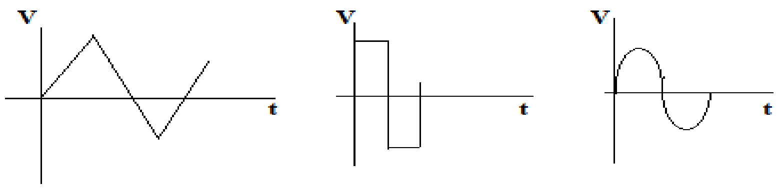
CHAPTER 2 : AC CIRCUITS

INTRODUCTION

All alternating waveform changes its magnitude and direction periodically. Figure shows various AC waveforms.



Many times alternating voltages and currents are represented by a sinusoidal waveforms. A sinusoidal voltage can be represented as

$$v = V_m \sin\theta$$
$$= V_m \sin\omega t$$
$$= V_m \sin 2\pi f t \text{ (wkt } f = 1/T)$$
$$\mathbf{v = V_m \sin\frac{2\pi}{T} t}$$

TERMS RELATED WITH ALTERNATING QUANTITIES

1. **Waveform**: A waveform is a graph in which the instantaneous value of any quantity is plotted against time.
2. **Cycle**: One complete set of positive and negative values of an alternating quantity is termed as cycle
3. **Frequency**: The number of cycles per second of an alternating quantity is known as frequency. It is denoted by 'f' and ins expressed in hertz (Hz) or cycles/sec (C/s)
4. **Time Period**: The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by 'T' and is expressed in seconds. ;T = $\frac{1}{f}$ sec
5. **Amplitude:** The max positive or negative values of an alternating quantity is called amplitude.
6. **Phase :** The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.
7. **Phase difference:** This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach the max and zero values at the same time. Their max value may be different in magnitude.

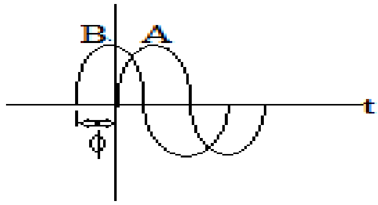
   A leading alternating quantity is one which reaches its max or zero value earlier as compared to the other quantity.

   A lagging alternating quantity is one which attains its max or zero value later as compared to the other quantity.

   A plus (+) sign which used in connection with the phase difference denotes leading, whereas minus (-) denotes lagging.

$$V_A = V_m \sin\omega t$$
$$V_B = V_m \sin(\omega t + \Phi)$$

Here quantity B leads A by a phase angle Φ



The average value of a waveform

In a.c. circuit applications we are interested in finding out the average value of a waveform, that wave could be sinusoidal, triangular trapezoidal or any other shape.

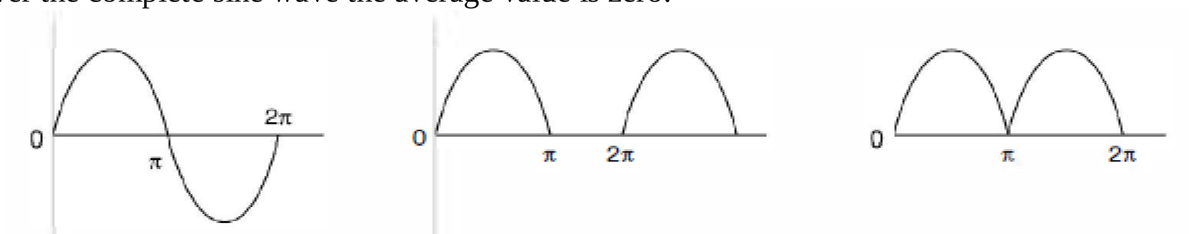
The average value of a cycle of a waveform is the area under the waveform divided by the length of one cycle.

$$V_{av} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{2\pi}\int_0^{2\pi} v\,dt$$

where T is the length of one cycle.

We first consider average value of sinusoidal waves for which we consider the following different shapes.

These are all sine waves in 1(a) since it is a full sine wave, the total area under the curve is zero as half of it is above ωt axis and equal half is below ωt-axis and hence the average area of the curve is zero and hence over the complete sine wave the average value is zero.



(a) (b) (c)

Fig. 1 (a) Full sine wave (b) Half-rectified wave (c) Full rectified wave

However, in Fig. 1(b) the average value is

$$V_{av} = \frac{1}{2\pi}\int_0^{\pi} v\,sin\omega t\,d\omega t$$

$$= \frac{1}{2\pi}[V_m cos\omega t]_0^{\pi}$$

$$= \frac{Vm}{2\pi}(\cos\pi - \cos 0);\ \mathbf{V_{av}} = \frac{\mathbf{Vm}}{\boldsymbol{\pi}}$$

For Fig. 1(c) the average value is j ust double of what we have for a half wave rectified sine wave as over the same period 2π, the area under the curve is double of what we have for the half wave.

i.e., For full rectified wave, average value is $\mathbf{V_{av}} = \frac{\mathbf{2Vm}}{\boldsymbol{\pi}}$

The effective or rms value of a wave

The rms or effective value of a wave is defined as that dc value which when allowed to flow through a particular resistance for a certain time would produce the same heating effect as that produced by the wave. This is represented mathematically as

$$I_{rms} = I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2}$$

As the name rms (root of mean of the squared values) suggests that we have to take the square of the various ordinates and find the area of the squared wave over the period T and find the mean value of this quantity and then obtain its square root. This is the effective value.

We again take first, various sinusoidal waves as shown in Fig. 1.3(a) to (c) and find out their rms values Fig. 1(a).As per the definition we have first to express the wave as per its instantaneous value as every instantaneous value is to be squared and then area under this curve is to be obtained.

Now, for this sine wave

$$I = I_m \sin \omega t$$

Since the wave is to be squared, the negative part of the wave during π to 2π would also become positive and hence rms value over the complete cycle will not be zero.

Hence $\mathbf{I_{rms} = I_{eff} = \frac{Im}{\sqrt{2}}}$

For Fig. 1(b) where it is a half rectified sine wave, the rms value over the complete cycle will be $\mathbf{I_{rms} = I_{eff} = \frac{Im}{2}}$

For Fig. 1.3(c) since it is a full rectified wave and the square of each ordinate will be similar to the full sine wave, the rms value will be same i.e. $\mathbf{I_{rms} = I_{eff} = \frac{Im}{\sqrt{2}}}$

Form factor : It is defined as the ratio of rms value of a wave and the average value of the wave.

$$\textbf{Form factor} = \frac{\textbf{rms value of the wave}}{\textbf{average value of the wave}}$$

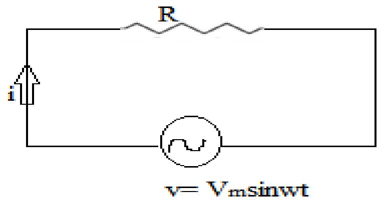
Peak factor or crest factor : It is the ratio of the peak value of the wave to its rms value.

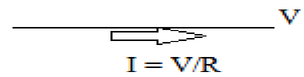
$$\textbf{Peak factor} = \frac{\textbf{rms value of the wave}}{\textbf{average value of the wave}}$$

	Form factor	**Peak factor**
Sinusoidal wave	1.11	$\sqrt{2} = 1.414$
Triangular wave	$\frac{2}{\sqrt{3}}$	$\sqrt{3} = 1.732$

SINGLE PHASE SERIES CIRCUITS:

1. **PURE RESISTIVE CIRCUITS :** A pure resistive or a non inductive circuit is a circuit which has inductance so small the at normal frequency its reactance is negligible as compared to its resistance. Ordinary filament lamps, water resistance etc.. are the examples of pure resistance. If the circuit is non inductive, no reactance emf is set up and whole of the applied voltage is utilized in overcoming the ohmic resistance of the circuit.





When an alternating voltage given by expression $\mathbf{v = V_m sin\omega t}$ is applied across a pure resistive circuit.

The alternating current flowing through the circuit is given as $\mathbf{i = I_m\ sin\omega t}$, where $I_m = V_m/R$

Power in pure resitive circuit is $\mathbf{P = V_{rms} * I_{rms} = VI}$ **watts**

Power factor of pure resistive circuit is unity.

2. **PURE INDUCTIVE CIRCUITS:** An inductive circuit is a coil with or without an iron core having negligible resistance. Practically pure inductance can never be only a inductive coil, but has small resistance. However a coil of thick copper wire wound on a laminated iron core has negligible resistance and is known as a choke coil.

When an alternating voltage given by expression $\mathbf{v = V_m sin\omega t}$ is applied across a Purely inductive circuit, the alternating current flowing through the circuit is given by

$$\mathbf{i = I_{max}\left(\omega t - \frac{\pi}{2}\right)}$$

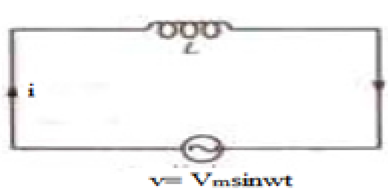
where, $I_{max} = \frac{V_{max}}{X_L}$; Inductive resistance X_L being equal to $2\pi fL$ (or) ωL ohms.

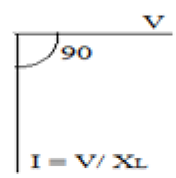
($X_L = 2\pi fL = \omega L$)

Current **lags** voltage by an angle 90^0

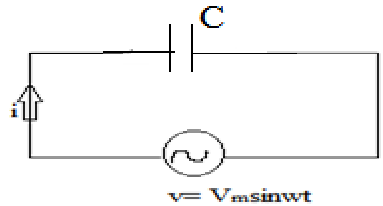
power absorbed by a pure inductor is zero.

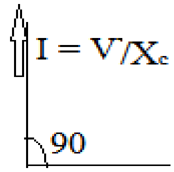
Power factor (PF) of a pure inductive circuit is zero.





3. **PURE CAPACITIVE CIRCUITS:** When DC voltage is impressed across the plates of a perfect condenser, it will become charged to full voltage almost instantaneously. The charging current will flow only during the period of ‘build up’ and will sease to flow as soon as the capacitor has attained the steady voltage of the source. This implies that for a direct current a capacitor is a break in the circuit or an infinite high resistance.





When a capacitor is connected to an AC supply source the capacitor is charged and discharged during alternate quarter cycles.

In a pure capacitive circuit, if $\mathbf{v = V_m sin\omega t}$

$$\mathbf{I = I_m sin(\omega t + \frac{\pi}{2})}$$

Where $I_m = V_m / X_c$; the capacitive resistance X_c being equal to $\frac{1}{\omega C}$ or $1/2\pi fC$ ohms.

$(x_c = \frac{1}{\omega C} = \frac{1}{2\pi f C})$

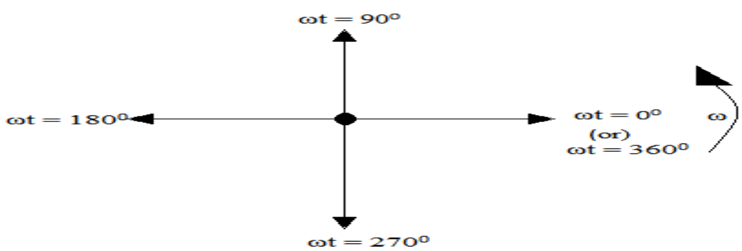
Current **leads** voltage by an angle 90^0

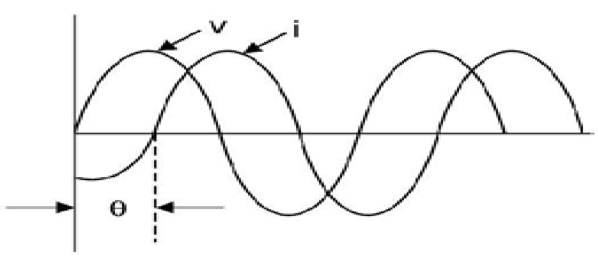
power absorbed by a pure capacitor is zero.

Power factor (PF) of a pure capacitive circuit is zero.

PHASOR DIAGRAMS:

- A geographical or pictorial representation of phases is known as phasor diagram.
- Phasor diagram is a pictorial representation of all the phase voltages and their respective currents in a network.
- Phasor is a **rotating vector**, which rotates in the **anticlockwise direction** with angular frequency 'ω' in the time domain.





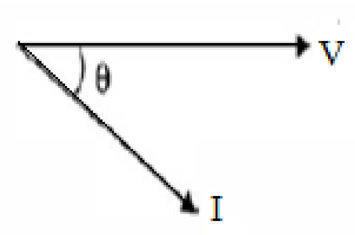


Fig: 2(a) VI sinusoidal wave 2(b) Phasor diagram of 2(a)

At each point on the time waveform, the angle of current lag is θ because the angle between the phasors V_m and I_m is at all time θ. Therefore, either the time waveform of the rotating phasors or the phasor diagram, can be used to describe the system. Since both the diagrams, the time diagram and the phasor diagram convey the same information, the phasor diagram being much more simpler, it is used for an explanation in circuit theory analysis. Since electrical data is given in terms of rms value, we draw phasor diagram with phasor values as rms rather than peak value used so far.

1) AC through series R-L circuit:

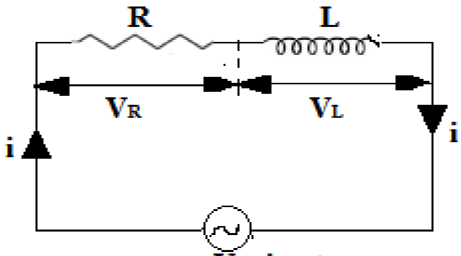


Fig 3(a): series RL circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in series with a pure inductance(L) in henry as shown in fig 3(a).

The series combination is connected across AC supply given by $v = V_m \sin\omega t$

Circuit draws current I, then the two voltage drops are given by

a) Drop across pure resistance $V_R = IR$

b) Drop across pure inductance $V_L = IX_L$

Where, $X_L = 2\pi fL$ and I, V_R , V_L are the rms values.

KVL can be applied to get , $V = \overline{V_R} + \overline{V_L}$ ……………………..(1)

$$V = \overline{IR} + \overline{IX_L}$$

Let us draw the phasor diagram, Current I is taken as reference as it is common in both the elements.

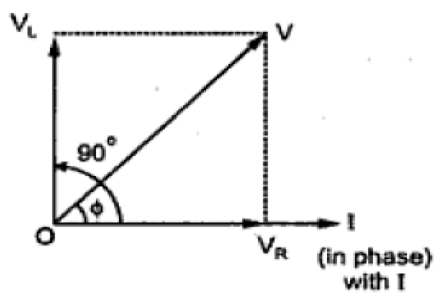


Fig 3(b) : Phasor diagram

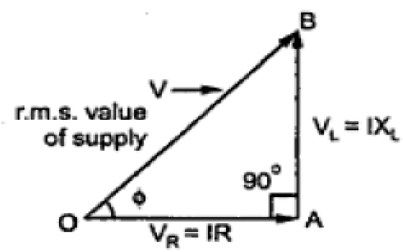


Fig 3(c) : Voltage triangle

❖ **Impedance:** Impedance is defined as the opposition in the circuit for the flow of alternating current. It is denoted by Z and its unit is ohms(Ω).

