UNIT - 3
Analysis of Trusses

Trusses:

A truss (or a frame) is a structure which consists of number of members connected at their ends by pin joints to support an external load system.

A frame in which all the members lie in a single plane, is called a plane truss (plane frame)
Ex: Roof trusses or Bridge trusses

If all the members of a truss don't lie in a single plane, then the truss is called a space truss (or space frame).
Ex: Tripods and Transmission towers

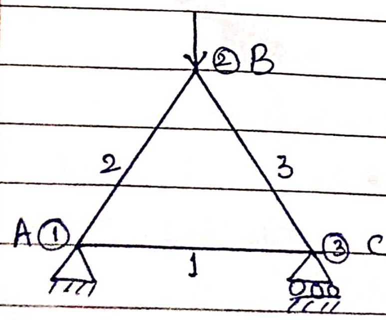
The members of the truss will be in either Tension (T) or compression (C).

Classification of trusses:

The trusses can be classified as follows.

1. Perfect Truss
2. Imperfect Truss → Deficient Truss
   → Redundant Truss

Perfect truss:



A perfect frame is that it should retain its shape when load is applied at any point in any direction.

The number of members in a perfect truss can be expressed as

$$n = (2j - 3)$$

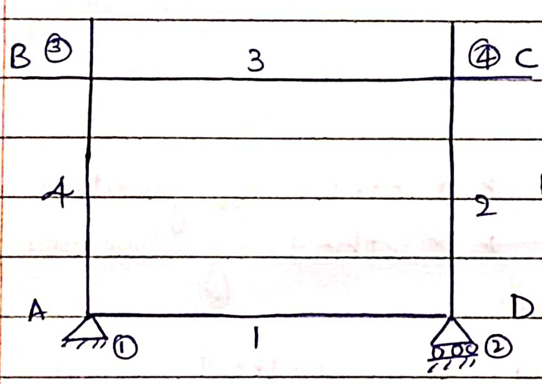
where n = number of members

j = number of joints.

Here $j = 3 \Rightarrow n = (2 \times 3 - 3) = 3$

Imperfect truss: An imperfect truss is that which doesn't satisfy the equation $n = (2j-3)$ i.e., $n \neq (2j-3)$.

1. **Deficient truss:** The number of members are less than $(2j-3)$ i.e., $n < (2j-3)$
   In this case, the frame won't be stable, such trusses can't retain their shape when loaded

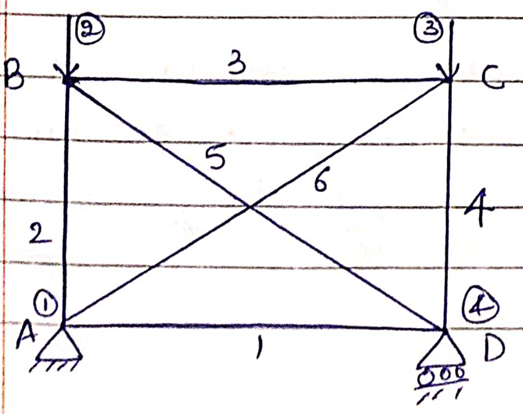


$n = 4, \ j = 4$

$n \Rightarrow (2j-3) = (2 \times 4 - 3) = 5$

$n < (2j-3)$

2. **Redundant frame (Truss):** The number of members are more than $(2j-3)$ i.e., $n > (2j-3)$
   Redundant truss can't be analysed by making use of equations of equilibrium
   (i.e., $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$)
   $\therefore$ A redundant truss is statically indeterminate.



$n = 6, \ j = 4$

$(2j-3) = (2 \times 4 - 3) = 5$

$n > (2j-3)$

Assumptions made in analysis of trusses

1. The ends of the members are pin connected (hinged).
2. The truss is a perfect truss i.e., it should satisfy the equation, $n = 2j - 3$.
3. The load act only at the joints
4. Self weights of the members are neglected.
5. Cross section of the members are uniform.

Analysis of trusses:

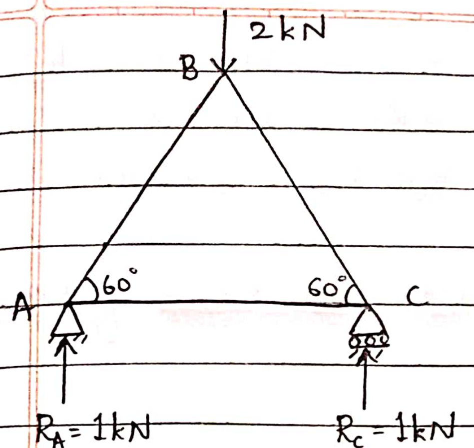
The trusses can be analysed by two methods. They are:

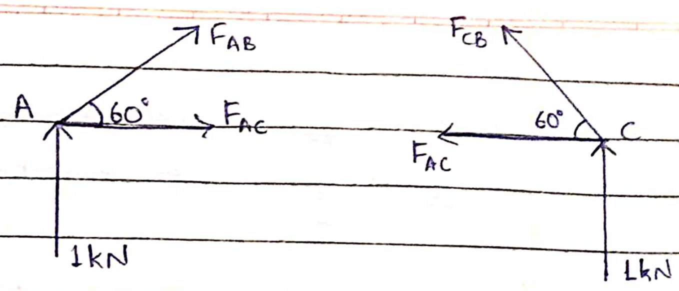
1. Method of joints
2. Method of sections

Analysis of trusses → Method of joints
→ Method of sections

1. Method of joints: In this method, each and every joint is selected and the unknown forces are then determined by the equations of equilibrium. The force meeting at a point and the loads acting, if any, constitute a system of concurrent forces. Therefore, the equations of equilibrium $\Sigma H = 0$ and $\Sigma V = 0$ are used to determine the forces in the members.

While selecting the joint care should be taken that at any instant, the joint shouldn't contain more than two members in which the forces are unknown.

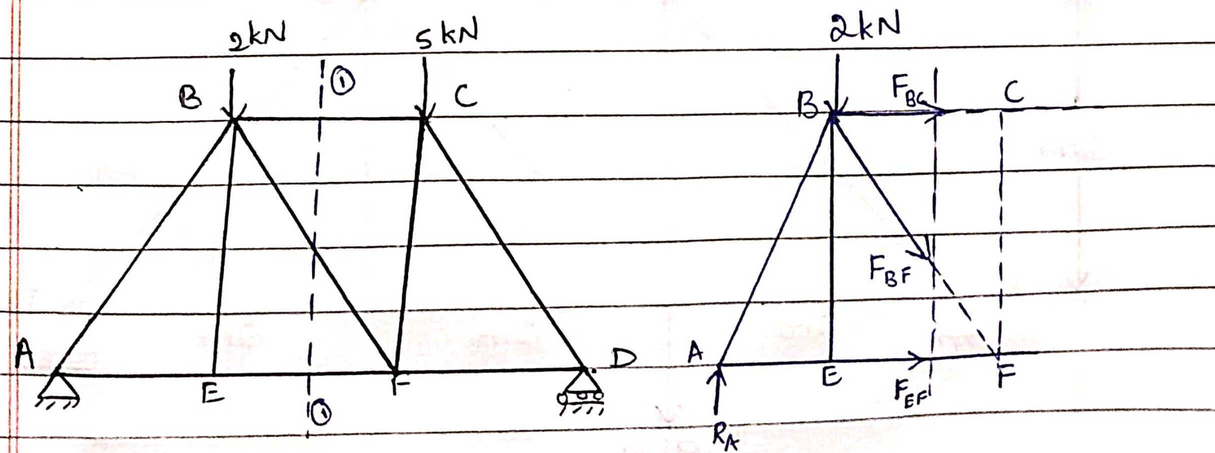




From FBD of joint A, F_{AB} can be found using $\Sigma V = 0$ and F_{AC} can be found using $\Sigma H = 0$.

