Introduction to Math/Physics Academy

Colin Ford

Math/Physics Academy

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- Layout a foundation of learning materials (literature), educational resources (e.g. interactive web apps), and tutoring services ("evangelism")

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- How can I better engage students unfamiliar with vectors or programming?

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VS1 for any two elements $\boldsymbol{u}, \boldsymbol{v}$ in V, $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$

VS2 for any three elements $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ in V, $\boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w}$

VS3 there is a *zero* vector $\mathbf{0}$ in V such that $\mathbf{0} + \boldsymbol{v} = \boldsymbol{v} + \mathbf{0} = \boldsymbol{v}$

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and *scalar multiplication* $\cdot$ (by real or complex numbers) satisfying

VS5 for $\boldsymbol{u}, \boldsymbol{v}$ in V and a real number a: $a(\boldsymbol{u} + \boldsymbol{v}) = a\boldsymbol{u} + a\boldsymbol{v}$

VS6 for $\boldsymbol{v}$ in V and two real numbers a, b: $(a + b)\boldsymbol{v} = a\boldsymbol{v} + b\boldsymbol{v}$

VS7 for $\boldsymbol{v}$ in V and two real numbers a, b: $a(b\boldsymbol{v}) = (ab)\boldsymbol{v}$

VS8 the real number 1 acts on $\boldsymbol{v}$ in V as $1 \cdot \boldsymbol{v} = \boldsymbol{v} \cdot 1 = \boldsymbol{v}$

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- They already know things like $1 + 2 = 2 + 1$, 0 and 1 are special, $1/0 =$ undefined, associated jargon like associativity or commutativity, and so on

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and, if $\star$ is commutative, the group $(G, \star)$ is called *abelian*.

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- Illustrate with examples like integers under addition $(\mathbb{Z}, +)$, real numbers under multiplication $(\mathbb{R} - \{0\}, \cdot)$

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- the elements of the group K are called *scalars*, and the group action $\cdot$ is called *scalar multiplication*

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  - for a in $\mathbb{R}$, and $\boldsymbol{u}, \boldsymbol{v}$ in $\mathbb{R}^3$, vector addition and scalar multiplication look like:

    $$a\boldsymbol{u}+\boldsymbol{v} = a(u_1, u_2, u_3)+(v_1, v_2, v_3) = (au_1+v_1, au_2+v_2, au_3+v_3)$$

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- colors form a 3-dimensional vector space, one dimension for each primary color (green is often used over yellow, e.g. RGB)

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- Helpful for the unfamiliar student

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- Ideally the programming is next to the definitions, such as a projector screen alongside a chalkboard or whiteboard

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- I welcome any and all feedback to my proposals and opinions

Thank you

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