For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle Φ. From the voltage triangle, we can write,

$$\tan\Phi = \frac{V_L}{V_R} = \frac{X_L}{R}; \quad \cos\Phi = \frac{V_R}{V} = \frac{R}{Z}; \quad \sin\Phi = \frac{V_L}{V} = \frac{X_L}{Z}$$

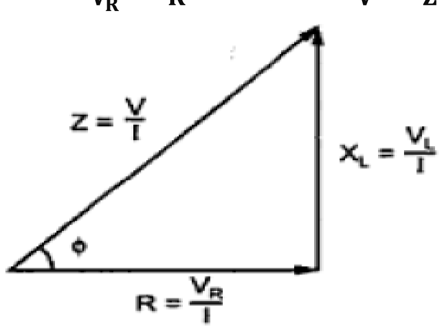


Fig 3(d) : Impedance triangle

If all the sides of voltage triangle are divided by current I, we get the triangle called impedance triangle as shown in fig 3(d).

Sides of this triangle are resistance R, inductive reactance X_L and an impedance Z.

From the impedance triangle, resistance R is given by $\mathbf{R = Z\cos\Phi}$

Inductive reactance X_L is given by $\mathbf{X_L = Z\sin\Phi}$

In rectangular form, impedance is given by $\mathbf{Z = R + jX_L\ \Omega}$

In polar form, impedance is given by $\mathbf{Z = |Z|\angle\Phi}$

$$\text{Where, } |\mathbf{Z}| = \sqrt{\mathbf{R^2 + X_L^2}}\ ;\ \Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

❖ **<u>Power and Power triangle</u>:**

- Average power $P_{av} = \frac{V_m I_m}{2}\cos\Phi = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} *\cos\Phi$

P = VI watts where V and I are rms values

If we multiply eq^n (1) by I, we get

$$\overline{V}I = \overline{IV_R} + \overline{IV_L}$$
$$\overline{V}I = \overline{V\cos\Phi I} + \overline{V\sin\Phi I}$$

From the above equation, power triangle can be obtained as shown in fig 3(e).

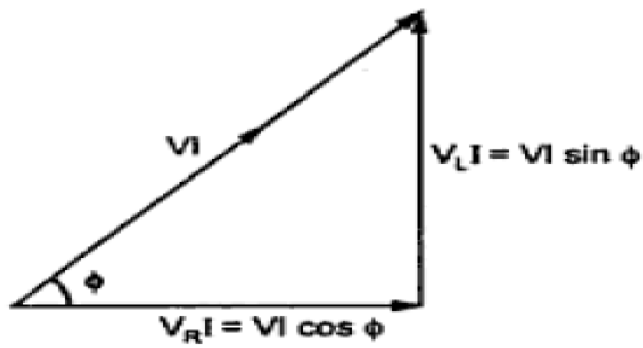


Fig 3(e) : Power triangle

So, the three sides of the triangle are (i) VI (ii) VIcosΦ (iii) VIsinΦ

❖ **<u>Apparent power (S):</u>** It is defined as the product of rms values of voltage(v) and current(I). It is denoted by S. Its unit is volt-amp(VA) or kilo volt-amp(KVA) **S = V*I VA**

❖ **<u>Real or True power (P):</u>** It is defined as the product of applied voltage and active component of current. It is the real component of apparent power. It is represented by 'P' and its unit is watts(W) or kilo watts(KW). **P = VIcos Φ watts**

❖ **<u>Reactive power (Q):</u>** It is defined as the product of applied voltage and reactive component of current. It is also defined as the imaginary component of apparent power. It is represented by 'Q' and its unit is volt-ampere reactive(VAR) or (KVAR). **Q = VIsin Φ VAR**

❖ **<u>Power factor (cos Φ):</u>** It is defined as a factor by which apparent power must be multiplied in order to obtain the true power.

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}} = \frac{\text{VI}\cos\Phi}{\text{VI}} = \cos\Phi$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. **It is always less than 1.**

It is also defined as the ratio of resistance to impedance. $\mathbf{\cos\Phi = \frac{R}{Z}}$

NOTE: *If current lags voltage, power factor is said to be lagging, if current leads voltage, power factor is said to be leading.*

So ,for pure inductance, the power factor is $cos(90°)$ *ie.,zero power factor(ZPF) lagging, while for pure capacitance, the power factor is* $\sin(90°)$ *ie.,zero power factor(ZPF) leading. For a purely resistive circuit volatge and current are in phase ie.,* $\Phi = 0°$*. Therefore, power factor is* $cos(0°) = 1$*. Such circuit is called unity power factor(UPF) circuit.*

$$\boldsymbol{Power\ factor\ = \cos\Phi}$$ *, where Φ is the angle between voltage and current.*

2) AC through series R-C circuit:

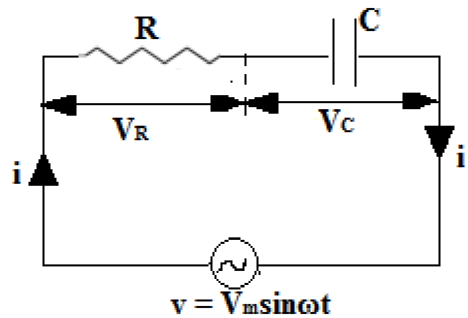


Fig 4(a): series RC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in series with a pure capacitance(C) in farad as shown in fig 4(a).

The series combination is connected across AC supply given by $v = V_m \sin\omega t$

Circuit draws current I, then the two voltage drops are given by

a) Drop across pure resistance $V_R = IR$

b) Drop across pure capacitance $V_C = IX_C$

Where, $X_C = \frac{1}{2\pi fC}$ and I, V_R , V_C are the rms values.

KVL can be applied to get , $V = \overline{V_R} + \overline{V_C}$ ……………………..(1)

$$V = \overline{IR} + \overline{IX_C}$$

Let us draw the phasor diagram, Current I is taken as reference as it is common in both the elements.

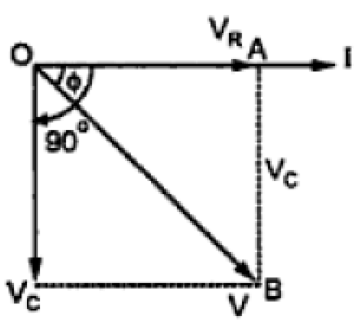


Fig 4(b) : Phasor diagram

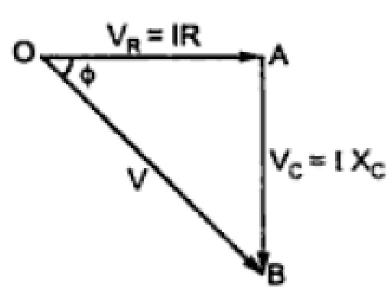
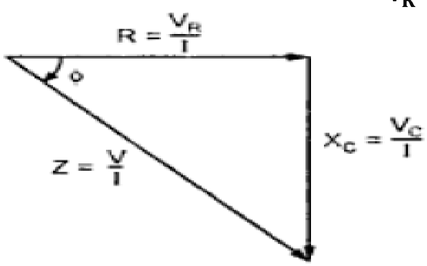


Fig 4(c) : Voltage triangle

❖ **Impedance:** Impedance is defined as the opposition in the circuit for the flow of alternating current. It is denoted by Z and its unit is ohms(Ω).

For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle Φ. From the voltage triangle, we can write,

$$\tan\Phi = \frac{V_L}{V_R} = \frac{X_L}{R}; \quad \cos\Phi = \frac{V_R}{V} = \frac{R}{Z}; \quad \sin\Phi = \frac{V_L}{V} = \frac{X_L}{Z}$$



If all the sides of voltage triangle are divided by current I, we get the triangle called impedance triangle as shown in fig 4(d).

Sides of this triangle are resistance R, inductive reactance X_C and an impedance Z.

Fig 4(d) : Impedance triangle

From the impedance triangle, resistance R is given by **$R = Z\cos\Phi$**

Inductive reactance X_L is given by **$X_L = Z\sin\Phi$**

In rectangular form, impedance is given by **$Z = R - jX_C\ \Omega$**

In polar form, impedance is given by **$Z = |Z|\angle -\Phi\ \Omega$**

Where, $|Z| = \sqrt{R^2 + X_C{}^2}$; $\Phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$

❖ **Power and Power triangle**:

Average power $P_{av} = \frac{V_m I_m}{2}\cos\Phi = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} * \cos\Phi$

P = VI watts where V and I are rms values

If we multiply eq[n] (1) by I, we get

$$\overline{VI} = \overline{IV_R} + \overline{IV_L}$$
$$\overline{VI} = \overline{V\cos\Phi I} + \overline{V\sin\Phi I}$$

From the above equation, power triangle can be obtained as shown in fig 4(e).

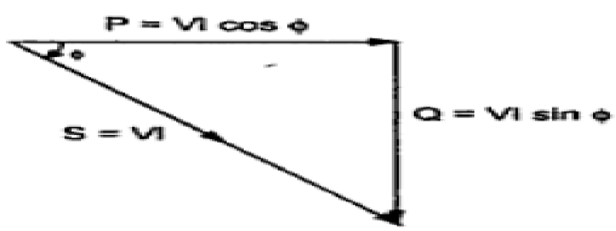


Fig 4(e) : Power triangle

Thus, the various powers are,

a. **Apparent power, $S = VI$ VA**

b. **True or avg power, $P = VI\cos\Phi$ watts**

c. **Reactive power, $Q = VI\sin\Phi$ VAR**

Note: *Remember that, X_L term appears to be positive in Z*

ie., $Z = R + jX_L\ \Omega = |Z|\angle\Phi$, where Φ is positive for Z.

X_C term appears to be negative in Z

ie., $Z = R - jX_C\ \Omega = |Z|\angle -\Phi$, where Φ is negative for Z.

3) **AC through series R-L-C circuit:**

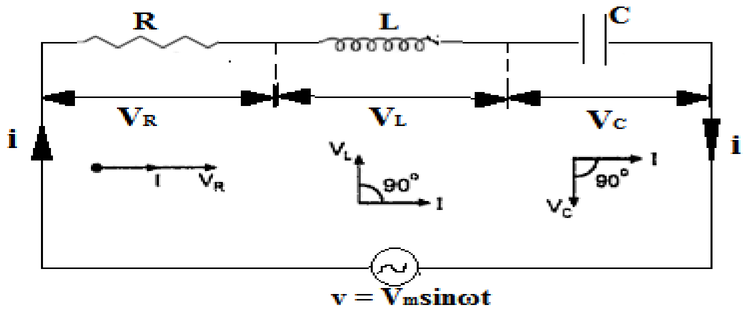


Fig 5(a) : Series RLC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in series with a pure inductance(L) in henry and a pure capacitance(C) in farad as shown in fig 5(a).

The series combination is connected across AC supply given by $v = V_m \sin\omega t$

Circuit draws current I, then the two voltage drops are given by

a) Drop across pure resistance $V_R = IR$

b) Drop across pure inductance $V_L = IX_L$; Where, $X_L = 2\pi fL$

c) Drop across pure capacitance $V_C = IX_C$ Where, $X_C = \frac{1}{2\pi fC}$

The values of I, V_R , V_L, V_C are the rms values.

The characteristics of three drops are

(a) V_R is in phase with current I.

(b) V_L leads current by 90°

(c) V_C lags current by 90°

According to kirchoff's law, we can write,

$$\bar{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

Let us draw the phasor diagram, Current I is taken as reference as it is common to all the elements.

The phasor diagram depends on the condition ofmagnitudes of V_L and V_C which ultimately depends on the valurs of X_L and X_C.

Let us consider the different cases,

(i) $X_L > X_C$: When $X_L > X_C$, obviously IX_L ie.,V_L is greater than IX_C ie., V_C. So the resultant of V_L and $V_C$will be directed towards V_L leading current I. Current I will lag the resultant of V_L and V_C ie., $(V_L – V_C)$.

The circuit is said to be inductive in nature. The phasor sum of V_R and $(V_L – V_C)$ gives the resultant supply voltage V. This is shown in fig 5(b).

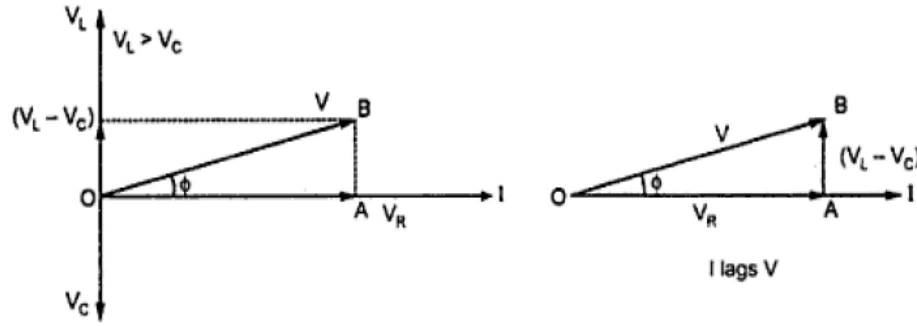


Fig 5(b) : Phasor diagram and voltage triangle for ($X_L > X_C$)

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where, $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$; $(\mathbf{X_L - X_C})$ **is positive**.

(ii) **$X_L < X_C$** : When $X_L < X_C$, obviously IX_C ie.,V_C is greater than IX_L ie., V_L. So the resultant of V_L and V_C will be directed towards V_C lagging current I. Current I will leads the resultant of V_L and V_C ie., $(V_L – V_C)$.

circuit is said to be capacitive in nature. The phasor sum of V_R and $(V_L – V_C)$ gives the resultant supply voltage V. This is shown in fig 5(c).

