MTL712 (Computational methods for diff. eq.) IITD MAJOR EXAM

Duration of Examination: 2 hours November 2023

Instructions

1. The total number of marks is 50 (marks are indicated in the margin).

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1. Prove that an LMM is convergent if, and only if, it is both consistent and zero-stable. [7]

2. Given the multistep method [8]

$$y_{n+3} + \frac{3}{2}y_{n+2} = 3y_{n+1} - \frac{1}{2}y_n + 3hf_{n+2}, \ n \geq 0,$$

with starting values y_0, y_1, y_2.

   (a) Find the order of accuracy of the method.

   (b) Comment on stability and convergence.

3. Solve the 2nd order initial value problem [7]

$$y'' - 2y' + 2y = 0, \ t > 0,$$

$$y(0) = -0.4, \ y'(0) = -0.6,$$

using the RK2 method and find the approximate solution at $t = 0.1$ with stepsize $h = 0.1$. Also, determine the region of absolute stability.

Note: The RK2 method for an IVP

$$z'(t) = f(t, z(t)), \ t > t_0,$$

$$z(t_0) = z_0,$$

is given by

$$z_{n+1} = z_n + \frac{h}{2}(k_1 + k_2),$$

$$k_1 = f(t_n, z_n), \ k_2 = f(t_n + h, z_n + hk_1),$$

4. For a difference scheme, discuss the following with advantages/disadvantages.

   (a) Pointwise consistency and consistency in norm. [2]

   (b) Von-Neumann stability for IVP and von-Neumann stability for IBVP. [2]

   (c) Convergence in ℓ_2 norm or $\ell_{2,\Delta x}$ norm. [2]

5. Discuss the consistency of the BTCS scheme applied to the IBVP [7]

$$v_t = \nu v_{xx},\ x \in (0,1),\ t > 0,$$

$$v(x,0) = f(x),\ x \in [0,1],$$

$$v(0,t) = 0 = v(1,t),\ t > 0.$$

6. Consider the following IBVP

$$v_t - v_x = 0,$$

$$v(x,0) = \sin(\pi x),\ x \in [0,1],$$

$$v(0,t) = \sin(\pi t),\ t \geq 0,$$

$$v(1,t) = -\sin(\pi t),\ t \geq 0.$$

   (a) Prove the stability/instability of the FTFS scheme on the above problem. [4]

   (b) Find the approximate solution of $v(0.5, 0.03)$ with FTFS scheme using $\Delta x = 1/4$ and $\Delta t = 0.015$. [4]

7. Determine the values of $\alpha \in [0,1]$ for which the following difference scheme [7]

$$-\alpha r u_{k-1}^{n+1} + (1+2\alpha r)u_k^{n+1} - \alpha r u_{k+1}^{n+1} = (1-\alpha)r u_{k-1}^n + [1 - 2(1-\alpha)r]u_k^n + (1-\alpha)r u_{k+1}^n,$$

when applied to the IVP

$$v_t = \nu v_{xx},\ x \in (-\infty,\infty),\ t > 0,$$

$$v(x,0) = f(x),$$

is unconditionally stable, where $r = \nu \frac{\Delta t}{\Delta x^2} \geq 0$.