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The End of Dirac Hydrogen Equation in One Dimension

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The angular quantum number and radial momentum operator of Dirac's electron theory belongs to the formal mathematical definition, rather than the result of standard mathematical calculation. Whether such formal mathematical definition that changes the fundamental nature of physical logic is reasonable or not can be judged according to whether the definitions of the radial momentum operator and angular momentum eigenvalue are consistent with the calculation results of standard mathematics. Selecting a specific physical model can find the exact answer to the problem, which forces us to seriously deal with the similarities and differences between the formal mathematical definition and the standard mathematical calculation results. Here I discuss the one-dimensional Dirac equation model of hydrogen-like atom and give the standard mathematical calculation conclusion: the one-dimensional Dirac equation has four non-equivalent first-order differential equation systems; the four first-order differential equation systems can also be converted into a system of differential equations composed of two second-order differential equations with the same energy parameter. The mathematical process of accurately solving the general solution of the second-order differential equations is beyond the existing mathematical basis. However, it can be proved that the exact solution of the ground state and a specified excited state of the first-order differential equations does not exist, It means that the first-order Dirac differential equations and the corresponding second-order differential equations of the hydrogen-like atom in the one-dimensional case have no energy quantized eigensolutions. The expression of the radial momentum operator here has a clear conclusion that is different from the definition of Dirac electron theory. The one-dimensional Dirac hydrogen equation is actually a special case of zero angular momentum. The specific zero angular momentum is intentionally avoided by Dirac electron theory because its existence exposes the serious non-consistency of the Dirac equation. This proves that a large number of mathematical calculations on the exact solution of the one-dimensional Dirac equation of the hydrogen atom, which is respected by various scientific documents, belong to the spurious calculation of the expected results. The one-dimensional Dirac equation is thus ended.

Keywords: Dongfang unitary principle; One-dimensional hydrogen atom; Dirac equation; Imaginary number energy; Existence and uniqueness theorems for solutions of differential equations.

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1 Introduction

Testing the reliability of mathematical computing processes in physics and the data provided by some well-known experimental reports has led to a large number of breakthrough discoveries that have subverted traditional knowledge, the most important of which is the widespread misconceptions in modern physics. Rather than continue to deify the beautiful and illusory stories that have been painstakingly written in the past, it is better to confront these facts directly, which is the fundamental reason why physics theory cannot achieve new breakthroughs.

The end[1,2] of Yukawa's nuclear meson theory[3-5], the end[6] of the Klein Gordon equation for Coulomb field[7-13], the end[14,15] of the teratogenicity theory for the Dirac hydrogen equation[16-32], the end[33] of the induced second order Dirac equation for hydrogen, the end [34]of the isomeric second order Dirac equation for hydrogen, the end[35] of the expected solution for the standard second order Dirac equation, the challenge solution[36] for the Dirac equation for hydrogen, the negation of the neutron state solution[37] and ground state solution[38] for the modified Dirac equation for hydrogen to the Dirac energy level formula, these new discoveries have made

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The mathematical calculations of all papers on the Dirac equation of one-dimensional hydrogen atom published in a large number of famous journals are distorted, and the so-called exact solutions given there are all pseudo solutions. This paper does not comment specifically on the pseudoscience theory unanimously advocated by famous journals one by one and only gives the inevitable mathematical solution and the final conclusion of the Dirac equation for one-dimensional hydrogen atom.

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it clear to us that relativistic quantum mechanics has evolved into a distorted logic that uses formal mathematics to piece together expectations, while relevant experimental reports have all claimed to verify the conclusions of actually distorted logic. Therefore, inferences that conform to standard mathematical operation rules are forced to be abandoned and denied because they do not meet expectations. From this perspective, modern physics not only hides many intentional or unintentional errors, but also hides serious ethical issues. From the physical perspective of whether an equation truly describes natural laws, we have to doubt the reliability of the Dirac equation. However, from a purely mathematical perspective, where differential equations can be written at will, the ingenious construction of Dirac equations and the introduction of interesting mathematical problems that are often unforeseen in writing differential equations still has great charm.

