3 mon

Vibrations

The to and fro motion of a body about its mean position is called oscillations or Vibrations.

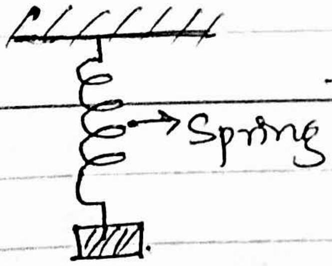
There are three parts of oscillations.

i) Free Oscillation
ii) Damped "
iii) Forced "

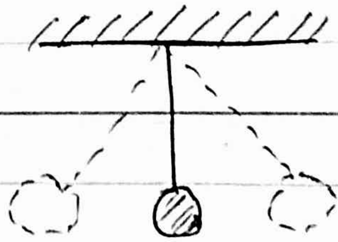
4 tue

i) Free Oscillation / vibration is defined as the body which vibrates with definite interval and with natural frequency.

Ex:- SHO. - Simple Harmonic Oscillation.



5 wed



A particle is said to execute SHM when it vibrates periodically in such a way that at any instant, the restoring force acting on it is proportional to its displacement from mean position and oppositely directed.

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Notes

6 thu

$\therefore F_{\text{Restoring}} \propto -x \longrightarrow (1)$

Where F is the restoring force acting on the oscillator when its displacement from equilibrium position is 'x'.

$$F_R = -kx. \text{ --- } (2)$$

7 fri

Now according to Newton's 2nd law.

$$F = ma. \longrightarrow (3).$$

From eqn (2) and (3).

$$ma = -kx.$$

8 sat

$$ma + kx = 0. \longrightarrow (4)$$

9 sun

We know that, $a = \frac{d^2x}{dt^2}$

Eqn (4) $\Rightarrow$ $m\frac{d^2x}{dt^2} + kx = 0.$

Divide by 'm' to above eqn.

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0.$$

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Notes

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Let us substitute $\frac{K}{m} = \omega^2$ in above eqn.

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \longrightarrow (5).$$

$\omega = \sqrt{\frac{K}{m}}$. Angular freq.

Eqn (5) represents the differential of Simple Harmonic oscillator / vibration.

11 tue

Solution of differential equation can be given as

$$x = e^{\alpha t} \longrightarrow (6)$$

where x is the displacement or amplitude

Differentiating above 'x'

$$\frac{dx}{dt} = \alpha . e^{\alpha t} \longrightarrow (7) \Rightarrow \frac{d^2x}{dt^2} = \alpha^2 . e^{\alpha t} \longrightarrow (8)$$

12 wed

Substituting (7) and (8) in eqn (5).

$$\alpha^2 . e^{\alpha t} + \omega^2 e^{\alpha t} = 0.$$

$$e^{\alpha t} (\alpha^2 + \omega^2) = 0.$$

$$e^{\alpha t} \neq 0 ; \quad \therefore \alpha^2 + \omega^2 = 0.$$

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Notes

$$\alpha^2 = -\omega^2.$$

$$\therefore \alpha = \pm i\omega.$$

General solution of eqn (5) is given as.

$$x = A.e^{i\omega t} + Be^{-i\omega t} \longrightarrow (9)$$

where A and B are constants

Now
$$e^{i\omega t} = \cos\omega t + i\sin\omega t.$$
$$e^{-i\omega t} = \cos\omega t - i\sin\omega t.$$

$$(9) \Rightarrow x = A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t).$$

$$\Rightarrow x = A\cos\omega t + iA\sin\omega t + B\cos\omega t - Bi\sin\omega t.$$

$$x = \cos\omega t\,(A+B) + i\sin\omega t\,(A-B). \longrightarrow (10)$$

Replacing $(A+B)$ and $i(A-B)$ by some constants

$$A+B = R\sin\phi \quad \text{and} \quad i(A-B) = R\cos\phi.$$

$$(10) \Rightarrow x = \cos\omega t\,.\,R\sin\phi + \sin\omega t\,.\,R\cos\phi.$$

$$x = R(\cos\omega t\,.\,\sin\phi + \sin\omega t\,.\,\cos\phi).$$

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Notes

$$\boxed{x = R \sin(\omega t + \phi)} \longrightarrow (11).$$

Eqn (11) represents the final solⁿ of SHM.

where $R \rightarrow$ Amplitude of the oscillatory system.
$(\phi + \omega t) \rightarrow$ Phase of vibration.
$x = R\sin(\omega t + \phi) \rightarrow$ represents the displacements of SHM (amplitude)

We now calculate the time period and frequency of SHM.

<u>Time period</u> is defined as the time required for one complete oscillation.

$$t = 2\pi/\omega \Rightarrow 2\pi/\sqrt{k/m} \qquad \because \omega^2 = \frac{k}{m}.$$
$$\omega = \sqrt{k/m}$$

<u>Frequency</u> of SHM:

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{\sqrt{k/m}}{2\pi}.$$

Significance of 'ω' in eqn (11).

If 't' is increased to $2\pi/\omega$

ie $t = t + \frac{2\pi}{\omega}$.

$\therefore$ (11) $\Rightarrow$ $x = R \sin(\omega(t + \frac{2\pi}{\omega}) + \phi)$.

