Thus conductivity and its units are Siemens per meter(S/m or mho)

$$\mathbf{G} = \frac{1}{R} = \frac{A}{\rho l} = \frac{1}{\rho}\frac{A}{l} = \sigma \frac{A}{l}$$

By using Ohm's Law, The power dissipated in a resistor can be expressed in terms of R as below

$$\mathbf{P = VI = V^2/R = I^2R}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$P = VI = V^2G = I^2/G$$

The energy lost in the resistor from time 0 to t is expressed as

$$\mathbf{W} = \int_0^t P = \mathbf{I^2Rt = V^2t/R}$$

Where V is in volts, I is in amperes, R is in ohms, and energy W is in joules

Example 9

For the circuit with voltage 0f 30V and resistance of 5KΩ, calculate the current i, the conductance G, the power P and energy lost in the resistor W in 2hours.

Solution:

The current is, $I = V/R = 30/5K = 6mA$

The conductance is, $G = 1/R = 1/5K = 0.2$ milli simens

We can calculate the power in various ways

$$P = VI = 30*6*10^{-3} = 180mW$$

$$P = V^2/R = 30^2/5*10^3 = 900/5k = 180mW$$

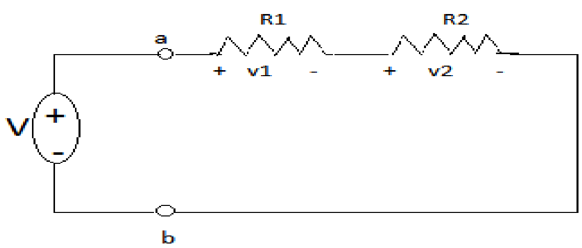
$$P = I^2R = (6*10^{-3})^2*(5*10^3) = (36*10^{-6})*(5*10^3) = 180mW$$

Energy lost in the resistor is $W = I^2Rt = (6*10^{-3})^2*(5*10^3)*(2) = (36*10^{-6})*(5*10^3)*(2) = 360mWhr =$ **360mJ**

❖ **RESISTIVE NETWORKS**

1) **SERIES RESISTORS AND VOLTAGE DIVISION**

Two or more resistors are said to be in series if the same current flows through all of them. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 1.18.



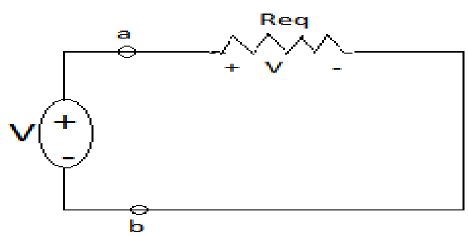


Fig.1.18 A single loop circuit with two resistors in series

Fig. 1.19 Equivalent Circuit of series resistors

The two resistors are in series, since the same current I flow in both of them. Applying Ohm's law to each of the resistors, we obtain

$$V_1 = I*R_1 \text{ , } V2 = I*R_2 \qquad \text{……..(1)}$$

$$I = \frac{V}{R_1+R_2} \qquad \text{………(2)}$$

$$V = Ir_{eq} \qquad \text{…………(3)}$$

$$\text{Total voltage, } V = V_1+V_2$$

$$Ir_{eq} = IR_1+IR_2$$

$$\mathbf{Req = R_1+R_2} \qquad \text{………..(4)}$$

Thus, Fig. 1.18 can be replaced by the equivalent circuit in Fig. 1.19. The two circuits in Fig 1.18 and 1.19 are the equivalent because they because they exhibit the same voltage-current relationships at the terminals a-b. An equivalent circuit such as the one in Fig. 1.19 is useful in simplifying the analysis of a circuit.

For N resistors in series then,

$$R_{eq} = R_1+R_2+R_3+\text{………..}+R_N = \sum_{n=1}^{N} Rn \qquad \text{……(5)}$$

Voltage Division

To determine the voltage across each resistor in Fig. 1.18, we substitute Eq. (2) into Eq. (1) and obtain

$$\mathbf{V_1} = \frac{V}{R_1+R_2}\boldsymbol{R_1} \quad ; \quad \mathbf{V_2} = \frac{V}{R_1+R_2}\boldsymbol{R_2} \qquad \text{………………..(6)}$$

The source voltage is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division, and the circuit in Fig. 1.18 is called a voltage divider. In general, if a voltage divider has N resistors (R_N) in series with the source voltage , the nth resistor (R_1,R_2,R_3,….R_N) will have a voltage drop of

$$V_n = \frac{V}{R_1+R_2+R_3+\cdots+R_n} \qquad \text{…………(7)}$$

2) **<u>PARALLEL RESISTORS AND CURRENT DIVISION</u>**

Two or more resistors are said to be in parallel if the same voltage appears across each element.Consider the circuit in Fig. 1.20, where two resistors are connected in parallel and therefore have the same voltage across them.

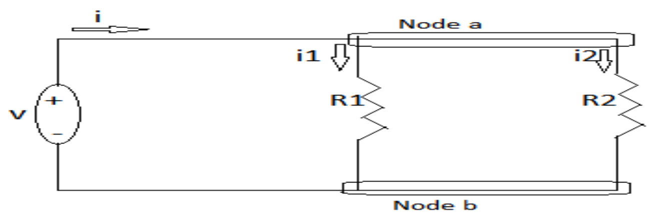


Fig. 1.20 Two resistors in parallel

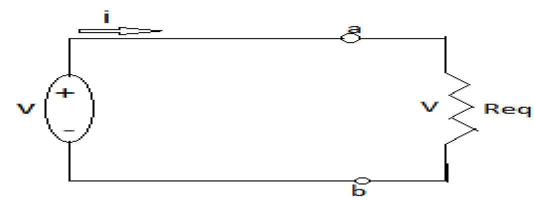


Fig. 1.21Equivalent circuit of Fig. 1.20

$$V=i_1*R_1 = i_2*R_2 \qquad \text{…………(1)}$$

$$i_1=\frac{V}{R_1} \;\; ; \;\; i_2 = \frac{V}{R_2} \qquad \text{……..(2)}$$

Applying KCL at node a gives the total current *i* as

$$i=i_1+i_2 \qquad \text{…………….(3)}$$

Substituting Eq. (2) into Eq. (3), we get

$$i =\frac{V}{R_1} + \frac{V}{R2} = V\left(\frac{V}{R_1} + \frac{V}{R2}\right) = \frac{V}{R_{eq}} \qquad \text{……………(4)}$$

where Req is the equivalent resistance of the resistors in parallel.

$$\frac{V}{R_1} + \frac{V}{R2} = \mathbf{\frac{1}{R_{eq}}} = \mathbf{\frac{R_1*R_2}{R_1+R_2}} \qquad \textbf{…………..(5)}$$

Thus, the equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum. It must be emphasized that this applies only to two resistors in parallel. From Eq. (5), if $R_1 = R_2$, then $\mathbf{R_{eq}=R_1/2}$

We can extend the result in Eq. (5) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \ldots . + \frac{1}{R_n} = \sum_{n=1}^{N} \frac{1}{R_n} \quad \text{…………..(6)}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots .... + \frac{1}{R_n}} \quad \text{.............(7)}$$

Thus, the equivalent Resistance of parallel-connected resistors is the reciprocal of the sum of the reciprocals of the individual resistances.

Current Division:

Given the total current i entering node a in Fig. 1.20, then how do we obtain currents *i1* and *i2* We know that the equivalent resistor has the same voltage, or

$$V = i*R_{eq} = \frac{\mathbf{i * (R_1 * R_2)}}{\mathbf{R_1 + R_2}} \quad \text{.....................(8)}$$

Substitute (8) in (2), we get

$$\mathbf{i_1} = \frac{\mathbf{i * R_2}}{\mathbf{R_1 + R_2}} \quad \text{................(9)}$$

$$\mathbf{i_2} = \frac{\mathbf{i * R_1}}{\mathbf{R_1 + R_2}} \quad \text{......................(10)}$$

This shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig.1.20 is known as a current divider. Notice that the larger current flows through the smaller resistance.

- **INDUCTOR**

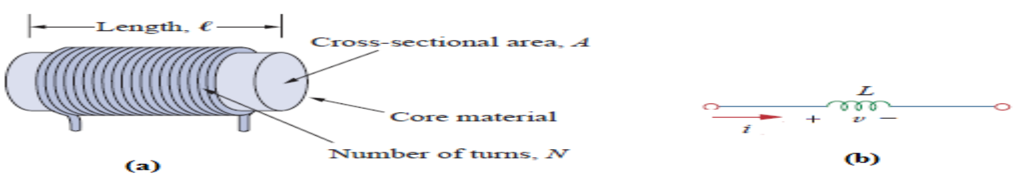


Fig. 1.14 (a) Typical Inductor, (b) Circuit symbol of Inductor

A wire of certain length, when twisted into a coil becomes a basic inductor. The symbol for inductor is shown in Fig.1.14 (b). The voltage across the inductor is directly proportional to the time rate of change of current.

Mathematically, $V \alpha \frac{di}{dt}$; $V = L\frac{di}{dt}$

Where L is the constant of proportionality called the inductance of an inductor. The unit of inductance is Henry (H).

The physical dimensions of the capacitor is $\mathbf{L} = \frac{\boldsymbol{\mu N^2 a}}{\boldsymbol{l}}$

Where, N is the number of coils, a is the area of the coilz l is the length of the coil and μ is the permeability (μ_0= Absolute permeability = $4\pi*10^{-7}$ H/m)

Current through an inductor is $\mathbf{i}=\frac{1}{L}\int_0^t \boldsymbol{v}\,\boldsymbol{dt}+\boldsymbol{i}(\mathbf{0})$

Voltage across the inductor is $\mathbf{V}=\mathbf{L}\frac{di}{dt}$

The power absorbed by the inductor is $\mathbf{P}=\mathbf{VI}=\mathbf{Li}\frac{di}{dt}$

The energy stored by the inductor is $\mathbf{w}=\mathbf{LI^2/2}$

From the above discussion, we can conclude the following.

- The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to DC.
- A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly i.e., the inductor opposes the sudden changes in currents.
- The inductor can store finite amount of energy. Even if the voltage across the inductor is zero
- A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

NOTE

- ***INDUCTIVE NETWORKS***

1) ***SERIES INDUCTORS:*** *Two or more inductors are said to be in series, if the same current flows through all of them.*

$$L_{eq}=L_1+L_2+L_3+\ldots\ldots+L_N$$

Thus, the equivalent inductance of series-connected inductors is the sum of the individual inductances.

2) ***INDUCTORS IN PARALLEL:*** *Two or more inductors are said to be in parallel, if the same voltage appears across each element.*

$$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\cdots\ldots+\frac{1}{L_n}$$

Thus, the equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances

- **CAPACITOR**

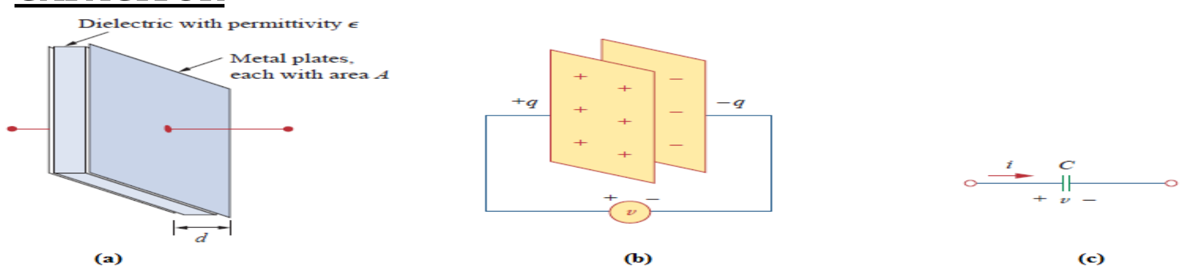


Fig. 1.15 (a) Typical Capacitor, (b) Capacitor connected to a voltage source, (c) Circuit Symbol of capacitor

The conducting surfaces are called electrodes, and the insulating medium is called dielectric. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric.

When a voltage source v is connected to the capacitor, as in Fig 1.15 (c), the source deposits a positive charge q on one plate and a negative charge — q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly pro-proportional to the applied voltage v so that **q=CV**

Where C is the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F).

The physical dimensions of the capacitor is $\mathbf{C} = \frac{\varepsilon A}{d}$

Where A is the surface area of each plate, d is the distance between the plates, and ε is the permittivity of the dielectric material between the plates. (ε_0 = Absolute Permitivity =8.854*10^{-12} F/m)

Current flowing through the capacitor is $\mathbf{i} = C\frac{dv}{dt}$

Voltage across the capacitor is $\mathbf{v(t)} = \frac{1}{C}\int_0^t i\,dt + v(0)$

Power absorbed by the capacitor is $\mathbf{P = VI} = CV\frac{dv}{dt}$

Energy stored by the capacitor is $\mathbf{W = CV^2/2}$

From the above discussion we can conclude the following,

- The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to DC.
- A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly. i.e., A capacitor will oppose the sudden changes in voltages.
- The capacitor can store a finite amount of energy, even if the current through it is zero.
- A pure capacitor never dissipates energy, but only stores it; that is why it is called non-dissipative passive element. However, physical capacitors dissipate power due to internal resistance.

