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Nuclear Force Constants Mapped by Yukawa Potential

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The nuclear meson theory evolved from Yukawa potential regards the observed mesons as its experimental test, which is far fetched. It goes without saying that there are no mesons in nuclei such as deuterons. However, it is the lack of scientific basis to construct the wave equation by using the quantum mechanical operator to act on the unknown function, so as to piece up the exact solution of the potential function. The relevant calculation results do not form an inevitable causal relationship with the existence of mesons. Even if a celebrity patchwork theory is maintained, meson theory should have at least one observable quantitative inference from the perspective of experimental test theory. Here, the ground state energy corresponding to the Yukawa potential expressed by the nuclear force constant and the mass of the PI meson is calculated by the variational method, and then the average binding energy of the atomic nucleus is connected with the ground state energy corresponding to the Yukawa potential to calculate the average nuclear force constant. From this result, it is estimated that the distribution range of the average nuclear force constant of 118 atomic nuclei in the periodic table of elements is from 1.093992 to 1.413981. The results also show that the estimated value of nuclear force constant up to 15 is unreliable. However, the nuclear force constant can not be predicted by other methods. Calculating the average nuclear force constant by using the ground state energy corresponding to Yukawa potential is only a dead cycle, and the desire to test meson theory through experimental observation can not be realized. This unexpected result shows that modern physical theory has long fallen into an unrealistically strange circle of formal logic shrouded in the halo of mathematics: although the conditions and conclusions of the theory are far from the true description of natural phenomena, the theory can be deduced infinitely, and is constantly beautified and advocated as a profound revelation of natural laws. The reason why this paper introduces the average nuclear force constant mapped by the unreasonable Yukawa potential is to disclose some unique calculation methods that need to be applied in establishing a reasonable nuclear force model in the future.

Keywords: Yukawa potential; Meson theory; nuclear force constant; ground energy.

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1 Introduction

The Yukawa potential[1, 2] is regarded as a phenomenological central potential between two nucleons[3] in theoretical nuclear physics[4]. Since its birth in 1935, Yukawa potential has been widely used, becoming an important model[5-7] for establishing the theoretical framework of nuclear force meson[8-16] and describing the intuitive physical image[17-19], and deriving many forms similar in form but different in essence. For examples, the Reid potential[20], the Green's form[21], the form with poles by Roriz-Delfino[22], the inverse Fourier transform by Garavelli-Oliveira[23, 24], the convergence form at the coordinate origin by Calvin Stubbins[25], the effective potential with the coupling constant "amplified:" by Stefano De Leo[26], Yukawa-like potentials by Flambaum-Shuryak[27] and Cordon-Arriola[28] respectively. Donoghue also gave an integrated form[29]. There are some more complex expressions of Yukawa potential, like the high-quality nucleon-nucleon(NN) potentials[30, 31] and Yukawa-Like tensor interactions[32] and so on. Most of these Yukawa-like potentials retain the features of central symmetry and simple form.

In fact, there has never been a meson in the nucleus. However, the interaction between nucleons is described as exchange through mesons, which is expressed by Yukawa potential. The meson theory established from this is still developing vigorously. This should be attributed to the material benefits and religiousness of modern physics. The fanatical belief and the eager pursuit of interests have led the theoretical creators to blindly follow the leader's singing to get more good things without asking whether what they say conforms to the natural law. We know that if a potential function is used to describe the bound system, there must be the corresponding ground state energy, which is the most important physical quantity. Is the result consistent with the nuclear structure? The original Yukawa potential is

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thought to come from a kind of pending particle relativistic Klein Gordon like equation:

$$\nabla^2\Phi = \frac{m^2c^2}{\hbar^2}\Phi quad\left(r>0\right)$$

However, this equation is not actually the Klein Gordon equation. Its derivation and hidden mathematical significance are intriguing. It can be said that it has enjoyed the glory of the theory of relativity that made physicists lose their minds at that time. The particle with the mass of m is identified as π meson, which is defined as the medium of strong interaction between nucleons.

