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The Morbid Equation of Quantum Numbers

X. D. Dongfang

Orient Research Base of Mathematics and Physics,
Wutong Mountain National Forest Park, Shenzhen, China

The lowest standard of natural science theory is logical self consistency. Specifically, its method and inference must conform to the Dongfang unitary principle. However, relativity and quantum mechanics, as the basis of modern physics, are not so, and the problems of quantum mechanics are hidden deeper because the substantive mathematical principles of quantum mechanics have not been revealed. The quantum model of valence electron generation orbital penetration of alkali metal elements with unique stable structure is investigated. The electric field outside the atomic kernel is usually expressed by the Coulomb field of the point charge mode, and the composite electric field in atomic kernel can be equivalent to the electric field inside the sphere with uniform charge distribution or other electric fields without divergence point. The exact solutions of two Schrödinger equations for the bound state of the Coulomb field outside the atom and the binding state of the equivalent field inside the atom determine two different quantization energy formulas respectively. Here I show that the atomic kernel surface is the only common zero potential surface that can be selected. When the orbital penetration occurs, the law of conservation of energy requires that the energy level formulas of the two bound states must have corresponding quantum numbers to make them equal. The result leads to an insoluble morbid equation of quantum numbers, indicating that the two quantum states of the valence electron are incompatible. This irreconcilable contradiction shows that the quantized energy of quantum mechanics cannot absolutely satisfy the law of conservation of energy, and quantum theory violates the unitary principle. Further, I list the morbid equations composed of various eigenvalues of the angular momentum of hydrogen atom described by Bohr theory, Schrödinger and Klein-Gordon theory, and Dirac theory, and point out that there is an irreconcilable contradiction between the minimum nontrivial angular momentum eigenvalue determined by the angular momentum eigenvalue equation in quantum mechanics and the minimum nontrivial angular momentum eigenvalue of Bohr theory.

Keywords: Dongfang unitary principle; Coulomb field; equivalent field; Schrödinger equation; quantized energy; incompatible quantum states.

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1 Introduction

In a broad sense, the so-called quantization law is essentially the discrete law of the motion and change of matter. It is generally believed that the macro discrete law is different from the microscopic discrete law. So do they have a common description method? Although the answer is usually considered negative[1], the problem deserves extensive and in-depth study. The key lies not only in whether it is interesting, but also in whether it can become a breakthrough point to reveal the essence of quantum mechanics. This is a very interesting question that deserves extensive and in-depth study Quantum mechanics has achieved great success in describing the law of micro motion, and quantum theory has been developing continuously. However, the scope of quantum theory has not been clearly defined[2], and the essence of quantum mechanics has been perplexing us. This is one of the reasons for the slow progress of the quantization theory of macro interaction. It is generally believed that quantum mechanics is the most accurate physical theory with perfect mathematical deduction. However, when we try to reveal the essence of quantum mechanics, we will come to a different conclusion.

Planck formula of blackbody radiation, Einstein equation of photoelectric effect and Bohr energy level formula of hydrogen atom gave birth to quantum mechanics based on Schrödinger equation. Born's statistical interpretation of wave functions, de Broglie's concept of the duality of matter waves and wave particles, Heisenberg's uncertainty principle, and the proposal of Klein Gordon and Dirac equations promote the mystery of the development of quantum mechanics. Quantum mechanics has almost developed into a new theology with the emergence of quantum technologies such as the so called photon entanglement communication. Because of

