

Precision in $W+c$ production at the LHC

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Outline:

→ Predictions for $W+c!$...

→ ... and why you want to compute them

- *NNLO QCD predictions for $W+c$ -jet production at the LHC*

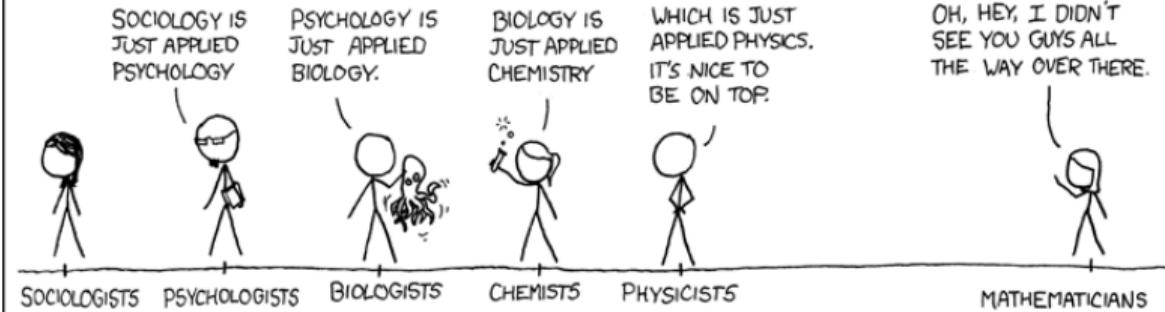
[Czakon, Mitov, MP, Poncelet; 2011.01011]

- *A detailed investigation of $W+c$ -jet at the LHC*

[Czakon, Mitov, MP, Poncelet; 2212.00467]

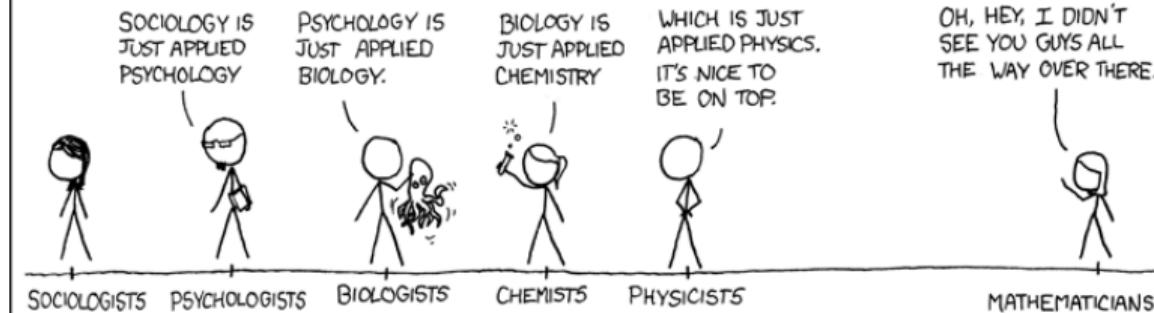
FIELDS ARRANGED BY PURITY

MORE PURE →

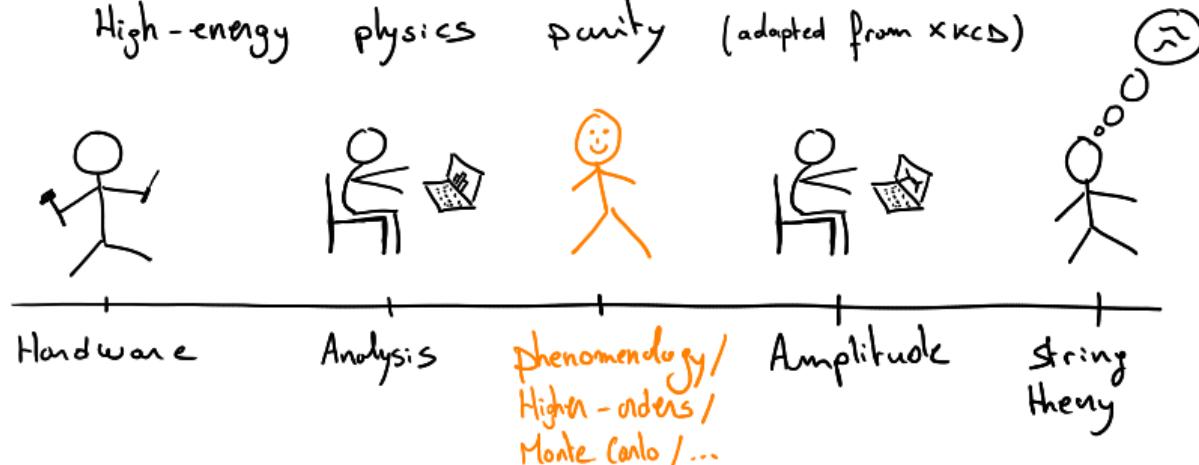


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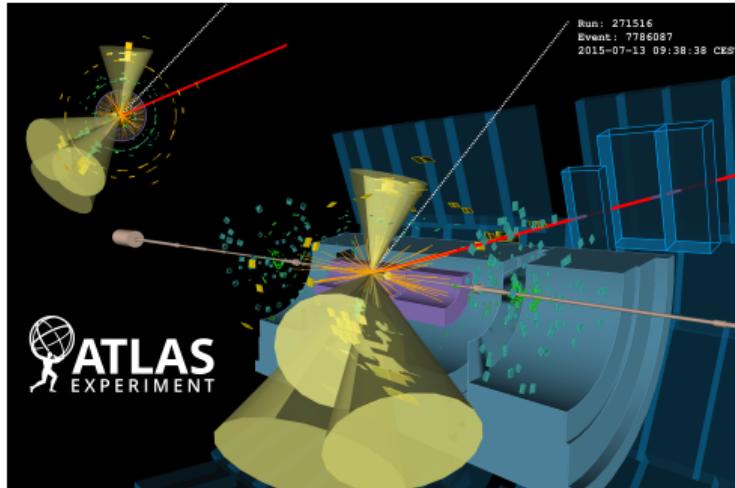
High-energy physics purity (adapted from XKCD)



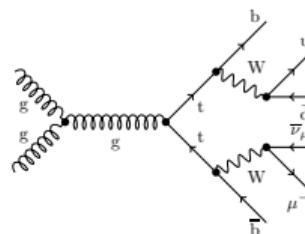


→ Illustration of Giordano Bruno's philosophical ideas (XVIth century)

LHC: Great tool to probe fundamental interactions at high energies
→ Cross talk between **experiment** and **theory**



↓
$$pp \rightarrow t^* \bar{t}^* \rightarrow (W^* \rightarrow \nu_\mu \mu^-) (W^* \rightarrow jj) b\bar{b}$$



- Greatest achievement of the LHC so far:

Discovery of the Higgs boson



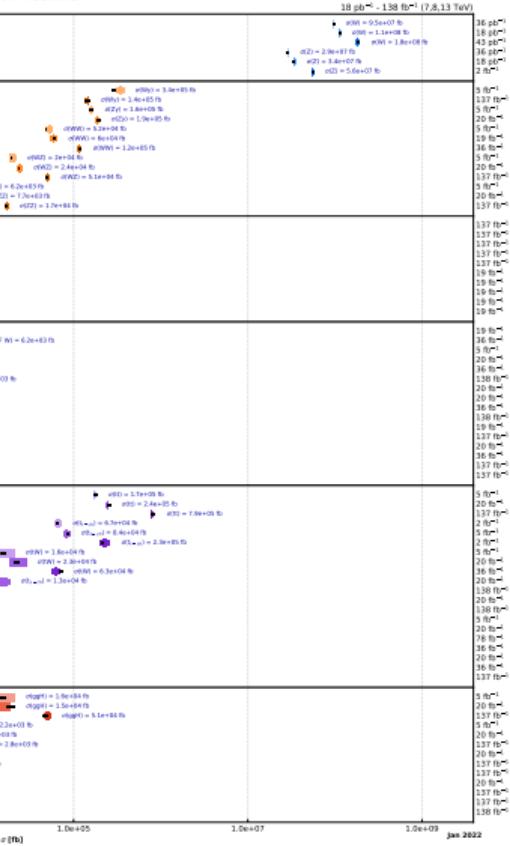
→ Great interest in measuring properties of the Higgs boson ...
... but there are also other interesting things

Overview of CMS cross section results

Measured cross sections and exclusion limits at 95% C.L.

Inner colored bars statistical uncertainty, outer narrow bars statistical+systematic uncertainty

Light-colored bars: ♀ 180. Dark
bars: ♂ 180. Hatched pattern:



→ Cross-sections measurements machine!

