MTL103. Major on 24/11/2023

Max Marks: 45 | Max Time: 2 hrs

Any unfair means will enforce disciplinary action.
Do all parts of the question at one place in the answer sheet.

1. Consider the nonlinear program [10

$$\begin{aligned} \min \quad & -x_1 + x_2 \\ \text{subject to} \quad & x_1^3 + x_2 \leq 0 \\ & x_1 - x_2^2 \geq 0 \end{aligned}$$

   (a) Identify (clearly) the feasible region S. Is S a convex set? Justify.
   (b) Determine the feasible direction set at $x^* = (1, -1)^T$ to S, with reasoning.
   (c) Does the basic constraint qualification hold at x^*? Justify.
   (d) The unique minimizer of the problem is x^*. Argue in favor or against this statement.
   (e) Is the claim that $\widehat{x} = (1/2, -1/8)^T$ not a local minimizer of the problem TRUE? Justify.

$A^T x = b$

$x \geq 0$

2. Consider the linear program (P): [10

max $\quad z = 5x_1 - 4x_2 + 3x_3$

subject to

$$\begin{aligned} 2x_1 + x_2 - 6x_3 &= 20 \\ 6x_1 + 5x_2 + 2x_3 &\leq 76 \\ 8x_1 - 3x_2 + 6x_3 &\geq 50 \\ x_1 \in \mathbb{R},\ x_2 \geq 0,\ x_3 &\leq 0. \end{aligned}$$

It is known that (P) has an optimal solution and x_1 is the only basic variable among x_1, x_2, x_3 in the optimal table of (P),

   (a) Find the optimal solution and the optimal value of (P).
   (b) Write the dual (D) of (P). Use the complementary slackness conditions to find the optimal solution of (D).
   (c) Suppose the right-hand side of the first and second constraints are changed respectively to 18 and 75, then without performing any additional simplex iterations or referring to the tableau, determine the effect of this change on solutions of (P) and (D)?
   (d) If the cost coefficient of x_1 and x_3 in (P) are changed to respectively 5.5 and 2.75, then without performing any additional simplex iterations or referring to the tableau, provides an upper bound on the optimal primal objective for the modified problem.

3. Solve the following transportation problem to maximize the profit, where the destinations can take more than the minimum demand b. [8

	D1	D2	D3	D4	a
O1	1	2	1	4	=30
O2	3	3	2	1	=50
O3	4	2	5	9	=20
b	≥ 20	≥ 30	≥ 20	≥ 10	

Consider the integer linear program:

max $z = -x_1 + 2x_2$

subject to

$$-4x_1 + 6x_2 \leq 9$$
$$x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0, \text{ both integers}$$

The optimal table of the associated relaxed linear problem is as follows:

c_B	v_B	x_B	y_1	y_2	y_3	y_4
2	x_2	5/2	0	1	1/10	2/5
-1	x_1	3/2	1	0	-1/10	3/5
	$z_j - c_j$		0	0	3/10	1/5

Starting from the above tableau and using the Gomory cutting plane technique, determine the optima solution for the given integer problem.

5. As a leader of a new industrial venture, you must inspect sites to determine four least-cost sites out of th eight available sites to build your industries. Suppose the available sites are labeled as $S_1, S_2, \ldots, S_8$, anc let the cost of these sites be $C_1, C_2, \ldots, C_8$. There are some regional development restrictions describec as follows:

i) Evaluating sites S_1 and S_4 will prevent you from exploring site S_8.
ii) Evaluating site S_2 or S_5 prevents you from assessing site S_6.
iii) Of the group S_1, S_3, S_5, S_7, only two sites may be assessed.
(iv) If S_4 is selected, then S_2 must also be selected.

Formulate (do not solve) an integer linear program to determine the minimum-cost scheme that satisfie all these restrictions. [4]

6. Write the dual of the general balanced assignment problem of minimizing the total cost of jobs assign ment to machines. Using the dual variables, explain the step (Step 5 of the algorithm) of updating th assignment matrix in an iteration of the Hungarian method.

[5