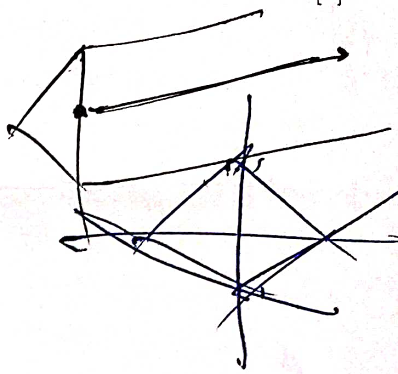
MTL103: Minor on 13/09/2023

Max Marks: 35 Max Time: 2 hrs

Any unfair means will cancel your exam.

1. Solve the following system of linear equations by the simplex method [5]

$$
\begin{aligned}
x + 2y + 3z &= 1\\
-x + 2y + 6z &= 6\\
4y + 9z &= 5\\
x,\ y,\ z &\geq 0.
\end{aligned}
$$



2. Using appropriate transformations, reformulate the following problem as an LPP

$$
(P) \qquad \begin{aligned} &\min\ 2x_1 + 3|x_2 - 10| \\ &\text{s.t.}\quad |x_1 + 2| + |x_2| \leq 5. \end{aligned}
$$

Clearly write the non-negativity restriction on the primal variables.

Write the dual of this LPP, clearly stating the dual constraints and non-negativity restriction on the dual variables. [6]

3. While solving a standard minimization form problem, we arrive at the following tableau:

v_B	x_B	y_1	y_2	y_3	y_4	y_5
x_3	4	-1	η	1	0	0
x_4	1	α	-4	0	1	0
x_5	β	γ	3	0	0	1
	$z_j - c_j \rightarrow$	δ	-2	0	0	0

For each one of the following statements, determine the parameter values making that statement true. [8]

(a) The current solution is non-degenerate optimal and there are multiple optimal solutions.

(b) The current solution is feasible but not optimal.

(c) The current solution is degenerate BFS but not optimal and in the next table we get a non-degenerate optimal solution.

(d) The problem is feasible but unbounded.

4. Let $S = \{(x, y) : 2x + y \geq 12,\ \ x - y \leq 3,\ \ 3y - x \geq 15\}$.

Clearly depict the region S graphically.

What are the extreme points and the extreme directions of S?

Express the point $(8, 11) \in S$ as a convex combination of the extreme points and a non-negative combination of the extreme directions. [8]

5. Consider the following LPP

$$
(P) \qquad \begin{aligned} &\min\ 7x_1 + 5x_2 + 4x_3 \\ &\text{s.t.}\quad -x_1 - x_2 + 2x_3 \geq 14, \\ &\qquad 2x_1 + x_2 \geq 10, \\ &\qquad 3x_1 + 2x_2 + x_3 \geq 28 \\ &\qquad x_1,\ x_2,\ x_3 \geq 0. \end{aligned}
$$

Write the dual of (P).

Given that $x^* = (6, 0, 10)$ is an optimal solution of (P), find the basic feasible solution corresponding to it.

Write the complete optimal simplex table with all entries of slack, surplus, and artificial variables.

Determine the optimal solution of the dual problem from it. Do you see any duality gap in the optimal values of the two problems? [8]