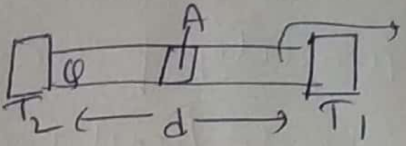
Thermal Conductivity (K)



Material having thermal conductivity K.

$T_2 > T_1$

The thermal conductivity of a material is a measure of its ability to conduct heat.

It is commonly denoted by 'K'.

$$\varphi \propto A \frac{dT}{dx} t \quad \text{or}$$

$$\varphi = +KA\left(\frac{dT}{dx}\right)t \quad , \quad \varphi = +K\frac{(T_2 - T_1)}{\lambda} A t$$

Heat flux

$\frac{dT}{dx}$ → Temperature Gradient

→ λ is separation b/w two ends

where 'K' is the co-efficient of thermal conductivity is defined as the quantity of heat flowing per second through a conductor of unit area of cross section, when there is a unit Temp gradient.

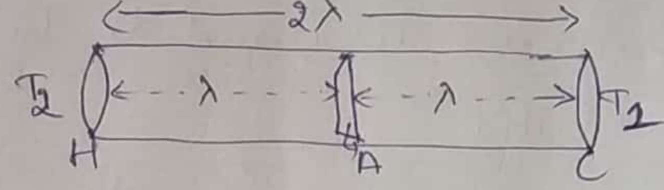
i.e, $\langle K = \varphi \rangle$ if A = 1 sq meter $\quad \frac{dT}{dx} = 1 \frac{\text{kelvin}}{\text{meter}}$

t = 1 second

Expression for thermal conductivity ('K'):-

consider a uniform rod H.C with the temperature of Hot end H as T_2 & the temperature of the cold end 'C' as T_1 as shown in the figure

Heat is flowing from the hot end 'H' to the cold end 'C'



Let 'A' be the area of cross section & 'λ' be the mean free path of the electrons between the two ends H & C. The Kinetic energy of the electrons at the hot end H is greater than that of the electrons at the colder end 'C'

Let 'Q' be the amount of heat flowing through the rod from end H to C whose length is '2λ'.

$$Q = \frac{KA(T_2 - T_1)}{2\lambda} t \longrightarrow (1)$$

where 'K' is coefficient of thermal conductivity, 't' is the time for conduction & 2λ is the length of the rod;

∴ thermal conductivity

$$K = \frac{Q(2\lambda)}{A(T_2 - T_1)t} \rightarrow (2)$$

Let 'n' be the no of available conduction electrons & 'v' be the root mean square velocity of the electrons

Let us assume that the free electrons in the metal are having equal probability to move in all six possible directions. Therefore an average of $\frac{1}{6}$ electrons can travel in any one direction

(Translation + Rotation)

WKT, the free electrons are assumed to be free moving gas molecules.

The average K.E of the electron at ~~cold~~ hot end 'H' of Temp T_2 is $3/_2\, K_B T_2 \rightarrow (3)$

ІІІ^{ly}, the average K.E of the electron at cold end 'C' of Temp T_1 is $3/_2\, K_B T_1 \rightarrow (4)$

where $K_B \rightarrow$ Boltzmann const

The no of electrons crossing the area at A per second

$$= \frac{1}{6} n v \rightarrow (5)$$

Therefore the resultant heat energy transferred per unit area per unit time from H to C =

No of electrons $\times$ Avg K.E of electrons moving from H to C.

i.e., $\varphi = \frac{1}{\cancel{6}_2} n v \times \cancel{3}/_2\, K_B (T_2 - T_1) \rightarrow (6)$

WKT. Temp gradient $\frac{dT}{dx} = \frac{T_2 - T_1}{2\lambda} \rightarrow (7)$

substituting eq^n (6) & (7) in (2)

$$\cancel{K}\, \varphi = \frac{n v}{4} K_B (T_2 - T_1), \quad K = \frac{\varphi (2\lambda)}{A (T_2 - T_1) t}$$

$$K = \frac{\left(\frac{n v}{\cancel{4}_2} K_B (T_2 - T_1)\right) (\cancel{2}\lambda)}{(T_2 - T_1)}$$

