Pre-Lab #03

Laplace Transform and Transfer Function

Objective

1) To learn how to convert a system from time domain to frequency domain using Laplace transform
2) To learn the frequency domain analysis of continuous-time systems using PSpice.

Introduction:

Laplace Transform

Laplace transform is one of the most powerful mathematical tools for analysis, synthesis, and design. Being able to look at circuits and systems in the s-domain can help us to understand how our circuits and systems really function.

We transform a circuit in the time domain to the frequency or s-domain by Laplace transforming each term in the circuit as shown in Fig.1. One advantage is that a complete response - transient and steady state - of a network is obtained.

If we assume zero initial conditions for the inductor and the capacitor, the voltagecurrent relationships for each circuit elements:

$$\begin{aligned} &\text{Resistor:} && V(s) = RI(s) \\ &\text{Inductor:} && V(s) = sLI(s) \\ &\text{Capacitor:} && V(s) = \frac{1}{sC}I(s) \end{aligned}$$

The impedances of the three circuit elements are:

$$\begin{aligned} &\text{Resistor:} && Z(s) = R \\ &\text{Inductor:} && Z(s) = sL \\ &\text{Capacitor:} && Z(s) = \frac{1}{sC} \end{aligned}$$

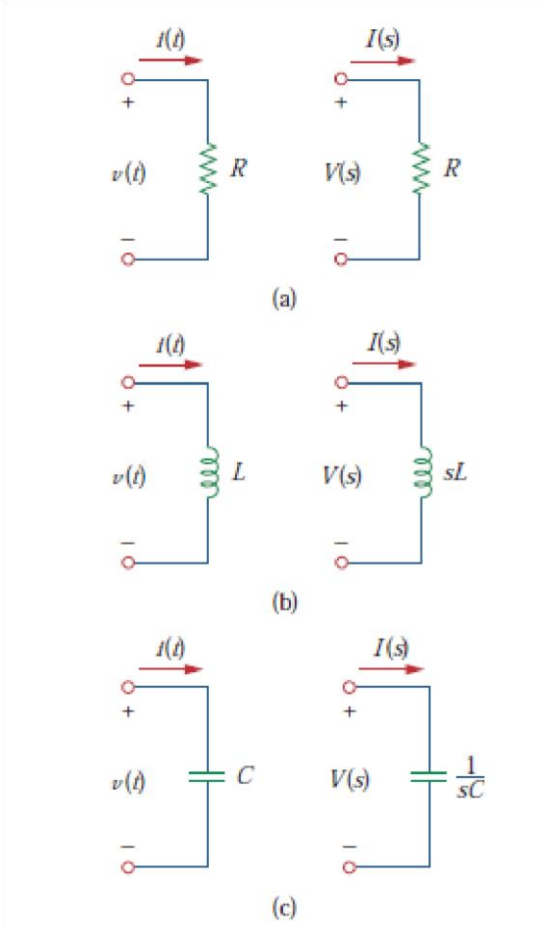
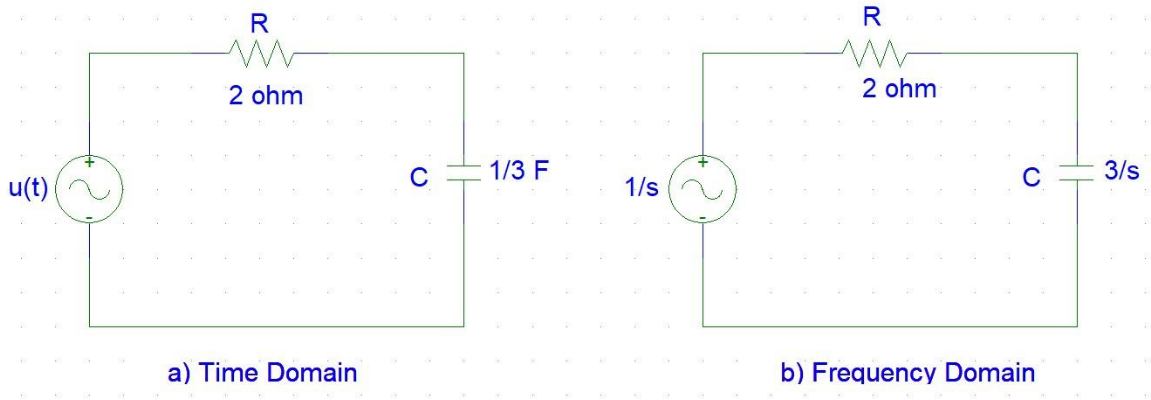