From FBD of joint C, F_{CB} can be found using $\Sigma V = 0$ and F_{AC} can be found using $\Sigma H = 0$.

2. Method of Sections: In this method, a section line is passed through the members in which the forces are to be determined.
The section line is drawn is which it shouldn't cut more than three members in which the forces are unknown.
The unknown forces in the members are then determined by using the equations of equidibrium $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$.
(∵ The system of forces acting on either part of the truss contributes a non-concurrent force system)
(The method of sections is used when force is only one member or the forces in very few members are to be determined)



To find force in member BC : $\sum M_F = 0 \therefore F_{BC} =$

To find force in member EF : $\sum M_B = 0 \therefore F_{EF} =$

To find force in member BF : $\sum V = 0$ or $\sum H = 0 \therefore F_{BF} =$

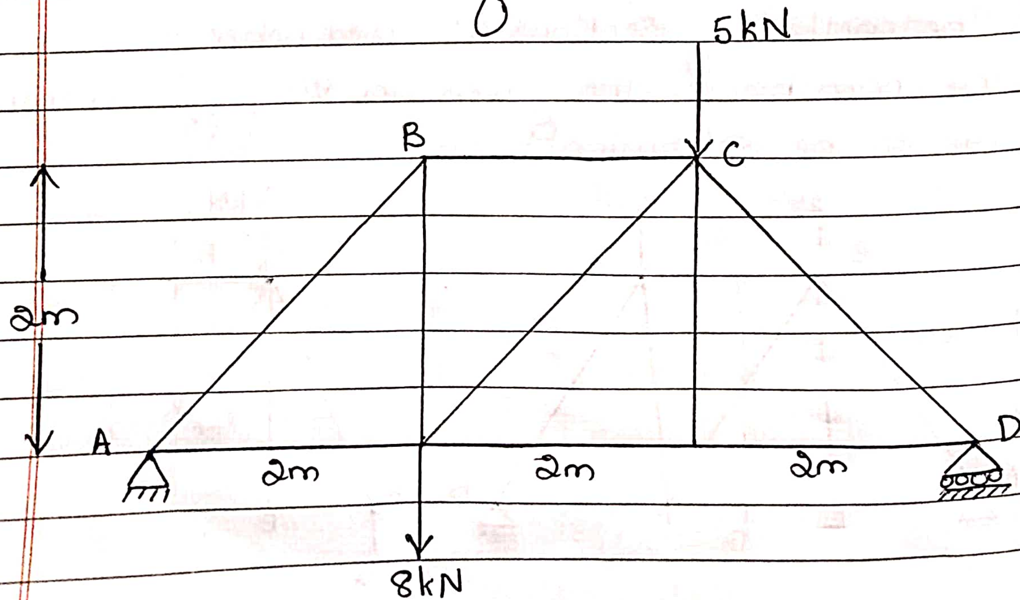
Analysis of forces in the members of truss using method of joints

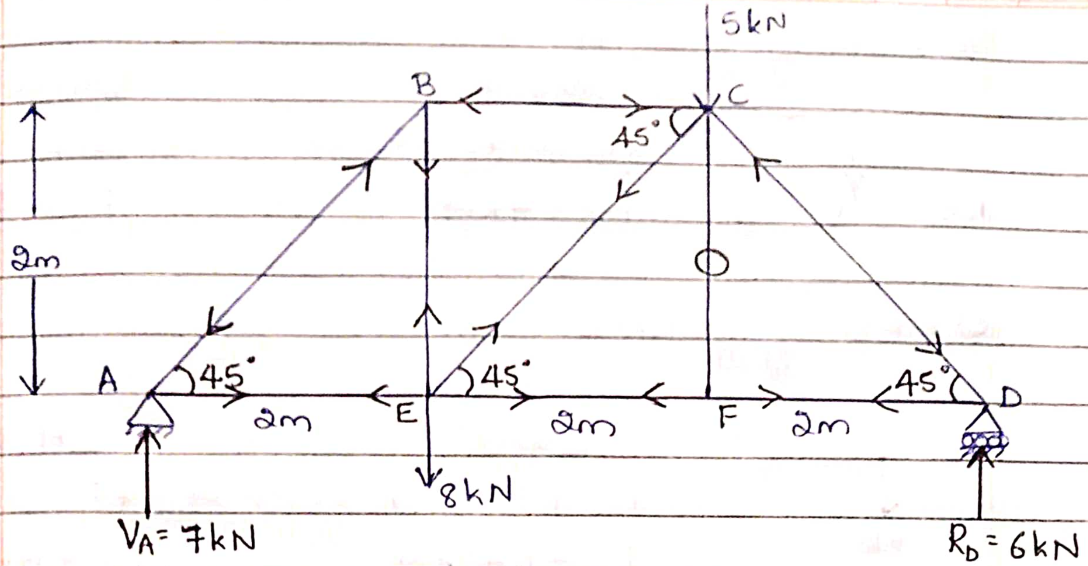
Procedure

1. Determine the support reactions
2. Determine the angle of inclination of the members with horizontal
3. Determine the internal forces acting in the members of frame by considering the free body diagram of the members of various points.
4. Tabulate the results with magnitude and direction
5. The FBD of the joints should be selected where the joint is having maximum of 2 unknown forces since only two equations $\sum H = 0$ and $\sum V = 0$ are available for the analysis.

Problems:

1. Find the internal forces in the members of frame





Let V_A and R_D be the support reactions at A and D respectively.

Taking moments of all forces about D (+ve, −ve)

$V_A \times 6 - 8 \times 4 - 5 \times 2 = 0$

$V_A = 7kN$

Taking moments of all forces about A (−ve, +ve)

$R_D \times 6 - 5 \times 4 - 8 \times 2 = 0$

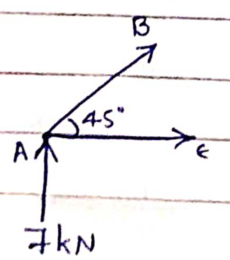
$R_D = 6kN$

Note: There are no horizontal or vertical forces acting on the frame.

$\therefore H_A = 0$

Method of joints

a) FBD of joint A.



