MATH1131/41 Calculus
Lab Test 2 Sample Solutions

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Question 1

Find, to 10 significant figures, the unique turning point x_0 of

$$f(x) = 5\sin\left(\frac{1}{2}x^2\right) - \sin\left(\frac{5}{2}x\right)^2$$

in the interval $[1, 2]$. Find, to 10 significant figures, the value of the second derivative $f''(x)$ at the turning point, that is $f''(x_0)$.

Solution: The answers are 1.580743250 and -9.652901450.
In Maple, assign the variable `f` to the function. Now assign a variable `p` to the derivative of the function

```
p := diff(f,x).
```

To solve for `p` on the interval [1,2], use the command

```
r := fsolve(p, x = 1..2).
```

This will output 1.580743250, which is the answer to the first part of the question.
Now differentiate p to get $f''(x)$ and assign it to a variable using the command

```
q := diff(p,x).
```

To find $f''(x_0)$, enter the command

```
evalf(subs(x=r,q)).
```

This will output -9.652901450, which is the answer to the second part of the question.

Question 2

The Maple expression for the constant π is ____. The Maple expression for ∞ is ____. Evaluate, to 10 significant figures,

$$\int_1^\infty \frac{e^{-x}\cos\left(\frac{x^2}{3}\right)}{3+x}\,dx.$$

Solution: The answers are Pi, infinity and 0.03239018036
To find the integral using Maple, assign the variable a to the fraction to be integrated, then enter the Maple commands

```
int(a,x=1..infinity)
```

then

```
evalf(%)
```

to get the answer to 10 significant figures.

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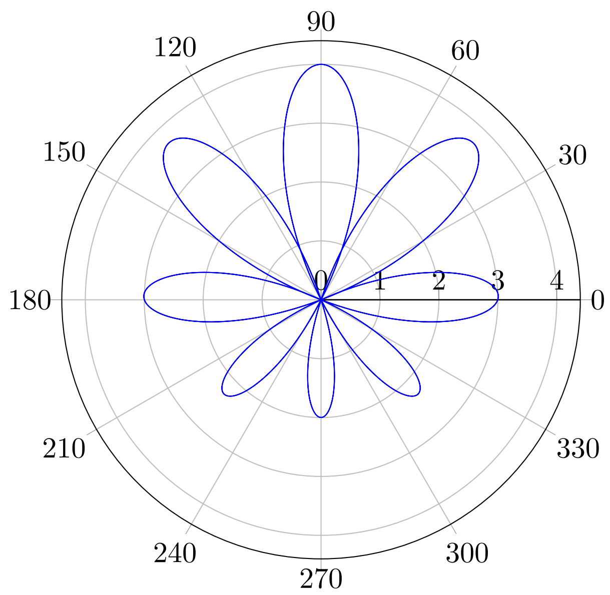
Question 3

Select the option below which is the plot of the polar curve $r = \sin(\theta) - 3\cos(4\theta)$ for $0 \le \theta \le 2\pi$.

Solution: In Maple, activate `with(plots)` then enter the equation of the curve and assign it to a variable `r`. Then enter the Maple command

```
polarplot([r], theta = 0..2*Pi)
```

which will plot the polar curve.



Question 4

Find the largest interval of the form $[a, b]$ *containing* -4 *on which the function* $f : \mathbb{R} \to \mathbb{R}$ *defined by the rule*

$$f(x) = 2x^3 + 9x^2 - 60x - 7$$

has an inverse.

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Solution: The answer is [-5,2].
A function is invertible if it is monotonically increasing or monotonically decreasing and continuous over an interval.
We can differentiate $f(x)$, either using Maple or by hand, resulting in

$$f'(x) = 6x^2 + 18x - 60.$$

To find the stationary points, set $f'(x) = 0$, which results in $x = 2, -5$. By testing points on either side of these x values, we can see that $x = -5$ is a max point and $x = 2$ is a min point. Since the curve is continuous between these two x values with a negative first derivative, we can deduce that it is monotonically decreasing over $[-5, 2]$.

