# Abdullah Gül University

## Department of Electrical and Electronics Engineering

# Pre-Lab 3

## Laplace Transform

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# Contents

# Chapter 1

# Overview

## 1.1 Objective

This pre-lab focuses on using the Laplace Transform to analyze circuits in the frequency domain. By applying these techniques, the transient and steady-state behavior of circuits can be fully understood. The experiment uses LTSpice to simulate and analyze RLC circuits and op-amp configurations in both the s-domain and time domain. These are the main objectives:

- To derive transfer functions from circuit elements.
- To validate analytical transfer functions through AC Sweep and transient simulations.
- To compare the performance of RLC circuits and op-amp designs using both analytical and simulated approaches.

# Chapter 2

# Procedure

## 2.1 RLC Circuit Analysis

### 2.1.1 Transfer Function Derivation

In order to derive the voltage gain transfer function ($H_V(s)$), we must first convert the circuit from time domain to the s-domain. The circuit is shown in Figure 2.1.

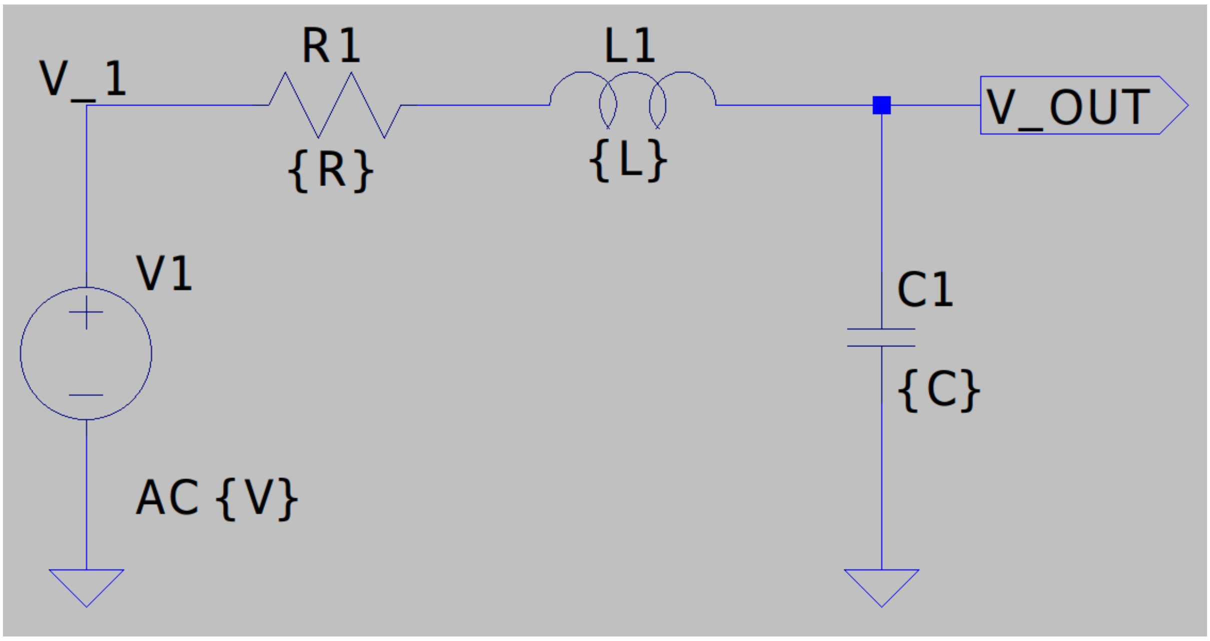


Figure 2.1: RLC Circuit

Where:

- $V_1$ is $1V_{pp}$ input voltage.
- $V_{out}$ is the output voltage.
- $R$ is $1k\Omega$ resistor.
- $L$ is $1mH$ inductor.
- $C$ is $0.1\mu F$ capacitor.

To find the transfer function, we first transform the circuit to the s-domain. The impedance of the resistor, inductor, and capacitor are given by:

$$Z_R = R$$
$$Z_L = sL$$
$$Z_C = \frac{1}{sC}$$

And the KVL equation for the circuit is:

$$V_1 = V_R + V_L + V_C$$
$$V_1 = IZ_R + IZ_L + IZ_C$$
$$V_1 = I(R + sL + \frac{1}{sC})$$

Where $I$ is the current through the circuit. The voltage gain transfer function is then:

$$H_V(s) = \frac{V_{out}}{V_1} = \frac{I\frac{1}{sC}}{I(R + sL + \frac{1}{sC})}$$
$$= \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$
$$\boxed{H_V(s) = \frac{1}{sRC + s^2LC + 1}}$$

### 2.1.2 AC Sweep Analysis Simulation

To validate the transfer function, we will perform an AC Sweep analysis in LTSpice. The AC Sweep analysis will sweep the frequency of the input voltage from $101Hz$ to $120kHz$. The output voltage will be measured and compared to the transfer function.