NOTE

- ***CAPACITIVE NETWORKS***

1. ***SERIES CAPACITORS***

Two or more capacitors are said to be in series, if the same current flows through all of them.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \ldots . + \frac{1}{C_n}$$

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

- *Note that the capacitors in series are combined in the same way as resistors in parallel.*

2. *PARALLEL CAPACITORS*

Two or more capacitors are said to be in parallel, if the same voltage appears across each element.

$$C_{eq} = C_1 + C_2 + C_3 + \ldots\ldots\ldots\ldots + C_N = \sum_{n=1}^{N} C_n$$

Thus ,the equivalent capacitance of parallel-connected capacitors is the sum of the individual capacitances.

- *Capacitors in parallel are combined in exactly the same way as resistors in series.*

NETWORK/CIRCUIT TERMINOLOGY

In the following section various definitions and terminologies frequently used in electrical circuit analysis are outlined.

1. **Network Elements:** The individual components such as a resistor, inductor, capacitor, diode, voltage source, current source etc. that are used in circuit are known as network elements.
2. **Network:** The interconnection of network elements is called a network**.**
3. **Circuit:** A network with at least one closed path is called a circuit. So, all the circuits are networks but all networks are not circuits.
4. **Branch:** A branch is an element of a network having only two terminals.
5. **Node:** A node is the point of connection between two or more branches. It is usually indicated by a dot in a circuit**.**
6. **Loop:** A loop is any closed path in a circuit. A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
7. **Mesh or Independent Loop:** Mesh is a loop which does not contain any other loops in it.

KIRCHHOFF'S LAWS

1. KIRCHOFF'S CURRENT LAW OR POINT LAW (KCL)

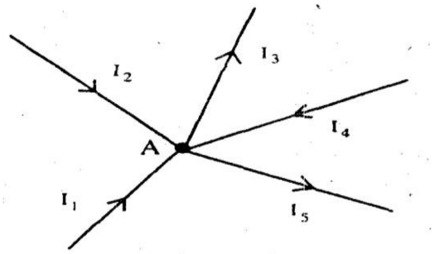
Statement:- In any electrical network, the algebraic sum of the currents at a junction or a node is zero.

$\sum I = 0$ ……………………at a junction or node

Assumption:- Incoming current = positive
Outgoing current = negative

NOTE: *KCL is based on conservation of charge*

***** Sum of the currents flowing towards a junction is equal to the sum of the currents flowing away from the junction ie. Sum of incoming currents = sum of outgoing currents.



In the above example, at node A, currents I_1 , I_2 ,I_4 are the incoming currents

Currents I_3 , I_5 are outgoing currents

Therefore, at node A, $I_1 + I_2 + I_4 = I_3 + I_5$ (or) $I_1 + I_2 + I_4 - I_3 - I_5 = 0$

2. KIRCHOFF'S VOLTAGE LAW OR MESH LAW (KVL)

Statement: Algebraic sum of all the voltages around a closed path or closed loop at any instant is zero.

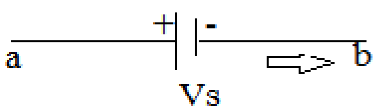
$$\sum V = 0 \text{……………………of a closed path}$$

NOTE: *KVL is based on conservation of energy.*

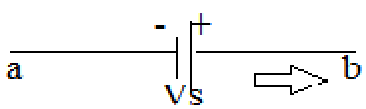
Sign Convention :

The polarity of voltage source is independent of the direction of current.
The positive (+) terminal is at higher potential and negative (-) terminal is at lower potential.

Polarity of voltage source

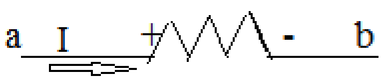


Fall in potential = -Vs (from 'a' to 'b' ie from '+' to '-')

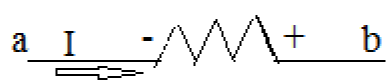


Rise in potential = +Vs (from 'a' to 'b' ie from '-' to '+')

Polarity of resistor

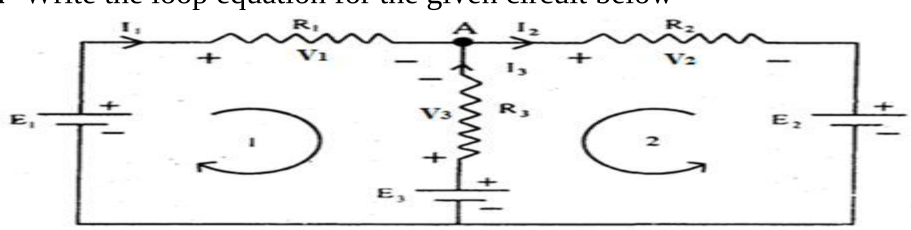


Fall in potential ie, -IR ('+' to '-')



Rise in potential ie, +IR ('-' to '+')

Explanation- Write the loop equation for the given circuit below



Solution : For loop1 it is considered around clockwise

$$+ E_1 - V_1 + V_3 - E_3 = 0$$

$$+ E_1 - I_1 R_1 + I_3 R_3 - E_3 = 0$$

$$\mathbf{E_1 - E_3 = I_1 R1 - I_3 R_3}$$

For loop2 it is considered anticlockwise

$$+ E_2 + V_2 + V_3 – E_3 = 0$$

$$+ E_2 + I_2 R_2 + I_3 R_3 – E_3 = 0$$

$$\mathbf{E_2 – E_3 = - I_2 R_2 - I_3 R_3}$$

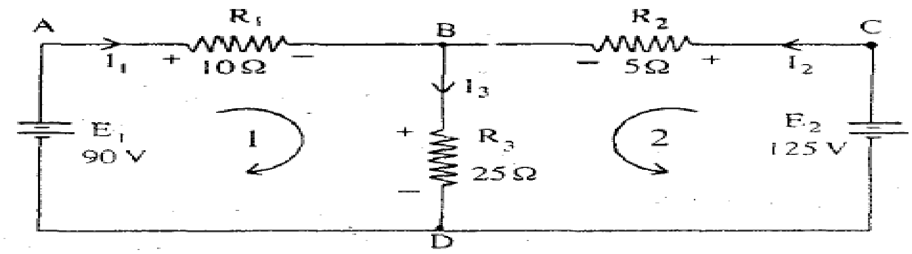
Two equations are obtained following Kirchhoff's voltage law.

The third equation can be written based on Kirchhoff's current law as

$$\mathbf{I_1 – I_2 + I_3 = 0}$$

With the three equations, one can solve for the three currents I_1, I_2, and I_3.

Example 10: Calculate the current supplied by two batteries in the circuit given below.



Solution: Applying KCL to junction B

$$I_3 = I_1 + I_2 \; \text{.....}\mathbf{(1)}$$

Applying KVL to loop 1

$$E_1 – I_1R_1 – I_3R_3 = 0$$

$$I_1R_1 + I_3R_3 = E_1$$

$$10I_1 + 25I_3 = 90 \; \text{......}\mathbf{(2)}$$

Substituting Eq. (1) in Eq. (2)

$$10I_1 + 25(I_1 + I_2) = 90$$

$$35I_1 + 25I_2 = 90 \; \text{.......} \mathbf{(3)}$$

Applying KVL to loop 2

$$E_2 – I_2R_2 – I_3R_3 = 0$$

$$I_2R_2 + I_3R_3 = E_2$$

$$5I_2 + 25I_3 = 125 \; \text{......} \mathbf{(4)}$$

Substituting Eq. (1) in Eq. (4)

$$5I_2 + 25(I_1 + I_2) = 125$$

$$25I_1 + 30I_2 = 125 \; \text{.......} \mathbf{(5)}$$

Multiplying Eq. (3) by 6/5 we get

$$42I_1 + 30I_2 = 108 \; \text{......} \mathbf{(6)}$$

Subtracting Eq. (6) from Eq. (5)

$$– 17I_1 = 17$$

$$I_1 = – 1 \text{ A}$$

Substituting the value of I_1 in Eq. (5) we get

$$I_2 = 5 \text{ A}$$

As the sign of the current I_1 is found to be negative from the solution, the actual direction of I_1 is from B to A to D i.e. 90 V battery gets a charging current of 1 A.

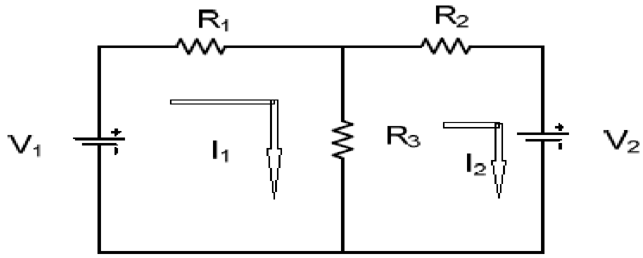
MESH ANALYSIS(MAXWELL'S LOOP CURRENT METHOD)

Statement:- **This method determines branch currents and voltages across the elements of a network.**

The following process is followed in this method:-

1) Identify the mesh and assign direction to it and assign the direction for unknown current in each mesh.
2) Assign polarities for voltages.
3) Apply KVL and Ohm's law to express branch voltages in terms of unknown mesh current and resistance.(ie in the form of V=IR)
4) Solve the equation for unknown mesh currents.

Explanation:- Consider a network as shown in Fig. below. It contains two meshes. Let I_1 and I_2 are the mesh currents of two meshes directed in clockwise.



Solution: Apply KVL to mesh-1,

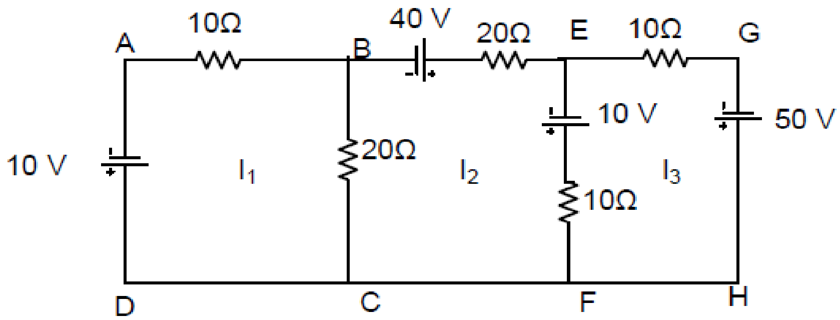
$$V_1 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

Apply KVL to mesh-2,

$$-I_2 R_2 - V_2 - (I_2 - I_1) R_3 = 0$$

When we consider mesh-1, the current I_1 is greater than I_2. So, current through R_3 is I_1-I_2. Similarly, when we consider mesh-2, the current I_2 is greater than I_1. So, current through R_3 is $I_2 - I_1$.

Example 11: Find I_1, I_2 and I_3 in the network shown in Fig below using loop current method.



Solution: For mesh ABCDA,

$$- I_1*10 - (I_1 - I_2)*20 - 10 = 0$$

$$3 I_1 - 2 I_2 = -1 \qquad \text{………….......} (1)$$

For mesh BEFCB,

$$40 - I_2*20 + 10 - (I_2 - I_3)*10 - (I_2 - I_1)*2\ 0 = 0$$

$$2 I_1 - 5 I_2 + I_3 = -5 \qquad \text{…………………} (2)$$

For mesh EGHFE,

$$-10I_3 + 50 - (I_3 - I_2)*10 - 10 = 0$$

$$I_2 - 2I_3 = -4 \qquad \text{………………………} (3)$$

Equation (2) x 2 + Equation (3)

$4I_1 - 9I_2 = -14$ ………………………. (4)

Solving eq^n (1) & eq^n (4)

$I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 3$ A

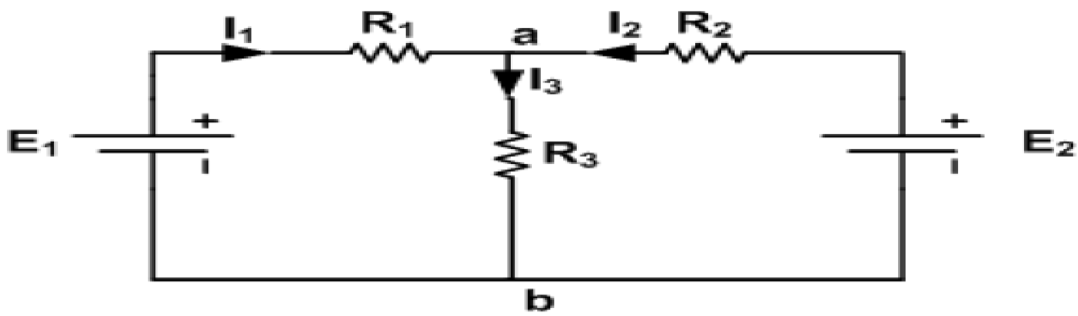
NODAL ANALYSIS

Statement: **This method determines branch currents in the circuit and also voltages at individual nodes.**

The following steps are adopted in this method:-

1) Identify all the nodes in the network.
2) One of these nodes is taken as reference node in at zero potential
3) The node voltages are measured w.r.t the reference node
4) KCL to find current expression for each node
5) This method is easier if all the current sources are present. If any voltage source is present, convert it to current source
6) The number of simultaneous equations to be solved becomes (n-1) where 'n' is the number of independent nodes.

