MATH1151 Algebra Test 1 2008 S1 v2b

January 28, 2015

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1. The plane is parallel to $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Note that we could have chosen any two other vectors. Let O be the origin.

$$\begin{aligned}
\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\
&= (3,3,-2) - (2,-1,5) \\
&= (1,4,-7) \\
\overrightarrow{AC} &= \overrightarrow{OC} - \overrightarrow{OA} \\
&= (-1,6,1) - (2,-1,5) \\
&= (-3,7,-4)
\end{aligned}$$

Hence, the vector form of the equation of the plane is $\boldsymbol{x} = (2,-1,5) + \lambda_1(1,4,-7) + \lambda_2(-3,7,-4)$ for all $\lambda_1, \lambda_2 \in \mathbb{R}$.

Note there are many other answers to this question, as long as you choose a position vector on the plane (either A, B or C) and add it to the span of two non-parallel directional vectors that the plane is parallel to.

2. You should draw a diagram to help you understand this question.

   Let O be the origin, $\overrightarrow{OA}$ be represented by $\boldsymbol{a}$, $\overrightarrow{OB}$ be represented by $\boldsymbol{b}$, $\overrightarrow{OC}$ be represented by $\boldsymbol{c}$ and $\overrightarrow{OD}$ be represented by $\boldsymbol{d}$.

$$\begin{aligned}\overrightarrow{OM} &= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}\\ &= \boldsymbol{a} + \frac{1}{2}(\overrightarrow{OB} - \overrightarrow{OA})\\ &= \boldsymbol{a} + \frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})\\ &= \frac{\boldsymbol{b}+\boldsymbol{a}}{2}\end{aligned}$$

   Similarly, $\overrightarrow{ON} = \dfrac{\boldsymbol{c}+\boldsymbol{d}}{2}$.

$$\begin{aligned}\therefore \overrightarrow{MN} &= \overrightarrow{ON} - \overrightarrow{OM}\\ &= \frac{\boldsymbol{c}+\boldsymbol{d}}{2} - \frac{\boldsymbol{b}+\boldsymbol{a}}{2}\\ &= \frac{(\boldsymbol{c}-\boldsymbol{b}) + (\boldsymbol{d}-\boldsymbol{a})}{2}\end{aligned}$$

   Now notice that $\overrightarrow{AD} = \boldsymbol{d} - \boldsymbol{a}$ and $\overrightarrow{BC} = \boldsymbol{c} - \boldsymbol{b}$. But since $\overrightarrow{AD} \parallel \overrightarrow{BC}$, we can let $\boldsymbol{c} - \boldsymbol{b} = \mu\,(\boldsymbol{d} - \boldsymbol{a})$ for some real μ.

$$\therefore \overrightarrow{MN} = \mu\left(\frac{\boldsymbol{d}-\boldsymbol{a}}{2}\right) + \frac{\boldsymbol{d}-\boldsymbol{a}}{2} = \left(\frac{\mu+1}{2}\right)(\boldsymbol{d}-\boldsymbol{a})$$

   Hence $\overrightarrow{MN} = \left(\frac{\mu+1}{2}\right)\overrightarrow{AD}$, which is a scalar mulitple of $\overrightarrow{AD}$, so $\overrightarrow{AD} \parallel \overrightarrow{MN}$.

   Now consider the distance:

$$\begin{aligned}|\overrightarrow{MN}| &= \left|\left(\frac{\mu+1}{2}\right)(\boldsymbol{d}-\boldsymbol{a})\right|\\ &= \left(\frac{\mu+1}{2}\right)|\boldsymbol{d}-\boldsymbol{a}|\\ &= \frac{\mu|\boldsymbol{d}-\boldsymbol{a}| + |\boldsymbol{d}-\boldsymbol{a}|}{2}\end{aligned}$$

   which is the average of $|\overrightarrow{AD}| = |\boldsymbol{d}-\boldsymbol{a}|$ and $|\overrightarrow{BC}| = \mu|\boldsymbol{d}-\boldsymbol{a}|$.

3.

$$\left(\begin{array}{cccc|c} -1 & 2 & 1 & 1 & 0 \\ 3 & -5 & -4 & -1 & 1 \\ 1 & 3 & -6 & 9 & 5 \\ -1 & 3 & 0 & 6 & 7 \end{array}\right)$$

Perform the following row operations: $R_2 = R_2 + 3R_1$, $R_3 = R_3 + R_1$ and $R_4 = R_4 - R_1$.

$$\left(\begin{array}{cccc|c} -1 & 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & 1 \\ 0 & 5 & -5 & 10 & 5 \\ 0 & 1 & -1 & 5 & 7 \end{array}\right)$$

$R_3 = R_3 - 5R_2$, $R_4 = R_4 - R_2$

$$\left(\begin{array}{cccc|c} -1 & 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 6 \end{array}\right)$$

Let $x_3 = \lambda$. We do this because column 3 is not a leading column. Now we start from the bottom and work upwards:

$$\begin{aligned} 3x_4 &= 6 \\ x_4 &= 2 \end{aligned}$$

$$\begin{aligned} x_2 - \lambda + 2x_4 &= 1 \\ x_2 - \lambda + 4 &= 1 \\ x_2 &= -3 + \lambda \end{aligned}$$

$$\begin{aligned} -x_1 + 2x_2 + \lambda + x_4 &= 0 \\ -x_1 + 2(-3+\lambda) + \lambda + 2 &= 0 \\ x_1 &= -4 + 3\lambda \end{aligned}$$

Hence:

$$\begin{aligned}
\boldsymbol{x} &= \begin{pmatrix} -4+3\lambda \\ -3+\lambda \\ \lambda \\ 2 \end{pmatrix} \\
&= \begin{pmatrix} -4 \\ -3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad \text{for } \lambda \in \mathbb{R}
\end{aligned}$$

MATH1151 Algebra Test 1 2009 S1 v2B

April 10, 2015

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1. Basically the same style as Test 1 2008 Version 2B.

$$\boldsymbol{x} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ -3 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}.$$

   Note that there are many forms for the answer to this question.

2. If we can express a vector through any two of the points (say for e.g. $\overrightarrow{BP}$) in terms of a vector through one of these points and the remaining point (so $\overrightarrow{BM}$ or $\overrightarrow{PM}$), then B, P and M are collinear.
   If we considered these vectors, then we would find $\overrightarrow{BP} = \frac{2}{3}\overrightarrow{BM}$ implying B, P and M are collinear.

3.

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix}.$$