State of the art: W+j

- NLO QCD:

[Giele et al.; hep-ph/9302225], [Arnold et al.; Nucl.Phys. B319 (1989) 37-71, Phys.Rev. D40 (1989) 912], [Campbell et al.; hep-ph/0202176, 0809.3003, 1107.3714]

- NNLO QCD:

[Boughezal et al.; 1504.0213, 1602.06965], [Gehrmann-De Ridder et al.; 1901.11041], [MP et al.; 2204.12394]

- NLO EW:

[Kühn et al.; hep-ph/0703283, 0708.0476], [Hollik et al.; 0707.2553], [Denner et al.; 0906.1656]

- Combinations of QCD and EW corrections:

[Kallweit et al.; 1412.5157, 1511.08692], [Lindert et al.; 1511.08692], [Biederman, MP et al.; 1704.05783]

- NLO QCD for polarised W-jet:

[Bern et al.; 1103.5445] [Stirling, Vryonidou; 1204.6427]

- NNLO QCD for polarised W-jet:

[MP, Popescu, Poncelet; 2109.14336]

State of the art: W+c

- NLO QCD for W+c-jet:

[Giele, Keller, Laenen; hep-ph/9511449] [Stirling, Vryonidou; 1203.6781]

- NLO QCD+PS for W+c-jet:

[Bevilacqua, Garzelli, Kardos, Toth; 2106.11261]

- Study of charm production in context of strange PDF:

[Lai et al.; hep-ph/0702268], [Yalkun, Dulat; 1908.00026], [Faura et al.; 2009.00014]

- NNLO QCD Z+b-jet - [Gauld et al.; 2005.03016]

- NNLO QCD W+bb - [Hartanto et al.; 2209.03280]

→ This work:

[Czakon, Mitov, MP, Poncelet; 2011.01011, 2212.00467]:

NNLO QCD computation for W+c-jet production

Tools



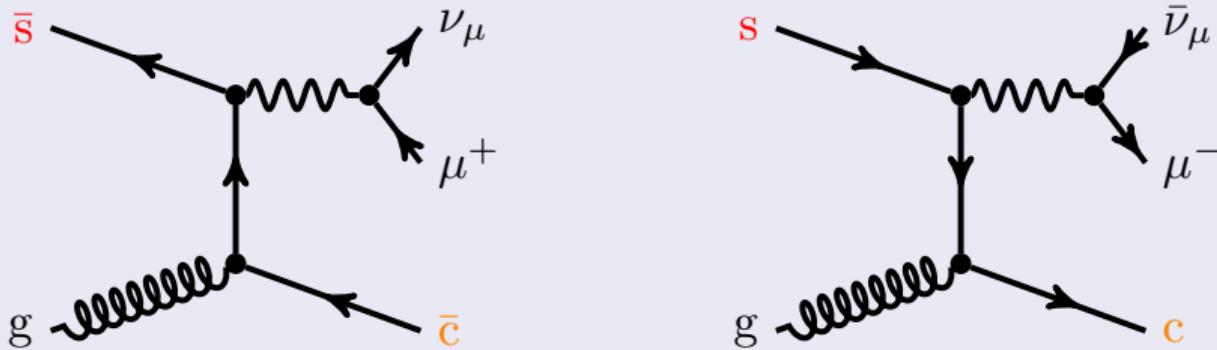
- Private Monte Carlo STRIPPER // MoCANLO
[Czakon, Heymes, Poncelet; 1005.0274, 1101.0642, 1408.2500] // [Denner, Feger, Lombardi, MP, Pelliccioli, Schmidt, Schwan]
- Tree level: AvH [Bury, van Hameren; 1503.08612]
- One-loop: OPENLOOP2 [Buccioni et al.; 1907.13071] // RECOLA [Actis et al.; 1211.6316, 1605.01090]
- Two-loop: [Gehrmann, Tancredi; 1112.1531]
→ using GINAC [Bauer, Frink, Kreckel], [Vollinga, Weinzierl; hep-ph/0410259]
- Complex-mass scheme [Denner et al.; hep-ph/9904472, hep-ph/0505042, hep-ph/0605312]
- PDF: LHAPDF [Buckley et al.; 1412.7420]

PART I

- *NNLO QCD predictions for $W+c$ -jet production at the LHC*

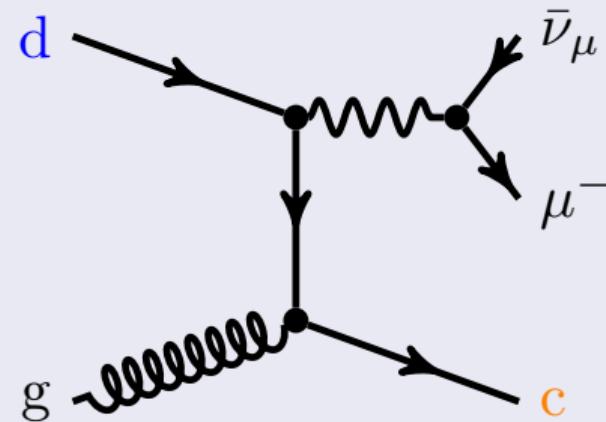
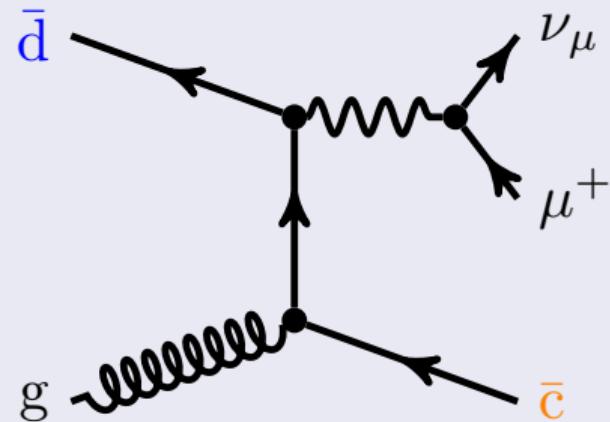
[Czakon, Mitov, MP, Poncelet; 2011.01011]

LO process (1)



- Direct link between $W+c$ measurements and strange PDF
 - main motivation to be interested in this process
- Test of (perturbative) QCD
 - $s-\bar{s}$ asymmetry predicted at 3-loop in QCD [Catani, de Florian, Rodrigo, Vogelsang; hep-ph/0404240]
- Study of flavour jets
- Further test of the Standard Model

LO process (2)



- With non-diagonal CKM matrix ($V_{cd} \neq 0$) ...
...more complicated interpretation in terms of strange PDF

LO process (3)

$$R_{W^\pm j_c} = \frac{\sigma_{W^\pm j_c}}{\sigma_{W^- j_c}} \sim (|V_{cs}|^2 \bar{s} + |V_{cd}|^2 \bar{d}) / (|V_{cs}|^2 s + |V_{cd}|^2 d)$$

PDF set	V_{cd}	$\sigma_{W^\pm j_c}$ [pb]	$\sigma_{W^- j_c}$ [pb]	$R_{W^\pm j_c}$
NNPDF31 LO	= 0	9.8395(4)	10.4654(4)	0.94020(5)
	$\neq 0$	12.0725(4)	14.2624(5)	0.84646(4)
NNPDF31 NLO	= 0	22.593(2)	23.718(2)	0.95260(6)
	$\neq 0$	24.500(9)	27.29(1)	0.8977(5)
CT18 NLO	= 0	21.675(2)	21.675(2)	1.0000(1)
	$\neq 0$	23.477(9)	25.252(8)	0.9297(5)

- Large differences between PDF sets ...

Inclusion of higher orders

$pp \rightarrow W^+ j_c$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	✓	✓	✓
sg	✗	✗	✓
$s\bar{s}$	✗	✓	✓
$\bar{s}\bar{s}$	✗	✓	✓
$\bar{s}q$	✗	✓	✓
qq'	✗	✓	✓
gq	✗	✗	✓
gg	✗	✓	✓

$pp \rightarrow W^- j_c$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	✗	✗	✓
sg	✓	✓	✓
$s\bar{s}$	✗	✓	✓
$\bar{s}\bar{s}$	✗	✓	✓
ss	✗	✓	✓
sq	✗	✓	✓
qq'	✗	✓	✓
gq	✗	✗	✓
gg	✗	✓	✓

- Higher-order corrections further complicates the picture
- Interpretation of $W+c$ -jet is not trivial

- Event selection

$$\begin{aligned} p_{T,\ell} &> 20 \text{ GeV}, & |\eta_\ell| &< 2.5 \\ p_{T,\text{miss}} &> 25 \text{ GeV}, & m_T^W &> 40 \text{ GeV}. \end{aligned}$$