Fig. 1. Time-domain and s-domain representations of passive elements under zero initial conditions

For example:



$$u(t) \rightarrow \frac{1}{s} \qquad 2\Omega \rightarrow 2\Omega \qquad \frac{1}{3}F \rightarrow \frac{1}{sC} = \frac{3}{s}$$

Transfer Functions

The transfer function is a key concept in signal processing because it indicates how a signal is processed as it passes through a network. It is a fitting tool for finding the network response, determining (or designing for) network stability, and network synthesis.

The transfer function of a network describes how the output behaves with respect to the input. It specifies the transfer from the input to the output in the s-domain, assuming no initial energy. The transfer function H(s) is the ratio of the output response Y (s) to the input excitation X(s), assuming all initial conditions are zero.

$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

The transfer function depends on what we define as input and output. Since the input and output can be either current or voltage at any place in the circuit, there are four possible transfer functions:

$$H(s) = \text{Voltage gain} = \frac{V_o(s)}{V_i(s)}$$

$$H(s) = \text{Current gain} = \frac{I_o(s)}{I_i(s)}$$

$$H(s) = \text{Impedance} = \frac{V(s)}{I(s)}$$

$$H(s) = \text{Admittance} = \frac{I(s)}{V(s)}$$

For example:

Obtain the transfer function V_o/V_i of the RC circuit in Fig.2.

Solution:

We first transform the circuit from the time domain to the *s*-domain.

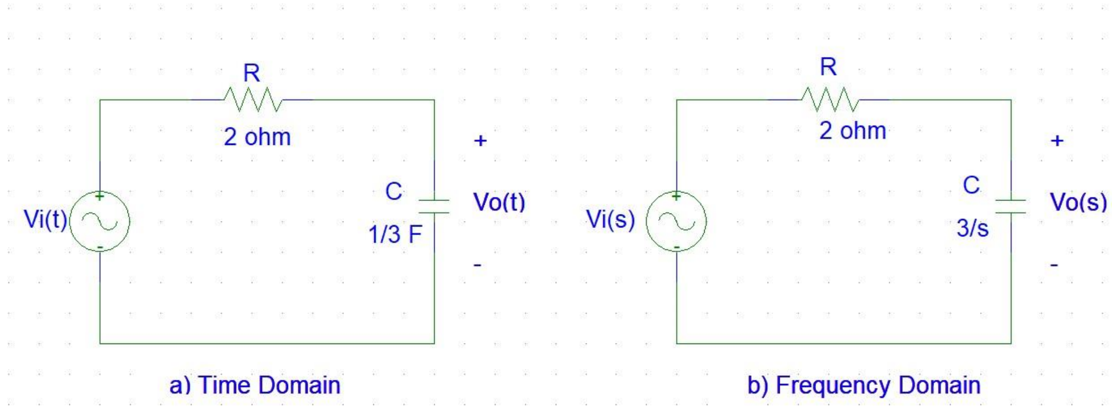
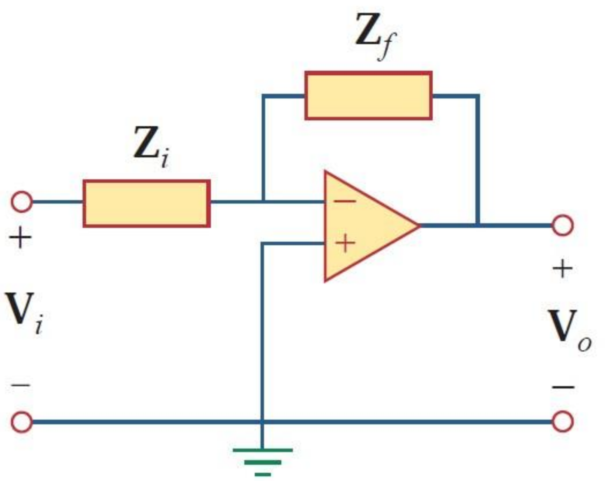


Fig.2. Circuit diagrams for example at time and frequency domains

The frequency-domain equivalent of the circuit is in Fig. 2. By voltage division, the transfer function is given by

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s(2)(1/3)} = \frac{1}{1 + s * 0.66}$$

<u>Opamp:</u>



The transfer function is:

$$H(s) = \frac{V_o(s)}{V_i(s)} = -\frac{Z_f}{Z_i}$$

<u>Pre-Lab:</u> ***Q1)***

Step1: Find transfer function (Vout/Vin) of Fig.3 in the s-domain using Laplace transform.

