If we let

$$L = \lim_{n\to\infty} x \ln\left(1 + \frac{1}{x}\right),$$

then we have

$$e^L = \exp\left(\lim_{n\to\infty} x \ln\left(1 + \frac{1}{x}\right)\right) = \lim_{n\to\infty} \left(1 + \frac{1}{x}\right)^x = e,$$

so by comparison of powers, $L = 1$. (solution by James Davidson)