It is generally believed that when the atomic nucleus is in a stable state, the nuclear forces between protons and protons, protons and neutrons, and neutrons and neutrons are equal: $f_{pp} = f_{pn} = f_{nn}$[48]. However, masses of π^0 meson and $\pi^{\pm}$ meson are different, and the corresponding Yukawa potentials are not exactly the same. Meson theory does not know how much and how to distribute pi0 and $\pi^{\pm}$ mesons in an atomic nucleus, but there are endless fairy tales of nucleon meson exchange. A question that readers often think of is that theories should be tested by experiments. Then, Yukawa potential and its countless derived potentials are regarded as the equivalent potentials of strong interaction forces between nucleons. Which form is consistent with the fact? Is there any inference in the meson theory consistent with the experimental observation?

In this paper, the average nuclear force constant σ is introduced to represent the average interaction intensity between nucleons in the nucleus described by Yukawa potential, and the average ground state energy of Yukawa potential mapping is calculated by the variational method. Then the average nuclear force constants of 118 nuclei in the periodic table of elements are calculated. Among them, the average meson force constant of ^{2}H nucleus with the lowest average binding energy is 1.093992, the average meson force constant of $^{62}_{28}$Ni nucleus with the highest average binding energy is 1.413981, and the average meson force constant of ^{295}Ei nucleus with the heaviest average binding energy is 1.360918. This new quantitative inference is the most realistic calculation result of Yukawa potential, and it seems to be a rare quantitative standard for experimental testing of meson theory. Then, can these results be verified experimentally to support the nuclear meson theory triggered by the Yukawa potential?

2 Variational parameters

The energy eigenvalue of quantum systems in bound states is one of the most important quantities of atomic and nuclear physics. They are often derived by solving the wave equation with a boundary condition. Yukawa potential includes an exponential function, solving precisely the Yukawa wave equation is difficult. It usually gives a rough result[33-36]. Experimental observations confirmed the existence of the nuclear energy levels[37]. There is warrantable supposition, if a changed Yukawa potential deviates from the original form not too far away, the approximate calculation should be within the range of error. It is unique for the most stable structure of any nucleus. Now that Yukawa potential is used to describe the nuclear force, the ground energy of Yukawa field[12,38,39] must uniquely exist. According to quantum mechanics, if the trial wave function of the ground state is appropriate, using the variational method to calculate the ground energy of a quantum system in the bound state will give accurate results.

As it is well known to us all that the fine-structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ represents the electromagnetic interaction strength, and the original Coulomb potential $U_e = \pm e^2/(4\pi\varepsilon_0 r)$ can be expressed as the form of $U_e = \pm\alpha\hbar c/r$. Where e is the elementary charge, $\hbar = h/2\pi$ is the Planck constant, and c is the speed of light in a vacuum[40]. Similarly, according to the gravitational potential energy $U_m = -Gm'm/r$(in which G is the gravitational constant.) for two bodies of mass m and m' respectively, we can define a constant $\mathrm{A} = Gm'm/\hbar c$ to represent the gravity strength, and express the Newton gravity potential as the form $U_m = -\mathrm{A}\hbar c/r$. As a corollary, we introduce the dimensionless physical constant σ to represent the nuclear force strength between two nucleons, thus the Yukawa potential can be expressed as the following unified form

$$\Phi_\pi = -\frac{\sigma\hbar c}{r}e^{-\frac{m_\pi c}{\hbar}r} \tag{1}$$

The nuclear force has both attractive and repulsive force. In the above equation, the minus sign expresses the attractive force.

