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Neutron State Solution of Dongfang Modified Dirac Equation

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The challenging solution of the Dirac equation of the Coulomb field satisfying exact boundary conditions is further studied. If the Dirac equation is effective, then the intrinsic angle quantum number determined by the exact solution of the equation must be introduced to modify the Dirac equation to make it self-consistent. The solution of the modified Coulomb field Dirac equation satisfying the exact boundary conditions leads to a variety of breakthrough conclusions that overturns the traditional physical thinking. 1) The modified Dirac equation of Coulomb field has a neutron state solution corresponding to the neutron structure, and the binding energy of the neutron has a certain value, while the calculation result of the intrinsic radius written in the accurate boundary condition is equivalent to the size of the atomic nucleus; 2) The energy eigenvalue formula of the modified Coulomb field Dirac equation contains only radial quantum numbers and is independent of the intrinsic angular quantum numbers, where the zero radial quantum number energy level corresponds to the neutron state, and the nonzero radial quantum number energy level corresponds to the atomic state, and the accuracy of each atomic state energy level is equivalent to the Bohr energy level, while the Dirac energy level formula as the expected solution no longer exists; 3) The intrinsic angular quantum number of the modified Coulomb field Dirac equation indirectly negates the Dirac algebra theory that constructs the Dirac angular quantum number beyond the mathematical calculation rules; 4) The neutron state wave function component of the modified Coulomb Dirac equation is the terminated Yukawa potential function, which reflects the physics dilemma that the wave function is wrongly described as a potential function to establish a Yukawa pseudo-scientific theory that can also be infinitely developed and admired by physicists around the world, exposing the false prosperity of modern physics. It is concluded that the Dirac equation of the Coulomb field defined by Dirac algebra is not self-consistent, and the exact boundary condition solution of the modified self-consistent Dirac equation of the angular quantum number regression intrinsic physical quantity negates the Dirac electronic theory of fabricating the energy level formula of the fine structure of the hydrogen atom spectrum, and the microscopic quantum theory urgently needs to find a more reasonable wave equation that describes the fine spectral structure of the hydrogen atom.

Keywords: Unitary principle; Dirac equation; inevitable solution; quantum neutron radius.

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1 Introduction

The Loren transformation in the sense of relativity [1-3]is not a set of spatiotemporal parameter transformations with complete concepts. The spatiotemporal transformation of the complete concept in the sense of relativity shows that the constant speed of light is only a mathematical definition applicable to the isolated reference frame. However, the theory of relativity based on the assumption that the speed of light is constant seems to have achieved irreplaceable and surprising results in many aspects. In particular, a large number of experimental reports claim to confirm various inferences of relativity. This forces us to try to break through the traditional thinking mode of either or, and to test the logical self consistency of each major developmental theory of relativity and the authenticity of those influential experimental reports in a more in-depth and detailed way. Theoretically, there is no essential difference between relativity and Newton's theory. If relativity is a scientific theory, the conclusion of relativity of all problems should be the same as that of Newtonian mechanics when the relative speed is far less than the speed of light.

The relativistic Dirac equation[4, 5] is the product of the combination of special relativity and quantum mechanics with morbid equation of quantum numbers[6]. Then, how can a theory with serious defects be combined with another theory with serious defects to achieve a historical breakthrough in physics? In fact, the traditional

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solution of the evolution Dirac equation for describing the hydrogen atom leads to various morbid problems, such as the virtual energy and the divergence of S-state wave function[7]. The latter obviously leads to the counterfactual conclusion of the collapse of the universe, which are unacceptable. But the Dirac equation pieced together the most expected energy level formula of the fine structure of the hydrogen atom in that era. This is one of the elusive and strange phenomena of quantum mechanics. Constructing equations that do not conform to mathematical and physical logic may get the expected energy level formula. In this case, we always try to find the beneficial factors in the distorted logic so as to correct the distorted logic. Replacing the rough boundary condition without considering the size of the atomic nucleus with the exact boundary condition with the size of the atomic nucleus, the challenge solution obtained by re solving the Dirac equation is self consistent, but the expected energy level formula can not be obtained. At the same time, it also brings more mathematical and physical problems to be solved. Perhaps only physicists who study the elementary proof of Fermat's final theorem can find the endless plausible and complex problems concealed in quantum mechanics. Not every physics paper or theory can reasonably describe the laws of nature. Only those physics papers or theories that can inspire people to break through the confinement of traditional incorrect ideas are worthy of in-depth study. Based on this idea, we further study the challenging solution of the Dirac equation of hydrogen like atom satisfying exact boundary conditions. The challenge solution may not be popular because it fails to piece together the expected formula of fine energy level structure of the hydrogen atom, but it can break through the confinement of traditional incorrect physical ideas better than almost all the conclusions about the Dirac equation in history.

There is another strange phenomenon in quantum mechanics. Some wave equations established from different angles are not exactly the same, although they have energy level formulas with the same accuracy but different expressions. This is not in line with the Dongfang unitary principle that natural science theories must follow. Whether testing the logical self consistency of the old theory or establishing a new self consistency theory, the unitary principle is very useful. The specific contents of this universal principle are: *There is a definite transformation relationship between different metrics describing the natural law, and the natural law itself does not change due to the selection of different metrics. When the mathematical expression of natural laws under different metrics is transformed into one metric, the result must be the same as the inherent form under this metric, 1=1, meaning the transformation is unitary*[3,8,9]. Its concise expression is as follows: *The result of the transformation of the mathematical form of the natural law in different metrics to the same metric is unique.* The unitary principle is the most effective principle to test the self consistency of theoretical and experimental reports. Conforming to the unitary principle is a necessary condition for the self consistency of theoretical logic. Although the theory that conforms to the unitary principle may not be the true description of natural laws or social laws, the theory that violates the unitary principle must be incorrect. If a theory used to describe the laws of nature does not conform to the unitary principle, it must contain principled mathematical calculation errors or philosophical errors. Nuclear meson theory[10], relativity[11-13] based on the pseudo proposition of constant speed of light, quantum mechanics[14] concealing morbid equations of quantum umbers, the thermodynamic theory with completely wrong basic equations[15] and Maxwell electromagnetic theory with wrong equation solution[16-18] do not accord with the unitary principle, so they must be treated[19,20]. Hendrik Schon experiment[21,22] and LIGO's spiral binary gravitational wave[23] cannot pass the test of unitary principle[24-26]. Now, there is a unified scientific method for exposing the truth that famous scientists or scientific groups fabricate experimental data or confuse different experimental data and publish the so-called major experimental reports. Just looking at the challenging problems such as the morbid equation of quantum numbers hidden in the Schrödinger equation, which is considered to be perfect, we know that quantum mechanics has not been really understood. The Bonn statistical interpretation of the wave function of quantum mechanics, which has always been mysterious, is abstract and vague. If the probability of electron particles appearing in the specified region in the Coulomb field is written independently, it will be found that the so-called probability density function does not satisfy the Schrödinger equation or the Dirac equation. God can't roll the dice[27,28]. That's right. Therefore, the concept of orbital density is proposed here, but these may not be the ultimate answer. From this corner, theoretical physics is still in its infancy, far from entering the view where its heyday can be predicted.

