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Dongfang Modified Equations of Electromagnetic Wave

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Systematically show the correct solution process of the initial value problem of Maxwell equations in rectangular coordinate system and cylindrical coordinate system, and clearly point out that the transverse electromagnetic wave equation described in the classical electromagnetic theory is the result of incorrect understanding and treatment of Maxwell equations: the definite solution conditions of magnetic field and electric field in the same phase that do not conform to the natural law are assumed in advance, and then the plane transverse electromagnetic wave mode is determined through the general solutions of the second-order Maxwell equations, and the conclusion violates the law of energy conservation, In particular, the mathematical process does not conform to the solving rules of differential equations. Through this simple example, I clarify my clear position: since the era of Maxwell's electromagnetic theory, theoretical physics has fallen into a strange circle in which the wrong calculation of celebrities is defined as standard methods and the assumption that celebrities cannot prove is defined as basic principles, resulting in a large number of wrong logic and wrong conclusions in physical theory, even some branches of theoretical physics are completely wrong from beginning to end, and all these errors are covered up by formal mathematical operations, the formal mathematics seriously distorts the laws of nature. However, all correctable and uncorrectable errors have not been found and recognized by mainstream physicists, and especially the pointing out of these errors has been rejected by mainstream scholars. A serious question arises: Can the future mainstream physics change this academic abnormal state?

Keywords: Maxwell Equations; Optimal differential equation; Initial conditions; Plane transverse electromagnetic wave; Cylindrical transverse electromagnetic wave.

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1 Introduction

Maxwell equations[1, 2] point out that the changing electric field and magnetic field can excite each other and propagate in space to form electromagnetic waves. However, Maxwell equations fail to indicate the time and space interval of the causal effect of the magnetic field excited by the changing electric field or the electric field excited by the changing magnetic field. Is the electric field near a point far away from the radiation source propagated by the electric field of the radiation source at the previous time, or is it excited by the magnetic field changed in the surrounding space at which time? The answer is unknown. It can be seen that the description of causality of an electromagnetic wave in Maxwell theory is actually very vague.

Reviewing the research on plane electromagnetic wave in classical electromagnetic wave theory: the expression forms of monochromatic plane electromagnetic wave with the same phase of magnetic field and electric field are predetermined in advance[3-8], $\mathbf{E}(\mathbf{x},t)=\mathbf{E}(\mathbf{x})e^{-i\omega t}$ and $\mathbf{B}(\mathbf{x},t)=\mathbf{B}(\mathbf{x})e^{-i\omega t}$, so as to determine that the relationship between electric field and magnetic field is $\mathbf{B}(\mathbf{x},t)=\sqrt{\mu\varepsilon}\mathbf{n}\times\mathbf{E}$, where $\mathbf{n}$ is the unit vector of propagation direction. The main formal conclusions are as follows: 1) the electromagnetic wave is shear wave, and both $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to the propagation direction; 2)$\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other, and $\mathbf{E}\times\mathbf{B}$ is along the wave propagation direction; 3) $\mathbf{E}$ and $\mathbf{B}$ are in phase, and the ratio of amplitudes in vacuum is equal to the speed of light. The electric field is in phase with the magnetic field, which breaks the law of conservation of energy. This kind of plane electromagnetic wave mode in which the electric field and magnetic field are in phase everywhere can not be generated or exist.

The expression of electric field and magnetic field characterizing electromagnetic wave mode naturally needs to meet Maxwell equations, but the expression of

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*The name of the author's ID card is Rui Chen, and the signature of his academic articles is X. D. Dongfang.

†This article integrates the regular solutions of plane transverse electromagnetic wave modes of Maxwell equations published in Chinese in 2000 and the regular solutions of cylindrical transverse electromagnetic wave modes of Maxwell equations published in English in 2011. Signatures of Chen in these two articles are my early signatures.

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the electric field and magnetic field satisfying Maxwell equations may not constitute an electromagnetic wave mode that can be realized or exist. Because Maxwell equations are differential and integral equations, the solution of differential and integral equations depends not only on the form of differential equations, but also on the definite solution conditions[9]. Different definite solution conditions will bring completely different solutions to the same differential and integral equation. The definite solution condition of Maxwell equations in unbounded space is the initial condition, that is, to solve the initial value problem. However, the process of determining the plane electromagnetic wave mode according to Maxwell equations in classical theory seems to indicate that the definite solution of Maxwell equations has nothing to do with the initial condition[10, 11], which is obviously a mistake. On the other hand, the changing electric field generates a magnetic field, and the changing magnetic field generates an electric field. If $\mathbf{E}$ and $\mathbf{B}$ are in phase, it will be difficult at zero time! Usually, the starting point of time is the time when the electric field or magnetic field begins to oscillate, that is, zero time. In the LC oscillation circuit, after the parallel plate capacitor is charged stably, the electric field in any region of the space is constant. At this time, the electric field does not excite the magnetic field. The electric field is maximum everywhere and the magnetic field is zero everywhere. On the contrary, if the magnetic field in any area of space does not change and the magnetic field does not excite the electric field, the magnetic field is maximum everywhere and the electric field is zero everywhere. It can be seen that $\mathbf{E}$ and $\mathbf{B}$ cannot always reach the maximum at the same time or disappear at the same time. The classical theory of electromagnetic wave with the same phase of the electric field and magnetic field leads to the difficulty of zero time. The two equations $\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})e^{-i\omega t}$ and $\mathbf{B}(\mathbf{x},t) = \mathbf{B}(\mathbf{x})e^{-i\omega t}$ used to describe plane electromagnetic wave mode in classical theory can not represent an objective or possible electromagnetic wave mode.