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https://orcid.org/0000-0002-3644-5170

a large number of mathematical descriptions, it is generally believed that quantum mechanics is the most accurate physical theory with perfect mathematical deduction. In fact, quantum mechanics is the most imprecise theory to describe physical phenomena in mathematics, although some quantitative inferences such as energy level formula seem to be valid. The reason why quantum mechanics brings mystery to physics readers is that its precise mathematical principle is not really known, and its existing mathematical description is very rough and vague. When we are really familiar with the establishment and solution of the wave equation of quantum force and try to reveal the mathematical essence of the deepest quantum mechanics, we will find that many inferences of quantum force are contradictory. For example, replacing the mechanical quantities in the laws of mechanics with the so-called Hermite operator acting on the wave function to construct the wave equation has become a decisive basic principle of quantum mechanics. So why can't quantum mechanics have third-order and fourth-order or even higher-order wave equations? To what extent can mechanical quantity be replaced by Hermite operators? There is no scientific conclusion. Blackbody radiation, photoelectric effect, the meaning of wave function and the principle of uncertainty can be given completely different explanations, so as to obtain inferences with the same accuracy. However, because modern physics advocates hegemony and sophistry, correct explanations and inferences often have no vitality. Maybe in 100 years, those superior physical theories will have a chance to survive and develop. Therefore, I am still interested in gradually introducing new universal scientific principles and new scientific inferences, and revealing the fatal logical contradictions hidden in modern physics.

The Dongfang unitary principle[3, 4] is a universal principle that can effectively test the logical self consistency of natural science theory. The expression of Dongfang unitary principle is as following: *There is a definite transformation relationship between different metrics, and the natural law itself will not change due to the selection of different metrics. When the mathematical expression form of natural law under different metrics is transformed into one metrics, the result must be the same as the inherent form under this metrics, 1 = 1, which the transformation is unified.* Bohr quantization angular momentum and quantum mechanics quantization angular momentum constitute two metric, but they do not meet the unitary principle. Based on the unitary principle, I propose the morbid equation of quantum numbers derived from the quantum model of valence electron orbital penetration of alkali metal elements. When the valence electron moves outside the atom kernel, the effective charge[5] of the atomic kernel is equivalent to the point charge, and the interaction force between the valence electron and the atomic kernel is described by the Coulomb force. In the case of orbital penetration, if the valence electron moves in the atomic kernel, the effective charge of the atomic kernel should not be equivalent to the point charge, otherwise the singularity will cause infinite electric field force and violate the natural law. Because in theory, valence electrons can reach the center of the atomic kernel, just as there is a particle in the center of a star. Although the electric field in the atomic kernel is complex, the charge of the atomic kernel can be equivalent to the spherical charge with uniform distribution. The formula of interaction force between the valence electron and the atomic kernel is similar to Hooke's law. The quantum behavior of valence electrons outside and inside the atomic kernel is described by two Schrödinger equations[6], and two different forms of quantized energy level formulas are given. Considering the conservation of energy, two kinds of correlated quantum states have at least one specific quantum number respectively, which makes the two kinds of quantized energy equal, resulting in the morbid equation of quantum numbers without any real number solution. Briefly speaking, when a particle moves between two different fields, two correlated quantum states constitute two metrics. However, the two quantized energy does not conform to the unitary principle, which means that the law of conservation of energy is broken.

In fact, there are some obvious but always ignored morbid problems in quantum mechanics. I enumerate an morbid equation composed of the eigenvalues of the angular momentum of the hydrogen atom described by different quantum mechanics theories. It is hoped that starting from solving this problem and one will expose other hidden logical contradictions in quantum mechanics, develop the mathematical theory required by quantum mechanics, and finally solve the problem that quantized energy destroys the law of conservation of energy.

2 The Morbid Equation of Quantum Numbers

If an isolated star body with uniform mass distribution has a straight hole through the center of the sphere, and a particle emits into the hole, the particle will move back and forth along the straight line where the hole is located. When a particle is outside the sphere body, the formula of the interaction force between the particle and the star is the universal gravitation between the particle and a mass point whose mass is equivalently concentrated at the center of the ball. It is the inverse square law. In the sphere, the interaction between the particle and the star is directly proportional to the distance from the particle to the center of the ball, similar to the spring force.

The Coulomb force is similar to gravitation, and it is also the inverse square law. Suppose that there is a charged sphere with uniformly distributed charges, and a particle with opposite charge moves through the sphere.

