MTL 783 (Theory of Computation), Major Exam

Instructions:

- A reminder of the IITD honor code which you are supposed to adhere to: **"I do hereby undertake that as a student at IIT Delhi: (1) I will not give or receive aid in examinations, and (2) I will do my share and take an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code."**
- This exam will be worth 30 points and time given will be 120 minutes.
- Please make your arguments concise. Unnecessarily long answers might attract negative marks.
- You do NOT need to write the questions on your answer-sheets. Please follow the instructions sent in the email previously.
- You can use any lemmas/theorems/intermediate results done in the class, but please clearly state them if you do.

Question 1: Give the definition of the following.

a) $NTIME(t(n))$

b) NP (in terms of certifiers)

c) Mapping reduction, i.e., $A \leq_m B$.

[3 points]

Question 2: For each of the following statements, answer **True**, **False** or **Open question** according to our current state of knowledge of theory of computation and complexity theory, as described in class. Give brief reasons for your answers.

i) $SAT \leq_m \overline{SAT}$

ii) $SAT \leq_p \overline{SAT}$

iii) If L is decidable then so is L^R (here $L^R := \{w \mid w^R \in L\}$)

[2 + 2 + 2 points]

Question 3: Prove or disprove: there exists a unary language $L \subseteq \Sigma^*$ with alphabet $\Sigma = \{1\}$ that is undecidable. [4 points]

Question 4: Assume P $\neq$ NP. Show that the following language is undecidable.

$$A = \{< M > \mid M \text{ is a deterministic TM such that } L(M) \in P\}$$

[4 points]

Question 5: The Unary Post Correspondence Problem (UPCP) is same as the Post Correspondence Problem (PCP) done in the class except for the fact that the upper and lower strings in each of the dominoes are unary. Is UPCP decidable? Justify your answer. [4 points]

Question 6: One of the following languages is Turing-recognizable and the other is not. Which is which? Give a proof for both. [5 points]

a) $A = \{< M > \mid |L(M)| \leq 42\}$

b) $B = \{< M > \mid |L(M)| \geq 42\}$

Question 7: In the SET COVER problem, you are given a universe $U = \{x_1, x_2, \ldots, x_n\}$, a collection of subsets of the universe $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$, and an integer k, and you are asked if there exists $\mathcal{S}' \subseteq S$ such that $|\mathcal{S}'| \leq k$ and $\cup_{S_i \in \mathcal{S}'} S_i = U$. Express the problem as a language and show that it is NP-complete. [4 points]