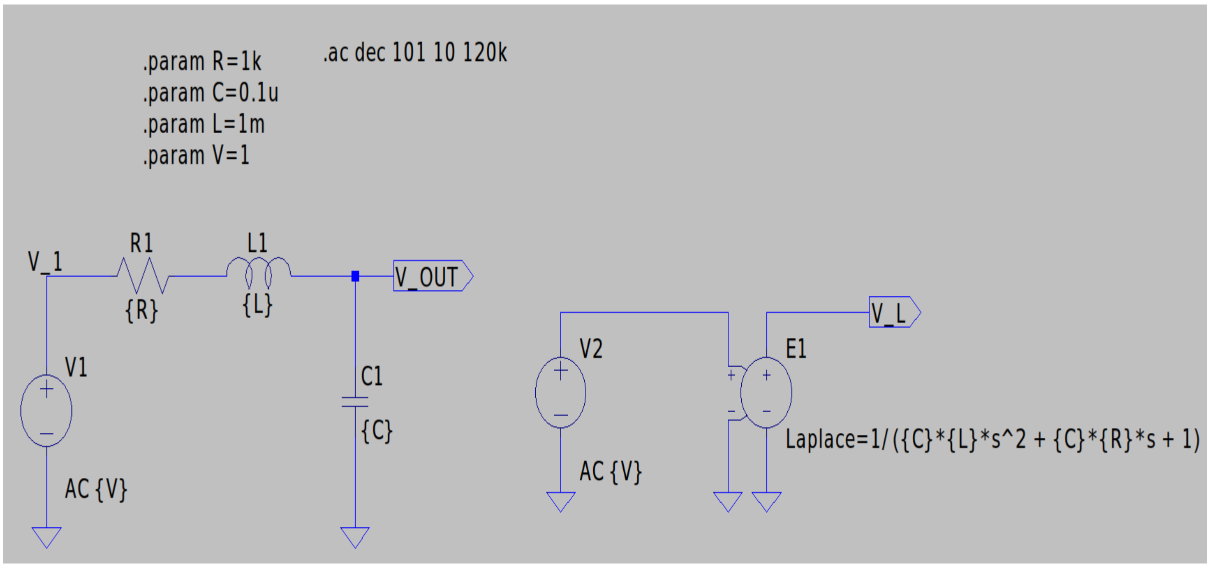


Figure 2.2: AC Sweep Analysis Simulation Setup

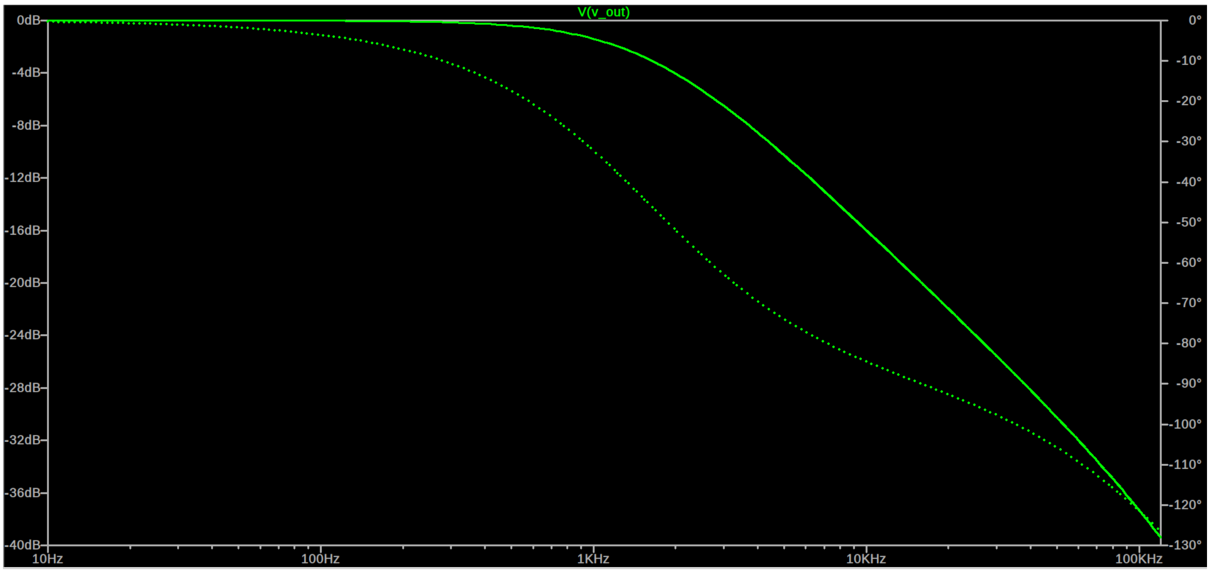


Figure 2.3: $V_{out}$ Bode Plot

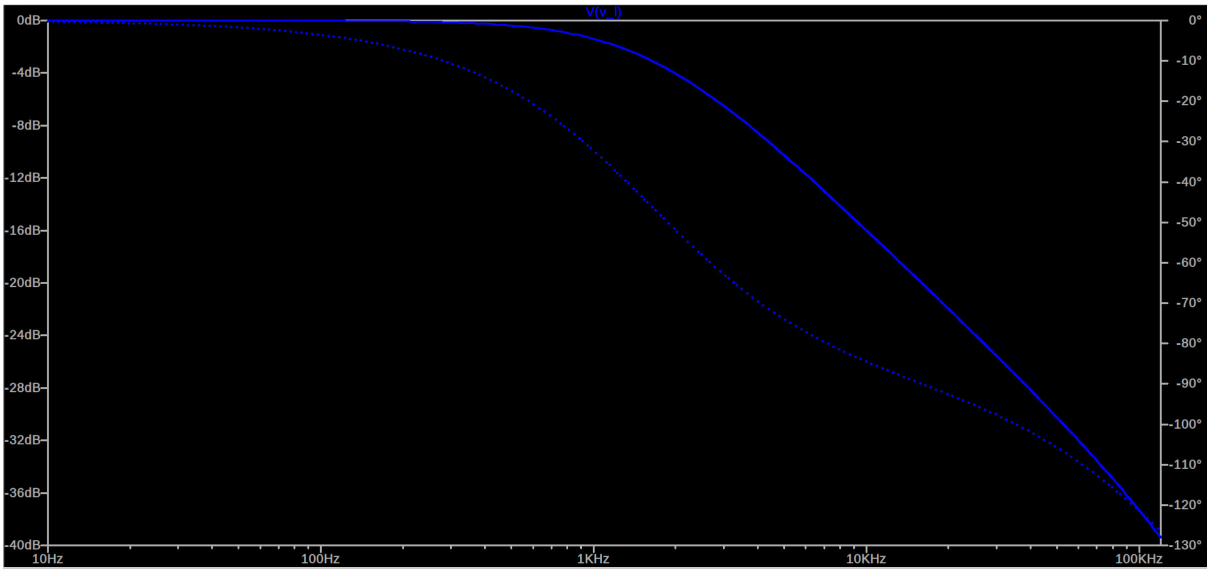


Figure 2.4: $H_V(s)$ Bode Plot

As shown in Figure 2.3, output voltage frequency response matches the transfer function $H_V(s)$ as shown in Figure 2.4.

### 2.1.3 Transient Analysis Simulation

To further validate the transfer function, we will perform a transient analysis in LTSpice. Changed the input voltage $V_{ac}$ to a $1V_{pp}$ sine wave at $10kHz$. The output voltage will be measured and compared to the transfer function.