As a system of differential equations, the solution of Maxwell's equations obviously depends on the initial value or boundary conditions of electromagnetic field[12, 13] and the determination of plane electromagnetic wave mode and cylindrical electromagnetic wave mode can be attributed to the initial value problem of Maxwell's equations. According to different initial conditions, the plane electromagnetic wave modes or cylindrical electromagnetic wave modes[14-18] obtained by solving Maxwell equations are different. Here, reasonable solutions of plane transverse electromagnetic wave in Chinese version in 2000[19] and cylindrical electromagnetic wave in an English version in 2011[20] are integrated, and the correct solution method of determining electromagnetic wave mode according to Maxwell equations is emphasized. The consistent conclusion is that both plane electromagnetic wave and cylindrical electromagnetic wave are transverse waves. The amplitude of the magnetic field excited by the changing electric field, or the amplitude of the electric field excited by the changing magnetic field is different at different positions, and the phase difference between the electric field and the magnetic field is also different.

2 Regular solution of plane transverse electromagnetic wave modes

In 2000, the author published a Chinese article on the regular solutions of the plane transverse electromagnetic waves of Maxwell equations with the signature Chen Rui[19], and gave the plane transverse electromagnetic wave which can be realized theoretically and can exist. However, this most basic and important discovery seems to have not attracted the attention of mainstream scholars. College education has always advocated the wrong electromagnetic wave mode of electric field and magnetic field in phase. This part is introduced again in English. One first determines the initial conditions of plane electromagnetic wave, then gives the general plane wave solution of Maxwell equations, and finally determines the special solution of Maxwell equations according to the initial conditions to obtain a correct plane transverse electromagnetic wave mode.

2.1 Physical background of generating plane electromagnetic wave

The determination of initial conditions depends on the specific physical background. Although the electromagnetic wave can be independent of the moving charge or current, the existence of the electromagnetic wave in the surrounding space depends on the accelerating charge or changing current of the emission source. Without the normal operation of the TV transmission tower, there will be no electromagnetic wave of a predetermined frequency in the nearby space. The charge variation law of the emission source determines the initial conditions of Maxwell equations in free space and the mode of an electromagnetic wave[21].

What is a plane electromagnetic wave? As shown in Figure 1, it is assumed that two rectangular metal plates P and M with finite width and sufficient length are parallel to each other and close to each other, the plane of the rectangular plate is perpendicular to the y-axis, the left end is at the coordinate origin, and a power supply with output voltage varying with time is connected. Therefore, the changing electric field propagates from left to right in the metal, resulting in the continuous change of charge distribution at any position on the two parallel metal plates. The changing electric field generated between the two plates is parallel to the y-axis, and the electric field propagates along the positive direction of the x-axis. When the rectangular plate is close together, it can be considered that the electric field is evenly distributed on the y-axis; Because the electric field propagates at the speed of light, the distribution change of the electric field to the finite width on the z-axis can be ignored; Considering the case of superconductivity, the energy of electromagnetic wave in the conduction process is not converted into Joule heat and lost. Therefore, the electric field between two parallel plates constitutes an ideal plane electric field wave mode, that is, plane free electric field wave.

The introduction of exponential complex number to express physical quantities often brings great convenience. However, this does not mean that the complex number has an inevitable causal relationship with physical theory. Some quantum force scholars have made mistakes in their understanding of this problem. The measurability of physical quantity originated in the field of real numbers and finally ended in the field of real numbers. Here, the changing electric field between parallel plates is expressed by a real function,

$$\mathbf{E} = \mathbf{e}_y E_y (x, t) \tag{1}$$

Where $\mathbf{e}_y$ represents the unit vector along the positive direction of the y-axis. The output voltage of the alternating power supply between the two parallel plates changes according to the sinusoidal law $u(t) = U_m \sin \omega t$. when $t = 0$, the magnetic field between two parallel plates as well as the surrounding space is zero. If the edge effect of parallel plates is not considered, this physical background gives the definite solution condition, that is, the initial condition is

$$\begin{aligned} &\mathbf{B}(t=0) = 0 \\ &\mathbf{E}(x,t) = \mathbf{e}_y E_m \sin \omega \left(t - \frac{x}{c}\right) \end{aligned} \tag{2}$$

The non-uniform changing electric field and magnetic field can excite each other. Different definite solution conditions determine the different modes of magnetic field generated by the changing electric field between parallel plates. Similarly, the changing magnetic field can be used as the electromagnetic wave source to determine a certain plane electromagnetic wave mode. In unbounded space, no matter whether the wave source is evacuated later or not, the electromagnetic field will not turn back, and the initial conditions cannot be changed.