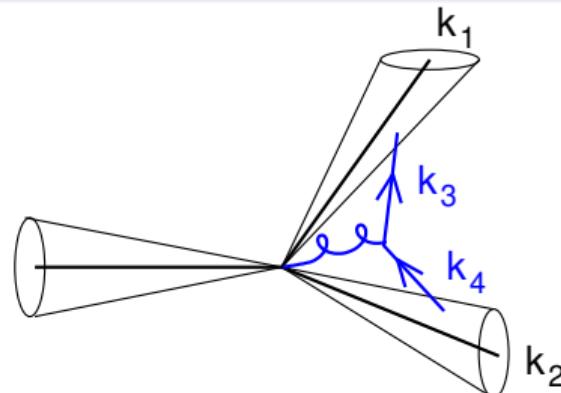
One and only one flavoured c-jet with:

$$p_{T,j_c} > 25 \text{ GeV}, \quad |\eta_{j_c}| < 2.5.$$

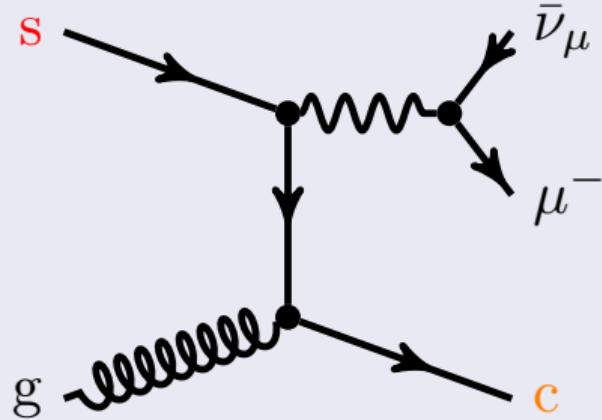
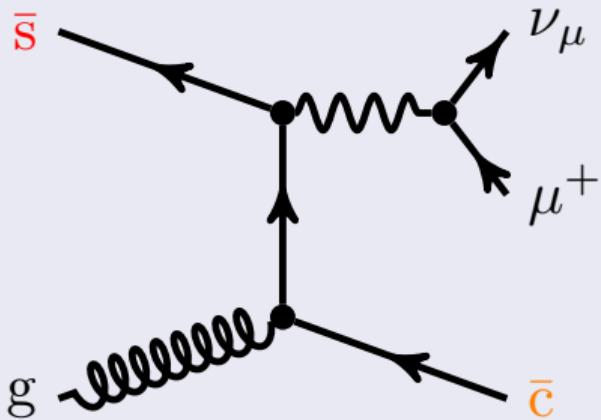
- NNPDF31 sets with $\alpha_s = 0.118$ [Ball et al.; 1706.00428]
- $\mu = \frac{1}{2} (E_{T,W} + p_{T,j_c})$ where $E_{T,W} = \sqrt{M_W^2 + (\vec{p}_{T,\ell} + \vec{p}_{T,\nu})^2}$
- flavour k_T algorithm with $R = 0.4$ [Banfi, Salam, Zanderighi; hep-ph/0601139]

Jet algorithm

- Beyond NLO, flavour jet algorithm is required
 - Otherwise not IR-safe definition of flavour jets
 - Large soft wide angle radiations are problematic
- flavour k_T algorithm with $R = 0.4$ [Banfi, Salam, Zanderighi; hep-ph/0601139]
 - Soft radiations are clustered first
 - rules:
 - $c + c = j$ or $c + \bar{c} = j$
 - $c + c + \bar{c} = j_c$ or $\bar{c} + c + \bar{c} = j_{\bar{c}}$



Features of the computation



- NNLO QCD computation to $pp \rightarrow \mu^+ \nu_\mu j_c$ and $pp \rightarrow \mu^- \bar{\nu}_\mu j_c$
- 5-flavour scheme
- PDF uncertainty computed at NNLO using [Carrazza et al.; 1602.00005]
- $V_{cd} \neq 0$ at LO when comparing against data

Th. vs. Exp. - cross section (1)

$V_{cd} \neq 0$

Order	$\sigma_{W^+ j_c}$ [pb]	$\sigma_{W^- j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+ j_c} / \sigma_{W^- j_c}$
LO	$12.0725(4)^{+11.6\%}_{-12.9\%}$	$14.2624(5)^{+11.6\%}_{-10.9\%}$	$0.84646(4)^{+1.48\%}_{-2.22\%}$
NLO	$35.164(9)^{+8.0\%}_{-7.0\%}$	$37.096(9)^{+7.5\%}_{-6.7\%}$	$0.9479(3)^{+0.49\%}_{-0.36\%}$
NNLO	$38.6(1)^{+2.2\% +3.8\%(PDF)}_{-3.2\% -3.8\%(PDF)}$	$39.3(1)^{+1.8\% +3.9\%(PDF)}_{-2.9\% -3.9\%(PDF)}$	$0.983(5)^{+0.45\% +2.7\%(PDF)}_{-0.37\% -2.7\%(PDF)}$

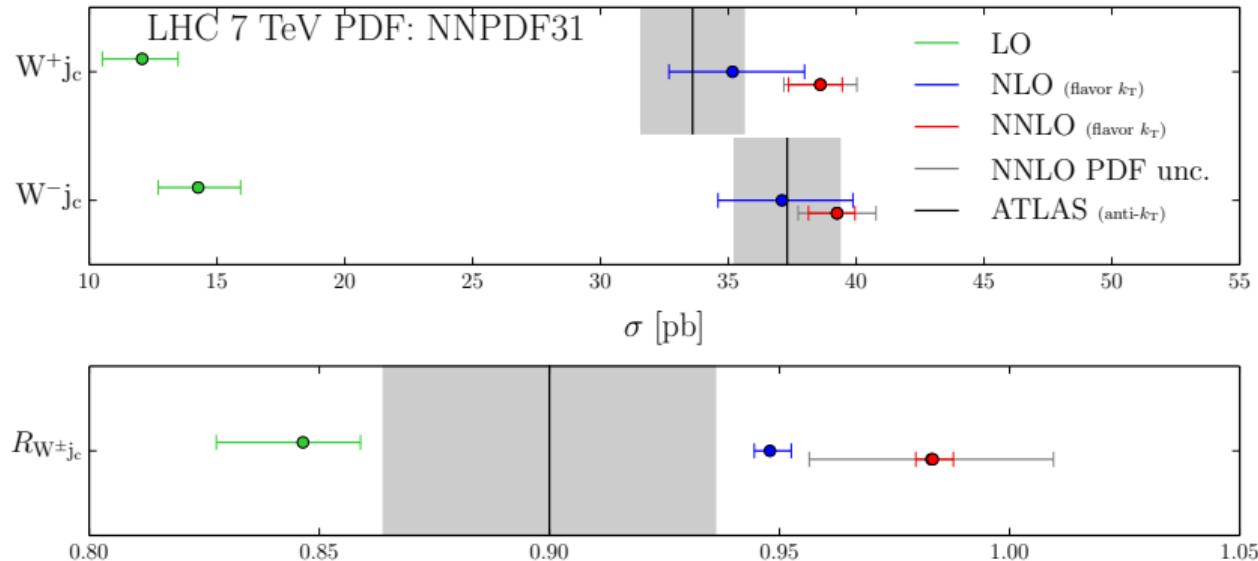
$$\sigma_{W^+ j_c}^{\text{ATLAS}} = 33.6 \pm 0.9 \text{ (stat)} \pm 1.8 \text{ (syst)} \text{ pb}$$

$$\sigma_{W^- j_c}^{\text{ATLAS}} = 37.3 \pm 0.8 \text{ (stat)} \pm 1.9 \text{ (syst)} \text{ pb}$$

$$R_{W^\pm j_c}^{\text{ATLAS}} = 0.90 \pm 0.03 \text{ (stat)} \pm 0.02 \text{ (syst)}$$