A wave function must satisfy the boundary condition $\psi\left(r\to\infty\right) = 0$. According to the characteristics of the equation (1), we choose the trial wave function of the ground state $\psi_0\left(r\right) = A\exp\left(-\lambda mcr/\hbar\right)$, where $\lambda > 0$ is the variational parameters. Using the normalized conditions $\int\limits_0^\infty \psi_0\left(r\right)\psi_0^*\left(r\right)r^2dr\int\limits_0^\pi \sin\theta d\theta r\int\limits_0^{2\pi} d\varphi = 1$ to calculate the normalization coefficients gives $A = \sqrt{\lambda^3 m_\pi^3 c^3/\pi\hbar^3}$. So the trial wave function takes the form

$$\psi_0\left(r\right) = \sqrt{\frac{\lambda^3 m_\pi^3 c^3}{\pi\hbar^3}}e^{-\frac{\lambda m_\pi cr}{\hbar}}$$

Hamilton without the rest energy is $H = E - m_\pi c^2 + \Phi_\pi$, where $E - m_\pi c^2 \approx p^2/2m_\pi$, p and E are the momentum and energy of the π meson respectively. From this it gives Hamilton with a Yukawa potential,

$$\hat{H} = -\frac{\hbar^2}{2m_\pi}\nabla^2 - \frac{\sigma\hbar c}{r}e^{-\frac{m_\pi c}{\hbar}r}$$

In the state described by wave function $\psi_0\left(r\right)$, the ground energy of the Yukawa field is $\bar{H} = \int\limits_0^\infty \psi_0^*\hat{H}\psi_0 r^2dr\int\limits_0^\pi \sin\theta d\theta\int\limits_0^{2\pi} d\varphi$, namely

$$\bar{H}=\int_0^\infty \frac{\lambda^3 m_\pi^3 c^3}{\pi\hbar^3} e^{-\frac{\lambda m_\pi c}{\hbar}r}\left(-\frac{\hbar^2}{2m_\pi}\nabla^2-\frac{\sigma\hbar c}{r}e^{-\frac{m_\pi c}{\hbar}r}\right)e^{-\frac{\lambda m_\pi c}{\hbar}r}r^2dr\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\varphi$$

the results of the above integration are as follows

$$\bar{H}=\left[\frac{\lambda^2}{2}-\frac{4\sigma\lambda^3}{(2\lambda+1)^2}\right]m_\pi c^2 \tag{2}$$

As $r\to\infty$, the potential energy of the system is zero, and the kinetic energy of the particles constituting the system is also zero, so the total energy without the rest energy of the system is zero. Since the nucleons combine to form a nucleus will release energy, the ground energy of Yukawa field must a negative number. By the equation (2), one obtains

$$\left[\frac{\lambda^2}{2}-\frac{4\sigma\lambda^3}{(2\lambda+1)^2}\right]m_\pi c^2<0$$

namely,

$$\sigma-\frac{1}{2}-\sqrt{\sigma(\sigma-1)}<\lambda<\sigma-\frac{1}{2}+\sqrt{\sigma(\sigma-1)}$$

Notice that $\sigma>0$, the quadratic radial $\sqrt{\sigma(\sigma-1)}$ of physically significant reads $\sigma\geqslant 1$. However, making use of the squeeze theorem[41], the condition of the inequality is

$$\sigma>1$$

so the nuclear force constant must be greater than 1. This result agrees with the experiments.

The exact ground wave function requires the variational parameters λ in the trial ground wave function making $\bar{H}$ the minimum energy. Let $d\bar{H}/d\lambda=0$, one obtains a cubic equation with one unknown,

$$8\lambda^3+4(3-2\sigma)\lambda^2+6(1-2\sigma)\lambda+1=0$$

Since the ground energy of a nucleus is the only one that exists, this equation has a unique positive root. From the solution of cubic equation $x^3+bx^2+sx+d=0$ in one variable, mark

$$p=s-\frac{b^2}{3},\quad q=d-\frac{bs}{3}+\frac{2b^3}{27}$$

the discriminant of the cubic equation with one unknown[42] is

$$\Delta=-108\left(\frac{p^3}{27}+\frac{q^2}{4}\right)=\frac{1}{4}\sigma^2\left(9\sigma^2+20\sigma-27\right)>0$$

for $\sigma>1$. So the cubic equation has three real roots,

$$\lambda_k=\sqrt{-\frac{4}{3}p}\cos\left(u+\frac{2}{3}k\pi\right)-\frac{b}{3}$$

where $k=0,\ 1,\ 2$ and

$$\cos 3u=-\frac{q}{2}\left(-\frac{p}{3}\right)^{-\frac{3}{2}}$$

Therefore, the variational parameters is determined by the following expressions,