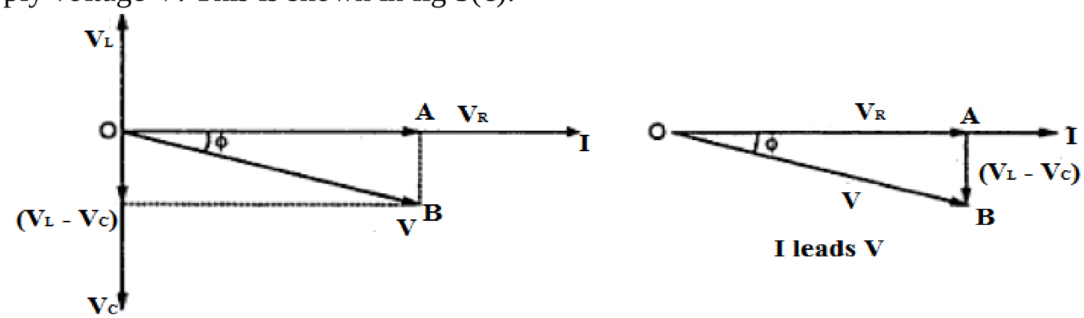


Fig 5(c) : Phasor diagram and voltage triangle for ($X_L < X_C$)

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

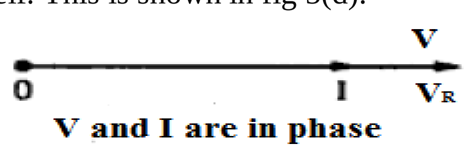
$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where, $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$; $(\mathbf{X_L - X_C})$ **is negative.**

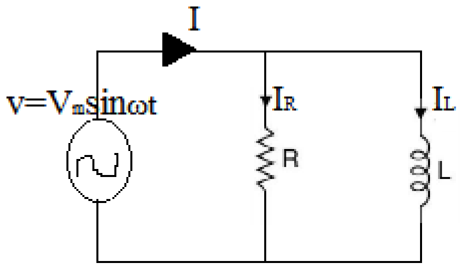
(iii) **$X_L = X_C$:** When $X_L = X_C$, obviously IX_C ie.,V_C is equal to IX_L ie., V_L ie., $(V_L – V_C) = 0$. Current I be in phase with the voltage V_R.

circuit is said to be resistive in nature. The phasor sum of V_R and $(V_L – V_C)$ gives the resultant supply voltage V ie., V_R itself. This is shown in fig 5(d).



$V = IZ$, where $Z = R$

4. AC through parallel R-L circuit



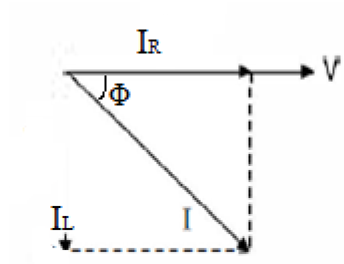


Fig :6(a) **Fig: 6(b)**

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure inductance(L) in henry as shown in fig 6(a).

We have taken here V as the reference quantity as it is the voltage which is common to both the elements R and L and not the current. Now current through R will be in phase with the voltage across it whereas current through inductor will lag by 90° the voltage across it. Hence, the phasor diagram in Fig. 6(b) follows and current I supplied by the source equals the phasor sum of the current I_1 and I_2.

Now, the current is given by , $I_R = \frac{V}{R}$; $I_L = \frac{V}{X_L} \angle -90°$; $I = I_R + I_L = \frac{V}{R} + \frac{V}{j\omega L}$

Here, $\frac{1}{R}$ is the reciprocal of resistance and is termed as conductance(G), whereas $\frac{1}{\omega L}$ is the reciprocal of the inductive reactance and is known as inductive susceptance(B_L).

From the above equation, we have

$$\frac{I}{V} = \frac{1}{R} + \frac{1}{j\omega L} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Now dimensionally $\frac{I}{V}$ is reciprocal of impedance and is known as admittance(Y). The above equation can be rewritten as **$Y = G - jB_L$**

Where, Impedance $Y = \frac{1}{Z}$; Conducatnce $G = \frac{1}{R}$; Susceptance $B_L = \frac{1}{\omega L}$

It is to be noted that whereas for an inductive circuit $Z = R + jX_L$, the admittance is $Y = G - jB_L$

The units of admittance is mho or siemens and denoted by ℧ or S respectively.

The power consumed by the circuit is again $P = VI\cos\Phi$ where I is the total current and Φ is the angle between V and I. $\Phi = \tan^{-1}\left(\frac{-B_L}{G}\right)$; Power factor , $\cos\Phi = \frac{G}{Y} = \frac{I_R}{I}$ (lag)

From eq^n (1), we have $\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L}$

If $Z_1 = R$ and $Z_2 = j\omega L$, we have

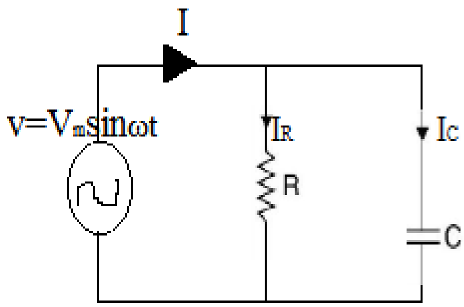
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \; ; \; Z = \frac{Z_1 * Z_2}{Z_1 + Z_2} \qquad \ldots\ldots\ldots(2)$$

i.e. if there are two impedances Z_1, Z_2 connected in parallel, their equivalent impedance equals the ratio of the product of the two impedance to the sum of the two impedances. Also from eq^n (2)

$$\mathbf{Y = Y_1 + Y_2}$$

i.e. if there are two admittances Y_1 and Y_2 connected in parallel, the equivalent admittance is the sum of the two admittances.

5. AC through parallel R-C circuit



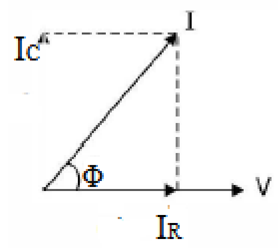


Fig 7(a) **Fig 7(b)**

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure capacitance(c) in farad as shown in fig 7(a).

We have taken here V as the reference quantity as it is the voltage which is common to both the elements R and C and not the current. Now current through R will be in phase with the voltage across it whereas current through capacitor will lead by 90° the voltage across it. Hence, the phasor diagram in Fig. 7(b) follows and current I supplied by the source equals the phasor sum of the current I_1 and I_2.

Now, the current is given by, $I_R = \frac{V}{R}$; $I_C = \frac{V}{X_C} \angle 90°$; $I = I_R + I_C = \frac{V}{R} + jV\omega C$

Here, $\frac{1}{R}$ is the reciprocal of resistance and is termed as conductance(G), whereas ωC is the reciprocal of the capacitive reactance and is known as capacitive susceptance(B_C).

From the above equation, we have

$$\frac{I}{V} = \frac{1}{R} + j\omega C \qquad \ldots\ldots\ldots\ldots(3)$$

Now dimensionally $\frac{I}{V}$ is reciprocal of impedance and is known as admittance(Y). The above equation can be rewritten as **$Y = G + jB_C$**

Where, Impedance $Y = \frac{1}{Z}$; Conductance $G = \frac{1}{R}$; Susceptance $B_C = \omega C$

It is to be noted that whereas for an inductive circuit $Z = R - jX_C$, the admittance is $Y = G + jB_C$

The units of admittance is mho or simens and denoted by ℧ or S respectively.

The power consumed by the circuit is again P = VI cos Φ where I is the total current and Φ is the angle between V and I. $\Phi = \tan^{-1}\left(\frac{B_C}{G}\right)$; Power factor , $\cos\Phi = \frac{G}{Y} = \frac{I_R}{I}$ (lead)

6. AC through parallel R-L-C circuit

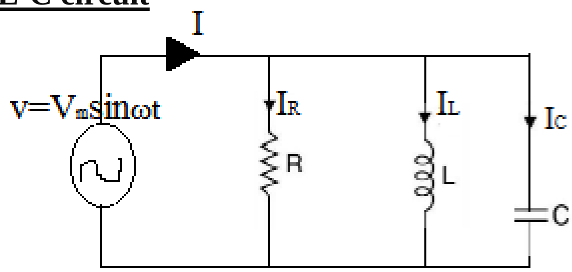


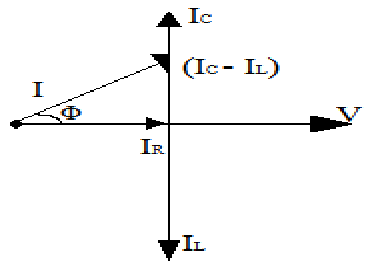
Fig 8(a) : parallel RLC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure inductance(L) in henry and a pure capacitance(C) in farad as shown in fig 8(a).

The parallel combination is connected across AC supply given by $v = V_m \sin\omega t$

The current is given by , $I_R = \frac{V}{R}$; $I_L = \frac{V}{X_L} \angle -90°$; $I_C = \frac{V}{X_C} \angle 90°$

I. $B_C > B_L$ ($I_C > I_L$)

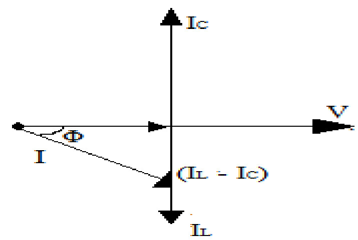


Current, $I = \sqrt{(I_R)^2 + (I_C - I_L)^2}$

Admittance angle, $\Phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right) = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$

Power factor, $\cos\Phi = \frac{I_R}{I}$ (PF lead)

II. **$B_C < B_L$ ($I_C < I_L$)**

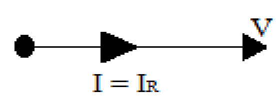


Current, $I = \sqrt{(I_R)^2 + (I_L - I_C)^2}$

Admittance angle, $\Phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right) = \tan^{-1}\left(\frac{B_L - B_C}{G}\right)$

Power factor, $\cos\Phi = \frac{I_R}{I}$ (ZPF lag)

III. $B_L = B_C$ ($I_L = I_C$)



Current, $I = I_R$; $\Phi = 0°$; $\cos\Phi = 1$ (UPF)

Circuit	Voltage and current relation	Impedance(Z) or Admittance(Y)	Phase angle Φ	Phasor relation	Power factor(PF) = cosΦ
Series RL circuit	$V = \sqrt{V_R^2 + V_L^2}$	Z = (R + jX_L)	$\tan^{-1}\left(\frac{X_L}{R}\right)$	I lags V by an angle Φ	$\frac{R}{z} = \frac{V_R}{V}$
Series RC circuit	$V = \sqrt{V_R^2 + V_c^2}$	Z = (R - jX_C)	$\tan^{-1}\left(\frac{-X_C}{R}\right)$	I leads V by an angle Φ	$\frac{R}{z} = \frac{V_R}{V}$
Series RLC circuit	$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$	Z=R + j(X_L-X_C)	$\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$	X_L>X_C; I lags V X_L<X_C; I leads V X_L=X_C;I in phase with V	$\frac{R}{z} = \frac{V_R}{V}$
Parallel RL circuit	$I = \sqrt{I_R^2 + I_L^2}$	Y = (G - jB_L)	$\tan^{-1}\left(\frac{-B_L}{G}\right)$	I lags V by an angle Φ	$\frac{G}{Y} = \frac{I_R}{I}$
Parallel RC circuit	$I = \sqrt{I_R^2 + I_c^2}$	Y = (G + jB_C)	$\tan^{-1}\left(\frac{B_C}{G}\right)$	I leads V by an angle Φ	$\frac{G}{Y} = \frac{I_R}{I}$
Parallel RLC circuit	$I = \sqrt{(I_R)^2 + (I_L - I_C)^2}$	Y=G + j(B_C-B_L)	$\tan^{-1}\left(\frac{B_C - B_L}{G}\right)$	B_L>B_C; I lags V B_L<B_C; I leads V B_L=B_C;I in phase with V	$\frac{G}{Y} = \frac{I_R}{I}$

R = Resistance ; X = Reactance; Z = Impedance

$G = \frac{1}{R}$ = Conductance; $B = \frac{1}{X}$ = susceptance; $Y = \frac{1}{Z}$ = Admittance

<u>Concept of time constant (τ):</u> **Time constant** is the duration in seconds during which the current through a capacities circuit becomes 36.7 percent of its initial value.

Type of circuit	**Formula for time constant (τ in seconds)**
RL circuit	$\tau = \frac{L}{R}$
RC circuit	τ = RC

Resonance: Resonance in electric circuit is because of the presence of both energy storing elements ie., inductor and capacitor.

Under resonance, for an electric circuit, if the applied voltage and resultant current, both are in phase ($\Phi = 0°$) , then the system is under resonance.

$$Z = \text{Resistive (ie., imaginary part = 0)}$$

(OR) The total impedance of any electrical circuit is only resistive (imaginary part = 0) then circuit is under resonance.

(OR) If any electrical circuit is operating at UPF(ie., cos $\Phi = 0$), then the circuit is under resonance.

Resonance

Series Resonance

Parallel Resonance

- R-L-C Parallel circuit
- R-L & C parallel circuit
- R-L & R-C parallel circuit

- **Series Resonance**

  - **R-L-C series circuit:**

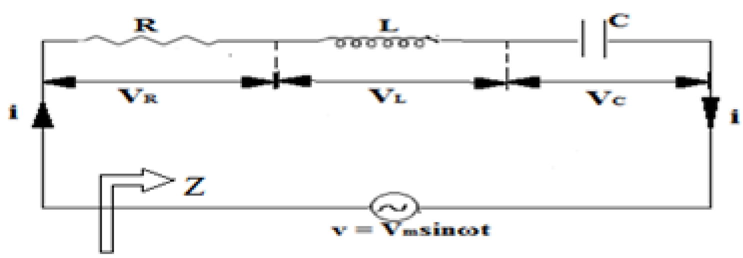


Fig 11: series RLC circuit for resonance

The impedance of the circuit at any frequency ω is given as

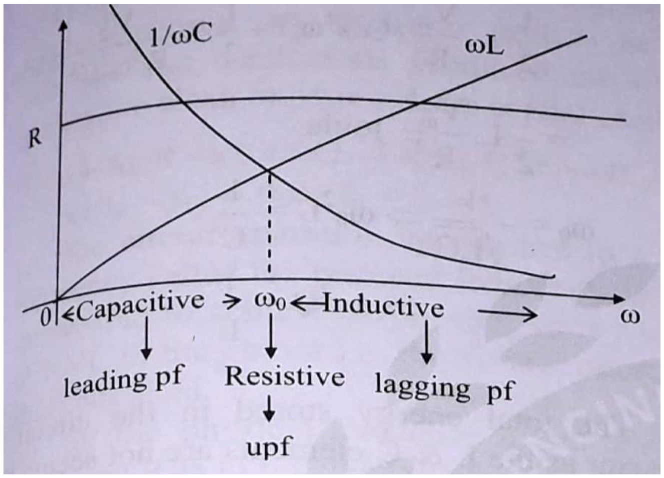
$$Z = Z_R + Z_L + Z_C$$

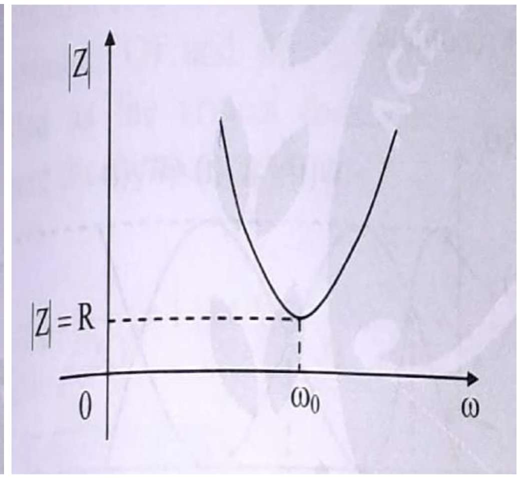
$$Z = R + j(\omega L - \frac{1}{\omega C})$$

Since resistance is independent of frequency the circuit will have minimum impedance at some frequency.