$= R \sin(\omega t + 2\pi + \phi)$.

$x = R \sin(2\pi + (\omega t + \phi))$

$\boxed{x = R \sin(\omega t + \phi)}$ $\because \sin(2\pi + \theta) = \sin\theta$.

which means displacement is same after time $t = 2\pi/\omega$.

So, the period $\boxed{t = \frac{2\pi}{\omega}}$ is the time period.

ii) Damped oscillation / vibration — For a free oscillation the energy remains constant and hence oscillator continues indefinitely.

But in reality, the amplitude of the oscillatory system gradually decreases due to the damping forces like friction (air) and resistance of the media.

"The oscillator whose amplitude, in successive oscillations goes on decreasing due to the presence of resistive forces are called as damped oscillators and oscillations are called damped oscillations (DHO)."

The damping force always act in a opposite directions to the motion of oscillatory body and velocity dependent.

$$F_{damping} \propto -v. \Rightarrow F_{damping} = -\gamma v.$$

$\therefore F_{damping} = -\gamma v. \rightarrow (1)$ $\gamma \rightarrow$ Damping constant. $v \rightarrow$ Damping force velocity.

Now $F_{restoring} = -kx. \rightarrow (2)$

According to Newton's 2nd law of motion.

$$F = ma. \rightarrow (3).$$

Total Force, $F = F_{Restoring} + F_{Damping}$.

$\therefore \; ma = -kx - r\frac{dx}{dt}$.

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = 0.$$

Divide by 'm' in the above eqn.

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0.$$

Substitute $r/m = 2k$ and $k/m = \omega^2$ in above eqn.

$$\therefore \boxed{\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2 x = 0.} \longrightarrow (4).$$

where "2k" is the damping coefficient and "ω" is the natural freq. of oscillating body.

Eqn (4) represents the differential eqn of Damped Harmonic Oscillator.

This is second order linear differential Homogeneous equation in x.

The general solution of the above eqn can be written as.

$$x = A.e^{\alpha t}. \qquad x \text{ is the amplitude}$$

$$\frac{dx}{dt} = A.\alpha.e^{\alpha t} \Rightarrow \frac{d^2x}{dt^2} = A.\alpha^2 e^{\alpha t}.$$

$$④ \Rightarrow A.\alpha^2.e^{\alpha t} + 2k.A\alpha e^{\alpha t} + w^2 A e^{\alpha t} = 0.$$

$$\Rightarrow Ae^{\alpha t}\left[\alpha^2 + 2k\alpha + w^2\right] = 0.$$

Since $Ae^{\alpha t} \neq 0$ but $\alpha^2 + 2k\alpha + w^2 = 0$

Quadratic eqn.

The two roots of the Quad. eqn. are.

$$\alpha = \frac{-2k \pm \sqrt{4k^2 - 4.1.w^2}}{2.1} = \frac{-2k \pm 2\sqrt{k^2 - w^2}}{2}$$

$$\alpha = -k \pm \sqrt{k^2 - w^2} \longrightarrow ⑤.$$

$\therefore$ α in eqn ⑤ has two values.

$$\alpha_1 = -k + \sqrt{k^2 - w^2} \quad \text{and} \quad \alpha_2 = -k - \sqrt{k^2 - w^2}$$

Therefore, general soln of differential eqn is.

$$x = A.e^{\alpha_1 t} + B.e^{\alpha_2 t}.$$

$$x = A.e^{(-k + \sqrt{k^2 - w^2})t} + B.e^{(-k - \sqrt{k^2 - w^2})t}. \longrightarrow ⑥$$

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Notes

Eqn (6) represents the amplitude of D.H.O.

The actual form of eqn (6) depends on whether

(a) $k^2 > \omega^2$ Heavy damping

(b) $k^2 < \omega^2$ Low "

(c) $k^2 = \omega^2$ Critical "

(a) Case - I $k^2 > \omega^2$ (Heavy damping)

If the damping is high, then $\sqrt{k^2-\omega^2} = \beta$ is a real quantity and positive.

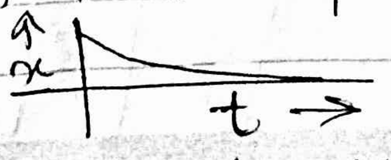
Then eqn (6) $\Rightarrow$

$$x = e^{-kt}\left[A.e^{(\sqrt{k^2-\omega^2})t} + B.e^{(-\sqrt{k^2-\omega^2})t}\right] \longrightarrow (7)$$

$$x = e^{-kt}\left[A.e^{\beta t} + B.e^{-\beta t}\right]$$

$$x = A.e^{-(k-\beta)t} + B.e^{-(k+\beta)t}$$

Since both the exponents are negative then the displacement 'x' decreases continuously with the time. That is if the particle when once displaced return to its equilibrium position quite slowly.



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Notes

Non oscillatory or aperiodic motion.

Ex:- Pendulum in viscous medium.

(b) <u>Case - II</u> $k^2 < w^2$ — Under damping / Low damping.
there $\sqrt{k^2-w^2}$ is imaginary.