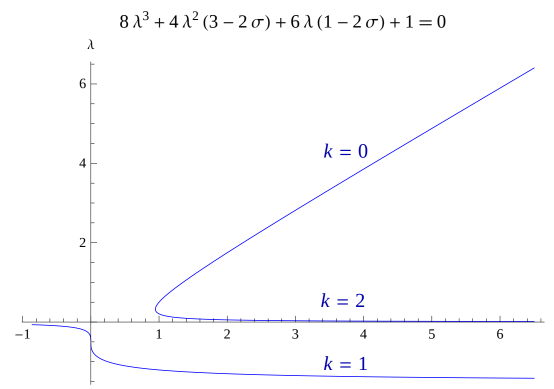

Figure 1 Three roots of the variational parameters

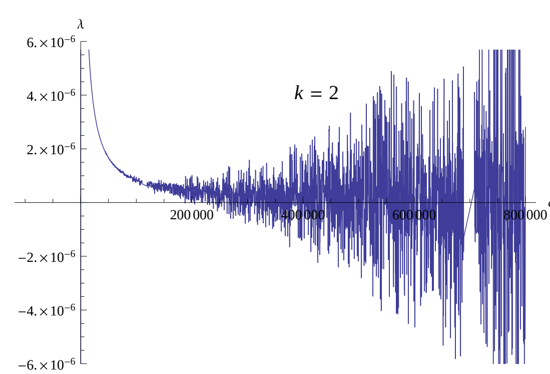

Figure 2 Formal root of the variational parameters with $k=2$

$$\lambda_k=\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left[\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma(2\sigma+3)^3}}+\frac{2k\pi}{3}\right]+\frac{2\sigma-3}{6} \tag{3}$$

The above results can also be obtained by solving equation $8\lambda^3+4(3-2\sigma)\lambda^2+6(1-2\sigma)\lambda+1=0$ with scientific calculation software such as Mathematica. In order to find the variational parameters of physical meaning, we draw the images of three solutions of the variational parameters, see the Fig.1, in which $\sigma>1$. $k=1$ reading $\lambda<0$ does not agree with the precondition, so it should be discarded. $k = 2$ reads that the variational parameters λ tends to zero rapidly with the increase of constant σ and then oscillates in the neighborhood of zero, resulting in that the total energy tends to zero rapidly then oscillates, neither is it in agreement with facts nor self-consistent, so it should be discarded too. $\lambda > 0$ only holds for $k = 0$, it determines the unique variational parameters as following,

$$\lambda = \frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left[\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma\left(2\sigma+3\right)^3}}\right]+\frac{2\sigma-3}{6} \tag{4}$$

Therefore, the approximate ground radial wave function takes the form

$$\begin{aligned} R_0\left(r\right) = & \sqrt{\frac{m_\pi^3 c^3}{\pi\hbar^3}\left\{\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left[\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma\left(2\sigma+3\right)^3}}\right]+\frac{2\sigma-3}{6}\right\}^3} \\ & \cdot\exp\left\{-\left[\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma\left(2\sigma+3\right)^3}}\right)+\frac{2\sigma-3}{6}\right]\frac{m_\pi c}{\hbar}r\right\} \end{aligned} \tag{5}$$