The electric field force on the particle outside the sphere is the Coulomb force, and the electric field force in the ball is proportional to the distance from the particle to the center of the ball. Alkali metal element is a stable structure with an atomic kernel and an extra valence electron in its outer layer. The atomic kernel can be regarded as a sphere with uniform charge distribution, and the valence electron motion may occur orbital penetration. The ratio β between the radius of the atomic kernel and the minimum radius of the atom satisfies the inequality $0 < \beta \leqslant 1$. The quantum number describing the quantization energy of the valence electron in the electric field inside the atomic kernel is a non negative integer l, and the quantum number n describing the quantization energy of the valence electron in the Coulomb field outside the atomic kernel is a positive integer. When the orbital penetration occurs, the energy is still conserved, and the two quantized energy formulas must be equal, which gives the algebraic equation

$$\left(3-\frac{\beta}{n^2}\right)\frac{\sqrt{\beta}}{2l+3}=1 \tag{1}$$

Where $0 < \beta \leqslant 1$, $n \geqslant 1$, $l \geqslant 0$.

However, no matter what the quantum numbers n and l take, equation (1) does not hold in the domain of definition. The equation (1) is transformed into a cubic equation about β expressed by the quantum numbers n and l,

$$\left(\sqrt{\beta}\right)^3 - 3n^2\sqrt{\beta} + (3+2l)\,n^2 = 0 \tag{2}$$

Cubic equation (2) has a real root and two imaginary roots. According to Fontana-Cardano formula[7, 8], the real number root is

$$\beta = 2n^2 + \frac{2^{\frac{2}{3}} + n^4\left[(3+2l)^2 - 2n^2 + (3+2l)\sqrt{(3+2l)^2-4n^2}\right]^{\frac{2}{3}}}{2^{\frac{1}{3}}\left[(3+2l)^2 - 2n^2 + (3+2l)\sqrt{(3+2l)^2-4n^2}\right]^{\frac{1}{3}}} \tag{3}$$

The definition domain of the quadratic radical in the above formula is $(3+2l)^2 \geqslant 4n^2$, that is, the two quantum numbers must satisfy the relation $n \leqslant l + 3/2$. Because β is not only the increasing function of n, but also the increasing function of l. In order to find the minimum value of β, when n takes the minimum value, l should also take the minimum value. Take the minimum quantum number $n = 1$ and $l = 0$ to get the minimum value of β

$$\beta_{\min} = 2 + \left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)^{\frac{1}{3}} + \left(\frac{7}{2}-\frac{3\sqrt{5}}{2}\right)^{\frac{1}{3}} \tag{4}$$

Namely, $\beta_{\min} = 4.425988757$. The result is inconsistent with the domain $\beta \in (0, 1]$.

Equation (1) is called the morbid equation of quantum numbers. It shows that when the same microscopic particle enters from one field to another, the quantized energy of two different bound states describing the same particle does not have the same value, even at the interface of two fields. In short, the quantized energy of quantum mechanics violates the unitary principle and destroys the law of conservation of energy. This is a logical contradiction. This contradiction is irreconcilable within the existing framework of quantum mechanics theory, and may need a new theory to solve it. The morbid equation of quantum numbers reveals that the great success of quantum mechanics has always implied sharp contradictions which must be corrected.

3 Derivation of morbid equation of quantum numbers

The effective charge number of alkali metal and other elements is represented by Z^*, and the elementary charge is represented by e. According to the description in the previous section, the electric field in the atomic kernel is equivalent to that of the positive charge Z^*e uniformly distributed in the sphere with radius δ. When the electron moves outside the atom kernel, the alkali metal atom forms the hydrogen like atom model. The radius δ of the atomic kernel does not exceed the Bohr radius $a_0 = \varepsilon_0 h^2 / \pi m Z^* e^2$ of the hydrogen like atom. Where ε_0 is the dielectric constant, h is the Planck constant, mm is the mass of the electron, $0 < \beta \leqslant 1$. The radius of the atom kernel is expressed as $\delta = \beta a_0$, and its concrete form is