So. eqn (6) $\sqrt{k^2-w^2} = \sqrt{-(w^2-k^2)} = i\sqrt{w^2-k^2} = i\omega$
where $\sqrt{-1} = i$ and $\omega = \sqrt{w^2-k^2}$

$\therefore$ Eqn (6) $\Rightarrow$ $x = e^{-kt}\left[A e^{i\omega t} + B e^{-i\omega t}\right]$

8 tue

$$x = e^{-kt}\left[A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t)\right]$$

$$x = e^{-kt}\left[(A+B)\cos\omega t + i(A-B)\sin\omega t\right]$$

$\Rightarrow$ $A+B = a_0 \sin\phi$; $i(A-B) = a_0\cos\phi$.

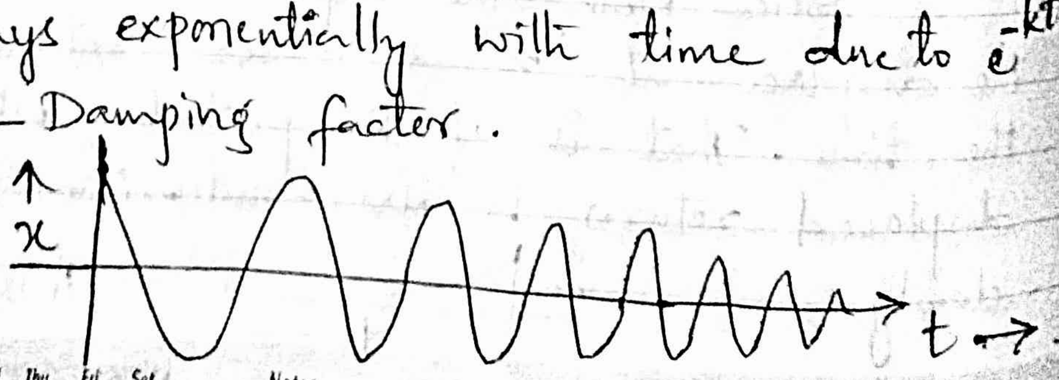
$$\therefore x = e^{-kt}\left[a_0\sin\phi\cos\omega t + a_0\cos\phi\sin\omega t\right].$$

9 wed

$$x = a_0 e^{-kt}\left[\sin\phi\cos\omega t + \cos\phi\sin\omega t\right]$$

$$\boxed{x = a_0 e^{-kt}\sin(\omega t + \phi).} \longrightarrow (8).$$

$a_0 e^{-kt}$ — represents amplitude of oscillation and it decays exponentially with time due to e^{-kt}.
e^{-kt} — Damping factor.



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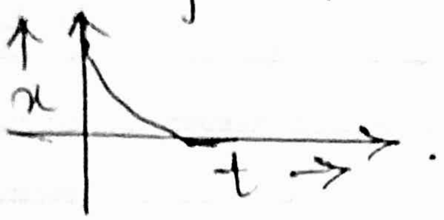
Notes

(c) Case III $k^2 = \omega^2$ (critical damping)

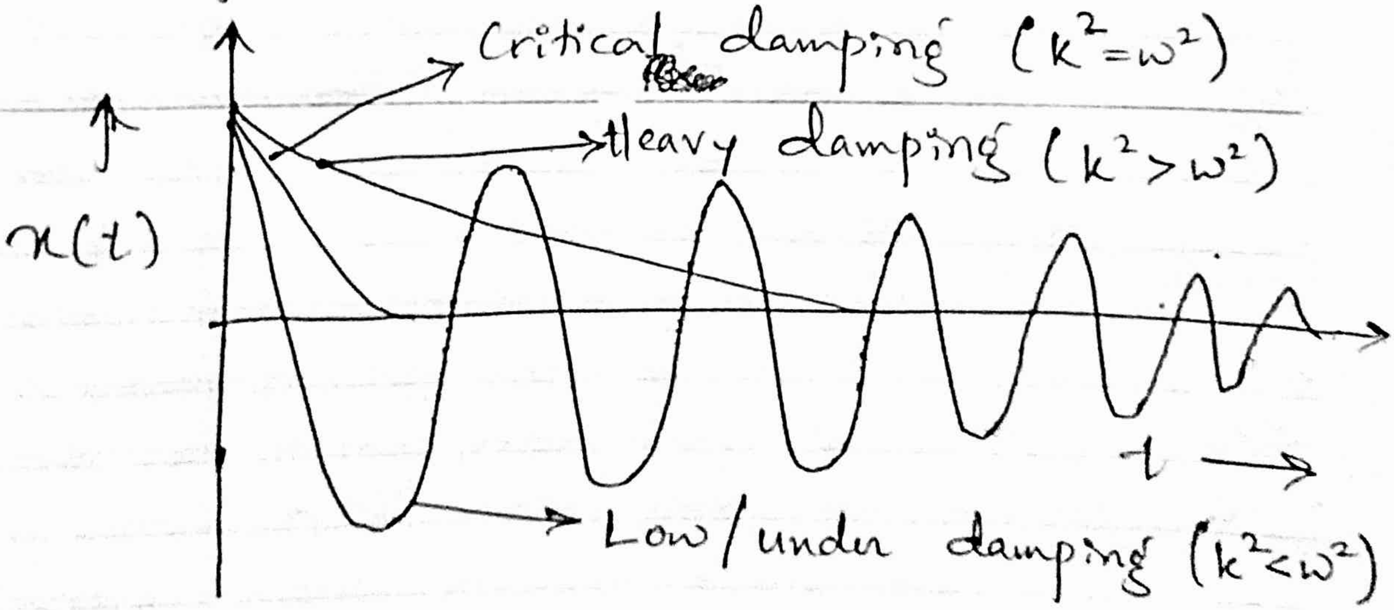
Eqn (6) will be like $x = (A + B)e^{-kt}$.