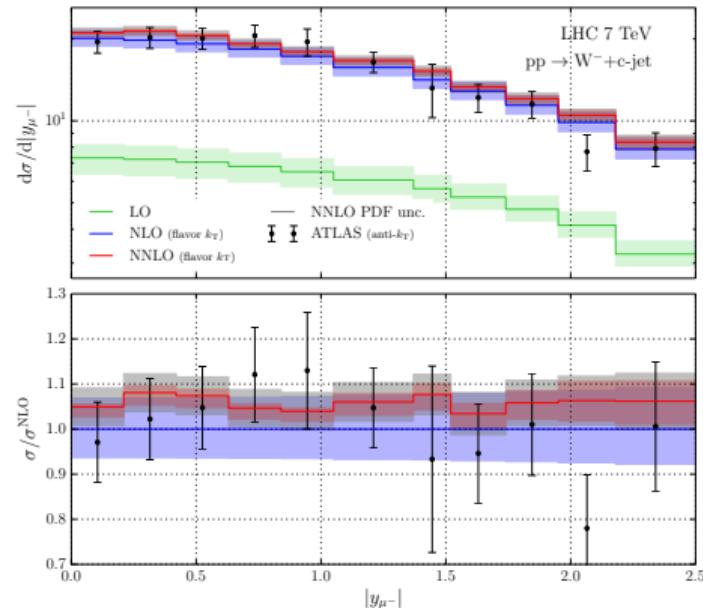
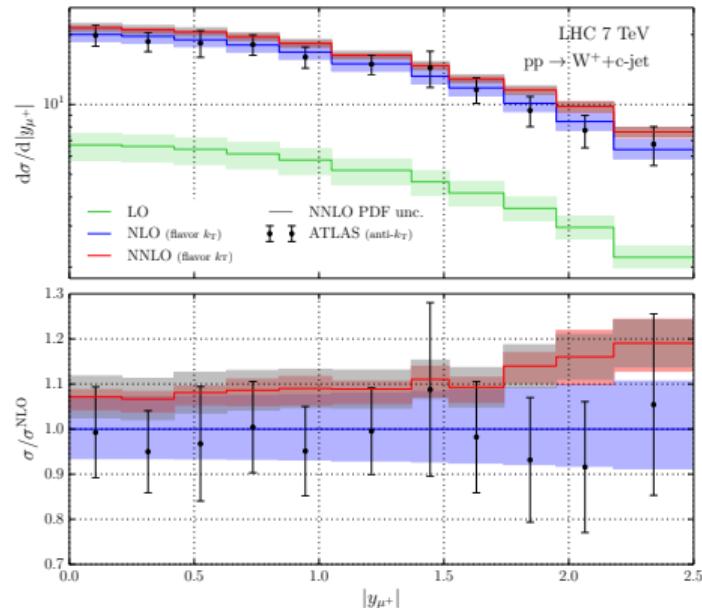
[ATLAS; 1402.6263]

Th. vs. Exp. - cross section (2)



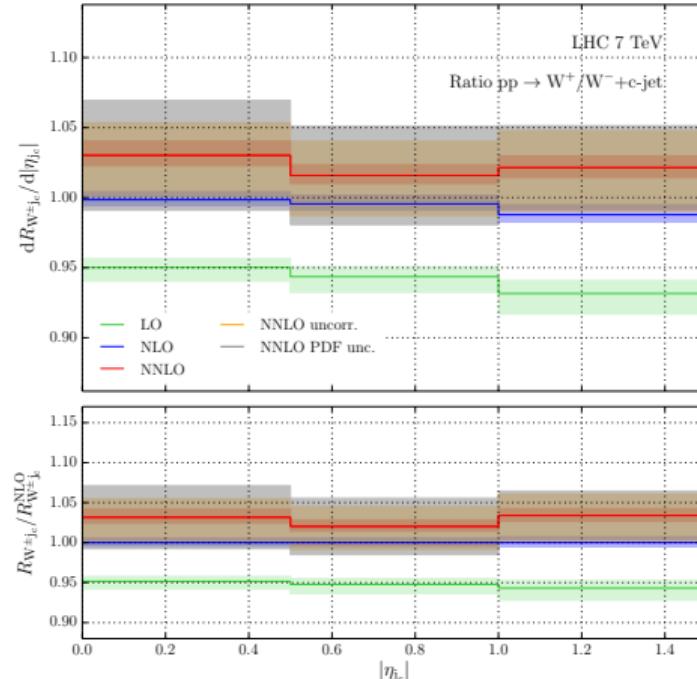
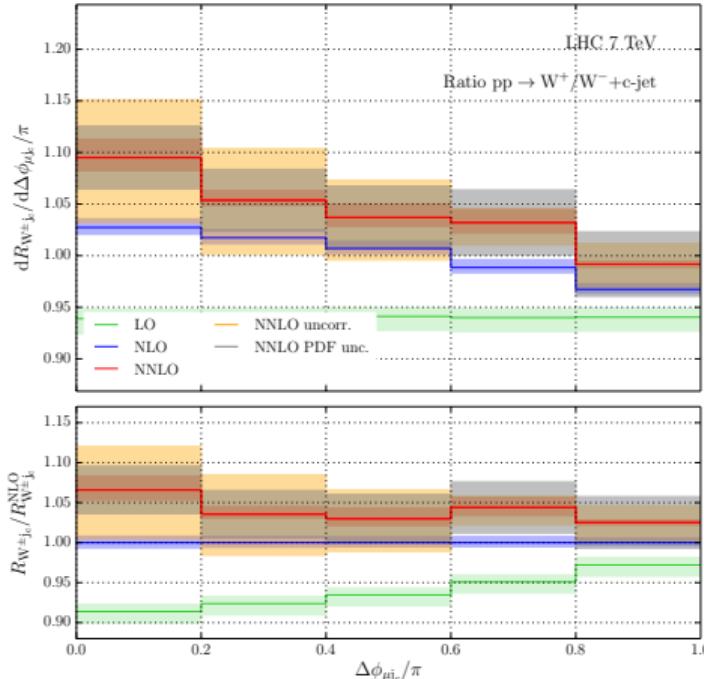
- PDF uncertainty dominant over NNLO scale uncertainty
- NNLO QCD prediction tends to be larger for the + signature
→ Not statistically relevant

Th. vs. Exp. - Differential distribution



Similar picture as for the total cross section
→ General good agreement

Differential distributions - ratio



As for total cross section, PDF uncertainty are dominant in ratios
 → Uncorrelated scale uncertainty more conservative

Discussion

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...
... but difficult to translate to $W + c$

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- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
 \sim few per cent

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 - ~ few per cent
- Absence of EW corrections ~ - few per cent

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 \sim few per cent
- Absence of EW corrections \sim - few per cent
- PDF uncertainty

Discussion

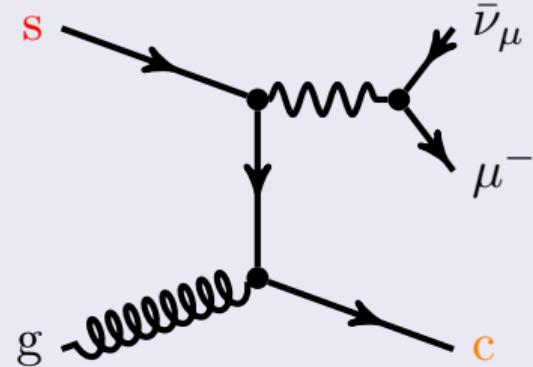
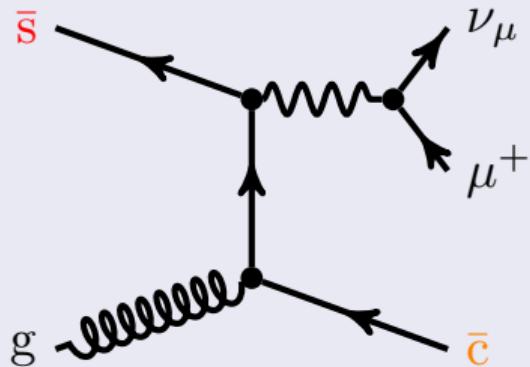
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... but difficult to translate to $W + c$
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ few per cent
- Absence of EW corrections ~ - few per cent
- PDF uncertainty
- Definition of the experimental measurement ?
 - D meson and not charm jet ?

PART II

- *A detailed investigation of $W+c$ -jet at the LHC*

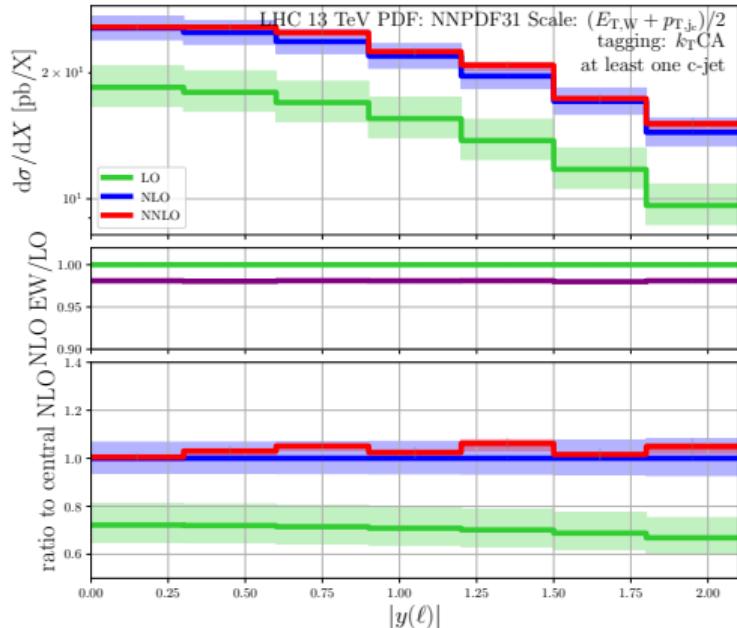
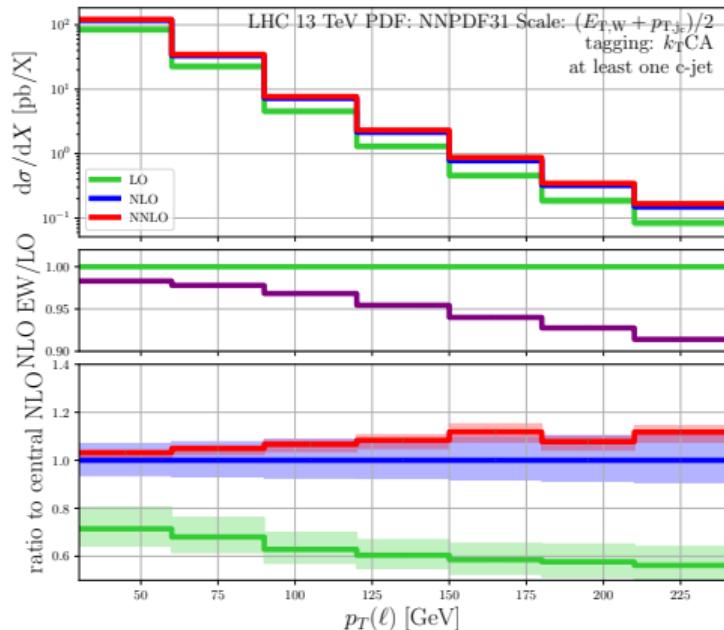
[Czakon, Mitov, MP, Poncelet; 2212.00467]

