MTL103: Quiz 1 on 21/08/2023

Max Marks: 20 Max Time: 45 minute + 10 minute for uploading

Any unfair means will cancel your exam to award you zero. Upload your answers on Gradescope with correct mapping. Each question carry 5 marks.

1. Consider the following linear programming problem and S be its feasible set:

$$\begin{aligned} \max\ & x_1 - x_2 \\ \text{s.t.}\ & -3x_1 + 2x_2 \geq -3, \\ & -x_1 + 3x_2 \geq 0, \\ & x_1,\ x_2 \geq 0. \end{aligned}$$

(i) Determine the set of feasible directions to the feasible set S.

(ii) Write the equivalent linear programming model involving only the extreme points of S and the extreme directions of S.

(iii) Use the equivalent model to identify the optimal solution of the given problem with appropriate justification.

2. Let $A = \begin{pmatrix} 3 & 4 & -2 \\ 5 & 3 & -1 \end{pmatrix}$ and $b = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$. Consider the linear program

$$\max\ c^T x \quad s.t.\ Ax = b,\ x \geq 0.$$

Find the conditions on $c = (c_1, c_2, c_3)^T$ which makes the basic feasible solution $x = (1, 1, 0)^T$ an optimal solution to the problem.

3. Let S be a non-degenerate polyhedron with four extreme points $(0, 0)$, $(2, 0)$, $(0, 2)$, $(4/3, 4/3)$. Describe the mathematical inequalities defining the set S. Find the BFS corresponding to the extreme points. What change is observed in the basic variables and basis matrices when we move from $(0, 2)$ to $(4/3, 4/3)$. Explain.

4. Which of the following sets is a convex set? Justify with correct proof or counter.

(i) $\{(x_1, x_2) : x_1 = 3,\ |x_2| \leq 4\}$

(ii) $\{(x_1, x_2, x_3) : x_3 = |x_2|,\ x_1 \leq 3\}$.