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The End of Expectation for True Second Order Dirac Equation

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The original Coulomb field radial Dirac equation is essentially a first order differential system of two-component wave functions. The second order differential equation of the original wave function component directly converted from the first order differential equation set is called the true second order Dirac equation. Relativistic quantum mechanics usually ignores the physical meaning of wave function and only focuses on the energy eigenvalue. So, the main expectation of solving the true second order Dirac equation of hydrogen-like atoms is that the Dirac energy level formula is the eigenvalue of the equation. Here I derive two true second order Dirac equations that are mutually independent in form but actually constrained by common energy parameters, and then use the traditional boundary conditions of hydrogen-like atoms to solve the true second order Dirac equation. The conclusion drawn from this is not exactly the same as the traditional understanding. 1) The formal solution of the true second order Dirac equation satisfying the traditional boundary conditions takes the Dirac hydrogen level formula as the energy eigensolution, which seems to meet the expectation; 2) However, when the radial quantum number is 0, regardless of the value of the angular quantum number, the complete expression of the wave function as the exact solution of the equation diverges at the coordinate origin, which does not meet the traditional boundary conditions, which means that the universe is collapsed and does not conform to the fact of the universe structure. From this it is concluded that the Dirac energy level formula is only the formal eigenvalue of the true second order Dirac equation that does not conform to the physical meaning. This announced the end of the expectation of using traditional boundary conditions to solve the true second order Dirac equation to naturally obtain the Dirac energy level formula. This result will promote the re-study of the exact solution of the original Dirac equation.

Keywords: Unitary principle, Dirac equation, Inevitable solution, Pseudo solution, Wave function divergence, Energy of imaginary number.

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1 Introduction

The relativistic Dirac equation[1-8] is a four-order matrix wave equation in three-dimensional rectangular coordinate system. It is essentially a differential equation system composed of four first order differential equations. The great influence of the Dirac equation has given birth to various specious of equations with the name of the Dirac equation, such as the teratogenic first order Dirac equation group and its corresponding teratogenic second order Dirac equation[9-12], which have been terminated[13]. Perhaps because of the difficulty in mathematical processing, several generations of physicists have avoided the true second order Dirac equation directly converted from the original Dirac equation, and instead put forward a variety of puzzling and fundamentally different heteromorphous second order Dirac equation[14-23] or heteromorphous first order Dirac equation[24-30], or even the title Dirac equation[31] with unclear mathematical process, to piece together the energy level formula of hydrogen-like atom that meets the expectation. What about the solution of the true second order Dirac equation transformed from the original first order Dirac equation?

From a mathematical point of view, people can write a differential equation at will to discuss its solution. However, the Dirac equation is not written randomly, and its construction idea is beyond that of the previous wave equation. If we do not discuss the applicable scope of the operator replacement rule of mechanical quantities, the matrix structure of the Dirac equation is quite creative and charming. However, from the physical point of view, the construction principle of the equation used for the specific physical model must conform to the basic laws of physics, and the real solution must be able to correctly describe the physical laws. In modern physics, when the true solution of the equation does not conform to the physical phenomenon, it often appears to distort

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the mathematical calculation to obtain the formal solution to replace the true solution, and even independently define the operation process that does not conform to the mathematical operation rules, so that the results are consistent with the experimental observation results, and then some people provide the unreal observation results that cannot be repeated, to confuse the false with the true, and to add fuel to the flames. Just like the gravitational wave of spiral double stars, the data obviously do not conform to the famous equation[32-34], but it has been confirmed by propaganda. The expected solution of the terminated deformed Dirac equation is not the inevitable solution of the equation. The correctness of such papers for mass production can be checked by using Dongfang's unitary principle, thus presenting its true background.

Dongfang's unitary principle[35-37], which is generally applicable to the logical test of natural and social sciences, abstracts a very simple logic. *There is a definite transformation relationship between different metrics describing the natural law, and the natural law itself does not change due to the selection of different metrics. When the mathematical expression of natural laws under different metrics is transformed into one metric, the result must be the same as the inherent form under this metric, 1=1, meaning the transformation is unitary.* We have made some important discoveries by using the unitary principle to test the logic of modern physics[38-43], and will make more and more important breakthroughs. The discovery of the law of angular motion[44] will make the unitary principle test of the basic principle of the operator construction of the wave equation of quantum mechanics fruitful. This is because the describing angular motion law is a system of equations composed of multiple equations, and the corresponding operator evolution equations constitute many different metrics of the same physical model, which leads to the conclusion that is difficult to find in the past is subversive enough problems. For example, is there any solution to the various equations of the evolution equations of angular motion law operator? Is the solution of each operator evolution equation consistent? How does the bound state model relate to the quantized energy? What are the eigensolutions of different operator evolution equations? The statistical interpretation of wave functions obviously requires that they meet the principle of normalization, so they are the same. Therefore, the statistical interpretation of wave function is also challenged. There are sufficient reasons to question the construction principle of the wave equation. The Dirac equation is also constructed by the method of constructing a wave equation with operators. So, is the Dirac equation unique?

The solution of the Dirac equation of the hydrogen atom seems to be perfect, and the result is considered to describe the fine spectral structure of the hydrogen atom. The Dirac equation is defined as having the causal relationship with the spin of antimatter and particle due to some additional explanations. However, when we try to demonstrate such causal relationships one by one, we know that some descriptions are popular science exaggerations that cannot be proved by scientific logic. It is a waste to comment one by one on the many second order Dirac equations generated by the first order Dirac hydrogen equation with a huge halo overhead. It is sufficient to have the terminated teratogenic Dirac equation as a representative. Now we know that the real purpose behind the definition of a new term "decoupling" for the teratogenic second order Dirac equation is to delete one solution that does not conform to the expected energy value and retain the other solution that meets the expectation, thus covering up the irreconcilable contradiction between the two solutions. The so-called "decoupling" is by no means a scientific principle but is cunningly used as a scientific principle. This is just a profile of the false prosperity of modern physics.

