

EASY Question

EASY QUESTION: Let  $a, b, d \in \mathbb{Z}$ , and suppose that

$$\gcd(a, b) = d.$$

Prove that

$$\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1.$$

Given,  $\gcd(a, b) = d$ , where  $a, b, d \in \mathbb{Z}$

Let  $a = kd$ ,  $b = ld$ , where  $k, l \in \mathbb{Z}$  and  $k \neq l$

$$k = \frac{a}{d}, l = \frac{b}{d}$$

Assuming that  $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) > 1$ ,

$$\gcd(k, l) = m > 1$$

Hence,  $k = \lambda_1 m$ ,  $l = \lambda_2 m$ , where  $\lambda_1, \lambda_2 \in \mathbb{Z}$

$$a = \lambda_1 md, b = \lambda_2 md$$

$$\gcd(a, b) = md > d$$

Contradicting the initial statement,  $\gcd(a, b) = d$ .

$$\therefore \gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1,$$