In fact, even assuming that the theory of relativity and quantum mechanics are correct, studying whether the construction of the Dirac equation conforms to the unitary principle will lead to many important problems that have not been discovered in the past[29,30]. So we return to the mathematics of the Dirac equation and gradually discuss the logic and inference of the Dirac equation that is rarely known. This paper introduces the advanced research results of the challenging solution of the Dirac equation for the hydrogen-like atom satisfying the exact boundary conditions. The intrinsic angular quantum number determined by the equation is introduced to replace Dirac's definition of angular quantum number to modify the Dirac equation of Coulomb field, and the precise boundary condition containing the size of atomic nucleus is used to replace the rough boundary condition without the size of atomic nucleus,

and then the modified Dirac hydrogen equation is solved accurately. The results show that the modified locally self-consistent Dirac equation of the hydrogen-like atom has the lowest energy state solution corresponding to the neutron state. However, the accuracy of other energy levels corresponding to the energy level formula associated with the exact solution of the modified Dirac equation is equivalent to that of the Bohr energy level formula. This is not the expected result of theoretical physics workers. To distort the logic of mathematics in order to obtain the desired results will not have lasting vitality. Any new subversive conclusion of the challenging solution of the Dirac equation for the hydrogen atom needs to be reprocessed. The neutron state solution of the modified Dirac equation of the Coulomb field is self-consistent; The nuclear radius used to write accurate boundary conditions belongs to the intrinsic parameter of the modified Dirac equation, which is equivalent to the radius of neutrons, so the binding energy of neutrons seems to have an exact calculation result; The angular quantum number is the intrinsic parameter of the modified Dirac equation, which is determined by the exact solution of the equation, which means that it is illogical to construct the angular quantum number through the user-defined Dirac algebra. So, is it possible that the neutron state of the modified Dirac equation that meets the exact boundary conditions is just another formal solution that seemingly conforms to the mathematical and physical logic? Is it another coincidence that the neutron state solution which seems to describe the neutron structure well[31, 32]?

2 Dongfang modified Dirac equation for hydrogen-like atoms

In Dirac electron theory, dealing the relativistic Dirac equation for hydrogen like atoms is finally reduced to solving the first-order radial differential equations about the wave functions of the upper and lower components

$$\left[c\alpha\cdot\hat{\mathbf{p}}+\begin{pmatrix}1&0\\0&-1\end{pmatrix}mc^2-\frac{Ze^2}{4\pi\varepsilon_0 r}\right]\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}=E\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} \tag{1}$$

where α is the Dirac matrix, $\hat{\mathbf{p}}=-i\hbar\nabla$, $\hbar=h/2\pi$ with the plank constant h, c the velocity of light in a vacuum, and $m$the rest mass of electron. $\alpha\cdot\tilde{\mathbf{p}}$ is defined by the Dirac algebra

$$\alpha\cdot\tilde{\mathbf{p}}=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\left[-i\hbar\frac{\partial}{\partial r}-\frac{i\hbar}{r}+\frac{i\hbar}{r}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\widehat{\kappa}\right] \tag{2}$$

Where the angular quantum number $\kappa=\pm1,\ \pm2,\ \pm3,\ \cdots$, which is constructed by Dirac algebra independent of mathematics, actually belongs to artificial angular quantum number The main reason is that Dirac algebra does not conform to the basic rules of mathematical operation. This part will be discussed separately as a special topic. Function transformation is usually introduced for two-component wave function

$$\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}=\frac{1}{r}\begin{pmatrix}F\\G\end{pmatrix} \tag{3}$$

In Dirac theory, the boundary conditions satisfied by the wave function

$$\left.\begin{aligned}&\lim_{r\to0}\frac{1}{r}\begin{pmatrix}F\\G\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix},\quad\lim_{r\to\infty}\frac{1}{r}\begin{pmatrix}F\\G\end{pmatrix}=0\\&\frac{1}{r}\begin{pmatrix}F\,(0<r<\infty)\\G\,(0<r<\infty)\end{pmatrix}\neq\begin{pmatrix}\pm\infty\\\pm\infty\end{pmatrix}\end{aligned}\right] \tag{4}$$

has been interpreted as

$$\begin{aligned}&\lim_{r\to0}\begin{pmatrix}F\\G\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix},\quad\lim_{r\to\infty}\begin{pmatrix}F\\G\end{pmatrix}=0\\&\begin{pmatrix}F\,(0<r<\infty)\\G\,(0<r<\infty)\end{pmatrix}\neq\begin{pmatrix}\pm\infty\\\pm\infty\end{pmatrix}\end{aligned}$$

This interpretation makes it difficult to detect the logical contradiction of the divergence of S-state Dirac wave function at the coordinate origin[7], and the divergence of wave function means the abnormal inference of the collapse of the universe, which is one of the fatal problems of Dirac hydrogen atom theory.

Now let us focus on the angular quantum number in the Dirac equation. One of the characteristics of quantum mechanics is that the angular quantum number is the eigensolution of the eigenequation of the angular momentum operator acting on the angular wave function, which exists in the wave equation. For example, solving the Schrodinger equation or the Klein Gordon equation naturally obtains the eigenvalue of angular momentum. However, Dirac theory does not write the angular wave equation corresponding to the angular momentum operator, but constructs the angular quantum number $\kappa=\pm1,\ \pm2,\ \pm3,\ \cdots$ through the self-defined Dirac algebra independent of mathematics. The so-called Dirac algebra does not conform to the basic mathematical operation rules. It must be reiterated that the important symbol of the success of quantum mechanics is that the quantized angular momentum and quantized energy of the bound state system are naturally derived from the exact solution of the wave equation satisfying the boundary conditions. It is futile and a waste to comment too much on Dirac algebra now. According to the unitary principle, we can inversely test whether the conclusion $\kappa=\pm1,\ \pm2,\ \pm3,\ \cdots$ of Dirac algebra is an inevitable corollary.

If the Dirac equation holds, first assume that the angular quantum number is an intrinsic parameter $\mathbb{C}$ of the Dirac equation, solve the Dirac equation and keep the calculation consistent from beginning to end, then the inevitable result of $\mathbb{C}$ can be obtained. This means that $\mathbb{C}$ should be obtained naturally in the process of solving

the Dirac equation. Whether the calculation result of $\mathbb{C}$ meets the expectation of Dirac algebra or not will make the problem suddenly clear. Equation (2) is reduced to

$$\alpha\cdot\hat{\mathbf{p}}=\begin{pmatrix}0&-i\\ i&0\end{pmatrix}\left[-i\hbar\frac{\partial}{\partial r}-\frac{i\hbar}{r}+\frac{i\hbar\mathbb{C}}{r}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right] \tag{5}$$

The potential energy of hydrogen like atom can be written as follows

$$V(r)=-\frac{Ze^2}{4\pi\varepsilon_0 r}=-\frac{\mathrm{Z}\alpha\hbar c}{r} \tag{6}$$