When $Z = R$; Or when, $\omega_o L - \frac{1}{\omega_o C} = 0$; $\boldsymbol{\omega_o = \frac{1}{\sqrt{L\,C}}}$; $\boldsymbol{f_o = \frac{1}{2\pi\sqrt{L\,C}}}$

Here f_o is the frequency of resonance i.e. if a circuit has fixed values of R, L and C, resonance will take place if the supply frequency $f_o = \frac{1}{2\pi\sqrt{L\,C}}$, when the impedance of the circuit is purely resistive i.e. the power factor of the circuit is unity, the supply voltage and current are in phase. However, it is to be noted that the phase relations between the voltage and current in the individual elements R, L and C are not same. The current in the inductor lags its voltage by 90° and in the capacitor it leads its voltage by 90° •





Since R is independent of frequency it is shown by a horizontal line Z = R. Also $X_L = \omega L$ the inductive reactance is a straight line passing through the origin and inductive reactance is taken as +ve, where as $X_C = \frac{1}{\omega C}$ the capacitive reactance as a function of ω is a rectangular hyperbola and the reactance is taken as -ve. The net impedance is shown a positive quantity. The resonance frequency is f_0 where ($\omega L - \frac{1}{\omega C}$) is zero and at this frequency the impedance curve has minimum value equal to R.

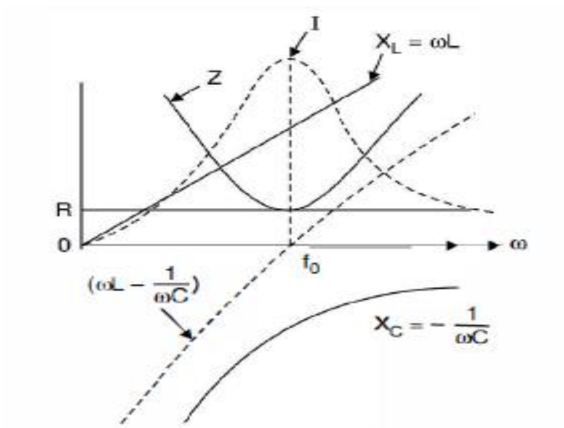


Fig 12: Z Vs ω for series RLC circuit

The variation of current is also shown in Fig 12 as a function of frequency and is maximum at f_0 whereas on either side the current decreases. It is to be noted that at $\omega = 0$, the current in the RLC series circuit is zero as the capacitor reactance is infinite and, therefore, the graph starts from origin whereas it is again zero at $\omega = \infty$ and hence the graph should not be passing through zero rather it should have some finite value as indicated in the diagram.

Again, it can be seen that the series circuit is capacitive for all frequency $\omega < \omega_0$, inductive for all frequency $\omega > \omega_0$ and at $\omega = \omega_0$ the circuit is resistive.

There are various applications of a series resonant circuit where the frequency is fixed and either L or C is varied to obtain the condition of resonance. A typical example is that of tuning a radio receiver to a particular desired station that is operating at a fixed frequency. Here a circuit or L and C is adjusted to resonance at the operating frequency of the desired station. The capacitor C (parallel plate) is variable in most portable radio receivers and the inductance of the coil is usually varied in tuning of an automobile radio receiver.

Under resonance, $I_o = \frac{V}{R}$

Therefore, $V_R = I_oR = \frac{V}{R}*R = V$ is the supply voltage

$V_L = I_o\omega L = \frac{V}{R}\omega L = V\frac{\omega_o L}{R}$

where Q is known as the quality factor of the series network. Usually Q >> 1, hence it is also known as voltage gain, as the voltage across the inductor is much greater than the supply voltage.

Also, $V_C = \frac{I_O}{\omega_O C} = V\frac{1}{\omega_O CR} = VQ;$ $V_L = V_c >> V$

Therefore, extreme care must be taken when working on series circuits that may become resonant when connected to power line sources.

Quality factor (Q):

In a practical circuit R is essentially resistance of the coil since practical capacitors have very low loss in comparison to practical inductor. Hence Q is a measure of the energy storage property (LI^2) in relation to the energy dissipation property (I^2R) of a coil or a circuit.

The Q is, therefore, defined as

$$\mathbf{Q = 2\pi \frac{maximum\ energy\ stored}{energy\ dissipated\ per\ cycle}}$$

In electric circuit energy is stored in the form of electromagnetic field in the inductance where as in electrostatic form of energy across a capacitance. It can be proved that at any instant at a certain frequency the sum of energy stored by the inductor and the capacitor is constant. At the extreme situation when the current through the inductor is maximum, the voltage across the capacitor is zero hence the total energy is

$$\frac{1}{2}L(\sqrt{2}\ I)^2 = LI^2$$

where $\sqrt{2}I$ is the instantaneous maximum value of the current. At this since V_C is zero, therefore maximum energy stored is LI^2. The power consumed per cycle is the energy per sec divided by f_0 under resonance condition. Therefore $P_R = \frac{I^2R}{f_o}$

Hence $Q = \frac{2\pi LI^2}{I^2R}$; $\mathbf{Q = \frac{\omega_o L}{R}}$ (Q for series RL circuit)

NOTE:

Q can also be looked as a ratio of

$$Q = \frac{Time\ rate\ of\ change\ of\ energy\ stored}{Time\ rate\ of\ change\ of\ energy\ dissipated} = \frac{Reactive\ power\ absored\ by\ inductor}{Active\ power\ consumed\ by\ resistor}$$

The great advantage of this definition of Q is that it is also applicable to more complicated lumped circuits, to distributed circuits such as transmission lines and to non-electrical circuits.

Q is also a measure of the frequency selectivity of the circuit. A circuit with high Q will have a very sharp current response curve as compared to one which has a low value of Q. To understand this let us consider Fig. 13 . Here we find that the current response is maximum at f_0 and on either side of f_0, the current decreases sharply.

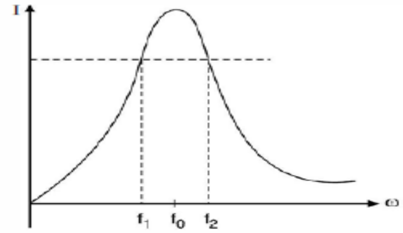


Fig 13 : Frequency selectivity

Frequencies f_1 and f_2 are known as half-power frequencies as at these frequencies the power dissipated by the circuit is half of that dissipated at f_o.

The band width of a resonant circuit is defined as the frequency range between the 70.7% current points. **Bandwidth, BW =** $f_2 - f_1$

Resonant frequency , $\omega_o = \sqrt{\omega_1 \omega_2}$

This means the resonance frequency is the geometric mean of the half-power frequencies.

$\frac{\omega_2 - \omega_1}{\omega_o} = \omega_o CR = \frac{1}{Q}$ (where, $Q = \frac{1}{\omega o CR}$ is the Q factor value for RC series circuit)

$$f_2 - f_1 = \frac{f_o}{Q}$$

Bandwidth is thus given by the ratio of the frequency of resonance to the quality factor and selectivity is defined as the ratio of resonant frequency to the bandwidth $f_0/(f_2 - f_1)$. This, therefore, shows that the larger the value of Q the smaller is $(f_2 - f_1)$ and hence sharper is the current response.

$$\textbf{Selectivity} \propto \textbf{Q-factor} \propto \frac{1}{\text{bandwidth}}; \; Q = \frac{f_o}{f_2 - f_1}\; ; \; Q = \frac{\omega_o}{\omega_2 - \omega_1}$$

NOTE

Q factor for

- *RL series circuit:* $Q = \frac{\omega_o L}{R}$
- *RC series circuit:* $Q = \frac{1}{\omega o CR}$
- *RLC series circuit:* $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

- **Parallel Resonance**
  - **Parallel R-L-C circuit**

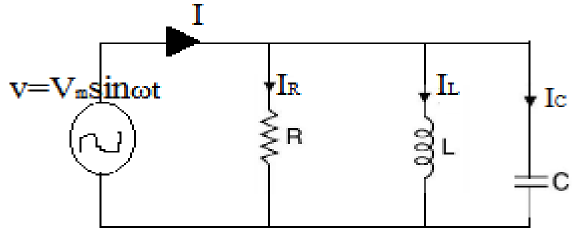


Fig 14 : Parallel RLC circuit

Consider a circuit consisting of a pure resistance(R) in ohms is connected in parallel across a pure inductance(L) in henry and a pure capacitance(C) in farad as shown in fig 14.

Wkt, $Y = G + j(B_C - B_L)$

Under resonance, $(B_C - B_L) = 0$; $B_C = B_L$

$$B_C = \frac{1}{X_C} = \omega_o C \; ; \qquad B_L = \frac{1}{X_L} = \frac{1}{\omega_o L}$$

$$\omega_o C = \frac{1}{\omega_o L}; \qquad \omega_o^2 = \frac{1}{LC} \; ; \qquad \boldsymbol{\omega_o = \frac{1}{\sqrt{LC}}} \text{ rad/sec } ; \qquad \boldsymbol{f_o = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

Therefore, the expression for resonant frequency is $\boldsymbol{f_o = \frac{1}{2\pi\sqrt{LC}}}$ **Hz**

NOTE:

Dynamic impedance at resonance (Z_D): *The impedance offered by the parallel circuit at resistance is denoted as dynamic resistance (Z_D). This is maximum at resistance.*

$$Z_D = \frac{L}{RC} = \text{dynamic resistance}$$

Quality factor of parallel circuit: Parallel circuit is used to magnify the current and hence known as current resonance circuit.

Q factor for

- RL parallel circuit: $Q = \frac{R}{\omega_o L}$
- RC parallel circuit: $Q = \omega_o CR$
- RLC parallel circuit: $Q = R\sqrt{\frac{C}{L}}$

Resonant frequency:

Series RLC	$f_o = \frac{1}{2\pi\sqrt{LC}}$ Hz	$\omega_o = \frac{1}{\sqrt{LC}}$ rad/sec
Parallel RLC	$f_o = \frac{1}{2\pi\sqrt{LC}}$ Hz	$\omega_o = \frac{1}{\sqrt{LC}}$ rad/sec
R-L & C in parallel	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz	$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ rad/sec
R-L & R-C in parallel	$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$ Hz	$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$ rad/sec

NOTE:

Series RLC resonant circuit – acceptor circuit

Parallel RLC resonant circuit – rejector circuit

	Q factor	

	Series	Parallel
R-L	$\frac{\omega_o L}{R} = \frac{X_L}{R}$	$\frac{R}{\omega_o L} = \frac{R}{X_L}$
R-C	$\frac{1}{\omega_o CR}$	$\omega_o CR = \frac{R}{X_C}$
R-L-C	$Q_S = \frac{1}{R}\sqrt{\frac{L}{C}}$	$Q_P = R\sqrt{\frac{c}{L}}$

Problems on RL, RC and RLC circuits:

1. A 50Hz, alternating voltage of 150V (rms) is applied independently to (i) Resistance of 10Ω (ii) Inducatance of 0.2H (iii) Capacitance of 50μF.Find the expression for instantaneous current in each case. Draw the phasor diagram in each case.

Solution: (i) Given : R = 10 Ω

$V = V_m \sin\omega t$

$V_m = \sqrt{2}\, V_{rms} = \sqrt{2} * 150 = 212.13V$

$I_m = \frac{V_m}{R} = \frac{212.13}{10} = 21.213A$

In a pure resistive circuit, current is in phase with voltage,ie., Φ = 0°

$i = I_m \sin\omega t = I_m \sin(2\pi ft)$

$\mathbf{i = 21.213\sin(100\pi t)\ A}$

(ii) **Given ; L = 0.2H**

Inductive reactance $X_L = 2\pi fL = 2\pi*50*0.2 = 62.83\ \Omega$

$I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.37A$

In a pure inductive circuit, current lags voltage by an angle 90° ie., $\Phi = 90° = \frac{\pi}{2}$

$i = I_m \sin(\omega t - \Phi)$

$\mathbf{i = 3.37\sin(2\pi ft - \frac{\pi}{2})\ A}$

(iii) **Given C = 50μF**

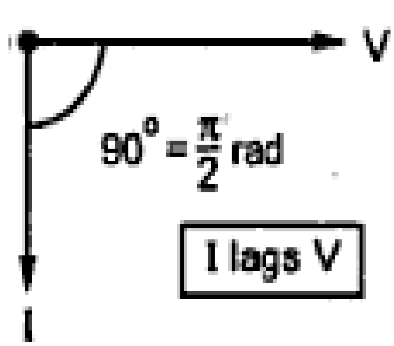
Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi*50*50*10^{-6}} = 63.66\ \Omega$

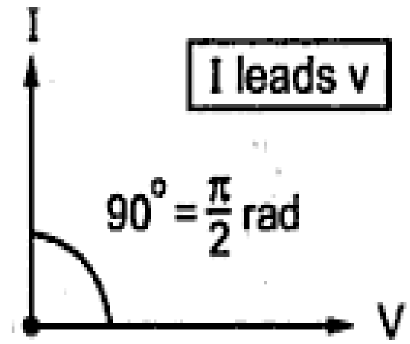
$I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33A$

In a pure capacitive circuit, current leads voltage by an angle 90° ie., $\Phi = 90° = \frac{\pi}{2}$

$i = I_m \sin(\omega t + \Phi)$ $\mathbf{i = 3.33\sin(2\pi ft + \frac{\pi}{2})\ A}$

Phasor diagrams

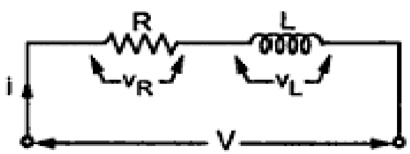




(i) Pure resistive circuit (ii) Pure inductive circuit (iii) Pure capacitive circuit

2. An alternating current, $i = 1.414\sin(2\pi*50*t)$A is passed through a series circuit consisting of a resistance of 100Ω and inductance of 0.31831H. Find the expression for instantaneous values of volatge across (i) resistance (ii)inducatabce (iii) combination

Sol^n :