Therefore, the displacement approaches to zero asymptotically for given value of initial position.

Here, the critically damped oscillator approaches to the equilibrium position more rapidly than compared to heavy damped.



Curves of all the three cases :



iii) Forced/Driven Oscillator (FHO):

When oscillator is subjected to external periodic force and it oscillates with the frequency of external periodic force, it is called as Forced Harmonic Oscillator (FHO).

15 tue

$$F = F_{Restoring} + F_{damping} + F_{ext\text{-}periodic\ force}$$

$$= -kx - \gamma v + F_0 \sin pt.$$

$$ma = -kx - \gamma \frac{dx}{dt} + F_0 \sin pt.$$

$$m\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = F_0 \sin pt.$$

16 wed Divide the above eqn by m.

$$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \left(\frac{k}{m}\right)x = \frac{F_0}{m} \sin pt.$$

$$\boxed{\frac{d^2x}{dt^2} + 2K\frac{dx}{dt} + \omega^2 x = f_0 \sin pt} \longrightarrow (1).$$

Eqn (1) is the differential eqn of FHO.

where $\frac{\gamma}{m} = 2K$; $\frac{k}{m} = \omega^2$; $\frac{F_0}{m} = f_0$.

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Notes

Let us assume the solution of eqn (1) as.

$$x = A\sin(pt-\theta)$$

where p is the driving frequency.

$\frac{dx}{dt} = Ap\cos(pt-\theta)$ $\frac{d^2x}{dt^2} = -Ap^2\sin(pt-\theta)$.

Substituting above values in eqn (1).

$$-Ap^2\sin(pt-\theta) + 2KAp\cos(pt-\theta) + w^2 A\sin(pt-\theta) = f_0\sin((pt-\theta)+\theta).$$

$\hookrightarrow \sin(A+B)$

$$\Rightarrow \sin(pt-\theta)(w^2 A - Ap^2) + 2KAp\cos(pt-\theta) = f_0\sin(pt-\theta)\cos\theta + f_0\cos(pt-\theta)\sin\theta.$$

$$\Rightarrow A(w^2-p^2)\sin(pt-\theta) + 2KAp\cos(pt-\theta) = f_0\sin(pt-\theta)\cos\theta + f_0\sin\theta\cos(pt-\theta) \rightarrow (2)$$

Now case-1 If $(pt-\theta)=0$ then $\because \sin 0 = 0$

eqn (2) $\Rightarrow 2KAp = f_0\sin\theta \rightarrow (3)$ $\cos 0 = 1$

case-2 If $(pt-\theta) = \pi/2$ then $\because \sin\pi/2 = 1$

eqn (2) $\Rightarrow (w^2-p^2)A = f_0\cos\theta \rightarrow (4)$ $\cos\pi/2 = 0$.

Squaring and adding eqn (3) and (4) we get

$$4k^2p^2A^2 + A^2(\omega^2-p^2)^2 = f_0^2(\sin^2\theta+\cos^2\theta)$$

$$A^2\left[4k^2p^2+(\omega^2-p^2)^2\right] = f_0^2.$$

$$\boxed{A = \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4k^2p^2}}} \rightarrow (5)$$

Now let us substitute eqn (5) in $x = A\sin(pt-\theta)$

$$\boxed{x = \frac{f_0}{\sqrt{(\omega^2-p^2)+4k^2p^2}}\sin(pt-\theta)} \rightarrow (6)$$

Eqn (6) is the General Solution eqn of FHO.

(a) Case - I When $\omega >> p$ and damping is low

ie., Driving frequency is very less than the natural frequency.