Substitute equation (5) into equation (1) to obtain the angular quantum number $\mathbb{C}$ as the intrinsic parameter equation, which is called Dongfang modified Dirac equation

$$\left\{c\begin{pmatrix}0&-i\\ i&0\end{pmatrix}\left[-i\hbar\frac{\partial}{\partial r}-\frac{i\hbar}{r}+\frac{i\hbar\mathbb{C}}{r}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right]+\begin{pmatrix}1&0\\0&-1\end{pmatrix}mc^2-\frac{Z\alpha\hbar c}{r}\right\}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=E\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \tag{7}$$

Although it is a historic breakthrough to revise the angular quantum number defined by Dirac to an intrinsic parameter, this revision contains an unclear causal relationship. The name of the equation (7) is only for distinguishing and attracting attention. It does not mean the end of the problem. Whether and how to construct a more reasonable equation to replace the equation is the key. The Dongfang modified Dirac equation (7) is locally self-consistent after introducing the intrinsic angular quantum number $\mathbb{C}$ determined by the exact solution of the equation. Expressed by the result of a matrix product, equation (7) is transformed into a first-order radial equation system of hydrogen-like atoms composed of two first-order differential equations

$$\begin{aligned}&\frac{d\psi_2}{dr}+\frac{\mathbb{C}+1}{r}\psi_2-\left(\frac{mc^2-E}{\hbar c}-\frac{Z\alpha}{r}\right)\psi_1=0\\ &\frac{d\psi_1}{dr}-\frac{\mathbb{C}-1}{r}\psi_1-\left(\frac{mc^2+E}{\hbar c}+\frac{Z\alpha}{r}\right)\psi_2=0\end{aligned} \tag{8}$$

From equation (2) to equation (8), readers are advised to take up the pen and deduce it again, and then deduce Dirac theory again, instead of just remembering and interpreting the traditional theory. Over the past 100 years, generations of theoretical physicists have only continued their high-intensity memory, but failed to find the mathematical and physical logic difficulties hidden in Dirac theory, which is more confusing than those enough puzzles left by the Dirac equation. Even for simple mathematics, the understanding of relevant mathematical logic in the process of personal derivation is often completely different from the unforgettable memory, not to mention finding and solving many less obvious mathematical and physical problems hidden in Dirac theory.

The Dirac electron theory has always adopted the two-component wave function transformation of form (3). In this way, the original two-component wave function should be returned after the accurate solution is obtained, so as to it is found that the solution (also called the Dirac wave function) of the Dirac equation satisfying rough boundary conditions is divergence at the coordinate origin, that is, the conclusion is inconsistent with the natural boundary conditions[7]. The transformation formula (3) of the upper and lower component wave functions is substituted into equation (8) to obtain the the induction equation of the modified first-order radial Dirac equation for hydrogen-like atoms

$$\begin{aligned}&\frac{dG}{dr}+\frac{\mathbb{C}}{r}G+\frac{Z\alpha}{r}F-\frac{mc^2-E}{\hbar c}F=0\\ &\frac{dF}{dr}-\frac{\mathbb{C}}{r}F-\frac{Z\alpha}{r}G-\frac{mc^2+E}{\hbar c}G=0\end{aligned} \tag{9}$$

Thus, the treatment of the modified Dirac equation for hydrogen-like atoms is left to a purely mathematical problem, and inference can always be given various physical meanings.

3 Statistical interpretation of orbital wave function and exact boundary conditions

What kind of physical quantity is the wave functions? Quantum mechanics has no definite answer. This has created the brilliance of Born's statistical interpretation, which holds that the wave function corresponds to the probability of electrons appearing in the space around the nucleus[33,34]. The Born statistical interpretation of wave function is generally accepted, forcing the concept of orbit of electron motion in the atom to be quietly denied, which is another misunderstanding hidden in quantum mechanics.

The Born statistical interpretation of the wave function means that electrons appear and disappear around the nucleus like ghosts, but the probability of electrons appearing at any position can be determined, which is calculated from the wave function. This does not conform to the unitary principle. The counter example in the experiment is that the trajectory of the electron beam in the cathode ray tube can be displayed by the fluorescence effect. According to the unitary principle, if the statistical interpretation of the wave function is the only correct, the trajectory of the electron beam can be determined by the wave function of the electron beam in the cathode ray tube. Actually not! The physical theory must satisfy the unitary principle. Otherwise it must be changed. Of course, we can also study the motion of electrons from other angles to further investigate the real physical meaning of wave function. For example, the probability of the electron appearing around the atomic nucleus can be directly calculated by the motion

law of the electron governed by the Coulomb force. So is its functional form or some agreed deformation form the solution of the wave equation in quantum mechanics, which will prove that the Born statistical interpretation of the wave function is only correct?

It is noted that the boundary condition (4), which is the definite solution condition of the quantum mechanical wave equation, obviously does not take into account the size of the nucleus, which is equivalent to the tacit consent that electrons can appear in the central neighborhood of the nucleus. For the hydrogen atom, it means that the electron can almost coincide with the proton, which has no scientific basis and does not accord with the fact. The boundary condition (4) thus written is a rough boundary condition. The modified boundary condition should be able to solve the problem of S-state Dirac wave function divergence[35].

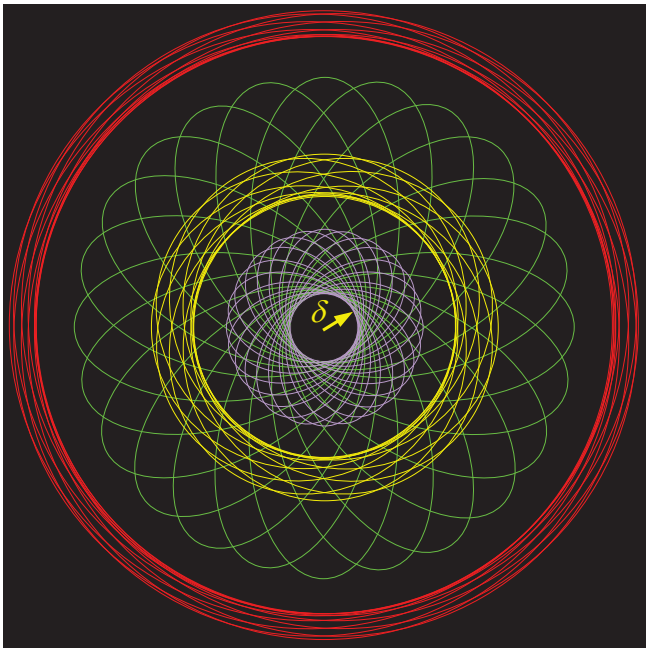

Figure 1 Schematic drawing for the electron's probability density and motion orbits. Each group of ellipse curves of different colours that express the orbit of the electron corresponds to the certain special energy states. The density of curves with the speed weight of moving electron is proportional to the magnitude of the probability density of the electron.