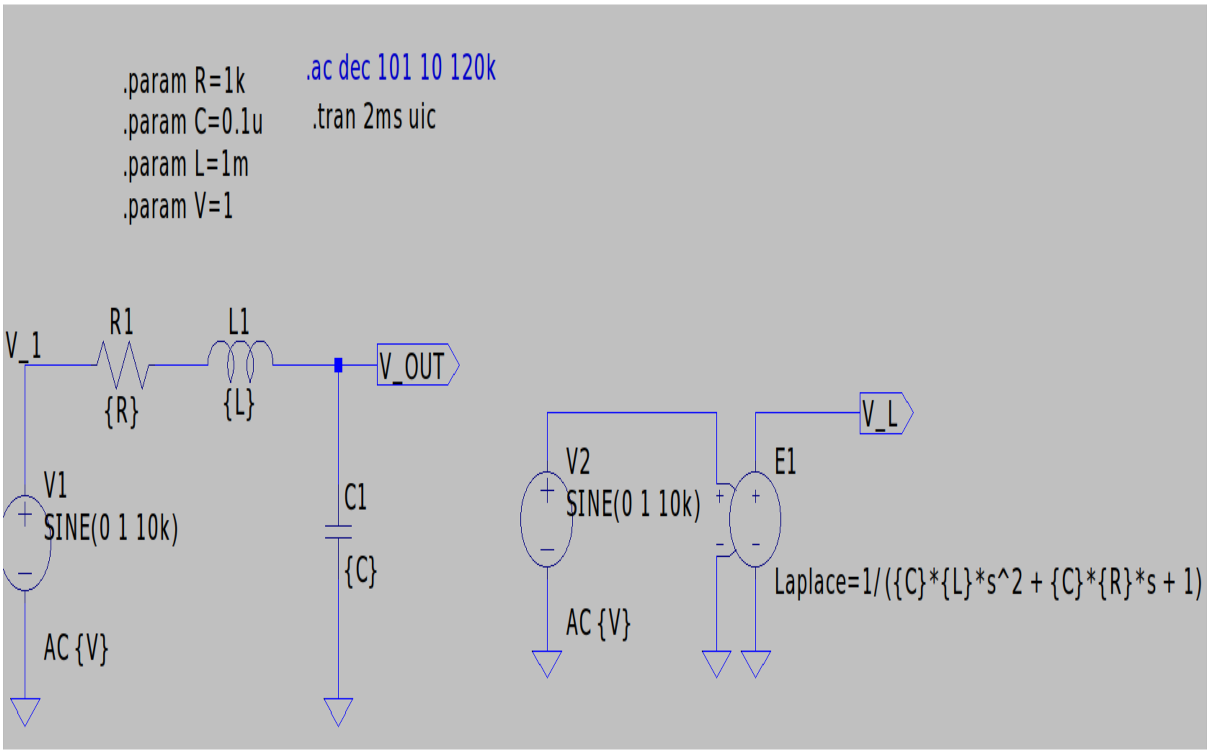


Figure 2.5: Transient Analysis Simulation Setup

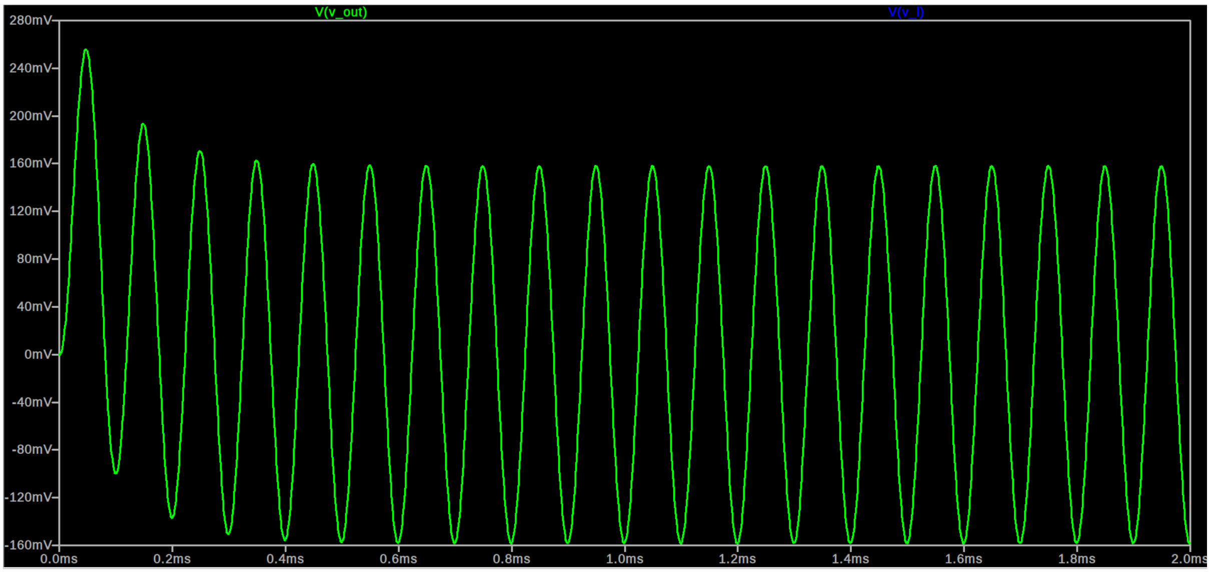


Figure 2.6: $V_{out}$ Transient Response

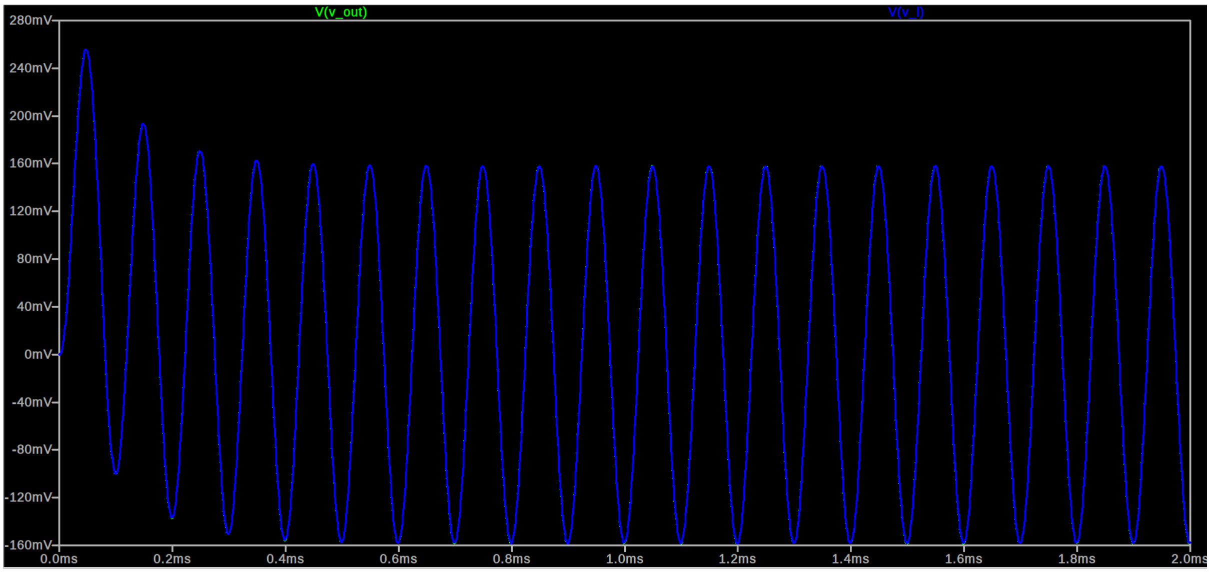


Figure 2.7: $H_V(s)$ Transient Response

As shown in Figure 2.6, output voltage matches the transfer function $H_V(s)$ as shown in Figure 2.7.

### 2.1.4 Unit Step Response

Unit step response can show the transient behavior of the circuit. In order to find unit step response, Heaviside step function ($u(t)$) is used as input voltage.