Question 5

Find the slope of the line tangent to

$$-y^6 + x^3 + 2x^2y = 1$$

at $(1,0)$ *and enter it as a fraction or integer.*

Solution: The answer is $-\dfrac{3}{2}$.
Differentiate the curve implicitly, either by hand or by setting the curve to the variable `eqn` then using the Maple command

```
implicitdiff(eqn,y,x).
```

This gives the output $\dfrac{-3x^2-4xy}{-6y^5+2x^2}$. Then substitute $(1,0)$ into this equation to find the answer.

Question 6

Consider the function $f:\mathbb{R}\to\mathbb{R}$*, defined by the rule* $f(x)=|x^3+5x^2|$.

(a) Find all critical points of f *on the interval* $[-5,-1]$ *as exact values.*

(b) Complete the following sentence. The function f *is guaranteed to have* ___ [choices: a max but no min / either a max or a min but not both / both a max and a min / stationary / min but no max] *value on* $[-5,-1]$ *because it is* ___ [choices: integrable / bijective / continuous / differentiable / surjective / bounded / injective] *on the interval* $[-5,-1]$ *which is* ___ [choices: invertible / both open and closed / continuous / real / closed / open] *and bounded.*

(c) The maximum value of f *on* $[-5,-1]$ *is* ___*. The minimum value of* f *on* $[-5,-1]$ *is* ___*.*

(d) At which type of critical point or points does the maximum occur? Tick all that apply. [Choices: An end point of $[-5,-1]$*; A stationary point of* f *on* $(-5,-1)$*; A point on* $(-5,-1)$ *where* f *is not differentiable].*

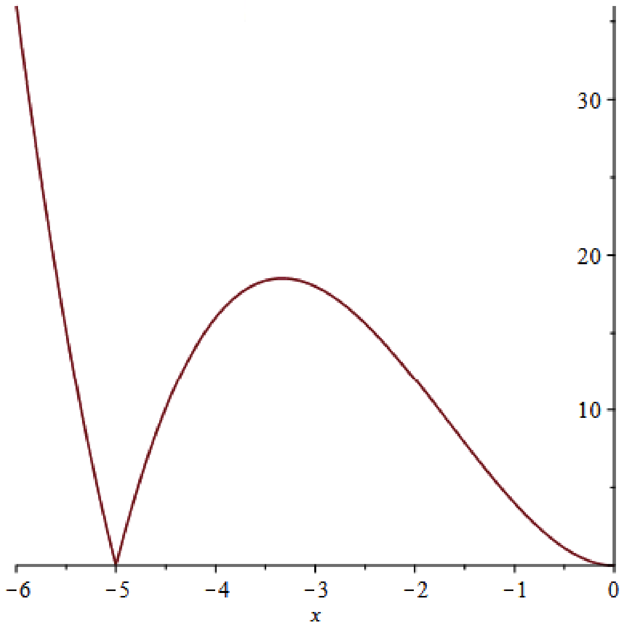
(e) At which type of critical point or points does the minumum occur? Tick all that apply. [Choices: An end point of $[-5,-1]$*; A stationary point of* f *on* $(-5,-1)$*; A point on* $(-5,-1)$ *where* f *is not differentiable]*

Solution:
(a) The answer is `{-5, -10/3, -1}`.
In Maple, assign the variable `f` to the function,

```
f := abs(x^3 + 5*x^2).
```

Then plot the graph on the interval $[-6,0]$ by using

```
plot(f, x=-6..0).
```



We can see from the graph that there is a point where f is not differentiable at $x = -5$, and there is a maximum point near $x = -3$.
To find this max, differentiate f and find where the first derivative is equal to 0, by using the command

```
g = diff(f, x)
```

which outputs $(3x^2 + 10x)|1, x^3 + 5x^2|$. To find the stationary point,

```
solve((3x^2 + 10x)*abs(1, x^3 + 5x^2) = 0)
```

which outputs $-\dfrac{10}{3}$.

(b) The answers are both a max and a min, continuous, closed. (By the Extreme Value Theorem.)

(c) The answers are $\dfrac{500}{27}$, 0.
We can see that the minimum is 0 from the graph we plotted for part (a). The maximum can be obtained by substituting $-\dfrac{10}{3}$ into f.

(d) The answer is: A stationary point of f on $(-5, 1)$.
We can see this from the graph we plotted in part (a).

(e) The answers are: An end point, a point where it is not differentiable.