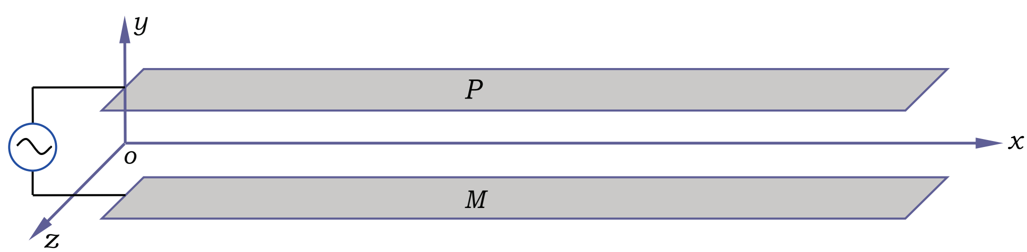

Figure 1 plane electromagnetic wave modes propagating along the positive direction of x-axis between sufficiently long parallel plate capacitors

2.2 General solution of plane wave modes for Maxwell equations

In a vacuum, electromagnetic waves travel at the speed of light. Maxwell's electromagnetic theory holds that in the process of electromagnetic wave propagation, the changing electric field wave and the changing magnetic field wave exist at the same time. In order to obtain the plane electromagnetic wave mode satisfying the initial condition (2), the Maxwell equations need to be solved first to determine the general magnetic field solution of the plane wave mode. The vector form of Maxwell equations is

$$\begin{aligned} &\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ &\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \\ &\nabla \cdot \mathbf{E} = 0 \\ &\nabla \cdot \mathbf{B} = 0 \end{aligned} \tag{3}$$

From this, the second-order wave equations for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ can be derived

$$\begin{aligned} \nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \\ \nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \end{aligned} \tag{4}$$

(3) and (4) are vector differential equations, which can be transformed into optimization differential equations. The mainstream literature directly writes the general solutions, $\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}) e^{-i\omega t}$ and $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x}) e^{-i\omega t}$, of the time harmonic electromagnetic waves of the two equations, and then writes the special solutions according to the shear wave conditions. This solution completely ignores the initial conditions of the wave equation and violates the solution rules of the wave equation. Strictly speaking, it lacks the mathematical basis for solving the differential equation, and the conclusion is naturally untenable.

⊗ Reference [9] uses the circulation integral to "prove" the result that is inconsistent with the initial condition: $H_z = \varepsilon_0 E_y v$ (page 613). Generally speaking, the magnitude of the electromagnetic field vector can be solved by flux integral or circulation integral only under the condition of spherical symmetry or axial symmetry, but plane electromagnetic wave does not have such symmetry.

If limited to the dogma of standard literature, it may involuntarily produce the sophistry of deviating from the inevitable logic and choosing the formal logic to maintain the wrong classical logic and conclusion. At this time, you might as well directly solve the first-order Maxwell equations to obtain the correct plane transverse electromagnetic wave mode, and all problems will become suddenly clear. The component form of the first-order Maxwell equations (3) is

$$\begin{aligned}
&\left(\mathbf{e}_x\frac{\partial}{\partial x}+\mathbf{e}_y\frac{\partial}{\partial y}+\mathbf{e}_z\frac{\partial}{\partial z}\right)\times\left(\mathbf{e}_xE_x+\mathbf{e}_yE_y+\mathbf{e}_zE_z\right)=-\frac{\partial}{\partial t}\left(\mathbf{e}_xB_x+\mathbf{e}_yB_y+\mathbf{e}_zB_z\right)\\
&\left(\mathbf{e}_x\frac{\partial}{\partial x}+\mathbf{e}_y\frac{\partial}{\partial y}+\mathbf{e}_z\frac{\partial}{\partial z}\right)\times\left(\mathbf{e}_xB_x+\mathbf{e}_yB_y+\mathbf{e}_zB_z\right)=\frac{1}{c^2}\frac{\partial}{\partial t}\left(\mathbf{e}_xE_x+\mathbf{e}_yE_y+\mathbf{e}_zE_z\right)\\
&\left(\mathbf{e}_x\frac{\partial}{\partial x}+\mathbf{e}_y\frac{\partial}{\partial y}+\mathbf{e}_z\frac{\partial}{\partial z}\right)\cdot\left(\mathbf{e}_xE_x+\mathbf{e}_yE_y+\mathbf{e}_zE_z\right)=0\\
&\left(\mathbf{e}_x\frac{\partial}{\partial x}+\mathbf{e}_y\frac{\partial}{\partial y}+\mathbf{e}_z\frac{\partial}{\partial z}\right)\cdot\left(\mathbf{e}_xB_x+\mathbf{e}_yB_y+\mathbf{e}_zB_z\right)=0
\end{aligned}\tag{5}$$

