Supplemental Material
Enclosure Algorithm for the Fixed Points of Order-Reversing Maps
Application to Polynomial Systems for Chemical Equilibrium

Gilles Gnacadja

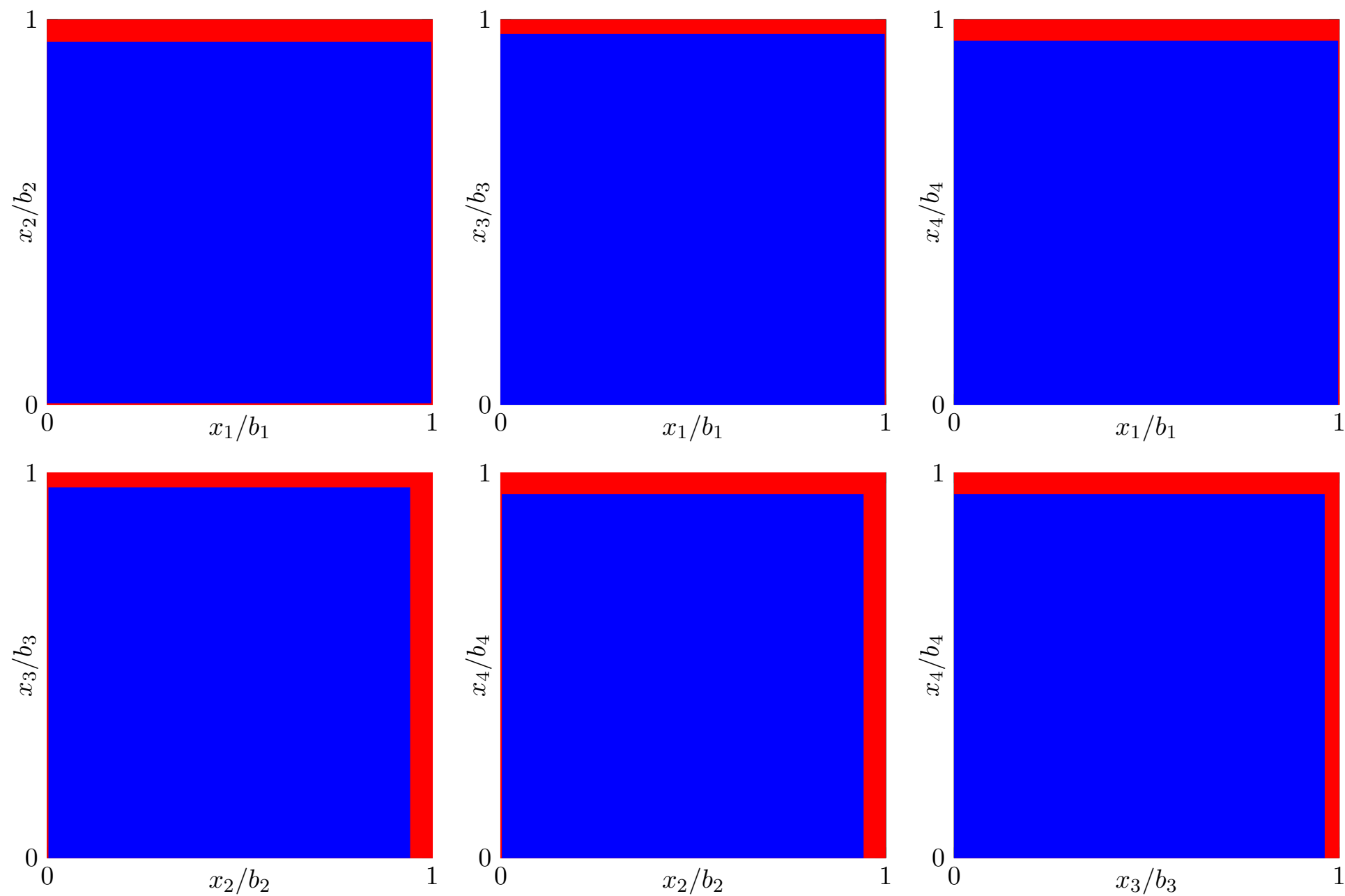Amgen, South San Francisco, California, USA

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We present additional illustrations of the calculations discussed in Section 6 of the main article. We show, for selected levels of box subdivision, the admissible and discarded regions in blue and red colors, respectively. We also highlight at each level the number of boxes that were examined, admitted and discarded.

