Solved Problems

Problem

Find the currents in all branch of the network shown in Figure 2.15 if the voltage drop in branch AB is 0.78 V with A at higher potential and the resistance of this branch is 0.02 Ω

(Feb. 1992, 6 marks)

Solution:

Voltage drop in branch AB, $V_{AB} =$ 0.78 V

Resistance of branch AB, $R_{AB} =$ 0.02 Ω

Therefore, current in branch AB is given by,

$$I_{AB} = \frac{V_{AB}}{R_{AB}} = \frac{0.78}{0.02} = 39\,\text{A}.$$

Assume the direction of current in all branches as shown in Figure 2.15 We assign a positive sign to the currents whose direction points towards the node and a negative sign to the currents whose direction points away from the node.

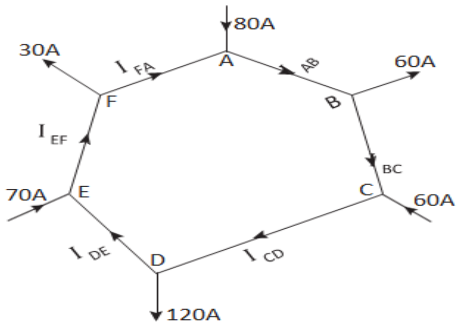
Applying KCL to node A we get,

$$I_{FA} + 80 - I_{AB} = 0$$
$$I_{FA} + 80 - 39 = 0$$

So,

$$I_{FA} = 39 - 80 = -41\ \text{A}.$$

The sign of I_{FA} is negative, which means the actual direction of current in branch FA is opposite to the assumed direction.



$$I_{FA} = 39 - 80 = -41\ \text{A}.$$

The sign of I_{FA} is negative, which means the actual direction of current in branch FA is opposite to the assumed direction.

Applying KCL to node F, is given by,

$$I_{EF} - 30 - I_{FA} = 0$$

therefore,

$$I_{EF} = 30 + I_{FA} = 30 + (-41) = -11\ \text{A}.$$

Applying KCL to node E is given by,

$$I_{DE} + 70 - I_{EF} = 0$$

therefore,

$$I_{DE} = I_{EF} - 70 = -11 - 70 = -81\ \text{A}$$

Applying KCL to node D is given by,

$$I_{CD} - 120 - I_{DE} = 0$$

therefore,

$$I_{CD} = 120 + I_{DE} = 120 + (-81) = 39\ \text{A}$$

Applying KCL to node C is given by,

$$I_{BC} + 60 - I_{CD} = 0$$

therefore,

$$I_{BC} = I_{CD} - 60 = 39 - 60 = -21\ \text{A}$$

Applying KCL to node B is given by,

$$I_{AB} - 60 - I_{BC} = 0$$
$$39 - 60 - (-21) = 0$$

□ Problem 2.

In the circuit shown in Figure 2.16 determine the values of E, R_1 and R_3.

(July 1993, 6 marks)

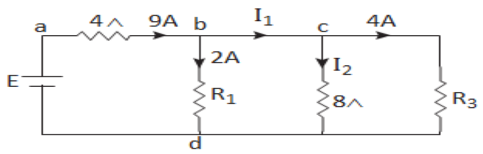


Figure 2.16

Solution:

Applying KCL to node b we get,

$$9 - 2 - I_1 = 0$$
$$7 - I_1 = 0$$
$$\therefore I_1 = 7 \text{ A}$$

Applying KCL to node c we get

$$I_1 - I_2 - 4 = 0$$
$$7 - I_2 - 4 = 0$$
$$\therefore I_2 = 3 \text{ A}$$

Voltage across 8 Ω resistor = $I_2 \times 8 = 3 \times 8 = 24$ V

Resistors R_1 and R_3 are in parallel with 8 Ω resistor, so,

$$V_{bd} = V_{cd} = 24 \text{V}$$

$$\text{Resistance} \quad R_1 = \frac{V_{bd}}{2} = \frac{24}{2} = 12\ \Omega (Ans)$$

$$\text{Resistance} \quad R_3 = \frac{V_{cd}}{4} = \frac{24}{4} = 6\ \Omega (Ans)$$

Voltage drop across 4 Ω resistor, is given by,

$$V_{ab} = 9 \times 4 = 36 \text{ V}.$$

Voltage drop across 4 Ω resistor, is given by,

$$V_{ab} = 9 \times 4 = 36 \text{ V}.$$

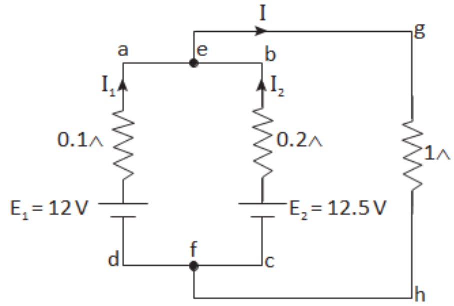
Applying KVL to loop $abda$ we get,

$$-V_{ab} - V_{bd} + E = 0$$
$$-36 - 24 + E = 0$$
$$-60 + E = 0$$
$$E = 60 \text{V} (Ans)$$

□ Problem 3

Two batteries of 12 V and 12.5 V with internal resistances of 0.1 Ω and 0.2 Ω are connected in parallel. When a resistance of 1 Ω is connected across the given circuit, find the current through it. (Jan 1990, 6 marks)

Solution:



Applying KCL to node *e*, we get,

$$I_1 + I_2 - I = 0 \qquad (2.1)$$

Applying KVL to loop *abcda*, we get,

$$I_2(0.2) - 12.5 + 12 - I_1(0.1) = 0$$
$$-0.1I_1 + 0.2I_2 = 0.5 \qquad (2.2)$$

Applying KVL to loop *fdaeghf*, we get,

$$12 - I_1(0.1) - I(1) = 0$$
$$12 - I_1(0.1) - (I_1 + I_2) = 0$$

Applying KVL to loop *abcda*, we get,

$$I_2(0.2) - 12.5 + 12 - I_1(0.1) = 0$$
$$-0.1I_1 + 0.2I_2 = 0.5 \quad \text{--------------} \qquad (2.2)$$

Applying KVL to loop *fdaeghf*, we get,

$$12 - I_1(0.1) - I(1) = 0$$
$$12 - I_1(0.1) - (I_1 + I_2) = 0$$
$$-1.1I_1 - I_2 = -12 \quad \text{-------} \quad (2.3)$$

Current through resistance of 1 Ω is given by,

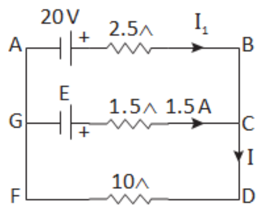
$$I = I_1 + I_2$$
$$I = 5.94 + 5.47 = 11.41 \text{ A.}$$

□ **Problem 4**

In the circuit shown in Figure , find E, I_1 and the energy dissipated in 1 minute in 10 Ω resistor

(March 1989, 5 marks)

Solution:



Applying KCL to node C we get,

$$I_1 + 1.5 - I = 0 \qquad (2.1)$$

Applying KVL to loop **ABCGA** weget,

$$20 - 2.5I_1 + 1.5 \times 1.5 - E = 0$$

or,

$$22.25 = 2.5I_1 + E \qquad (2.2)$$

Applying KVL to the loop **GCDFG** ve get,

$$E - 1.5 \times 1.5 - 10I = 0$$

or

$$E - 10I = 2.25 \qquad (2.3)$$

From Eqn. (2.1),

$$I = I_1 + 1.5$$

Eqn. (2.3) becomes,

$$E - 10(I_1 + 1.5) = 2.25$$

or

$$E - 10I_1 - 15 = 2.25$$

or

$$E - 10I_1 = 17.25 \qquad (2.4)$$

Subtracting equation (2.2) from (2.4) we get,

$$E \pm 2.5I_1 = 22.25$$
$$-12.5I_1 = -5.00$$

Therefore,

$$I_1 = \frac{-5.00}{-12.5} = 0.4\,\mathrm{A}(Ans)$$

From Eqn. (2.2) $E = 22.25 - 2.5I_1 = 22.25 - 2.5 \times 0.4 = 21.25$ V (Ans)

From Eqn. (2.1) $I = I_1 + 1.5 = 0.4 + 1.5 = 1.9$ A.