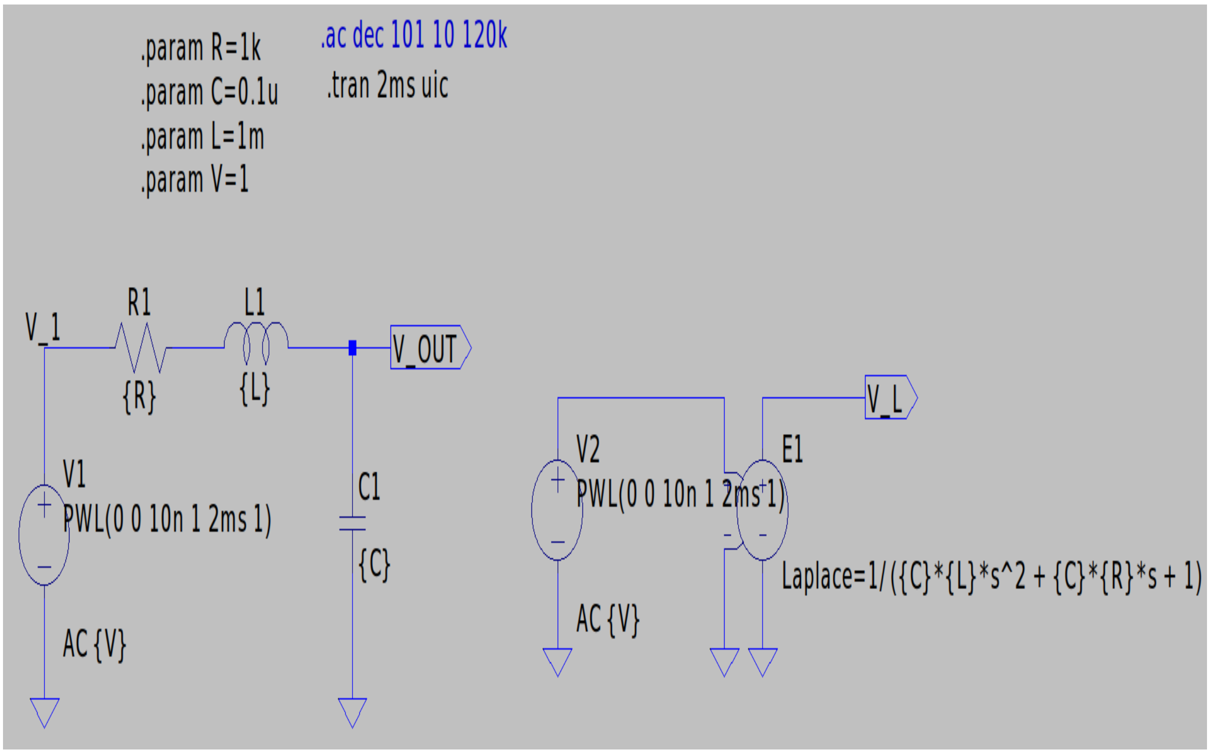


Figure 2.8: Unit Step Response Simulation Setup

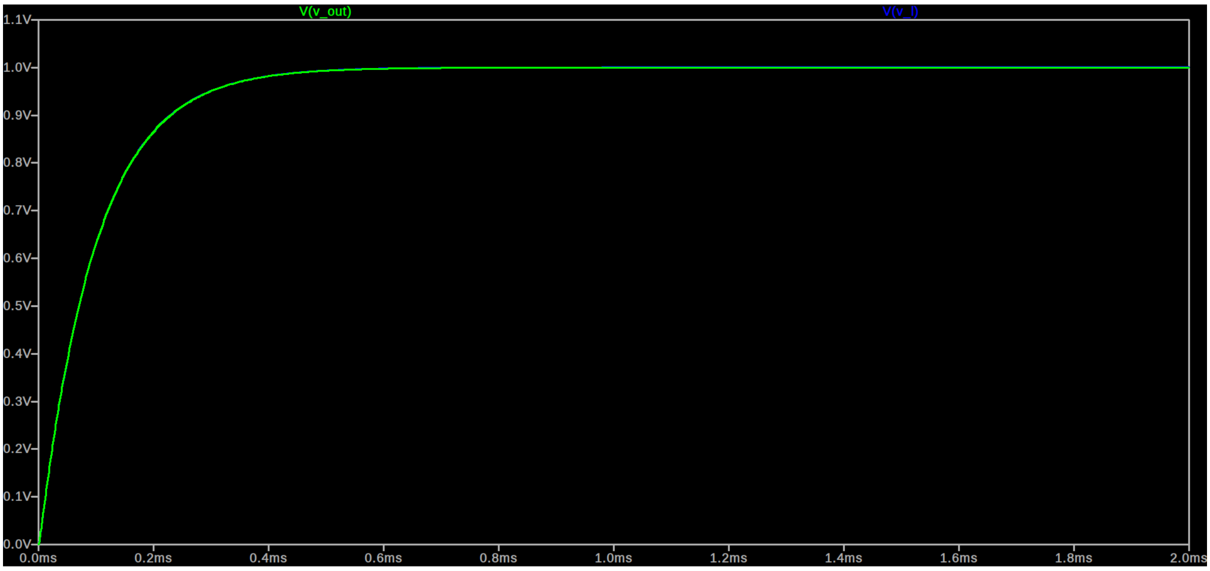


Figure 2.9: $V_{out}$ Unit Step Response

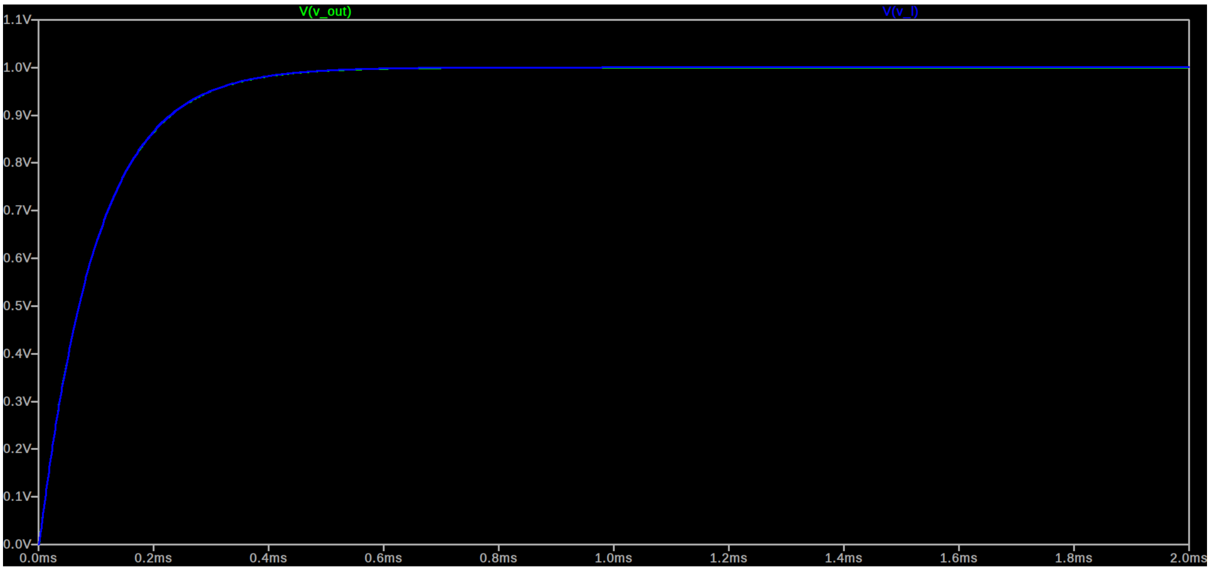


Figure 2.10: $H_V(s)$ Transient Response

As shown in Figure 2.9, output voltage matches the transfer function $H_V(s)$ as shown in Figure 2.10.