Explanation:-



Solution:

At node 'a' $I_1 + I_2 = I_3$

By ohms law, $I_1 = \dfrac{E_1 - V_a}{R_1}, I_2 = \dfrac{E_2 - V_a}{R_2}, I_3 = \dfrac{V_a}{R_3}$

Therefore, $$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{E_2}{R_2}=0$$

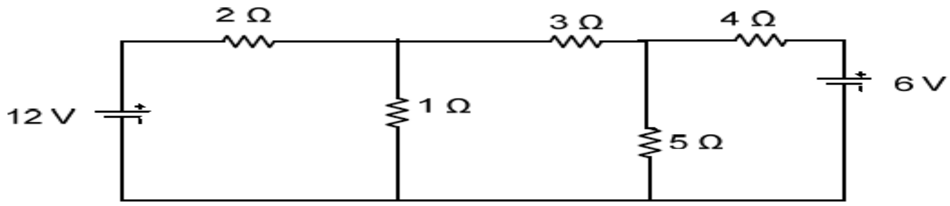
or, $$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{E_2}{R_2}=0$$

or, $$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{E_2}{R_2}=0$$

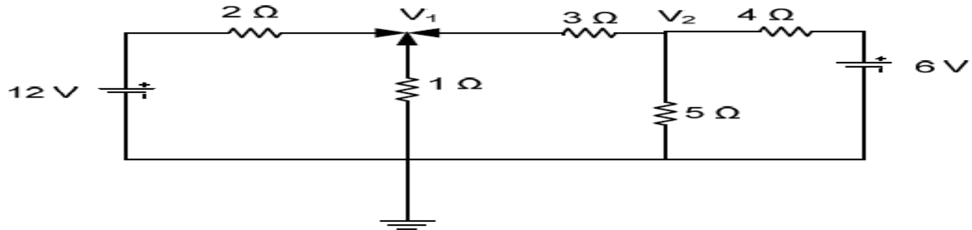
Hence,

- ❖ Node voltage multiplied by sum of all the conductance connected to this node. This term is positive
- ❖ The node voltage at the other end of each branch (connected to this node multiplied by conductance of this branch). This term is negative.

Example 12:- Use nodal analysis to find currents in the different branches of the circuit shown below.



Solution: Let V_1 and V_2 are the voltages of two nodes as shown in Fig below



Solution: Applying KCL to node-1, we get

$$\frac{12-V_1}{2} + \frac{0-V_1}{1} + \frac{V_2-V_1}{3} = 0$$

$$36 - 3V_1 - 6V_1 + 2V_2 - 2V_1 = 0$$

$$-11V_1 + 2V_2 = -36 \text{ .... } (1)$$

Again applying KCL to node-2, we get

$$\frac{V_1-V_2}{3} + \frac{0-V_2}{5} + \frac{6-V_2}{4} = 0$$

$$20V_1 - 20V_2 - 12V_2 + 90 - 15V_2 = 0$$

$$20V_1 - 47V_2 + 90 = 0$$

$$20V_1 - 47V_2 = -90 \text{ .... } (2)$$

Solving Eq^n (1) and (2) we get V_1 = 3.924 Volt and V_2 = 3.584 volt

Current through 2Ω resistance = $\frac{12-V_1}{2} = \frac{12-3.924}{2} = 4.038\ A$

Current through 1Ω resistance = $\frac{0-V_1}{1} = \frac{0-3.924}{1} = -3.924$ A

Current through 3Ω resistance = $\frac{V_1-V_2}{3} = \frac{3.924-3.584}{3} = 0.1133$ A

Current through 5Ω resistance = $\frac{0-V_2}{5} = \frac{0-3.584}{5} = -0.7168\ A$

Current through 4Ω resistance = $\frac{6-V_2}{4} = \frac{6-3.584}{4} = 0.604\ A$

As currents through 1 Ω and 5 Ω are negative, so actually their directions are opposite to the assumptions.

STAR - DELTA TRANSFORMATION

When a given circuit cannot be reduced using series parallel reduction technique, then star-delta transformation can be used. Complicated networks can be simplified by successively replacing delta mesh to star equivalent system and vice-versa.

• In delta network, three resistors are connected in delta fashion (Δ) and in star network three resistors are connected in wye (Y) fashion.

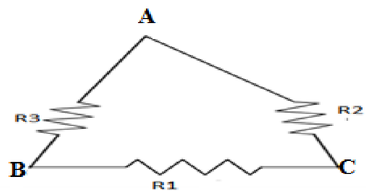


Fig (1)- Delta

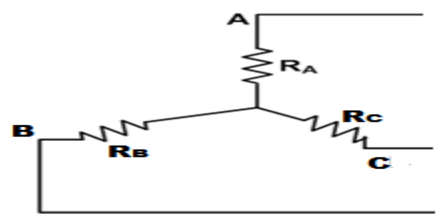
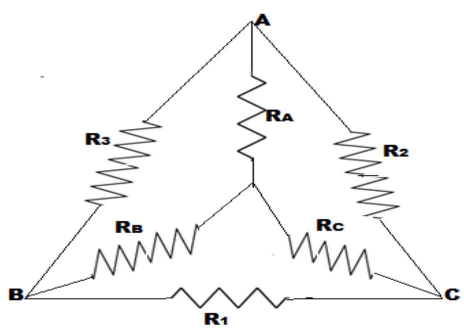


Fig (2)- Star / Wye

Fig (1) shows three resistors R_1 , R_2 , R_3 connected in delta connection.
Fig (2) shows three resistors R_A , R_B , R_C connected in star connection.

If the above two figures are to be same , then the resistance between any two pairs of terminals (AB, BC or CA) has to be same, when the third line is open.



1. DELTA – STAR

Keeping A open , equate resistance between B and C , $R_B + R_C = \frac{R_1\,(R_2 + R_3)}{R_1+R_2+R_3}$ ………(1)

Keeping B open , equate resistance between C and A , $R_C + R_A = \frac{R_2\,(R_1 + R_3)}{R_1+R_2+R_3}$ ………(2)

Keeping C open , equate resistance between A and B , $R_A + R_B = \frac{R_3(R_1 + R_2)}{R_1+R_2+R_3}$ ………(3)

Adding (1) , (2) , (3) we get , $2R_A + 2R_B + 2R_C = \frac{2R1R2+2R2R3+2R1R3}{R1+R2+R3}$ …………...(4)

Dividing eq^n (4) by 2, we get, $R_A + R_B + R_C = \frac{R1R2+R2R3+R1R3}{R1+R2+R3}$ …………………....(5)

Now, to obtain the value of R_A . R_B , R_C

Eq^n (5) – (1) , $R_A + R_B + R_C - (R_B + R_C) = \frac{R1R2+R2R3+R1R3-R1R2-R1R3}{R1+R2+R3} = \mathbf{R_A}\ \frac{R_2R_3}{R_1+R_2+R_3}$ **....(6)**

Eq^n (5) –(2) , $R_A + R_B + R_C -(R_C + R_A) = \frac{R1R2+R2R3+R1R3-R1R2-R2R3}{R1+R2+R3} = \mathbf{R_B} = \frac{R_1R_3}{R_1+R_2+R_3}$ **....(7)**

Eq^n (5) –(3) , $R_A + R_B + R_C - (R_B + R_A) = \frac{R1R2+R2R3+R1R3-R1R23-R2R3}{R1+R2+R3} = \mathbf{R_C} = \frac{R_1R_2}{R_1+R_2+R_3}$ **..(8)**

Equations (6) , (7) , (8) are the set of equations to transform from delta to star.

Any arm of star connection = $\frac{\textbf{Product of two adjacent arms of delta}}{\textbf{sum of arms of delta}}$

❖ ***NOTE***

If the values of R_1 , R_2 , R_3 are equal ie., $R_1 = R_2 = R_3 = R$, then $\mathbf{R_A} = \frac{R}{3}$

2. STAR – DELTA

Multiply eq^n (6) & (7), $R_AR_B = \frac{R1R2R3^2}{(R1+R2+R3)^2}$ ………….(A)

Multiply eq^n (7) & (8), $R_BR_C = \frac{R2R3R1^2}{(R1+R2+R3)^2}$ ………….(B)

Multiply eq^n (6) & (8), $R_AR_C = \frac{R1R3R2^2}{(R1+R2+R3)^2}$ ………….(C)

Adding (A),(B),(C) we get, $R_AR_B + R_BR_C + R_CR_A = \frac{R1R2R3^2+R2R3R1^2+R1R3R2^2}{(R1+R2+R3)^2} = \frac{R1R2R3\ (R1+R2+R3)}{(R1+R2+R3)^2}$

$$R_AR_B + R_BR_C + R_CR_A = \frac{R1R2R3}{R1+R2+R3} \quad \text{………….(D)}$$

Eq^n (D) can be written as $R_1(\frac{R2R3}{R1+R2+R3}) = R_1R_A$

Therefore, $\mathbf{R_1 = R_B + R_C} + \frac{R_B*R_C}{R_A}$ **………..(9)**

Similarlly, $\mathbf{R_2 = R_A + R_C} + \frac{R_A*R_C}{R_B}$ **………..(10)**

$\mathbf{R_3 = R_A + R_B} + \frac{R_A*R_B}{R_C}$ **………..(11)**

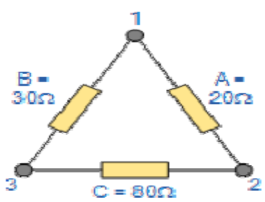
Equations (9) , (10) , (11) are the set of equations to transform from star to delta.

Resistance between two terminals of delta=sum of star resistance connected to those terminals + $\frac{\textbf{product of the same two resistance}}{\textbf{Remaining resistance}}$

❖ ***NOTE***

If the values of R_A , R_B , R_C *are equal ie.,* $R_A = R_B = R_C = R$, *then* $\boldsymbol{R_Y = 3R}$

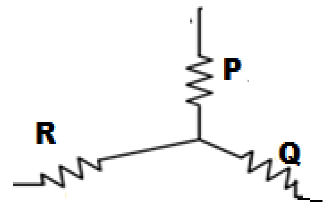
Example 13(delta to star):- Convert the following Delta Resistive Network into an Star Network.



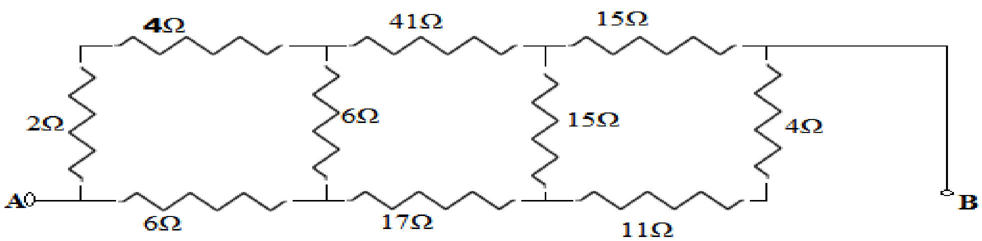
$$Q = \frac{AC}{A+B+C} = \frac{20\times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20\times 30}{130} = 4.61\Omega$$

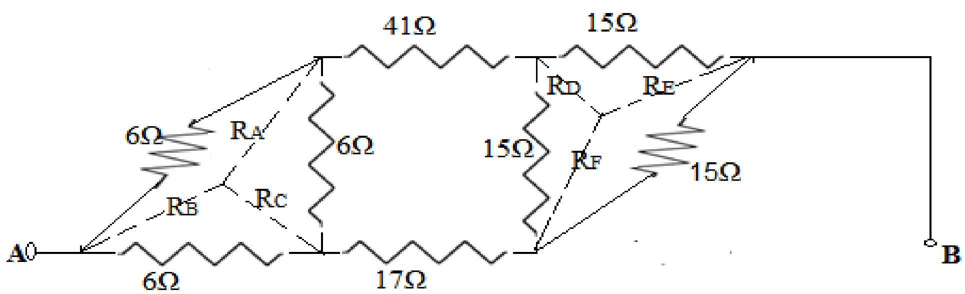
$$R = \frac{BC}{A+B+C} = \frac{30\times 80}{130} = 18.46\Omega$$



Example 14: Find the equivalent resistance between A & B



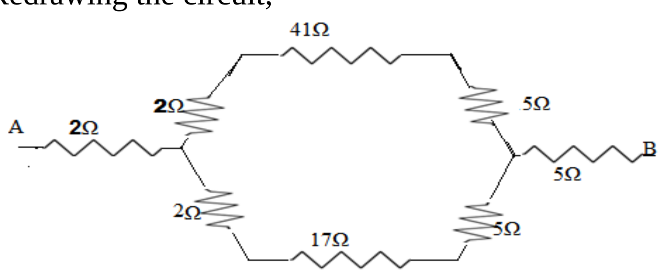
Solution: Resistance 2Ω , 4Ω and 11Ω, 4Ω are in series

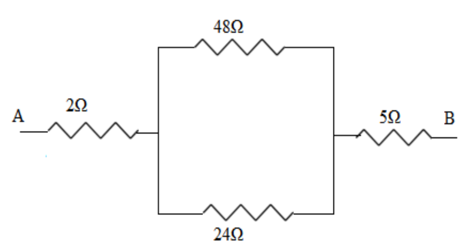


Converting **Δ** 6Ω , 6Ω, 6Ω and **Δ**15Ω , 15Ω, 15Ω into star ,

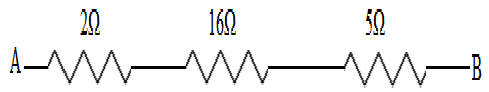
$R_A = R_B = R_C = \frac{6*6}{6+6+6} = \frac{36}{18} = 2\Omega$; $R_D = R_E = R_F = \frac{15*15}{15+15+15} = \frac{15*15}{3*15} = 5\Omega$