Energy dissipated in 1 minute in 10 Ω resistor = $I^2RT = (1.9)^2 \times 10 \times 60 = 2166$ joules.

□ **Problem**

In the circuit shown in Figure 2.20, find E_1, E_2 and I when the power dissipated in 5 Ω resistor is 125 W.

(July 1990, 6 marks)

Solution:

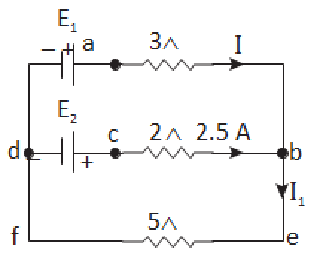


Figure 2.20

$$I_1^2 5 = 125$$

$$I_1 = \frac{125}{5} = \sqrt{25} = 5\text{A}.$$

Applying KCL to node b, we get,

$$I + 2.5 - I_1 = 0$$

$$I + 2.5 - 5 = 0 \quad \text{or} \quad I = 2.5\ \text{A}(Ans)$$

Applying KVL to the $abefda$, we get,

$$-3I - 5I_1 + E_1 = 0$$

$$-3 \times 2.5 - 5 \times 5 + E_1 = 0 \quad \text{or} \quad E_1 = 32.5\ \text{V}(Ans)$$

Applying KVL to the $cbefdc$, we get

$$-2 \times 2.5 - 5 \times I_1 + E_2 = 0$$

$$-5 - 5 \times 5 + E_2 = 0 \quad \text{or} \quad E_2 = 30\ \text{V}(Ans)$$

□ Problem 6

Resistances are arranged in a network in the following manner $R_{AB} = 8\ \Omega$, $R_{BC} = 16\ \Omega$, $R_{CD} = 10\ \Omega$, $R_{DA} = 4\ \Omega$, $R_{BD} = 200\ \Omega$. A voltage of 6 V is applied across AC with A positive. Find the current in each resistor.

(Jan 1993, 10 marks)

Solution:

The magnitude and direction of the currents flowing through each resistor is assumed as shown in the Figure 2.26

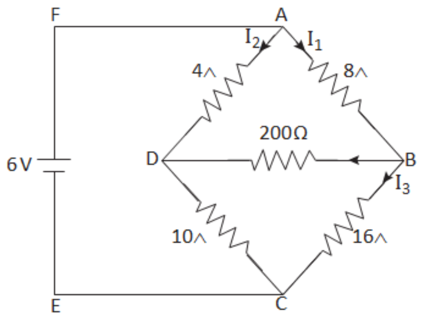


Figure 2.26

Applying KCL to node B we get,

$$I_1 - I_{BD} - I_3 = 0$$

Therefore,

$$I_{BD} = (I_1 - I_3)$$

Applying KCL to node D, we get,

$$I_2 + I_{BD} - I_{DC} = 0$$

Therefore,

$$I_{DC} = I_2 + I_{BD}$$
$$= (I_2 + I_1 - I_3).$$

Applying KVL to the loop $ABDA$, we get,

$$-8I_1 - 200(I_1 - I_3) + 4I_2 = 0$$

Applying KVL to the loop $ABDA$, we get,

$$-8I_1 - 200(I_1 - I_3) + 4I_2 = 0$$

or,

$$-208I_1 + 4I_2 + 200I_3 = 0 \qquad (2.1)$$

Applying KVL to the loop $BCDB$, we get,

$$-16I_3 + 10(I_2 + I_1 - I_3) + 200(I_1 - I_3) = 0$$

or

$$210I_1 + 10I_2 - 226I_3 = 0 \qquad (2.2)$$

Applying KVL to the loop $ABCEFA$, we get,

$$-8I_1 - 16I_3 + 6 = 0 \qquad (2.3)$$

SOLVING 3- EQUATIONS WE GET

$I_1 = 0.249$ A.

$I_2 = 0.4295$ A

$I_3 = 0.2505$ A

□ Problem 7

In the circuit shown in Figure 2.28 find

(i) the values of currents I_1, I_2, I_3 and I
(ii) voltage V_{AB}
(iii) power in 5 Ω resistor.

(Feb 1994)(10 Marks)

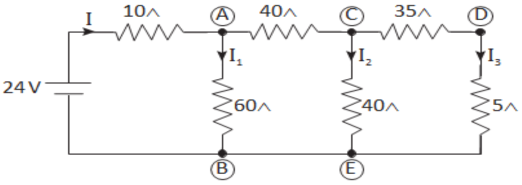
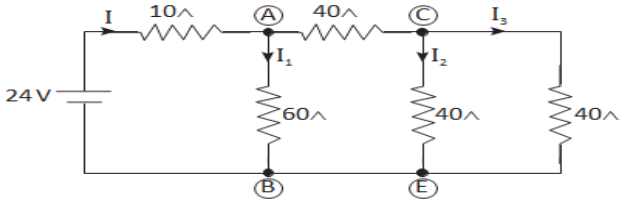


Figure 2.28

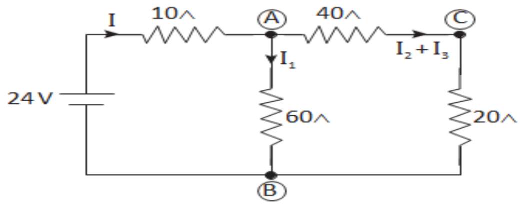
Solution:

Between nodes C and E the resistors 35 Ω and 5 Ω are in series. So they can be replaced by single resistor of 40 Ω.

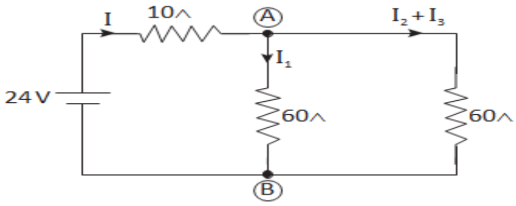
Now the two 40 Ω resistors are in parallel between the nodes C and E. They can be replaced by a single resistor of $\frac{40}{2} = 20\Omega$. [Figure 2.29(a)]



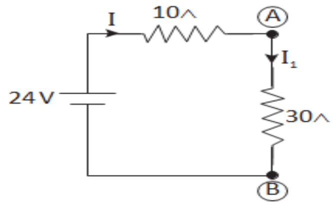
(a)



(b)



(c)



(d)

$$R_{eq} = 10 + 30 = 40\ \Omega \quad \text{[Figure 2.29(d)]}$$

(i) Total current from the battery is given by,

$$I = \frac{24}{R_{eq}} = \frac{24}{40} = 0.6\ \text{A}$$

(a) Current through 60 Ω resistor connected between nodes A and B as shown in Figure 2.29(c)

$$I_1 = \left(\frac{60}{60+60}\right) I = \frac{1}{2} I = \frac{0.6}{2} = 0.3\ \text{A}.$$

Applying KCL at node A, we get Figure 2.29(a)

$$I - I_1 - I_{AC} = 0$$
$$0.6 - 0.3 - I_{AC} = 0$$
$$I_{AC} = 0.3\ \text{A}.$$

(b) Current through 40 Ω resistor connected between nodes C and E as shown in Figure 2.29(a)

$$I_2 = \left(\frac{40}{40+40}\right) I_{AC} = \frac{I_{AC}}{2} = \frac{0.3}{2} = 0.15\ \text{A}$$

Applying KCL at node C, we get from Figure 2.29(a)

$$I_{AC} - I_2 - I_3 = 0$$
$$0.3 - 0.15 - I_3 = 0$$
$$I_3 = 0.15\ \text{A}.$$

(ii) Voltage $V_{AB} = I_1 \times 60 = 0.3 \times 60 = 18$ V.

(iii) Power in 5 Ω resistor $= I_3^2 \times 5 = (0.15)^2 \times 5 = 0.1125$ W.

1.6 SOURCE TRANSFORMATIONS

It is possible to transform an independent voltage source in series with a resistor into a current source in parallel with a resistance or vice versa.

A source transformation is a procedure for transforming one source into another while retaining the terminal characteristics of the original source.