$\Sigma V = 0$

$7 + F_{AB} \sin 45° = 0$

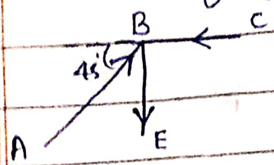
$F_{AB} = -9.9kN$

$\Sigma H = 0$

$F_{AE} + F_{AB} \cos 45° = 0$

$F_{AE} = 7kN$

b) FBD of joint B



$\Sigma V = 0$

$F_{AB} \sin 45° - F_{BE} = 0$

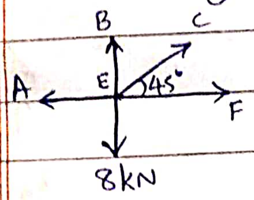
$F_{BE} = 7 kN$

$\Sigma H = 0$

$F_{AB} \cos 45° - F_{CB} = 0$

$F_{CB} = 7 kN$

c) FBD of joint E



$\Sigma V = 0$

$F_{BE} + F_{EC} \sin 45° - 8 = 0$

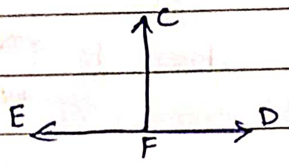
$F_{EC} = 1.41 kN$

$\Sigma H = 0$

$-F_{AE} + F_{EC} \cos 45° + F_{EF} = 0$

$F_{EF} = 6 kN$

d) FBD of joint F



$\Sigma V = 0$

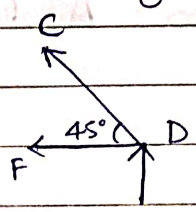
$F_{FC} = 0$

$\Sigma H = 0$

$F_{FD} - F_{EF} = 0$

$F_{FD} = 6 kN$

e) FBD of joint D



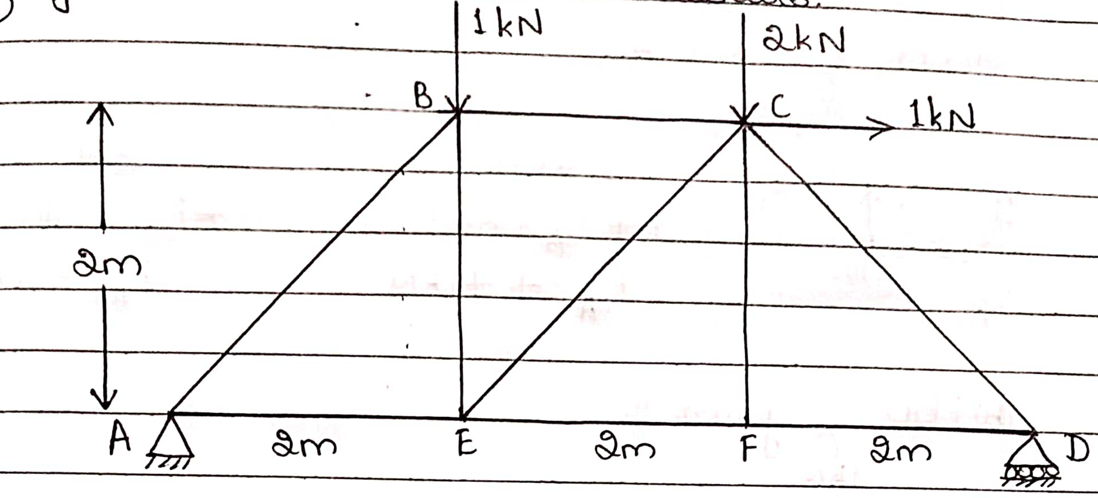
$\Sigma H = 0$

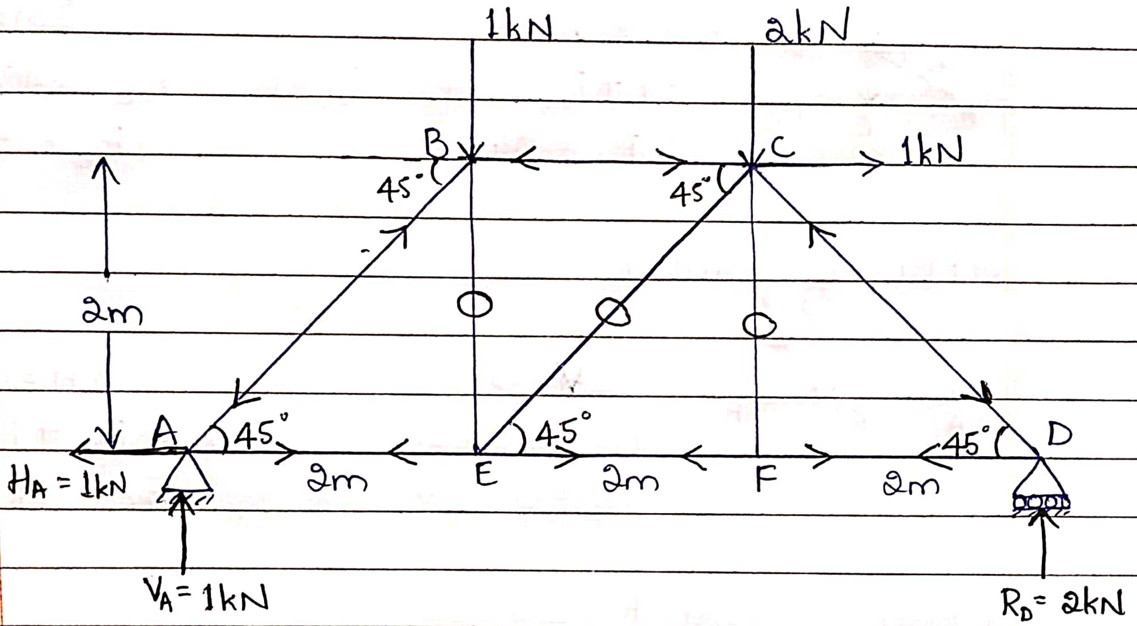
$-F_{FD} - F_{DC} \cos 45° = 0$

$F_{DC} = -8.48 kN$

Member	Magnitude of force in kN	Nature of force (Direction)
AB	9.9	C
BC	7	C
CD	8.48	C
DF	6	T
FE	6	T
EA	7	T
BE	7	T
CF	0	-
EC	1.41	T

2. Analyse the frame shown in figure by method of joints and tabulate the results.





Let V_A, R_D and H_A be the support reactions at at A and D respectively.

Taking moments of all forces at D   +ve   −ve

$V_A \times 6 - 1\times 4 - 2\times 2 + 1\times 2 = 0$

$V_A = 1kN$

Taking moments of all forces at A   −ve   +ve

$R_D \times 6 - 2\times 4 - 1\times 2 - 1\times 2 = 0$

$R_D = 2kN$

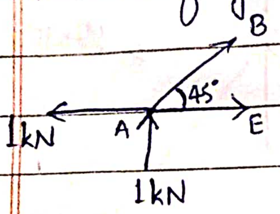
$\Sigma H = 0$

$-H_A + 1 = 0$

$H_A = 1kN$

Method of joints

a) FBD of joint A



$\Sigma V = 0$

$1 + F_{BA} \sin 45^\circ = 0$

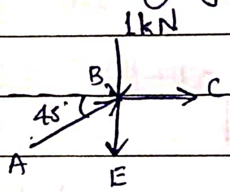
$F_{BA} = -1.41 kN$

$\Sigma H = 0$

$-1 + F_{AB} \cos 45^\circ + F_{AE} = 0$

$F_{AE} = 2 kN$

b) FBD of joint B



$\Sigma V = 0$

$-1 + F_{AB} \sin 45^\circ - F_{BE} = 0$

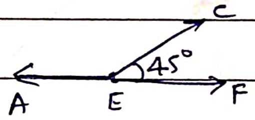
$F_{BE} = 0 kN$

$\Sigma H = 0$

$F_{AB} \cos 45^\circ + F_{BC} = 0$

$F_{BC} = -1 kN$

c) FBD of joint E



$\Sigma M = 0$

$F_{EC} \cos 45^\circ = 0$

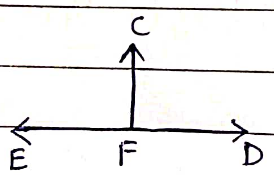
$F_{EC} = 0$

$\Sigma H = 0$

$-F_{AE} + F_{EF} + F_{EC} \cos 45^\circ = 0$

$F_{EF} = 2 kN$

d) FBD of joint F



$\Sigma V = 0$

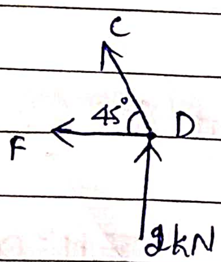
$E_{FC} = 0$

$\Sigma H = 0$

$E_{FD} - E_{FE} = 0$

$E_{FD} = 2 kN$

e) FBD of joint D.