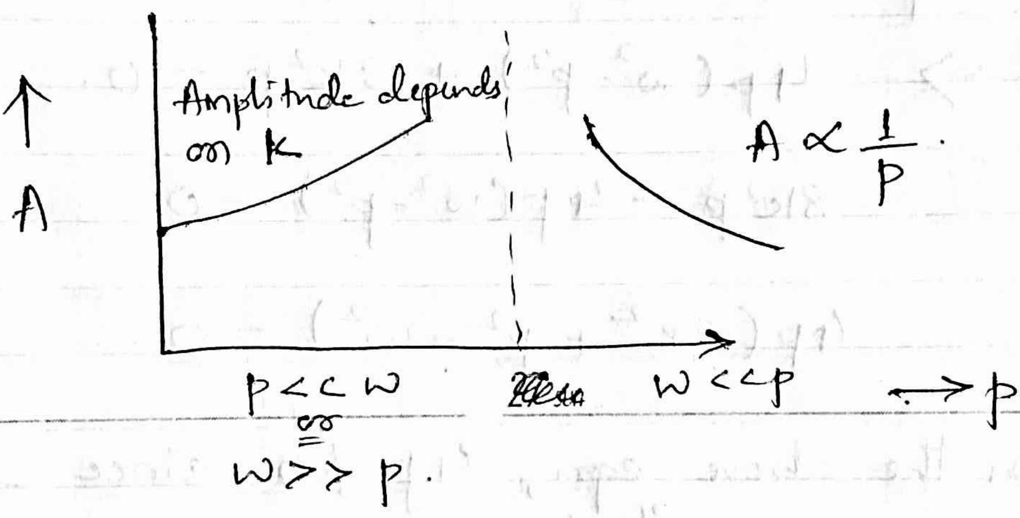
Eqn (5) $\Rightarrow$ $A = \frac{f_0}{\omega^2} = \frac{F_0/m}{k/m} = \frac{F_0}{k}$.

In this case amplitude depends on 'k' force constant only but not on the mass, damping and frequency of driving force.

(b) Case - II When $\omega << p$ and damping is low.

then Eqn (5) $\Rightarrow$ $A \cong \frac{f_0}{p^2}$

$\Rightarrow$ If 'p' increases then amplitude 'A' will decrease



(c) Case - III When $\omega = p$ (Resonance case).

In this case the amplitude will be maximum.

Eqn (5) $\Rightarrow$ $A = \frac{f_0}{\sqrt{4k^2p^2}}$ $\Rightarrow A = \frac{f_0}{2kp}$.

but in order to get 'A' maximum then we should get denominator of eqn (5) be minimum.

Consider $U = (\omega^2 - p^2)^2 + 4k^2p^2$ should be minimum.

also $\frac{du}{dp}$ should be minimum.

$$\therefore \frac{dU}{dp} = 2(\omega^2 - p^2)(-2p) + 8k^2p = 0.$$

$$\Rightarrow -4p(\omega^2 - p^2) + 8k^2p = 0.$$

$$8k^2p - 4p(\omega^2 - p^2) = 0.$$

$$4p(2k^2 + p^2 - \omega^2) = 0.$$

In the above eqn, $4p \neq 0$ since driving frequency can't be zero.

$$\therefore p^2 - \omega^2 + 2k^2 = 0$$

$$p^2 = \omega^2 - 2k^2$$

$$p = \sqrt{\omega^2 - 2k^2} = p_r \rightarrow \text{Resonant frequency}$$

At this frequency 'p_r' the amplitude will be maximum.

Substitute 'p_r' in the above expression for amplitude

$$A = \frac{f_0}{\sqrt{(\omega^2-p^2)^2 + 4k^2p^2}}$$

Replace $p \longrightarrow$ as $p_r = \sqrt{\omega^2-2k^2}$

$$\therefore A = \frac{f_0}{\sqrt{(\omega^2-\{\sqrt{\omega^2-2k^2}\}^2)^2 + 4k^2(\sqrt{\omega^2-2k^2})^2}}$$

$$A_{max} = \frac{f_0}{\sqrt{(2k^2)^2 + 4k^2(\omega^2-2k^2)}}$$

$$\boxed{A_{max} = \frac{f_0}{2k\sqrt{\omega^2-k^2}}}$$

For a certain value of driving frequency the amplitude will be maximum which is called as amplitude maximum Resonance.

Velocity Resonance :

The displacement of a driven oscillation is given by

$$x = A \sin(pt - \theta) \longrightarrow (1).$$

Substitute for A in eqn (1).

8 tue

$$x = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4K^2p^2}} \sin(pt - \theta)$$

∴ The velocity of the driven oscillator at that instant is.

$$V = \frac{dx}{dt} = pA\cos(pt - \theta) \longrightarrow (2).$$

9 wed

$$\therefore V = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4K^2p^2}} p\cos(pt - \theta)$$

$$\langle V = V_0 \sin(\underbrace{pt - \theta + \pi/2}) \rangle \longrightarrow (3).$$

$\sin(\pi/2 + \theta) = \cos\theta$.
∴ sin is positive in 2nd Quadrant.

where $V_0 = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4K^2p^2}} \longrightarrow (4)$ and $\theta = \tan^{-1}\left(\frac{2kp}{\omega^2 - p^2}\right) \longrightarrow (5)$.

WKT Eqn (3) ⇒ $2KAp = f_0 \sin\theta$.

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Notes

Eqn (4) ⇒ $(\omega^2 - p^2)A = f_0 \cos\theta$.

Dividing eqn ③ by ④.

$$\frac{\cancel{f_0}\sin\theta}{\cancel{f_0}\cos\theta} = \frac{2k\cancel{A}p}{(\omega^2-p^2)\cancel{A}}$$

$$\tan\theta = \frac{2kp}{(\omega^2-p^2)} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{2kp}{\omega^2-p^2}\right)}$$

v_0 is known as the velocity amplitude. Thus the velocity amplitude v_0 varies with p.

when $p=0$, $v=0$ and maximum when $p=\omega$. Hence at the frequency $p=\omega$ (driving frequency is equal to the Natural frequency). of the impressed force, the velocity has the maximum

value and we call it ~~Res~~ as velocity resonance.