Because the problem is four-dimensional, what we actually show are the six canonical two-dimensional projections. A collection of such six projections is not a faithful representation, in the sense that one cannot in general reconstitute the projected four-dimensional region from it. These graphics nevertheless provide an informative visualization of the progression of the algorithm.

For the implementation based on the geometric long-edge bisection, we first show the priming-stage fixed-point iteration, and then the first 7 of the 18 levels of box subdivision. At levels 8 through 18, the progression is not visually perceptible. We then proceed similarly for the implementation based on the arithmetic long-edge bisection. The priming-stage fixed-point iteration is the same, and following that are the first 18 of the 29 levels of box subdivision; at levels 19 through 29, the progression is not visually perceptible.
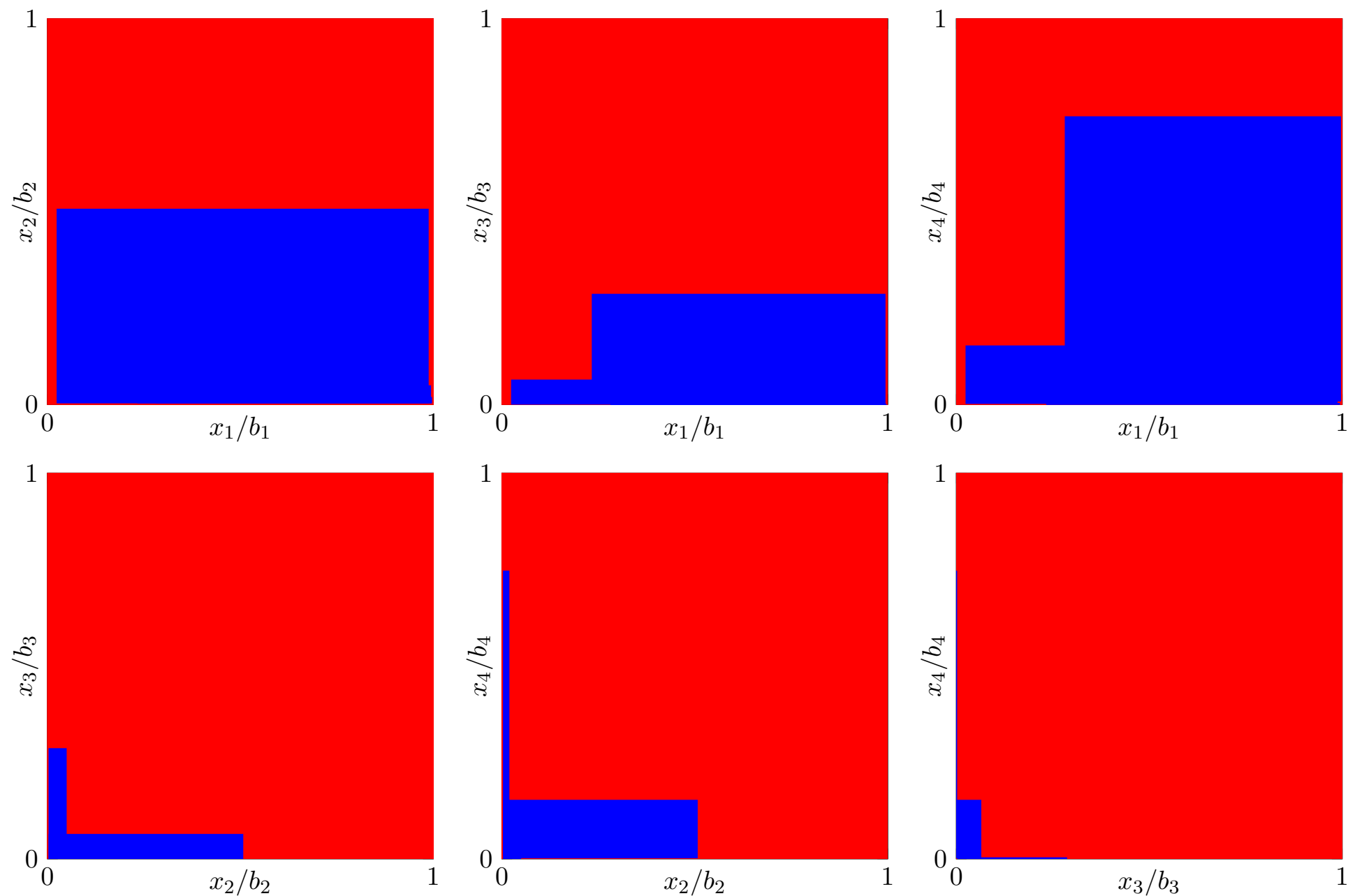
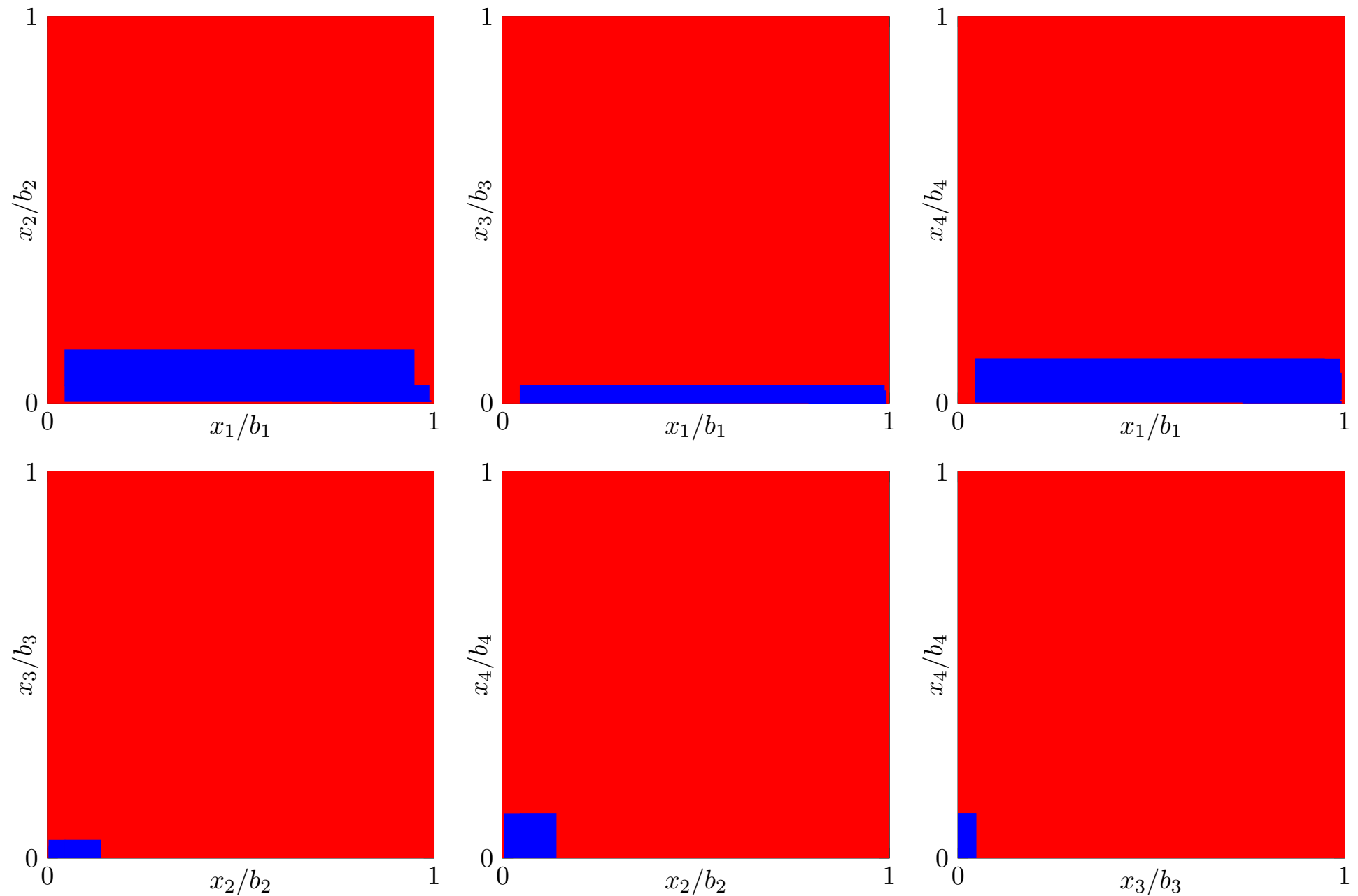## Priming fixed-point iteration – Stalled

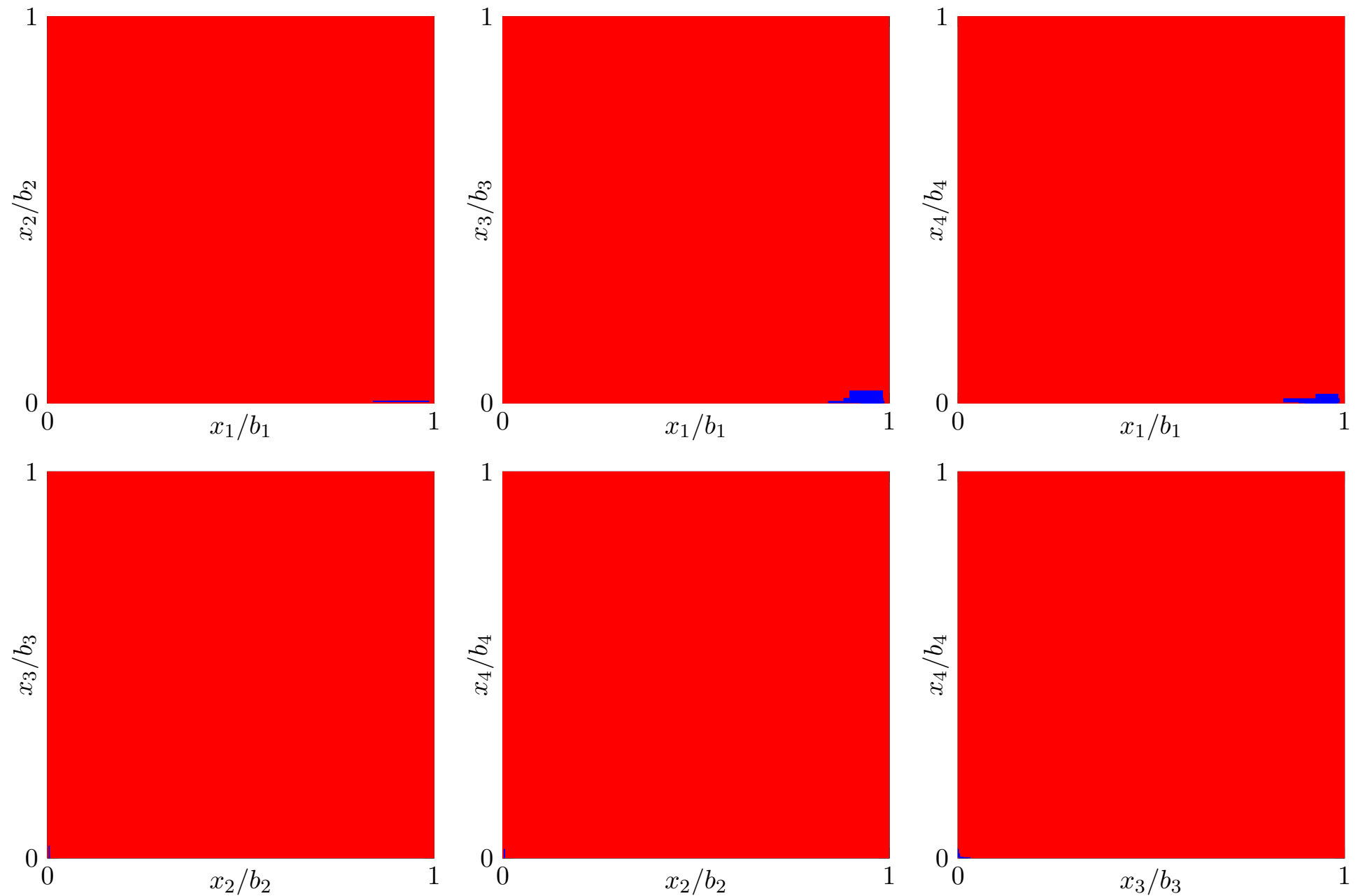**Geometric long-edge bisection – Subdivision level 1 of 18 – Number of boxes examined/admitted/discarded : 2/2/0**
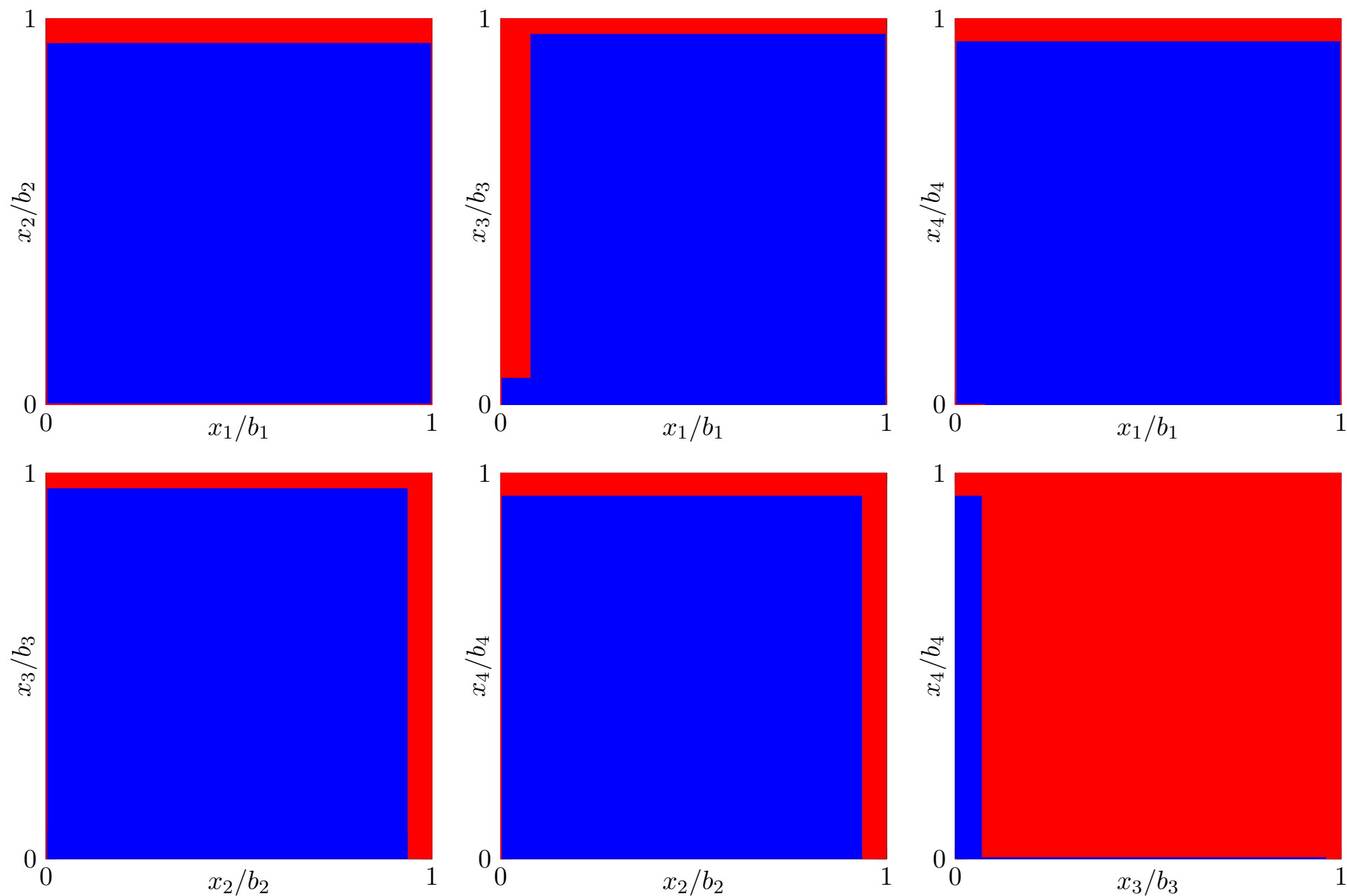
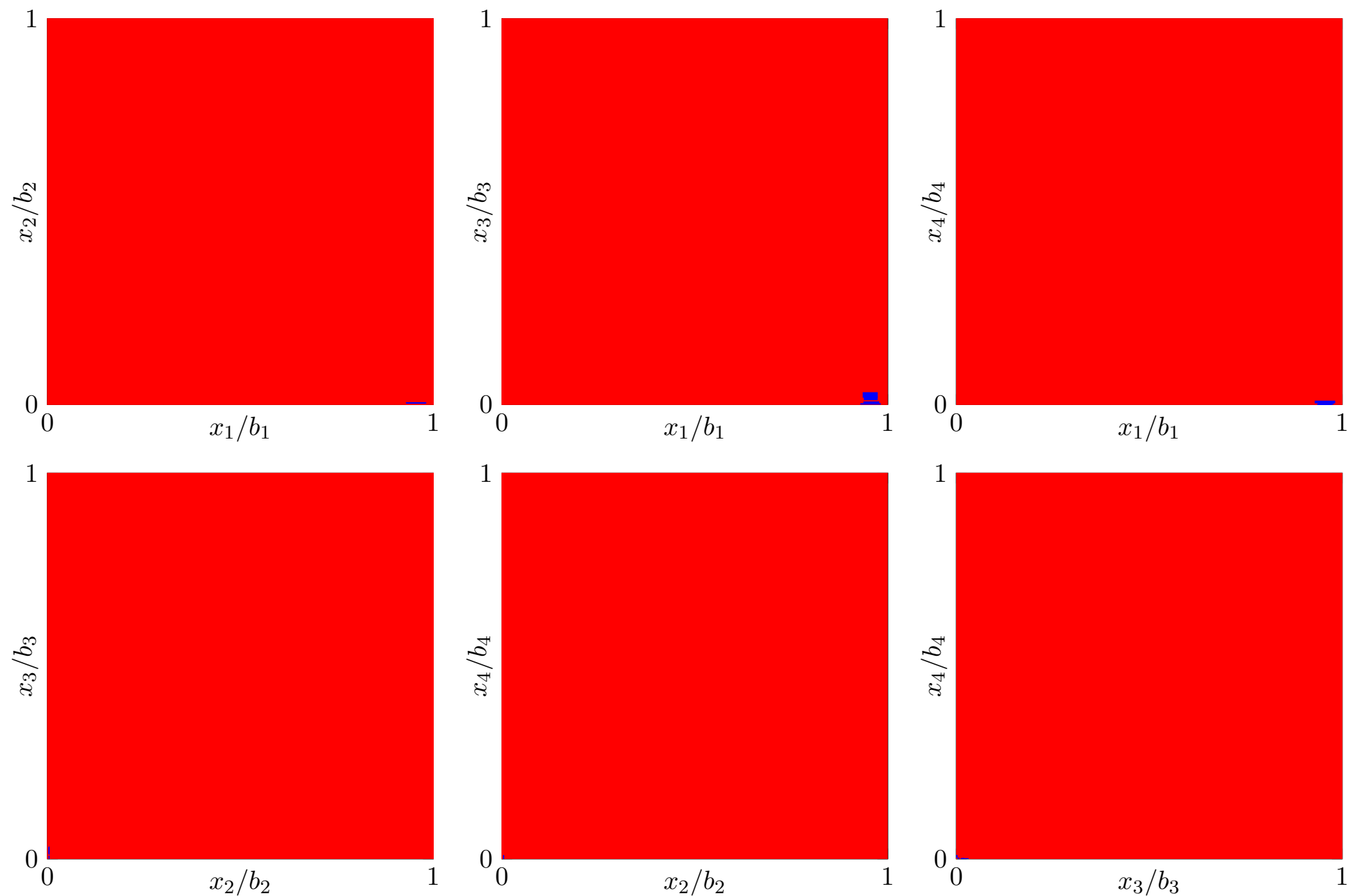**Geometric long-edge bisection – Subdivision level 2 of 18 – Number of boxes examined/admitted/discarded : 4/3/1**

Geometric long-edge bisection – Subdivision level 3 of 18 – Number of boxes examined/admitted/discarded : 6/4/2