Here we discuss the true second order Dirac equation which is directly transformed from the original first order radial differential equation of the hydrogen atom. It has two second order differential equations. The boundary conditions for transforming the first order differential equations into the second order differential equations remain unchanged. We will prove that these two second order Dirac differential equations have the same quantized energy eigenvalues as the first order Dirac differential equation, which is consistent with the Dirac energy level formula, and seems to meet expectations. However, the solution of the equation determined by the bounded boundary conditions of the wave function does not completely meet the boundary conditions. When the radial quantum number is 0, it diverges at the coordinate origin, which means that the universe is collapsed and does not conform to the fact of the universe structure. This announced the end of the expectation of solving the true second order Dirac equation satisfying the traditional boundary conditions to naturally obtain the Dirac energy level formula of hydrogen-like atoms.

2 Conclusions and comments

The mathematical treatment process of the main equations of relativistic quantum mechanics was tested by the unitary principle, and the Yukawa nuclear force meson theory was ended, the relativistic Klein-Gordon equation of Coulomb field was ended, the teratogenic first-order Dirac equation group and its corresponding teratogenic second-order Dirac equation were ended, and the isomeric second-order Dirac equation of Coulomb field was also ended. Here I end the expected solutions of the two true second-order differential equations transformed from the original first-order radial Dirac equations of the Coulomb field or hydrogen-like atom.

The solution of the true second-order Dirac equation of the Coulomb field satisfying the traditional bound-

ary conditions is similar to the solution of the first-order Dirac equation system, and the energy level formula is the same, which seems to meet the expectations. However, the wave function as the expected solution hides the case of divergence at the origin of coordinates and does not meet the boundary conditions, so the expected solution is only a pseudosolution. All kinds of second-order Dirac equations introduced by famous scientific journals are meaningless in fact. Relevant calculations distort mathematical operation rules or physical logic. According to the unitary principle, since the expected solution of the standard second-order Dirac equation is only a pseudo-solution, the expected solution of all other second-order Dirac equations also belongs to pseudo-solution, unless the constructed second-order Dirac equation has nothing to do with the original Dirac first-order differential equation system, but only uses Dirac's fame to increase the appeal of the article. The end of the expectation of the true second-order Dirac equation means the end of the distorted mathematical logic and reasoning of the so-called second-order Dirac equation and the so-called higher-order Dirac equation[47].

It is puzzling that all kinds of incoherent formal solutions of the real and false Dirac equation contain the same so-called fine structure energy level expectation. The Dirac equation has the challenge of thinking not only in physical logic but also in mathematical logic, so it is worthy of further study. However, the focus of this paper is to determine the standard form of the true second-order Dirac equation. The purpose is to develop the research of relevant mathematical theories to directly deal with such second-order differential equations, thus revealing the conclusions of mathematics and physics that are rarely known. Are there any solutions to the two true second-order Dirac equations of hydrogen-like atoms that conform to the mathematical and physical meanings? What ends here is only the expected solution of the true second-order Dirac equation, not the second-order Dirac equation itself. Dirac's hydrogen atom theory only focuses on the energy level formula, which has caused many authors to follow suit. In fact, even if an unreasonable wave equation is constructed so that some term in the series solution part is zero, the same so-called quantized energy formula can be obtained. Although the essence of quantum mechanics is the quantized energy formula, obtaining the quantum energy formula through wave equation may not conform to the unitary principle. A theory that conforms to the unitary principle may not be correct, but a theory that does not conform to the unitary principle must be wrong. Therefore, one can foresee how the unitary principle will bring about changes in scientific theory.

The expected solution of the first-order Dirac equation system of a hydrogen-like atom, which is respected by the standard course, is recognized as accurately describing the spectral fine structure of the hydrogen atom. How reliable is the accepted conclusion? We can use the unitary principle to test it in depth and get a conclusion. It is one of the basic principles that quantum mechanics has not been strictly proved to construct wave equations by replacing mechanical quantity with operators and acting on wave functions. Attempts to prove or deny this principle will have unexpected results. Dongfang's law of angular motion puts forward requirements for the scope of application of this basic principle of quantum mechanics. The angular motion law of the same physical model is a set of equations containing multiple equations. If the principle of constructing wave equations by quantum mechanics operators is universally established, a large number of equations representing the angular motion law will lead to a large number of wave equations of the same physical model. The solutions of these equations are not necessarily consistent. Therefore, the angular motion law has subversive significance for the unitary test of quantum mechanics.

As a physical equation, Dirac equation needs to undergo a comprehensive logical test. However, in the field of mathematics, differential equations can be constructed at will. After constructing the first order differential equation, transforming it into the second order differential equation will produce enough additional roots. According to the unitary principle, the solution of the second order differential equation from the first order differential equation system should contain the special solution of the first order differential equation system, otherwise the conversion calculation from the first order differential equation to the second order differential equation will be prohibited, which will make the mathematical rules not conform to the unitary principle. Here, the original first-order Dirac radial differential equations of the hydrogen atom are transformed into standard second-order Dirac equations. So, can we obtain the expected solution of the first order differential equation system from the second order differential equation? The mutual transformation and exact solution of the first order differential equation and the second order differential equation must conform to the unitary principle, otherwise it will constitute a mathematical paradox. The conclusions of many mathematical problems in physics, especially many calculations in theoretical physics, are not reliable or questionable, and may require the participation of mathematicians to be reasonably corrected and improved.

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