INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
SEMESTER I 2023 – 24
MTL 104 (Linear Algebra and its Applications)
Quiz 1

Date: 23/08/2023 Timing: 3:10 PM to 3:45 PM

Question 1: Examine if the set S with usual addition and multiplication is a field.

(i) $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, (ii) $S = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. (1 + 3)

Question 2: Let V be an n-dimensional vector space over the field $\mathbb{R}$. Let W be a subspace of V such that Dim $(W) = m$ and $m < n$. Prove that there is a subspace U of V such that $V = W \oplus U$. What is Dim (U)? (4 + 1)

Question 3: Let V, W be two finite dimensional vector spaces and $T \in \mathcal{L}(V, W)$.

1. Prove that $V/(\text{Null } T)$ is isomorphic to Range (T) (by showing there is an isomorphism between them). (3)
2. Prove that Null $T^* = (\text{Range T})^0$. (3)