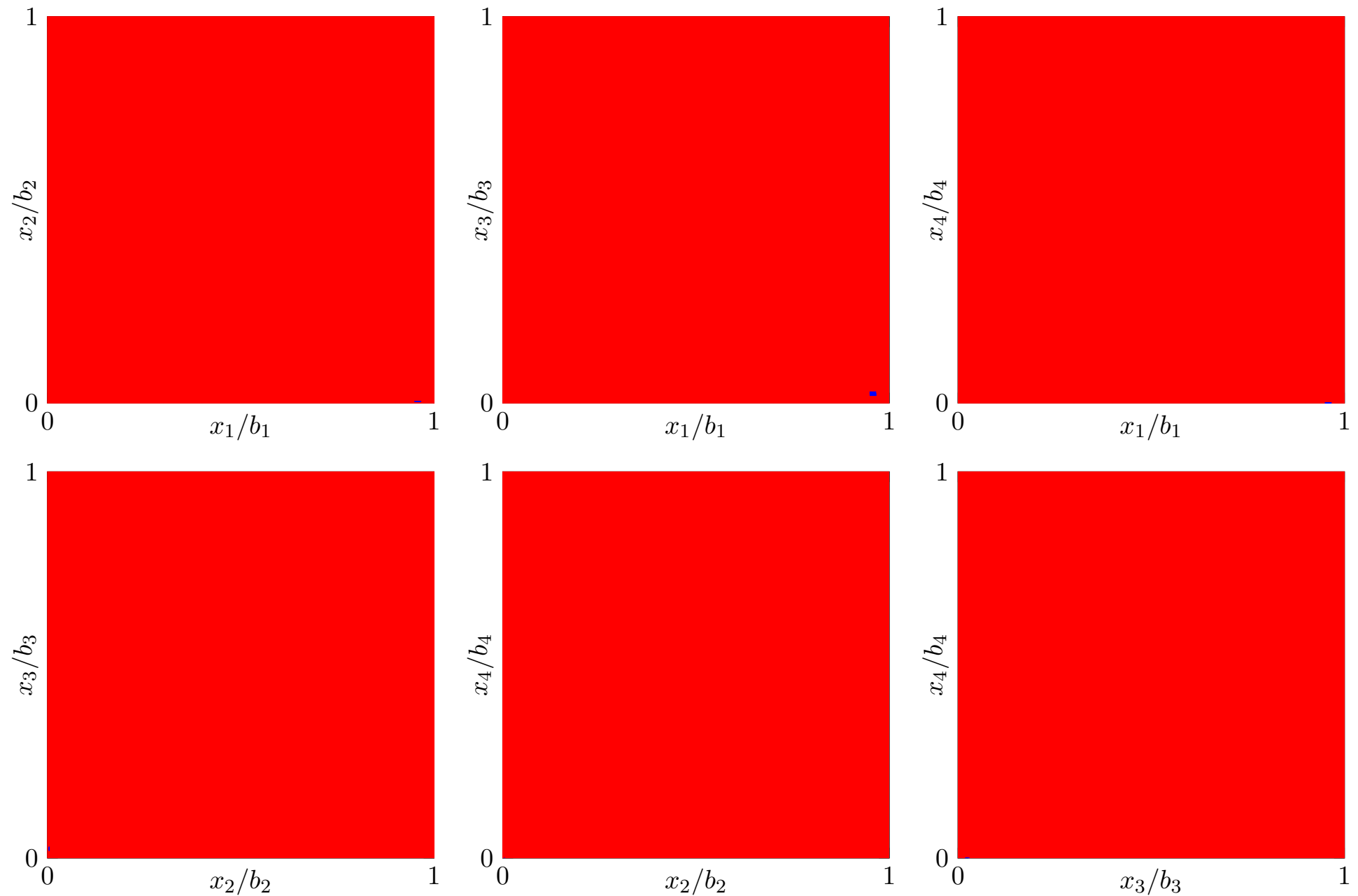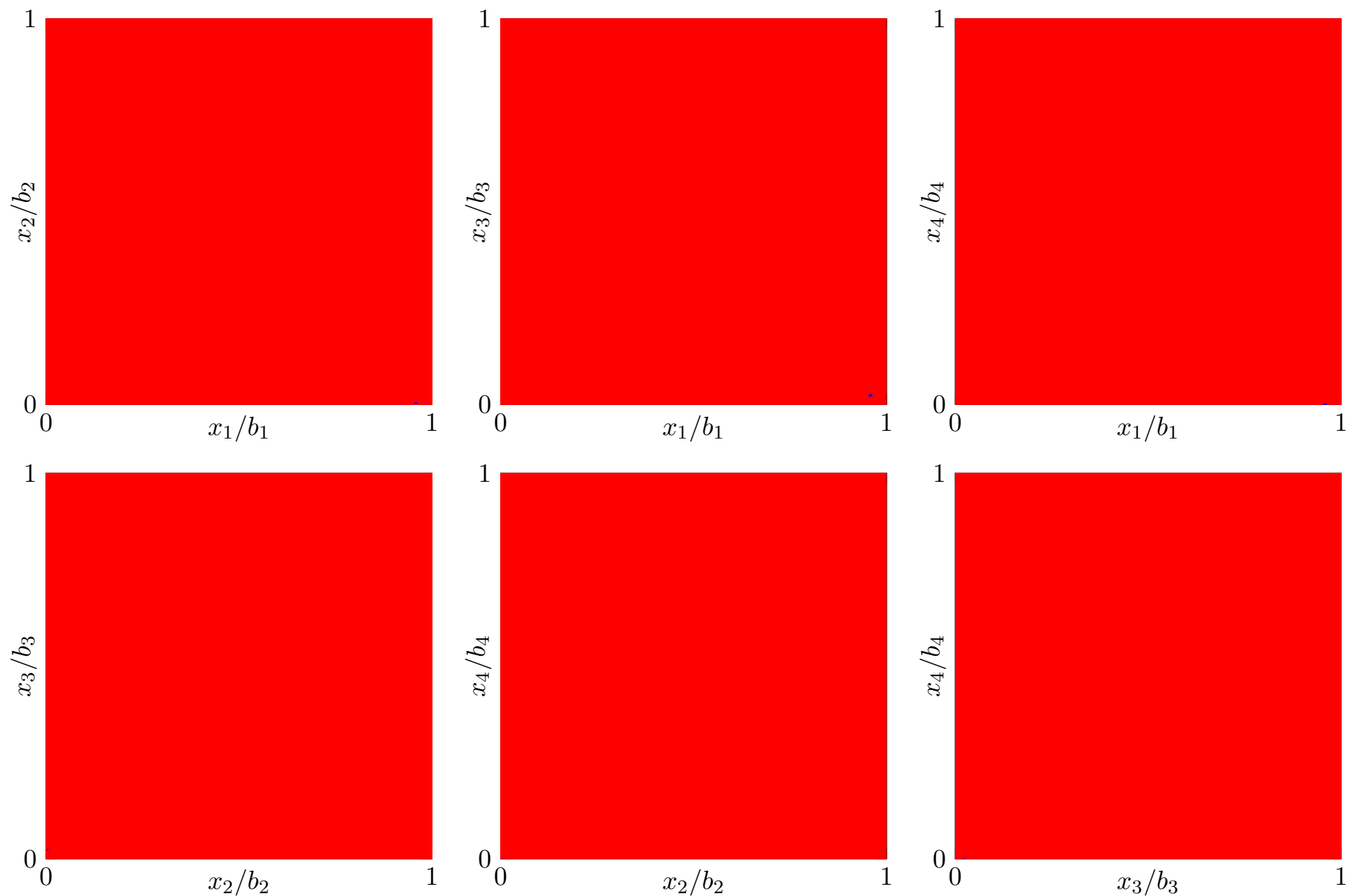
Geometric long-edge bisection – Subdivision level 4 of 18 – Number of boxes examined/admitted/discarded : 8/4/4

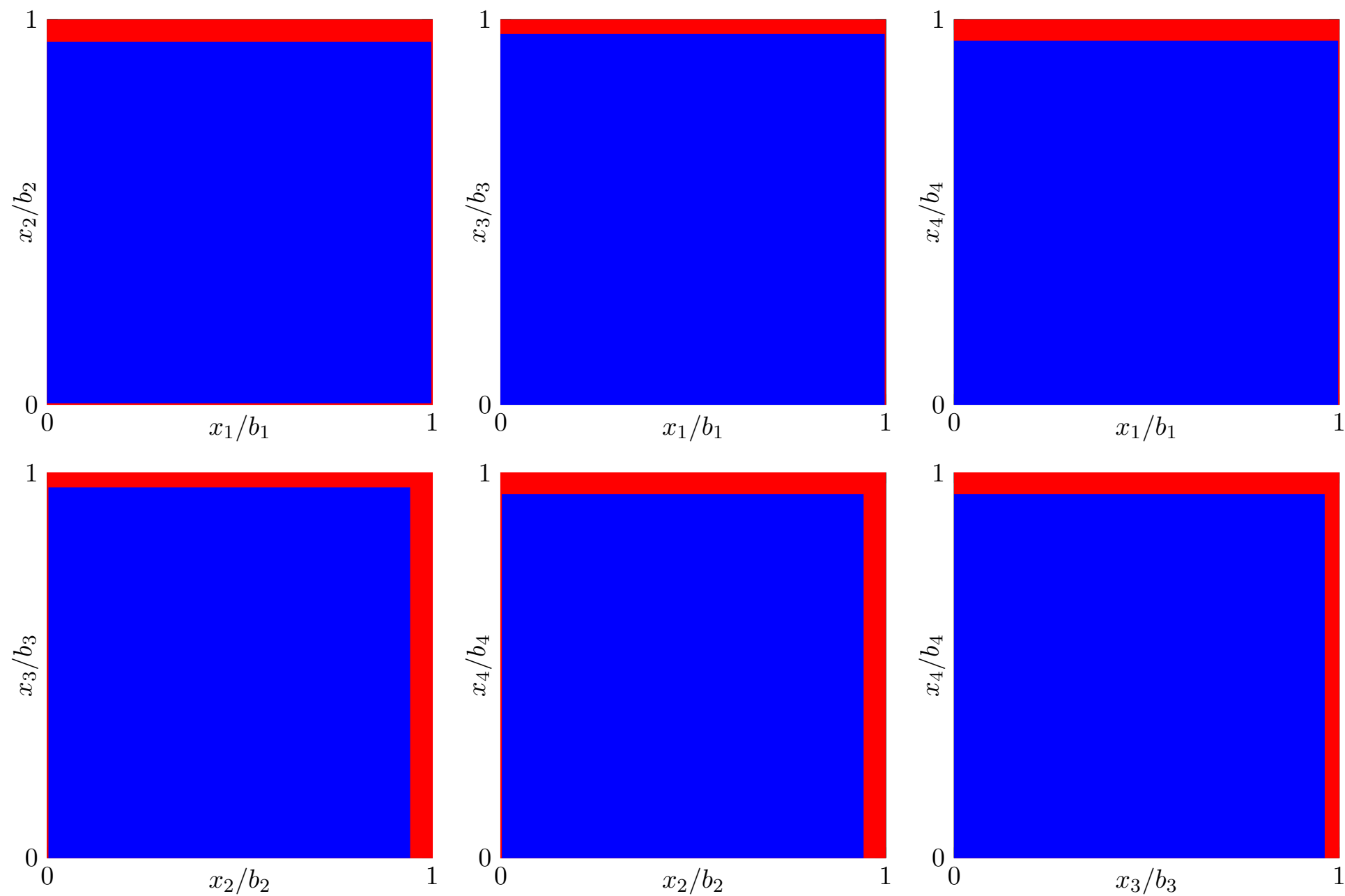**Geometric long-edge bisection – Subdivision level 5 of 18 – Number of boxes examined/admitted/discarded : 8/4/4**

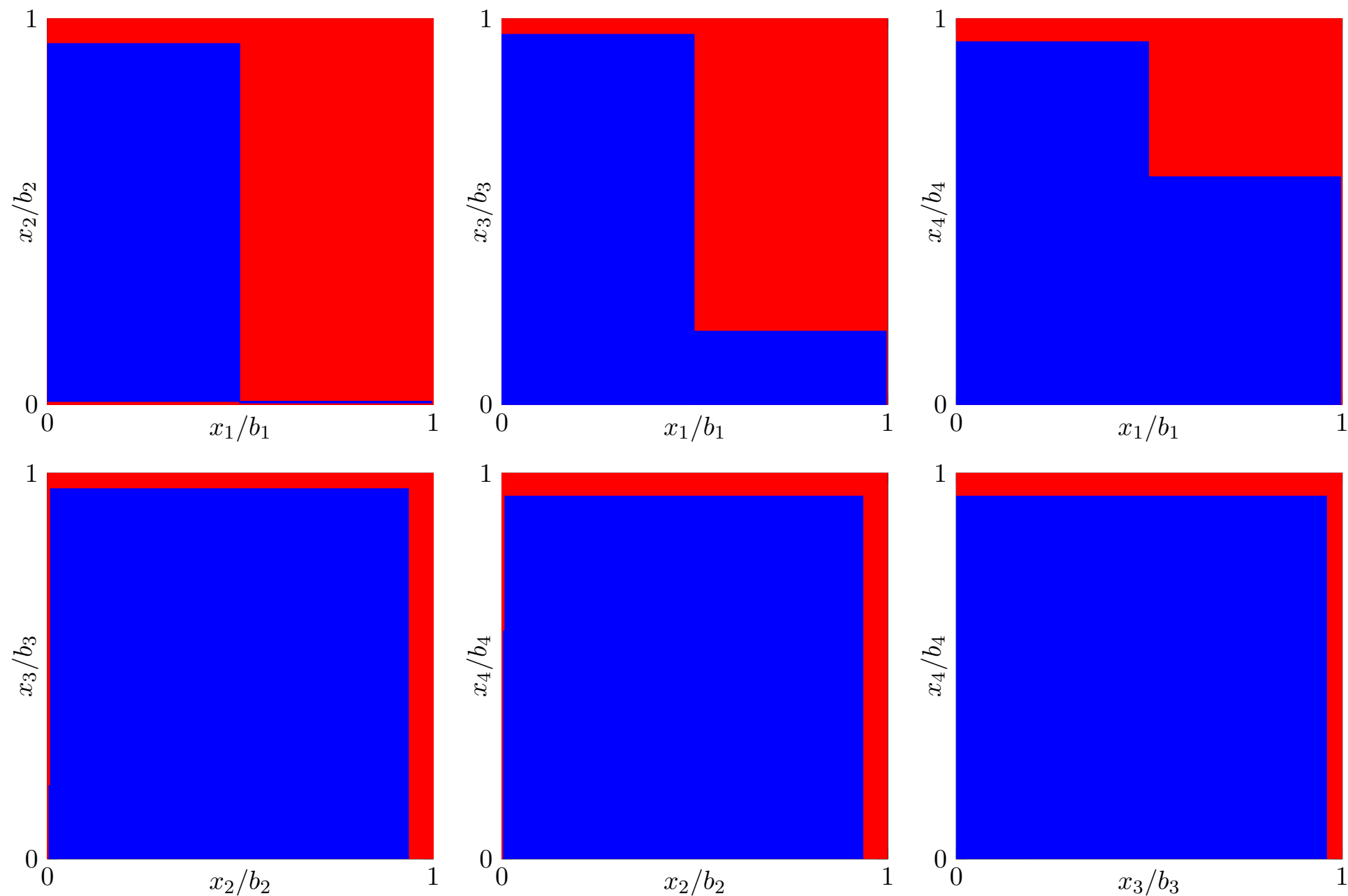**Geometric long-edge bisection – Subdivision level 6 of 18 – Number of boxes examined/admitted/discarded : 8/2/6**

Geometric long-edge bisection – Subdivision level 7 of 18 – Number of boxes examined/admitted/discarded : 4/2/2