$$\delta = \frac{\beta \varepsilon_0 h^2}{\pi m Z^* e^2} \tag{5}$$

The effective charge density of the uniformly distributed effective charge of the equivalent sphere is $\rho = Z^*e\left(4\pi\delta^3/3\right)^{-1}$, and the effective charge of the concentric sphere with $r \leqslant \delta$ is $q = \rho\left(4\pi r^3/3\right) = Z^* e r^3/\delta^3$. According to the Gauss theorem of electrostatic field, the electric field inside the atomic kernel of $r \leqslant \delta$ is $E_1 = Z^* e r / 4\pi\varepsilon_0\delta^3$, and that of the atom with $r \geqslant \delta$ is $E_2 = Z^* e / 4\pi\varepsilon_0 r^2$. The electric field force on the electron is $F = eE$, and its direction points to the center of the sphere. Considering the penetration of valence electron orbit, let the common zero potential surface outside and inside the atomic kernel be a concentric sphere surface with $r = a_c$. according to the definition of potential energy $U = \int_r^{a_c} \mathbf{F} \cdot d\mathbf{r}$, the potential energies of the electron and the atomic kernel are calculated as follows

$$\begin{aligned} U_1 &= \frac{Z^* e^2 r^2}{8\pi\varepsilon_0 a_c^3} - \frac{Z^* e^2}{8\pi\varepsilon_0 a_c} \quad (r \leqslant \delta) \\ U_2 &= \frac{Z^* e^2}{4\pi\varepsilon_0 a_c} - \frac{Z^* e^2}{4\pi\varepsilon_0 r} \quad (r \geqslant \delta) \end{aligned} \tag{6}$$

When the velocity of the valence electron arriving at the sphere surface of atomic kernel ($r=\delta$) is v, it is obtained according to the energy conservation law of classical mechanics

$$\frac{1}{2}mv^2+\frac{Z^*e^2\delta^2}{8\pi\varepsilon_0 a_c^3}-\frac{Z^*e^2}{8\pi\varepsilon_0 a_c}=\frac{1}{2}mv^2+\frac{Z^*e^2}{4\pi\varepsilon_0 a_c}-\frac{Z^*e^2}{4\pi\varepsilon_0\delta}$$

This formula is simplified to obtain $2a_c^3-3\delta a_c^2+\delta^3=0$, that is, $(a_c-\delta)^2(2a_c+\delta)=0$, is obtained by simplification. This cubic equation has two equal positive roots $a_c=\delta$ and one negative root $a_c=-\delta/2$. The solution satisfying the physical meaning $a_c>0$ is $a_c=\delta$. Therefore, the zero potential energy surface can only be selected on the equivalent sphere surface of the atomic kernel. Thus, the two potential energy expressions (6) are transformed into

$$\begin{aligned} U_1&=\frac{Z^*e^2r^2}{8\pi\varepsilon_0\delta^3}-\frac{Z^*e^2}{8\pi\varepsilon_0\delta} \quad (r\leqslant\delta)\\ U_2&=\frac{Z^*e^2}{4\pi\varepsilon_0\delta}-\frac{Z^*e^2}{4\pi\varepsilon_0 r} \quad (r\geqslant\delta)\end{aligned} \tag{7}$$

The wave functions ψ_1 and ψ_2 are used to describe the quantum states of valence electrons in and out of the atomic kernel respectively. The Schrödinger wave equations for ψ_1 and ψ_2 are respectively as following

$$\begin{aligned}&\left(-\frac{\hbar^2}{2m}\nabla^2+\frac{Z^*e^2r^2}{8\pi\varepsilon_0\delta^3}-\frac{Z^*e^2}{8\pi\varepsilon_0\delta}\right)\psi_1=E_1\psi_1 \quad (r\leqslant\delta)\\ &\left(-\frac{\hbar^2}{2m}\nabla^2+\frac{Z^*e^2}{4\pi\varepsilon_0\delta}-\frac{Z^*e^2}{4\pi\varepsilon_0 r}\right)\psi_2=E_2\psi_2 \quad (r\geqslant\delta)\end{aligned} \tag{8}$$