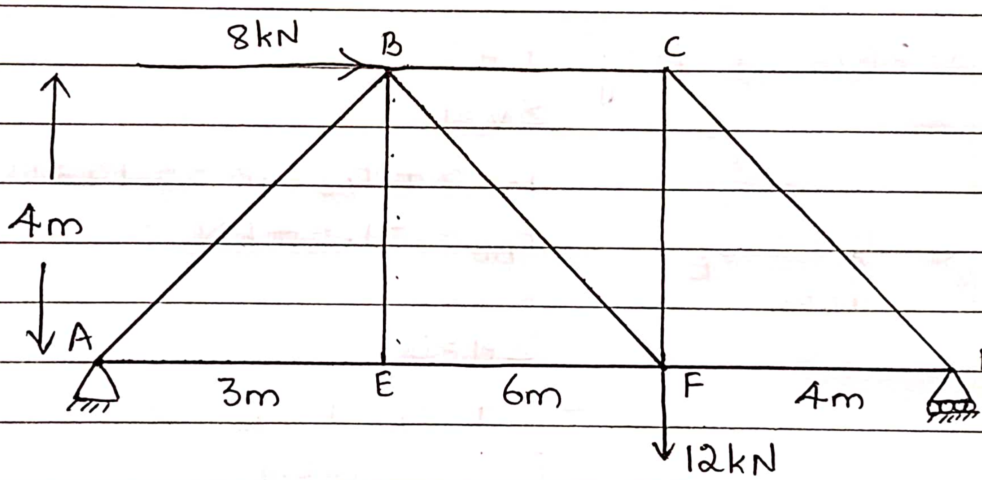
$\Sigma V = 0$

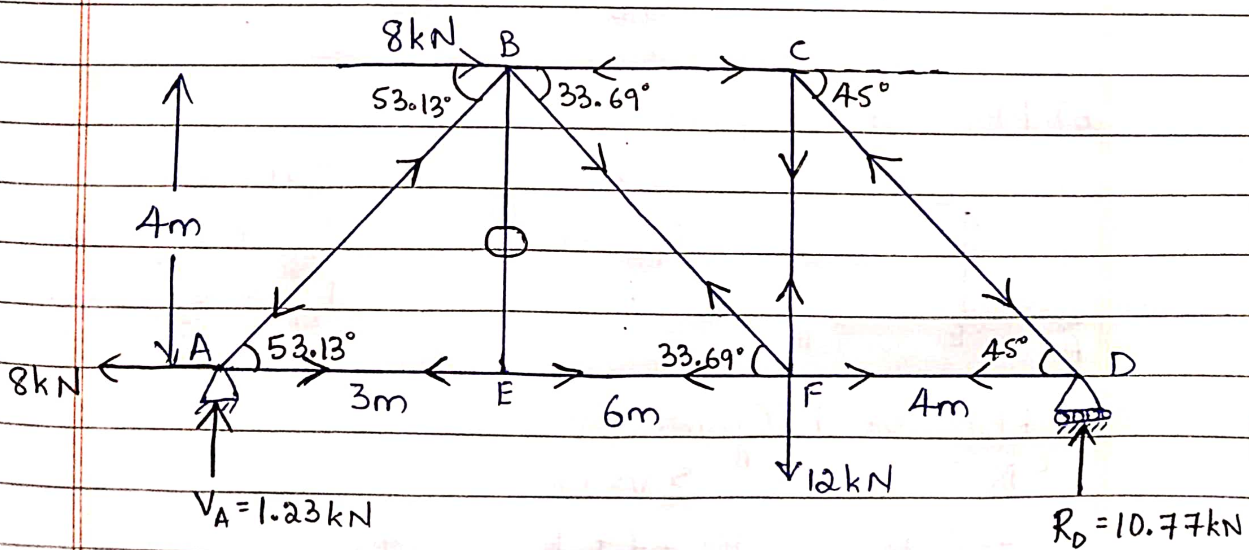
$2 + F_{DC} \sin 45^\circ = 0$

$F_{DC} = -2.82 kN$

Members	Internal forces (in kN)	Nature of forces (Direction)
AB	1.41	compression
BC	1	compression
DC	2.82	compression
AE	2	Tension
EF	2	Tension
FD	2	Tension
EB	0	-
EC	0	-
FC	0	-

3. Analyse the truss shown in figure by the method of joints and tabulate the results.





$\Sigma H = 0 \Rightarrow 8 - H_A = 0 \Rightarrow H_A = 8kN$

Let V_A and R_D be the vertical reactions at A and D respectively.

Taking moments of all forces about D ↻ +ve ↺ −ve

$V_A \times 13 + 8 \times 4 - 12 \times 4 = 0$

$V_A = 1.23\ kN$

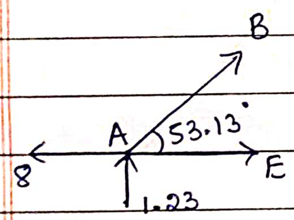
Taking moments of all forces about A ↻ −ve ↺ +ve

$R_D \times 13 - 8 \times 4 - 12 \times 9 = 0$

$R_D = 10.77\ kN$

Method of joints:

a) FBD of A (joint A)



$\Sigma V = 0$

$1.23 + F_{AB} \sin 53.13^\circ = 0$

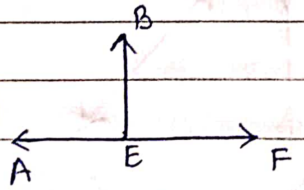
$F_{AB} = -1.54\ kN$

$\Sigma H = 0$

$-8 + F_{AE} + F_{AB} \cos 53.13^\circ = 0$

$F_{AE} = 8.92\ kN$

b) FBD of E (joint E)



$\Sigma V = 0$

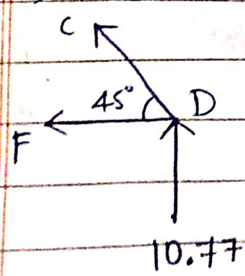
$F_{EB} = 0$

$\Sigma H = 0$

$-F_{AE} + F_{EF} = 0$

$F_{EF} = 8.92\ kN$

c) FBD of D (joint D)



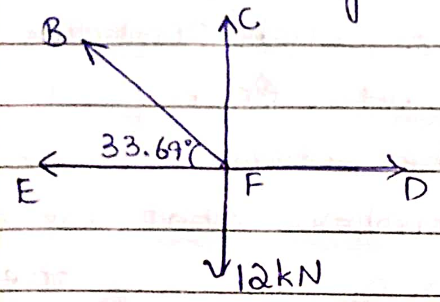
$\Sigma V = 0$

$10.77 + F_{DC} \sin 45^\circ = 0$

$F_{DC} = -15.23\ kN$

$$\Sigma H = 0$$
$$-F_{FD} - F_{DC} \cos 45° = 0$$
$$F_{FD} = 10.77 \text{ kN}$$

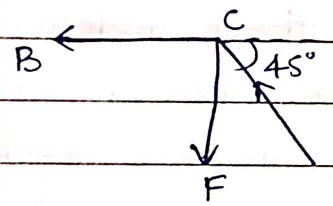
d) FBD of F (joint F)



$$\Sigma H = 0$$
$$-F_{EF} + F_{FD} - F_{BF} \cos 33.69° = 0$$
$$F_{BF} = 2.22 \text{ kN}$$

$$\Sigma V = 0$$
$$-12 + F_{FC} + F_{BF} \sin 33.69° = 0$$
$$F_{FC} = 10.77 \text{ kN}$$

e) FBD of C (joint C)



$$\Sigma H = 0$$
$$-F_{BC} - F_{CD} \cos 45° = 0$$
$$F_{BC} = -10.77 \text{ kN}$$