At velocity Resonance $v_0 = \frac{f_0\,\omega_0}{\sqrt{(\omega^2-\omega^2)+4k^2\omega^2}}$

$$v_0 = \frac{f_0}{2K} = f_0\tau$$

where $\tau = \frac{1}{2K}$.

$$\theta = \tan^{-1}\frac{2kp}{(\omega^2-\omega^2)} = \tan^{-1}(\infty)$$

$$= \frac{\pi}{2}.$$

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Notes

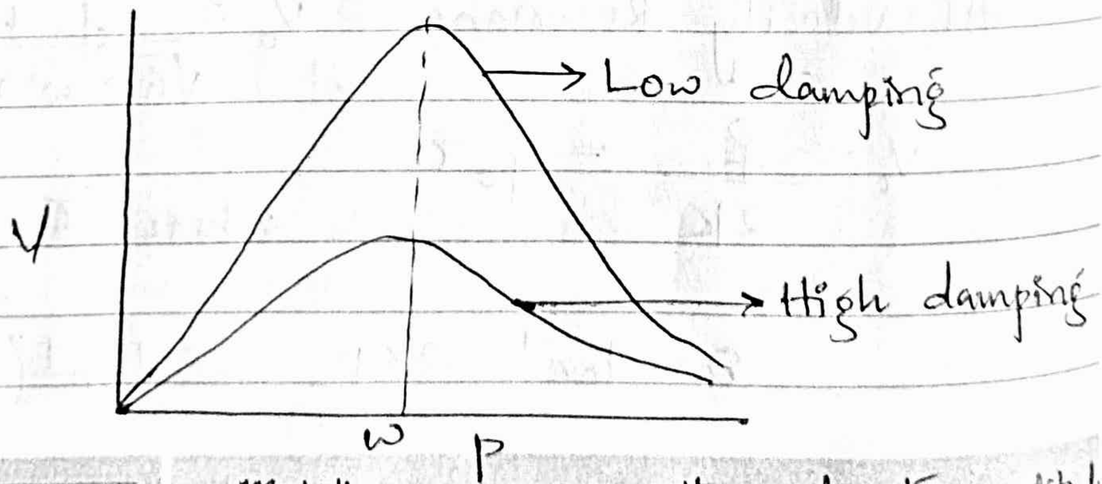
$$\therefore \theta = \frac{\pi}{2}.$$

Consider eqn (3) $v = v_0 \sin\left(pt \underbrace{- \theta + \frac{\pi}{2}}\right)$

$\therefore$ Velocity phase constant $= -\theta + \frac{\pi}{2} = -\frac{\pi}{2} + \frac{\pi}{2} = 0.$

Now, as we know that the displacement at resonance lags behind in phase by $\pi/2$, behind the driving force and as we have just seen velocity then leads the displacement in phase by $\pi/2$.

Therefore, at resonance, the velocity of driven oscillation is in phase with the driving force. This is the most favourable situation for transfer of energy from the applied force to the oscillator, because the rate of work done on the oscillator by the impressed force is Fv which is always positive for F and v in phase.



When $p \gtrless \omega$, the velocity amplitude is smaller than $p = \omega$.

Sharpness of Resonance :

At resonance, the amplitude of the oscillating system becomes maximum. It decreases from the maximum value with change in frequency of imposed force.

The term sharpness of resonance refers to rate of fall of amplitude with change in

forced frequency on either side of Resonant frequency.

The rate at which the amplitude changes corresponding to small change in frequency of applied external force.

change in Amp $= A_{max} - A$.

$$= \frac{f_0}{2k\omega} - \frac{f_0}{\sqrt{(\omega^2-p^2)+4k^2p^2}}$$

$$= \frac{f_0}{2k\omega} - \frac{f_0}{2kp}$$

$\therefore$ $\omega^2-p^2=0$ and since $\omega = p$. small change in applied freq.