3 Ground energy distribution

The stable structure of nuclei corresponds to a minimum energy state. Using the Yukawa potential to describe the forces between nucleons, the minimum energy of the nucleus is just the ground energy. Substituting equation (4) into the equation (2) gives the relationship that the ground energy E_0 changes with the nuclear force constant σ, which is called the nuclear Yuakwa ground energy distribution function,

$$E_0 = -\left\{\begin{array}{l} \dfrac{4\sigma\left[\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma(2\sigma+3)^3}}\right)+\frac{2\sigma-3}{6}\right]^3}{\left(2\left[\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma(2\sigma+3)^3}}\right)+\frac{2\sigma-3}{6}\right]+1\right)^2} \\ -\dfrac{1}{2}\left[\dfrac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\dfrac{1}{3}\arccos\dfrac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma\left(2\sigma+3\right)^3}}\right)+\dfrac{2\sigma-3}{6}\right]^2 \end{array}\right\} m_\pi c^2 \tag{6}$$

This energy of the ground state does not contain the static energy of the π meson, so it is negative. Yukawa ground state energy (6) is the exact result of calculating the binding energy of atomic nuclei according to the first principle[43], and is an inevitable quantitative inference of meson theory. The variational method is an approximate method but also a reliable method for calculating the ground energy. Although choosing different trial ground wave function will cause the different calculation of the ground energy, there should be no obvious errors. The Yukawa potential describes forces in short distances within 1.7fm or so, and its accurate experimental verification is difficult. However, observing outside the nucleus, the interactions between nucleons can always be treated as an equivalent of force that has a center of symmetry. This is the meaning of Yukawa potential.

Linked to the binding energies of the nucleus, the Yukawa ground energy should have the experimental observation effect. It is the basis for experimental tests of meson theory. Let A be the mass number and B the binding energy of a nucleus, Yukawa ground state energy describes the minimum energy of the interaction between two nucleons, and its absolute value is equivalent to twice the average binding energy of nucleons, so $2B/A = E_\infty - E_0$. Where $E_\infty = 0$. So the nuclear force constant $\sigma$of nucleus with multiple nucleons implies the average value. It can be calculated by the following equation

$$\frac{2B}{A} = \left\{\begin{array}{l} \dfrac{4\sigma\left[\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma(2\sigma+3)^3}}\right)+\frac{2\sigma-3}{6}\right]^3}{\left\{2\left[\frac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\frac{1}{3}\arccos\frac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma(2\sigma+3)^3}}\right)+\frac{2\sigma-3}{6}\right]+1\right\}^2} \\ -\dfrac{1}{2}\left[\dfrac{\sqrt{4\sigma^2+6\sigma}}{3}\cos\left(\dfrac{1}{3}\arccos\dfrac{4\sigma^2+9\sigma-27}{\sqrt{2\sigma(2\sigma+3)^3}}\right)+\dfrac{2\sigma-3}{6}\right] \end{array}\right\} m_\pi c^2 \tag{7}$$

In principle, from the expressions (6) and (7), one can get the average binding energies E_0

$$B = A\,|E_0| \tag{8}$$

How to obtain the average nuclear force constant[44] in theory may be a puzzle. In the standard literature, the nuclear force constant is usually expressed as a dimensional constant $g^2/4\pi$, and some estimations achieve up to $g^2/4\pi > 16.6$[44]. If we use the π^0 meson to calculate, the binding energy of this nucleus is at least 16453.897 MeV. There are other mesons participating in nuclear force, and the masses of different mesons are different. Since any ground energy corresponds to the binding energy and has a certain value, the calculated average force constants are not the same. The π meson is the lightest meson, switching over to other heavier mesons in the Yukawa potential to describe the nuclear force will result in that the corresponding nuclear force constants become small. There is some important difference between many reported[30, 44, 45] nuclear force constants, and there seems to be no relation to each other. An accurate nuclear force constant should ensure that the absolute value of the average ground energy is just the average binding energy of the nucleus. Now, as one of the quantitative standards, the Yukawa Nuclear Ground Energy distribution function can unify the various deductions of the meson theory.