Even if the Born statistical interpretation of the wave function is considered to be the only correct one, it does not contradict the concept of the classical orbit of particles. Considering an artificial satellite turning around the earth, for example, its probability density can be translated as the density of the orbit number (the orbit number per unit volume). In fact, classical mechanics theory and quantum mechanics theory are just two metrologies for describing the order of nature. The concept of the classical orbit will not suddenly disappear because of quantum mechanics theory. According to the unitary principle, using the probability amplitudes for the position of the particle to determine the orbit of the particle is necessarily equivalent to using the density of the orbit number to determine the probability of finding the particle. FIG.1 shows symbolically the responding relation between the wave function's probability distribution and the motion orbit of an electron outside the nucleus of an atom. For the same energy level, an electron may have the different circle or the ellipse orbits. Its orbit plane is varying continuously because of the electromagnetic disturbance. The farther the electron is away from the nuclear, the smaller the probability of the electron crossing a given surface appearing to be. The electrons absorb and radiate the energy to produce the orbit transition. On the surface of the atomic nucleus, the probability of electron appearance must be a nonzero finite value and cannot go to infinity. From this macroscopic picture of statistical interpretation on the wave function, it is not difficult to find that the quantum mechanics should not give the meaning of God play dice.

The initial-boundary value conditions play an important role for determining the logical solution of a wave equation from its general solutions. In order to overcome the divergence of relativistic wave functions at the origin[35], we should consider the quantum radius of the atomic nucleus. As one of the reliable treatments, we assume that equation (5d) holds only for $r \geqslant \delta$. Inside δ, the potential has to be modified from a Coulomb $Ze^2/4\pi\varepsilon_0\delta$ potential to one corresponding to a spread-out charge distribution. Therefore, to do a completely correct calculation, one solves the Dirac equation separately outside of and inside of δ, with two different potentials, and then matches, at $r = \delta$, the outside solution (i.e., the original Dirac-Coulomb one) to the inside solution. The inside solution is the finite constant; its effect is to modify the energy-level formula by a small correction that takes into account the finite radius of the nuclear. This idea just supports introducing the exact boundary condition to the wave equations for hydrogen-like atom.

$$\lim_{r\to\delta}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \neq \begin{pmatrix}\pm\infty\\ \pm\infty\end{pmatrix}, \quad \lim_{r\to\infty}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = 0 \tag{10}$$

where $\delta > 0$ is the quantum radius of the atomic nucleus.

Around 1996, I replaced the rough boundary without considering the size of the atomic nucleus with the exact boundary condition written in the size of the atomic nucleus, and re solved the Schrödinger equation of the hydrogen atom, but the energy level formula as the eigenvalue of the equation did not change. It can be inferred that the energy level formula corresponding to the solution of the Klein Gordon equation satisfying exact boundary conditions is also invariant, including the solution of the real number wave equation[9] which gives the meaning of a steady state again. To solve Schrödinger equation and the Klein Gordon equation with exact boundary conditions, it is necessary to generalize the

relevant theorems of optimization differential equations. However, in the past many years, efforts to publish relevant papers in mainstream journals have failed.

The challenge solution of the hydrogen atom Dirac equation meeting the exact boundary condition[7] eliminates the divergence and virtual energy of the hydrogen atom Dirac wave function, but the corresponding energy level formula is ignored by physicists because it does not meet expectations. The age of interest causes people who study scientific theories to care less about the correctness of theories. Theoretical physics has always been indulged in the impetuous atmosphere of fabricating expected results with the wrong calculations and has developed to the extreme stage of fabricating observation data to conjecture imaginary results and then making wild speculation to achieve unprecedented success. Scientific journals generally refuse to disclose famous scientific theories with false calculations and serious logical contradictions, and famous scientific experiment reports with untrue data. The responsibility of scientific journals is more committed to maintaining the reputation of famous scientists and constantly developing their theories. The main reason is that scientific journals are widely controlled by the makers and their successors of false theories and untrue experimental reports. For relativistic quantum mechanics, abandoning the precise boundary conditions and desalinating the fatal logic problems hidden in the traditional solution of Dirac equation to maintain Dirac theory will not only keep the quantum theory stuck in the wrong ideological framework forever, but also seriously hinder the revelation of the mathematical essence of quantum mechanics and thus hinder the unification of macroscopic and microscopic quantum theories. However, in any era, there is always a need for someone to adhere to the truth.

4 The neutron state solution of the modified Dirac equation

Under the same solution conditions, is it possible that the exact solution of a differential equation or system of differential equations may be different due to different methods of solving the equations, thus violating the unitary principle? If the answer is yes, the solution of the induced equation of the modified Dirac equation (9) satisfying the exact boundary condition (10) is a local self consistent solution. The local self consistent definition leads to new problems, such as whether the global self consistent solution of the Dirac equation exists or even what form the global self consistent Dirac equation is, which are left to the reader to study.

Now let's look for the exact solution of the induced equation of the modified Dirac equation (9) that satisfies the exact boundary condition (10). For the convenience of writing and calculation, a new variable $\xi = r - \delta$ is introduced, where $\xi \geqslant 0$, and the exact boundary condition (10) becomes

$$\lim_{\xi\to 0}\begin{pmatrix}F\\G\end{pmatrix} \neq \begin{pmatrix}\pm\infty\\\pm\infty\end{pmatrix}, \quad \lim_{\xi\to\infty}\begin{pmatrix}F\\G\end{pmatrix} = 0 \tag{11}$$

Equation group (9) is equivalent to the following differential equations

$$\begin{aligned}&\frac{dG}{d\xi} + \frac{\mathbb{C}}{\xi+\delta}G + \frac{\mathrm{A}}{\xi+\delta}F - \frac{mc^2-E}{\hbar c}F = 0\\&\frac{dF}{d\xi} - \frac{\mathbb{C}}{\xi+\delta}F - \frac{\mathrm{A}}{\xi+\delta}G - \frac{mc^2+E}{\hbar c}G = 0\end{aligned} \tag{12}$$

For the convenience of later calculation, we write the differential equation in the following form

$$\begin{aligned}&(\xi+\delta)\frac{dG}{d\xi} + \mathbb{C}G + \mathrm{A}F - \frac{mc^2-E}{\hbar c}(\xi+\delta)F = 0\\&(\xi+\delta)\frac{dF}{d\xi} - \mathbb{C}F - \mathrm{A}G - \frac{mc^2+E}{\hbar c}(\xi+\delta)G = 0\end{aligned} \tag{13}$$

The undetermined quantum number $\mathbb{C}$ and the undetermined nuclear equivalent radius δ in the equation are intrinsic parameters. Let the analytical solution of equation (13) be

$$\begin{aligned}F &= e^{-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\xi}\sum_{\nu=0}^{\infty} b_\nu \xi^{\sigma+\nu}\\G &= e^{-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\xi}\sum_{\nu=0}^{\infty} d_\nu \xi^{\sigma+\nu}\end{aligned} \tag{14}$$

Their first derivatives are

$$\begin{aligned}\frac{dF}{d\xi} &= e^{-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\xi}\sum_{\nu=0}^{\infty}\left((\sigma+\nu+1)\,b_{\nu+1} - \frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_\nu\right)\xi^{\sigma+\nu}\\\frac{dG}{d\xi} &= e^{-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\xi}\sum_{\nu=0}^{\infty}\left((\sigma+\nu+1)\,d_{\nu+1} - \frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_\nu\right)\xi^{\sigma+\nu}\end{aligned} \tag{15}$$