The plane wave modes satisfying equation (1) are implied in the following equation

$$E_x=E_z=0,\quad \frac{\partial E_y(x,t)}{\partial z}=0\tag{6}$$

It can be seen that the third formula of equation (5) is valid, and the other three formulas are simplified to

$$\begin{aligned}
&\mathbf{e}_z\frac{\partial E_y}{\partial x}=-\mathbf{e}_x\frac{\partial B_x}{\partial t}-\mathbf{e}_y\frac{\partial B_y}{\partial t}-\mathbf{e}_z\frac{\partial B_z}{\partial t}\\
&\mathbf{e}_x\left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)+\mathbf{e}_y\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)+\mathbf{e}_z\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)=\mathbf{e}_y\frac{1}{c^2}\frac{\partial E_y}{\partial t}\\
&\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0
\end{aligned}\tag{7}$$

therefore

$$\begin{aligned}
&\frac{\partial B_x}{\partial t}=0,\quad \frac{\partial B_y}{\partial t}=0,\quad \frac{\partial E_y(x,t)}{\partial x}=-\frac{\partial B_z}{\partial t}\\
&\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}=0,\quad \frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}=\frac{1}{c^2}\frac{\partial E_y(x,t)}{\partial t},\quad \frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=0\\
&\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0
\end{aligned}\tag{8}$$

The general solutions of the three equations in the first line of (8) are

$$B_x=B_x(x,y,z),\quad B_y=B_y(x,y,z),\quad B_z=a(x,y,z)-\int\frac{\partial E_y(x,t)}{\partial x}dt\tag{9}$$

Where $a(x,y,z)$ is the partial integral "constant". So the expression of the magnetic field is

$$\mathbf{B}=\mathbf{e}_xB_x(x,y,z)+\mathbf{e}_yB_y(x,y,z)+\mathbf{e}_z\left[a(x,y,z)-\int\frac{\partial E_y(x,t)}{\partial x}dt\right]\tag{10}$$

Of course, the expression of magnetic field B must satisfy the third line equation of (8). It can be seen from the definite solution condition (2)

$$B_x(y,z)=0;\quad B_y(x,z)=0\tag{11}$$

Therefore, the general expression for the magnetic field wave mode is obtained from equation (10),

$$\mathbf{B}=\mathbf{e}_z\left[a(x,y,z)-\int\frac{\partial E_y(x,t)}{\partial x}dt\right]\tag{12}$$

It should be noted that using equation (11) to integrate the second equation in the second line of (8), one gets

$$B_z=b(y,z,t)-\frac{1}{c^2}\int\frac{\partial E_y(x,t)}{\partial t}dx+\int\frac{\partial B_x}{\partial z}dx$$

Where $b(y, z, t)$ is also the "constant" of the partial integral. The expression for the magnetic field can be expressed as

$$\mathbf{B} = \mathbf{e}_x B_x(y, z) + \mathbf{e}_y B_y(x, z) + \mathbf{e}_z \left[b(x, y, z) - \frac{1}{c^2} \int \frac{\partial E_y(x, t)}{\partial t} dx + \int \frac{\partial B_x}{\partial z} dx \right] \tag{13}$$

It is also obtained from $B_x(y, z) = 0$ and $B_y(x, z) = 0$ given in (11)

$$\mathbf{B} = \mathbf{e}_z \left[b(y, z, t) - \frac{1}{c^2} \int \frac{\partial E_y(x, t)}{\partial t} dx \right] \tag{14}$$

(12) and (14) are equivalent and represent the general solution of plane electromagnetic wave modes of Maxwell equations.

The first-order Maxwell differential equations or integral equations are sufficient to give the correct solution of the electromagnetic wave mode, while transforming the first-order differential equations into second-order differential equations will produce additional roots and unreasonable roots, which are mathematical common sense. $\mathbf{E} \perp \mathbf{v}$ has been explained earlier, and these two results show that $\mathbf{B} \perp \mathbf{E}$ and $\mathbf{B} \perp \mathbf{v}$, so the plane electromagnetic wave is a transverse wave. According to (8), the scalar form of the electric and magnetic field wave equation can also be derived, and the solution of an initial value problem in unbounded space is usually d'Alembert solution. Similarly, if the plane magnetic field wave mode is given, the integral expression of the plane electric field wave mode can be obtained.

