MTL712 (Computational methods for differential equations) November 2023

Lab Examination

Instructions:

(i). 20 marks are assigned to the LAB exam.
(ii). Duration of LAB exam is 1:30 hours.
(iii). Login to the systems using "Guest" account. Perform the coding in offline MATLAB.
(iv). Make a .zip file which includes all the MATLAB (.m) files. Send this .zip file to the email "Shweta.Kumari@maths.iitd.ac.in" no later than 3:40 pm.
(v). Heavy penalty would be given to each student whose code is found to be copied from others.
(vi). The Name of your .zip file must be your name_entry number.

1. Consider the following initial value problem (IVP) [8]

$$y' = 2x - y, \quad x \in (0, 4),$$

$$y(0) = 1.$$

Write a Matlab code based on the TS(2) method with automatic step size selection for the above IVP and plot the global error versus x_n for two different tolerances 10^{-2} and 10^{-3}. The exact solution of the above IVP is $y(x) = 2x - 2 + 3e^{-x}$.

2. Consider the following initial-boundary value problem

$$\begin{aligned} v_t &= \nu v_{xx}, \quad x \in (0,1), \ t > 0, \\ v(x,0) &= 6\sin(\pi x), \quad x \in [0,1], \\ v(0,t) &= 0 = v(1,t), \quad t \geq 0. \end{aligned}$$

Write a Matlab code for solving the above problem using the BTCS scheme on the spatial grid

$$\{x_k \ : \ x_k = k\Delta x, \ k = 0, 1, \ldots, M\} \ \text{ where } \Delta x = 1/M$$

at $t = 1$, by taking $M = 20$, $\nu = 1/6$, $\Delta t = 0.2$. The exact solution to this problem is $v(x,t) = 6\sin(\pi x)e^{-\nu(\pi^2)t}$.

1. Plot the numerical solution over the exact solution at $t = 1$. [4]
2. Check the convergence of the scheme numerically in any norm. [4]
3. Check the stability of the scheme numerically. [4]