Members	Magnitude of Internal forces (kN)	Nature (Direction)
AB	1.54	Compression
BC	10.77	Compression
CD	15.23	Compression
DF	10.77	Tension
FE	8.92	Tension
EA	8.92	Tension
BE	0	–
CF	10.77	Tension
BF	2.22	Tension

Method of Sections

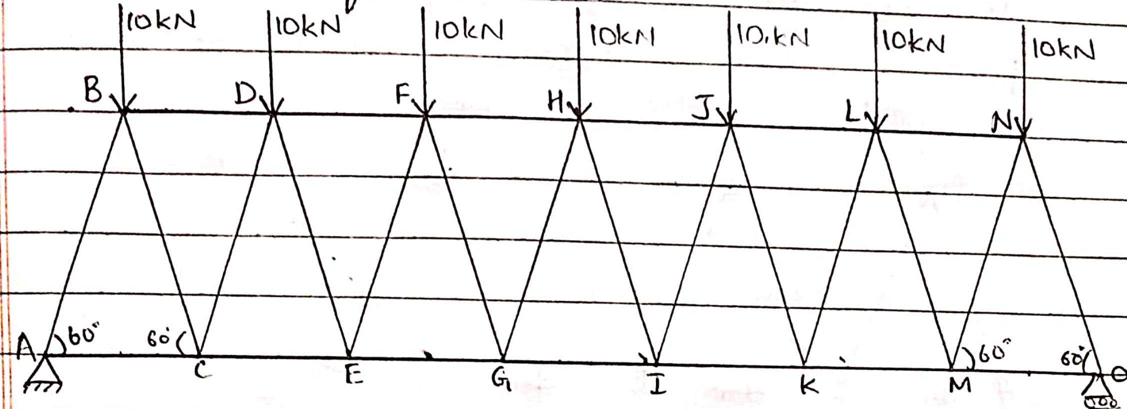
In the method of section, after determining the support reactions, a section line is drawn passing through not more than three members in which forces are not known, such that the frame is cut into two seperate parts. Each part should be in equilibrium under the action of loads, reaction and the forces in the members that are cut by the section line. Equilibrium of anyone of these two parts is considered and the unknown forces in the members cut by the section line are determined. The system of forces acting on either part of truss constitutes a non-concurrent force system. Since there are only three independent equations of equilibrium, there should be only three unknown forces. Hence, in this method it is an essential requirement that the section line should pass through not more than three members in which forces are not known and it should seperate the frame into two parts.

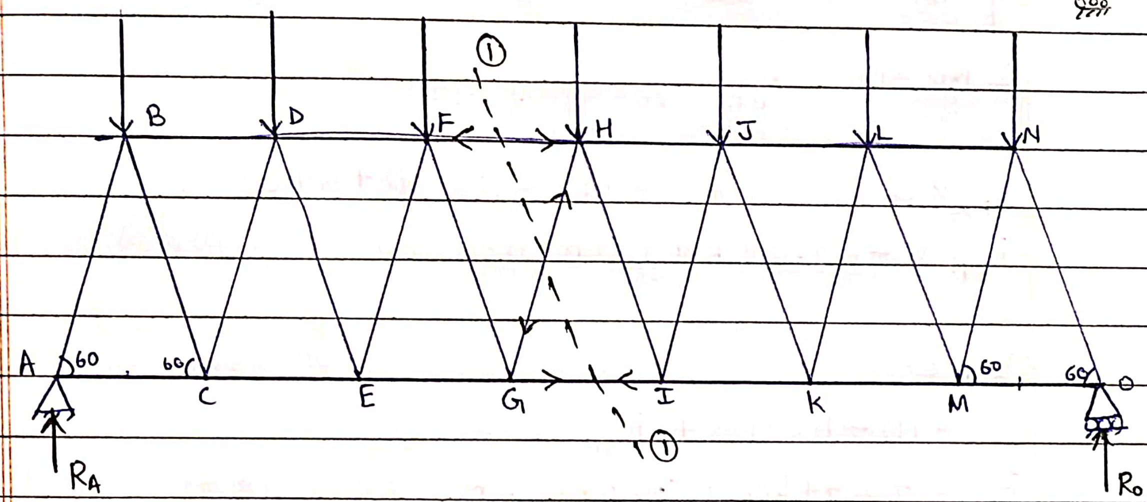
Under the following two situations the method of sections is preferred over the method of joints:

(i) In the analysis of large truss in which forces in only few members are required.

(ii) If method of joints fails to start or proceed with analysis for not getting a joint with only two unknown forces.

1. Determine the forces in the members FG, GH and GI. in the truss shown in figure. Each Load is 10kN and all triangles are equilaterals with sides equal to 4m.





Due to symmetry,

Let R_A and R_O be the vertical reactions at A and O respectively

Taking moments of all forces about O   +ve   −ve

$R_A \times 28 - 10\times(26+22+18+14+10+6+2)=0$

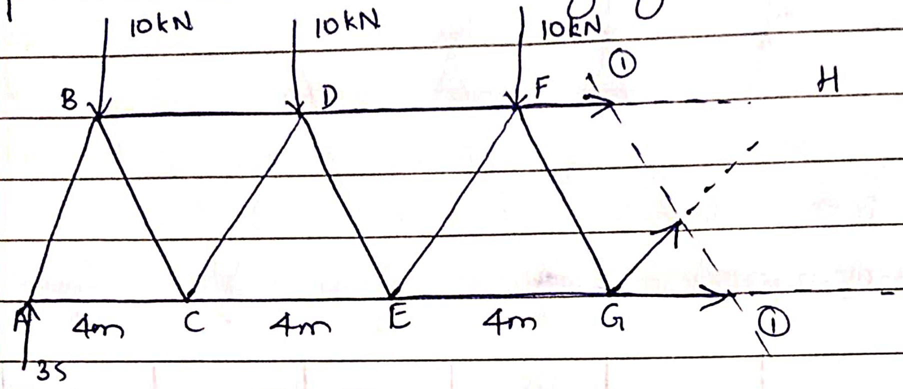
$R_A = 35\,kN$

Taking moments of all forces about A   −ve   +ve

$R_O \times 28 - 10\times(26+22+18+14+10+6+2)=0$

$R_O = 35\,kN$

There is no horizontal component of reaction.
Take section (1)-(1) which cuts FH, GH and GI and seperate the truss into two parts. Consider the equilibrium of LHS part as shown (Prefer the part in which number of forces are less)