## 2.2 Op-Amp Circuit Analysis

### 2.2.1 Transfer Function Derivation

In order to derive the voltage gain transfer function ($H_V(s)$), we must first convert the circuit from time domain to the s-domain. The circuit is shown in Figure 2.11.

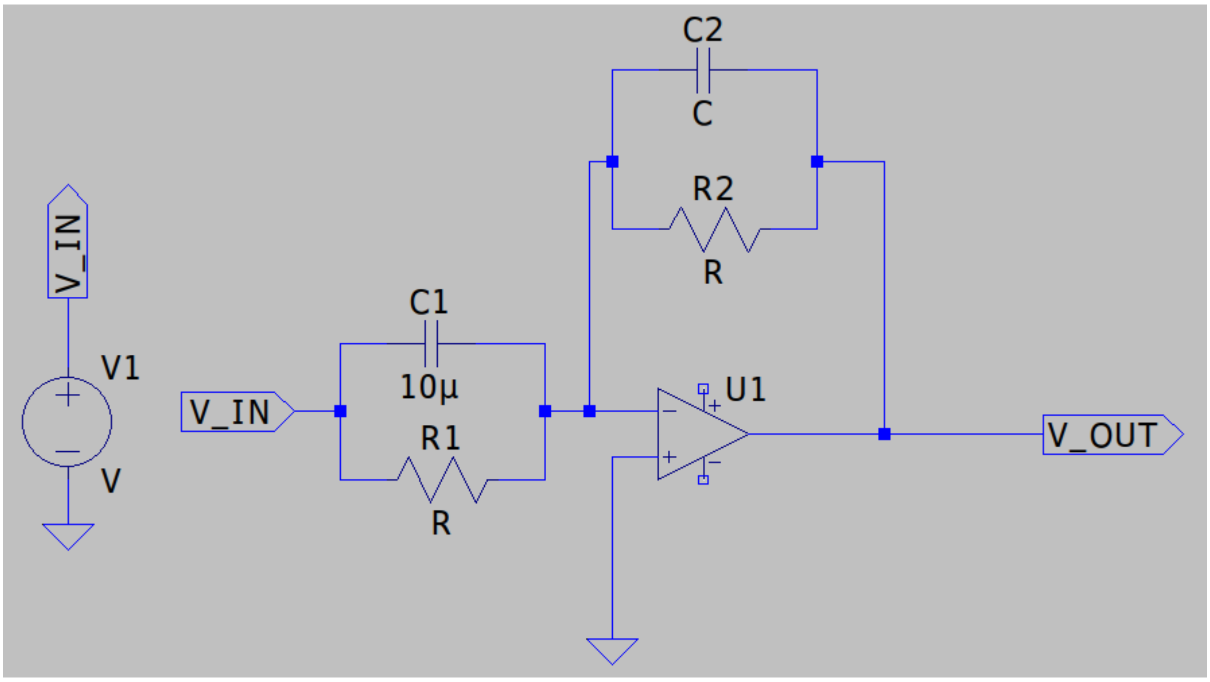


Figure 2.11: Op-Amp Circuit

Expected transfer function is $H_V(s) = \frac{V_o}{V_i} = -\frac{s+1000}{2(s+4000)}$:

$$\begin{aligned}
\text{let } Z_1 &= \frac{R_1}{1 + C_1 R_1 s} \\
\text{let } Z_2 &= \frac{R_2}{1 + C_2 R_2 s} \\
\frac{V_{in}}{Z_1} &= -\frac{V_{out}}{Z_2} \\
V_{out} &= -\frac{Z_2}{Z_1} V_{in} \\
\implies H_V(s) &= -\frac{Z_2}{Z_1} = -\frac{s+1000}{2(s+4000)} \\
\frac{s+1000}{2(s+4000)} &= \frac{C_1}{C_2} \cdot \left[\frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}}\right] \\
&\implies C_1 = 10\mu F, C_2 = 20\mu F, R_1 = 100\Omega, R_2 = 12.5\Omega
\end{aligned}$$

Using these values, bode plot of the circuit and transfer function is shown below:

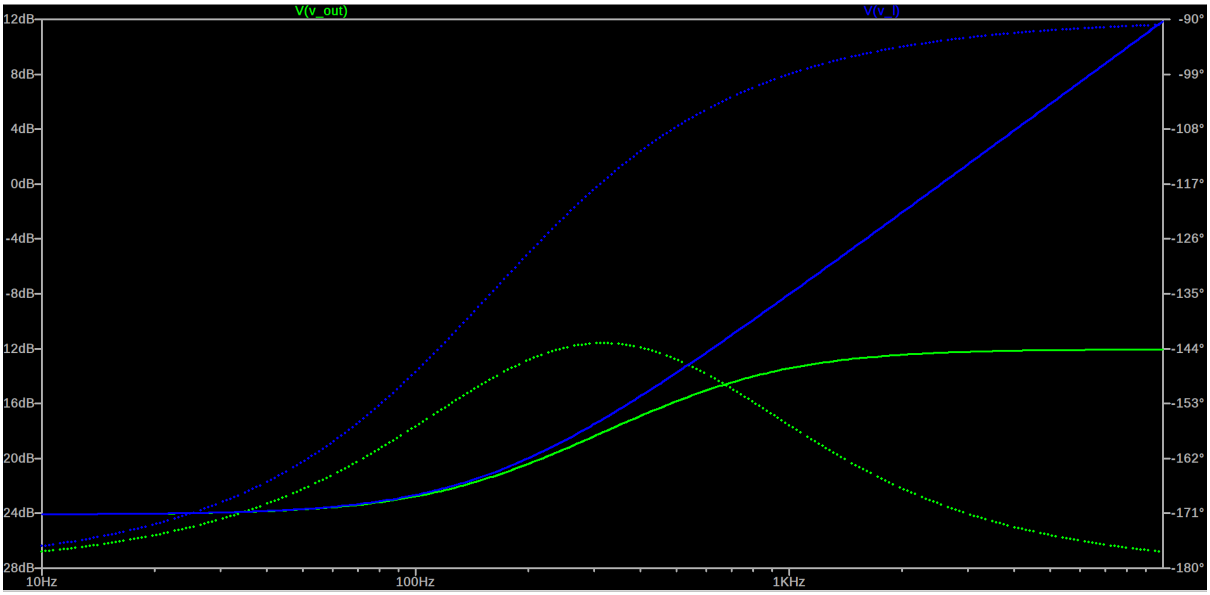


Figure 2.12: $V_{out}$ & $H(s)$ Bode Plot

As can be seen in Figure 2.12, output bode plot does not match the transfer function. This is due to pole and zero locations.

- **Pole at** $s = -4000$**:**
  - Pole has negative real part, meaning system is stable. The output will decay over time rather than growing unbounded.
  - The pole at $s = -4000$ ensures the output responds quickly to changes.
- **Zero at** $s = -1000$**:**
  - Zero has negative real part, meaning it adds a notch or dip in the frequency response.
  - The zero at $s = -1000$ might suppress or attenuate frequencies associated with noise or other disturbances.
- **Stability:** The system is stable as all poles have negative real part.