Given : $i = 1.414\sin(2\pi*50*t)$A , f = 50Hz , $\omega = 2\pi f = 2*\pi*50$, R = 100Ω , L = 0.31831H ,
$X_L = 2\pi fL = 2*\pi*50*0.31831 = 100\Omega$

(i) The voltage across resistance is,

$V_R = iR = 1.414\sin(2\pi*50*t)*100$

$\mathbf{V_R = 141.4\sin(2\pi*50*t)\ V}$

(ii) The voltage across inductor leads current by an angle 90°,

$V_L = iX_L = 1.414\sin(2\pi*50t + \Phi) * 100$

$\mathbf{V_L = 141.4\sin(2\pi*50t + 90°)V}$

(iii) From the expression of V_R, we can write,

$V_{R(rms)} = \frac{V_R}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100V$, $\Phi = 0°$

$V_{R(rms)} = 100\angle 0° = 100 + j0$ V

$V_{L(rms)} = \frac{V_L}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100V$, $\Phi = 90°$

$V_{L(rms)} = 100\angle 90° = 0 + j100$ V

$V = 100 + j0 + 0 + j100$

$\quad = 100 + j100 = 141.42\angle 45°$

$V_m = \sqrt{2} * 141.42 = 200V$

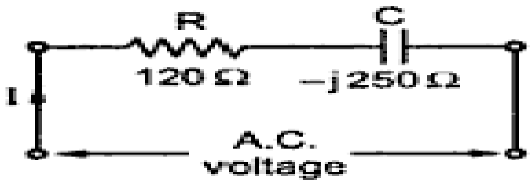
Hence the expression for instantaneous value of resultant voltage is,

$\mathbf{V = 200\sin(2\pi*50t + 45°)V}$

3. A resistance of 120Ω and a capacitive reactance of 250 Ω are connected in series across a AC voltage source. If the current of 0.9A is flowing in a circuit, then find (i) Power factor (ii) Supply voltage

(iii) Voltage across resistance and capacitance (iv) Active and reactive power

Sol^n :



Given : R = 120 Ω , X_C = 250 Ω , I = 0.9∟0°A

Z = R – jX_C = 120 – j250 Ω = 277.308∟-64.358°

(i) Power factor = cosΦ = cos(-64.358°) ; **PF = 0.4327 leading**

(ii) Supply voltage, V = I*Z = (0.9∟0°)*(277.308∟-64.358°)

V = 249.5772∟-64.358° V

(iii) V_R = IR = 0.9*120 = 108V

V_C = IX_C = 0.9*250 = 225V

(iv) P = VIcosΦ = 249.5772*0.9*0.4327

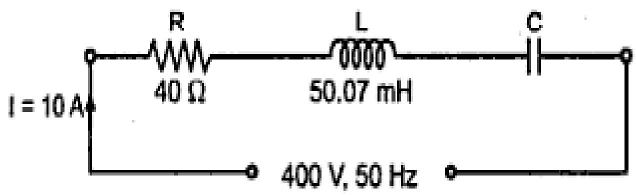
P = 97.1928W

Q = VIsinΦ = 249.5772*0.9*sin(-64.358°)

Q = -202.49VAR

4. A series circuit having pure resistance of 40Ω, pure inductance of 50.07mH and a capacitor is connected across a 400V, 50Hz, AC supply. This R, L , C combination draws acurrent of 10A. Calculate (i) Power factor of the circuit (ii) Capacitor value

Sol^n:



Given: R = 40 Ω , L = 50.07mH , I = 10A

$X_L = 2\pi fL = 2\pi * 50*50.07*10^{-3} = 15.73\ \Omega$

$Z = R + j(X_L – X_C)$

$|Z| = \sqrt{(R)^2 + (X_L - X_C)^2} = \sqrt{(40)^2 + (15.73 - X_C)^2}$

$|Z| = \frac{|V|}{|I|} = \frac{400}{10} = 40\ \Omega$

$40 = \sqrt{(40)^2 + (15.73 - X_C)^2} = 1600 = 40^2 + (15.73 – X_C)^2$

$(15.73 – X_C)^2 = 0$; $X_C = 15.73\ \Omega$

$X_C = \frac{1}{2\pi fC}$; $C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi*50*15.73} = 2.02358*10^{-4}$

C = 202.358μF

Z = 40 + j(15.73 – 15.73) = 40 + j0 Ω = 40∟0°

PF = cosΦ = cos(0°); **PF = 1 (unity power factor)**

(iv) **W_1 = -2000 watts, W_2 = 2000 watts**)

① A coil having a resistance of 7 Ω and an inductance of 31.8 mH is connected to 230V, 50 Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor (d) power consumed (v) voltage drop across resistor and inductor.

Soln:- (i) Inductive Reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 3.18 m = 10$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$$

$$\text{Current},\ I = \frac{V}{Z} = \frac{230}{12.2} = 18.85\ A \qquad \boxed{I = 18.85 A}$$

(ii) Phase angle, $\phi = \tan^{-1}\left[\frac{X_L}{R}\right] = \tan^{-1}\left[\frac{10}{7}\right]$

$$\boxed{\phi = 55^\circ \text{ lag}}$$

(iii) Power factor. $\cos\phi = \cos(55^\circ) \Rightarrow \boxed{\cos\phi = 0.573 \text{ lag}}$

(iv) Power consumed, $P = V I \cos\phi = 230 \times 18.85 \times 0.573$

$$\boxed{P = 2484.24 \text{ watts}}$$

(v) Voltage drop across $R \Rightarrow V_R = IR = 18.85 \times 7$

$$\boxed{V_R = 131.95 \text{ volts}}$$

Voltage drop across $L \Rightarrow V_L = I X_L = 18.85 \times 10$

$$\boxed{V_L = 188.5 \text{ volts}}$$

② An inductor coil is connected to a supply of 250V at 50Hz and takes a current of 5A. The coil dissipates 750W. Calculate (i) power factor (ii) resistance of coil (iii) inductance of coil.

Soln:- (i) Power consumed, $P = VI\cos\phi$

$$\text{Power factor},\ \cos\phi = \frac{P}{VI} = \frac{\cancel{750}^{3}}{\cancel{250}_{1} \times 5} = \frac{3}{5}$$

$$\boxed{\cos\phi = 0.6 \text{ lag}}$$

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(ii) Impedance of coil, $Z = V/I = \frac{250}{5} = 50\,\Omega$

Resistance of coil $\Rightarrow R = Z\cos\phi \Rightarrow 50 \times 0.6$

$$\boxed{R = 30\,\Omega}$$

(iii) Reactance of the coil, $X_L = \sqrt{Z^2 - R^2} = \sqrt{50^2 - 30^2} = 40\,\Omega$

Inductance of the coil, $L = \frac{X_L}{2\pi f} = \frac{40}{2\pi \times 50}$

$$\boxed{L = 0.127\,H}$$

(3) It is desired to operate a 100W, 120V electric bulb at its current rating from a 240V, 50Hz supply. Give details of the simplest manner in which this could be done using (i) a resistor (ii) a capacitor (iii) an inductor having a resistance of 10Ω. What power factor would be presented to the supply in each case and which method is the most economical of power?

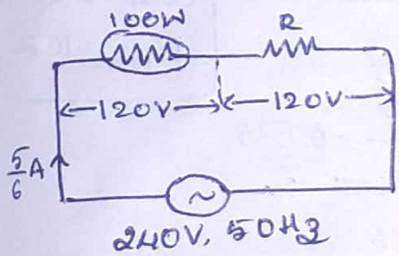


Fig (i)

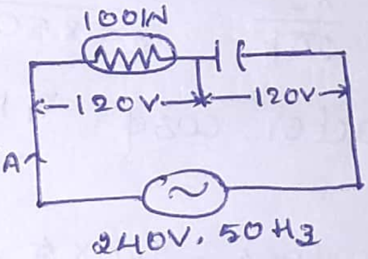


Fig (2)

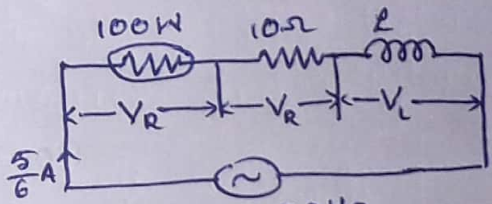


Fig (3)

Soln: Rated current of the bulb, $I = \frac{100}{120} = \frac{5}{6}$ A. For the proper operation of the bulb, the rated voltage (= 120V) and the rated current (= 5/6 A) of the bulb must be same in all the three cases.

(i) Fig (i) shows the circuit conditions for the first case

Voltage across R, $V_R = 240 - 120 = 120$ volts

Value of $R = \frac{120}{5/6} = 144\,\Omega$

Power loss $= 240 \times \frac{5}{6} \Rightarrow 200W$

Power factor is unity.

$$\boxed{\begin{array}{l} R = 144\,\Omega \\ P_{loss} = 200W \end{array}}$$

$$\boxed{\cos\phi = 1}$$

(ii) Fig (ii) shows the circuit conditions for the second case

Voltage across C, $V_c = \sqrt{240^2 - 120^2}$ $\Rightarrow$ $V_c = 207.5$ volts

Reactance, $X_c = \frac{V_c}{I} = \frac{207.5}{5/6}$ $\Rightarrow$ $X_c = 249\ \Omega$

Value of C $= \frac{1}{2\pi f X_c} = \frac{1}{2\pi \times 50 \times 249}$ $\Rightarrow$ $C = 12.8\ \mu F$

power factor $= \frac{120}{240} = 0.5$ lead

Power loss $= 240 \times \frac{5}{6} \times 0.5 = 100W$

$$\boxed{\begin{aligned} V_c &= 207.5 \text{ volts} \\ X_c &= 249\ \Omega \\ C &= 12.8\ \mu F \\ \cos\phi &= 0.5 \text{ lead} \\ P_{loss} &= 100W \end{aligned}}$$

(iii) Fig (iii) shows the circuit conditions for the third case.

Here $V_R = IR = \frac{5}{6} \times 10 = \frac{25}{3}$ volts

Voltage across L $= \sqrt{240^2 - (120 + 25/3)^2}$ $\Rightarrow$ $V_L = 203V$

$X_L = \frac{203}{5/6} = 243.6\Omega$

$L = \frac{X_L}{2\pi f} = \frac{243.6}{2\pi \times 50} = 0.775H$

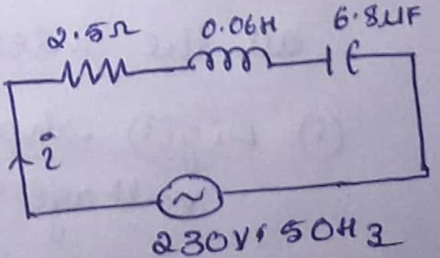
Power factor, $\cos\phi = \frac{120 + \frac{25}{3}}{240} = 0.535$

Power loss $= 240 \times \frac{5}{6} \times 0.536$

$\boxed{P_{loss} = 107W}$

$$\boxed{\begin{aligned} V_L &= 203V \\ X_L &= 243.6\Omega \\ L &= 0.775H \\ \cos\phi &= 0.535 \text{ lag} \\ P_{loss} &= 107W \end{aligned}}$$

(4) A 230V, 50Hz AC supply is applied to a coil of 0.06 henry inductance and 2.5Ω resistance connected in series with a 6.8μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) Power factor and (v) power consumed.



Soln: $X_L = 2\pi \times 50 \times 0.06 \Rightarrow X_L = 18.85\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 6.8\mu} \Rightarrow X_C = 468\Omega$

(i) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.5)^2 + (18.85 - 468)^2}$

$\boxed{Z = 449.2\Omega}$

(ii) circuit current, $I = \frac{V}{Z} = \frac{230}{449.2} \Rightarrow \boxed{I = 0.512\,A}$

(iii) $\tan\phi = \frac{X_L - X_C}{R} \Rightarrow \phi = \tan^{-1}\left[\frac{18.85 - 468}{2.5}\right] = -89.7^\circ$

$\boxed{\phi = 89.7^\circ \text{ lead}}$

The -ve sign with ϕ shows that current is leading the voltage.

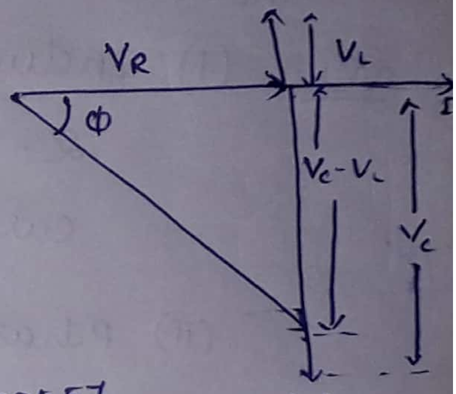


Fig (2)

(iv) Power factor, $\cos\phi = \frac{R}{Z} = \frac{2.5}{449.2}$

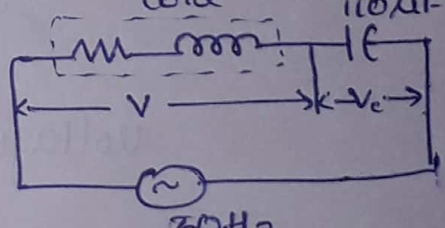
$\boxed{\cos\phi = 0.00557 \text{ lead}}$

(v) Power consumed, $P = VI\cos\phi$

$= 230 \times 0.512 \times 0.00557$

$\boxed{P = 0.656\,W}$

⑤ The coil of PF of 0.8 is connected in series with a $110\mu F$ capacitor. The supply frequency is 50Hz. The potential difference across the coil is found to be equal to the potential difference across the capacitor. Calculate the resistance and inductance of the coil.



Soln: Reactance, $X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 110}$

$\boxed{X_C = 29\Omega}$

$I Z_{coil} = I X_C \quad \therefore Z_{coil} = Z_C = 29\Omega$

$\cos\phi = \frac{R}{Z_{coil}} \quad \therefore R = Z_{coil} \times \cos\phi = 29 \times 0.8$

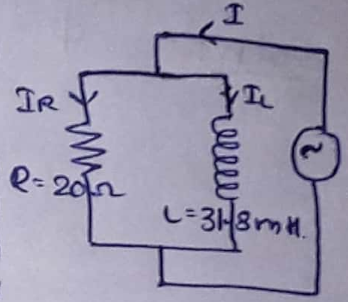
$\boxed{R = 23.2\Omega}$

Reactance, $X_L = \sin\phi \times Z_{coil} = 29 \times 0.6 \Rightarrow X_L = 17.4\Omega$

$L = \frac{X_L}{2\pi f} = \frac{17.4}{2\pi \times 50} \Rightarrow \boxed{L = 0.055 \text{ henry}}$

Problems on parallel R, L, C circuits

① A resistance of 20Ω and a coil of inductance of 31·8 mH. and negligible resistance are connected in a parallel across 230 V, 50Hz supply. Find (i) line current (ii) power factor and power consumed by the circuit.