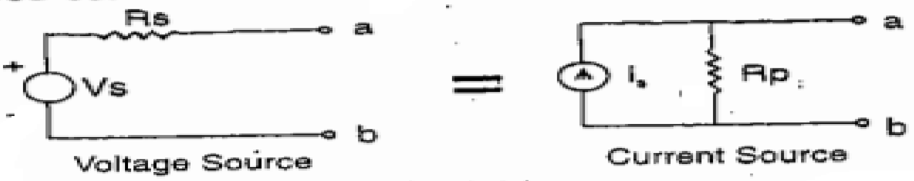


Fig. 1.11

In the fig. 1.11, voltage source is converted into current source.

The voltage of $I_s = \frac{V_s}{R_s}$ and parallel resistor $R_p = R_s$.

A current source can also be converted into voltage source.

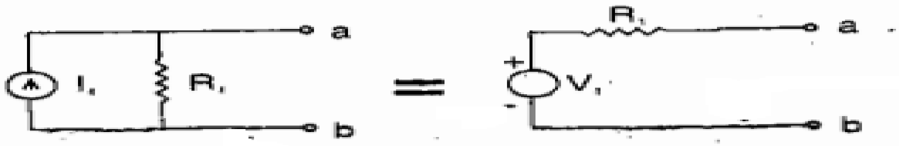
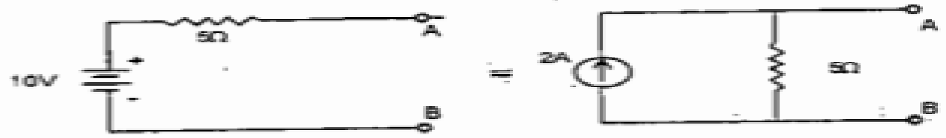


Fig. 1.12

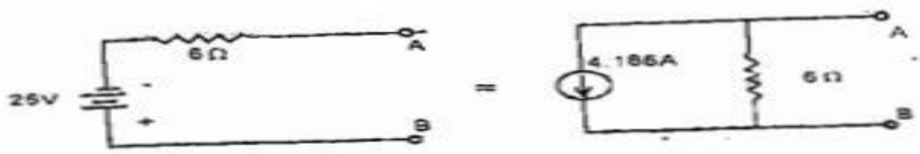
In the fig. 1.12, current source is converted into voltage source. The value $V_1 = I_1 R_1$ and series resistor R of voltage source is same as that of current source.

(a) $I = \frac{10}{5} = 2$ Amps



(b) $I = \frac{25}{6} = 4.166$ Amps

The Direction of current source should be same as that of the current direction of the voltage source.



current source into
voltage source

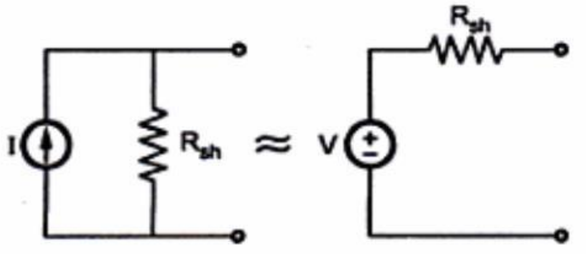


Fig. 2.24 (c) $V = I \times R_{sh}$

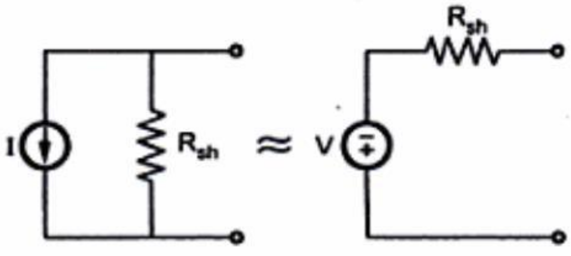
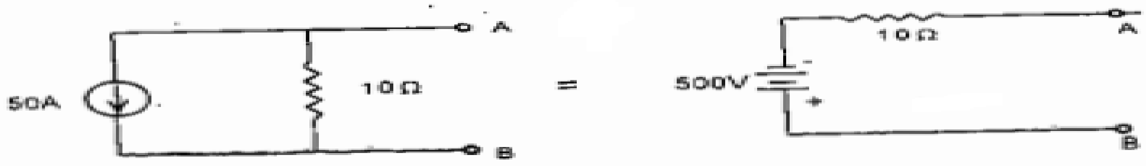


Fig. 2.24 (d) $V = I \times R_{sh}$

Voltage acress AB = I × R = 50 × 10 = 500V

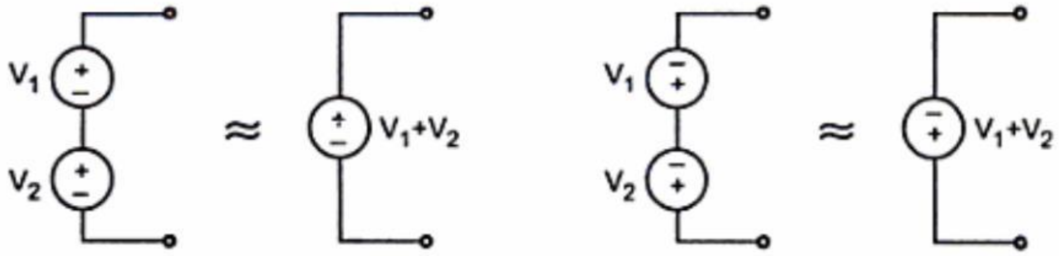
Equivalent voltage source is,



Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources.

Consider the two sources as shown in the Fig. 2.27.



(a) Fig. 2.27 (b)

Voltage Sources in Parallel

Consider the two voltage sources in parallel as shown in the Fig. 2.29.

The equivalent single source has a value same as V_1 and V_2.

It must be noted that at the terminals open circuit voltage provided by each source must be equal as the sources are in parallel.

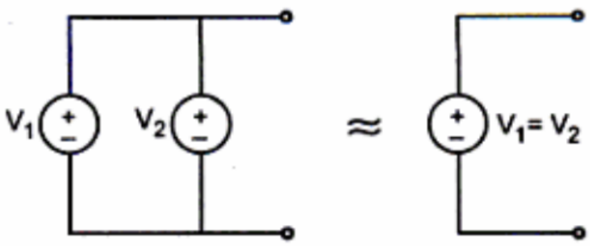
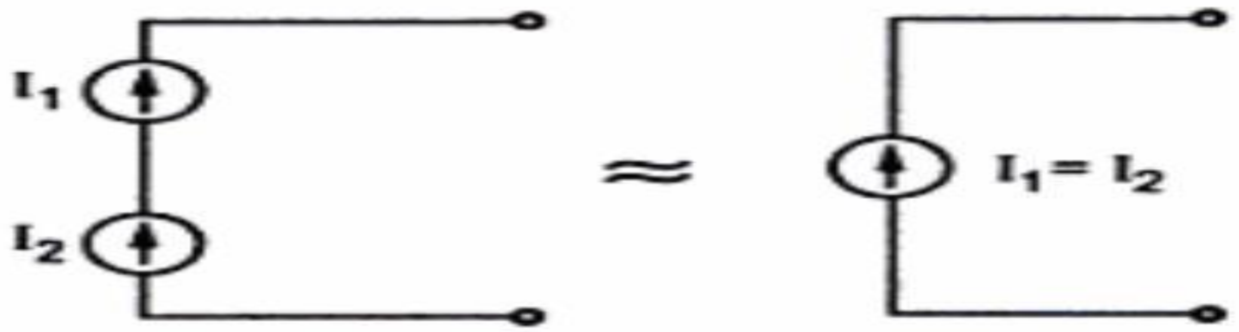


Fig. 2.29

Current Sources in Series



Current Sources in Parallel

Consider the two current sources in parallel as shown in the Fig. 2.31.