Redrawing the circuit,



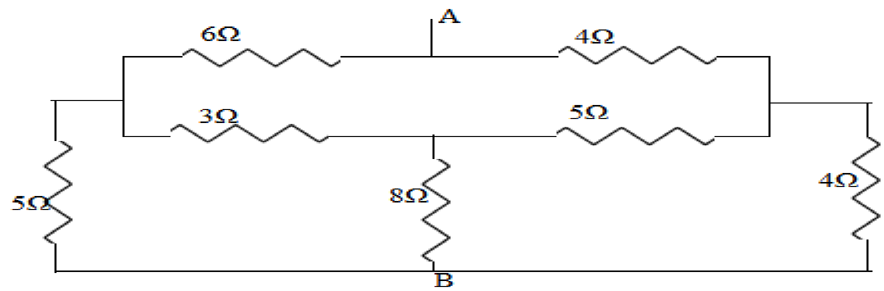


$R_{AB} = 2 + (48 \| 24) + 5 = 2 + \frac{24*48}{24+48} + 5 = 2 + 16 + 5 = 23\Omega$

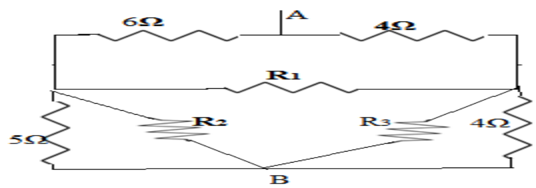


The equivalent resistance between A & B is **R_{AB} = 23 Ω**

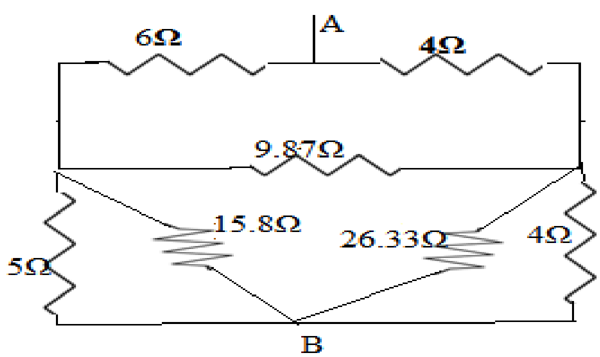
Example 15: The equivalent resistance between A & B

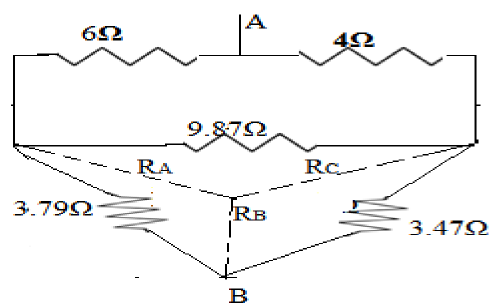


Solution: Converting 3Ω, 5Ω and 8Ω from star to delta



$R_1 = 3 + 5 + \frac{3*5}{8}$ **$= R_1 = 9.87\Omega$** ; $R_2 = 3 + 8 + \frac{3*8}{5}$ **$= R_2 = 15.8\Omega$** ; $R_3 = 8 + 5 + \frac{8*5}{3}$ **$= R_3 = 26.33\Omega$**



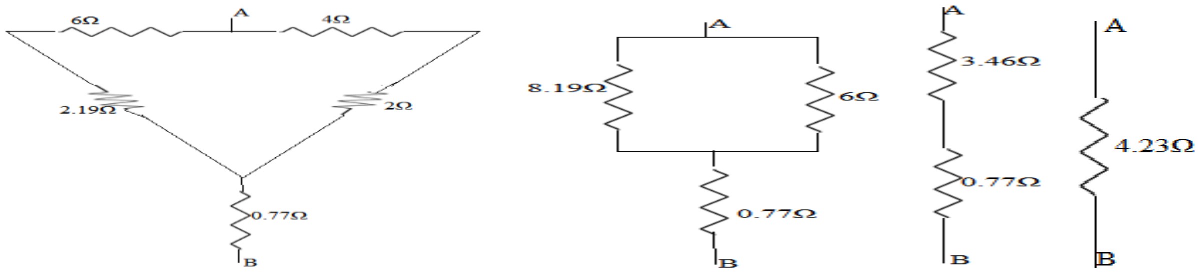


In the above circuit, we have 15.8Ω , 5Ω and 26.33Ω, 4Ω in parallel,

$$\text{ie., } \frac{15.8*5}{15.8+5} = 3.79\Omega \ ; \ \frac{26.33*4}{26.33+4} = 3.47\Omega$$

Now converting 9.87 Ω , 3.79 Ω and 3.47 Ω into star

We get, $R_A = \frac{3.79*9.87}{9.87+3.79+3.47} = 2.19\Omega$; $R_B = \frac{3.79*3.47}{9.87+3.79+3.47} = 0.77\Omega$; $R_C \ \frac{3.47*9.87}{9.87+3.79+3.47} = 2\Omega$



= 6 Ω , 2.19 Ω in series and 4 Ω, 2 Ω are in series

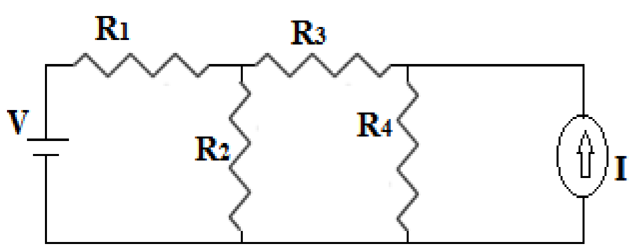
Now, 8.19Ω and 6 Ω are in parallel, so $\frac{8.19*6}{8.19+6} = 3.46\Omega$

Finally, 3.46 Ω and 0.77Ω are in series. Therefore, **R_{AB} = 4.23 Ω**

NETWORK THEORMS

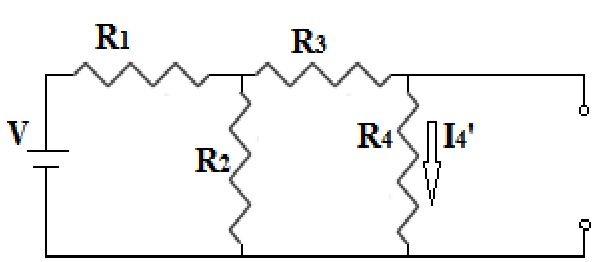
1. SUPERPOSTION THEOREM

Statement: ***In a linear network containing more than one independent source and dependent sources, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone. All the other independent sources being represented meanwhile by their respective internal resistances.***



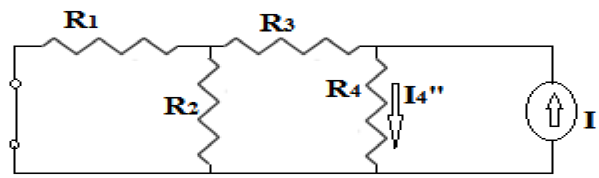
Steps to be followed: To find the current through R_4 ,

Step 1: When only voltage source is acting, current source is removed by making it open circuit.



$I_4^{|}$ can be calculted by using $I_4^{|} = \frac{V}{R_{eq}}$, where $R_{eq} = (R_3+R_4)||(R_1+R_2)$

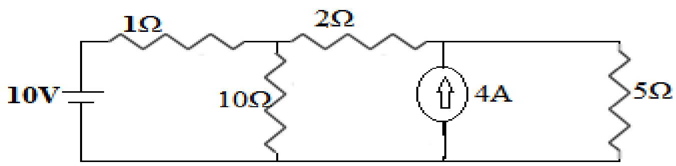
Step 2 : When only current source is acting , voltage source is replaced by short circuit.



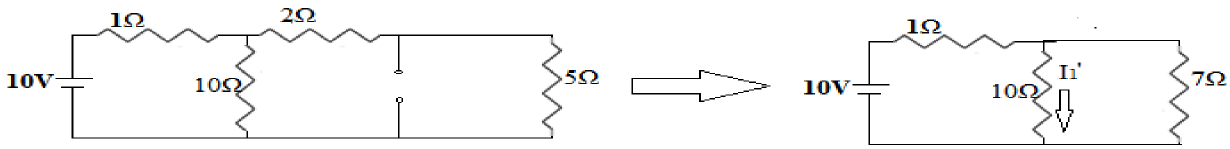
$I_4^{||}$ can be calculated using current division rule

Step 3 : The resultant current I_4 through R_4 is $\mathbf{I_4 = I_4^{|} + I_4^{||}}$

Example 16: Determine the current through 10Ω resistor using superposition theorem



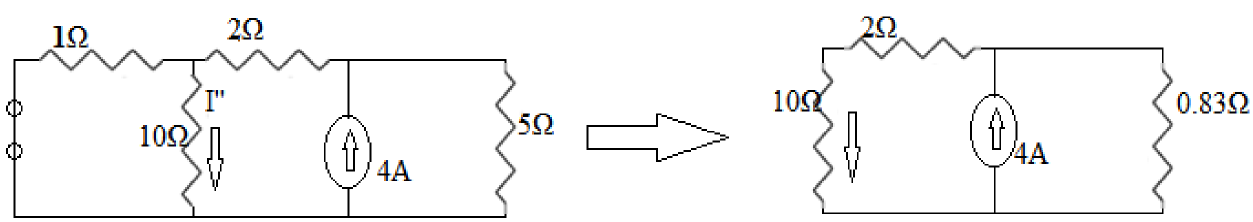
Solution: **Step 1:** When 10V source is acting alone, replace current source by an open circuit



$I_T = \frac{V}{R_{eq}}$; $R_{eq} = (10\|7) + 1 = \frac{10*7}{10+7} + 1 = 5.11\Omega$; $I_T = \frac{V}{R_{eq}} = \frac{10}{5.11} = 1.95A$;

$I_1^{|} = \frac{7}{10+7} * 1.95 = 0.80A$; $\mathbf{I_1^{|}=0.80A}$

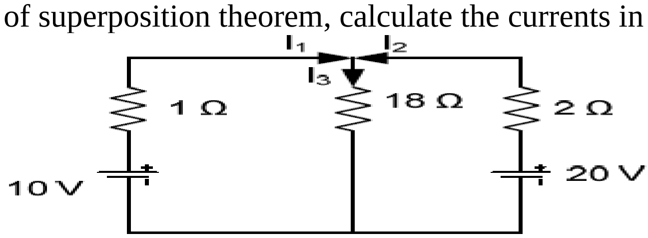
Step 2: When 4A source is acting alone, replace voltage source by an short circuit



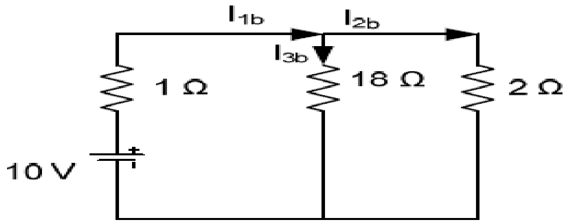
$R_{eq} = (1\| 5) = 0.833\Omega$; $I^{||} = \frac{0.833}{0.833+12} * 4 = 0.25A$; $\mathbf{I^{||} = 0.25A}$

By superposition theorem, $I_{10\Omega} = I^{|} + I^{||} = 0.80 + 0.25 = 1.05$; $\mathbf{I_{10\Omega} = 1.05A}$

Example 17: By means of superposition theorem, calculate the currents in the network shown.