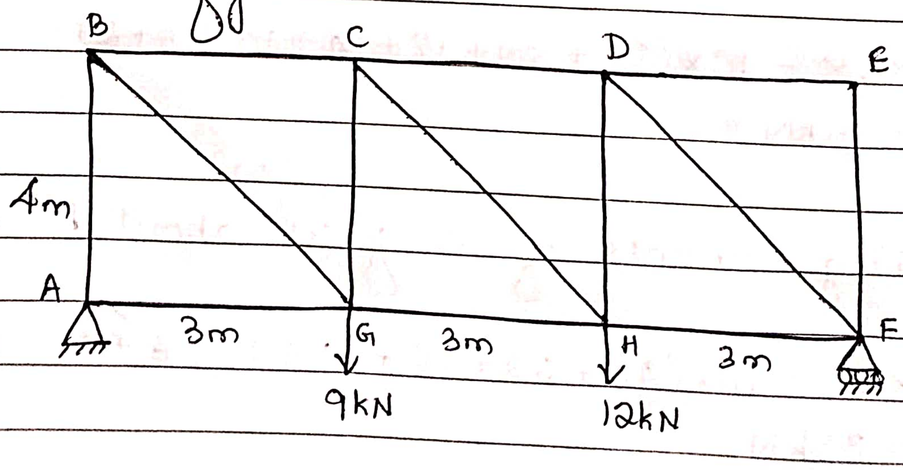
$\Sigma M_G = 0$ +ve -ve

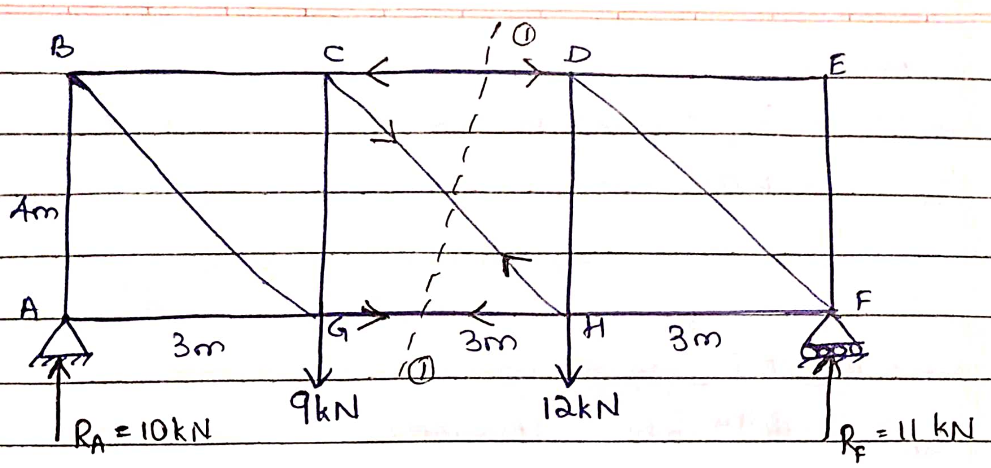
$F_{FH} \times 4 \sin 60 + 35 \times 12 - 10(10+6+2) = 0$
$F_{FH} = -69.28$ kN (Compression)

$\Sigma V = 0$
$35 - 10 - 10 - 10 + F_{GH} \sin 60^\circ = 0$
$F_{GH} = -5.77$ kN (Compression)

$\Sigma H = 0$
$F_{GI} + F_{FH} + F_{GH} \cos 60^\circ = 0$
$F_{GI} = 72.166$ kN (Tension)

2 Determine the forces in the members CD, CH and GH in the figure.





Let R_A and R_F be the support reactions at A and F respectively

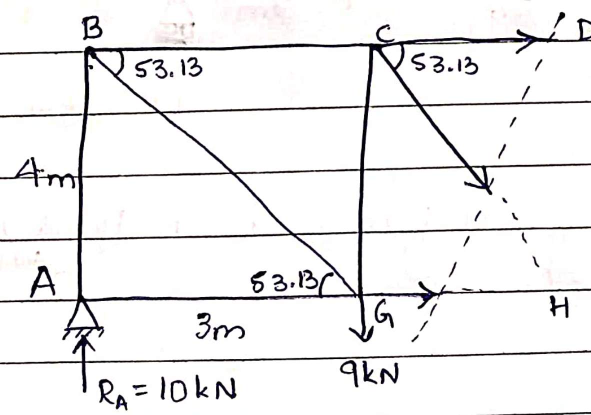
Taking moments of all forces about F (+ve, −ve)

$R_A \times 9 - 9 \times 6 - 12 \times 3 = 0$
$R_A = 10kN$

Taking moments of all forces about A (−ve, +ve)

$R_F \times 9 - 12 \times 6 - 9 \times 3 = 0$
$R_F = 11kN$

Draw the section ①-① passing through the members CD, CH and GH as shown in figure.



$\Sigma M_H = 0$
$10 \times 6 - 9 \times 3 + F_{CD} \times 4 = 0$
$F_{CD} = -8.25kN$ (compression)

$\Sigma V = 0$

$10 - 9 - F_{CH} \sin 53.13^\circ = 0$

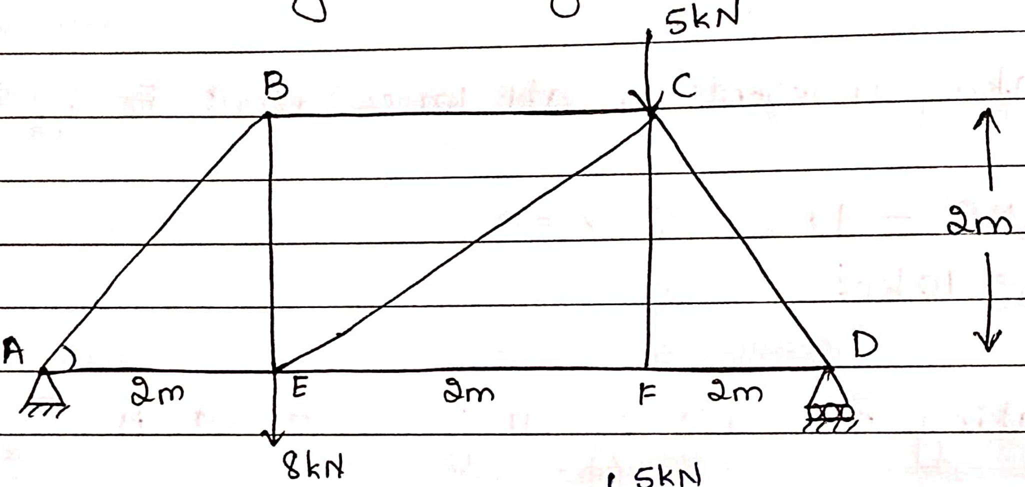
$F_{CH} = 1.25 kN$ (Tension)

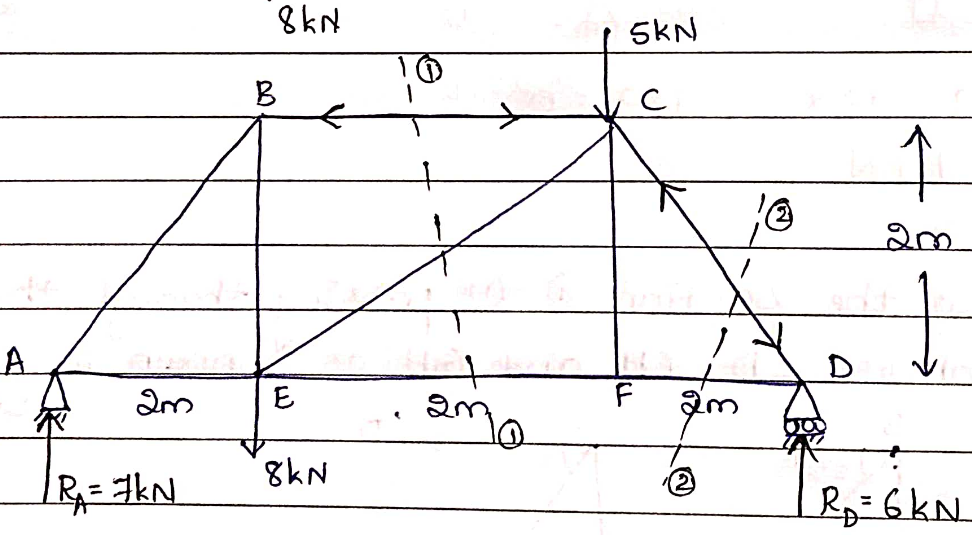
$\Sigma H = 0$

$F_{CD} + F_{GH} + F_{CH} \cos 53.13^\circ = 0$

$F_{GH} = 7.499 kN$ (Tension)