Choose R= 1kΩ L= 1mH C=0.1µF Vp-p=1V

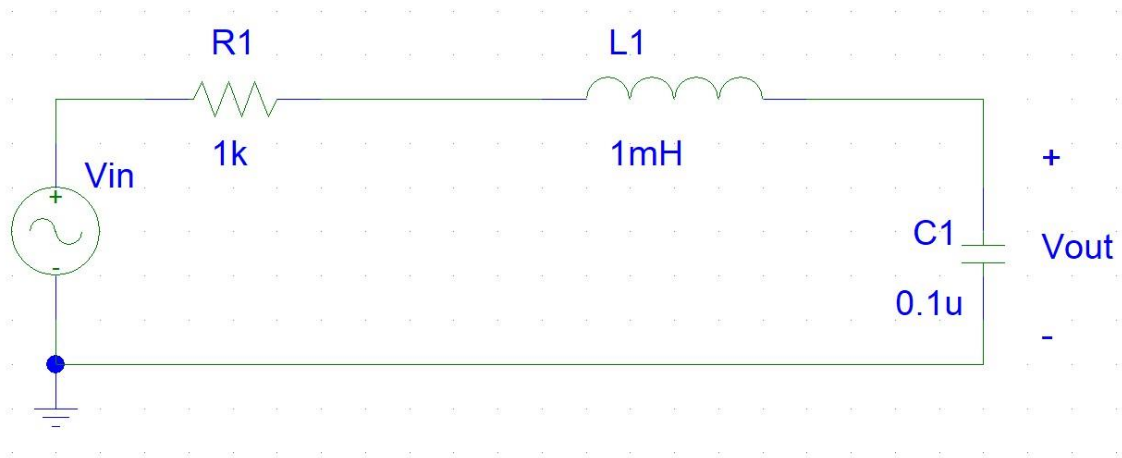


Fig.3.

Step2: Build the circuit on PSpice (There are guides on internet for how to build Laplace circuit on LTSpice. You can check those if you don't have PSpice installed) and using AC Sweep, observe output voltage (Vout) versus frequency and phase difference between Vin-Vout. Use Sweep parameters as shown Fig. 4:

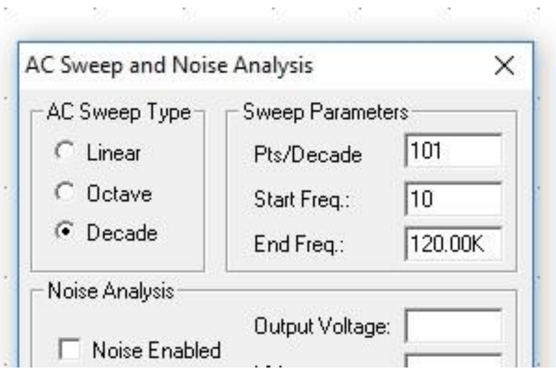


Fig.4. Analysis Setup Parameters

Step3: By using LAPLACE part from PSpice (get new part → Laplace), build the circuit as shown in Fig 5. Enter the transfer function you found into LAPLACE part and observe output signal (Vout). Then compare the voltage waveform with Step 2.

Note: In schematic, you can write exponential numbers into Laplace part according to Table 1:

