INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
SEMESTER I 2023 – 24
MTL 104 (Linear Algebra and its Applications)
Re-Quiz

Date: 8/11/2023 Timing: 9:10 AM to 9:50 AM

Question 1: Let V be a finite-dimensional inner product space and $f : V \to V$ be a function such that $f(0) = 0$ and $\|f(x) - f(y)\| = \|x - y\|$. Prove that f is a linear transformation. (5)

Question 2: Let V be a finite-dimensional complex inner product space and $T : V \to V$ be self-adjoint. Prove that $T - iI$ is invertible, where I is the identity linear map. (5)

Question 3: Let T be a normal linear map on a finite-dimensional complex inner product space. Prove that there is a polynomial f, with complex coefficients, such that $T^* = f(T)$. (5)

Question 4: Let V be a finite-dimensional inner product space and $T : V \to V$ be a linear map. Prove that T is positive if and only if there exists a linear map $R : V \to V$ such that $T = R^*R$. (2 + 3)