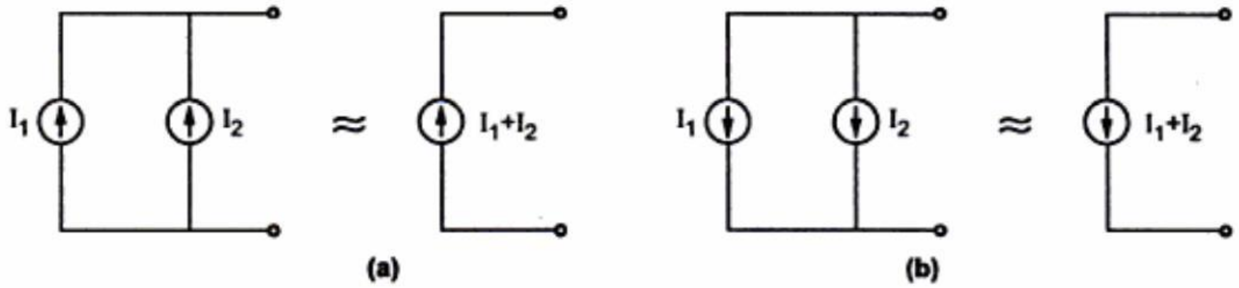


Fig. 2.31

Thus if the directions of the curents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources.

Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the nework plays an important role.

Short Circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire, the two points are said to be short circuited. **The resistance of such short circuit is zero.**

Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

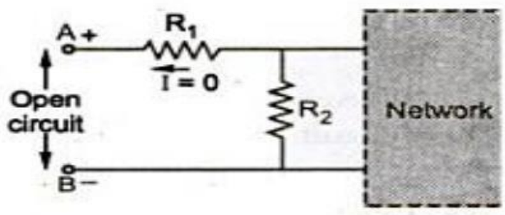


Fig. 2.16

As there is no direct connection in an open circuit, **the resistance of the open circuit is ∞.**

The part of the network which is open circuited is shown in the Fig. 2.16. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{OC} = \infty\,\Omega$.

There exists a voltage across the points AB called open circuit voltage, V_{AB} but $R_{OC} = \infty\,\Omega$.

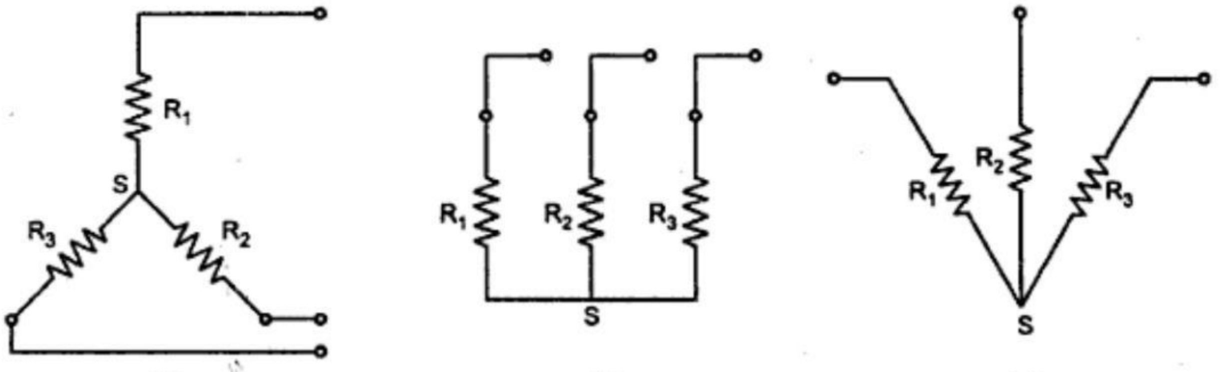
According to Ohm's law,

$$I_{OC} = \frac{V_{AB}}{R_{OC}} = \frac{V_{AB}}{\infty} = 0\ \text{A}$$

Star and Delta Connection of Resistances

In the complicated networks involving large number of resistances, Kirchhoff's laws give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case application of Star-Delta or Delta-Star transformation, considerably reduces the complexity of the network and brings the network into a very simple form. This reduces the number of unknowns and hence network can be analysed very quickly for the required result. These transformations allow us to replace three star connected resistances of the network, by equivalent delta connected resistances, without affecting currents in other branches and vice-versa.

The Fig. 2.38 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 2.38 (a) and (b) are exactly identical. The Fig. 2.38 (b) can be redrawn as Fig. 2.38 (a) or vice-versa, in the circuit from simplification point of view.



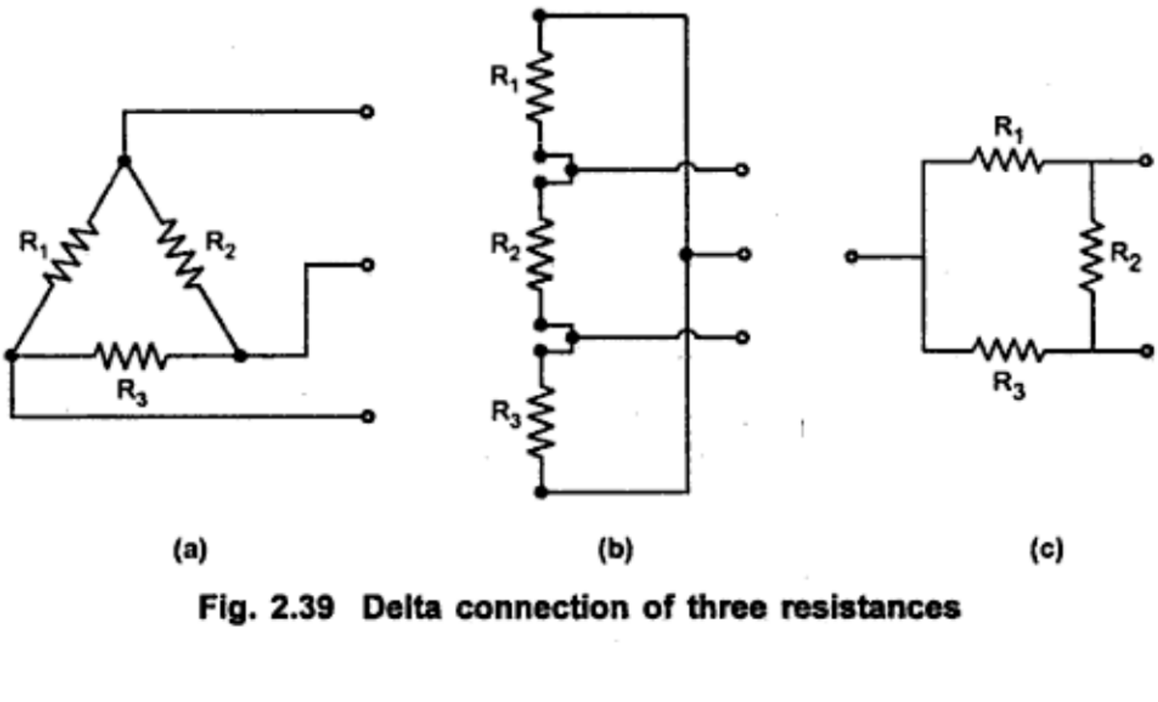
(a) (b) (c)

Fig. 2.38 Star connection of three resistances

Let us see what is delta connection ?

If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in Delta.

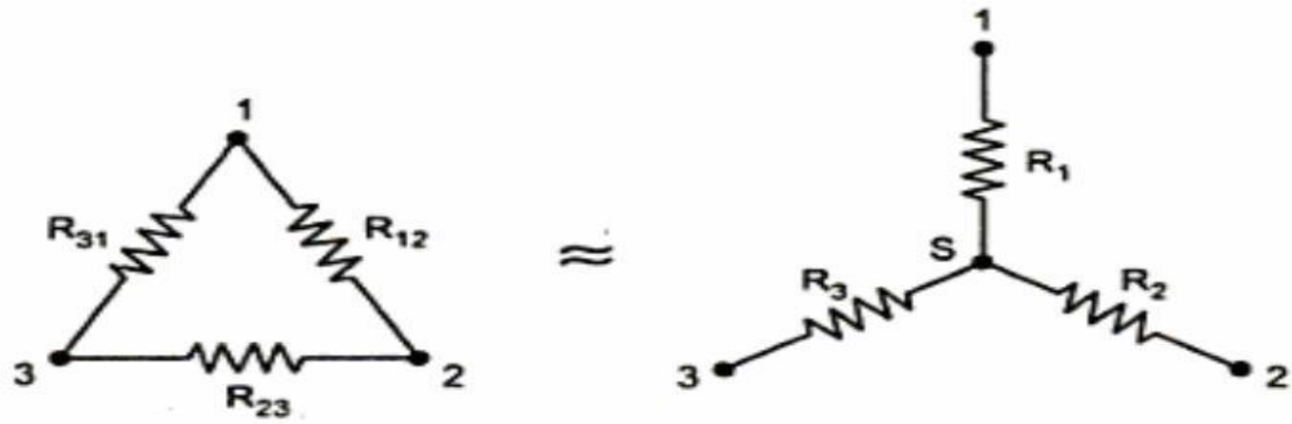
The Fig. 2.39 (a) and (b) show delta connection of three resistances. The Fig. 2.39 and (b) are exactly identical.