3. Determine the forces in the members of the frame CD and BC by method of sections.





Taking moments of all forces about D +ve -ve

$R_A \times 6 - 8 \times 4 - 5 \times 2 = 0$

$R_A = 7kN$

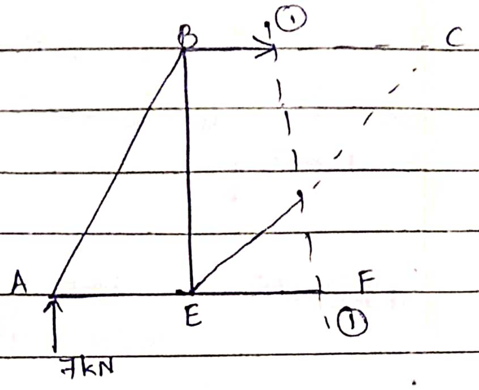
Taking moments of all forces about A -ve +ve

$R_D \times 6 - 5 \times 4 - 8 \times 2 = 0$

$R_D = 6kN$

Draw a section ①-① and ②-② as shown in fig.
To determine the forces in the members BC and CD, the sections ①-① and ②-② are drawn as shown

To find BC

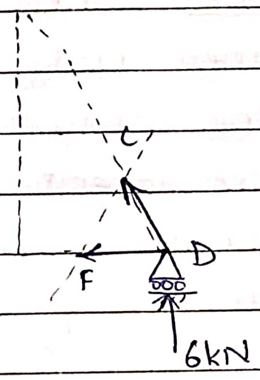


$\Sigma M(E) = 0$

$7 \times 2 + F_{BC} \times 2 = 0$

$F_{BC} = -7\,kN$ (compression)

To find DC

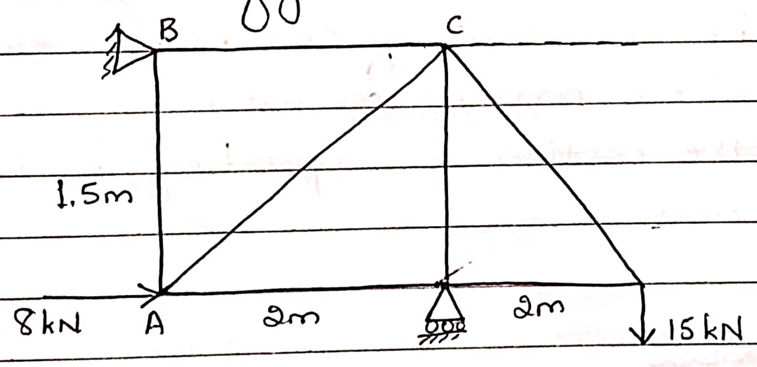


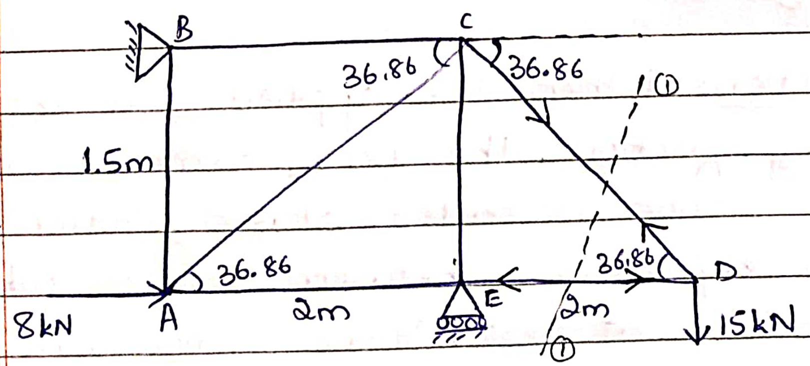
$\Sigma V = 0$

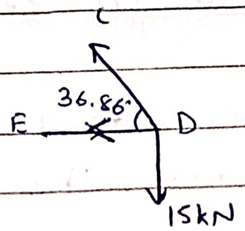
$6 + F_{DC} \sin 45^\circ = 0$

$F_{DC} = -8.48\,kN$ (compression)

4. Determine the forces in the members CD, ED as shown in figure.







$\Sigma V = 0$

$-15 + F_{CD} \sin 36.86^\circ = 0$

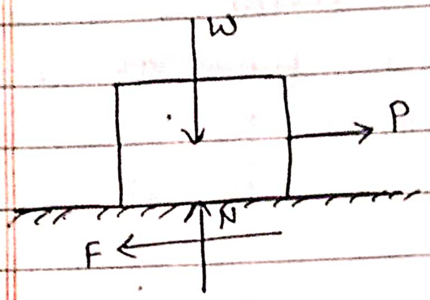
$F_{CD} = 25\,kN$ (Tension)

$\Sigma H = 0$

$-F_{DE} - F_{CD} \cos 36.86^\circ = 0$

$F_{DE} = -20\,kN$ (compression)

Friction



Note: Weigth of the body will act perpendicular to the ground. Normal reaction 'N' will be acting perpendicular to the surface. Frictional force will be acting opposite to the applied force.

Friction: When a body moves or tend to move over another body, a force opposing the motion develops at the contact surfaces. The force which opposes the movement or tendency of movement is called frictional force. The resistance offerent to motion at the surface of the bodies is called frictional force.

Limiting Friction: If the applied tangential force is ~~less than~~ more than the maximum frictional force, there will be movement of one body over the other body. This maximum value of frictional force, when the motion is impending is known as limiting friction (Fmax).

Types of friction

1. Static friction: When the applied force is less than the limiting friction, the body remains at rest and such friction is called static friction. It is the friction experienced between bodies when both the bodies are at rest. Static friction will have any value between zero and limiting friction.

2. Kinetic friction: If the applied force exceeds the limiting friction, the body starts to move over

another body and its frictional resistance experienced while moving is known as kinetic friction.

Friction experienced between two bodies when one body moves over the other body. The magnitude is found to be less than limiting friction.

Kinetic / Dynamic friction → Sliding friction
Kinetic / Dynamic friction → Rolling friction

* Sliding friction - It is a friction experienced by a body when it slides over the other body

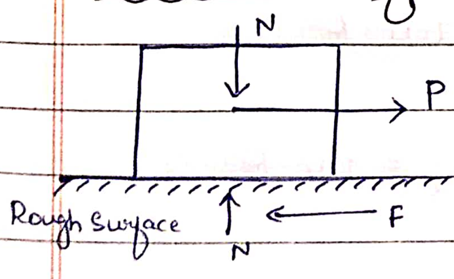
* Rolling friction - It is the friction experienced by a body when it rolls over the other body.

Based of contact surface

* Dry friction - If the contact surfaces between the two bodies are dry, then the friction between such bodies is dry friction.

* Fluid friction - The friction between two fluid layers or the friction between a solid and a fluid is known as fluid friction.

Co efficient of friction



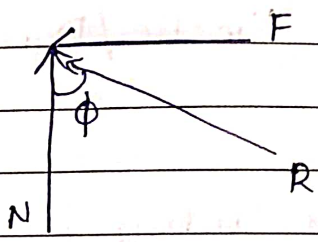
Co-eff of friction = $\frac{\text{Limiting Fric (F)}}{\text{Normal react (N)}}$

$$\boxed{\mu = \frac{F}{N}}$$

It is defined as the ratio of the limiting friction (F) to the normal reaction (N) between the two bodies.

For smooth surfaces $\boxed{\mu = 0}$.