$$= \frac{f_0 p - f_0 \omega}{2k\omega p}$$

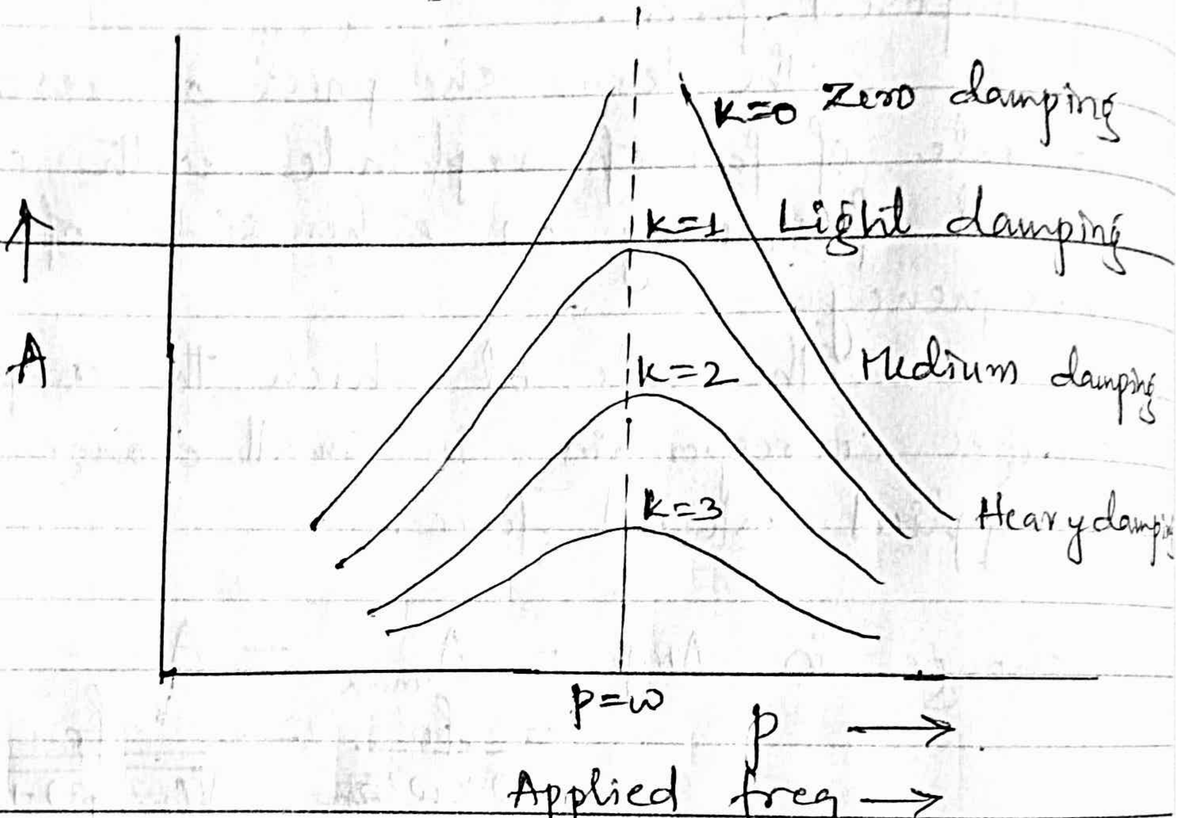
$$= \frac{f_0(p-\omega)}{2k\omega p}$$

Sharpness $= \dfrac{\text{change in Amp}}{\text{Change in freq of Applied force}}$

$$= \frac{\frac{f_0}{2k\omega p}\cancel{(p-\omega)}}{\cancel{(p-\omega)}}$$

$$\text{Sharpness} = \frac{f_0}{2K\omega p}.$$

Sharpness is inversely proportional to K.



When the K is large, curves are flat = Resonance is flat

If K is small → curves are sharp peak.
If K is zero → A is infinity.

When the damping is low, the amplitude falls very rapidly on either side of Resonant frequency for which the resonance is sharp.

On the other hand, for high damping the amplitude falls of very slowly on either side of resonance.

Resonance is either flat or sharp depending
25 fri on the damping for the oscillating system is large or small.

Example for flat resonance is observed in the resonance of an air coloumn of a aspirator bottle with turing fork. Due to its large damping the air column respond with turning fork over a wide range in the neighbourhood of resonance. Thus in this case, it is actually dificult to
26 sat predict the exact point of resonance. and hence the resonance is said to flat.

While in the case of sonometer wire, the damping is small and responds only with one particular frequency i.e its own natural frequency, hence the resonance is sharp.

April 2005

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Notes

Quality factor

Q factor is a measure of sharpness of resonance, which explains how fast energy decays in an oscillatory system. Its a dimensionless quantity.

The mathematical expression is

$$Q = 2\pi \frac{\text{Energy stored in oscillator}}{\text{Energy lost per cycles}} \rightarrow (1).$$

Energy stored $= KE + PE$

$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$x = A\sin(pt-\theta)$

$\frac{dx}{dt} = Ap\cos(pt-\theta).$

$= \frac{1}{2}m[A^2p^2\cos^2(pt-\theta)] + \frac{1}{2}k[A^2\sin^2(pt-\theta)]$

$= \frac{1}{2}m \cdot A^2p^2\cos^2(pt-\theta) + \frac{1}{2}m\omega^2A^2\sin^2(pt-\theta) \rightarrow (2)$

where $\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m.$

Energy loss per cycle = damping force × velocity.

$= T\gamma v \times v.$

$= T\gamma \frac{dx}{dt} \times \frac{dx}{dt}.$

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February 2005

Notes

$= T\gamma[A^2p^2\cos^2(pt-\theta)]$

Where $T = \frac{2\pi}{\omega} = \frac{2\pi}{p}$

$2k = \frac{\gamma}{m}$ and $\tau = \frac{1}{2k} \Rightarrow \gamma = 2km = m \times \frac{1}{\tau}$

Damping coeff.