Soln:

$I_R = \frac{V}{R} = \frac{230}{20} \Rightarrow \boxed{I = 11 \cdot 5A}$ in phase with V

$X_L = 2\pi f L = 2\pi \times 50 \times 31 \cdot 8m \Rightarrow \boxed{X_L = 10\Omega}$

$I_L = \frac{V}{X_L} = \frac{230}{10} \Rightarrow \boxed{I_L = 23A}$ lags V by 90°

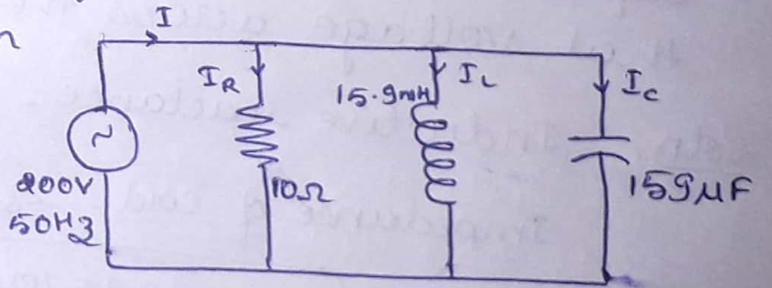
(i) $I = \sqrt{I_R^2 + I_L^2} = \sqrt{(11 \cdot 5)^2 + (23)^2} \Rightarrow \boxed{I = 25 \cdot 71A}$

(ii) Power factor, $\cos\phi = \frac{I_R}{I} = \frac{11 \cdot 5}{25 \cdot 71} \Rightarrow \boxed{\cos\phi = 0 \cdot 447 \text{ lag}}$

(iii) Power consumed, $P = VI\cos\phi = 230 \times 25 \cdot 71 \times 0 \cdot 447$

$\boxed{P = 2643 \text{ watts}}$

② A 10Ω resistor, a 15·9mH inductor and 159μF capacitor are connected in parallel to a 200V, 50Hz source. Calculate the supply current and power factor.



Soln: $X_L = 2\pi f L = 2\pi \times 50 \times 15 \cdot 9m$

$\boxed{X_L = 5\Omega}$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 159 \times 10^{-6}}$

$\boxed{X_C = 20\Omega}$

$I_R = \frac{V}{R} = \frac{200}{10} \Rightarrow I_R = 20A$ → in phase with V

$I_L = \frac{V}{X_L} = \frac{200}{5} \Rightarrow I_L = 40A$ → lags V by 90°

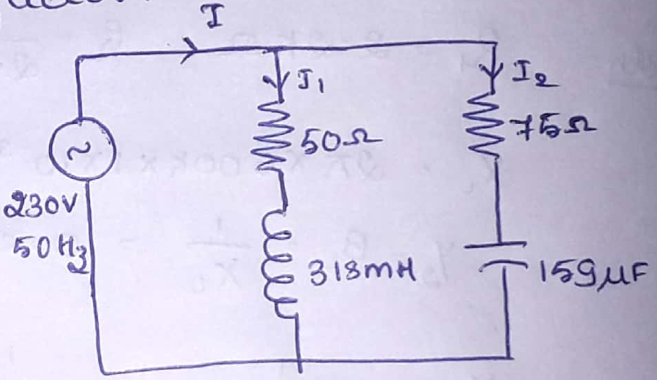
$I_C = \frac{V}{X_C} = \frac{200}{20} \Rightarrow I_C = 10A$ → leads V by 90°

Supply current, $I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{20^2 + (40-10)^2} \Rightarrow 36\,A$

$\boxed{I = 36\,A}$

Power factor, $\cos\phi = \frac{I_R}{I} = \frac{20}{36} \Rightarrow \boxed{\cos\phi = 0.56 \text{ lag}}$

(3) A coil of resistance 50Ω and inductance 318 mH is connected in parallel to a circuit consisting of a 75Ω resistor in series with a 159μF capacitor. The circuit is connected to a 230V, 50Hz supply. Determine the supply current and circuit power factor.



Soln: Given $V = 230\angle 0^\circ$

$X_L = 2\pi f L = 2\pi \times 50 \times 318 \times 10^{-3}$

$\boxed{X_L = 100\Omega}$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 159 \times 10^{-6}}$

$\boxed{X_C = 20\Omega}$

$Z_1 = R_1 + jX_L = (50 + j100)\Omega = 112\angle 63.5^\circ\ \Omega$

$Z_2 = R_2 - jX_C = (75 - j20)\Omega = 77.6\angle -15^\circ\ \Omega$.

$I_1 = \frac{V}{Z_1} = \frac{230\angle 0^\circ}{112\angle 63.5} = 2.05\angle -63.5 = (0.91 - j1.83)A$

$I_2 = \frac{V}{Z_2} = \frac{230\angle 0^\circ}{77.6\angle -15^\circ} = 2.96\angle 15^\circ = (2.86 + j0.766)\,A$

Total circuit current, $I_T = I_1 + I_2$

$I_T = (0.91 - j1.83) + (2.86 + j0.766)$

$\boxed{I_T = (3.77 - j1.06)A = 3.92\angle -15.7^\circ\ A}$

Power factor, $\cos\phi = \cos(15.7^\circ) = 0.963$ lag.

$\boxed{\cos\phi = 0.963 \text{ lag}}$

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Three Phase (3 φ) Circuits and systems

INTRODUCTION

The earliest application of ac current was for heating the filament of an electric lamp. For this purpose, the single-phase system was quite satisfactory. Some years later, ac motors were developed. It was found that single-phase ac supply was not very satisfactory for this application. For instance, the single-phase induction motor—the type most commonly used—was not self-starting unless it was fitted with an auxiliary winding. It was found that by using two separate windings with currents differing in phase by 90° or three windings with currents differing in phase by 120°, the induction motor became self-starting, had better efficiency and power factor.

The system utilising two windings is referred to as a *two-phase system* and that utilising three windings is referred to as a *three-phase system* .

Almost all the electrical power used in the country is generated and distributed in the form of three-phase ac supply. The single-phase ac supply used in homes, offices, factories, etc., originates as a part of 3-phase system.

Advantages of Three-Phase System

The advantages of using 3-ϕ systems over 1-ϕ systems are as follows:

1. Three-phase transmission lines require much less conductor material. Since the phasor sum of currents in all the phases is zero, there is substantial saving by eliminating the return conductor or replacing it by a single neutral conductor of comparatively small size.
2. For a given frame size, a three-phase machine gives a higher output than a single-phase machine.
3. The power in a single-phase system pulsates at twice the line frequency. However, the sum of powers in the three phases in a three-phase system remains constant. Therefore, a three-phase motor develops a uniform torque, whereas a single-phase motor develops pulsating torque.
4. Since the three-phase supply can generate a rotating field, the three-phase induction motors are self-starting.
5. The three-phase system can be used to supply domestic as well as industrial (or commercial) power.
6. The voltage regulation in three-phase system is better than that in single-phase supply.

STAR (Y) CONNECTED THREE-PHASE SYSTEM

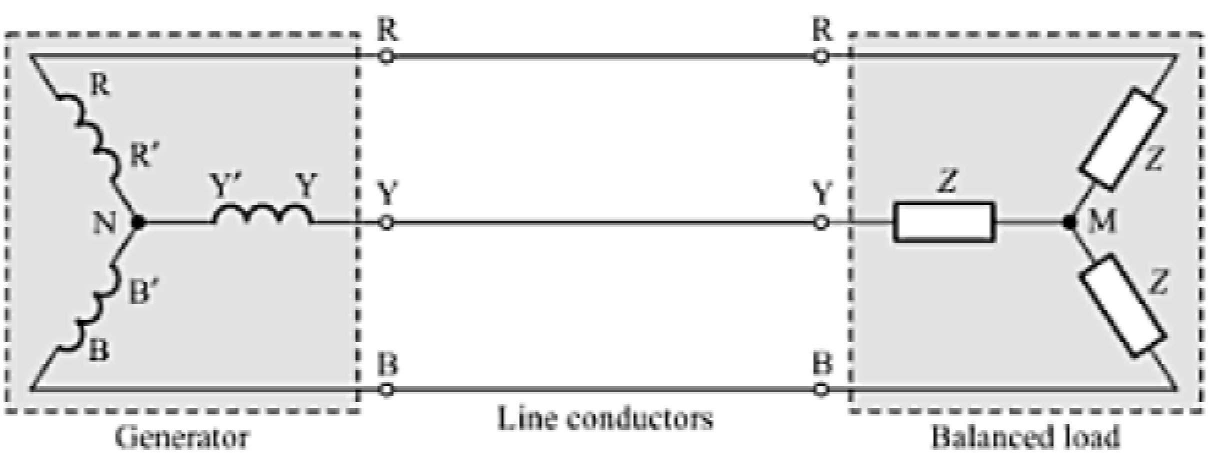
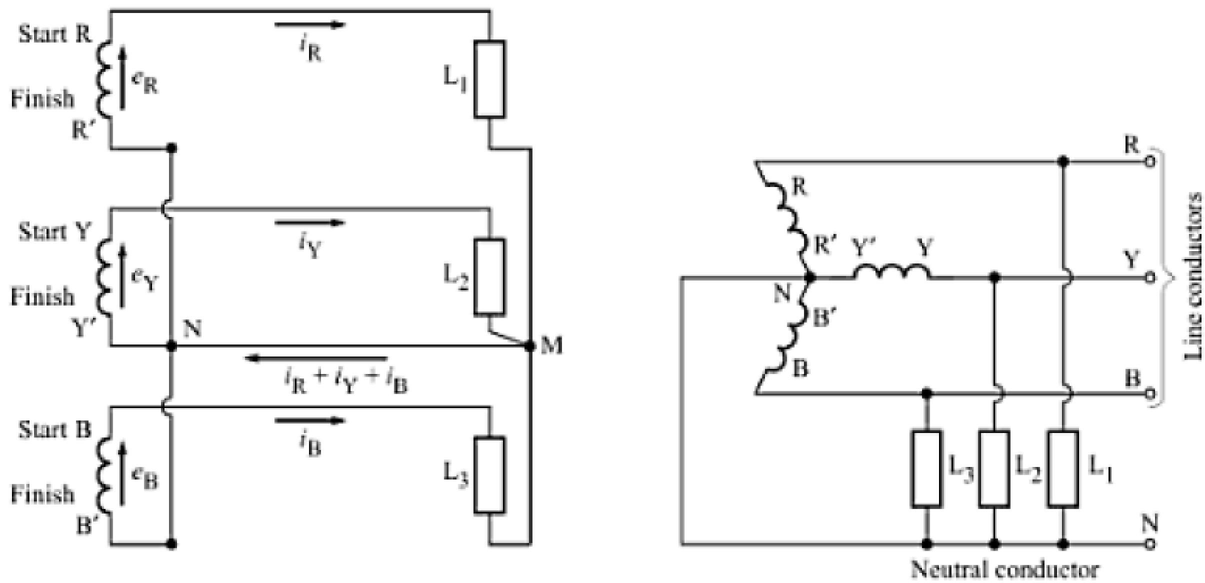


Fig. *Three-wire star-connected voltage system with balance load.*



(*a*) The three-phase windings connected in star.

(*b*) Conventional representation of star-connected system.

Fig. *Star-connected four-wire three-phase voltage system.*

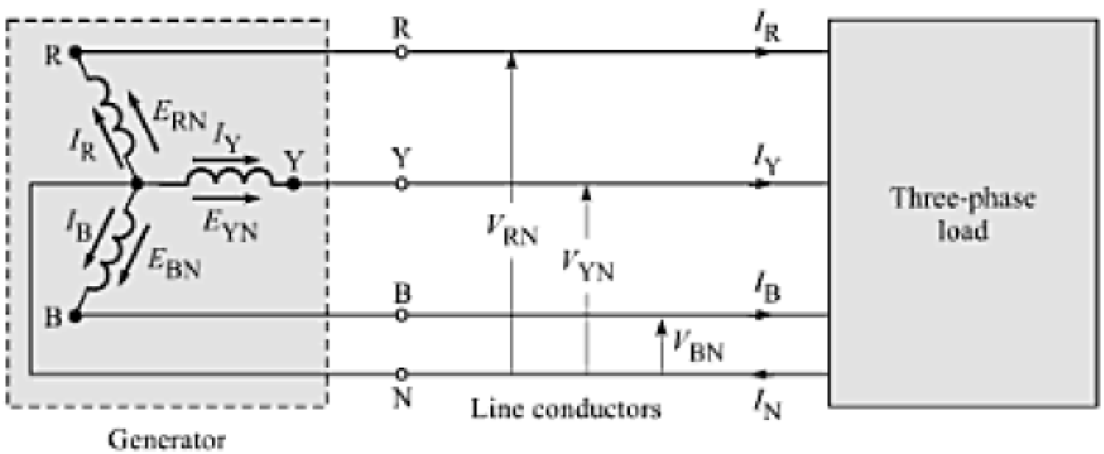


Fig. *A star-connected generator supplying power to a three-phase load.*

Analytical Analysis In a balanced system, each phase voltage has the same magnitude. So, we can write

$$|\mathbf{V}_{RN}| = |\mathbf{V}_{YN}| = |\mathbf{V}_{BN}| = V_{ph} \text{ (say)}$$

The three phasors representing the set of phase voltages can be written as

$$\mathbf{V}_{RN} = V_{ph}\angle 0°; \quad \mathbf{V}_{YN} = V_{ph}\angle{-120°}; \quad \mathbf{V}_{BN} = V_{ph}\angle{-240°} = V_{ph}\angle 120°$$

Equation 12.5 can then be written as

$$\mathbf{V}_{RY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = V_{ph}\angle 0° - V_{ph}\angle{-120°} = V_{ph} - V_{ph}(\cos 120° - j\sin 120°)$$
$$= V_{ph} - V_{ph}(-1/2 - j\sqrt{3}/2) = V_{ph}(3/2 + j\sqrt{3}/2)$$

Thus, the magnitude of $\mathbf{V}_{RY}$ is given as

$$V_{RY} = V_{ph}\sqrt{(9/4 + 3/4)} = \sqrt{3}\,V_{ph}$$

And the phase angle of $\mathbf{V}_{RY}$ with respect to the reference phasor $\mathbf{V}_{RN}$ is given as