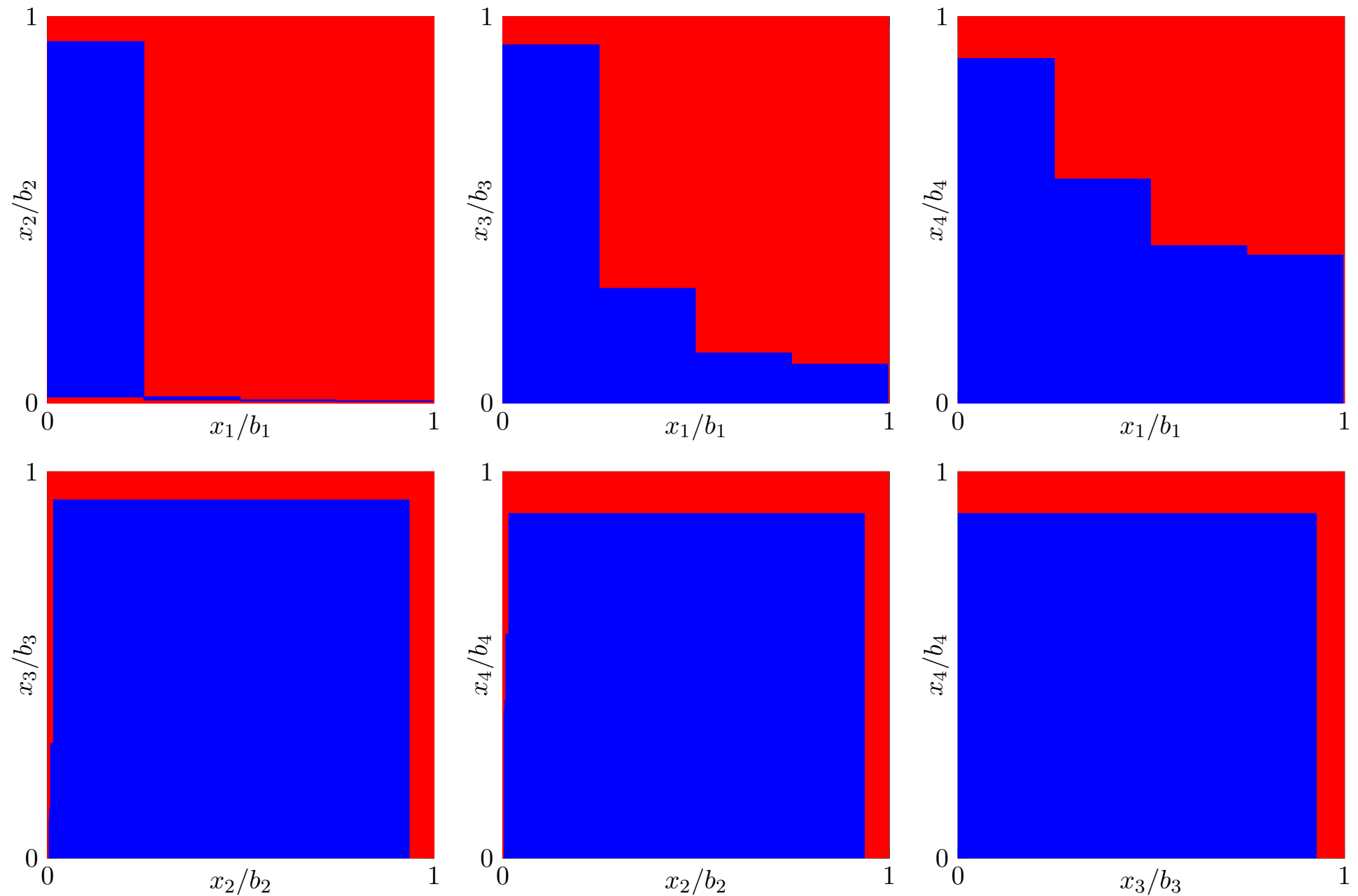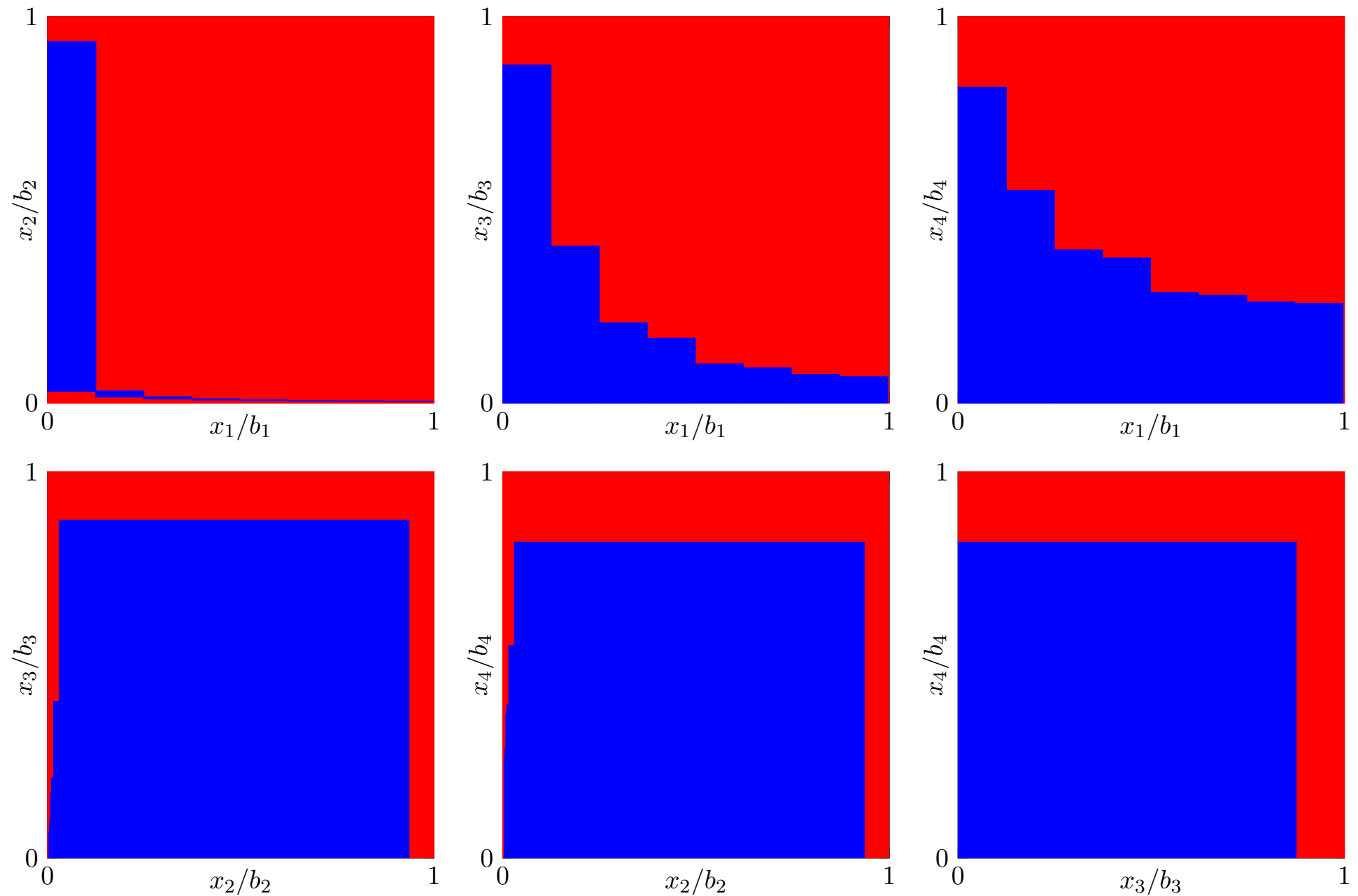
Priming fixed-point iteration – Stalled

Arithmetic long-edge bisection – Subdivision level 1 of 29 – Number of boxes examined/admitted/discarded : 2/2/0
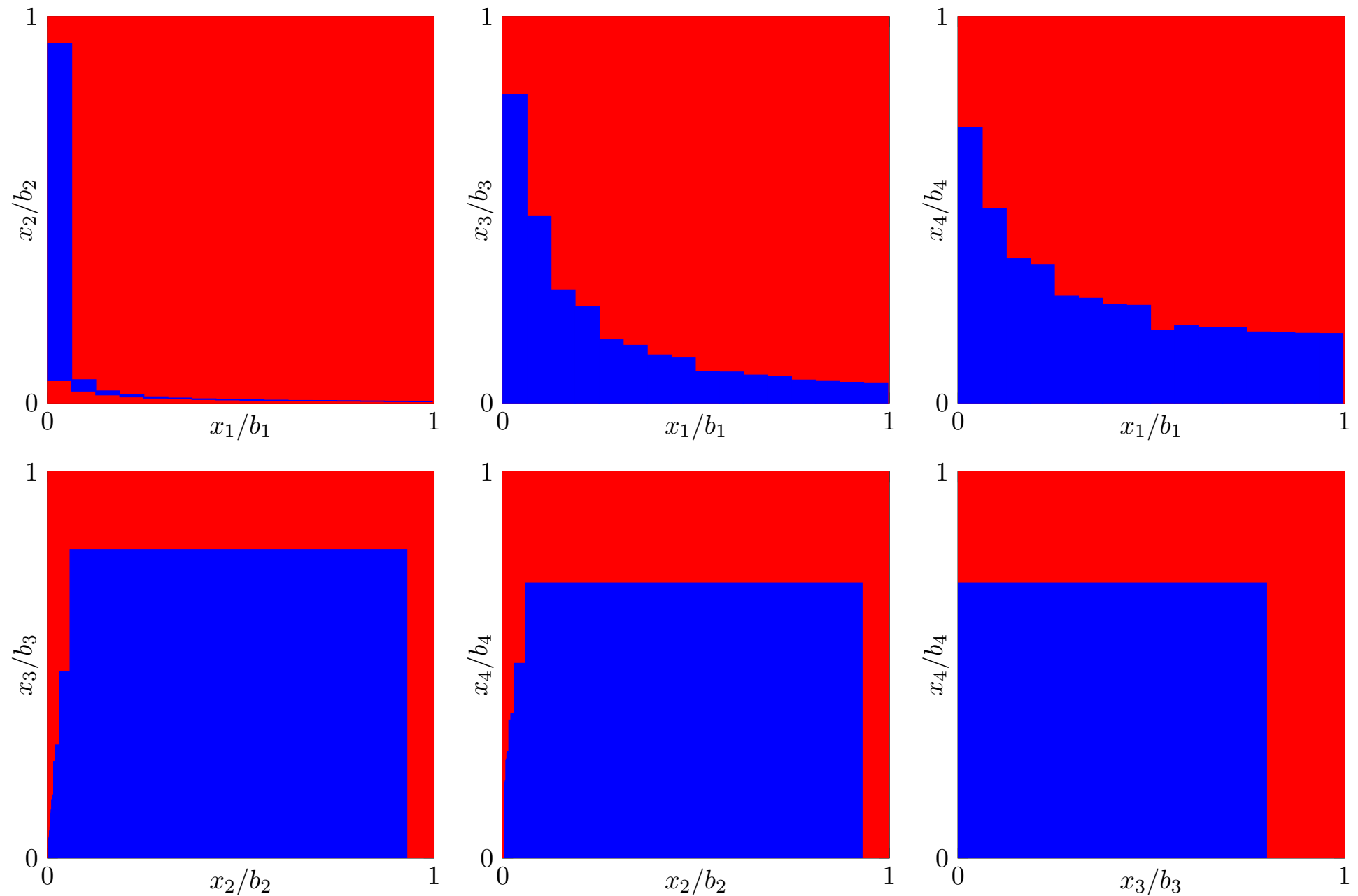
Arithmetic long-edge bisection – Subdivision level 2 of 29 – Number of boxes examined/admitted/discarded : 4/4/0
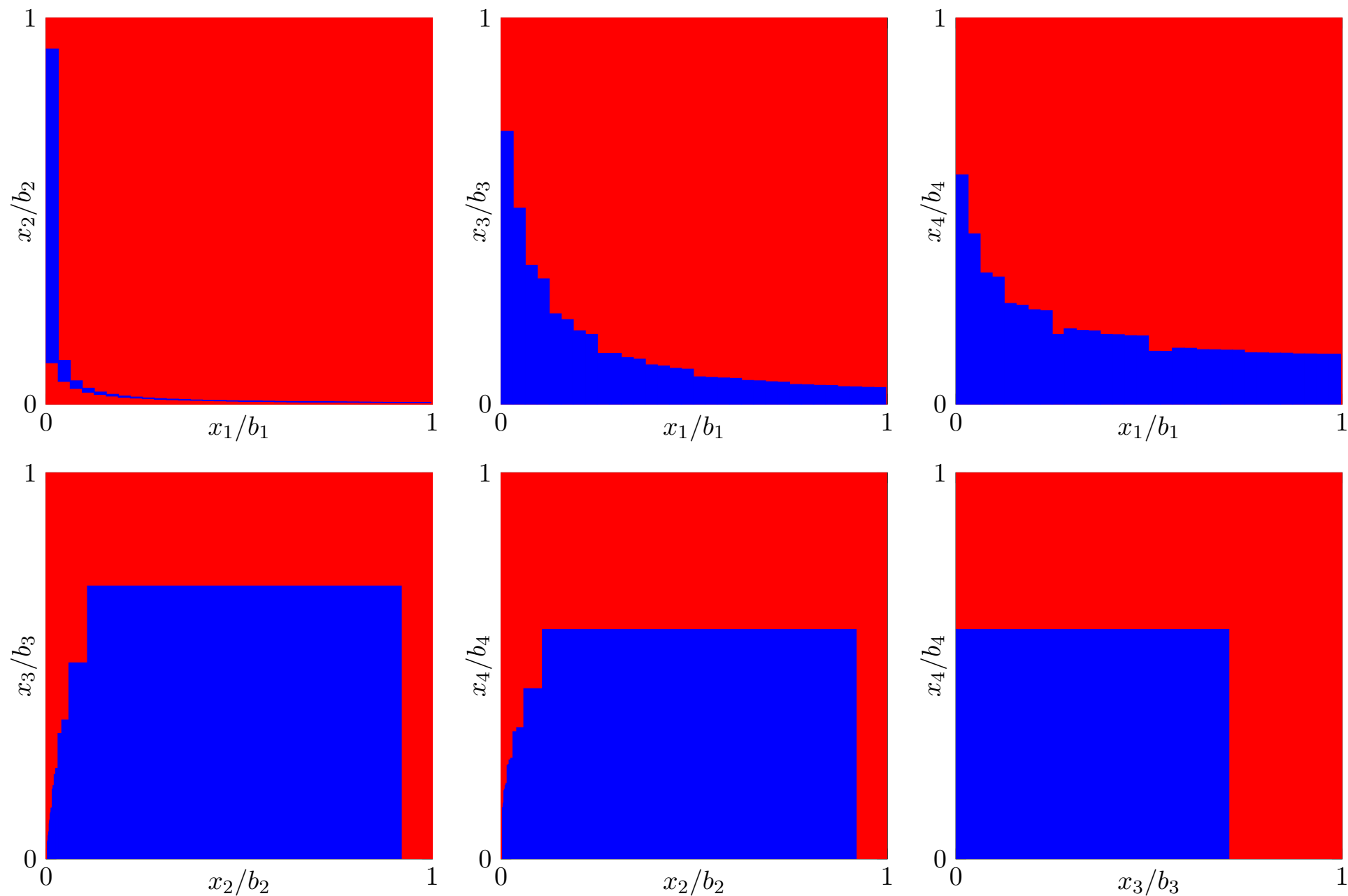
Arithmetic long-edge bisection – Subdivision level 3 of 29 – Number of boxes examined/admitted/discarded : 8/8/0
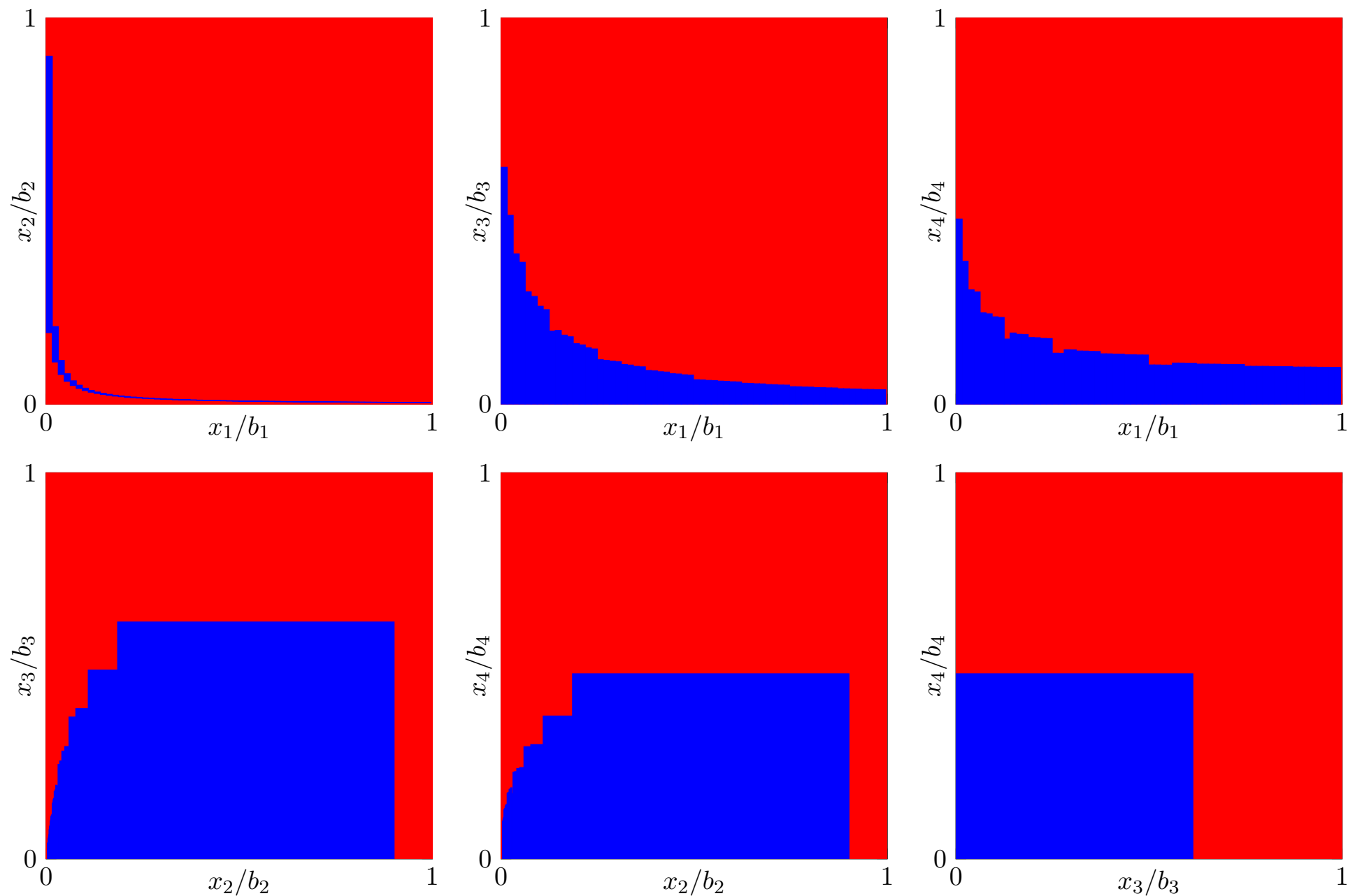
Arithmetic long-edge bisection – Subdivision level 4 of 29 – Number of boxes examined/admitted/discarded : 16/16/0
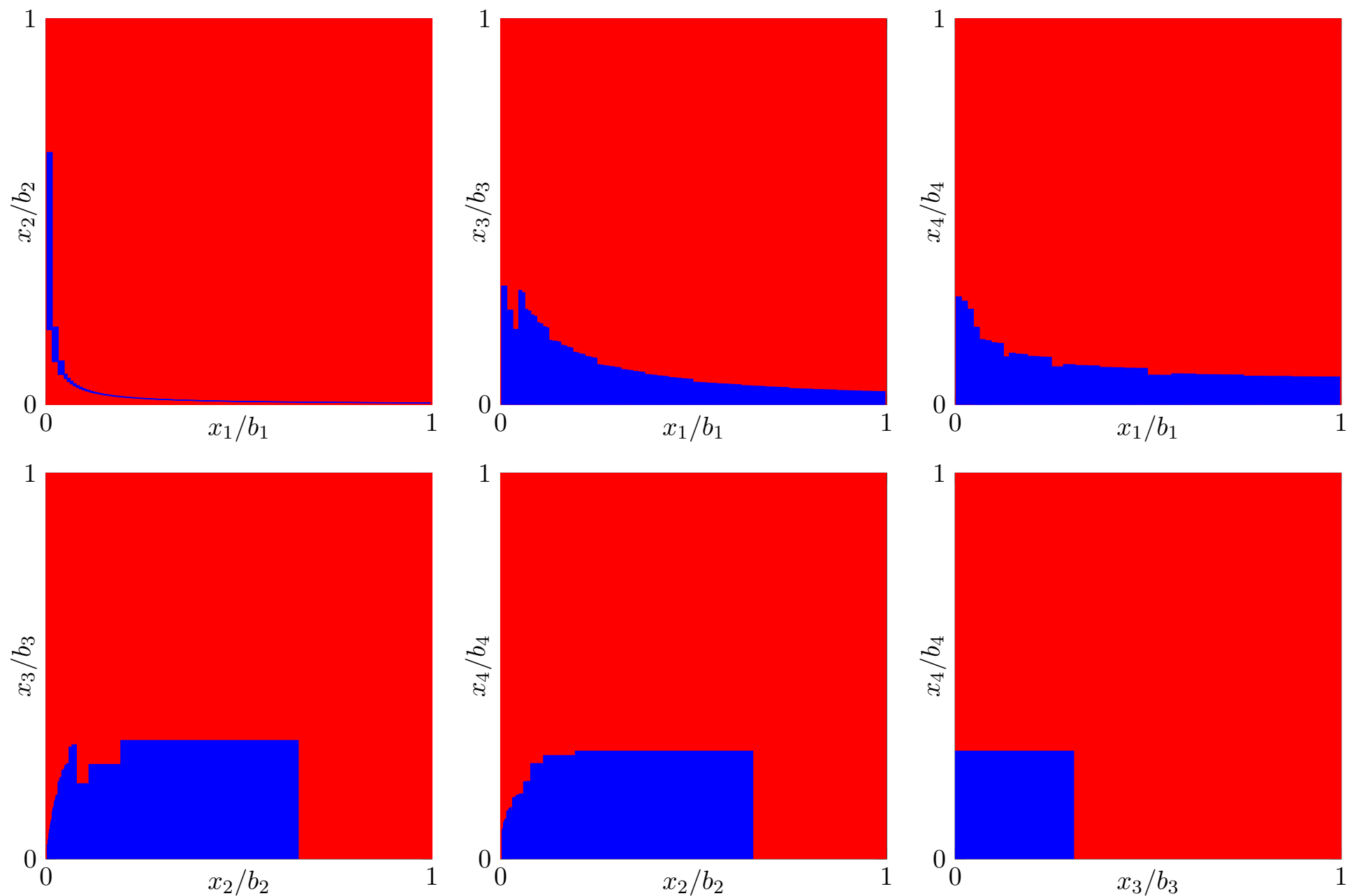