Substituting (14) and (15) into equation (13) gives a recursive relationship group

$$\begin{aligned}&(\sigma+\nu+2)\,\delta d_{\nu+2}+\left(\sigma+\nu+1-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\delta+\mathbb{C}\right)d_{\nu+1}+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_{\nu+1}-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_\nu-\frac{mc^2-E}{\hbar c}b_\nu=0\\&(\sigma+\nu+2)\,\delta b_{\nu+2}-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_{\nu+1}+\left(\sigma+\nu+1-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\delta-\mathbb{C}\right)b_{\nu+1}-\frac{mc^2+E}{\hbar c}d_\nu-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_\nu=0\end{aligned}\tag{16}$$

From $\delta\neq 0$, $b_0\neq 0$, $d_0\neq 0$, $b_{-1}=b_{-2}=\cdots=0$, $d_{-1}=d_{-2}=\cdots=0$, let $\nu=-2$, $\sigma=0$ is obtained. The above recursive relationship is simplified to

$$\begin{aligned}&(\nu+2)\,\delta d_{\nu+2}+\left(\nu+1-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\delta+\mathbb{C}\right)d_{\nu+1}+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_{\nu+1}-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_\nu-\frac{mc^2-E}{\hbar c}b_\nu=0\\&(\nu+2)\,\delta b_{\nu+2}-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_{\nu+1}+\left(\nu+1-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\delta-\mathbb{C}\right)b_{\nu+1}-\frac{mc^2+E}{\hbar c}d_\nu-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_\nu=0\end{aligned}\tag{17}$$

The boundary condition (11) requires the series interruption in the radial wave function (14) to be polynomial. Let the highest power of the polynomial be n, so the general solution of Dirac radial equations (8) for hydrogen like atoms is

$$R=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\begin{pmatrix}\frac{1}{r}F\\ \frac{1}{r}G\end{pmatrix}=\begin{pmatrix}e^{-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}(r-\delta)}\frac{1}{r}\sum_{\nu=0}^{n}b_\nu(r-\delta)^\nu\\ e^{-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}(r-\delta)}\frac{1}{r}\sum_{\nu=0}^{n}d_\nu(r-\delta)^\nu\end{pmatrix},\quad(n=0,\ 1,\ 2,\ \cdots)\tag{18}$$

The expansion form of recursive relation group (16) is as follows

$$\begin{aligned}&\delta d_1+\left(-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}+\mathbb{C}\right)d_0+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_0=0\\&\delta b_1-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_0+\left(-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}-\mathbb{C}\right)b_0=0\\&2\delta d_2+\left(1-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}+\mathbb{C}\right)d_1+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_1-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_0-\frac{mc^2-E}{\hbar c}b_0=0\\&2\delta b_2-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_1+\left(1-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}-\mathbb{C}\right)b_1-\frac{mc^2+E}{\hbar c}d_0-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_0=0\\&\vdots\\&(\nu+2)\,\delta d_{\nu+2}+\left(\nu+1-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}+\mathbb{C}\right)d_{\nu+1}+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_{\nu+1}-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_\nu-\frac{mc^2-E}{\hbar c}b_\nu=0\\&(\nu+2)\,\delta b_{\nu+2}-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_{\nu+1}+\left(\nu+1-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}-\mathbb{C}\right)b_{\nu+1}-\frac{mc^2+E}{\hbar c}d_\nu-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_\nu=0\\&\vdots\\&n\delta d_n+\left(n-1-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}+\mathbb{C}\right)d_{n-1}+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_{n-1}-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_{n-2}-\frac{mc^2-E}{\hbar c}b_{n-2}=0\\&n\delta b_n-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_{n-1}+\left(n-1-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}-\mathbb{C}\right)b_{n-1}-\frac{mc^2+E}{\hbar c}d_{n-2}-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_{n-2}=0\\&\left(n-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}+\mathbb{C}\right)d_n+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)b_n-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_{n-1}-\frac{mc^2-E}{\hbar c}b_{n-1}=0\\&-\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)d_n+\left(n-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}-\mathbb{C}\right)b_n-\frac{mc^2+E}{\hbar c}d_{n-1}-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_{n-1}=0\\&-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_n-\frac{mc^2-E}{\hbar c}b_n=0\\&-\frac{mc^2+E}{\hbar c}d_n-\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_n=0\end{aligned}\tag{19}$$

Multiply the penultimate formula by $\left(mc^2+E\right)/\hbar c$ and the penultimate formula by $-\sqrt{m^2c^4-E^2}/\hbar c$ to get

$$\left(n-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}+\mathbb{C}\right)\frac{mc^2+E}{\hbar c}d_n+\left(\mathrm{A}-\frac{mc^2-E}{\hbar c}\delta\right)\frac{mc^2+E}{\hbar c}b_n-\frac{\left(mc^2+E\right)\sqrt{m^2c^4-E^2}}{\hbar^2c^2}d_{n-1}-\frac{m^2c^4-E^2}{\hbar^2c^2}b_{n-1}=0$$
$$\left(\mathrm{A}+\frac{mc^2+E}{\hbar c}\delta\right)\frac{\sqrt{m^2c^4-E^2}}{\hbar c}d_n-\left(n-\delta\frac{\sqrt{m^2c^4-E^2}}{\hbar c}-\mathbb{C}\right)\frac{\sqrt{m^2c^4-E^2}}{\hbar c}b_n+\frac{\left(mc^2+E\right)\sqrt{m^2c^4-E^2}}{\hbar^2c^2}d_{n-1}+\frac{m^2c^4-E^2}{\hbar^2c^2}b_{n-1}=0$$

The left and right sides of the two recursive relations are added correspondingly, and finally a simplified equation is obtained

$$\left[(n+\mathbb{C})\frac{mc^2+E}{\hbar c}+\mathrm{A}\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\right]d_n+\left[\mathrm{A}\frac{mc^2+E}{\hbar c}-(n-\mathbb{C})\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\right]b_n=0 \tag{20}$$

The condition that the homogeneous system of equations composed of the above equation and the last equation of the recursive relationship group (19) has a nontrivial solution is

$$\begin{vmatrix}(n+\mathbb{C})\frac{mc^2+E}{\hbar c}+\mathrm{A}\frac{\sqrt{m^2c^4-E^2}}{\hbar c} & \mathrm{A}\frac{mc^2+E}{\hbar c}-(n-\mathbb{C})\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\\ -\frac{mc^2+E}{\hbar c} & -\frac{\sqrt{m^2c^4-E^2}}{\hbar c}\end{vmatrix}=0$$

The result is $\sqrt{m^2c^4-E^2}-\mathrm{A}E/n=0$. Because $\mathrm{A}>0$, $n>0$, this equation shows that $E>0$. The energy eigenvalues containing only radial quantum numbers are obtained

$$E=\frac{mc^2}{\sqrt{1+\frac{\mathrm{A}^2}{n^2}}}=\frac{mc^2}{\sqrt{1+\frac{Z^2\alpha^2}{n^2}}}\quad(n=0,\ 1,\ 2,\ 3,\ \cdots) \tag{21}$$