The exact binding energies of almost all nucleuses have been confirmed by experiments. There is one-to-one correspondence between the ground energy and the binding energy. According to the equation (7), one can work out conversely the average nuclear force constants of all nucleuses by numerical computation. To make use of the masses $m_{\pi^0}c^2 = 134.9766$MeV and $m_{\pi^\pm} = 139.57018$MeV[46], we calculated the average nuclear force constants of the lightest ^{2}H with average binding energy 1112.283Mev, and the results are $\sigma_D = 1.054487$ and 1.052947 respectively. In Table 1, it is listed for the average nuclear force constants corresponding to π^0 and $\pi^\pm$ respectively for the 118 stable nucleuses in the periodic table of the elements. Its range of distribution is 1.052947 to 1.264019. In calculation, the values of the binding energy of the nucleuses are taken from the 2012 atomic mass table[47]. These average nuclear force constants are obtained one by one by image solution method. The specific calculation is very time-consuming. Readers can check the accuracy of the calculation results. However, the accuracy of calculation does not mean that the conclusion is scientific, because the construction of Yukawa potential function is not strict scientific logic, and because the existence of meson and the construction of Yukawa potential are not necessarily causal, but just a coincidence. So can the experimental test these nuclear force constants to confirm the meson theory evolved by Yukawa potential? Let's look forward to the answer.

Table 1: Average nuclear force constants of nuclei mapped by Yukawa potential

Z	Nuclide	B/A (MeV)	σ_{π^0}	$\sigma_{\pi^\pm}$	Z	Nuclide	B/A (MeV)	σ_{π^0}	$\sigma_{\pi^\pm}$
1	^{2}H	1.112283	1.096527	1.093992	59	^{141}Pr	8.353998	1.400850	1.392488
1	^{3}H	2.827266	1.194528	1.189953	60	^{144}Nd	8.326922	1.400032	1.248724
2	^{4}He	7.073915	1.360720	1.353066	61	^{145}Pm	8.302656	1.399298	1.390964
3	^{7}Li	5.606439	1.310294	1.303546	62	^{150}Sm	8.261617	1.398055	1.389742
4	^{9}Be	6.462668	1.340364	1.333074	63	^{152}Eu	8.226577	1.396991	1.388697
5	^{11}B	6.927716	1.355928	1.348360	64	^{157}Gd	8.2035	1.396289	1.388088
6	^{12}C	7.680144	1.380115	1.372118	65	^{159}Tb	8.188796	1.395842	1.387568
7	^{14}N	7.475614	1.373654	1.365772	66	^{162}Dy	8.173449	1.395374	1.387108
8	^{16}O	7.976206	1.389573	1.381459	67	^{165}Ho	8.14696	1.394566	1.386315
9	^{19}F	7.779018	1.383210	1.365772	68	^{167}Er	8.131743	1.394101	1.385858
10	^{20}Ne	8.03224	1.391051	1.382868	69	^{169}Tm	8.114475	1.393573	1.385339
11	^{23}Na	8.111493	1.385250	1.393482	70	^{173}Yb	8.087433	1.392745	1.384526
12	^{24}Mg	8.260709	1.398227	1.389960	71	^{175}Lu	8.069151	1.392185	1.383975
13	^{27}Al	8.331548	1.400180	1.391822	72	^{178}Hf	8.049456	1.391580	1.383381
14	^{28}Si	8.447744	1.403671	1.395260	73	^{181}Ta	8.023418	1.390780	1.382595
15	^{31}P	8.481167	1.404673	1.396245	74	^{184}W	8.005088	1.390216	1.382041
16	^{32}S	8.493129	1.405031	1.396597	75	^{186}Re	7.981288	1.389483	1.381321
17	^{36}Cl	8.521932	1.405893	1.397443	76	^{190}Os	7.962112	1.388891	1.380740
18	^{40}Ar	8.595259	1.408080	1.399592	77	^{192}Ir	7.93901	1.388178	1.380039
19	^{39}K	8.557025	1.406941	1.398473	78	^{195}Pt	7.926565	1.387793	1.379661
20	^{40}Ca	8.551304	1.406770	1.398305	79	^{197}Au	7.91566	1.387456	1.379330
21	^{45}Sc	8.618915	1.408784	1.400284	80	^{201}Hg	7.897561	1.386895	1.378779
22	^{48}Ti	8.722986	1.411870	1.403315	81	^{204}Tl	7.880021	1.386352	1.378245
23	^{51}V	8.742096	1.412434	1.403870	82	^{207}Pb	7.869864	1.386036	1.377935

