

For the sake of a contradiction, suppose neither T nor U are closed under multiplication.

Since T isn't closed under multiplication, there exist $t_1, t_2 \in T$ such that $t_1 t_2 \notin T$. However, as $T \subseteq S$, it follows $t_1, t_2 \in S$ and since S is closed under multiplication, $t_1 t_2 \in S$. Because $t_1 t_2 \notin T$ and $t_1 t_2 \in S$, then $t_1 t_2 \in U$ as $T \cup U = S$.

Hence there exist $t_1, t_2 \in T$ such that $t_1 t_2 = u$ and similarly there exist $u_1, u_2 \in U$ such that $u_1 u_2 = t$. Multiplying the equations together we get,

$$t_1 t_2 t = u_1 u_2 u$$

which is our desired contradiction, since $t_1 t_2 t \in T$ and $u_1 u_2 u \in U$, but T and U are disjoint, so in fact $t_1 t_2 t \neq u_1 u_2 u$. Hence at least one of T or U are closed under multiplication.