(a) (b) (c)

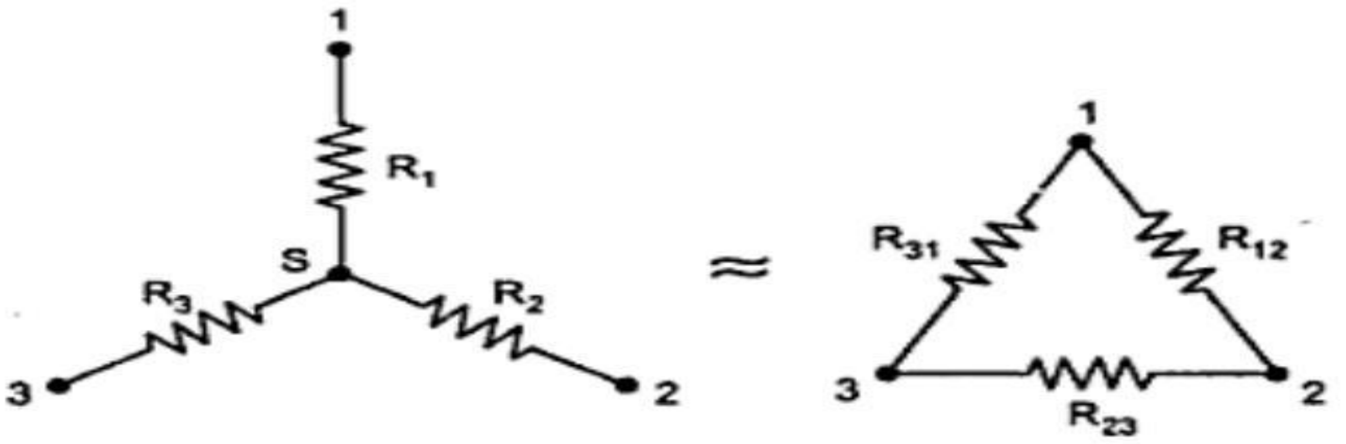
Fig. 2.39 Delta connection of three resistances

2.16.1 Delta-Star Transformation



Given Delta **Fig. 2.40** **Equivalent Star**

2.16.2 Star-Delta Transformation



Given Star **Fig. 2.44** **Equivalent Delta**

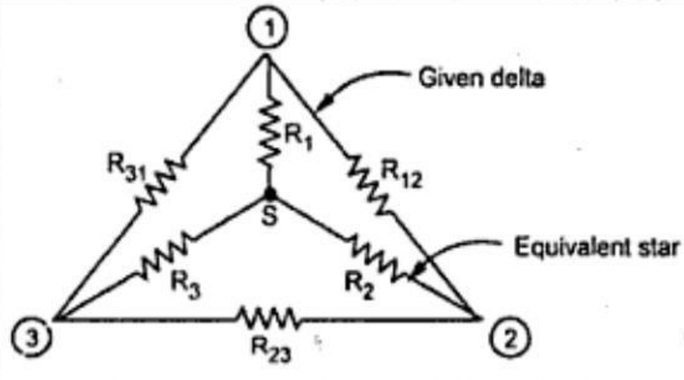
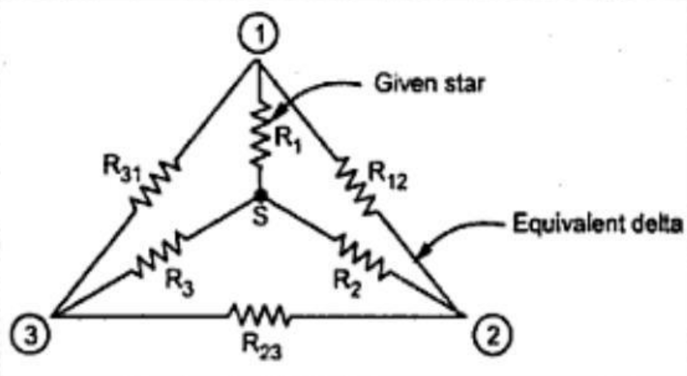
	
$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$	$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$
$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$	$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$
$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$	$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$

Table 2.1 Star-Delta and Delta-Star Transformations

Example 1 *Find equivalent resistance between points A-B.*

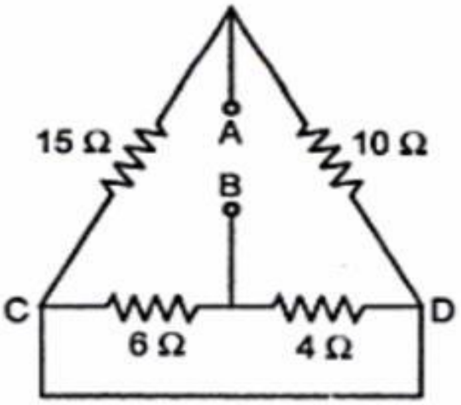
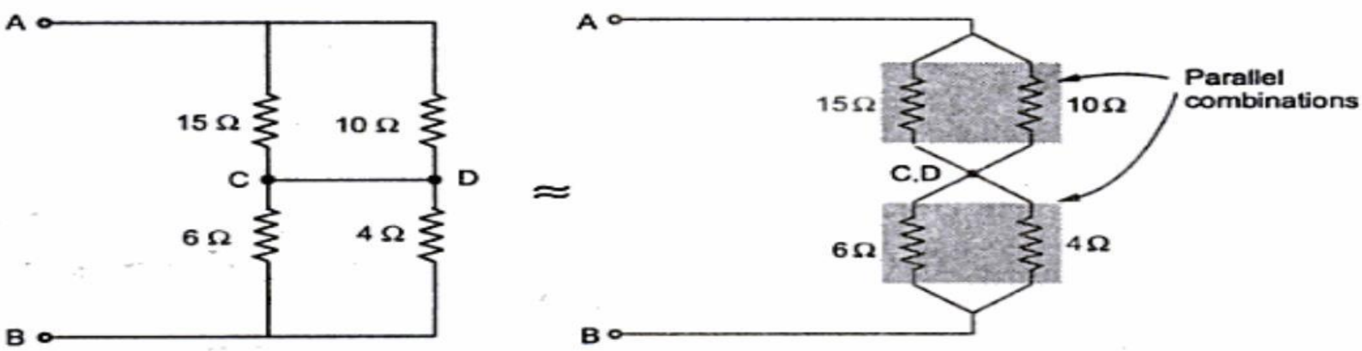
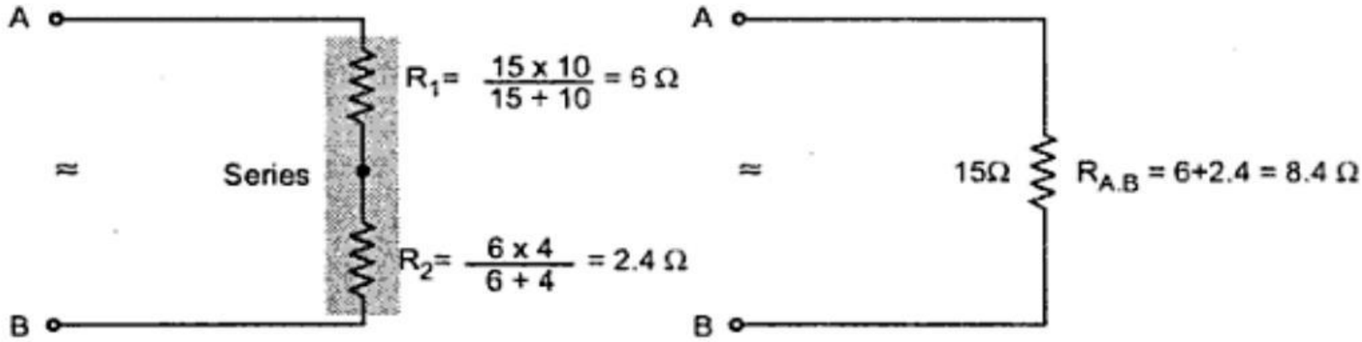


