

# DEPARTMENT OF MATHEMATICS

## MTL 122: Real and Complex Analysis

Major

Marks - 40

All questions are compulsory. Assume everything done in class.

(1.) Determine the limits:

(i)  $\lim_{z \rightarrow -1} \frac{iz+3}{z+1}$

(ii)  $\lim_{z \rightarrow \infty} \frac{2z+i}{z+1}$

(iii)  $\lim_{z \rightarrow i} \frac{iz^3-1}{z-i}$

[1+2+2 Marks]

(2.) Find the principal value of  $(1-i)^{4i}$ .

[5 marks]

(3.) Use antiderivative to evaluate  $\int_C z^{\frac{1}{2}} dz$ , where  $C$  is any contour joining 3 and -3, but otherwise lies above the Real axis.

[5 Marks]

(4.) State Morera's theorem.

[2 Marks]

(5.) Let  $P(z)$  be a polynomial of degree  $n$ . Prove that  $|P(z)| < 2|a_n||z|^n$  whenever  $|z| > R$ , for some large  $R$  with  $a_n$  being the leading coefficient of  $P(z)$ .

[4 Marks]

(6.) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = \sin x$ . Note that  $f(x)$  is analytic on whole  $\mathbb{R}$  and  $|\sin(x)| \leq 1$  on  $\mathbb{R}$ . Does this contradict Liouville's theorem? Explain in brief.

[4 Marks]

(7.) Compute the Laurent series expansion of the following around the prescribed point of singularity, thence determine the nature of each singularity:

(a)  $\frac{e^{2z}}{(z-1)^3}$  around  $z = 1$ .

(b)  $(z-3) \sin\left(\frac{1}{z+2}\right)$  around  $z = -2$ .

(c)  $\frac{z^2}{e^z-1}$  around  $z = 0$ .

[9 Marks]

(8.) Compute the integrals:

(p)  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2)} dx$

(q)  $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta}$ , where  $a > |b|$ .

[6 Marks]