Feature of the (new) computation



- Full CKM dependence up to NNLO QCD
- NLO EW
- Study of flavour-jet algorithm
- Study of experimental definition
- 13 TeV setup

Best predictions @ 13 TeV - Differential distributions



- Good perturbative behaviour for QCD corrections
- Sudakov logarithm for EW corrections

Best predictions @ 13 TeV - cross sections

Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
LO	$113.817(2)^{+12.4\%}_{-9.87\%}$	$119.711(2)^{+12.4\%}_{-9.88\%}$	$0.95076(2)^{+0.013\%}_{-0.021\%}$
NLO	$162.4(1)^{+7.2\%}_{-6.6\%}$	$168.1(1)^{+6.9\%}_{-6.4\%}$	$0.9659(9)^{+0.29\%}_{-0.21\%}$
NNLO	$168.6(8)^{+0.7\% +3.8\%(PDF)}_{-2.1\% -3.8\%(PDF)}$	$173.9(1.9)^{+0.6\% +3.7\%(PDF)}_{-1.8\% -3.7\%(PDF)}$	$0.96(1)^{+0.2\% +2.1\%(PDF)}_{-0.3\% -2.1\%(PDF)}$

- PDF uncertainty dominant at NNLO QCD

Best predictions @ 13 TeV - cross sections

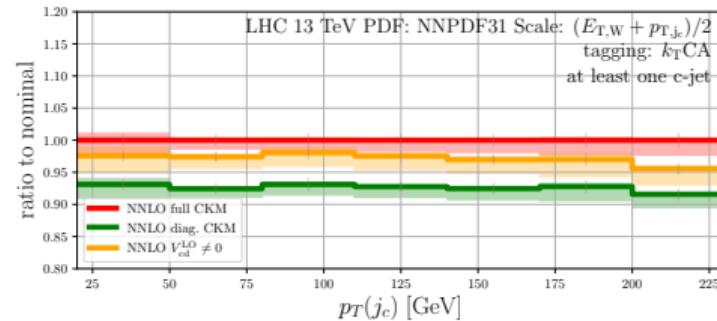
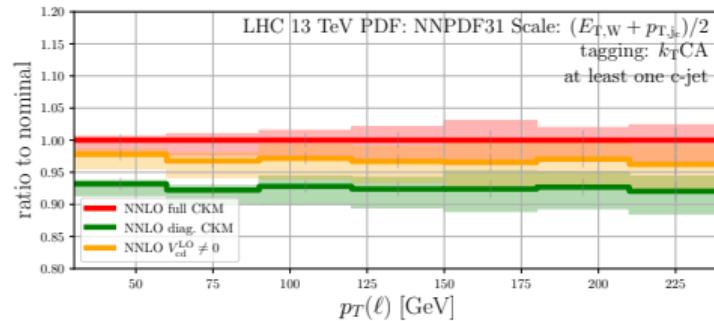
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NNLO	$168.6(8)^{+0.7\% + 3.8\%(\text{PDF})}_{-2.1\% - 3.8\%(\text{PDF})}$	$173.9(1.9)^{+0.6\% + 3.7\%(\text{PDF})}_{-1.8\% - 3.7\%(\text{PDF})}$	$0.96(1)^{+0.2\% + 2.1\%(\text{PDF})}_{-0.3\% - 2.1\%(\text{PDF})}$

- PDF uncertainty dominant at NNLO QCD

Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
NLO EW	$117.399(2)$	$111.627(2)$	$0.95084(2)$
$\delta_{\text{NLO EW}} [\%]$	-1.93	-1.92	-0.01

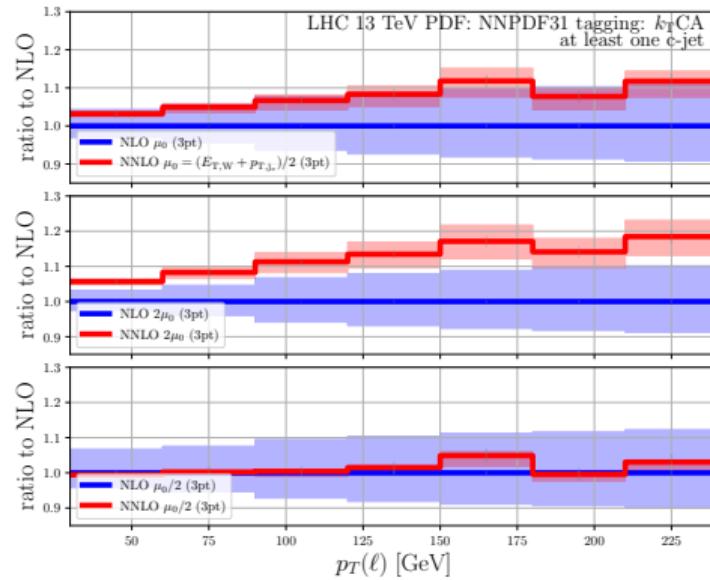
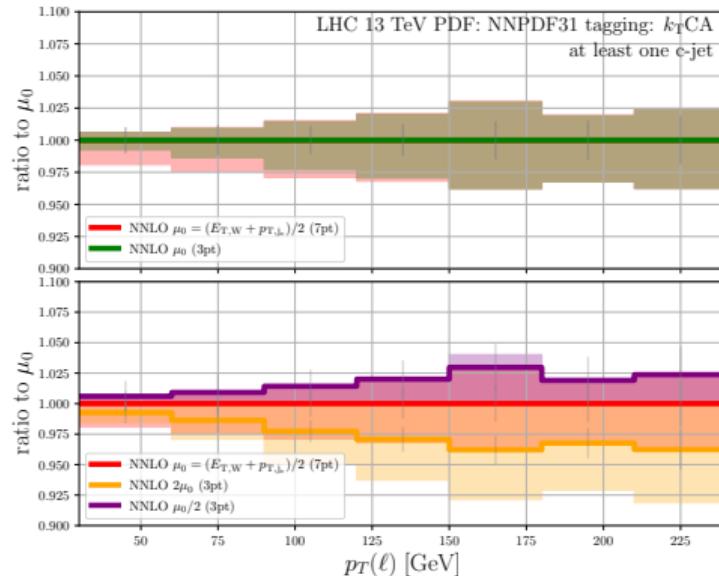
- EW corrections null in the ratio