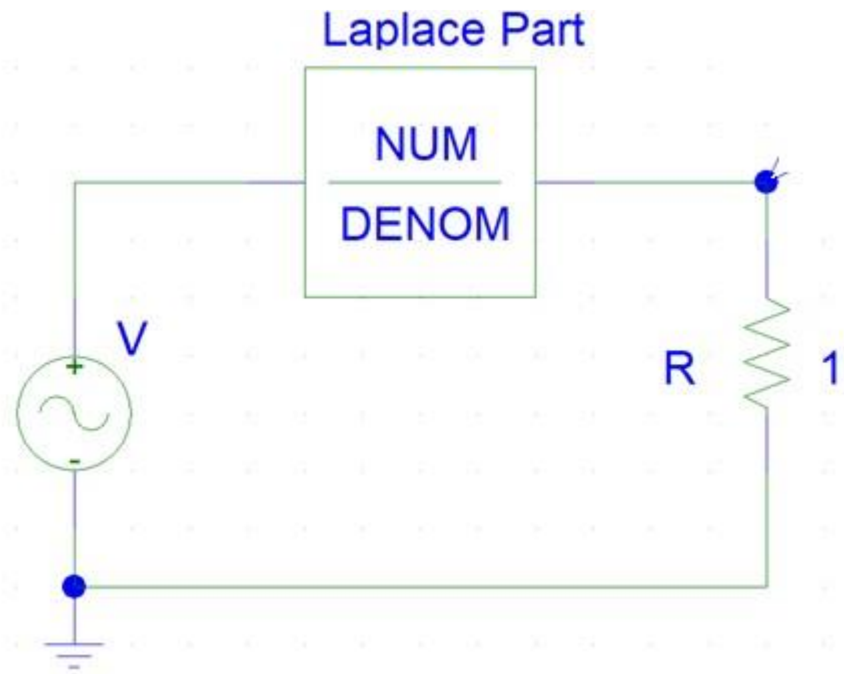


Fig.5.

Table 1: Symbolic Scale Factors

Symbol	Exponential Form	Value
F (or f)	1E-15	10^{-15}
P (or p)	1E-12	10^{-12}
N (or n)	1E-9	10^{-9}
U (or u)	1E-6	10^{-6}
M (or m)	1E-3	10^{-3}
K (or k)	1E3	10^{3}
MEG (or meg)	1E6	10^{6}
G (or g)	1E9	10^{9}
T (or t)	1E12	10^{12}

For example: 10^6 → 1MEG, $3*10^{10} = 30$G

Step 4: By changing V_{AC} with Vsin, repeat Step2 and Step 3 in transient analysis.

Step 5: To investigate unit step response of circuit use VPWL source to generate a unit step function. Construct the circuit for obtaining the step response and set the transient analysis parameters as shown in Fig 6 and Fig 7.