If $A = 1$ sq m, $t = 1$ sec

$$K = \frac{1}{2} n v K_B \lambda \rightarrow (8)$$

$$\boxed{K = \frac{1}{2} n v K_B \lambda}$$

From the classical free electron theory wkT, the electronic specific heat capacity of the metal is

$$C = \frac{3}{2} R$$

where $R = n K_B$ $R \rightarrow$ gas const

$$n K_B = \frac{2}{3} C \rightarrow (10)$$

substituting (10) in (8)

$$K = \frac{1}{2} v \lambda (n K_B)$$

$$K = \frac{1}{2} v \lambda \left(\frac{2}{3} C\right)$$

$$\therefore \boxed{K = \frac{1}{3} C v \lambda} \rightarrow (11)$$

eqn (8) & (11) represents the expression for thermal conductivity of a metal. depending

Wiedemann - Franz Law:-

The Law states that the ratio of the thermal conductivity & electrical conductivity of a metal is directly proportional to the Absolute Temp of the metal,

i.e., $$\frac{K}{\sigma} \propto T \longrightarrow (1)$$

$$\frac{K}{\sigma} = L T \longrightarrow (2)$$

where 'L' is the propotionality const called Lorentz const whose value $L = 1.12 \times 10^{-8}\ W\Omega K^{-2}$

WKT thermal conductivity, $K = \frac{1}{2} n v k_B \lambda \rightarrow (3)$

Electrical conductivity, $\sigma = \frac{n e^2 \tau}{m} \rightarrow (4)$

$$\frac{K}{\sigma} = \frac{\cancel{n} v k_B \lambda}{2\left(\frac{\cancel{n} e^2 \tau}{m}\right)} = \frac{v k_B \lambda m}{2 e^2 \tau}$$

$$\frac{K}{\sigma} = \frac{1}{2} v \left(\frac{\lambda}{\tau}\right) m \left(\frac{k_B}{e^2}\right)$$

WKT $v = \frac{\lambda}{\tau}$

$$\frac{K}{\sigma} = \underbrace{\tfrac{1}{2} m v^2}\left(\frac{k_B}{e^2}\right) \rightarrow (5)$$

By kinetic theory of gases

K.E of electrons $= \frac{3}{2} k_B T$

$\frac{1}{2} m v^2 = \frac{3}{2} k_B T$

Eqn (5) $\Rightarrow$ $\frac{K}{\sigma} = \frac{3}{2} k_B T \left(\frac{k_B}{e^2}\right)$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T \rightarrow (6)$$

from eqn (2) $\frac{K}{\sigma} = LT$

$\frac{K}{\sigma} = LT$

where $L = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 \rightarrow (7)$

substituting the value of 'L' is found to be

$$L = \frac{3(1.38 \times 10^{-23})^2}{2(1.6 \times 10^{-19})^2} = 1.1 \times 10^{-8}\ W\Omega/K^2$$

$L = 1.11 \times 10^{-8}\ W\Omega/K^2$

NOTE! the exptl value of 'L' is found to be $2.44 \times 10^{-8}\ W\Omega/K^2$. This is due to the failure of the classical theory to give the correct value of the Thermal conductivity of metals.

Acc to Quantum Theory $K = \frac{n\pi^2 k_B^2 T\tau}{3m}$

$L = \frac{K}{\sigma T}$ $\langle L = 2.44 \times 10^{-8}\ W\Omega/K^2 \rangle$

thus it confirms that W-F law is verified using the quantum theory & it supports that it is not applicable for low Temperature

Failures of classical free electron theory

1. It fails to explain the electric specific heat & the specific heat capacity of metals.
2. It fails to explain superconducting properties of metals
3. It fails to explain new phenomena like photo-electric effect, compton effect, Black body radiation etc.
4. classical theory states that electrons absorbs all the energy. But all the electrons will not absorb energy $(E = h\nu)$
5. classical theory will not explain semi-conductor & Insulators.