Effect of non-diagonal CKM @ NNLO QCD



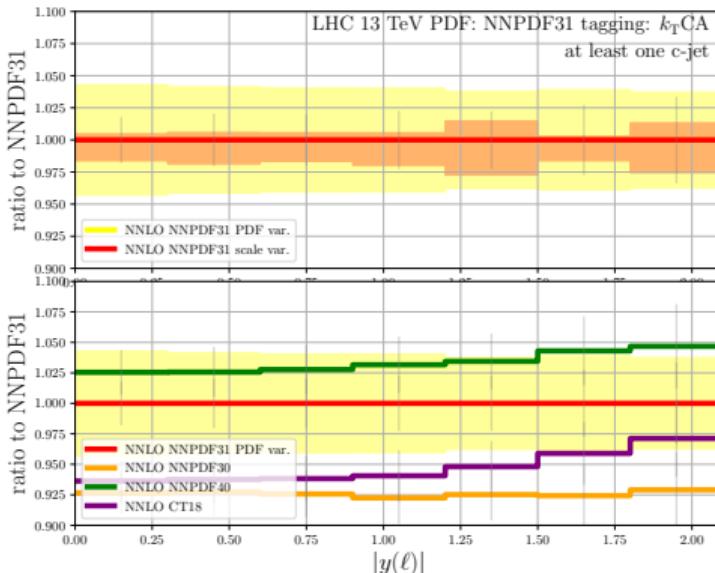
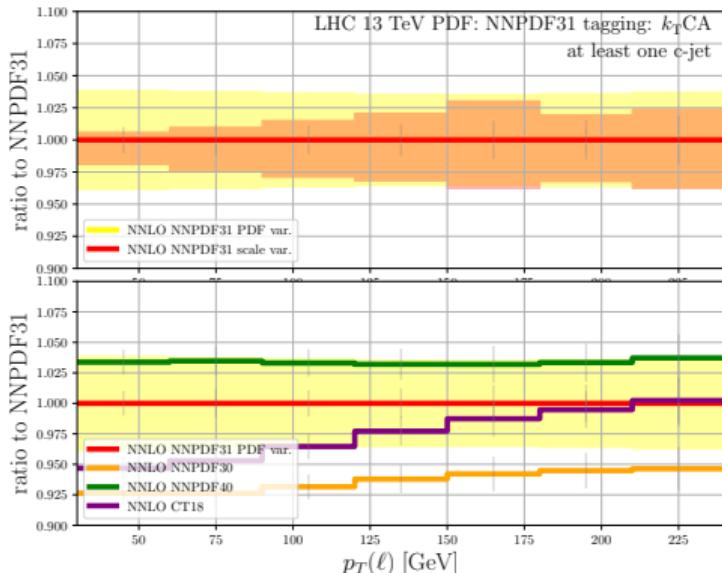
- full CKM / no CKM $\sim 7.5\text{--}11\%$
- full CKM / $V_{cd}^{\text{LO}} \neq 0 \sim 3\%$
 - Original approximation rather good
 - Full CKM dependence up to NNLO QCD for precise predictions

Scale setting



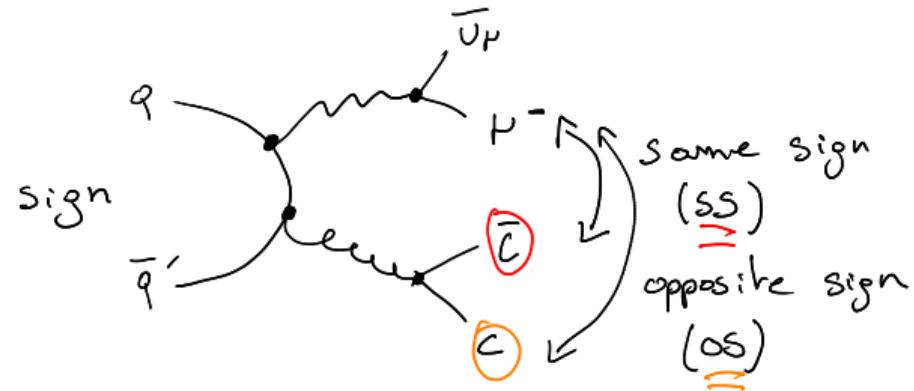
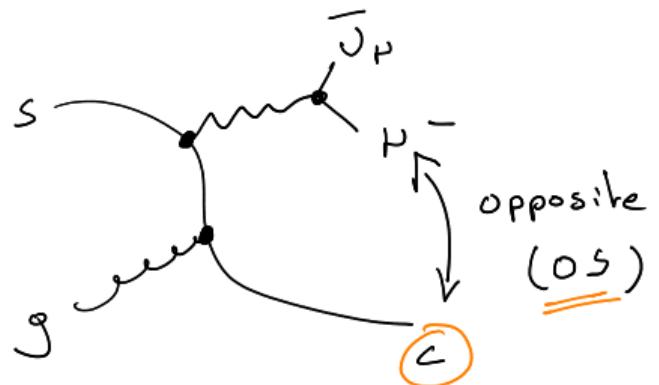
- $\mu_0 = \frac{1}{2} (E_{T,W} + p_{T,j_c})$
- For $p_{T,\ell}$, $\mu_0/2$ best / For p_{T,j_c} , μ_0 best / For cross section, $2\mu_0$ best
→ μ_0 good choice with good perturbative convergence

PDF uncertainty

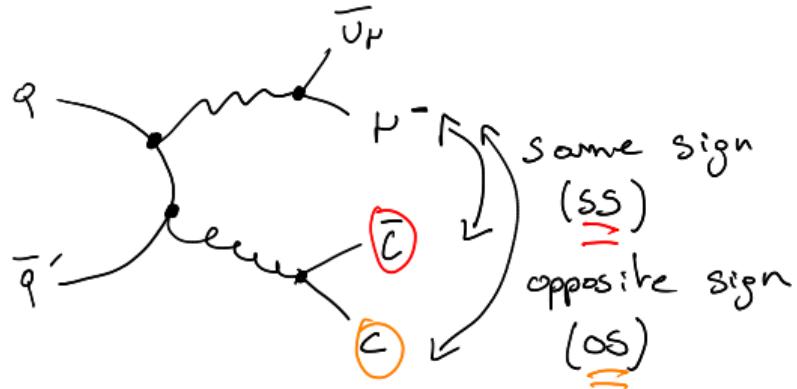
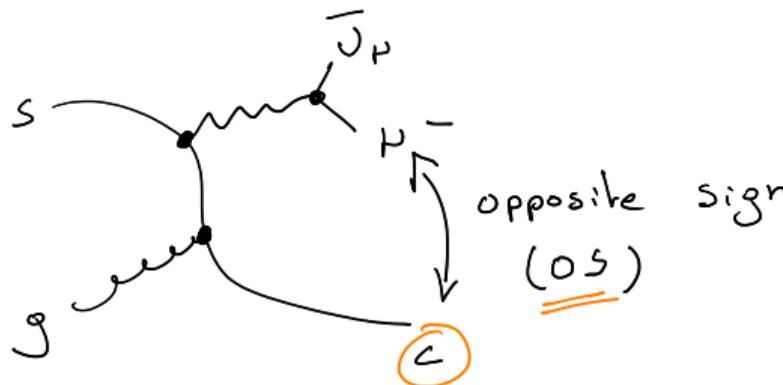


- NNPDF3.1 variation $\sim \pm 4\%$
- Spread of various PDF sets $\sim 10\%$
 - PDF error is the largest theoretical uncertainty

Event selection(s)

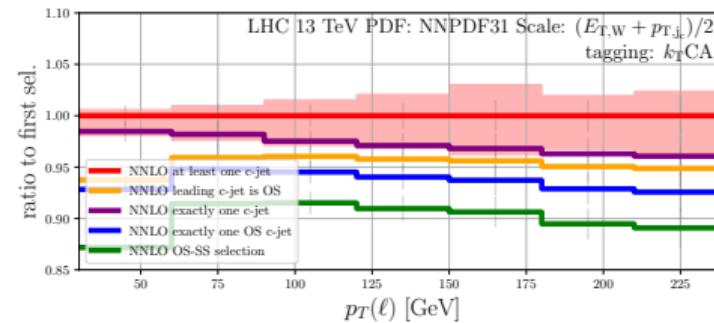
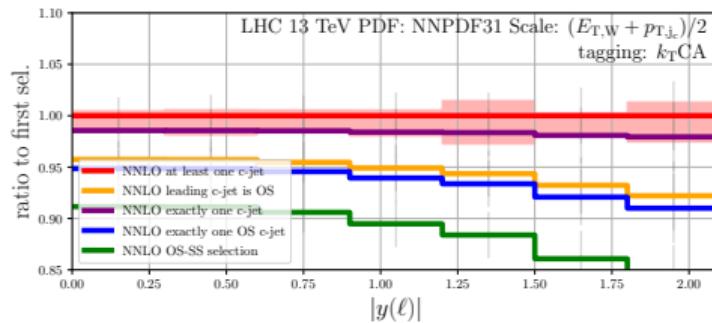
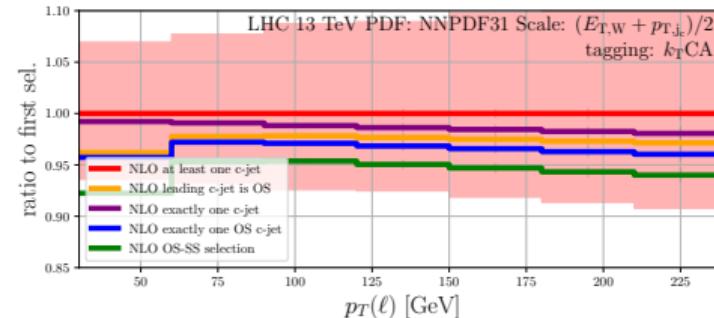
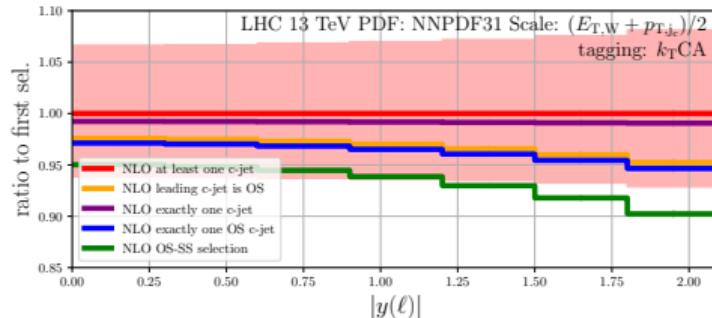