The first equation is the Schrödinger equation of three-dimensional like harmonic oscillator, and the second equation is the Schrödinger equation of three-dimensional like hydrogen atoms. The natural boundary condition is $\psi(r\to\infty)=0,\quad |\psi|(0<r<\infty)\neq\infty$. The exact solutions of the two wave equations satisfying the natural boundary conditions determines that the corresponding energies in the equations are quantized, and the two energy eigenvalues[9-11] are respectively

$$\begin{aligned} E_1&=\left(l+\frac{3}{2}\right)\frac{h}{2\pi}\sqrt{\frac{Z^*e^2}{4\pi\varepsilon_0\delta^3 m}}-\frac{Z^*e^2}{8\pi\varepsilon_0\delta} \quad (l=0,1,\cdots)\\ E_2&=\frac{Z^*e^2}{4\pi\varepsilon_0\delta}-\frac{mZ^{*2}e^4}{8n^2\varepsilon_0^2h^2} \quad (n=1,2,\cdots)\end{aligned} \tag{9}$$

These two quantized energy expressions of alkali metal elements are different from the common expressions in textbooks, and both have a constant term. This is because the classical law of conservation of energy determines that the common zero potential energy surface can only be selected on the surface of the atomic kernel when the orbital penetration occurs.

When the valence electrons of an atom in a certain energy state switch between the outer and inner atomic orbits, the quantized energy is bound to be conserved. The simplest model is that when the valence electron is on the surface of the atom, the two quantized energies must be the same, that is, $E_1=E_2$. According to formula (9), the equation is obtained

$$\left(l+\frac{3}{2}\right)\frac{h}{2\pi}\sqrt{\frac{Z^*e^2}{4\pi\varepsilon_0\delta^3 m}}=\frac{3Z^*e^2}{8\pi\varepsilon_0\delta}-\frac{\mu Z^{*2}e^4}{8n^2\varepsilon_0^2h^2} \tag{10}$$

By substituting formula (5) into the above formula, we can get

$$\frac{2l+3}{\sqrt{\beta^3}}=\frac{3}{\beta}-\frac{1}{n^2} \tag{11}$$

In other words, the morbid equation of quantum numbers (1) is obtained.

In mathematics form, the limit cases of $\beta=1$, $l=0$ and $n\to\infty$ can make equation (1) hold, but the solution of this limit case does not conform to the physical meaning. The two forms of quantized energy are monotone increasing functions of n and l, and they are conserved. The two forms of corresponding energy are equal. When $n\to\infty$, there must be $l\to\infty$. However, $n\to\infty$ is beyond the atomic range, and $l\to\infty$ cannot be explained. Moreover, only when$n\to\infty$ and $l=0$, there is a limit solution $\beta=1$ in mathematical form. These are all contradictory. Therefore, the limit cases of $n\to\infty$, $l=0$ and $\beta=1$ are not special solutions of equation (1) in accordance with physical meaning. This detail also indirectly reminds us that there may be some differences between physical mathematics and pure mathematics that are usually ignored. On the other hand, when $0<\beta<1$, the left side of equation (1) is the fraction while the right side is natural number 1, which is obviously contradictory. Therefore, equation (1) has no physical solution in the domain of definition. If there is a sphere with uniformly distributed charges, the quantum behavior of a particle with opposite charge passing through the uniformly charged sphere also has various quantum morbid equations. The quantum morbid equation shows that the Schrödinger equations of the different bound states of the microscopic particles in the sphere and outside the sphere are incompatible, which naturally includes the Dirac equation.