2.3 Special solution of plane wave mode for Maxwell equations

The definite solution condition (2) and equation (12) determine a plane transverse electromagnetic wave mode:

$$\begin{aligned} &\mathbf{E} = \mathbf{e}_y E_m \sin \omega (t - x/c) \\ &\mathbf{B} = \mathbf{e}_z \left[a_z(x, y, t) - \int \frac{\partial E_y(x, t)}{\partial x} dt \right] \\ &\mathbf{B}(t = 0) = 0 \end{aligned} \tag{15}$$

The solution of the above second equation is

$$\mathbf{B} = \mathbf{e}_z \frac{E_m}{c} \left[\sin \frac{\omega x}{c} + \sin \omega \left(t - \frac{x}{c} \right) \right]$$

It can be seen that the amplitude of the time harmonic magnetic field excited by the time harmonic electric field is different at different positions in space. The plane transverse electromagnetic wave mode is

$$\begin{aligned} &\mathbf{E} = \mathbf{e}_y E_m \sin \omega \left(t - \frac{x}{c} \right) \\ &\mathbf{B} = \mathbf{e}_z \frac{E_m}{c} \left[\sin \frac{\omega x}{c} + \sin \omega \left(t - \frac{x}{c} \right) \right] \end{aligned} \tag{16}$$

The phase difference between electric field and magnetic field is also a function of position.

Obviously, the above plane electromagnetic wave is a transverse wave. Although the amplitude of the electric field is the same everywhere, the amplitude of the magnetic field is different at different positions. Although the frequency of the electric field and magnetic field oscillation is the same, they are not in phase everywhere. The ratio of electric field to the magnetic field is not a constant, and the maximum ratio of electric field amplitude to magnetic field amplitude is c and the minimum is $c/2$. It can be seen that the plane electromagnetic wave modes of $\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}) e^{-i\omega t}$ and $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x}) e^{-i\omega t}$ described by the classical theory is not tenable. The discussion of plane electromagnetic wave should be based on the objectively existing wave modes.

3 Regular solution of cylindrical transverse electromagnetic wave mode

In 2011, the author cooperated with Xijun Li and published an English article on regular solutions of cylindrical transverse electromagnetic modes[20] of Maxwell equations with the signature Rui Chen. Here, the relevant mathematical process and conclusion are introduced again to emphasize the correct treatment of the initial value problem of Maxwell equations. Firstly, the general solution of vector Maxwell equations is solved in cylindrical coordinate system, and then the cylindrical transverse electromagnetic wave mode is determined according to the initial value conditions. The results show that the phase of electric field and magnetic field of cylindrical transverse electromagnetic mode is not the same everywhere, and the ratio of amplitude is also a function of time and space, which has the same characteristics as the plane transverse electromagnetic wave mode that can be generated and existing in practice.

3.1 Initial conditions of cylindrical transverse electromagnetic wave

Theoretically, the coaxial transmission line device composed of a hollow conductor tube and core wire as shown in Figure 2 can realize cylindrical transverse electromagnetic wave. Let the radius of the core wire be R_1 and the radius of the coaxial hollow conductor tube be R_2. By using the distribution formula of electrostatic field, when the core wire and hollow conductor tube have the equal amounts of different kinds of charges respectively, let the charged amount on the unit length of the surface along the axis be λ_m, and the electrostatic field in the transmission line can be obtained by using

Gauss theorem

$$\mathbf{E}=\frac{\lambda_m}{2\pi\varepsilon_0 r}\mathbf{e}_r \quad (R_1 \leqslant r \leqslant R_2) \tag{17}$$

Among them, $\mathbf{e}_r$ is the radial unit vector, and r is the distance from a certain point in the vacuum of the coaxial transmission line to the axis.

When the periodically changing power supply is connected at the left end so as to the periodically changing charge distribution state is excited to propagate from left to right, a periodically changing electric field is generated in the coaxial transmission line and propagates from left to right to form an electric field wave. The propagation direction is along the positive direction of the z-axis and perpendicular to the direction of the electric field Assuming that the hollow conductor tube is close to the core wire and the distribution of electric field to the radial direction is stable, the form of electric field wave can be deduced according to the electrostatic field distribution formula (17). This result can also be determined by Maxwell's equations. Naturally, they are the same, and the specific form is

Figure 2 realization of cylindrical transverse electromagnetic wave with coaxial transmission line

$$\begin{aligned}&\mathbf{E}=\mathbf{e}_r\frac{\lambda_m}{2\pi\varepsilon_0 r}f(z,t) \qquad (R_1 \leqslant r \leqslant R_2)\\&\mathbf{B}(t=0)=0\end{aligned} \tag{18}$$

$f(z,t)$ can be a sine function or a cosine function, which depends on the change of charge distribution on the conductor surface, and the corresponding wave mode is expressed in a complex number as

$$\begin{aligned}&\mathbf{E}=\mathbf{e}_r\frac{\lambda_m}{2\pi\varepsilon_0 r}e^{i\omega\left(t-\frac{z}{c}\right)} \qquad (R_1 \leqslant r \leqslant R_2)\\&\mathbf{B}(t=0)=0\end{aligned} \tag{19}$$

At zero time, the electric field in the transmission line is zero, so the magnetic field is zero everywhere, which is the initial condition.