Arithmetic long-edge bisection – Subdivision level 5 of 29 – Number of boxes examined/admitted/discarded : 32/32/0
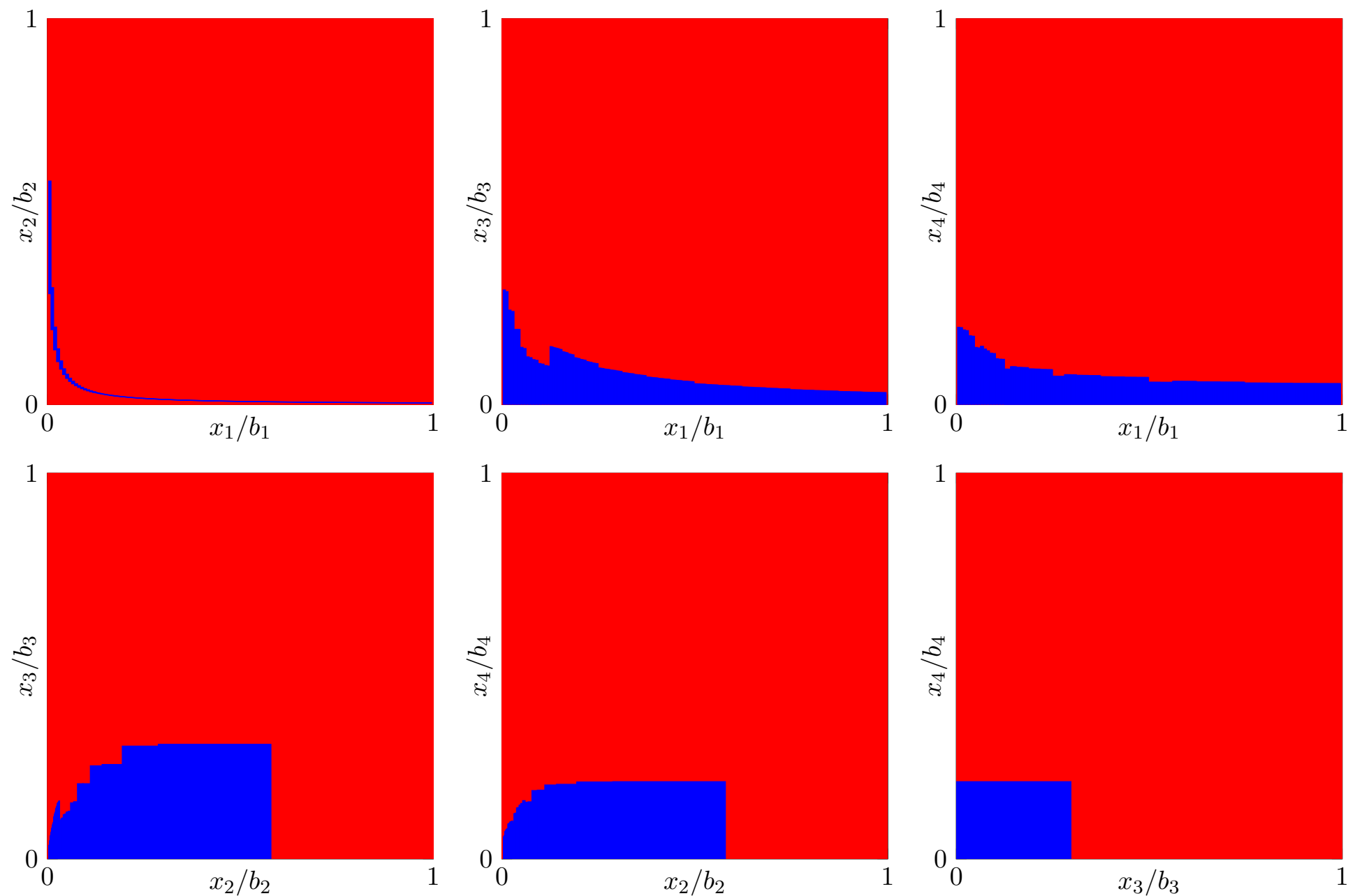
Arithmetic long-edge bisection – Subdivision level 6 of 29 – Number of boxes examined/admitted/discarded : 64/64/0
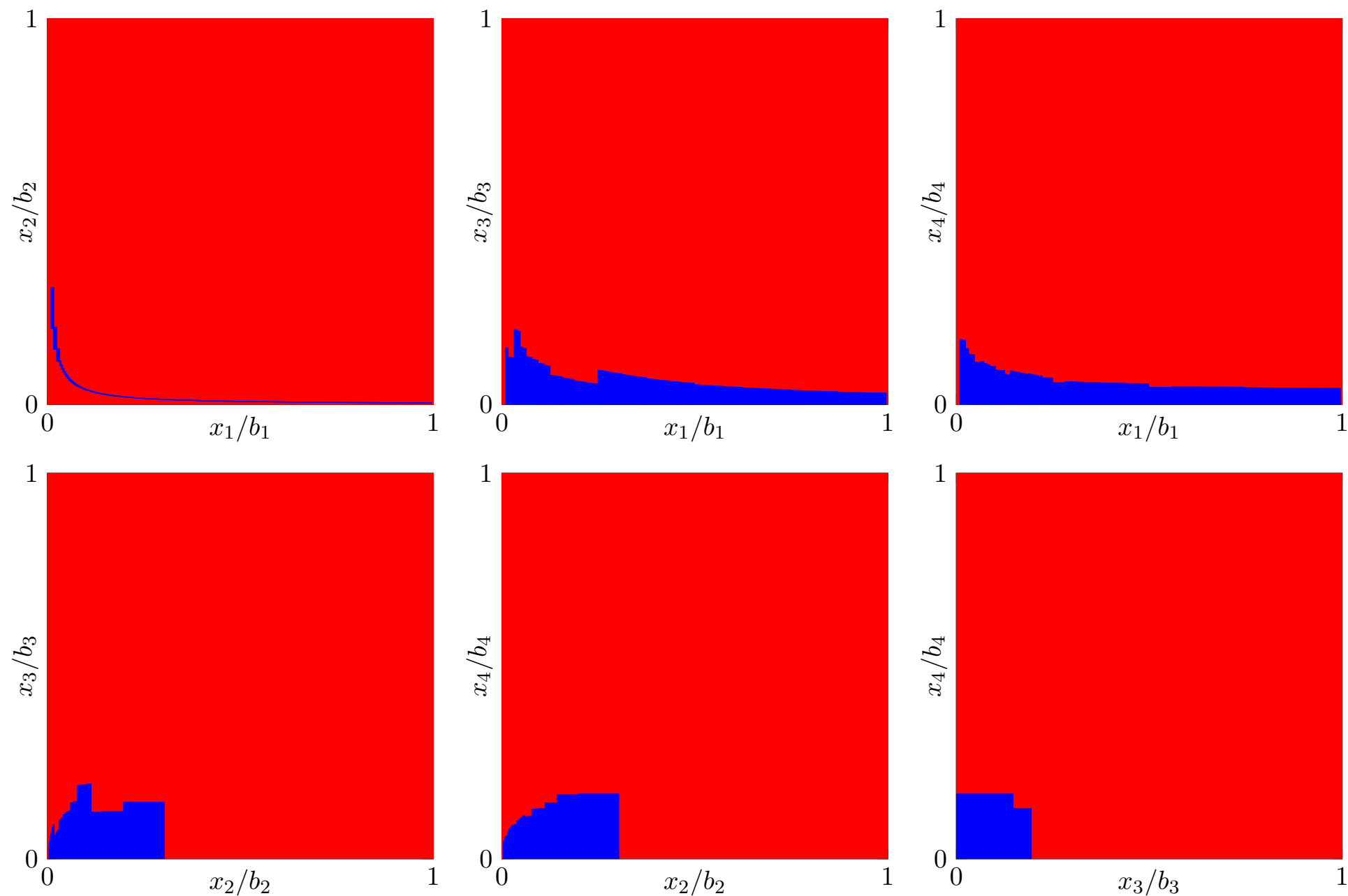
Arithmetic long-edge bisection – Subdivision level 7 of 29 – Number of boxes examined/admitted/discarded : 128/125/3
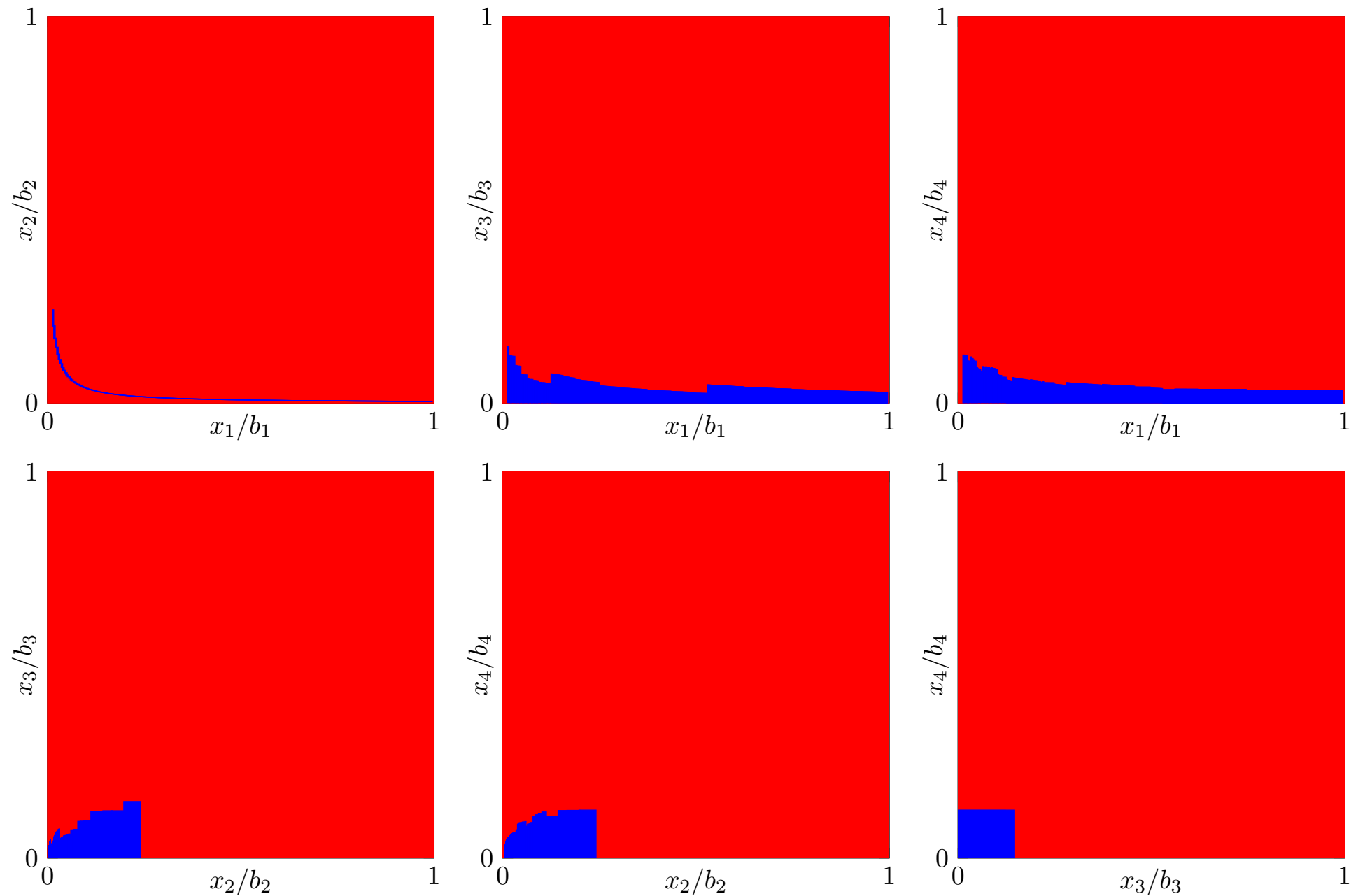
Arithmetic long-edge bisection – Subdivision level 8 of 29 – Number of boxes examined/admitted/discarded : 250/240/10
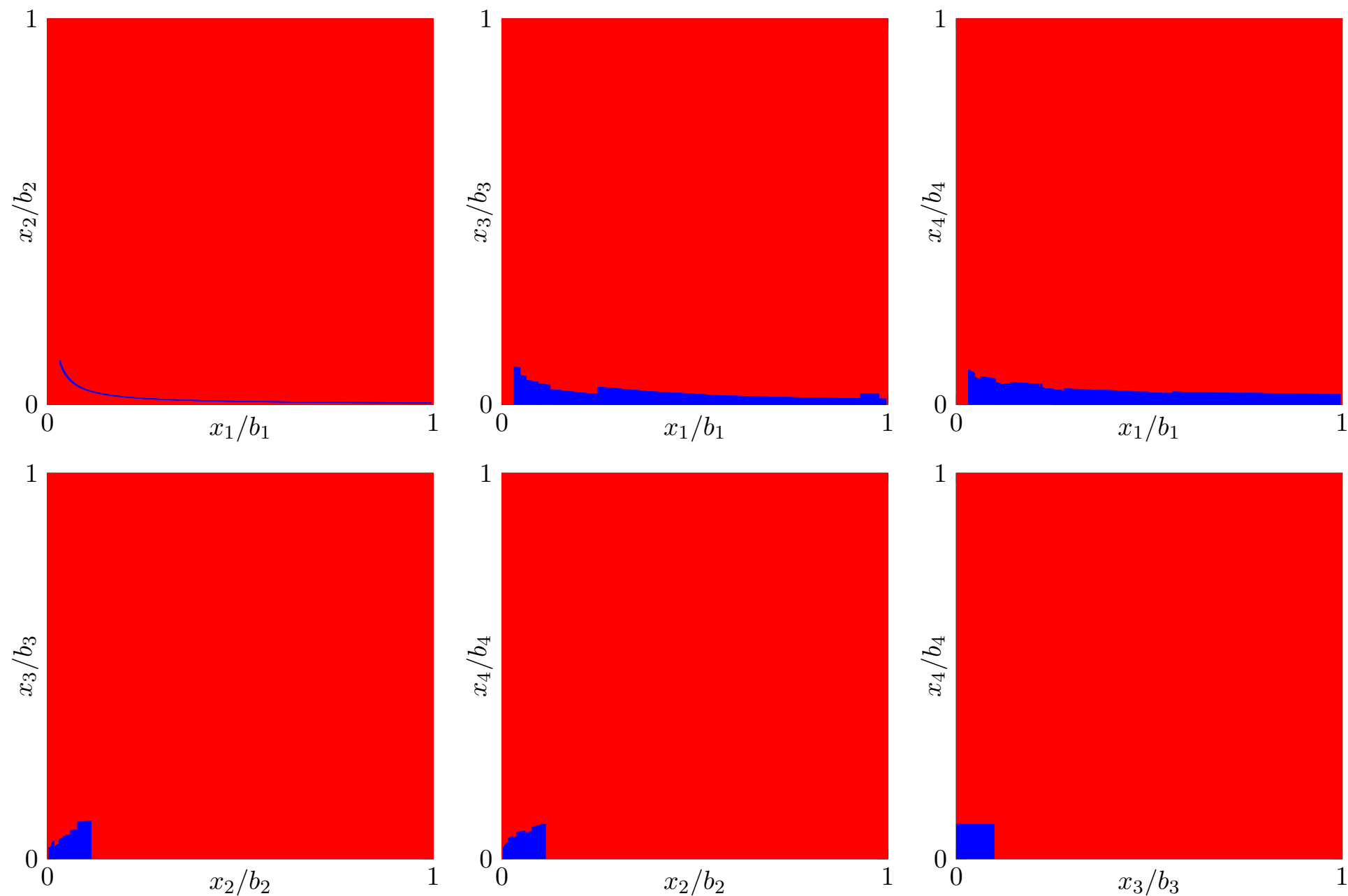
Arithmetic long-edge bisection – Subdivision level 9 of 29 – Number of boxes examined/admitted/discarded : 480/443/37