Z	Nuclide	B/A (MeV)	σ_{π^0}	$\sigma_{\pi^\pm}$	Z	Nuclide	B/A (MeV)	σ_{π^0}	$\sigma_{\pi^\pm}$
24	^{52}Cr	8.775967	1.414434	1.404852	83	^{209}Bi	7.847984	1.385357	1.377268
25	^{55}Mn	8.765009	1.413110	1.404534	84	^{210}Po	7.834344	1.384933	1.376851
26	^{56}Fe	8.790342	1.413857	1.405268	85	^{210}At	7.811661	1.384227	1.376158
27	^{59}Co	8.768025	1.413200	1.404622	86	^{222}Rn	7.694489	1.380565	1.372560
28	^{59}Ni	8.736578	1.412271	1.403710	87	^{223}Fr	7.683657	1.380225	1.372227
28	^{62}Ni	8.794546	1.413981	1.405390	88	^{226}Ra	7.661954	1.379544	1.371557
29	^{64}Cu	8.739068	1.412345	1.403782	89	^{227}Ac	7.650701	1.379190	1.371210
30	^{66}Zn	8.75963	1.412952	1.404378	90	^{232}Th	7.615024	1.378067	1.370106
31	^{70}Ga	8.70928	1.411464	1.402917	91	^{231}Pa	7.618419	1.378174	1.370211
32	^{73}Ge	8.705049	1.411339	1.402794	92	^{238}U	7.57012	1.376650	1.368714
33	^{75}As	8.700874	1.411216	1.402672	93	^{237}Np	7.574981	1.376803	1.368865
34	^{79}Se	8.695591	1.411059	1.402519	94	^{239}Pu	7.56031	1.368409	1.376340
35	^{80}Br	8.677653	1.410528	1.402519	95	^{243}Am	7.530168	1.375386	1.367472
36	^{84}Kr	8.717446	1.411706	1.403154	96	^{247}Cm	7.501926	1.374490	1.366593
37	^{86}Rb	8.696901	1.411098	1.402557	97	^{247}Bk	7.498935	1.374395	1.366499
38	^{88}Sr	8.732592	1.412154	1.403594	98	^{251}Cf	7.470495	1.373491	1.365612
39	^{89}Y	8.713987	1.411604	1.403054	99	^{252}Es	7.45724	1.373070	1.365197
40	^{91}Zr	8.69332	1.410992	1.402453	100	^{257}Fm	7.422191	1.371953	1.364100
41	^{93}Nb	8.664186	1.410129	1.402453	101	^{258}Md	7.409668	1.371553	1.363708
42	^{96}Mo	8.653974	1.409826	1.401307	102	^{259}No	7.400	1.371245	1.363405
43	^{99}Tc	8.613599	1.400128	1.408626	103	^{262}Lr	7.374	1.370414	1.362588
44	^{102}Ru	8.607392	1.408442	1.399947	104	^{261}Rf	7.371372	1.370329	1.362506
45	^{103}Rh	8.584157	1.407750	1.399267	105	^{262}Db	7.352	1.369709	1.361896
46	^{107}Pd	8.560893	1.407056	1.398586	106	^{266}Sg	7.332	1.369068	1.361267
47	^{108}Ag	8.542025	1.406493	1.398033	107	^{264}Bh	7.315	1.368522	1.360731
48	^{112}Cd	8.544738	1.406574	1.398112	108	^{277}Hs	7.255	1.366592	1.358834
49	^{115}In	8.516546	1.405732	1.397285	109	^{268}Mt	7.271	1.367107	1.359341
50	^{119}Sn	8.499449	1.405221	1.396782	110	^{281}Ds	7.220	1.365462	1.357725
51	^{122}Sb	8.468317	1.404288	1.395866	111	^{272}Rg	7.227	1.365688	1.357967
52	^{128}Te	8.448752	1.403701	1.395290	112	^{285}Cn	7.185	1.364330	1.356613
53	^{127}I	8.445487	1.403603	1.395193	113	^{284}Ed	7.174	1.363974	1.356263
54	^{131}Xe	8.423736	1.402950	1.394551	114	^{289}Fl	7.149	1.363163	1.355466
55	^{133}Cs	8.409978	1.402536	1.394145	115	^{288}Ef	7.136	1.362741	1.355052
56	^{137}Ba	8.391828	1.401990	1.393608	116	^{292}Lv	7.117	1.362123	1.354445
57	^{139}La	8.378043	1.401575	1.393200	117	^{283}Eh	7.097	1.361472	1.353805
58	^{140}Ce	8.376339	1.401523	1.393150	118	^{294}Ei	7.080	1.360918	1.353261