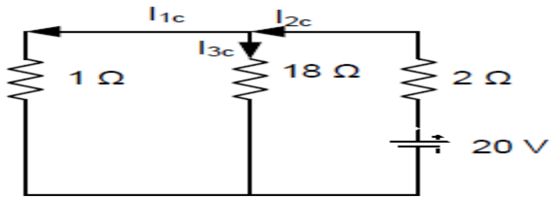
Solution: **Step 1:** Consider 10V source



$$R_{eq} = \frac{18*2}{18+2} + 1 = 2.8\Omega$$

$$I_{1b} = \frac{10}{2.8} = 3.57A \ ; \quad I_{2b} = 3.57 * \frac{18}{20} = 3.21A \ ; \quad I_{3b} = I_{1b} - I_{2b} = 3.57 - 3.21 = 0.36A$$

Step 2: Consider 20V source



$$R_{eq} = \frac{18*1}{18+1} + 2 = 2.95\Omega$$

$$I_{2c} = \frac{20}{2.95} = 6.78A \ ; \quad I_{1c} = 6.78 * \frac{18}{19} = 6.42A \ ; \quad I_{3c} = I_{2c} - I_{1c} = 6.78 - 6.42 = 0.36A$$

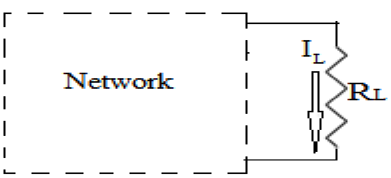
$$I_1 = I_{1b} - I_{1c} = 3.57 - 6.42 = -2.58A$$

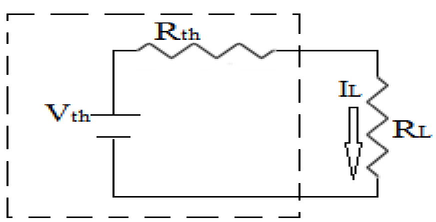
$$I_2 = I_{2c} - I_{2b} = 6.78 - 3.21 = 3.57A$$

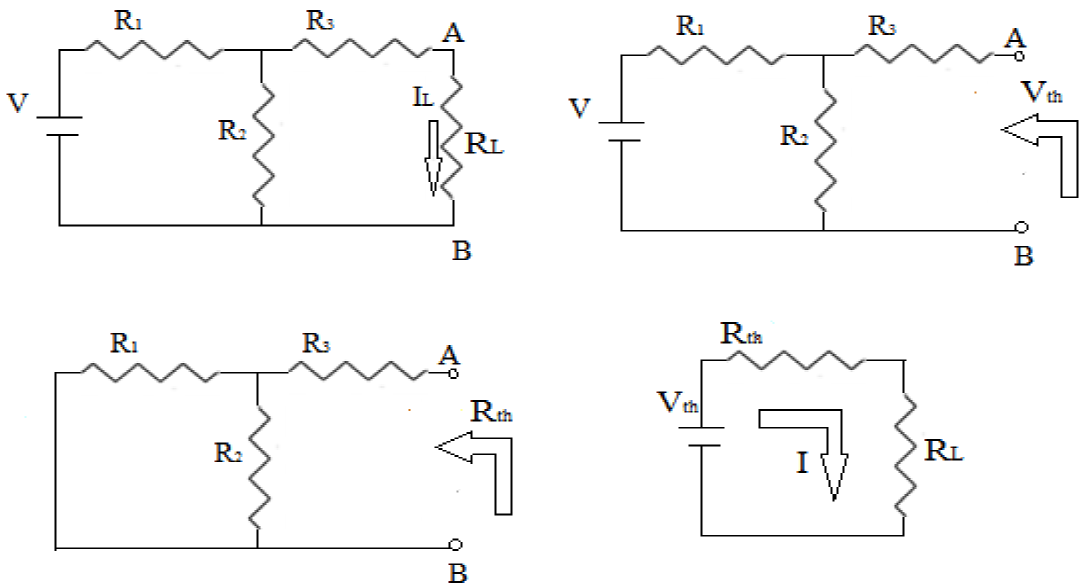
$$I_3 = I_{3b} + I_{3c} = 0.36 + 0.36 = 0.72A$$

2. THEVENIN'S THEOREM

Statement: *Any two terminals of a network can be replaced by an equivalent source and an equivalent resistance. The voltage source is the equivalent voltage across the two terminals of the load, if any removed. The equivalent resistance is the resistance of the network measured between two terminals with load removed and the constant voltage source replaced by its internal resistance (if not given, then by zero resistance ie., by a short circuit) and the current source is replaced by its infinite resistance ie., open circuit.*



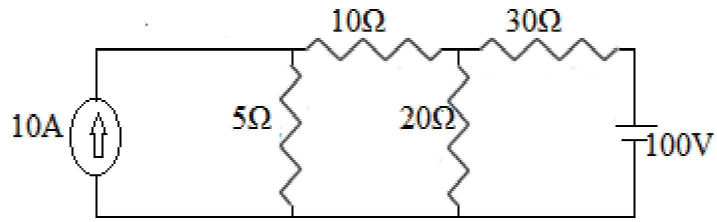




Procedure:

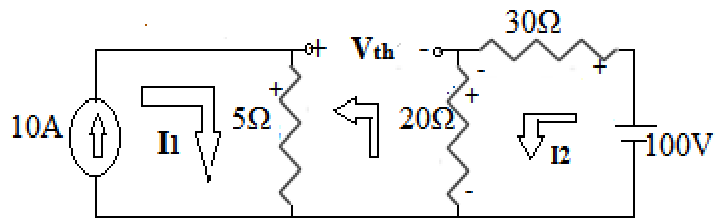
1) Remove the load resistance R_L
2) Find the open circuit voltage V_{th} across A & B
3) Find the resistance R_{th} as seen from A & B with voltage source V replaced by short circuit
4) Replace the network by voltage source V_{th} in series with resistance R_{th}
5) Find the current through R_L by using ohm's law $I_L = \frac{V_{TH}}{R_{TH}+R_L}$

Example 18: Find the current through 10Ω resistance using thevenin's theorem



Solution:

Step 1: Remove the load resistance R_L ie., 10-



From figure, $I_1 = 10A$

Apply KVL to find I_2, $100 - 30I_2 - 20I_2 = 0$

$100 = 50I_2$; $\mathbf{I_2 = 2A}$

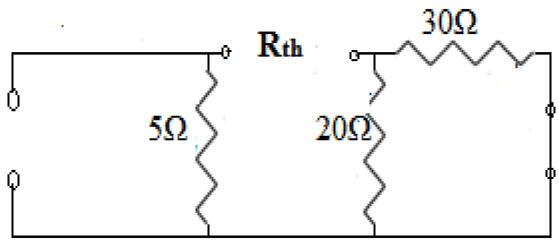
Step 2: To find open circuit voltage V_{th}, apply KVL, $V_{th} - 5I_1 + 20I_2 = 0$

$$V_{th} = 5I_1 - 20I_2 = 5(10) - 20(2) = 10V$$

$$\mathbf{V_{th} = 10V}$$

Step 3: To find R_{th},

Replace current source by open circuit and voltage source by short circuit

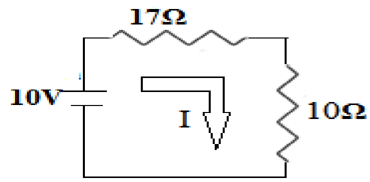


$$R_{th} = 20||30 + 5 = \frac{20*30}{20+30} + 5 = 17$$

$$\mathbf{R_{th} = 17\Omega}$$

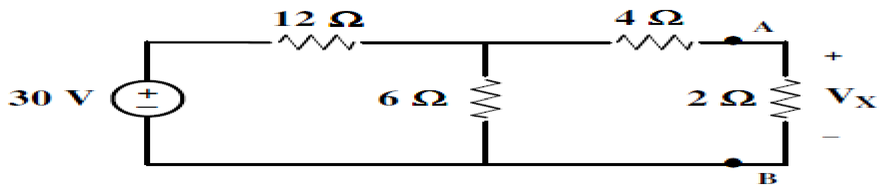
Step 4: To find current through load ie., I_L

Draw thevenin's equivalent circuit



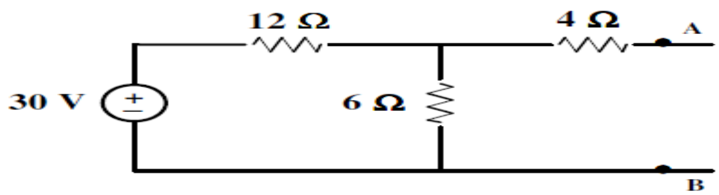
$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{10}{17+10} = 0.37A \; ; \; \mathbf{I_L = 0.37A}$$

Example 19: Find V_X by first finding V_{TH} and R_{TH} to the left of A-B



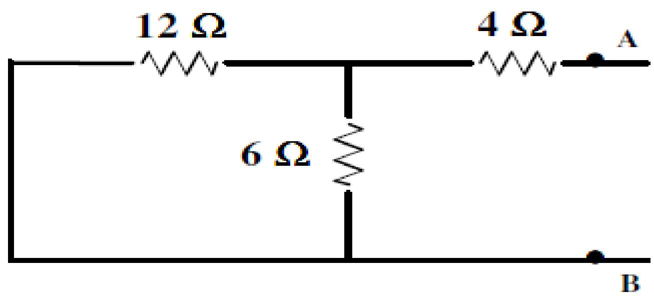
Solution:

Step 1: To find V_{th}, remove load resistance 2Ω



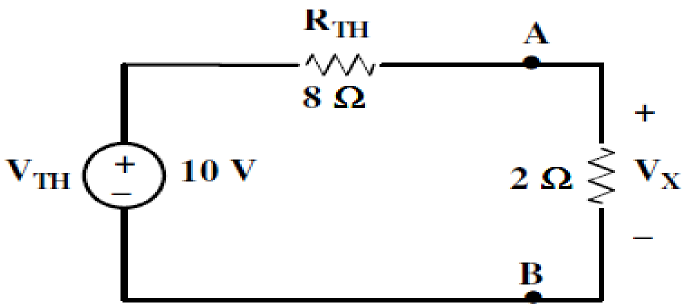
$$V_{th} = \frac{30*6}{6+12} = 10V \; ; \; \mathbf{V_{th} = 10V}$$

Step 2: To find R_{th}, voltage source is short circuited



R_{th} = (12 ∥ 6) +4 =8 ; $\mathbf{R_{th} = 8\Omega}$

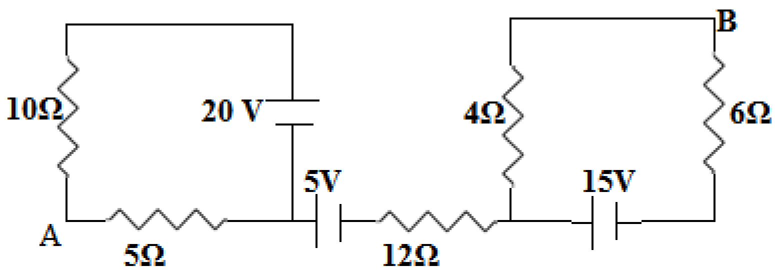
Step 3: To find V_X, draw thevenin's equivalent circuit,



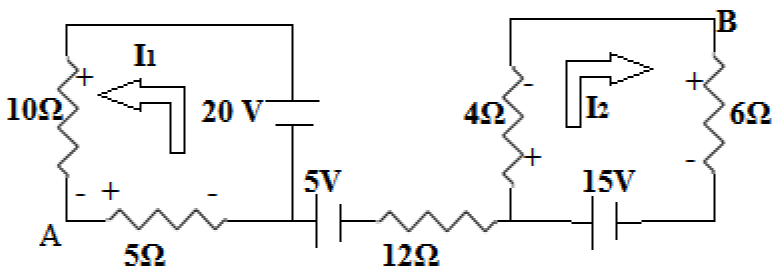
$V_x = \frac{10*2}{8+2} = 2$; $\mathbf{V_x = 2V}$

<u>Numericals:</u>

Example 20: Find the voltage between A & B



Solution:



$I_1 = \frac{20}{10+5} = 1.33A$; $I_2 = \frac{15}{6+4} = 1.5A$

$$\text{Voltage between A \& B is } V_{AB} = V_A - V_B$$

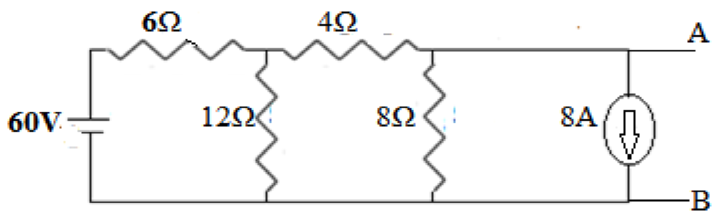
Apply KVL equation for the path A to B,

$$V_A - 5I_1 - 5 - 15 + 6I_2 - V_B = 0$$

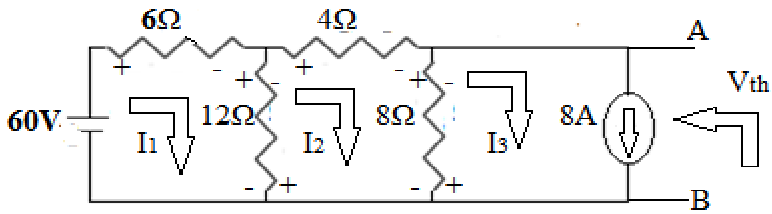
$$V_A - 5(1.33) - 5 - 15 + 6(1.5) - V_B = 0$$

$$\mathbf{V_{AB} = V_A - V_B = 17.65V}$$

Example 21: Find the Thevenin's equivalent between the terminals A & B in the circuit shown in figure



Solution: Redraw the circuit , apply mesh analysis to find I_1 , I_2 , I_3



Mesh1: $60 - 6I_1 - 12(I_1 - I_2) = 0$

$$8I_1 - 12I_2 = 60\text{.....(1)}$$

Mesh 2: $-4I_2 - 8(I_2 - I_3) - 12(I_2 - I_1) = 0$

$$12I_1 - 24I_2 + 8(6) = 0$$

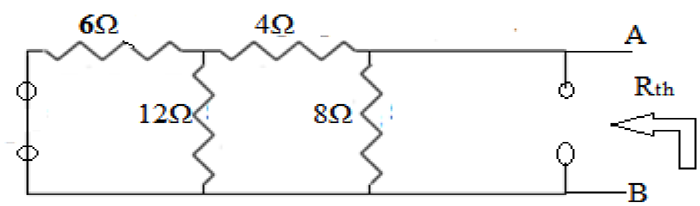
$$12I_1 - 24I_2 = -64\text{.......(2)}$$

Solving eq^n 1 & 2, we get $\mathbf{I_1 = 7.661A}$; $\mathbf{I_2 = 6.5A}$; $\mathbf{I_3 = 8A}$