The law of conservation of quantized energy requires that there is a corresponding quantum number l for any quantum number n, which makes equation (1) hold. In fact, there is no set of quantum numbers n and l that have physical significance to satisfy this equation. The morbid equation of quantum numbers in one-dimensional motion is different. In addition, considering that the values of the two wave functions on the real surface of atoms should be equal, the morbid equation of wave functions can be derived. If there is a sphere of uniformly distributed charges, the quantum behavior of a particle with opposite charge passing through the uniformly charged sphere also has various quantum morbid equations. The quantum morbid equation shows that the Schrödinger equations of the different bound states of the microscopic particles in the sphere and outside the sphere are incompatible, which naturally includes the Dirac equation[12-14]. In fact, the Dirac equation of harmonic oscillator has been avoided. The mathematical derivation methods of all kinds of quantum morbid equations are the same. If the morbid equation of quantum numbers is solved, other problems will not exist. All in all, only the morbid equation of quantum numbers is introduced here.

4 Morbid equation of quantized angular momentum

The solution of some neglected difficult problems of quantum theory is the key to reveal the essence of quantum mechanics, and it is also the prelude to solving the morbid equation of quantum numbers that represents the destruction of the law of conservation of energy by quantized energy. Completely solving the morbid equation of quantum number requires new mathematical principle, which is the prerequisite for unifying the macro and micro quantum theory[15]. Today's theoretical physicists may not be able to successfully complete this work for the time being. Here let me introduce the morbid problem of eigenvalue set of angular momentum, which is obvious in quantum mechanics but has been ignored.

From Bohr theory to the Schrödinger equation, and then from the Klein Gordon equation to the Dirac equation, the eigenvalues of angular momentum in quantum mechanics are inconsistent. Bohr-Sommerfeld quantization conditions are:

$$\oint pdq = nh \tag{12}$$

The eigenvalue set of angular momentum of Bohr's hydrogen atom theory is

$$L = n\hbar, \quad n = 1,\ 2,\ 3,\ \cdots \tag{13}$$

The eigen equation of the square of angular momentum in quantum mechanics is

$$-\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] = L^2 Y \tag{14}$$

The eigenvalue set of angular momentum of Schrödinger and Klein Gordon's hydrogen atom theory is

$$\begin{aligned} &L = \sqrt{l\,(l+1)}\hbar, \quad l = 0,\ 1,\ 2,\ \cdots \\ &L_z = m\hbar, \quad m = 0,\ \pm1,\ \pm2,\ \cdots,\ \pm l \end{aligned} \tag{15}$$

The Dirac electron theory constructs an eigenequation of angular momentum

$$\hbar\hat{\kappa}\psi = \pm\left(j+\frac{1}{2}\right)\hbar\psi,\ j = l\pm\frac{1}{2} \tag{16}$$

The eigenvalue set of angular momentum of Dirac's electron theory of the hydrogen atom is

$$L = \kappa\hbar, \quad \kappa = \pm\,1,\ \pm2,\ \pm3,\ \cdots \tag{17}$$

Later, I will prove that this set of angular momentum eigenvalues does not actually satisfy the Dirac equation for the hydrogen atom.

Bohr Sommerfeld angular momentum eigenvalue set, Schrödinger Klein Gordon angular momentum eigenvalue set and Dirac angular momentum eigenvalue set of the hydrogen atom are inconsistent, which construct the morbid angular momentum eigenvalue problem of quantum mechanics:

$$L = \begin{cases} n\hbar, \quad n = 1,\ 2,\ 3,\ \cdots \quad \text{(Bohr - Sommerfeld)} \\ \begin{cases} \sqrt{l\,(l+1)}\hbar, \quad l=0,\ 1,\ 2,\ \cdots \\ m\hbar, \quad m=0,\ \pm1,\ \pm2,\ \cdots,\ \pm l \end{cases} \text{(Schrödinger)} \\ \kappa\hbar, \quad \kappa = \pm\,1,\ \pm2,\ \pm3,\ \cdots \quad \text{(Dirac)} \end{cases} \tag{18}$$

Different quantum mechanics theories actually constitute multiple metrics to test quantum mechanics. The inference of angular momentum eigenvalues under different metrics is very different, so it does not meet the principle of normalization, which poses a challenge to quantum theory. If the result of the experimental test chooses one kind of inference, then the theory of other inferences is actually denied. At this time, Bohr theory seems to have a superior position. This is completely inconsistent with the expectation of modern physics to be eager for quick success and instant benefits.