Event selection(s)



- Experiments measure OS-SS
 - More sensitivity to strange PDF
- Many possibilities...
 - most inclusive: at least one c-jet

Event selection(s)



- At NLO QCD, differences covered by scale uncertainty
- At NNLO QCD, differences $> 10-15\%$

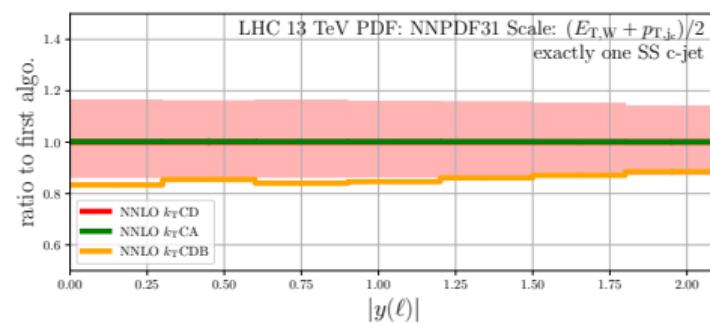
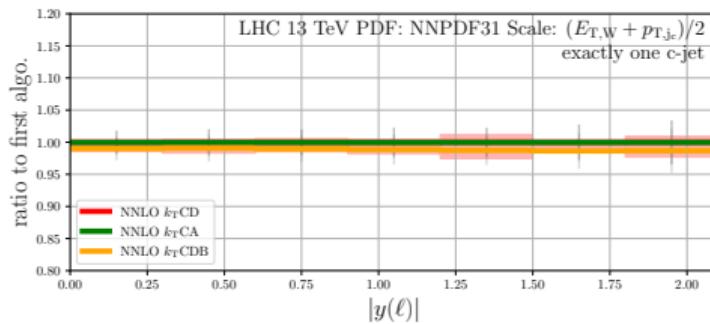
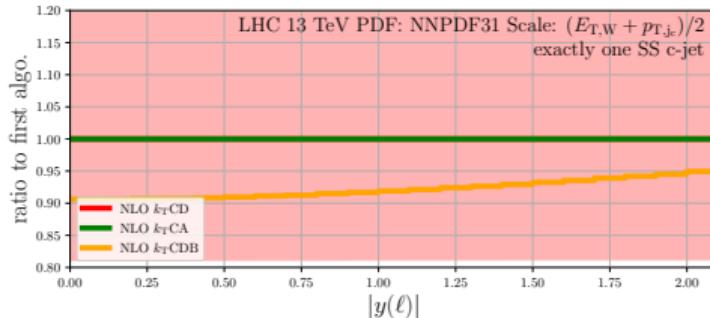
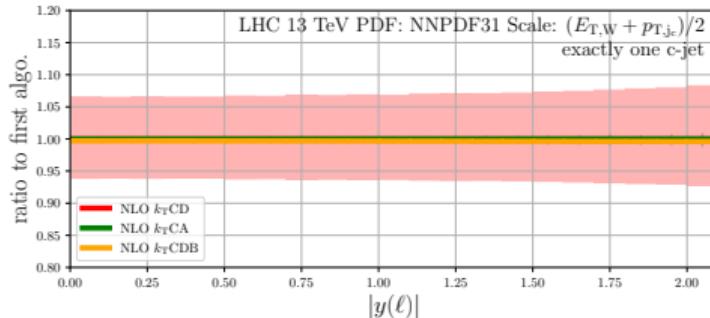
Jet algorithms - definitions (1)

→ Freedom in choosing whether cc and $\bar{c}\bar{c}$ are flavoured

Variation of flavour k_T algorithm [Banfi, Salam, Zanderighi; hep-ph/0601139]

- flavoured k_T algorithm, charge dependent ($k_{T\text{CD}}$)
- flavoured k_T algorithm, charge agnostic ($k_{T\text{CA}}$)
- flavoured k_T algorithm, charge dependent, with beam definition including W momenta ($k_{T\text{CDB}}$)

Jet algorithm (1)



- No difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet

Jet algorithms - definitions (2)

→ Flavoured anti- k_T algorithm

$$d_{ij}^{(\text{flavored})} = d_{ij}^{(\text{standard})} \times \begin{cases} S_{ij}, & \text{if both } i \text{ and } j \text{ have non-zero flavor of opposite sign,} \\ 1, & \text{otherwise.} \end{cases}$$

where

$$S_{ij} = 1 - \theta(1 - \kappa_{ij}) \cos\left(\frac{\pi}{2} \kappa_{ij}\right) \quad \text{with} \quad \kappa_{ij} \equiv \frac{1}{a} \frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}.$$

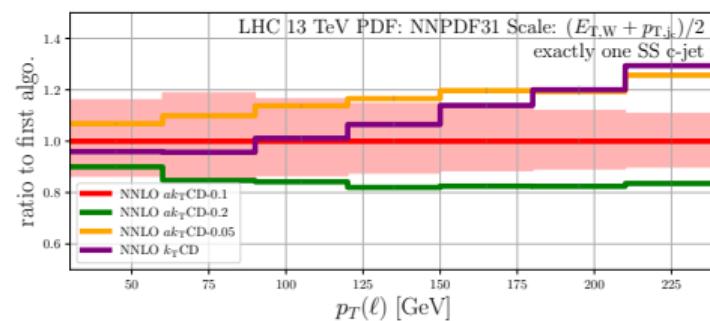
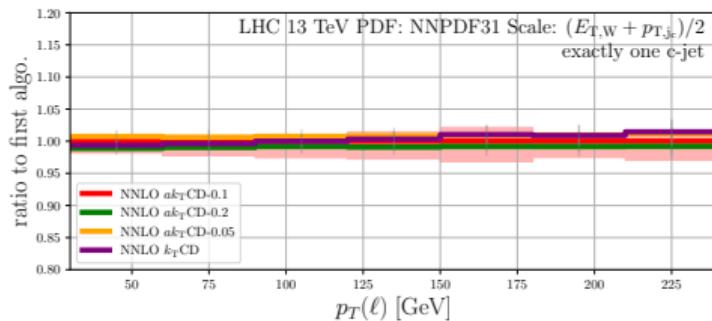
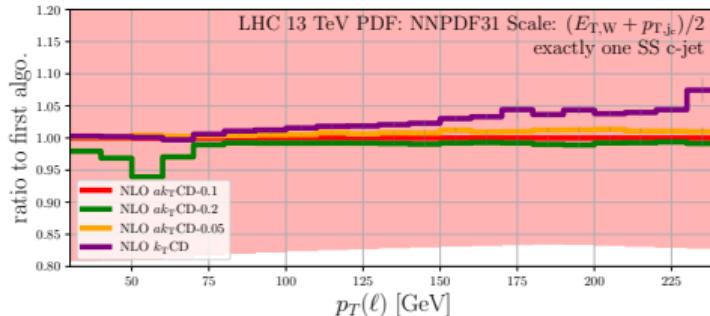
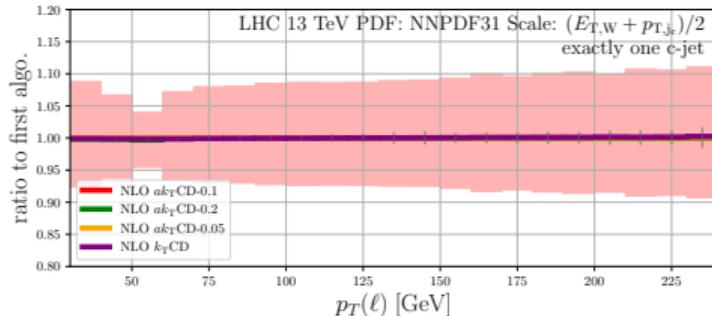
[Czakon, Poncelet, Mitov; 2205.11879]

Variation of anti- k_T algorithm

- flavoured anti- k_T algorithm, charge dependent, with $a = 0.2, 0.1, 0.05$
(ak_T CD-0.2, ak_T CD-0.1, ak_T CD-0.05)
- flavoured anti- k_T algorithm, charge agnostic, with $a = 0.1$ (ak_T CA-0.1).