4 Conclusions and comments

As we all know, there is no meson in the nucleus, so how can the nuclear force maintain the stable structure of the nucleus through the meson? On the other hand, to establish the potential energy function of nucleon interaction through the solution of a differential equation, it is necessary to demonstrate the scientific rationality of this differential equation. The purpose of my calculation of Yukawa's ground state energy here does not mean to support Yukawa's theory, but only to supplement relevant calculation methods and explain the basic content of this theory so as to facilitate testing. Whether we establish or develop the meson theory of nuclear force, we should always consider that since the Yukawa potential is adopted, there is a ground state energy corresponding to this theory, and the ground state energy calculated by other methods must have the same accuracy as the result of the variational method (6).

According to the ground state energy and the binding energy of the atomic nucleus, the average interaction intensity of two adjacent nucleons in 118 stable elements is estimated, which is only the result corresponding to the meson theory. The gap between the results and the experiment shows the limitations of the meson theory For the derived Yukawa like potential, the corresponding ground state energy can be derived theoretically However, the average ground state energy derived from different nuclear force potential energy functions is different Since the average binding energy of atomic nucleus is unique, the results of calculating the average nuclear force constant are different by changing the form of Yukawa potential energy function An accurate average nuclear force constant must ensure that the ground state energy is consistent with the average binding energy of atoms The ground state energy of Yukawa is a standard for the unification of meson theory, and should be the object of experimental testing of Yukawa meson theory.

However, the average nuclear force constant of various nuclei can not be measured by other methods, or there is no other reasonable definition of the average nuclear force constant. The experiment cannot verify whether the ground state energy corresponding to Yukawa potential is true. From a theoretical point of view, Yukawa constructed a wave equation by applying an operator of quantum mechanics to an unknown function, and defined the exact solution of such a wave equation as a potential function. This operation is extremely lacking in scientific basis. The nuclear force constant is defined by the Yukawa potential, and the average nuclear force constant calculated by the ground state energy of the Yukawa potential belongs to a dead cycle. It should be pointed out that the construction of Yukawa potential belongs to far fetched mathematical calculation, and its infinite deduction and development just shows the spiritual essence of formal modern physics under the cover of gorgeous mathematics. The factual basis that best illustrates this view is that there are no so-called mesons in the light nucleus, otherwise the mass of the atomic nucleus is greater than known, or the mass of the nucleon is less than known.

There are no mesons in the nucleus. It can be seen that the so-called meson theory of nuclear force is just imagination and patchwork. Mesons have been vividly described as the medium of interaction between nucleons for nearly 100 years. Yukawa's meson theory of nuclear force is a microcosm of modern physical theory. The mass production method of modern physical theory, which has almost become witchcraft, has even been applied to the production of famous experimental reports[49-51]. It takes a lot of energy and time to expose such theories and experimental reports. However, for the reasonable nuclear force model to be established in the future, the relevant calculation methods introduced in this paper are applicable, which is another reason why I write this article carefully.

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