To find V_{th} , voltage across 8Ω, $-V_{th} - 8(I_3 - I_2) = 0$

$$V_{th} = -8(8 - 6.5); \mathbf{V_{th} = -12V}$$

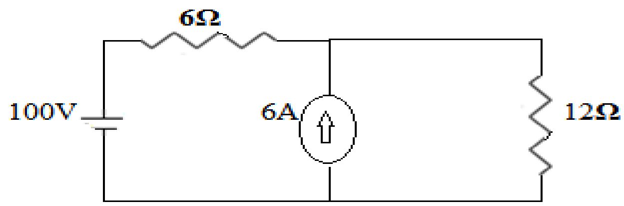
To find thevenin's resistance R_{th}



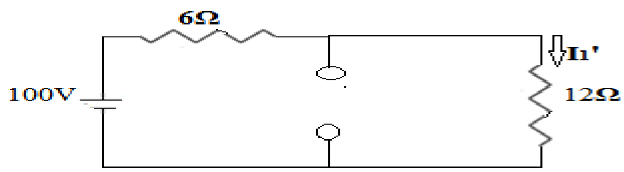
$$(6||12) = 4\Omega ; (4 + 4) = 8\Omega$$

$$R_{th} = 8||8 = 4; \mathbf{R_{th} = 4\Omega}$$

Example 22: Using superposition theorem, find the power absorbed by 12Ω resistor

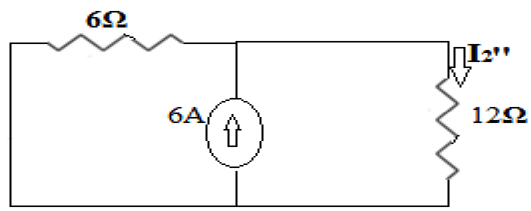


Solution: When 100V source is acting , Current source is open circuited,



$$I_1^{|} = \frac{100}{12+6} = 5.55\text{A}$$

When 6A source is acting , voltaget source is short circuited

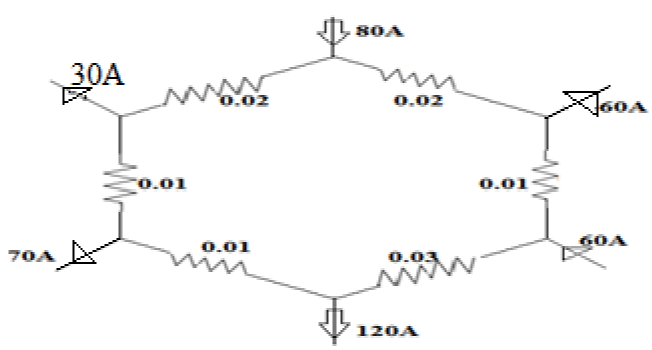


$$I_2^{||} = = \frac{6}{12*6} * 6 = 2\text{A}$$

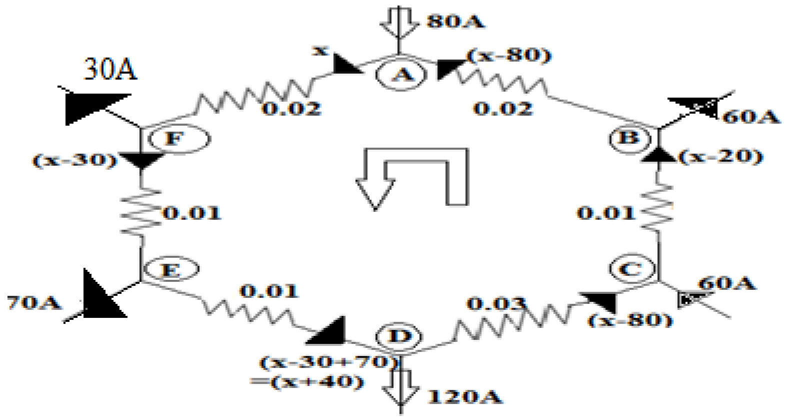
Total current through 12Ω resistor is $I_{12\Omega} = I_1^{|} + I_2^{||} = 5.55 + 2 = 7.55$A ; $\mathbf{I_{12\Omega} = 7.55A}$

Power absorbed by 12Ω resistor is $P_{12\Omega} = (\mathbf{I_{12\Omega}})^2R = (7.55)^2 * 12 = 684$W ; $\mathbf{P_{12\Omega} = 684W}$

Example 23: Find the current through all the branches of the network shown. All resistors are in ohms.



Solution: Let us consider, $I_{AF} = x$; $I_{FE} = (x - 30)$; $I_{ED} = (x+40)$; $I_{DC} = (x-80)$; $I_{CB} = (x-20)$; $I_{BA} = (x-80)$



Apply KVL to the loop AFEDCBA,

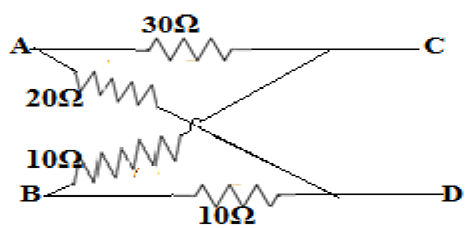
$$-0.2x - 0.01(x-30) - 0.01(x+40) - 0.03(x-80) - 0.01(x-20) - 0.02(x-80) = 0$$

$$\mathbf{x = 41A}$$

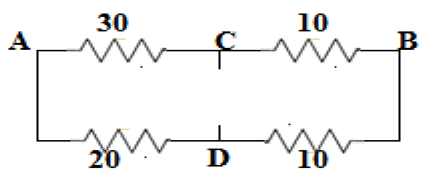
Substituting for x,

$$\mathbf{I_{AF} = 41A \; ; I_{FE} = 11A \; ; I_{ED} = 81A \; ; I_{DC} = -39A \; ; I_{CB} = 21A \; ; \; I_{BA} = -39A}$$

Example 24: Find the input resistance of AB when terminals CD (i) open circuited
(ii) short circuited

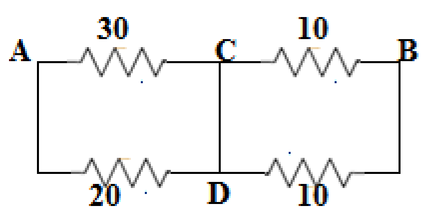


Solution: When C & D is open circuited,



$$R_{AB} = \frac{30*40}{30+40} = 17.14\Omega \; ; \; \mathbf{R_{AB} = 17.14\ \Omega}$$

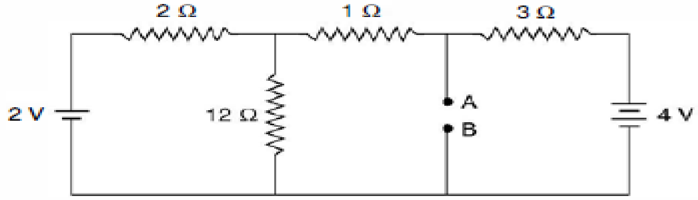
When C & D is short circuited,



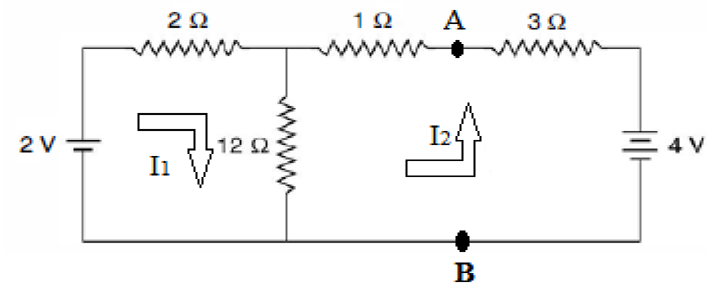
$$R_{AB} = (30 \| 20) + (10 \| 10) = \frac{30*20}{30+20} + \frac{10*10}{10+10} = 12 + 5 = 17\Omega$$

$$\mathbf{R_{AB} = 17\Omega}$$

Example 25: Determine the current through 2Ω resistor connected between A and B in the circuit shown using Thevenin theorem.



Solution: To obtain V_{th},



$$2I_1 + 12(I_1 + I_2) = 2; \qquad 4I_2 + 12I_1 + 12I_2 = 4$$

$$14I_1 + 12I_2 = 2 \text{ or } 7I_1 + 6I_2 = 1 \;; \qquad 12I_1 + 16I_2 = 4 \text{ or } 3I_1 + 4I_2 = 1 \ldots\ldots(1)$$

$$7I_1 + 6I_2 = 3I_1 + 4I_2$$

$$4I_1 = -2I_2$$

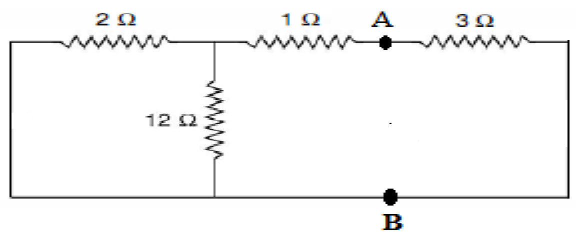
$$I_1 = \frac{-I_2}{2}$$

Substituting the value of I_1 in eq^n (1), $\frac{-3I_2}{2} + 4I_2 = 1$

$$\frac{5I_2}{2} = 1; \; I_2 = \frac{2}{5}A$$

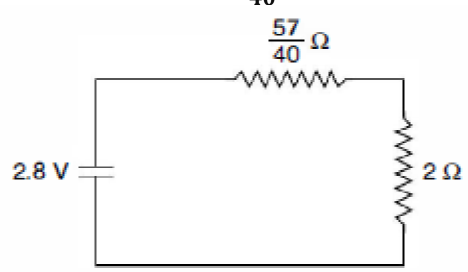
Therfore, $V_{th} = 4 - 3I_2 = 4 - 3(\frac{2}{5})$; $\mathbf{V_{th} = 2.8V}$

To obtain R_{th},



$$R_{th} = ((2\|12) + 1)\|3 \;; \left(\frac{2*12}{2+12} + 1\right)\|3$$

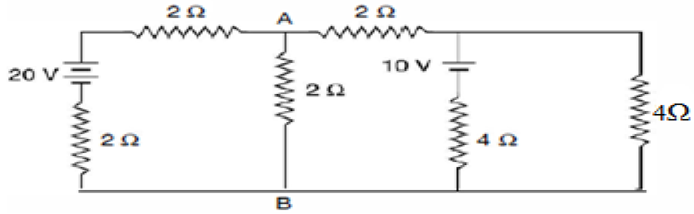
$$\mathbf{R_{th} = \frac{57}{40}\Omega}$$



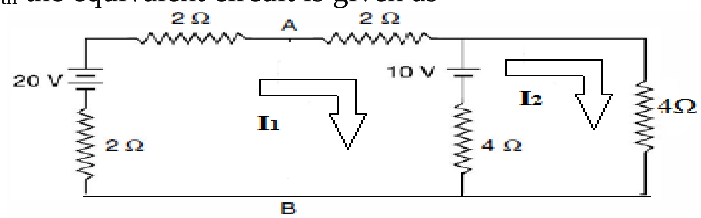
Therefore, current through 2Ω resistor,

$$I = \frac{2.8}{\frac{57}{40} + 2} \;; \mathbf{I = 0.82A}$$

Example 26: Determine the current through the branch AB of the network shown here using Thevenins equivalent.



Solution: To obtain V_{th} the equivalent circuit is given as



Mesh 1: $2I_1 - 20 + 4I_1 + 10 + 4(I_1 - I_2) = 0$

$10I_1 - 4I_2 = 10$

$5I_1 - 2I_2 = 5$ …….(1)

Mesh 2: $4(I2 - I_1) - 10 + 4I_2 = 0$

$-4I_1 + 8I_2 = 10$

$-2I_1 + 4I_2 = 5$ ………..(2)

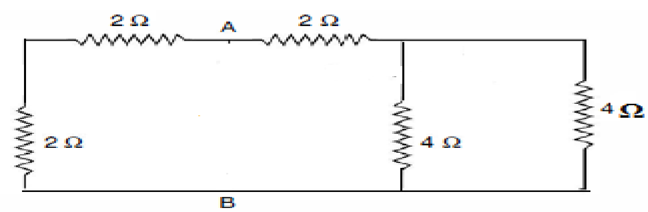
Solving eq^n (1) & (2) , we get ,

$$I_1 = \frac{15}{8}\text{ A} ; I_2 = \frac{35}{16}\text{ A}$$

$$V_{th} + 4I_1 = 20$$

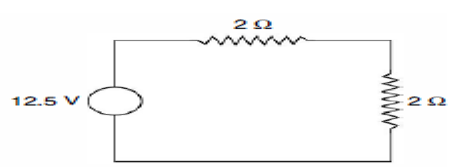
$$V_{th} + 4\left(\frac{15}{8}\right) = 20 ; \;\mathbf{V_{th} = 12.5V}$$

To obtain R_{th} the equivalent circuit is given as



$$R_{th} = ((4\|4) + 2)\|(2+2) = (2 + 2)\|(2+2) = 4\|4 = 2\Omega ; \;\mathbf{R_{th} = 2\ \Omega}$$

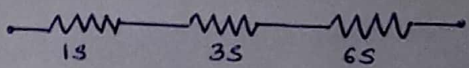
Current through branch AB is



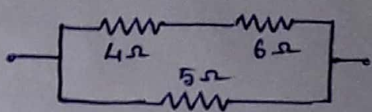
$$I = \frac{12.5}{4} = 3.125A ; \;\mathbf{I_{AB} = 3.125A}$$