Applying Dongfang unitary principle can reveal more and fatal logical contradictions hidden in modern physics. The Dongfang unitary principle points out: *There is a definite transformation relationship between different metrics describing the natural law, and the natural law itself does not change due to the selection of different metrics. When the mathematical expression of natural laws under different metrics is transformed into one metric, the result must be the same as the inherent form under this metric, 1=1, meaning the transformation is unitary.*[39-43] Mathematical equations are one of the best tools for accurately describing the natural laws. According to Dongfang unitary principle, the results of the transformation of mathematical forms of natural laws from different metrics to the same metric must be unique. For example, the rectangular coordinate system, the cylindrical coordinate system, and the spherical coordinate system naturally constitutes three different metrics of wave equation theory. The wave equations of hydrogen atoms are handled in spherical coordinates. What new things does processing in cylindrical coordinates bring to mathematics? I have been using the Dongfang unitary principle to test complex and huge physical systems and some mathematical theories, and have made significant progress. Although attempts to publish these research results have encountered unprecedented resistance and failed in the past 40 years, today, with the advent of the Internet, any researcher has been able to freely publish their research and accept the test of readers. Truth, although often insignificant to individuals and groups who only pursue or maintain fame and fortune, has infinite charm for those who love it.

Since the birth of quantum mechanics, the mathematical essence of quantum mechanics has been deeply hidden. This is why it is difficult to solve those most fundamental quantum mechanical problems such as the morbid[42] equation of quantum numbers. Establishing wave equations and solving them accurately are the essence of quantum mechanics. Handling wave equations without considering the exact boundary conditions of atomic nucleus size and the boundary conditions written in atomic nucleus size constitute two metrics of wave equation theory. The unitary principle requires that wave equations either have consistent solutions under two boundary conditions, or make a choice between different exact solutions under two boundary conditions. We systematically tested the Dirac theory of hydrogen atoms using the unitary principle and obtained two clear conclusions: 1) The exact solution of the Dirac equation of hydrogen atoms under rough boundary conditions implies difficulties that cannot be eliminated, such as wave function divergence at the coordinate origin and imaginary energy, and therefore belongs to a formal solution; 2) The exact solution of the Dirac equation for hydrogen atoms under precise boundary conditions is an inevitable solution, which eliminates the difficulties of wave function divergence and imaginary energy. However, the accuracy of the obtained energy level formula is comparable to that of the Bohr energy level formula, which does not meet expectations. This requires looking for reasons from the origin of the establishment of wave equations, rather than writing seemingly profound qualitative explanations that can satisfy some people's wishes.

The mainstream solutions of the Klein-Gordon Coulomb field equation, coupled second-order Dirac hydrogen equation, and Dirac hydrogen equation of relativistic quantum mechanics has all been ended. Here, the Dirac equation for one-dimensional hydrogen atoms is further ended. If we choose three-dimensional wave equation and the one-dimensional wave equation as two metrics to test the theory of quantum mechanics wave equation, then according to the unitary principle, the exact solution of three-dimensional wave equation must contain the exact solution of the corresponding one-dimensional wave equation. I discussed in detail the Dirac equation for one-dimensional hydrogen atoms. Its exact solution cannot be obtained from the Dirac equation for three-dimensional hydrogen atoms, and the exact solution does not exist. This declared the end of the Dirac equation for one-dimensional hydrogen atoms.

2 Conclusions and comments

I have listed the standard one-dimensional Dirac equations for hydrogen like atoms and combined them into six systems of one-dimensional first order Dirac differential equation. Some one-dimensional first order Dirac differential equations have virtual coefficients, while some one-dimensional first order Dirac equations have real coefficients. Six one-dimensional first order Dirac differential equation systems correspond to two one-dimensional second order Dirac equations. Then it is proved in turn that the one-dimensional second order Dirac equation

has no expected solution, while the one-dimensional first order Dirac equation with imaginary and real coefficients has no expected solution, declaring the end of the one-dimensional Dirac hydrogen equation. The expression of the one-dimensional radial angular motion operator naturally occurring in the one-dimensional Dirac equation is contrary to the definition of the radial motion operator in the Dirac electron theory. It should be noted that other articles on the one-dimensional Dirac equation[59-84] actually have a wrong understanding of one-dimensional Dirac equation.