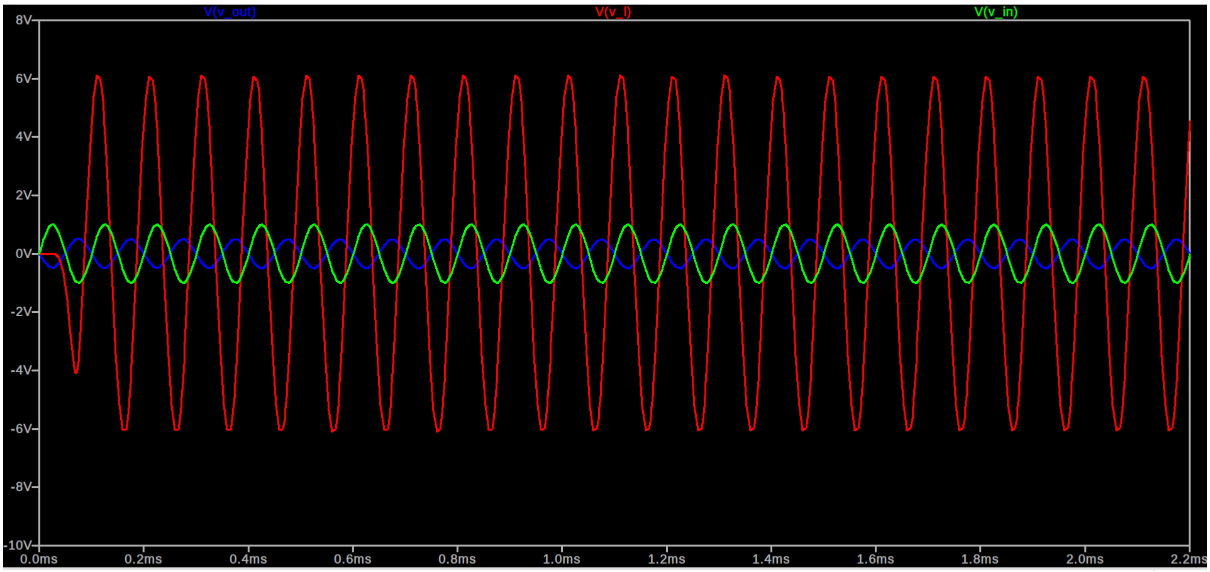


Figure 2.13: $V_{in}$, $V_{out}$ & $H(s)$ Plot @ $10kHz$

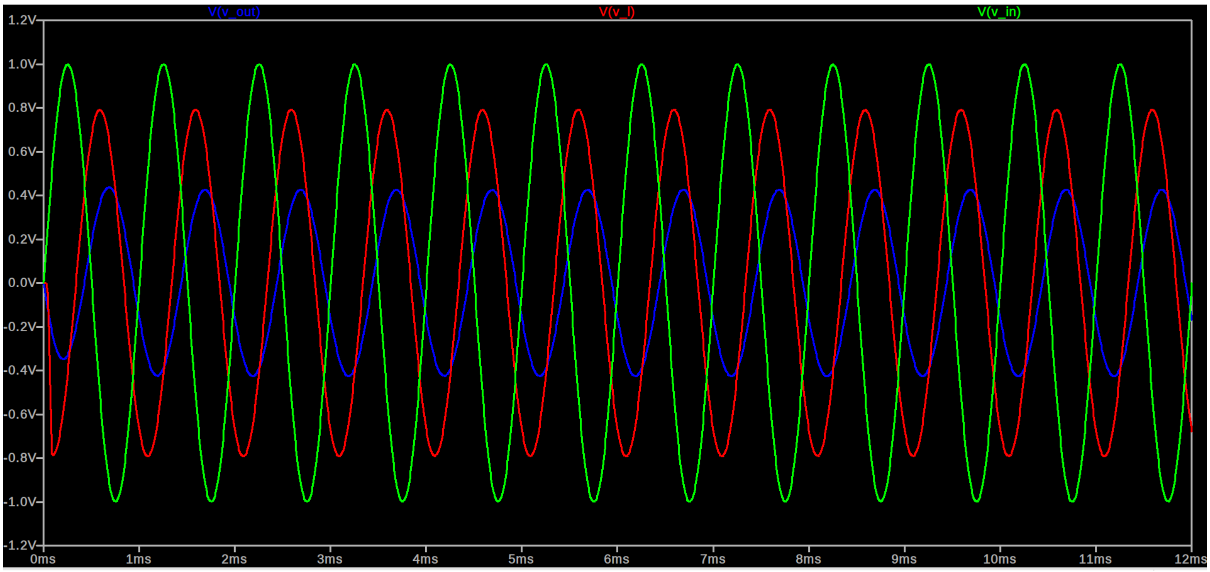


Figure 2.14: $V_{in}$, $V_{out}$ & $H(s)$ Plot @ $1kHz$

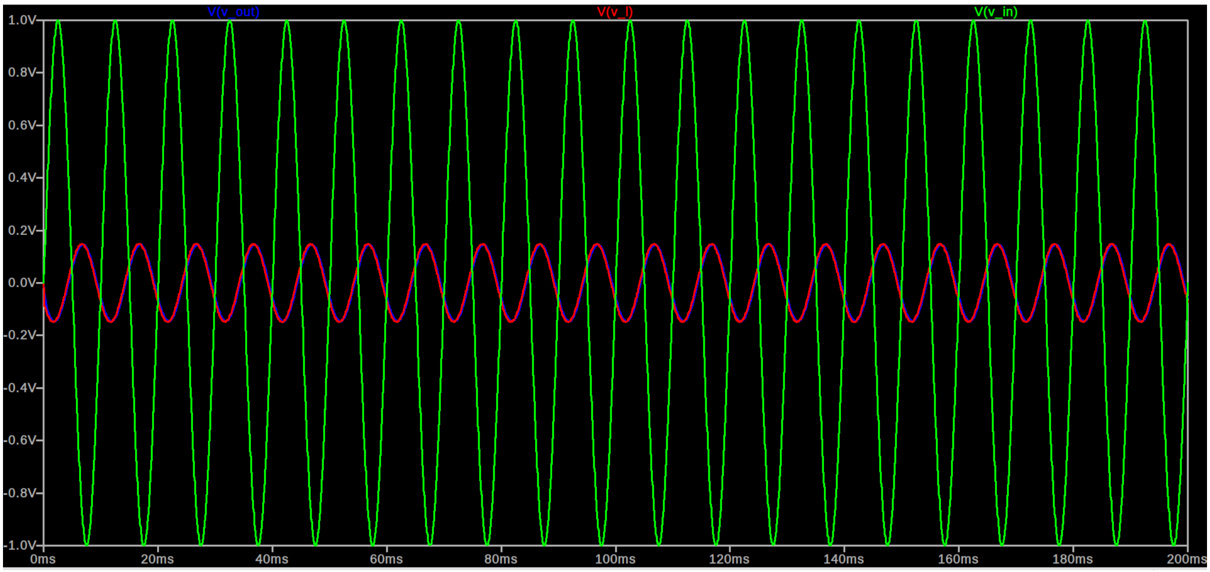


Figure 2.15: $V_{in}$, $V_{out}$ & $H(s)$ Plot @ $100Hz$

Plots seen in Figures 2.13, 2.14, and 2.15 show the input and output voltages at $10kHz$, $1kHz$, and $100Hz$ respectively. These graphs confirms the bode plot in Figure 2.12.

# Chapter 3

# Discussion

- **Comparison of Analytical and Simulated Results:** The derived transfer function $H(s)$ accurately predicted the frequency and transient responses observed in LTSpice simulations.
- **Frequency Response:** The AC Sweep analysis showed that Laplace transform and transfer functions are effective tools for analyzing frequency response that simplifies the process of determining the circuit's behavior at different frequencies.
- **Step Response:** The transient response of the circuit was analyzed using the step response of the circuit. The step response showed that the circuit is stable and has a fast response time.
- **Op-Amp Circuit Behavior:** The op-amp circuit was analyzed using the transfer function $H(s)$ and the transient response of the circuit. The circuit was found to be stable and had a fast response time. Poles and zeros of the transfer function were used to analyze the stability and characteristics of the circuit and it matched with the results.

# Chapter 4

# Conclusion

This pre-lab demonstrated the effectiveness of the Laplace Transform and LTSpice simulations in analyzing and designing circuits. The step-by-step procedure validated theoretical predictions through simulation. The approach provided valuable insights into frequency and time-domain responses for RLC and op-amp circuits, highlighting the practical application of Laplace Transform in circuit analysis.