Excluding the so-called static energy, this energy level formula is equivalent to the energy eigenvalue of the Schrodinger equation, that is, the Bohr energy level formula, which is independent of the angular quantum number $\mathbb{C}$, and there is no fine structure prediction. However, the solution of the Dirac equation of hydrogen atom satisfying the exact boundary is locally self consistent, while the so-called spectral fine structure prediction of the traditional solution is inseparable from the collapse of the atom, which leads to the collapse of the universe, hiding fatal mathematical difficulties and disastrous physical conclusions. The Dirac theory of the so-called energy level formula of hydrogen atom fine structure takes into account one thing and loses the other, and violates the unitary principle. Any theory should at least ensure logical self consistency, and should not aim at patching up the expected conclusions. When the radial quantum number $n=0$, the self-consistent energy level formula (21) of the induced equation (9) of the modified Dirac equation becomes

$$E_0=0 \tag{22}$$

This is a unique conclusion that the traditional solution of the Dirac equation for the hydrogen atom has never appeared. It belongs to the unique solution of the modified Dirac equation (7), that is, (8). When $\mathrm{Z}=1$, the lowest energy state corresponding to $E_0=0$ is the neutron state, and the values of the minimum self-consistent angle quantum number $\mathbb{C}_0$ and the radius δ of the nucleus are determinable.

5 Radius and angular quantum number of the neutron

The Dirac equation is the product of the combination of energy dynamic relationship of special relativity and quantum mechanics. Although it can be proved theoretically that relativity is not a correct theory, if modern physics with many mathematical errors[7] is regarded as an empirical theory, it is understandable that its logic has problems and the conclusion is consistent with the observed results. It is inferred that the Dirac equation may be locally effective, which means that there are at least some unknown factors in line with natural phenomena. We need to calculate the radius and self consistent angular quantum number C corresponding to the atomic nucleus, so as to obtain the local self consistent complete solution of the Dirac equation satisfying the exact boundary conditions. Readers can also directly find the neutron state solution of Dirac equation (13), take

$$\begin{aligned}F&=b_0e^{-\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}\xi}\\ G&=d_0e^{-\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}\xi}\end{aligned} \tag{23}$$

Substitute into equation (9) to obtain the neutron state recursive relation group for b_0, d_0, δ and $\mathbb{C}_0$, which can also be obtained by making making $d_0\neq 0$ and

$d_1 = d_2 = \cdots = 0$ in the recursive relation group (19).

$$\begin{aligned}
&\left(-\delta\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}+\mathbb{C}_0\right)d_0+\left(\mathrm{A}-\frac{mc^2-E_0}{\hbar c}\delta\right)b_0=0\\
&\left(\mathrm{A}+\frac{mc^2+E_0}{\hbar c}\delta\right)d_0+\left(\delta\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}+\mathbb{C}_0\right)b_0=0\\
&-\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}d_0-\frac{mc^2-E_0}{\hbar c}b_0=0\\
&-\frac{mc^2+E_0}{\hbar c}d_0-\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}b_0=0
\end{aligned} \tag{24}$$

This homogeneous system of equations has a unique solution, but its solution method is unusual.

The latter two equations of equation set (24) are linearly related, and only the third equation is taken in the following calculation. The necessary and sufficient condition for the homogeneous equations composed of the first two equations to have nontrivial solutions is

$$\begin{vmatrix} -\delta\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}+\mathbb{C}_0 & \mathrm{A}-\frac{mc^2-E_0}{\hbar c}\delta \\ -\left(\mathrm{A}+\delta\frac{mc^2+E_0}{\hbar c}\right) & -\delta\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}-\mathbb{C}_0 \end{vmatrix}=0$$

The formula for calculating δ is thereby obtained

$$\delta=\frac{\hbar c\left(\mathbb{C}_0^2-\mathrm{A}^2\right)}{2\mathrm{A}E_0} \tag{25}$$

The necessary and sufficient condition for a homogeneous system of equations composed of the first equation and the third equation to have nontrivial solutions is

$$\begin{vmatrix} -\delta\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}+\mathbb{C}_0 & \mathrm{A}-\delta\frac{mc^2-E_0}{\hbar c} \\ -\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c} & -\frac{mc^2-E_0}{\hbar c} \end{vmatrix}=0$$

The formula for calculating $\mathbb{C}_0$ or E_0 is thereby obtained

$$\begin{aligned}
\mathbb{C}_0&=\mathrm{A}\sqrt{\frac{mc^2+E_0}{mc^2-E_0}}>0\\
E_0&=\frac{\left(\mathbb{C}_0^2-\mathrm{A}^2\right)mc^2}{\mathbb{C}_0^2+\mathrm{A}^2}
\end{aligned} \tag{26}$$

The necessary and sufficient condition for the system of homogeneous equations composed of the second equation and the third equation to have nontrivial solutions is

$$\begin{vmatrix} -\left(\mathrm{A}+\delta\frac{mc^2+E_0}{\hbar c}\right) & \left(-\delta\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c}-\mathbb{C}_0\right) \\ -\frac{\sqrt{m^2c^4-E_0^2}}{\hbar c} & -\frac{mc^2-E_0}{\hbar c} \end{vmatrix}=0$$

Thus, another calculation formula of $\mathbb{C}_0$ or E_0 is obtained

$$\begin{aligned}
\mathbb{C}_0&=\mathrm{A}\sqrt{\frac{mc^2-E_0}{mc^2+E_0}}>0\\
E_0&=\frac{\left(\mathrm{A}^2-\mathbb{C}_0^2\right)mc^2}{\mathbb{C}_0^2+\mathrm{A}^2}
\end{aligned} \tag{27}$$

The only solution satisfying equations (26) and (27) simultaneously is

$$\mathbb{C}_0=\mathrm{A}=Z\alpha,\quad E_0=0 \tag{28}$$

The energy eigenvalue is the same as the above inference (22). According to the above results, equation (25) for calculating P has the significance of limit calculation. Substituting E in (26) and (27) into (25) respectively, two results of nuclear radius are obtained

$$\delta=\pm\mathrm{A}\frac{\hbar c}{mc^2} \tag{29}$$

But the radius of the nucleus can't be negative. Discard the negative root and keep the positive root, so

$$\delta=\mathrm{A}\frac{\hbar}{mc}=\frac{Ze^2}{4\pi\varepsilon_0 mc^2} \tag{30}$$

Finally, by substituting the above results and $\xi=r-\delta$ into equation (23), the zero energy state wave function is obtained

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\begin{pmatrix}\frac{b_0}{r}e^{-\frac{mc}{\hbar}(r-\delta)}\\ -\frac{b_0}{r}e^{-\frac{mc}{\hbar}(r-\delta)}\end{pmatrix}=\begin{pmatrix}\frac{B_0}{r}e^{-\frac{mc}{\hbar}r}\\ -\frac{B_0}{r}e^{-\frac{mc}{\hbar}r}\end{pmatrix} \tag{31}$$

Where B_0 is the constant to be normalized. How should we explain that the two-component wave functions of the neutron state of the Dongfang modified Dirac equation of the hydrogen-like atom are opposite to each other? Now we find that the so-called Yukawa potential function is the zero energy state wave function hidden in the modified Dirac hydrogen equation, which surprisingly derives the nuclear meson theory[36-39]. How to evaluate the relationship between the two or how to re understand Yukawa potential function[10, 40], requires a bit of real mathematics and physics. Readers may wish to prove by themselves that Yukawa potential function is the product of completely distorting the principles of quantum mechanics and brutally dismembering the relativistic momentum energy equation[41], while the mesons that do not exist in the atomic nucleus are vividly described as the medium of nucleon interaction, which makes the theory continue to grow. This far fetched subjective logic is unprecedented, and it is also a microcosm of the development process of modern physics.