3.2 General solution of cylindrical wave for Maxwell equations

In order to obtain the cylindrical electromagnetic wave mode satisfying the initial condition (18), the general magnetic field solution of Maxwell equations in cylindrical coordinate system[22] should be solved. The component form[23] of Maxwell equations (3) in the cylindrical coordinate system is

$$\begin{aligned}&\left(\frac{1}{r}\frac{\partial E_z}{\partial\theta}-\frac{\partial E_\theta}{\partial z}\right)\mathbf{e}_r+\left(\frac{\partial E_r}{\partial z}-\frac{\partial E_z}{\partial r}\right)\mathbf{e}_\theta+\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)-\frac{1}{r}\frac{\partial E_r}{\partial\theta}\right]\mathbf{e}_z=-\frac{\partial\mathbf{B}}{\partial t}\\&\left(\frac{1}{r}\frac{\partial B_z}{\partial\theta}-\frac{\partial B_\theta}{\partial z}\right)\mathbf{e}_r+\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\mathbf{e}_\theta+\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)-\frac{1}{r}\frac{\partial B_r}{\partial\theta}\right]\mathbf{e}_z=\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}\\&\frac{1}{r}\frac{\partial}{\partial r}(rE_r)+\frac{1}{r}\frac{\partial E_\theta}{\partial\theta}+\frac{\partial E_z}{\partial z}=0\\&\frac{1}{r}\frac{\partial}{\partial r}(rB_r)+\frac{1}{r}\frac{\partial B_\theta}{\partial\theta}+\frac{\partial B_z}{\partial z}=0\end{aligned} \tag{20}$$

For the electric field wave mode (18), because $E_r=E_r(r,z,t)$, $E_\theta=0$ and $E_z=0$, the system of equations (20) is reduced to

$$\begin{aligned}&\frac{\partial E_r}{\partial z}\mathbf{e}_\theta=-\mathbf{e}_r\frac{\partial B_r}{\partial t}-\mathbf{e}_\theta\frac{\partial B_\theta}{\partial t}-\mathbf{e}_z\frac{\partial B_z}{\partial t}\\&\left(\frac{1}{r}\frac{\partial B_z}{\partial\theta}-\frac{\partial B_\theta}{\partial z}\right)\mathbf{e}_r+\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\mathbf{e}_\theta+\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)-\frac{1}{r}\frac{\partial B_r}{\partial\theta}\right]\mathbf{e}_z=\mathbf{e}_r\frac{1}{c^2}\frac{\partial E_r}{\partial t}\\&\frac{1}{r}\frac{\partial}{\partial r}(rE_r)=0\\&\frac{1}{r}\frac{\partial}{\partial r}(rB_r)+\frac{1}{r}\frac{\partial B_\theta}{\partial\theta}+\frac{\partial B_z}{\partial z}=0\end{aligned} \tag{21}$$

According to the third line of equation (21), the electric field under the condition of axis-symmetry must satisfy the equation $rE_r = g(z,t)$, so

$$E_r = \frac{g(z,t)}{r} \tag{22}$$

Consistent with the result inferred from the electrostatic field (18), it can be seen that the third line of (21) is true.

The components on both sides of the vector equation are equal, so the first, second and fourth equations in (21) are reduced to

$$\begin{aligned}
&\frac{\partial B_r}{\partial t} = 0, \quad \frac{\partial B_\theta}{\partial t} = -\frac{\partial E_r}{\partial z}, \quad \frac{\partial B_z}{\partial t} = 0 \\
&\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z} = \frac{1}{c^2}\frac{\partial E_r}{\partial t}, \quad \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0, \quad \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{1}{r}\frac{\partial B_r}{\partial \theta} = 0 \\
&\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0
\end{aligned} \tag{23}$$

According to the three equations in the first row, one gets

$$B_r = B_r(r,\theta,z), \quad B_\theta = -\int \frac{\partial E_r}{\partial z} dt + a(r,\theta,z), \quad B_z = B_z(r,\theta,z) \tag{24}$$

Of course, this result must satisfy the last two equations in (23). Thus, the expression of the magnetic field is

$$\mathbf{B} = \mathbf{e}_r B_r(r,\theta,z) + \mathbf{e}_\theta \left[-\int \frac{\partial E_r}{\partial z} dt + a(r,\theta,z)\right] + \mathbf{e}_z B_z(r,\theta,z) \tag{25}$$

From the initial value condition $\mathbf{B}(t=0) = 0$, it can be seen that

$$B_r(r,\theta,z) = 0; \quad B_z(r,\theta,z) = 0 \tag{26}$$

Replace the corresponding magnetic field component in (25) with the above results, so there is

$$\mathbf{B} = \mathbf{e}_\theta \left[-\int \frac{\partial E_r}{\partial z} dt + a(r,\theta,z)\right] \tag{27}$$

On the other hand, using equation (26), the integration result of the first equation in the second line of (23) is

$$\mathbf{B} = \mathbf{e}_\theta \left[-\int \frac{1}{c^2}\frac{\partial E_r}{\partial t} dz + b(r,\theta,t)\right] \tag{28}$$