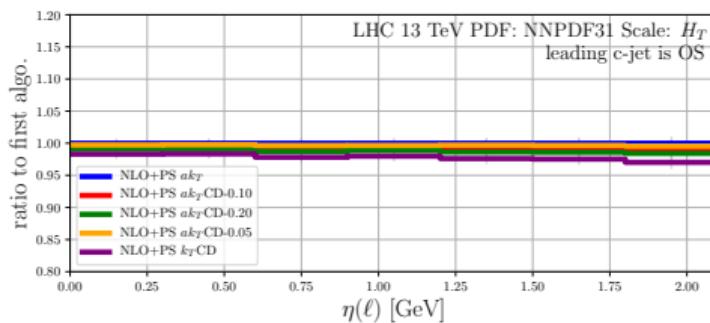
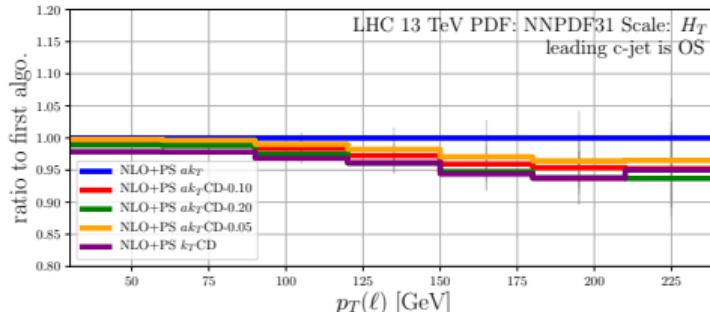
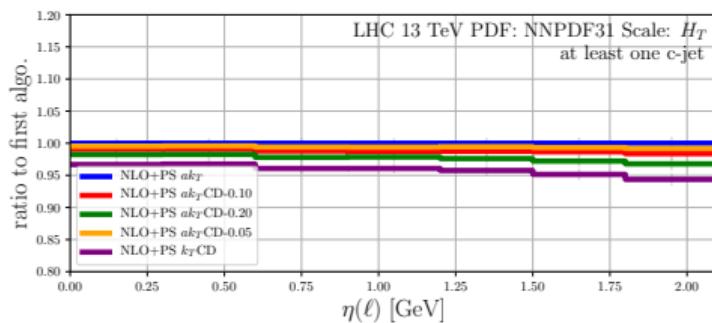
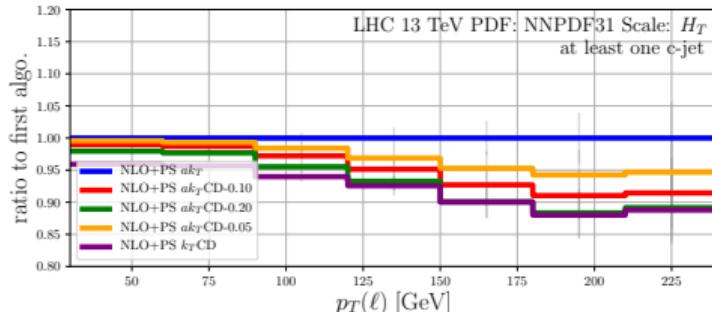
NB: Alternatives [Caletti, Larkoski, Marzani, Reichelt; 2205.01117, 2205.01109], [Gauld, Huss, Stagnitto; 2208.11138]

Jet algorithm (2)



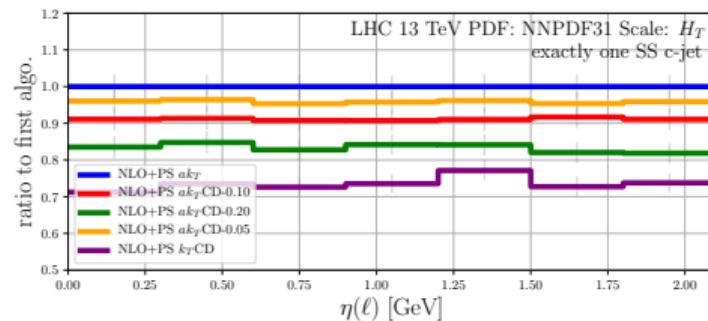
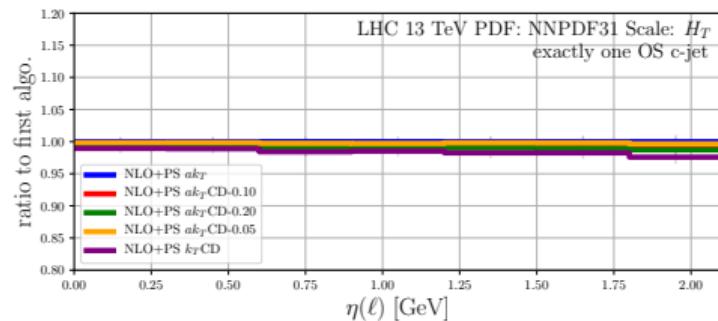
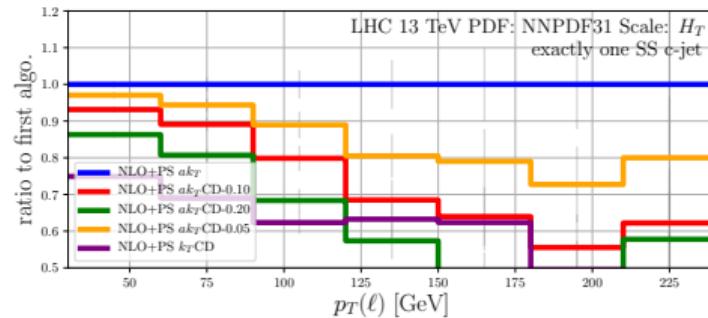
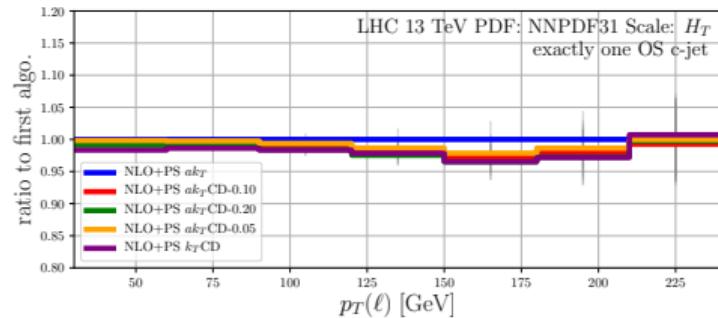
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Jet algorithm (3) at NLO+PS



- 5–10% differences for at least one c-jet (inclusive)
- Below 5% differences for leading c-jet is OS

Jet algorithm (4) at NLO+PS



- < 3% differences for exactly one OS c-jet
- Huge differences for exactly one SS c-jet
 - exactly one OS c-jet is preferred in this respect

Discussion reloaded

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gault et al., 2005.03016] ...
 - ... but difficult to translate to $W + c$ ✓ → < 3% for OS selection

Discussion reloaded

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Side remark: OS-SS rather artificial definition of the cross section
→ with higher orders, simple LO picture is broken

Summary - Detailed study of W+c

[Czakon, Mitov, MP, Poncelet; 2011.01011, 2212.00467]

- W+c at NNLO QCD+EW
- non-diagonal CKM effects
- PDF uncertainty
- Jet algorithm
- Experimental definition

→ All theoretical aspects under control apart from PDF

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→ All theoretical aspects under control apart from PDF

- Decisive information for SM measurements
 - Precision programme at the LHC
- Crucial interplay between theory and experiment
 - Big impact on physics results

BACK-UP

Cuts @ 13 TeV

- Charged lepton

$$p_{T,\ell} > 30 \text{ GeV}, \quad |\eta_\ell| < 2.5.$$

- At least one c-tagged jet

$$p_{T,j_c} > 20 \text{ GeV}, \quad |\eta_{j_c}| < 2.5.$$

CKM effect

$\sigma_{\text{NNLO}} \text{ [pb]}$	full CKM	$V_{cd}^{\text{LO}} \neq 0$	no CKM
+	$168.6(8)^{+0.7\%}_{-2.1\%} {}^{+3.8\%(\text{PDF})}_{-3.8\%(\text{PDF})}$	$164.4(8)^{+1.0\%}_{-2.4\%} {}^{+3.9\%(\text{PDF})}_{-3.9\%(\text{PDF})}$	$156.7(8)^{+0.7\%}_{-2.1\%} {}^{+4.2\%(\text{PDF})}_{-4.2\%(\text{PDF})}$
-	$173.9(1.9)^{+0.6\%}_{-1.8\%} {}^{+3.7\%(\text{PDF})}_{-3.7\%(\text{PDF})}$	$168.5(1.9)^{+1.0\%}_{-2.2\%} {}^{+3.8\%(\text{PDF})}_{-3.8\%(\text{PDF})}$	$156.7(1.9)^{+0.5\%}_{-1.6\%} {}^{+4.2\%(\text{PDF})}_{-4.2\%(\text{PDF})}$