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{3/2}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Hence the phasor $\mathbf{V}_{RY}$ can be written as

$$\mathbf{V}_{RY} = \sqrt{3}V_{ph}\angle 30°$$

Similarly, we can get

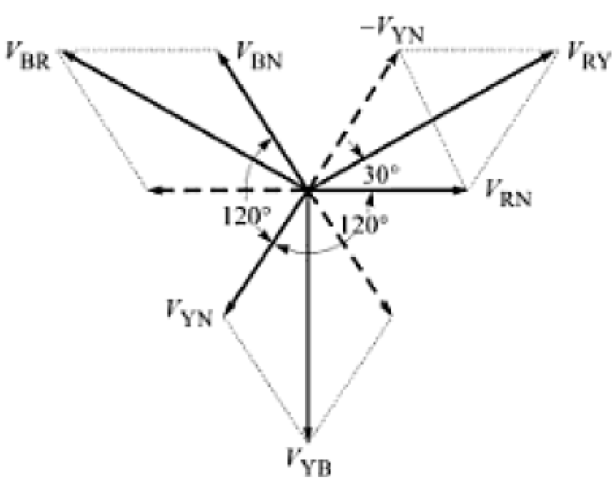
$$\mathbf{V}_{YB} = \sqrt{3}V_{ph}\angle -90° \qquad \text{and} \qquad \mathbf{V}_{BR} = \sqrt{3}V_{ph}\angle 150°$$

Thus, we can say that the magnitude of the line voltage V_L for star connection is given as

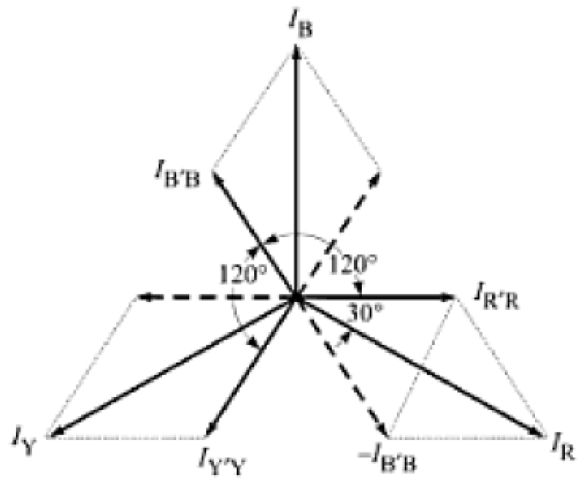
$$\boxed{V_L = \sqrt{3}V_{ph}}$$

It is obvious that any current that flows out of the line terminal R must be the same as that which flows due to the phase source voltage appearing between terminals R and N. Therefore, for star-connection, we have

$$\boxed{I_L = I_{ph}}$$



(*a*) Star-connected system.



(*b*) Delta-connected system.

Delta-Connected System

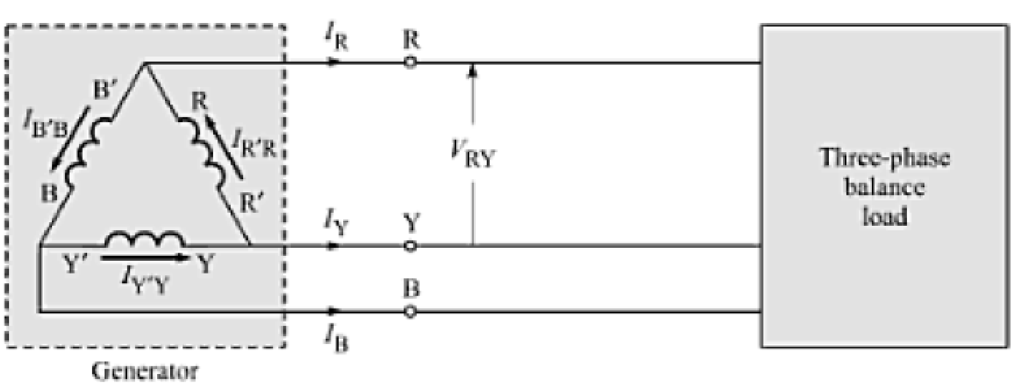
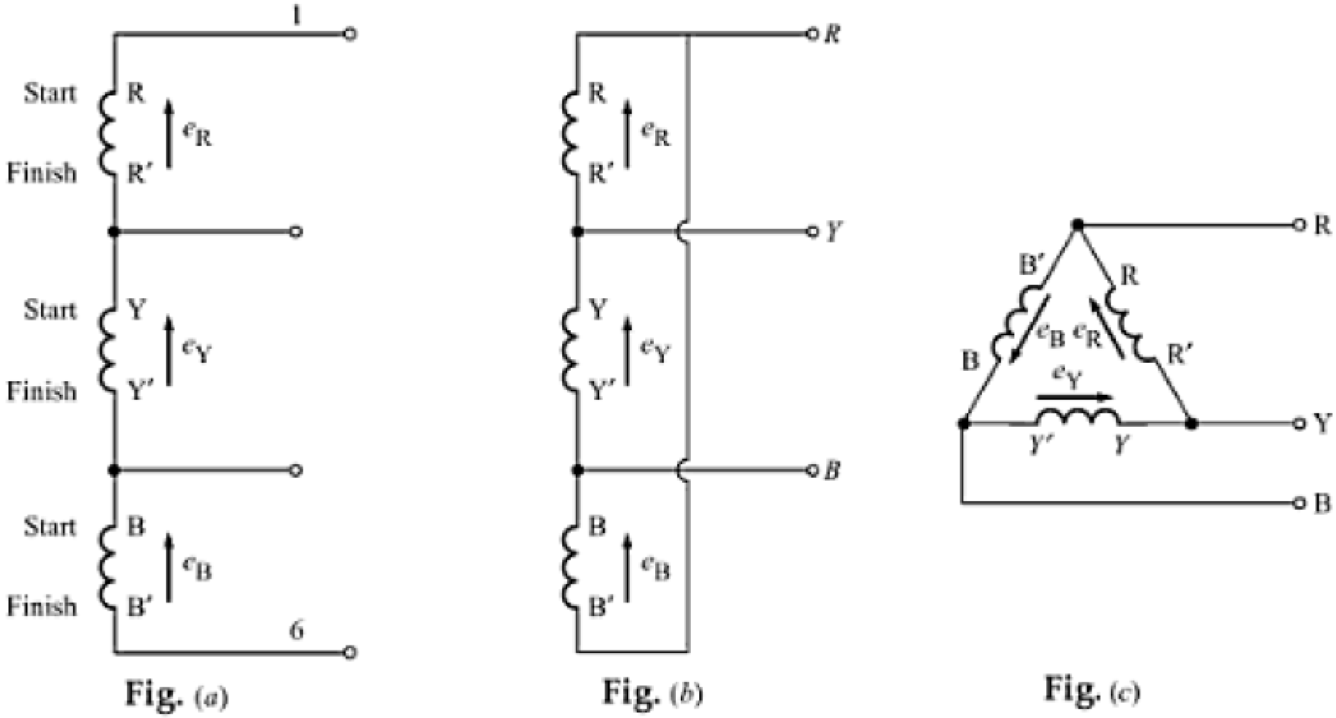


Fig. A Delta *-connected generator supplying power to a 3-Φ load.*



Connecting the three phase windings to make delta (Δ) connected three-phase system.

Let $I_{R'R}$, $I_{Y'Y}$, $I_{B'B}$, be the rms values of the phase currents in the three windings of the generator. Their assumed positive directions are indicated by arrows in Fig. Since the load is assumed balanced, these currents are equal in magnitude and differ in phase by 120° Therefore, we can write

$$|\mathbf{I}_{R'R}| = |\mathbf{I}_{Y'Y}| = |\mathbf{I}_{B'B}| = I_{ph} \text{ (say)}$$

The three phasors representing the set of phase currents can be written as

$$\mathbf{I}_{R'R} = I_{ph}\angle 0°; \quad \mathbf{I}_{Y'Y} = I_{ph}\angle -120°; \quad \mathbf{I}_{B'B} = I_{ph}\angle -240° = I_{ph}\angle 120°$$

From Fig. it can be seen that the phase current $I_{R'R}$ flows towards the line conductor R, whereas the phase current $I_{B'B}$ flows away from it. Applying KCL at the terminal R, we can write

$$\mathbf{I}_R = \mathbf{I}_{R'R} - \mathbf{I}_{B'B}$$

Above vector addition of $\mathbf{I}_{R'R}$ and $-\mathbf{I}_{B'B}$ is shown in the phasor diagram of Fig. . From the symmetrical geometry of the diagram, it is evident that the line currents are equal in magnitude and differ in phase by 120°. Also

$$I_R = 2(I_{ph}\cos 30°) = \sqrt{3}\, I_{ph}$$

Hence, for a delta-connected system with balanced load, the magnitudes of line current and of phase current are related as

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

From Fig. it is obvious that the line voltage V_{RY} is same as the phase voltage $V_{RR'}$. Hence, for a delta-connected system, we have

$$\boxed{V_L = V_{ph}}$$

Important Points about Three-Phase Systems

Following important points should be noted while dealing with three-phase systems:

(*i*) For a three-phase system, unless otherwise mentioned, it is normal practice to specify the values of the line voltages and line currents. Thus, when we say that a 3-phase, 11-kV circuit is carrying a current of 500 A, it implies that $V_L = 11$ kV and $I_L = 500$ A.

(*ii*) The current in any phase can be determined by dividing the phase voltage by its impedance. That is, $I_{ph} = V_{ph}/Z$. The power factor of Z is the same as the cosine of the phase difference between V_{ph} and I_{ph}.

A 400-V, 3-ϕ supply is connected across a balanced load of three impedances each consisting of a 32-Ω resistance and 24-Ω inductive reactance. Determine the current drawn from the power mains, if the three impedances are (*a*) Y-connected, and (*b*) Δ-connected.

Solution Each impedance, $\mathbf{Z} = R + jX = (32 + j24)\ \Omega$.

$$\therefore \qquad Z = \sqrt{R^2 + X^2} = \sqrt{32^2 + 24^2} = 40\ \Omega$$

(*a*) Y-*connection*: $V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}}\text{ V} \Rightarrow I_{ph} = \dfrac{V_{ph}}{Z} = \dfrac{400/\sqrt{3}}{40} = \dfrac{10}{\sqrt{3}}\text{ A}$

$$\therefore \qquad I_L = I_{ph} = \frac{10}{\sqrt{3}} = \mathbf{5.78\ A}$$

(*b*) For Δ-*connection*: $V_{ph} = V_L = 400\text{ V} \Rightarrow I_{ph} = \dfrac{V_{ph}}{Z} = \dfrac{400}{40} = 10\text{ A}$

$$\therefore \qquad I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 10 = \mathbf{17.32\ A}$$

POWER IN THREE-PHASE SYSTEM WITH A BALANCED LOAD

Consider one phase only. For this load, the voltage is V_{ph} and the current is I_{ph}. The average active power consumed by this load is given by

$$P_1 = V_{ph} I_{ph} \cos\phi \quad \text{where } \phi \text{ is the phase angle of the load.}$$

As the load is balanced, the power in other two phase circuits will also be the same. Hence, the total power consumed is

$$P = 3P_1 = 3V_{ph} I_{ph} \cos\phi$$

Above expression for the total power is in terms of phase voltage and phase current. However, it is a normal practice to mention line voltage and line current in a three-phase system. Hence, let us determine the expression for the total power in terms of V_L and I_L.

For a *star-connected system*, we have $V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$. Hence,

$$P = 3(V_L/\sqrt{3})\, I_L \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

For a *delta-connected system*, we have $V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$. Hence,

$$P = 3V_L (I_L/\sqrt{3}) \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

Thus, it follows that, for *any balanced load* (connected in either Y or Δ), the total power is given as

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

While using above equation, it is important to note that ϕ is the angle of the load impedance per phase and not the angle between V_L and I_L.

Table *Comparison between star- and delta-connected systems.*

S. No.	*Star-Connected System*	*Delta-Connected System*
1.	Similar ends are joined together.	Dissimilar ends are joined.
2.	$V_L = \sqrt{3}\,V_{ph}$ and $I_L = I_{ph}$	$V_L = V_{ph}$ and $I_L = \sqrt{3}\,I_{ph}$
3.	Neutral wire is available.	Neutral wire is not available.
4.	4-wire, 3-ϕ system is possible.	4-wire, 3-ϕ system is not possible.
5.	Both domestic and industrial loads can be handled.	Only industrial loads can be handled.
6.	By earthing the neutral wire, relays and protective devices can be provided in alternators for safety.	Due to absence of neutral wire, it is not possible.

MEASUREMENT OF POWER

The method of measurement of total power in three-phase depends upon the type of system and that of the load. There exists the following methods.

(i) Three-Wattmeter Method This is the simplest and straight forward method. One wattmeter is inserted in each of the phases. The power consumed by the load is the algebraic sum of the three wattmeter-readings.

(ii) One-Wattmeter Method This can be used to determine the total power consumed by a star-connected balanced load, with neutral point accessible. The current coil of the wattmeter is connected in one line and the potential coil is connected between that line and the neutral point, as shown in Fig. *a*.
The reading of the wattmeter gives the power per phase. Therefore,

$$\text{Total power} = 3 \times \text{wattmeter reading}$$

(iii) Two-Wattmeter Method This can be used for any balanced or unbalanced load, star- or delta-connected. Details of this method are explained below.

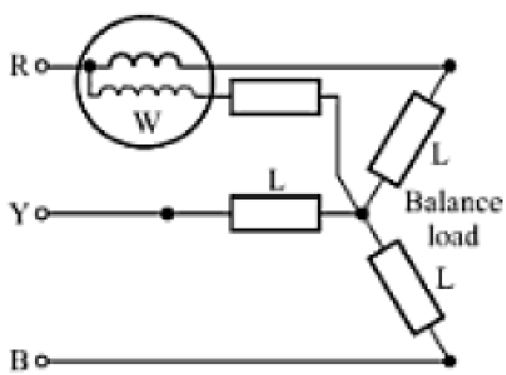


Fig. (*a*) Star-connected balanced load.

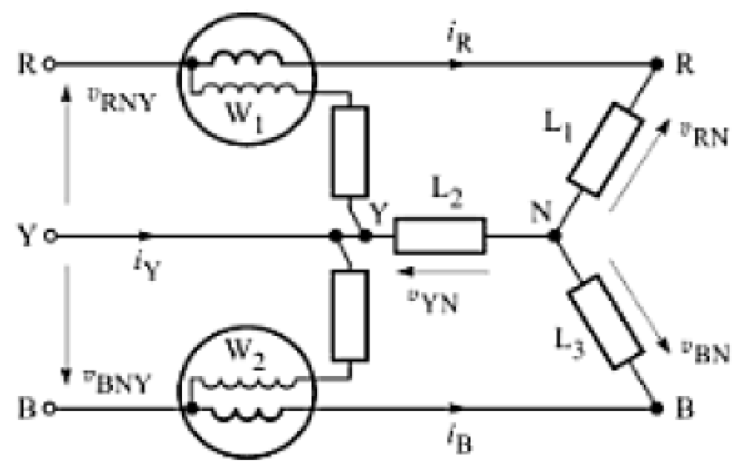


Fig. (*b*) Star-connected unbalanced load.