The general expression of cylindrical electromagnetic wave mode given by synthesizing (18) and (27) is

$$\begin{aligned}
&\mathbf{E} = \mathbf{e}_r \frac{\lambda_m}{2\pi\varepsilon_0 r} f(z,t) \quad (R_1 \leqslant r \leqslant R_2) \\
&\mathbf{B} = \mathbf{e}_\theta \left[-\int \frac{\partial E_r}{\partial z} dt + a(r,\theta,z)\right] \\
&\mathbf{B}(t=0) = 0
\end{aligned} \tag{29}$$

The general expression of cylindrical electromagnetic wave mode given by synthesizing (18) and (28) is

$$\begin{aligned}
&\mathbf{E} = \mathbf{e}_r \frac{\lambda_m}{2\pi\varepsilon_0 r} f(z,t) \quad (R_1 \leqslant r \leqslant R_2) \\
&\mathbf{B} = \mathbf{e}_\theta \left[-\int \frac{1}{c^2}\frac{\partial E_r}{\partial t} dz + b(r,\theta,t)\right] \\
&\mathbf{B}(t=0) = 0
\end{aligned} \tag{30}$$

Equations (29) and (30) are equivalent. Within the framework of Maxwell's electromagnetic theory that the transformed electric field produces a changed magnetic field and the changed magnetic field produces a transformed electric field, equations (29) and (30) represent the general solution of Maxwell's equations of cylindrical magnetic field excited by a radial changing electric field.

3.3 Special solutions of cylindrical waves for Maxwell equations

According to (29) or (30), the expression of the magnetic field can be obtained from the expression of the electric field, that is, the special solution of the cylindrical wave of Maxwell equations. Substituting (19) into (29) , one obtains

$$\begin{aligned}
&\mathbf{E} = \mathbf{e}_r \frac{\lambda_m}{2\pi\varepsilon_0 r} e^{i\omega\left(t-\frac{z}{c}\right)} \qquad (R_1 \leqslant r \leqslant R_2) \\
&\mathbf{B} = \mathbf{e}_\theta \left[-\int \frac{\partial E_r}{\partial z} dt + a(r,\theta,z)\right] \\
&\mathbf{B}(t=0) = 0
\end{aligned} \tag{31}$$

The partial derivative of electric field E is obtained from the first equation above,

$$\frac{\partial E_r}{\partial z} = -\frac{i\omega}{c}\frac{\lambda_m}{2\pi\varepsilon_0 r} e^{i\omega\left(t-\frac{z}{c}\right)} \tag{32}$$

Substitute it into the second equation of (31) and integrate it to obtain

$$\mathbf{B} = \mathbf{e}_\theta \left[\frac{1}{c}\frac{\lambda_m}{2\pi\varepsilon_0 r} e^{i\omega\left(t-\frac{z}{c}\right)} + a(r,\theta,z)\right] \quad (R_1 \leqslant r \leqslant R_2) \tag{33}$$

Using the initial condition $\mathbf{B}(t=0)=0$, it is determined that the result of the undetermined function is

$$a(r,\theta,z)=-\frac{1}{c}\frac{\lambda_m}{2\pi\varepsilon_0 r}e^{-i\omega\frac{z}{c}}\qquad (R_1\leqslant r\leqslant R_2)\quad (34)$$

Then substitute it into equation (33) to obtain

$$\mathbf{B}=\mathbf{e}_\theta\frac{\lambda_m}{2\pi c\varepsilon_0 r}\left[e^{i\omega\left(t-\frac{z}{c}\right)}-e^{-i\omega\frac{z}{c}}\right]\qquad (R_1\leqslant r\leqslant R_2)\quad (35)$$

Therefore, the specific form of (31) is

$$\begin{aligned}\mathbf{E}&=\mathbf{e}_r\frac{\lambda_m}{2\pi\varepsilon_0 r}e^{i\omega\left(t-\frac{z}{c}\right)} && (R_1\leqslant r\leqslant R_2)\\ \mathbf{B}&=\mathbf{e}_\theta\frac{\lambda_m}{2\pi c\varepsilon_0 r}\left[e^{i\omega\left(t-\frac{z}{c}\right)}-e^{-i\omega\frac{z}{c}}\right] && (R_1\leqslant r\leqslant R_2)\end{aligned}\quad (36)$$

It can be verified that the expression (36) of the above electromagnetic wave satisfies each equation of the component form (7) of Maxwell's equations, that is, it satisfies Maxwell's equations. (36) is a special solution of Maxwell's equations that meets the initial conditions. It expresses a cylindrical time harmonic electromagnetic wave mode that can exist in theory. Take the imaginary part of formula (36) to obtain the cylindrical electromagnetic wave mode excited by sinusoidal electric field wave[24-28]

$$\begin{aligned}\mathbf{E}&=\mathbf{e}_r\frac{\lambda_m}{2\pi\varepsilon_0 r}\sin\omega\left(t-\frac{z}{c}\right) && (R_1\leqslant r\leqslant R_2)\\ \mathbf{B}&=\mathbf{e}_\theta\frac{\lambda_m}{2\pi c\varepsilon_0 r}\left[\sin\omega\left(t-\frac{z}{c}\right)+\sin\frac{\omega z}{c}\right] && (R_1\leqslant r\leqslant R_2)\end{aligned}\quad (37)$$