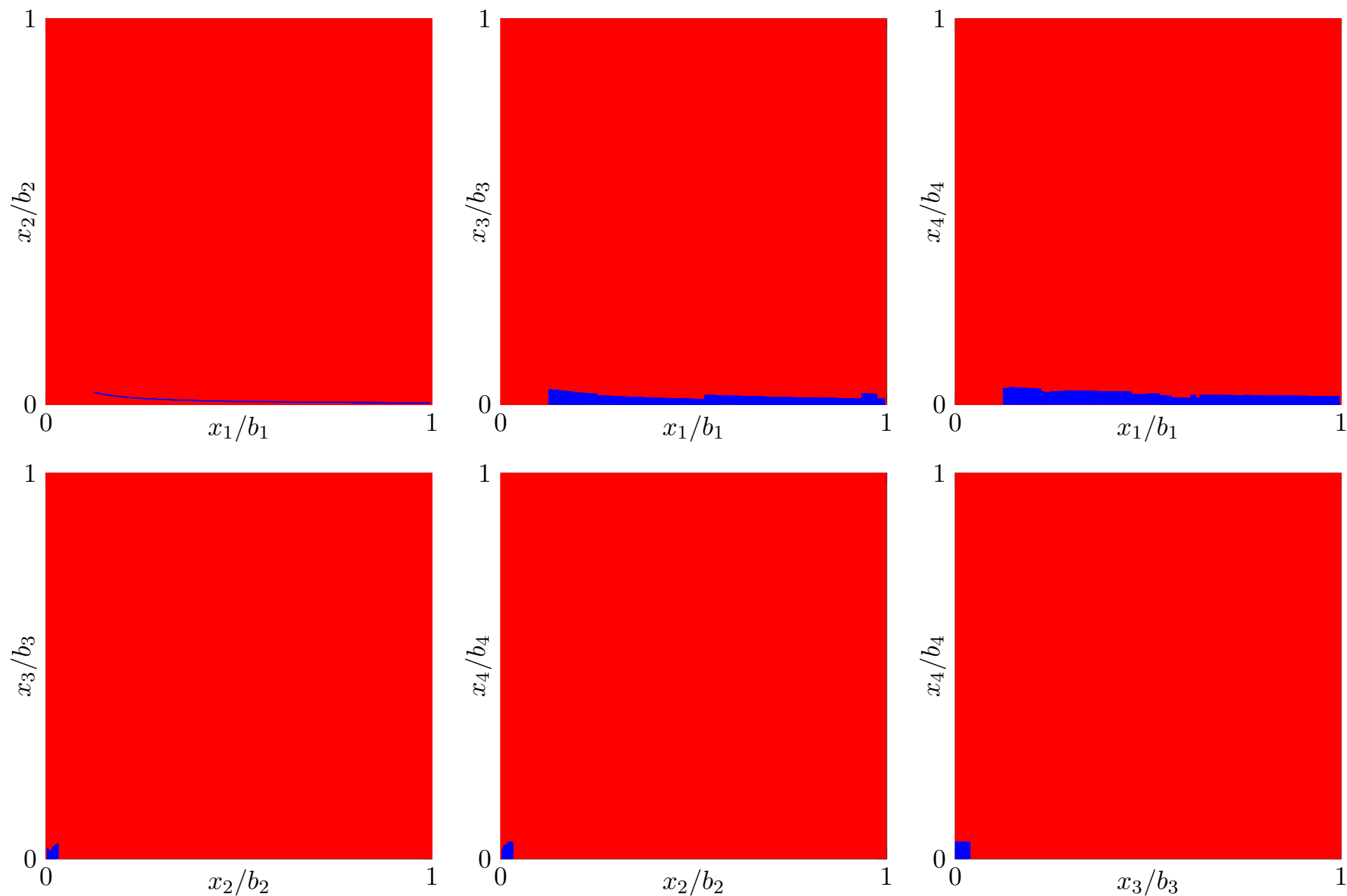
Arithmetic long-edge bisection – Subdivision level 10 of 29 – Number of boxes examined/admitted/discarded : 886/717/169
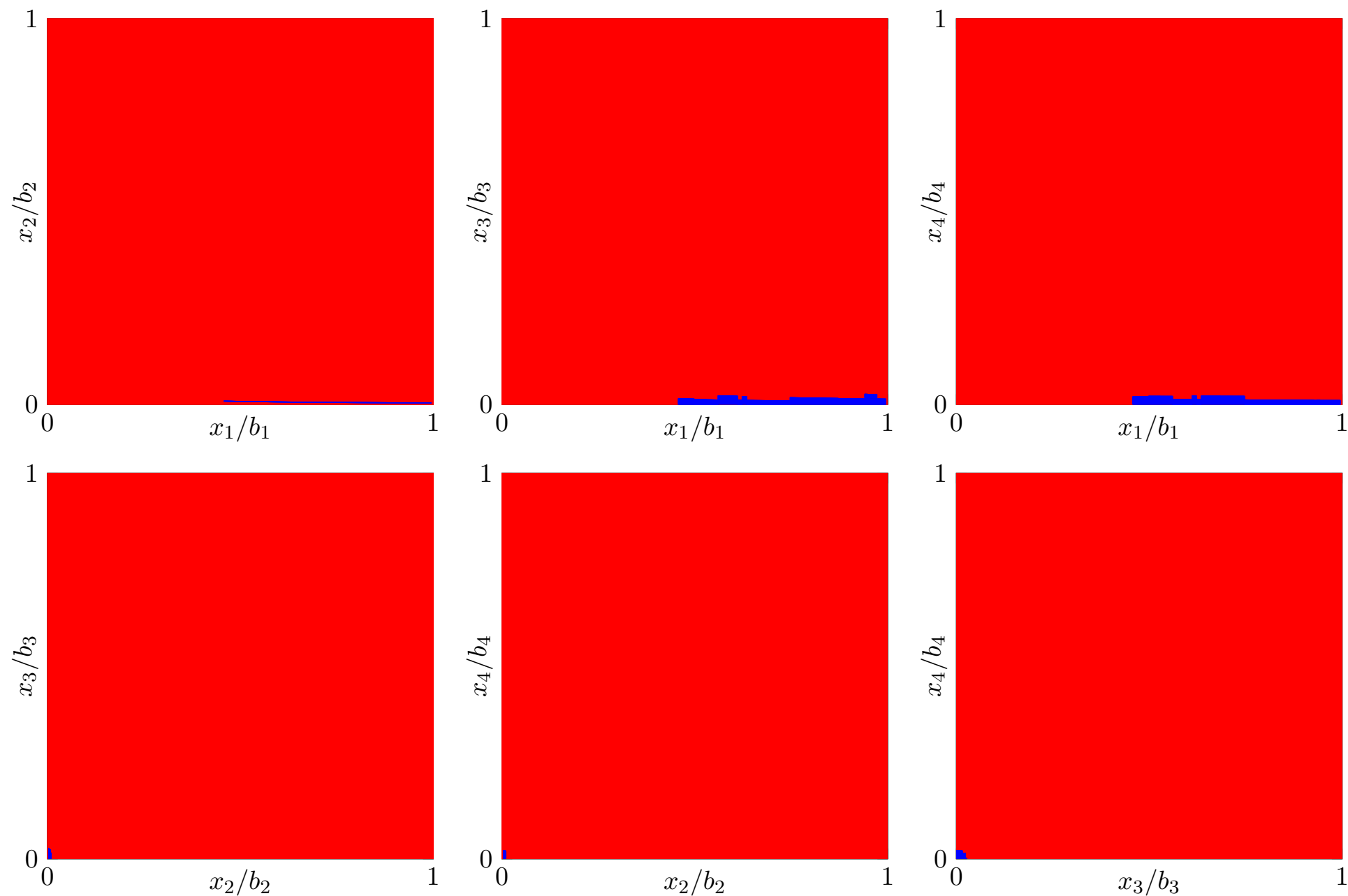
Arithmetic long-edge bisection – Subdivision level 11 of 29 – Number of boxes examined/admitted/discarded : 1434/927/507
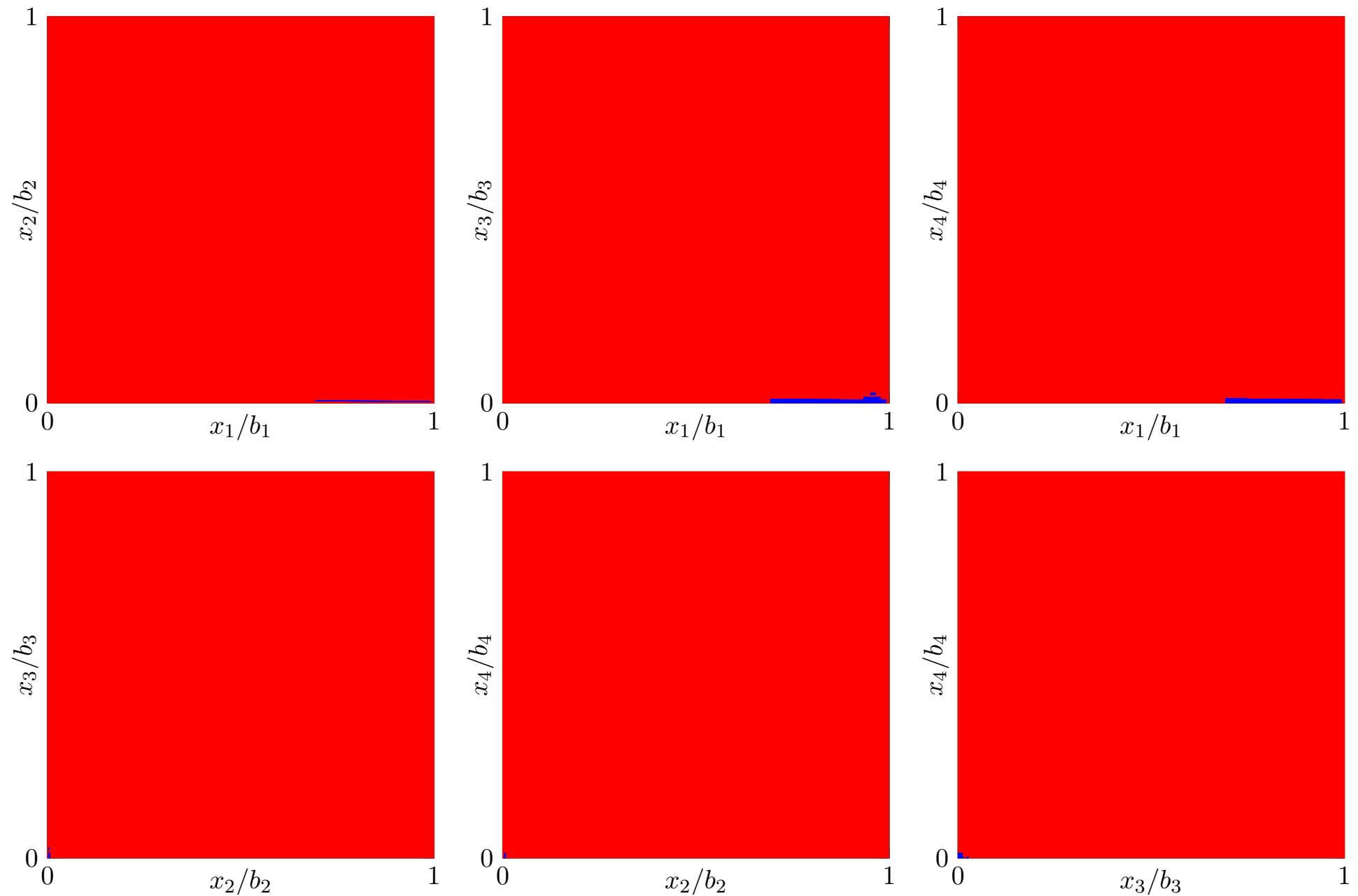
Arithmetic long-edge bisection – Subdivision level 12 of 29 – Number of boxes examined/admitted/discarded : 1854/1389/465
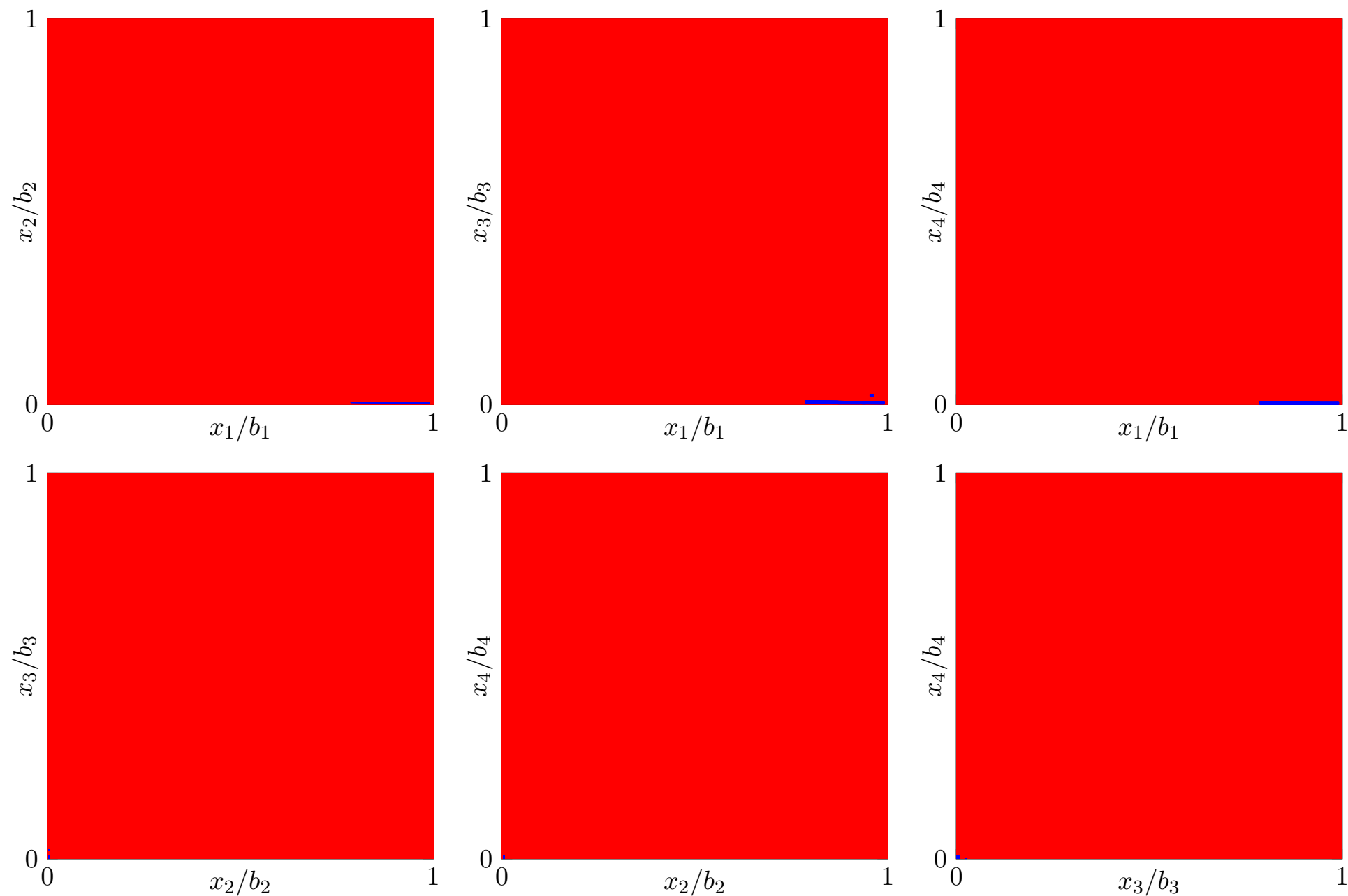
Arithmetic long-edge bisection – Subdivision level 13 of 29 – Number of boxes examined/admitted/discarded : 2778/1234/1544