If we review the literature after independently completing the main calculations of the Dirac equation for one-dimensional hydrogen atoms, we will find that the independently derived conclusions are inconsistent with the conclusions of the papers on the relativistic wave equation for one-dimensional hydrogen atoms that have been published continuously since a long time ago. However, the process of independent calculation is rigorous! At this point, we began to change our cognition and noticed that the calculations in many papers actually seriously distorted the rules of mathematical operations. Compared to our learning habits of trying to memorize mathematical steps in textbooks and then dealing with some related exercises, closing textbooks to learn a new physics model and strictly following mathematical operation rules to independently deduce and then reading textbooks will always yield more and significantly different results. The latter allows us not to be distracted by generally accepted conclusions that are not actually true when trying to discover the essence of a problem, and in particular allows us to maintain the advantage of independent thinking to discover those undiscovered but crucial subversive conclusions. The educational system determines the criteria for talent selection, and a large number of talented people with strong memories have been given opportunities. As a result, textbooks and well-known mainstream journals have been cloned from generation to generation. The development of physics today, including numerous experimental reports and so-called achievements in experimental engineering, is fraught with errors and lies that we cannot discern and clarify using ordinary research methods.

Whether the Dirac equation has truly achieved brilliant achievements in physics is not the most important issue. According to Dongfang's unitary principle, the most important question is why the Dirac equation and Klein Gordon equation based on the same relativistic momentum and energy relationship are so different, regardless of whether the relativistic momentum relationship can be proven to be actually not true. Even assuming that there is a causal relationship between the Dirac matrix and the intrinsic spin of the electron that cannot be proven, the wave equation constructed by the Dirac matrix is actually a set of partial differential equations. So, why can't we directly construct a first order differential equation system that describes the hydrogen atom, or can we construct other different first order partial differential equations? The conclusion of every question is important. Then, what are the essential differences between the first order partial differential equations and the second order partial differential equations, so that we have to choose Dirac's first order partial differential equations and abandon the second order partial differential equations when dealing with the so-called relativistic effects of hydrogen atoms? Whether viewed from a physical or mathematical perspective, the problems posed by the Dirac equation may be enough to change the traditional cognition of mathematicians and physicists. Regarding the Dirac equation for hydrogen atoms, the three-dimensional Dirac equation still has a mathematically meaningful formal eigensolution. So why is the one-dimensional Dirac equation forced to end because it has no formal eigensolution due to complex number energy? In fact, one-dimensional wave equations belong to the special case where the traditional angular quantum number $l = 0$ in quantum mechanics, while the angular quantum number defined by Dirac is $\kappa = \pm 1, \pm 2, \cdots$, excluding the traditional zero angular quantum number, which just avoids the serious logic difficulties hidden in one-dimensional equations. The intrinsic solution of the three-dimensional Dirac equation cannot contain the intrinsic solution of the one-dimensional Dirac equation, which is a logical contradiction in itself.

Numerous papers on the Dirac equation[59-84] for one-dimensional hydrogen atoms advocated by various scientific journals hide false calculations, absurd logic, and erroneous conclusions. The readers do not have to comment on intentional or unintentional errors one by one, but only to explore the correct handling and inference of the one-dimensional Dirac equation for hydrogen atoms. Here, I prove that the Dirac equation for one-dimensional hydrogen atoms does not have an intrinsic solution for energy quantization, thereby ending the Dirac equation for one-dimensional hydrogen atoms. It is well known that the energy quantized form eigensolutions of the Dirac equation for three-dimensional hydrogen atoms are in line with expectations. According to Dongfang's unitary principle, differences in the conclusions of mathematical equations of one-dimensional, two-dimensional, and three-dimensional wave equations of the same physical model cannot lead to essential differences between the existence and non-existence of quantized energy eigensolutions. Therefore, the termination of the Dirac equation for one-dimensional hydrogen atoms has an important impact on the systematic testing of the theory of Dirac for hydrogen atoms.

From the Bohr model of the hydrogen atom, to the Schrodinger equation of the hydrogen atom, to the Dirac equation of the hydrogen atom, to quantum computation and quantum information, behind the flourishing phenomena presented by quantum mechanics, there are not only a large number of principled errors in cognition and computation, but also lies hidden by gorgeous math-

ematical forms. Scientific theories and experimental reports can tolerate computational flaws and cognitive errors, but they should not tolerate lies that intentionally distort mathematics and fabricate observational data to create physical achievements. Famous scientific journals should take the lead in ending the behavior of distorting mathematical calculations and fabricating observational data, and making real contributions to the development of strict scientific theories, rather than trying to stifle those great discoveries[85-87].

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