For hydrogen atom, the angular quantum number $\mathrm{C}_0=\mathrm{A}=\alpha$ of the neutron state is a very small value, which has not appeared in quantum theory before. It is generally believed that quantum mechanics breaks through the classical theory, which is a flashy bias. Quantum mechanics is just a different description. According to classical theory, the angular quantum number $\mathrm{C}_0=\alpha$ of hydrogen atom corresponds to the angular momentum

of electrons moving around protons at the speed of light. But because of Lorentz transformation, relativity denies that the relative speed of matter reaches or exceeds the speed of light, including the relative propagation speed of light. We can find the physical model that the theory of the limit of the speed of light of the matter does not accord with the unitary principle within the framework of relativity. Of course, this deviates from the theme of this paper. In fact, $\mathrm{C}_0 = \mathrm{A} = \alpha$ has a reasonable explanation within the framework of quantum theory, and this problem is left to the readers for the time being. The angular quantum number C, which is the self consistent solution of the proxy equation of the modified Dirac equation, is consistent with the logic determined by the exact solution of the Schrodinger equation, and is different from the angular quantum number constructed by Dirac algebra. It inversely shows that the logically distorted Dirac algebra does not accord with the basic rules of mathematics and the inference does not accord with the physical meaning.

The radius of the hydrogen-like atomic neutron state is c $\delta = Ze^2/4\pi\varepsilon_0 mc^2$, and it seems to correspond to neutron-like quantum radius. The special eigenvalue $E_0 = 0$ denotes the unique energy state of the neutron-like. For $Z = 1$, the formula (13) reads the neutron binding energy

$$\Delta E = E_\infty - E_0 = mc^2 \tag{32}$$

where E_∞ is the energy of the hydrogen atom corresponding to $n_r = \infty$ in the formula (13), and E_0 is the energy of a neutron. Physics is a theory that describes natural phenomena in mathematical language. The solution of the equation, the method of solving the equation and the popular science interpretation of inference must comply with the unitary principle. (30) and (32) form the self consistent solution of the radial Dirac equation with a Coulomb potential. The neutron binding energy implies that the neutron can be broken up by a photon of the energy $m_e c^2$, or perhaps an electron and a proton could combine into a neutron and emit a photon of the energy $m_e c^2$ at the same time. Usually, $\delta = e^2/4\pi\varepsilon_0 mc^2$ (=2.8117940285 fm) is regarded as the classical electron radius. The above analysis shows that it should be the quantum neutron radius. This is about triplication of the neutron radius, which the recent value is reported to be 0.8418 fm, the early results are 0.805(12)[42], 0.861(26)fm[43], 0.862(13)fm [44], 0.8768 fm, 0.88014 fm.[45], 0.89014 fm and 0.895±0.018 fm.[46] and so on. These data are actually calculated by Lamb shift [47-49].

6 Test of neutron state wave function for modified Dirac equation

Subversive conclusions about famous traditional theories are always doubted, even though mathematical inferences can be easily tested by repeated or non repeated calculations in different orders. The neutron state wave function (31) satisfying the exact boundary condition (10) is the true solution of the modified Dirac equation. The reader can verify that the neutron state wave function (31) satisfies the modified Dirac equation (8) of hydrogen-like atoms. Substitute $E = 0$ and $\mathbb{C} = \mathrm{A} = Z\alpha$ into equation (8) to obtain

$$\begin{aligned} &\frac{d\psi_2}{dr} + \frac{Z\alpha+1}{r}\psi_2 - \left(\frac{mc^2}{\hbar c} - \frac{Z\alpha}{r}\right)\psi_1 = 0 \\ &\frac{d\psi_1}{dr} - \frac{Z\alpha-1}{r}\psi_1 - \left(\frac{mc^2}{\hbar c} + \frac{Z\alpha}{r}\right)\psi_2 = 0 \end{aligned} \tag{33}$$

Substitute the wave function (31) into the above two equations successively, and the results

$$\begin{aligned} &\frac{d\psi_2}{dr} + \frac{Z\alpha+1}{r}\psi_2 - \left(\frac{mc^2}{\hbar c} - \frac{Z\alpha}{r}\right)\psi_1 \\ &= \frac{d}{dr}\left(-B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right) + \frac{Z\alpha+1}{r}\left(-B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right) - \left(\frac{mc^2}{\hbar c} - \frac{Z\alpha}{r}\right)B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r} \\ &= \left(\frac{\mathrm{e}^{-\frac{mcr}{\hbar}}B_0}{r^2} + \frac{mc\mathrm{e}^{-\frac{mcr}{\hbar}}B_0}{\hbar r}\right) + \frac{Z\alpha+1}{r}\left(-B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right) - \left(\frac{mc^2}{\hbar c} - \frac{Z\alpha}{r}\right)B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r} \\ &= B_0\mathrm{e}^{-\frac{mcr}{\hbar}}\left[\left(\frac{1}{r^2} + \frac{mc}{\hbar r}\right) - \frac{Z\alpha+1}{r^2} - \left(\frac{mc}{\hbar} - \frac{Z\alpha}{r}\right)\frac{1}{r}\right] \\ &= 0 \end{aligned} \tag{34}$$

and

$$\begin{aligned}
&\frac{d\psi_1}{dr}-\frac{Z\alpha-1}{r}\psi_1-\left(\frac{mc^2}{\hbar c}+\frac{Z\alpha}{r}\right)\psi_2\\
&=\frac{d}{dr}\left(B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right)-\frac{Z\alpha-1}{r}\left(B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right)-\left(\frac{mc^2}{\hbar c}+\frac{Z\alpha}{r}\right)\left(-B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right)\\
&=\left(-\frac{\mathrm{e}^{-\frac{cmr}{\hbar}}B_0}{r^2}-\frac{c\mathrm{e}^{-\frac{cmr}{\hbar}}mB_0}{r\hbar}\right)-\frac{Z\alpha-1}{r}\left(B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right)-\left(\frac{mc^2}{\hbar c}+\frac{Z\alpha}{r}\right)\left(-B_0\frac{1}{r}e^{-\frac{mc}{\hbar}r}\right)\\
&=B_0\mathrm{e}^{-\frac{cmr}{\hbar}}\left[\left(-\frac{1}{r^2}-\frac{mc}{\hbar r}\right)-\frac{Z\alpha-1}{r^2}+\left(\frac{mc^2}{\hbar c}+\frac{Z\alpha}{r}\right)\frac{1}{r}\right]\\
&=0
\end{aligned}\tag{35}$$

The test result of neutron state wave function is valid. The introduction of the intrinsic angular quantum number $\mathbb{C}$ to replace the Dirac electron theory and the angular quantum number $\kappa=\pm1\ \pm2\ \pm3\cdots$ defined by the unique mathematical operation rules are consistent with the logic that the quantized angular momentum is the intrinsic solution of the angular equation of the wave equation. The Dirac equation for the hydrogen-like atom modified by the intrinsic angular quantum number is self-consistent, while the angular quantum number constructed by Dirac does not conform to the logic and should be abandoned. The exact boundary condition including nuclear size is the actual boundary condition of the wave equation.