Substitute $T = \frac{2\pi}{p}$ and $\gamma = \frac{m}{\tau}$.

$$\text{Energy loss} = T\gamma\left[A^2p^2\cos^2(pt-\theta)\right].$$

$$= \frac{2\pi}{p}\cdot\frac{m}{\tau}\left[A^2p^2\cos^2(pt-\theta)\right] \longrightarrow (3).$$

Substitute eqn (2) and (3) in eqn (1).

$$Q = 2\pi\cdot\frac{\frac{1}{2}mA^2p^2\cos^2(pt-\theta) + \frac{1}{2}m\omega^2A^2\sin^2(pt-\theta)}{\frac{2\pi}{p}\cdot\frac{m}{\tau}\left[A^2p^2\cos^2(pt-\theta)\right]}$$

WKT $\sin^2\theta + \cos^2\theta = 1$; Avg value of $\cos^2\theta$ or $\sin^2\theta = \frac{1}{2}$

$$\therefore Q = 2\pi\cdot\frac{\frac{1}{2}mA^2p^2\cos^2(pt-\theta) + \frac{1}{2}m\omega^2A^2\sin^2(pt-\theta)}{\frac{2\pi}{p}\cdot\frac{m}{\tau}A^2p^2\cos^2(pt-\theta)}$$

$$\left[\begin{array}{l}\cos^2(pt-\theta) = \frac{1}{2} \\ \sin^2(pt-\theta) = \frac{1}{2}\end{array}\right]$$

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Notes

4 mon

$$Q = \cancel{2\pi} \cdot \frac{\frac{1}{4} m A^2 p^2 + \frac{1}{4} m \omega^2 A^2}{\frac{\cancel{2\pi}}{p} \cdot \frac{m}{\tau} A^2 p^2 \cdot \frac{1}{2}}$$

$$= \frac{\frac{1}{4} \cancel{m} \cancel{A^2} (p^2 + \omega^2) \cdot 2 \cancel{p} \tau}{\cancel{m} \cancel{A^2} p^{\cancel{2}}}$$

$$= \frac{1}{2} \frac{(p^2 + \omega^2)}{p} \cdot \tau$$

5 tue

At resonance $\omega = p$;

$$\therefore Q = \frac{1}{\cancel{2}} \left(\frac{\cancel{2} \omega^{\cancel{2}}}{\cancel{\omega}} \right) \tau .$$

$$\boxed{Q = \omega \tau}$$

6 wed

March 2005

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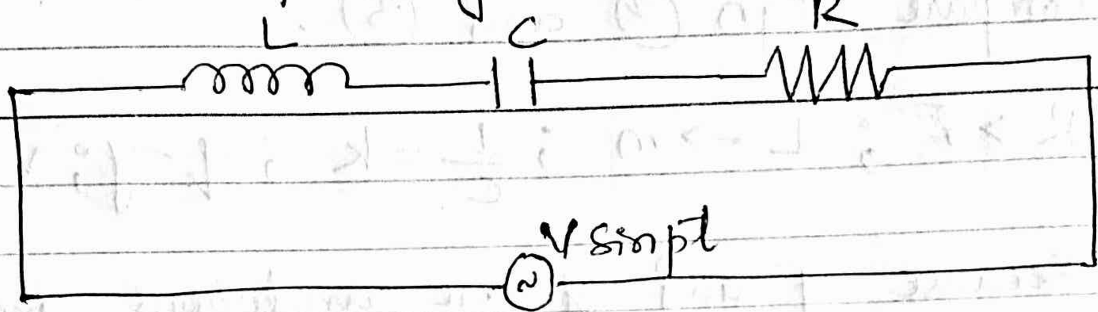
Notes

LCR Resonance

Consider a series LCR circuit in which $V\sin pt$ is the applied AC voltage. The voltage drop across the inductor L, Resistor R and capacitor C respectively.



$$V_L = L\frac{di}{dt} \qquad V_R = iR \qquad V_C = \frac{q}{C}.$$

Since the applied voltage must be equal to the sum of the voltage drop across each of the elements.

$$V_L + V_R + V_C = V\sin pt \longrightarrow (1).$$

$$L\frac{di}{dt} + iR + \frac{1}{C}q = V\sin pt.$$

But $i = \frac{dq}{dt}$

$$\therefore L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V\sin pt.$$

Divide above eqn by L.

$$\frac{d^2q}{dt^2} + \left(\frac{R}{L}\frac{dq}{dt}\right) + \left(\frac{1}{LC}\right)q = \frac{V}{L}\sin pt$$

$$\longrightarrow (2)$$

11 mon

The above eqn resembles the eqn of motion of the vibrating system given by.

$$\frac{d^2y}{dt^2} + \left(\frac{r}{m}\right)\frac{dy}{dt} + \left(\frac{k}{m}\right)y = \frac{F}{m}\sin pt \rightarrow (3).$$

Compare eqn (2) and (3).

12 tue

$R \rightarrow r$; $L \rightarrow m$; $\frac{1}{C} = k$; $q - y$; $V \rightarrow F$;

Because q and y are analogous quantity.

Thus $\frac{1}{LC} = \frac{k}{m} = \omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}}$.

13 wed

Resonance condition, $p = \omega = \sqrt{\frac{k}{m}}$.

$$\boxed{\frac{1}{LC} = \omega^2}$$

∴ In LCR circuit at Resonance ; $\boxed{\omega^2 = \frac{1}{LC}}$

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Notes