Physicists who lack mathematical knowledge or understanding of the essence of physical concepts, and therefore cannot really solve problems that could have been solved, can always sophisticate and give logical contradictions various usually unspeakable conclusions. Just like understating that the relativistic motion clock is expanded and the motion length is shortened, abandoning scientific argument, authoritative physicists only need to declare that the eigenvalues of angular momentum of different quantum theories are different, which seems to have become or will become a scientific conclusion. However, using the unitary principle to test, the minimum nontrivial angular momentum of Bohr's hydrogen atom theory is [16], and the minimum nontrivial angular momentum of quantum mechanics is $\sqrt{2}\hbar$[17]. It is impossible to take the median value between $\hbar$ and $\sqrt{2}\hbar$ for the experimental observation of the minimum nontrivial angular momentum of the hydrogen atom. The Bohr model and Schrödinger equation of quantum mechanics need these two contradictory angular momentum values respectively to derive the same energy level formula of hydrogen atom [18]. This is the most intuitive difficult problem of quantum mechanics, and sophistry no longer seems magical. This shows the power of the unitary principle. Perhaps abstract and vague knowledge such as the so-called semiclassical quantum theory can easily be chosen as the reason to avoid the above contradiction, but in fact, the minimum non-zero angular momentum of the hydrogen atom happens to be $\hbar$. So, can we solve the minimum nontrivial angular momentum difficulty of quantum mechanics from the essence of quantum theory? The seemingly accurate and complete quantum mechanics theory actually hides enough subversive logical inconsistencies. Only by discovering and eliminating the hidden logical contradictions in scientific theory can we promote the correct development of theory.

5 Conclusions and comments

The unitary principle is of great significance to test and establish scientific theory. A complete physical theory should not only conform to the unitary principle locally, but also conform to the unitary principle globally. Using the unitary principle, we can find many important problems hide in modern physics theory. The difficulties of quantization angular momentum and the morbid equation of quantum numbers show that the conservation law of energy is destroyed because of quantized energy. These are actually computational problems rather than philosophical speculative problems. Quantum mechanics is a computational science, and its paradoxes should be expressed by mathematical equations. It is not helpful for the progress and development of physics theory to express the questions of quantum mechanics abstractly or to limit the philosophical speculation to weaken the logical difficulties of quantum mechanics. Physical logic difficulties inevitably contain new physics mysteries, which need to be discovered by scientific calculation. Quantum mechanics has puzzled some great physicists. Feynman famously declared "I think I can safely say that nobody understands quantum mechanics."[19-21] Einstein put forward the theory of light quantum, but evaluated quantum mechanics "God does not play dice with the universe."[22] Nevertheless, the success of quantum mechanics is undeniable. Almost all physics problems are interrelated. If one of the most basic physics problems is solved, other physics problems can be solved well. Over the past 30 years, I have devoted ourselves to the discovery and solution of the difficult basic problems in theoretical physics. We have learned that the effective breakthrough of physical theory must come from the discovery and solution of the contradictions implied in the past theories that can not be reconciled logically. There must be systematic solutions to seemingly intractable physical problems. It seems that quantum mechanics is often misunderstood. Some reports on the progress of quantum theory are even based on metaphysical thinking and tend to develop towards metaphysics. The main reason is that the essence of quantum mechanics has not been known. It is one of the effective breakthroughs to reveal the essence of quantum mechanics and realize the unification of macroscopic and microscopic quantization theories by eliminating the logical difficulties of quantum mechanics such as the inconsistency of quantization angular momentum and the morbid equation of quantum numbers.

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