Basic Electrical Engineering

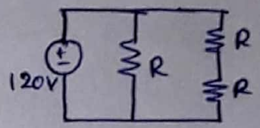
Assignment - 01

01. Determine the value of Req in 

02. Consider the given network. The power in 5Ω resistor is 10W. Determine the power in 4Ω resistor.

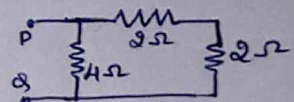


03. Determine the value of R, if the power delivered by the source is 1440 watts.



04. A metal has a resistance of 20Ω and it is heated to make double the length, then the new value of resistance is —

05. The equivalent resistance of the given circuit at terminals P-Q is



06. Find the current through 5Ω resistor in fig 6 (a)

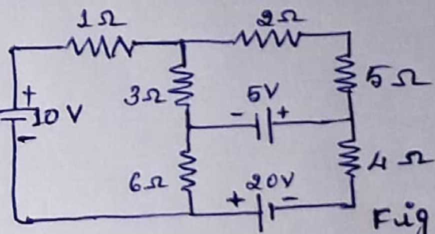


Fig 6 (a)

07. Find the current supplied by the battery for the network shown in fig 7(a)

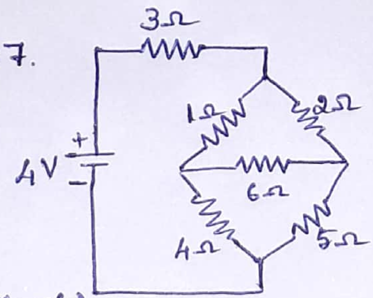


Fig 7(a)

08. In the circuit shown in fig 8(a), the current I through resistor of 30Ω is ____

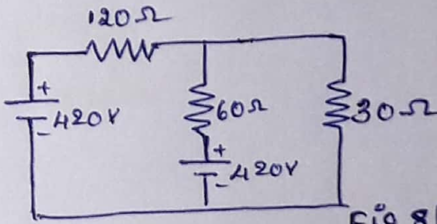


Fig 8(a)

09. The value of resistance R, for the network shown in fig 9(a) is ____

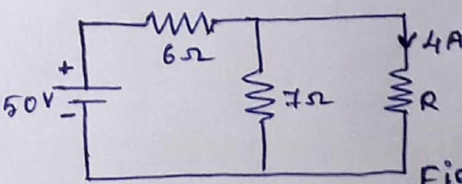


Fig 9(a)

10. For the circuit shown in fig 10(a), determine the values of currents i_1, i_2 and i_3.

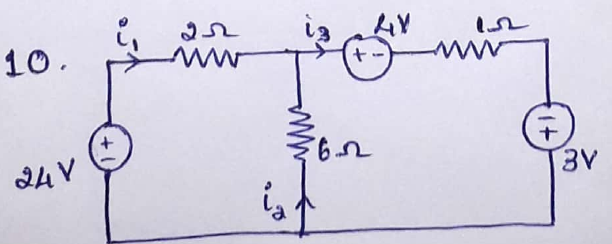


Fig 10(a)

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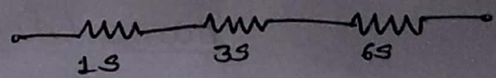
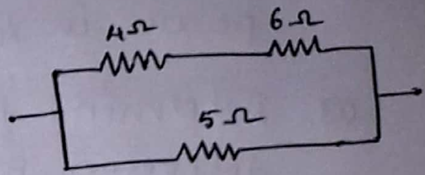
① Determine the value of Req in 

Soln: Given $G = 1S, 3S, 6S$

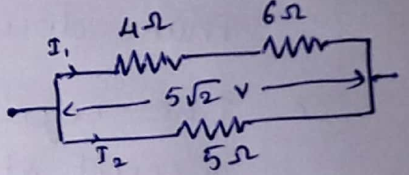
wkt $R = \frac{1}{G}$ $\therefore R_{eq} = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} = \frac{6+2+1}{6} \Rightarrow \frac{\cancel{9}^3}{\cancel{6}_2} = \frac{3}{2}$

$\therefore \boxed{R_{eq} = \frac{3}{2}\,\Omega}$

② Consider the given n/w, the power in 5Ω resistor is 10W. Determine the power in 4Ω resistor.



Soln: Given $P_{5\Omega} = 10W = \frac{V^2}{R} \Rightarrow V^2 = 10 \times R$



$\Rightarrow V^2 = 10 \times 5 = 50$

$V = \sqrt{50} = 5\sqrt{2}$ volts

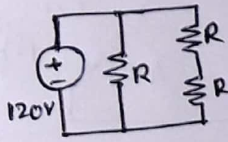
wkt, V is same when connected in parallel

$\therefore$ voltage across 4Ω & 6Ω is also $5\sqrt{2}$ V

$I_{4\Omega} = \frac{V}{R} = \frac{5\sqrt{2}}{4+6} = \frac{\cancel{5}\sqrt{2}}{\cancel{10}_2} \Rightarrow \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ amps

$\therefore P_{4\Omega} = I^2 R = \left(\frac{1}{\sqrt{2}}\right)^2 \times 4 = \frac{1}{2} \times 4 \Rightarrow \boxed{P_{4\Omega} = 2\text{ watts}}$

③ Determine the value of R, if power delivered by the source is 1440 watts



Soln:- $P = \frac{V^2}{R} \Rightarrow 1440 = \frac{(120)^2}{R_{eq}}$

$\therefore R_{eq} = \frac{\cancel{120}^{10} \times \cancel{120}^{1}}{\cancel{1440}_{\cancel{12}\,1}} \Rightarrow R_{eq} = 10\,\Omega$

Now, $(R+R)||R = 2R||R = \frac{2R \times R}{2R+R} = \frac{2R^2}{3\cancel{R}} = \frac{2R}{3} = R_{eq}$

$R = \frac{3 \times R_{eq}}{2} = \frac{3 \times \cancel{10}^5}{\cancel{2}} \Rightarrow \boxed{R = 15\,\Omega}$

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④ A metal has a resistance of 20Ω and it is heated to make double the length, then the new value of resistance is —

Soln: wkt $R = \frac{\rho l}{a}$

Given that $l_2 = 2l_1$, volume is constant

$\Rightarrow l_1 a_1 = l_2 a_2$

$\cancel{l_1} a_1 = 2\cancel{l_1} a_2$

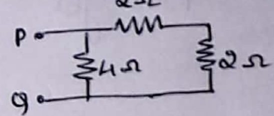
$a_2 = a_1/2$

$\therefore R_2 = \frac{\rho l_2}{a_2} = \frac{\rho(2l_1)}{(a_1/2)} \Rightarrow \frac{\rho l_1 (4)}{a_1} = 4\left[\frac{\rho l_1}{a_1}\right]$

$R_2 = 4(R_1) \quad \left[\because R_1 = \frac{\rho l_1}{a_1}\right]$

$\therefore R_2 = 4(20) \Rightarrow \boxed{R_2 = 80\Omega}$

⑤ The equivalent resistance of the given ckt at terminals P-Q is



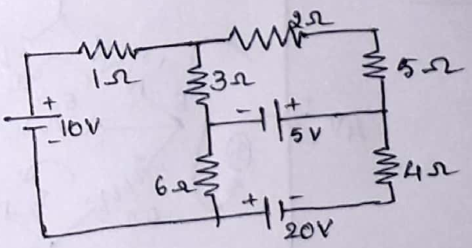
Soln: In the given ckt, 2Ω is in series with 2Ω

$\therefore 2 + 2 = 4\Omega \Rightarrow R_1$

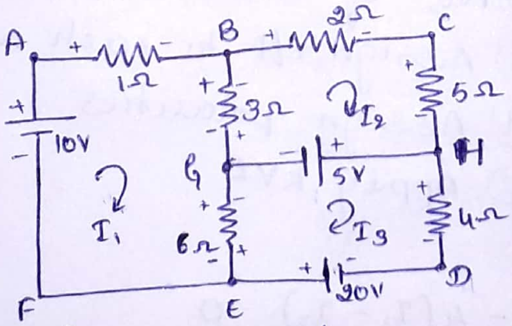
R_1 is in parallel with 4Ω resistor

$\therefore R_{eq} = R_1 \| 4 = \frac{4R_1}{R_1 + R_1} = \frac{4 \times 4}{4+4} = \frac{16}{8} \Rightarrow \boxed{R_{eq} = 2\Omega}$

⑥ Find the current through 5Ω resistor.



Soln:



① we have 3 meshes

② 3 unknown currents

③ Assign current either in clockwise (or) anticlockwise

④ Assign KVL by assigning polarities.

Mesh① → ABEFA → $10 - 1I_1 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$

$\Rightarrow 10 - 10I_1 + 3I_2 + 6I_3 = 0$

$\Rightarrow \boxed{10I_1 - 3I_2 - 6I_3 = 10} \longrightarrow ①$

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Now KVL to mesh (2) → BCHGB

$$-2I_2 - 5I_2 - 5 - 3(I_2 - I_1) = 0$$

$$-10I_2 - 5 + 3I_1 = 0$$

$$\boxed{-3I_1 + 10I_2 = -5} \longrightarrow (2)$$

Now KVL to mesh (3) → GHDEG

$$-4I_3 + 20 - 6(I_3 - I_1) + 5 = 0$$

$$-10I_3 + 6I_1 + 25 = 0$$

$$\boxed{-6I_1 + 10I_3 = 25} \longrightarrow (3)$$

Solving equation (1), (2) & (3).

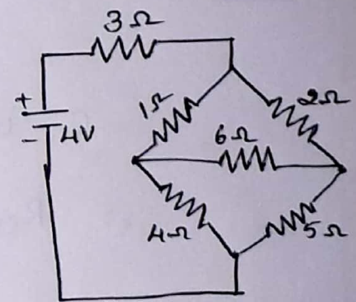
we get $I_1 = 4.27$ A

$I_2 = 0.78$ A

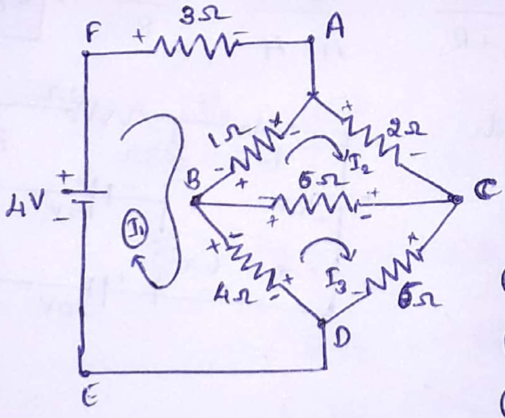
$I_3 = 5.06$ A.

Now $\boxed{I_{5\Omega} = I_2 = 0.78A}$

(7) Find the c/t supplied by the battery for the network shown.



Soln:-



(a) No of meshes = 3

(b) No of unknown currents = 3

(c) Assign c/t to each mesh

(d) Assign polarities

(e) Apply KVL

Mesh (1) → ABDEFA

$$-4 - 3I_1 - 1(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$-4 - 8I_1 + I_2 + 4I_3 = 0$$

$$\boxed{8I_1 - I_2 - 4I_3 = 4} \longrightarrow (1)$$

Mesh (2) → ACBA → $-2I_2 - 6(I_2 - I_3) - 1(I_2 - I_1) = 0$

$$-9I_2 + 6I_3 + I_1 = 0$$

$$\boxed{I_1 - 9I_2 + 6I_3 = 0} \rightarrow (2)$$

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Mesh ③ (BCDB)

$\rightarrow -5I_3 - 4(I_3 - I_1) - 6(I_3 - I_2) = 0$

$$\boxed{4I_1 + 6I_2 - 15I_3 = 0} \longrightarrow ③$$

Solving eq^n ①, ② & ③,

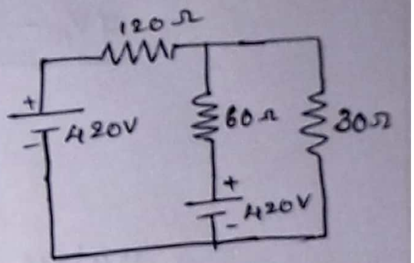
we get $I_1 = 0.67\,A$

$I_2 = 0.26\,A$

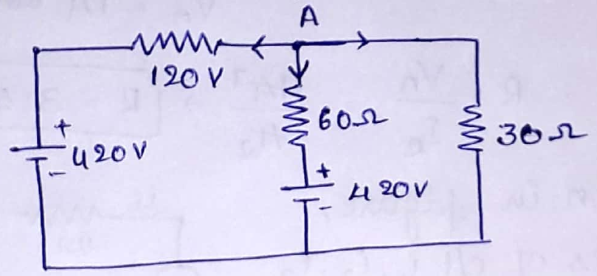
$I_3 = 0.28\,A$

Now, c/t supplied by battery $= \boxed{I_1 = 0.67\,A}$

⑧ In the ckt shown in the figure, current I through resistor of 30Ω is



Soln:



Applying nodal analysis at point A,

we get, $$\frac{V_A - 420}{120} + \frac{V_A - 420}{60} + \frac{V_A - 0}{30} = 0$$

$$\frac{V_A - 420 + 2V_A - 840 + 4V_A}{120} = 0$$

$$7V_A - 1260 = 0$$

$$7V_A = 1260$$

$$V_A = \frac{1260}{7} = 180\,V$$

Now, c/t through 30Ω resistor is

$$I_{30\Omega} = \frac{V_A}{30} = \frac{180}{30} \qquad \boxed{I_{30\Omega} = 6\,A}$$

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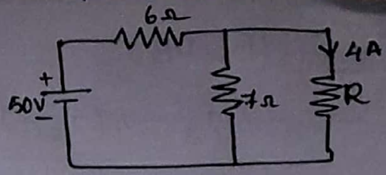
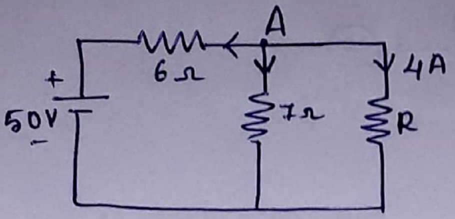
⑨ The value of resistance R, for the n/w shown in figure is



Soln:



S. Applying nodal analysis at point A,

$$\frac{V_A - 50}{6} + \frac{V_A - 0}{7} + 4 = 0$$

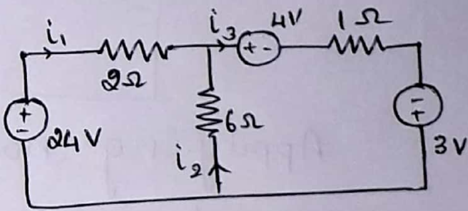
$$\frac{7V_A - 350 + 6V_A + 168}{42} = 0 \Rightarrow 13V_A - 182 = 0$$

$$13V_A = 182$$

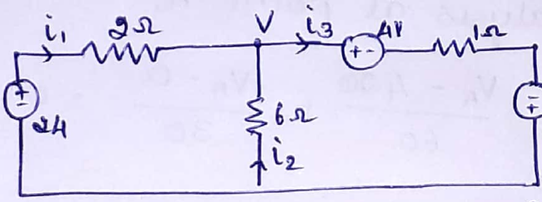
$$V_A = 14 \text{ volts}$$

$\Rightarrow$ Value of R is, $R = \frac{V_A}{I_R} = \frac{\cancel{14}^{7}}{\cancel{4}_{2}} \Rightarrow \boxed{R = 3.5\,\Omega}$

⑩ For the ckt shown in figure, determine the values of c/t i_1, i_2, i_3



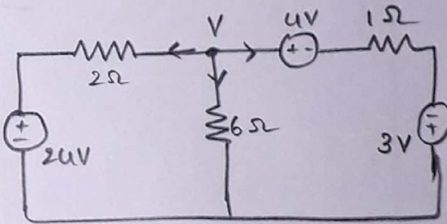
Soln:



By KCL at node V $\Rightarrow i_3 = i_1 + i_2$

By applying nodal analysis at point V

For nodal analysis



$$\frac{V - 24}{2} + \frac{V}{6} + \frac{V - 4 + 3}{1} = 0$$

$$3V - 72 + V + 6V - 6 = 0$$

$$10V = 78 \Rightarrow \boxed{V = 7.8 \text{ volts}}$$

Now $i_1 = \frac{24 - V}{2} = \frac{24 - 7.8}{2} \Rightarrow i_1 = 8.1\,A$

$i_2 = \frac{0 - V}{6} = \frac{0 - 7.8}{6} \Rightarrow i_2 = -1.3\,A$

$i_3 = \frac{V - 4 + 3}{1} = \frac{7.8 - 1}{1} \Rightarrow i_3 = 6.8\,A$

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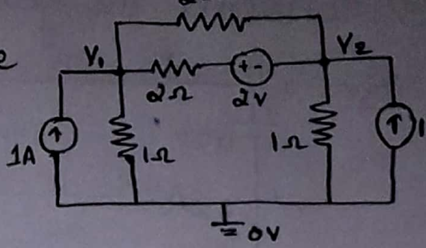
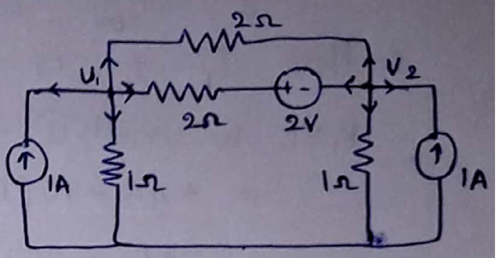
Numericals

① Consider the following ckt, determine the node voltages V_1 and V_2.



Soln: Apply nodal analysis at V_1.



$$-1 + \frac{V_1}{1} + \frac{V_1 - V_2 - 2}{2} + \frac{V_1 - V_2}{2} = 0$$

$$-2 + 2V_1 + V_1 - V_2 - 2 + V_1 - V_2 = 0$$

$$\boxed{4V_1 - 2V_2 = 4} \longrightarrow ①$$

Apply nodal analysis at V_2

$$-1 + \frac{V_2}{1} + \frac{V_2 - V_1 + 2}{2} + \frac{V_2 - V_1}{2} = 0$$

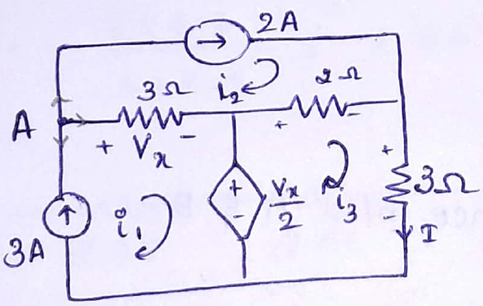
$$-\cancel{2} + 2V_2 + V_2 - V_1 + \cancel{2} + V_2 - V_1 = 0$$

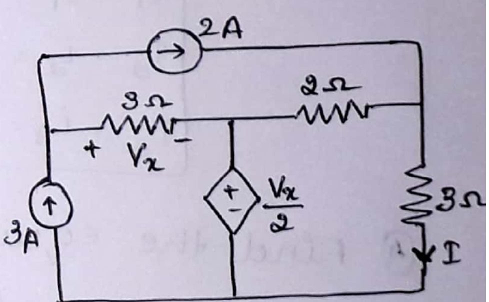
$$\boxed{-2V_1 + 4V_2 = 0} \longrightarrow ②$$

Solving eqⁿ ① & ②, we get $\boxed{V_1 = \frac{4}{3} \text{ volts} \quad V_2 = \frac{2}{3} \text{ volts}}$

② Consider the ckt & determine the value of current I.

Soln:





Apply KCL at node A, $-3 + 2 + \frac{V_x}{3} = 0$

$$\frac{V_x}{3} = 1 \Rightarrow \boxed{V_x = 3 \text{ volts}}$$

So here, $i_1 = 3A$, $i_2 = 2A$ & $i_3 = I$

Apply KVL to mesh ③,

$$\frac{V_x}{2} - 2(i_3 - i_2) - 3i_3 = 0$$

$$\frac{3}{2} - 2(i_3 - 2) - 3i_3 = 0$$

$$1.5 - 2i_3 + 4 - 3i_3 = 0$$

$$5i_3 = 5.5$$

$$\boxed{I = i_3 = 1.1 \text{ A}}$$

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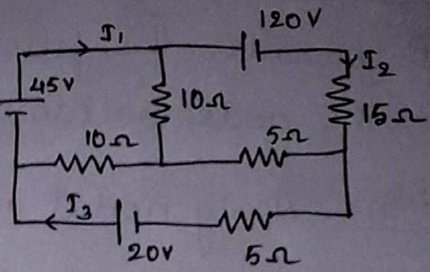
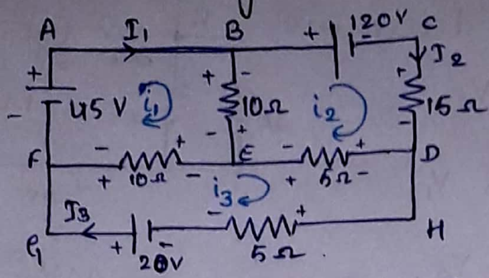
③ Consider the given n/w, determine I_1, I_2, I_3



Soln:



Here $i_1 = I_1$, $i_2 = I_2$, $i_3 = I_3$

KVL to mesh ① → (ABEFA)

$\Rightarrow +45 - 10(i_1 - i_2) - 10(i_1 - i_3) = 0$

$45 - 20i_1 + 10i_2 + 10i_3 = 0$

$\boxed{+20i_1 - 10i_2 - 10i_3 = 45} \rightarrow ①$

Mesh ② (BCDEB) $\rightarrow -120 - 15i_2 - 5(i_2 - i_3) - 10(i_2 - i_1) = 0$

$-120 - 30i_2 + 5i_3 + 10i_1 = 0$

$\boxed{10i_1 - 30i_2 + 5i_3 = 120} \rightarrow ②$

Mesh ③ (DHGFD) $\rightarrow 20 - 10(i_3 - i_1) - 5(i_3 - i_2) - 5i_3 = 0$

$20 - 20i_3 + 10i_1 + 5i_2 = 0$

$\boxed{10i_1 + 5i_2 - 20i_3 = -20} \rightarrow ③$

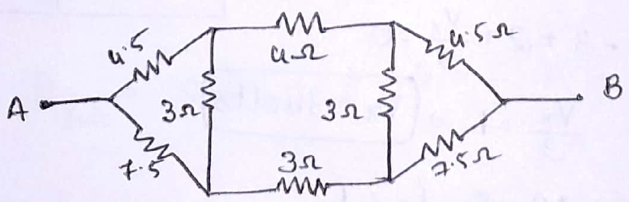
Solving eq^n ①, ② & ③, we get

$I_1 = i_1 = 0.522A$

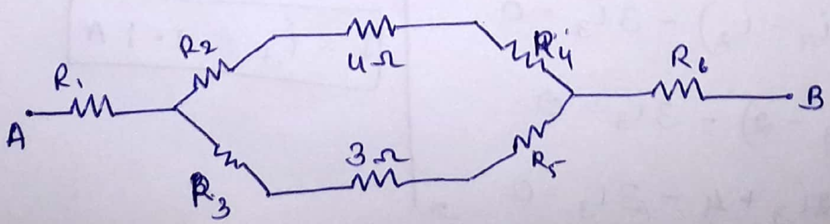
$I_2 = i_2 = -3.77A$

$I_3 = i_3 = 0.318A$

④ Find the equivalent resistance b/w A & B.



Soln: Converting two delta n/w formed by 7.5Ω, 3Ω & 4.5Ω into star n/w.



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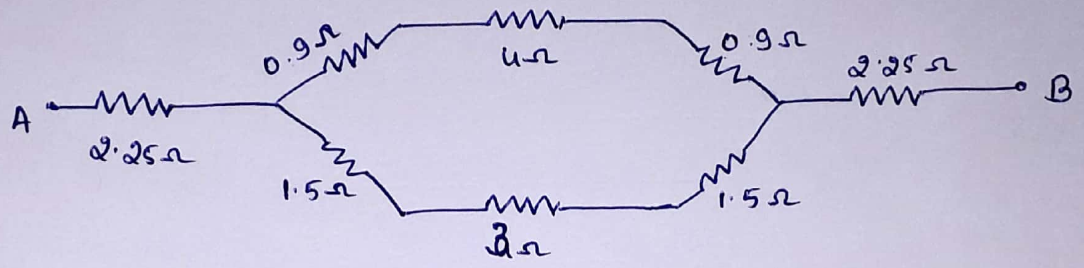
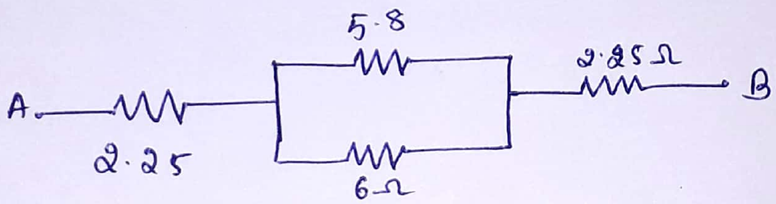
$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\,\Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\,\Omega$$

$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\,\Omega.$$

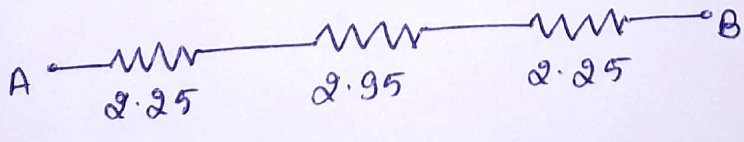


$\Rightarrow$ $0.9\Omega + 4\Omega + 0.9\Omega = 5.8\,\Omega$
$1.5\Omega + 3\Omega + 1.5\Omega = 6.0\Omega$ } because they are in series

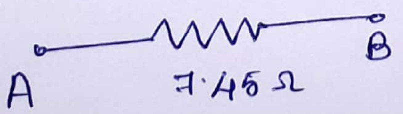


$\Rightarrow$ $6\,\Omega$ and $5.8\,\Omega$ are in parallel

$$\therefore \frac{6 \times 5.8}{6 + 5.8} = 2.95\,\Omega$$



$$\Rightarrow R_{AB} = 2.25 + 2.95 + 2.25 = 7.45\,\Omega$$



$$\therefore \boxed{R_{AB} = 7.45\,\Omega}$$

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