7 Conclusions and comments

The Dirac electron theory is unreasonable to define the angular quantum number through formal mathematics. We know that the angular quantum number of Schrödinger equation is the intrinsic solution of the angular equation of the equation. From Schrödinger equation to the Dirac equation, if the angular quantum number changes from the eigensolution of the equation to a definition, the theory does not conform to the unitary principle. The intrinsic angular quantum number is used to replace the angular quantum number defined by Dirac electron theory to modify Dirac equation, which solves the inconsistent difficulty hidden in the Dirac equation. To solve the problem fundamentally, we need to analyze the reason why Dirac electron theory defines the angular quantum number, test the mathematical logic of Dirac algebra, and prove the existence of an intrinsic angular quantum number. The self-consistency of the Dirac equation modified by the Coulomb field and the existence of the neutron state solution not only have the demonstrative mathematical significance of proving whether the wave equation constructed is self-consistent and whether the exact solution of the equation is true, but also have the demonstrative physical significance of explaining the true and false relationship between the various forms of solution of the wave equation and the natural law with scientific logic.

Since Einstein's assumption of constant speed of light has been proved to be untenable, relativity based on the assumption of constant speed of light is naturally incorrect, and the relationship between momentum and energy of relativity has no real scientific significance. Relativistic quantum mechanics based on the relationship between relativistic momentum and energy should be abandoned. The dirac equation is a representative equation of relativistic quantum mechanics. However, Dirac theory of the hydrogen atom, which was considered to be very successful in the past, contains fatal mathematical and physical difficulties such as the collapse of the universe and virtual energy, so it does not belong to scientific theory. Many negative results prove that relativistic quantum mechanics deviates from scientific logic. However, we have been trying to find the factors that the Dirac equation can retain. Therefore, we first consider how to eliminate the divergence and virtual energy of the original Dirac wave function, and then put forward the exact boundary conditions written into the nuclear radius, and give the challenging solution of the Dirac equation for the hydrogen atom. This is done in order to avoid possible bias and draw unreasonable conclusions. The solution of the Schrodinger equation and the solution Klein Gordon equation of the hydrogen atom satisfying the exact boundary condition (10) and satisfying rough boundary condition (4) corresponds to the same energy level formula[1]. However, the challenging solution of the Dirac equation of the hydrogen atom satisfying the exact boundary condition (10) corresponds to a completely different energy level formula from the traditional solution satisfying the rough boundary condition (4). The energy level formula corresponding to the exact boundary condition solution is equivalent to the energy level formula of the Schrodinger equation, and the intention of the Dirac equation is to find an energy level formula that is more in line with the expectation than the Klein Gordon equation. This does not conform to the unitary principle.

The existence problem of the solution of the linear recursive relation system caused by the challenge solution of the Dirac equation requires the introduction of the intrinsic angular undetermined quantum number C instead of the artificial angular quantum number $\kappa=\pm1\ \pm2\ \cdots$ to modify the Dirac hydrogen equation.

The energy parameter E, undetermined angular quantum number C and the nuclear radius δ constitute three intrinsic parameters of Dongfang modified Dirac hydrogen equation. The radius and minimum angular quantum number of the hydrogen like nuclei are obtained by solving the lowest energy state solution. The lowest energy state solution of the hydrogen atom model seems to exactly describe the structure of hydrogen-like neutrons. This is the best result of conservative treatment of Dirac electron theory. The result of non-conservative treatment will be to establish a new accurate wave equation. From the perspective of energy, the result of removing the so-called static energy from relativistic energy is within the same accuracy range as Newtonian mechanical energy. This shows that the establishment of the Dirac equation and the Klein Gordon equation based on the relationship between relativistic momentum and energy does not seem so outrageous, but belongs to an additional operation showing new ideas. Therefore, qualitatively speaking, relativistic Klein Gordon equation and the Dirac equation should not have so-called fine energy level structure expectation, otherwise it would violate the unitary principle. Quantitatively, Dirac energy level formula is a patchwork formula that distorts mathematics. The divergence of S-state Dirac wave function hidden behind it and the Dirac energy level formula that cover up the virtual energy confirms this inference of the unitary principle. Here we should initially realize the powerful logical power of the unitary principle.

Dirac theory has been developed to quantum field theory[50]. Many contents of modern physics are the product of blindly promoting wrong theories. The essence of quantum mechanics has always been hidden, but quantum mechanics has been developing bravely, and even the so-called quantum communication, quantum computer, quantum food and other concepts have emerged. Of course, the modified Dirac equation is not the ultimate answer. The modified Dirac equation without the virtual energy solution of Dirac electron theory and the divergent S-state wave function is locally self consistent, which is much more reasonable than the logic of Dirac electron theory. We are always used to try to defend famous theories that do not match the name. The starting point of modifying the Dirac hydrogen equation is still to maintain the widely accepted theory. However, the solution of the modified Dirac hydrogen equation that meets the exact boundary conditions cannot fit the fine structure energy level formula, which exposes the systematic defects of Dirac electron theory. There are enough problems in relativity, quantum mechanics and relativistic quantum mechanics that need to be clarified step by step. In the final analysis, the problems of relativistic quantum mechanics belong to relativistic problems. The speed of light is the limit of Lorentz transformation, but the so-called superluminal phenomenon[51-53] may not be the best solution to the problem. Mathematical problems caused by the Dirac equation are worthy of further study. To find the exact wave equation[54-56] containing the energy level formula of fine structure, whether we can jump out of the logical cycle of relativity or not, we must ensure that the mathematical rules are not destroyed.

The Dirac equation is just a product of the combination of quantum mechanics and relativity. The former hides the morbid problem of quantum numbers that needs to be solved urgently, while the latter is based on the untenable assumption that the speed of light is constant. There are enough mathematical loopholes in quantum mechanics and relativity, which need to be radically corrected, and those parts that cannot be corrected should be discarded. However, it is also one of the most beautiful landscapes in physics to trace the famous physical theories that pieced together the expected conclusions, uncover the magic veil that whitewashed the anomalous logic, and finally destroy the magic illusion[7, 57-60]. Perhaps the reader will soon be able to find the exact wave equation to achieve the desired results. The neutron state solution of the modified Dirac equation is an ideal result that meets the requirements of mathematical self-consistency. So, does the intrinsic angular quantum number of the atomic ground state and excited state of the modified Dirac equation exist? What are the intrinsic solutions of the ground and excited states of atoms?

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