Measurement of power in 3-ϕ load.

Power Measurement by Two-Wattmeter Method

Suppose that the three loads L_1, L_2 and L_3 are connected in star, as shown in Fig *b*. The current coils (CC) of the two wattmeters W_1 and W_2 are connected in any two lines, say, the 'red' and 'blue' lines. The potential coils (PC) of the wattmeters are connected between these lines and the third line. The sum of the wattmeter readings gives the average value of the total power absorbed by the three phases.

Proof Let v_{RN}, v_{YN} and v_{BN} be the instantaneous values of the voltages across the loads, with the positive direction marked by arrows in the diagram. Let i_R, i_Y and i_B be the corresponding instantaneous values of the line (and phase) currents.

$\therefore$ Total instantaneous power $= i_R v_{RN} + i_Y v_{YN} + i_B v_{BN}$.

Since the current through the current coil of W_1 is i_R, and the potential across its potential coil is $v_{RN} - v_{YN}$, we have

the instantaneous power measured by W_1, $p_1 = i_R(v_{RN} - v_{YN})$

Similarly, the instantaneous power measured by W_2, $p_2 = i_B(v_{BN} - v_{YN})$

Hence, the sum of the instantaneous powers of W_1 and W_2 is

$$p_1 + p_2 = i_R(v_{RN} - v_{YN}) + i_B(v_{BN} - v_{YN}) = i_R v_{RN} + i_B v_{BN} - (i_R + i_B)v_{YN}$$

From KCL, the algebraic sum of the instantaneous currents at N is zero, i.e.,

$$i_R + i_Y + i_B = 0 \quad \Rightarrow \quad (i_R + i_B) = -i_Y$$

$$\therefore \quad p_1 + p_2 = i_R v_{RN} + i_B v_{BN} - (i_R + i_B)v_{YN} = i_R v_{RN} + i_B v_{BN} + i_Y v_{YN}$$

$$= \text{total instantaneous power}$$

Actually, the power measured by each wattmeter varies from instant to instant, but due to the inertia of the moving system the pointer stays at the average value of the power.

Since the above proof does not assume a balanced load or a sinusoidal waveform, it follows that *the sum of the two wattmeter readings gives the total power under all conditions.* The above proof was derived for a star-connected load. One could derive the same conclusion for a delta-connected load.

Power Factor Measurement by Two-Wattmeter Method

Consider a balanced three-phase inductive load at a power factor $\cos\phi$ (lagging), connected to a 3-wire, 3-ϕ system, as shown in Fig. 12. *a*. The phase sequence is R Y B. The current coils of the two wattmeters W_1 and W_2 are connected in the line conductors R and Y, respectively. Their potential coils are connected between the corresponding line conductor and the third line conductor B.

Let I_R, I_Y and I_B be the three line currents, and V_{RN}, V_{YN} and V_{BN} be the three phase-voltages . Since the load is balanced, the three line currents and the three line voltages will have same magnitude, i.e.,

$$I_R = I_Y = I_B = I_L \text{ (say)} \qquad \text{and} \qquad V_{RN} = V_{YN} = V_{BN} = V_{ph} \text{ (say)}$$

Each line current lags by angle ϕ its corresponding voltage as shown in the phasor diagram of Fig. 12.*b*. Since $\mathbf{V}_{RB} = \mathbf{V}_{RN} - \mathbf{V}_{BN}$, and $\mathbf{V}_{YB} = \mathbf{V}_{YN} - \mathbf{V}_{BN}$, we can determine the line voltages V_{RB} and V_{YB} by phasor method. It is seen from Fig. 12. *b* that the line voltage V_{RB} lags the phase voltage V_{RN} by 30° and V_{YB} leads V_{YN} by 30°. Thus, the phase angle between the line voltage V_{RB} and the line current I_R is $(30° - \phi)$. Similarly,

the phase angle between the line voltage V_{YB} and the line current I_Y is $(30° + \phi)$. Therefore, the readings of the two wattmeters are

$$P_1 = V_{RB} I_R \cos(30° - \phi) = V_L I_L \cos(30° - \phi) \qquad (1)$$

and

$$P_2 = V_{YB} I_Y \cos(30° + \phi) = V_L I_L \cos(30° + \phi) \qquad (2)$$

Dividing Eq. 1 by Eq. 2 , we get $\dfrac{P_1}{P_2} = \dfrac{\cos(30° - \phi)}{\cos(30° + \phi)}$

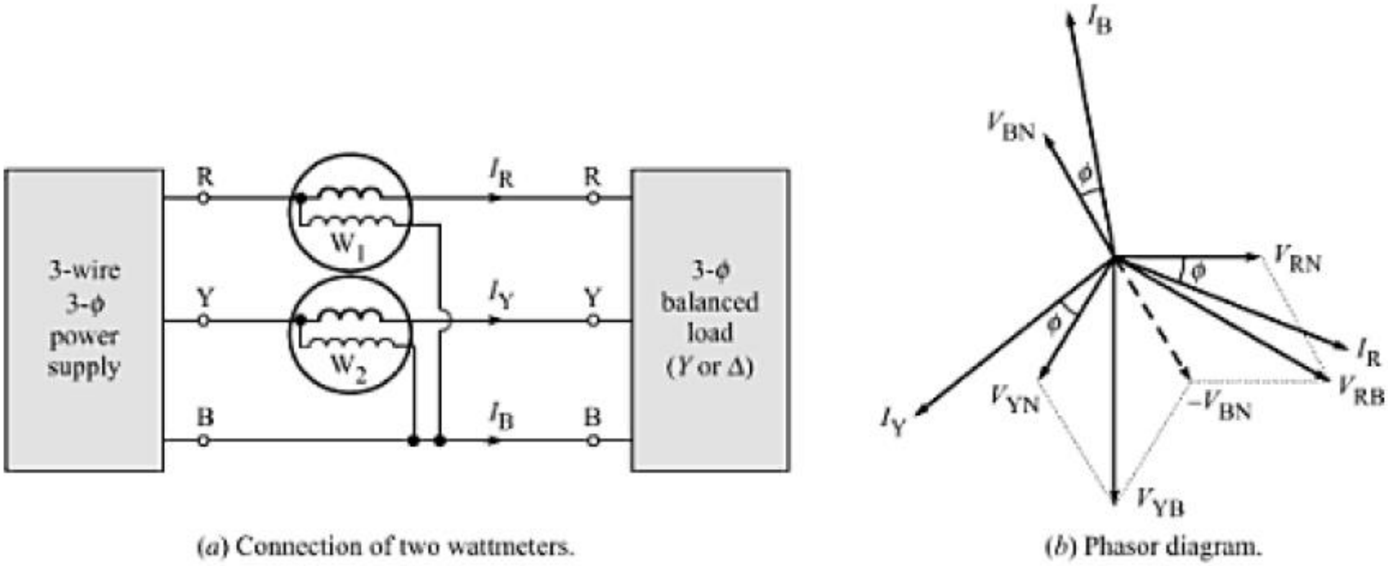


Fig. 12 *Measurement of power factor by two-wattmeter method.*

By applying componendo and dividendo to the above, we get

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\cos(30° - \phi) - \cos(30° + \phi)}{\cos(30° - \phi) + \cos(30° + \phi)} = \frac{2\sin 30° \sin\phi}{2\cos 30° \cos\phi} = \tan 30° \tan\phi$$

or
$$\boxed{\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right]} \qquad \textbf{(3)}$$

We can calculate the phase angle ϕ from the above relation, and then determine the power factor $\cos\phi$.

Let us check whether we get total power consumed by the load by adding P_1 and P_2. Using Eqs. 1 and 2 , we have

$$P_1 + P_2 = V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi) = V_L I_L[\cos(30° - \phi) + \cos(30° + \phi)]$$
$$= V_L I_L[2\cos 30° \cos\phi] = V_L I_E[2(\sqrt{3}/2)\cos\phi] = \sqrt{3}\, V_L I_L \cos\phi$$

The right hand side of the above equation gives the total power consumed by a balanced three-phase load. Hence, we have

$$\text{Total power consumed} = P_1 + P_2$$

This is an ***alternative method*** of proving that the sum of the two wattmeter-readings gives the total power. But, note that this proof assumed a balanced load.

Which is higher-reading wattmeter ? How can we tell which wattmeter reads higher (*i.e.* W_2) and which reads lower (*i.e.* W_1) ?

(*i*)
$$W_2 = V_L I_L \cos(30° - \phi)$$
$$W_1 = V_L I_L \cos(30° + \phi)$$

Since the value of load p.f. can vary from 0 to 1 (*i.e.* ϕ can vary from 90° to 0°), it is clear that wattmeter whose deflection is proportional to (30° − ϕ) is *always positive and is always the higher reading wattmeter* (*i.e.* W_2 in this case).

(ii) Since the phase sequence is known, we can even tell the higher-reading wattmeter from the circuit diagram. Thus referring to the circuit , the phase sequence is *RYB* and that current coils of wattmeters W_1 and W_2 are connected in *R* and *B* lines. Since V_{BN} leads V_{RN} (of course by 120°), the wattmeter in line *B* (*i.e.* W_2) is the higher reading wattmeter. Had the current coils of W_1 and W_2 been connected in *R* and *Y* lines, V_{RN} leads V_{YN} by 120°, the wattmeter in line *R* (*i.e.,* W_1) would have been the higher-reading wattmeter.

Effect of Load p.f. on Wattmeter Readings

We have seen that for *lagging* load (balanced) power factor of cos ϕ, the two wattmeter readings are :

$$W_2 = V_L I_L \cos(30° - \phi)$$
$$W_1 = V_L I_L \cos(30° + \phi)$$

It is clear that readings of the two wattmeters depend upon load p.f. angle ϕ.

(i) **When p.f. is unity** (*i.e.* $\phi = 0°$).

$$W_2 = V_L I_L \cos 30°$$
$$W_1 = V_L I_L \cos 30°$$

Both wattmeters indicate equal and positive (*i.e.* upscale) readings.

(ii) **When p.f. is 0.5** (*i.e.* $\phi = 60°$).

$$W_2 = V_L I_L \cos 30°$$
$$W_1 = V_L I_L \cos 90° = 0$$

Hence total power is measured by wattmeter W_2 alone.

(iii) **When p.f. is less than 0.5 but greater than 0** (*i.e.* $90° > \phi > 60°$).

$$W_2 = \text{positive reading}$$
$$W_1 = \text{negative reading}$$

The wattmeter W_2 reads positive (*i.e.* upscale) because for the given conditions (*i.e.* $90° > \phi > 60°$), the phase angle between voltage and current will be less than 90°. However, for wattmeter W_1, the phase angle between voltage and current shall be more than 90° and hence this wattmeter gives negative (*i.e.* downscale) reading. In order to obtain upscale reading on wattmeter W_1, reverse its potential or current coil. The reading obtained after reversal of coil connection should be taken as negative.

$$\text{Total power} = W_2 + (-W_1) \qquad \text{...algebraic sum}$$
$$= W_2 - W_1$$

Note that total power is obtained by subtracting the reading of W_1 from W_2. We arrive at a very important conclusion that *if load p.f. is less than 0.5, then lower reading wattmeter (i.e. W_1 in this case) will give negative reading.*

(iv) **When p.f. is zero** (*i.e.* $\phi = 90°$). Such a case will occur when the load consists of pure inductance and/or capacitance.

$$W_2 = V_L I_L \cos(30° - 90°) = V_L I_L \sin 30°$$
$$W_1 = V_L I_L \cos(30° + 90°) = -V_L I_L \sin 30°$$

Thus the two wattmeters will read equal and opposite. $W_1 + W_2 = 0$

The above facts are summarised below in the tabular form.

ϕ	0°	60°	More than 60°	90°
$\cos\phi$	1	0.5	< 0.5	0
W_2	positive	positive	positive	positive
W_1	positive	0	negative	negative
Conclusion	$W_1 = W_2$ Total power $= W_1 + W_2$	$W_1 = 0$ Total power $= W_2$	Total power $= W_2 - W_1$	$W_2 = -W_1$ Total power $= 0$

The following points may be noted carefully :

(*i*) *The wattmeter whose deflection is proportional to (30° – ϕ) (i.e. W_2 in this case) is always positive and is the higher-reading wattmeter.*

(*ii*) *The wattmeter whose deflection is proportional to (30° + ϕ) (i.e., W_1 in this case) is the lower-reading wattmeter.*

(*iii*) *The negative reading will only be obtained on the lower-reading wattmeter (i.e. W_1 in this case) and that too when the load p.f. is less than 0.5.*

NUMERICAL

Example *Find the reading on the wattmeter when the network shown in Fig. is connected to a symmetrical 440 V, 3-phase supply. The phase sequence is RYB. Neglect electrostatic effects and instrument losses.*

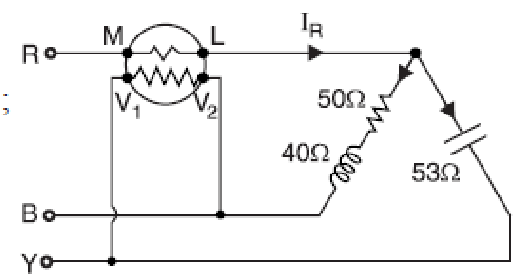


Fig.

Solution. The phase sequence is *RYB*.

$\therefore \quad V_{RY} = 440 \angle 0° \text{ V} ; V_{YB} = 440 \angle -120° \text{ V} ;$

$V_{BR} = 440 \angle 120° \text{ V}$

Current in current coil,

$$I_{ML} = I_R = \frac{V_{RB}}{50 + j\,40} + \frac{V_{RY}}{53\angle -90°}$$

$$= -\frac{440\angle 120°}{64\angle 38.7°} + \frac{440\angle 0°}{53\angle -90°}$$

$$= j\,8.3 - 6.875 \angle 81.3°$$

$$= j\,8.3 - (1.03 + j\,6.796)$$

$$= (-1.03 + j\,1.5) \text{ A} = 1.825 \angle 124.7° \text{ A}$$

Voltage across potential coil, $V_1V_2 = V_{YB} = 440 \angle -120° \text{ V}$

$\therefore$ Wattmeter reading, $W = I_{ML} \times V_1V_2 = I_{ML} \,.\, V_{YB}$

$$= 1.825 \times 440 \times \cos 244.7°$$

$$= -0.343 \times 10^3 \text{ W} = \mathbf{-0.343 \text{ kW}}$$

****