Fig. 2.50

Solution : Redraw the circuit,





Example 2 : *Calculate the effective resistance between points A and B in the given circuit in Fig. 2.51.* **(Dec. - 97)**

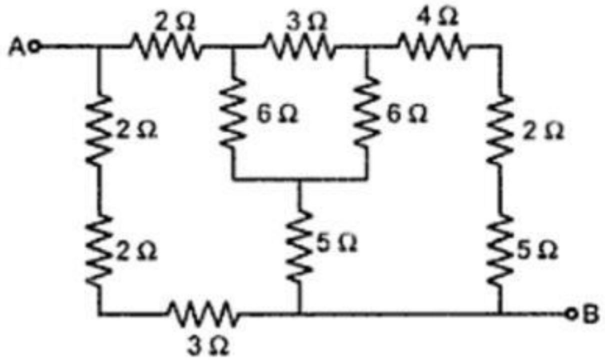


Fig. 2.51

Solution : The resistances 2, 2 and 3 are in series while the resistances 4, 2, and 5 are in series.

$\therefore \quad 2 + 2 + 3 = 7\ \Omega$

and $\quad 4 + 2 + 5 = 11\ \Omega$

The circuit becomes as shown in Fig. 2.51 (a).

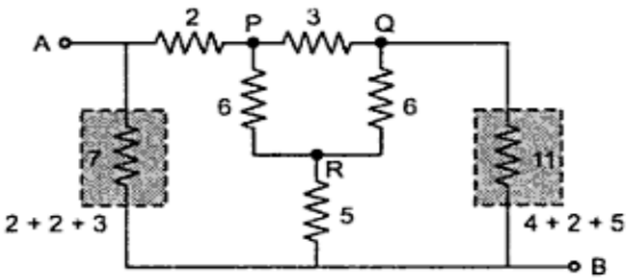


Fig. 2.51 (a)

Converting Δ PQR to equivalent star,

$$R_{PN} = \frac{6\times 3}{6+3+6} = 1.2\ \Omega$$

$$R_{PN} = \frac{6\times 6}{6+3+6} = 2.4\ \Omega$$

$$R_{QN} = \frac{6\times 3}{6+3+6} = 1.2\ \Omega$$

Hence the circuit becomes as shown in the Fig. 2.51 (b).

The resistances 2 and 1.2 are in series.

1.2 and 11 are in series.

5 and 2.4 are in series.

∴ Circuit becomes after simplification as shown in the Fig. 2.51 (c).

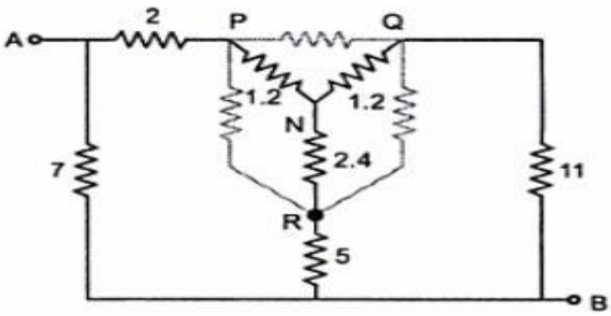
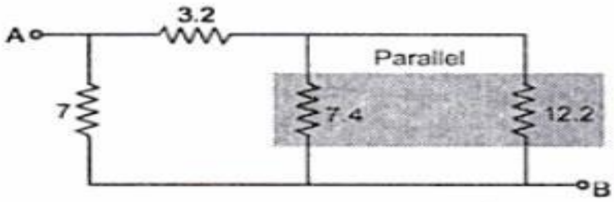


Fig. 2.51 (b)

The resistances 7.4 and 12.2 are in parallel.

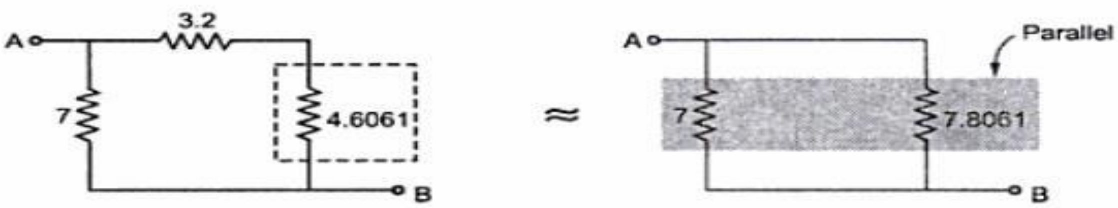
$$\therefore \quad 7.4 \parallel 12.2 = \frac{7.4\times 12.2}{7.4+12.2} = 4.6061\ \Omega$$

So circuit becomes,



Now the two resistances are in parellel as shown in the Fig. 2.51(e).

$$\therefore \quad R_{AB} = \frac{7\times 7.8061}{7+7.8061} = 3.69\ \Omega$$



➡ **Example 3 :** *Find the equivalent resistance between terminals B and C of the circuit shown in the fig. 2.54.* **(May - 99)**

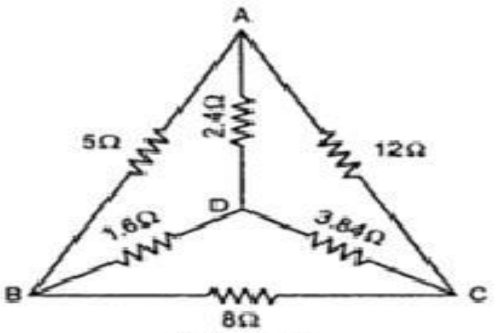


Fig. 2.54

Solution : Solution is also possible by converting Delta to Star which gives solution in less steps.

Converting star ADCB to delta ACB.

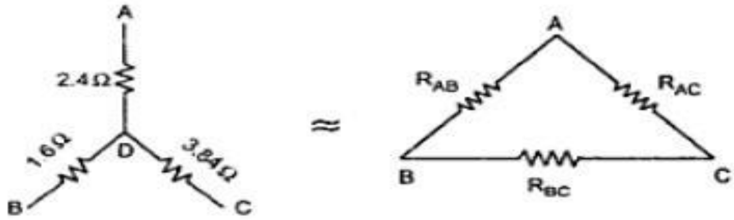


Fig. 2.54 (a)

$$R_{AB} = 2.4 + 1.6 + \frac{2.4 \times 1.6}{3.84} = 5\ \Omega$$

$$R_{AC} = 2.4 + 3.84 + \frac{2.4 \times 3.84}{1.6} = 12\ \Omega$$

$$R_{BC} = 1.6 + 3.84 + \frac{1.6 \times 3.84}{2.4} = 8\ \Omega$$

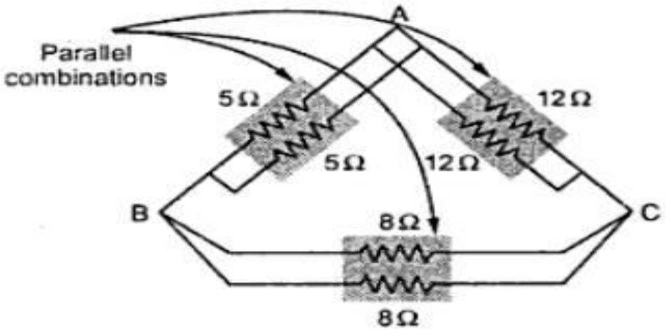


Fig. 2.54 (b)