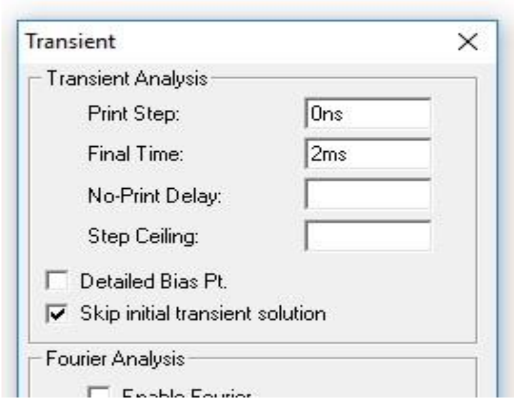


Fig.6. Transient analysis parameters

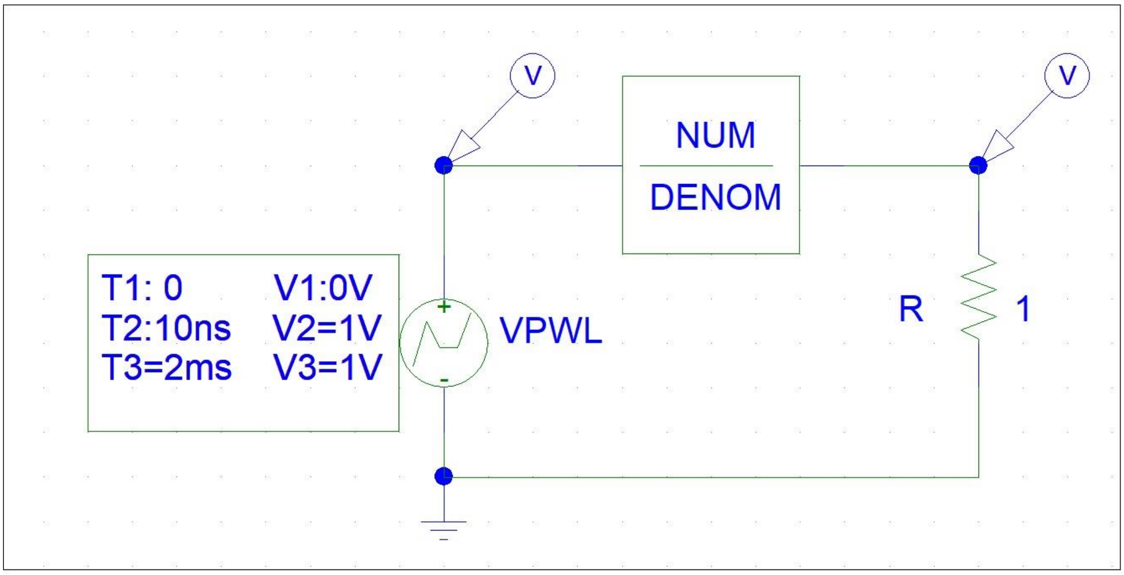


Fig.7. Step response testing

Q2)

Step1: Design an op-amp circuit, using Fig. 8, which will realize the following transfer function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = -\frac{s + 1000}{2(s + 4000)}$$

Choose C1=10µF; determine R1, R2, and C2.

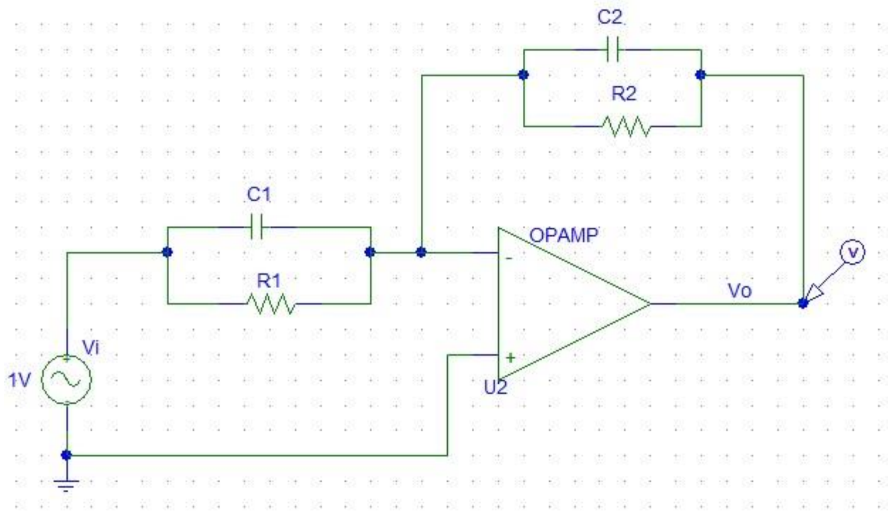


Fig. 8

Step2: Then, build the circuit on PSpice with the values you found. Observe output and input voltage waveforms**.**

Step3: By using LAPLACE part from PSpice (get new part → Laplace), build the following circuit and observe output signal.

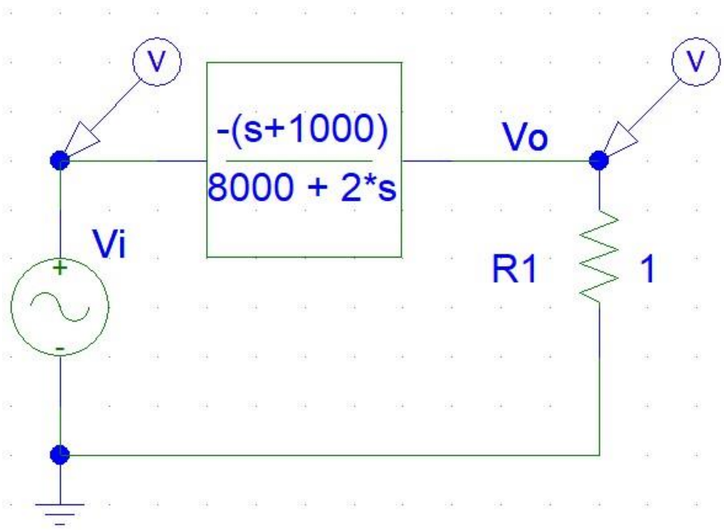


Fig 9

Step4: Compare Step2 and Step3 output waveforms.

Reference:

Charles K. Alexander, Matthew N. O. Sadiku, *"Fundamentals of Electric Circuits",* Fourth Edition.