Taking the real part of equation (36), the cylindrical electromagnetic wave mode excited by cosine electric field wave is obtained,

$$\begin{aligned}\mathbf{E}&=\mathbf{e}_r\frac{\lambda_m}{2\pi\varepsilon_0 r}\cos\omega\left(t-\frac{z}{c}\right) && (R_1\leqslant r\leqslant R_2)\\ \mathbf{B}&=\mathbf{e}_\theta\frac{\omega\lambda_m}{2\pi c\varepsilon_0 r}\left[\cos\omega\left(t-\frac{z}{c}\right)-\cos\frac{\omega z}{c}\right] && (R_1\leqslant r\leqslant R_2)\end{aligned}\quad (38)$$

4 Conclusions and comments

Since the classical theory first preconcert the forms of solutions before solving the Maxwell equations, the obtained transverse electromagnetic wave mode shows a most incredible characteristic that the electric field and magnetic field are in phase everywhere. Now that the periodic changing electric field generates periodically changing magnetic field, the process of the transmission of an electromagnetic wave is that of energy radiation, and the electromagnetic energy is transmitted with the electromagnetic waves. If the maximum or minimum value of the electric field and magnetic field in an electromagnetic wave are always obtained simultaneously, how do the electric energy and magnetic energy interchange? Consequently, it can be concluded that the classical electromagnetic wave in which the electric field and magnetic field are in phase everywhere violates the law of conservation of energy. In both mathematical and physical sense, the plane transverse electromagnetic wave determined by the classical electromagnetic theory is not tenable. Theoretically, the solution of the initial value problem of Maxwell equations is the regularity of plane or cylindrical transverse electromagnetic wave modes.

From Maxwell' theory, if the electric field in space changes, the magnetic field at the same place would change and these changing electric field and magnetic field will generate new changing electric field and magnetic field in farther space. Consequently, the changed electric field and magnetic field are not confined to a region but propagate from near site to farther places. The propagation of electromagnetic field forms electromagnetic waves. The characteristics of the electromagnetic wave are actually described by the solutions of the Maxwell equations. The electromagnetic wave is a transverse wave, which is just determined by the transverse wave conditions $\nabla\cdot\mathbf{E}=0$ and $\nabla\cdot\mathbf{B}=0$ of the Maxwell equations. We strive to maintain the logic and conclusion of classical electromagnetic theory, but this does not mean that Maxwell's electromagnetic wave theory is perfect. The important problems mentioned at the beginning of the article and the deeper problems hidden in Maxwell's electromagnetic field theory are easy to solve. We will discuss them later. At present, the most important problem that should attract our attention is the hidden problem of physics fame and wealth thought and degree system, which is the fundamental reason for the confusion of physics theory and a large number of errors.

Celebrities in theoretical physics often regard personal very random calculation as new mathematics, and are good at explaining the contradictory logic as physical mathematics different from pure mathematics. In fact, it is a distorted understanding of mathematical rules and natural laws due to the lack of due mathematical basis. By systematically testing the logic and basic mathematical calculation of theoretical physics with the unitary principle, it will be found that famous theoretical physicists often make mistakes in physical concepts, physical logic and mathematical calculation[29-35], especially in the calculation of elementary calculus and elementary algebra. The electromagnetic wave mode solution of Maxwell equations is only a simple example. Theoretical physics is mixed with a large number of similar low-level errors, which have not been found and recognized by mainstream physics scholars, and non mainstream scholars who find errors have been excluded, slandered and denied. Since mainstream theoretical physics scholars have made a large number of very serious logical and computational errors on very simple problems for a long time, and have been unaware of these errors for more than 100 years, we should not firmly refuse to test the logical and mathematical calculations of theoretical physics with too much confidence, let alone ignore or even slander the rigorous arguments of non mainstream physics scholars, Unless we lose the awe of scientific truth and abandon the dignity of human nature for personal fame and wealth.

The reason why theoretical physics is limited to this embarrassing situation is that the desire for fame and wealth drives researchers to excessively pursue celebrities and degree system, resulting in generations of students having to clone textbooks. Theoretical physics scholars who grew up in this environment have long lost their basic consciousness and ability to independently judge the right and wrong of physical logic. The determination of the transverse wave mode equation is an initial value problem of Maxwell equation, but it has been completely misunderstood. Most of the wrong mathematical operations in theoretical physics are unknown to later scholars, which just reflected the serious defects of the degree system in the past. Perhaps changing the academic monopoly system of fame and wealth and the degree system of cloning celebrities are the key to the breakthrough and major development of theoretical physics in the future.

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