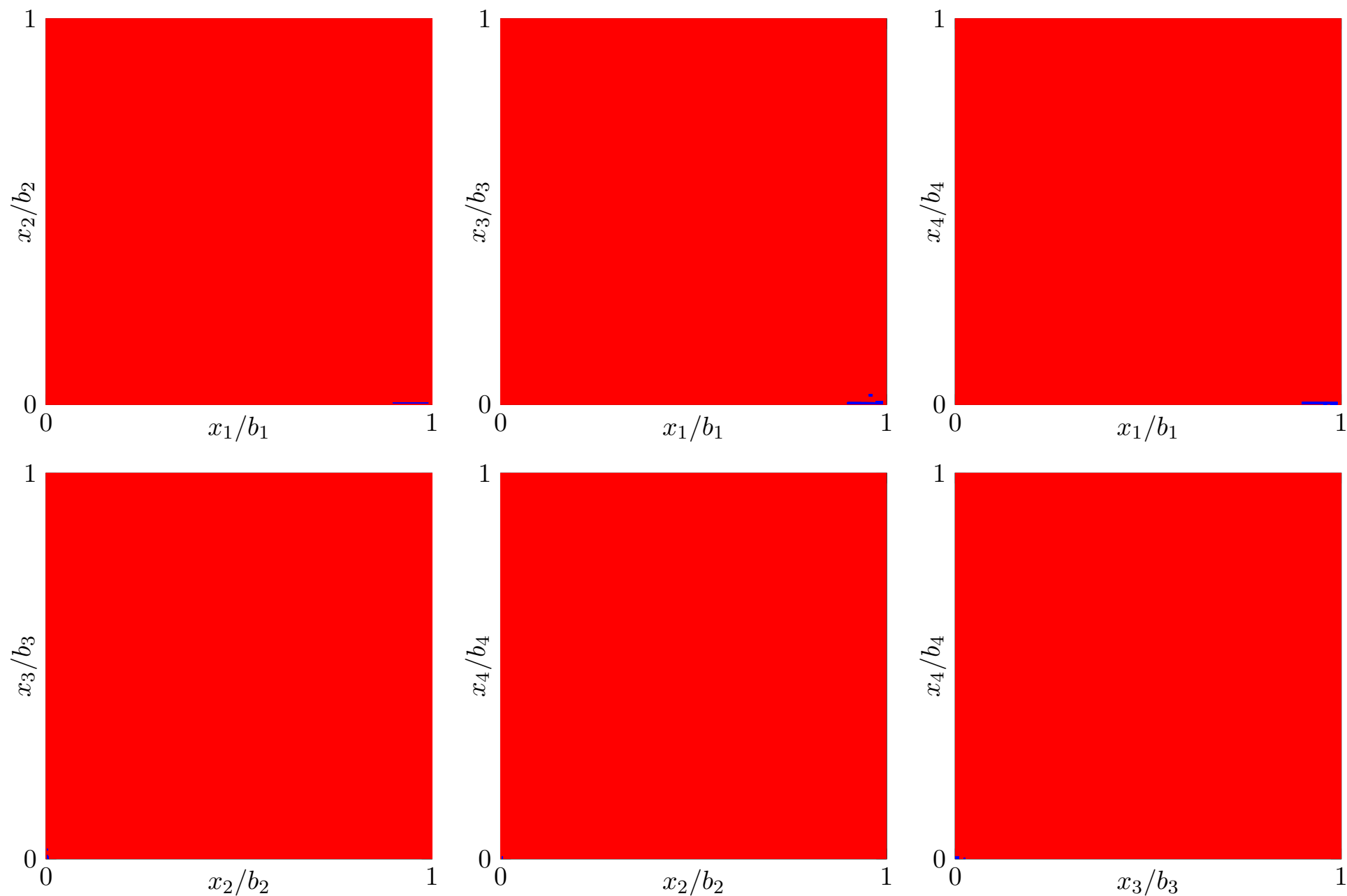
Arithmetic long-edge bisection – Subdivision level 14 of 29 – Number of boxes examined/admitted/discarded : 2468/747/1721
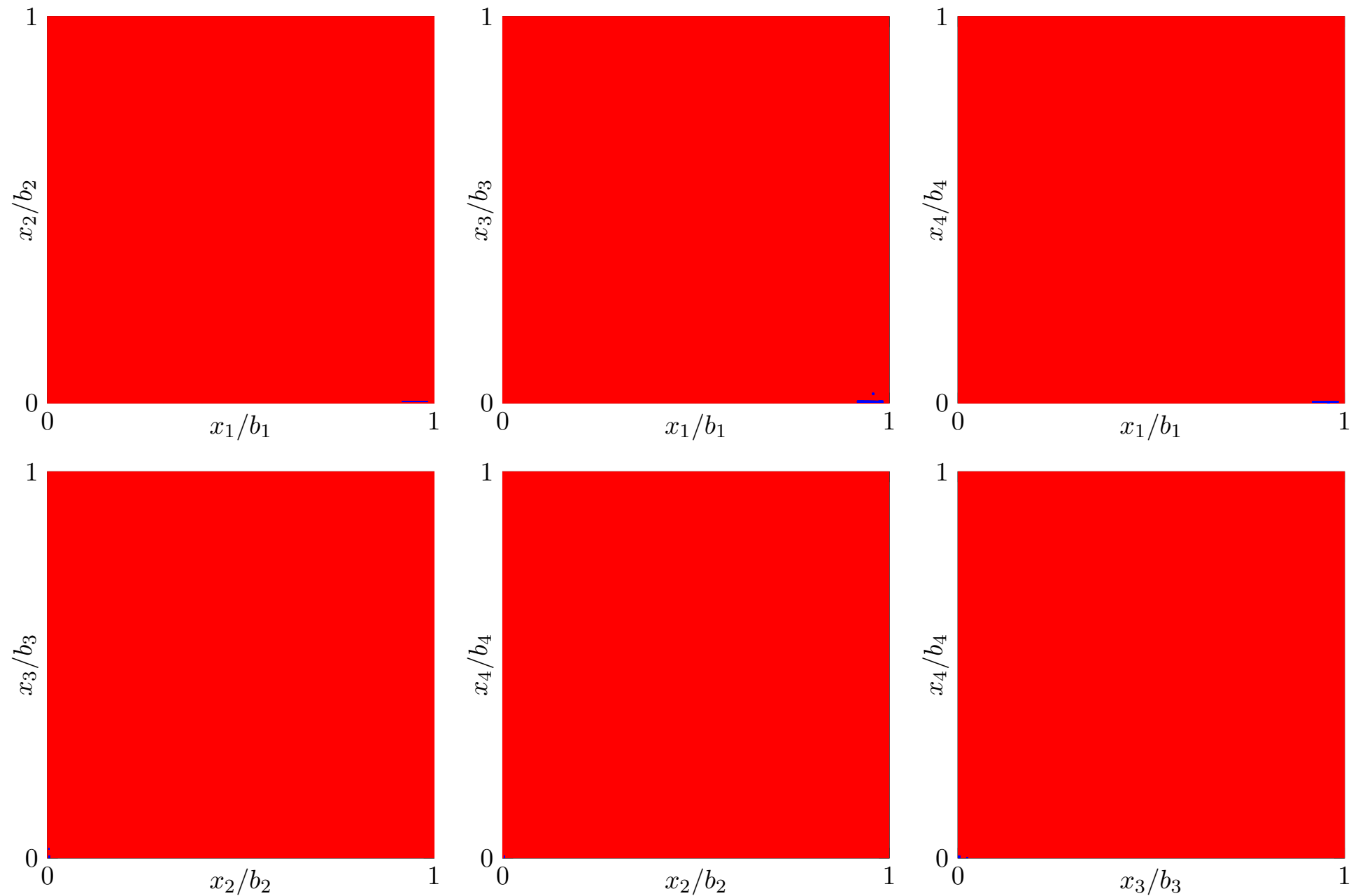
Arithmetic long-edge bisection – Subdivision level 15 of 29 – Number of boxes examined/admitted/discarded : 1494/914/580

Arithmetic long-edge bisection – Subdivision level 16 of 29 – Number of boxes examined/admitted/discarded : 1828/559/1269

Arithmetic long-edge bisection – Subdivision level 17 of 29 – Number of boxes examined/admitted/discarded : 1118/637/481

Arithmetic long-edge bisection – Subdivision level 18 of 29 – Number of boxes examined/admitted/discarded : 1274/139/1135