≈

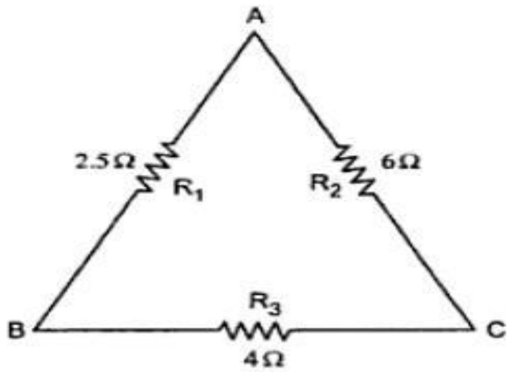


Fig. 2.54 (c)

$$R_1 = \frac{5 \times 5}{5+5} = 2.5\ \Omega, \quad R_2 = \frac{12 \times 12}{12+12} = 6\ \Omega, \quad R_3 = \frac{8 \times 8}{8+8} = 4\ \Omega$$

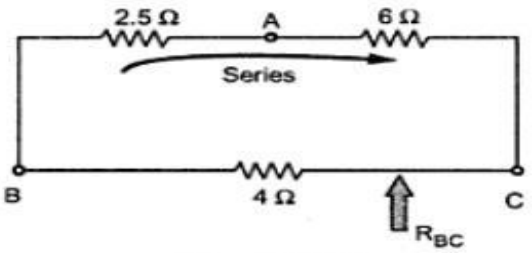


Fig. 2.54 (d)

≈

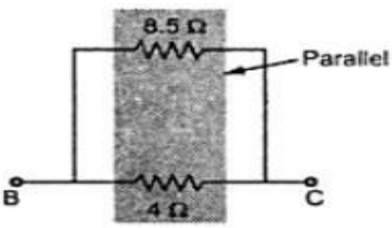


Fig. 2.54 (e)

$$R_{BC} = \frac{4 \times 8.5}{4+8.5} = 2.72\ \Omega$$

Example 3 *Find the resistance between (1) B & C and (2) A & C in the netw shown in the Fig. 2.56.* **(Dec. - 99, Dec. - 200**

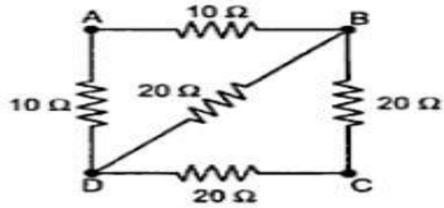


Fig. 2.56

Solution : (i) Between B and C

As looking through B and C, 10 Ω and 10 Ω are in series, as both carry same curren

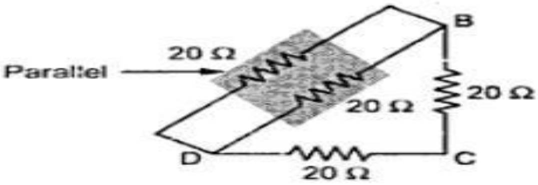


Fig. 2.56 (a)

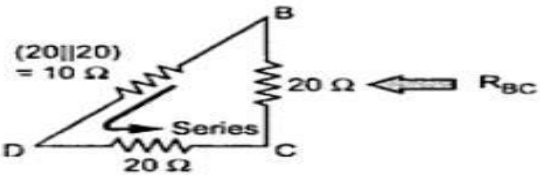


Fig. 2.56 (b)

Again, 10 Ω and 20 Ω are in series.

$$R_{BC} = 20 || 30 = \frac{20 \times 30}{20 + 30}$$

$$= 12\ \Omega$$

Example 4 *Find the resistance between (1) B & C and (2) A & C in the networ shown in the Fig. 2.56.* **(Dec. - 99, Dec. - 2000)**

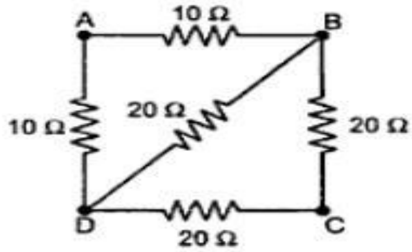


Fig. 2.56

Solution : (i) Between B and C

As looking through B and C, 10 Ω and 10 Ω are in series, as both carry same current.

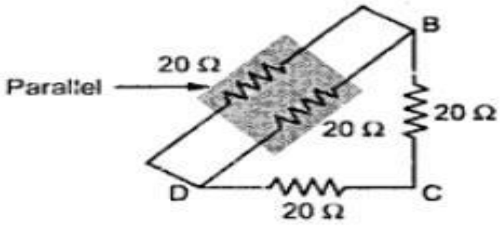


Fig. 2.56 (a)

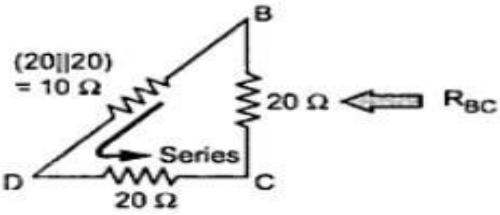
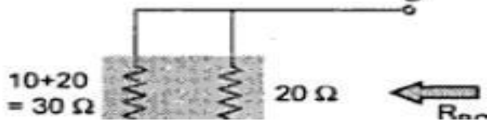


Fig. 2.56 (b)

Again, 10 Ω and 20 Ω are in series.

$$R_{BC} = 20 || 30 = \frac{20 \times 30}{20 + 30}$$

$$= 12\ \Omega$$



(ii) Between A and C

Converting delta BCD to equivalent star,

$$R_{BS} = R_{CS} = R_{DS} = \frac{20 \times 20}{(20 + 20 + 20)}$$

$$= 6.57\ \Omega$$

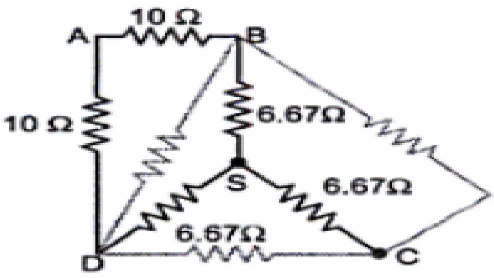


Fig. 2.56 (d)

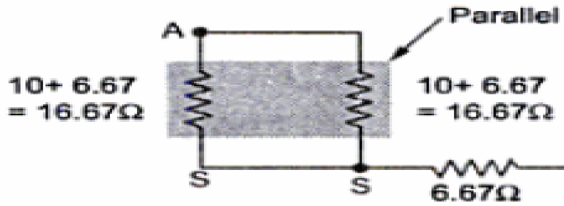
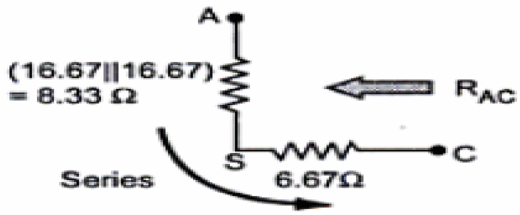


Fig. 2.56 (e)

$$R_{AC} = 8.33 + 6.67$$

$$= 15\ \Omega$$



Example 5 . *Calculate the equivalent resistance between the terminals (X) and (Y) for the circuit shown in Fig. 2.58.* **(Dec. - 2007)**

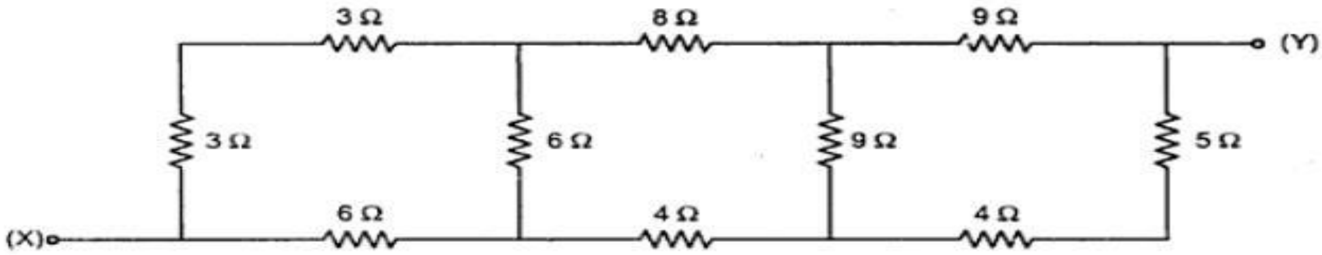


Fig. 2.58

Fig. 2.58 (a)

Convert Δ ABFE to equivalent star.