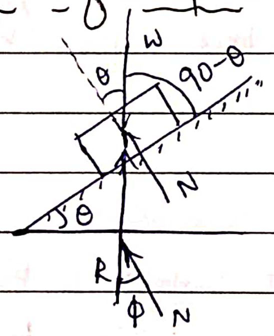
Angle of friction: (ϕ): It is defined as the angle made by resultant (R) of the normal reaction (N) and the force of friction (F) with the normal reaction (N).



$\tan\phi = \frac{F}{N}$ But $\mu = \frac{F}{N}$

$\therefore \boxed{\mu = \tan\phi}$

Angle of repose (θ): Consider a body placed in an inclined plane as shown in fig. Let the angle of inclination (θ) be gradually increased till the body just starts sliding down the plane.



The angle of inclined plane at which the body just begins to slide down the plane is called the angle of repose (θ)

$\Sigma F_{\perp} = 0$ $N - W\sin(90-\theta) = 0$ $\therefore N = W\cos\theta$

$\Sigma F_{\parallel} = 0$ $F - W\cos(90-\theta) = 0$ $\therefore F = W\sin\theta$

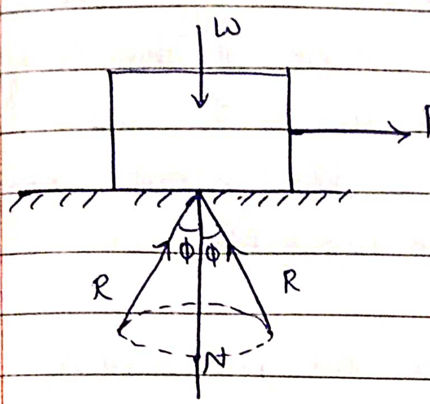
WKT, $\mu = \frac{F}{N} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta$

But $\mu = \tan\phi \Rightarrow \tan\phi = \tan\theta$

$$\phi = \theta$$

Angle of friction = Angle of Repose

Cone friction : The force P corresponding to limiting friction as in figure, remains constant in magnitude P but revolves in a horizontal plane, the force R will move along the surface of a cone as shown in figure. This cone is called cone of friction.



Laws of friction

1. The frictional force always acts in the direction opposite to that in which the body tends to move

2. The limiting force of friction bears a constant ratio to the normal reaction between the two surfaces i.e, $\mu = F/N$.

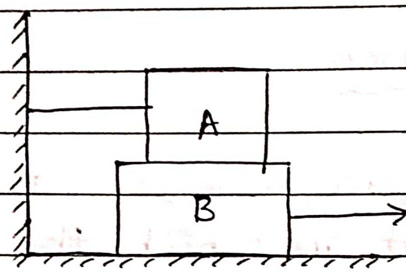
3. The force of friction is independent of the area of contact between the two surfaces.

4. The force of friction depends upon the roughness/smoothness of the surfaces.

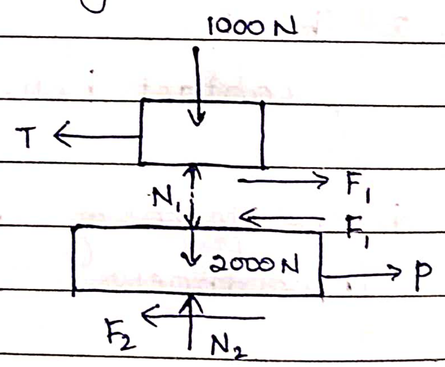
5. Till the limiting value is reached, the magnitude of frictional force is exactly equal to the tangential force which tends to move the body.

6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio to the normal force. This ratio is called coefficient of dynamic friction or kinetic friction.

1 A block weighing 1000N rests over another block B. which weighs 2000N as shown in figure. A is tied to wall with a horizontal stone. If the coefficient of friction between blocks A and B is 0.25 and between block B and floor is 1/3. What should be the value of P to move the block (B), if
a) P is horizontal
b) P acts at 30° upwards to the horizontal.



a) When P is horizontal -
The FBD is shown in fig. A. It may be noted that the frictional force F_1 and F_2 are to be marked in opposite directions of impending relative motion.
Considering Block A.



$\Sigma V = 0$

$N_1 - 1000N = 0$

$N_1 = 1000N$

Since $\frac{F_1}{N_1} = \mu_1 \Rightarrow F_1 = \mu_1 N_1$

$F_1 = 0.25(1000)$

$F_1 = 250N$

$\Sigma H = 0$

$F_1 - T = 0$

$T = 250N$

Considering Block B,

$\Sigma V = 0$

$N_2 - N_1 - 2000 = 0$

$N_2 = 3000N$

$F_2 = \mu_2 N_2$

$F_2 = \frac{1}{3} \times 3000$

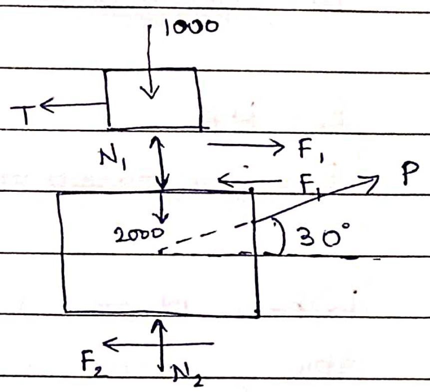
$F_2 = 1000N$

$\Sigma H = 0$

$P - F_2 - F_1 = 0$

$P = 1250N$

b) When P is ~~inclid~~ inclined



Considering block B

$\Sigma V = 0$

$N_2 - N_1 - 2000 + P\sin 30° = 0$

$N_2 = 3000 - P\sin 30$

$\Sigma H = 0$

$-F_2 + P\cos 30° - F_1 = 0$

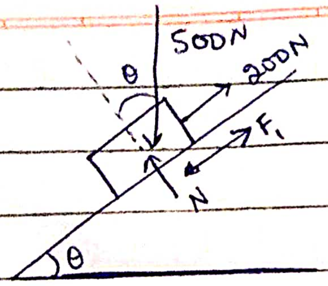
$F_2 = -250 + P\cos 30°$

WKT, $F_2 = \mu_2 N_2$

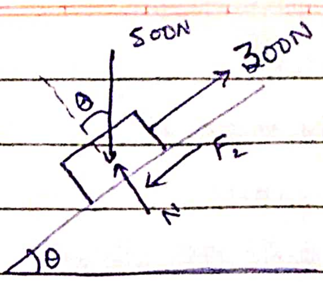
$$-250 + P\cos 30° = \frac{1}{3}(3000 - P\sin 30°)$$

$$P = 1210.4\,N$$

2. A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. The same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coefficient of friction between plane and block.



A



B

$\Sigma F_\perp = 0$

$N - 500\cos \cancel{30}\,\theta = 0$

$N = 500\cos\theta \rightarrow (1)$

$F_1 = \mu N$

$F_1 = 500\mu\cos\theta \rightarrow (2)$

$F_{\parallel} = 0$

$200 + F_1 - 500\sin\theta = 0$

$200 = 500\sin\theta - 500\mu\cos\theta \rightarrow (3)$

When block starts moving up when 300N is applied, the friction force acting is F_2 down (refer B).

$\Sigma F_\perp = 0$

$N - 500\cos\theta = 0$

$N = 500\cos\theta \rightarrow (4)$

$F_2 = \mu N$

$F_2 = 500\mu\cos\theta \rightarrow (5)$

$\Sigma F_{\parallel} = 0$

$300 - F_2 - 500\sin\theta = 0$

$300 = 500\mu\cos\theta + 500\sin\theta \rightarrow (6)$

Adding (3) and (6)

$500 = 1000\sin\theta$

$\theta = 30^\circ$

Consider (3) $\Rightarrow$ $200 = 500\sin 30^\circ - 500\mu\cos 30^\circ$

$\Rightarrow \mu = 0.11547$