Convert Δ CDHG to equivalent star.

As all the resistance of Δ ABFE are equal, all the resistances of equivalent star are also equal, given by,

$$R = \frac{6\times 6}{6+6+6} = 2\ \Omega$$

Similarly in equivalent star of CDGH, each resistance is equal say R′ given by,

$$R' = \frac{9\times 9}{9+9+9} = 3\ \Omega$$

The circuit reduces as shown in the Fig. 2.58 (d).

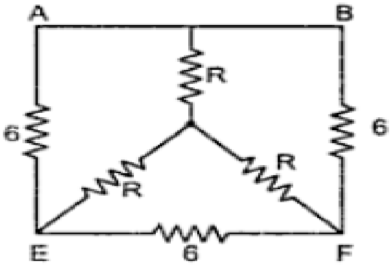


Fig. 2.58 (b)

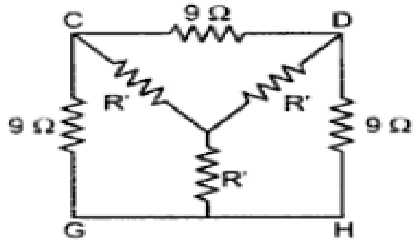


Fig. 2.58 (c)

The resistances 2 Ω, 8 Ω and 3 Ω are in series. The resistances 2 Ω, 4 Ω and 3 Ω are in series.

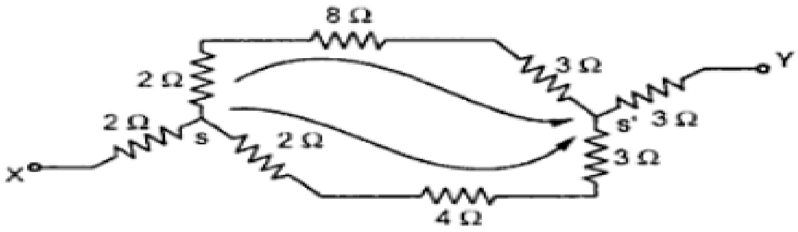


Fig. 2.58 (d)

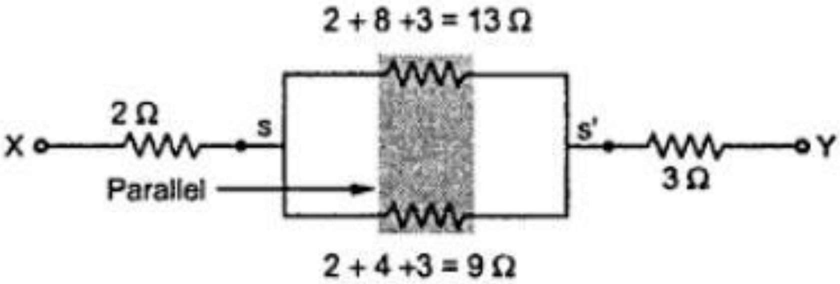


Fig. 2.58 (e)

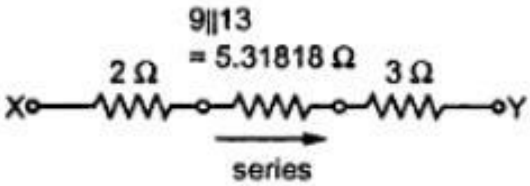
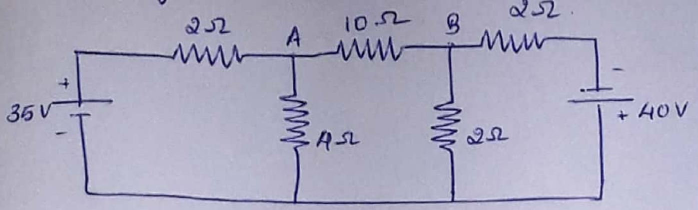


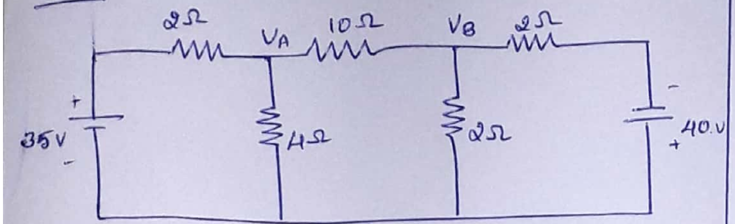
Fig. 2.58 (f)

$$\therefore \quad R_{XY} = 2 + 5.31818 + 3 = 10.31818\ \Omega$$

① Determine voltages at nodes A & B by node voltage technique/nodal Analysis technique.



Soln:



At node V_A,

$$\frac{V_A - 35}{2} + \frac{V_A - 0}{4} + \frac{V_A - V_B}{10} = 0$$

$$\Rightarrow \frac{10V_A - 350 + 5V_A + 2V_A - 2V_B}{20} = 0$$

$$\Rightarrow \boxed{17V_A - 2V_B = 350} \longrightarrow ①$$

At node V_B,

$$\frac{V_B - V_A}{10} + \frac{V_B - 0}{2} + \frac{V_B + 40}{2} = 0$$

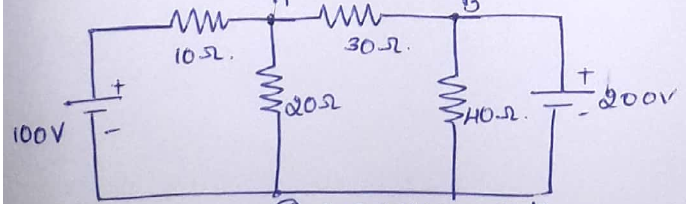
$$\frac{V_B - V_A + 5V_B + 5V_B + 200}{10} = 0$$

$$\boxed{11V_B - V_A = -200} \longrightarrow ②$$

Solving eq^n ① & eq^n ②,

we get, $\boxed{\begin{matrix} V_A = 18.64 \text{ volts} \\ V_B = -16.48 \text{ volts} \end{matrix}}$

② Find the node voltages V_A & V_B



© → Reference node

Soln: As -ve terminal of 200V battery is at reference node, the potential at node B is 200V $\therefore$ $\boxed{V_B = 200 \text{volts}}$

Apply nodal equations for node A.

$$\frac{V_A - 100}{10} + \frac{V_A}{20} + \frac{V_A - V_B}{30} = 0$$

$$\frac{6V_A - 600 + 3V_A + 2V_A - 2V_B}{60} = 0$$

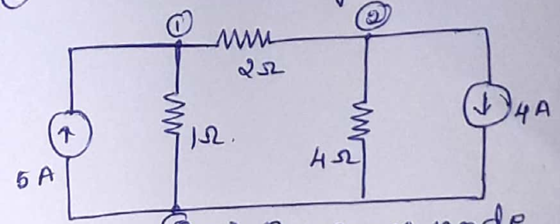
$$11V_A - 2V_B = 600$$

Substitute $V_B = 200v$ in the above equation.

$$11V_A - 2(200) = 600$$

$$11V_A = 1000 \Rightarrow \boxed{V_A = 90.91 \text{ volts}}$$

③ Find V_1 & V_2 by nodal analysis



③ → Reference node

Soln: At node①

$$\frac{V_1 - 0}{1} + \frac{V_1 - V_2}{2} - 5 = 0$$

$$2V_1 + V_1 - V_2 - 10 = 0$$

$$\boxed{3V_1 - V_2 = 10} \longrightarrow ①$$

At node②

$$4 + \frac{V_2 - V_1}{2} + \frac{V_2}{4} = 0$$

$$8 + 2V_2 - 2V_1 + V_2 = 0$$

$$\boxed{-2V_1 + 3V_2 = -8} \longrightarrow ②$$

Solving equation ① & ②

we get, $\boxed{\begin{matrix} V_1 = 2 \text{ volts} \\ V_2 = -4 \text{ volts} \end{matrix}}$