

MATH1031 Mastery Lab Test 1

Solutions to Samples

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We cannot guarantee that our answers are correct - please notify us of any errors or typos at unswmathsoc@gmail.com, or on our [Facebook page](#). There are sometimes multiple methods of solving the same question. Remember that in the real class test, you will be expected to explain your steps and working out.

Question 2

Let

$$A = \begin{pmatrix} -7 & -3 & -7 \\ -1 & 2 & -1 \\ 4 & 13 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} -9 & -8 & 17 \\ 14 & -3 & 6 \end{pmatrix},$$
$$C = \begin{pmatrix} -1 & -3 & -1 \\ 6 & 5 & -3 \end{pmatrix}, \quad E = \begin{pmatrix} -7 & -8 \\ 2 & 6 \\ -8 & 3 \end{pmatrix},$$

and I is the 3×3 identity matrix. Some of the matrix expressions below are undefined. Select all the undefined expressions.

[Choices: $EC + 3I$, $B + 2C$, BC^T , $AE - 2I$, BC , BE , AB , $B - E$].

Solution: The answers are $AE - 2I$, BC , AB , $B - E$.

Recall that matrix addition (and subtraction) is only defined for matrices of the same size.

Whereas for the product of two matrices F and G of size $m_1 \times n_1$ and $m_2 \times n_2$ respectively, matrix multiplication FG is only defined when $m_2 = n_1$, and furthermore FG has size $m_1 \times n_2$. Using these rules, we can check the size of the resulting matrix in each option to find the answer.

- EC : E has 2 columns and C has 2 rows, so EC is well defined, and is in particular a 3×3 matrix. This certainly can be added with a 3×3 matrix.
- Clearly B and $2C$ are both 2×3 .
- Note that C^T is a 3×2 matrix, so BC multiplies a 2×3 by a 3×2 which is valid.
- A has 3 columns and E has 3 rows so AE is well defined. However AE will be a 3×2 matrix, which cannot be added to a 3×3 matrix.
- B has 3 columns but C has 2 rows, so BC is not defined.
- B has 3 columns and E has 3 rows, so BE is defined.
- A has 3 columns but B has 2 rows, so AB is not defined.
- Clearly B is 3×2 whilst E is 2×3 , so their difference is not defined.

Question 3

Given $A = \begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix}$, compute $3A - B$.

Solution: The answer is $\begin{pmatrix} -1 & -4 \\ 11 & 0 \end{pmatrix}$.

Steps:

$$\begin{aligned} 3A - B &= 3 \begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix} \\ &= \begin{pmatrix} -3 & -3 \\ 6 & 3 \end{pmatrix} - \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -4 \\ 11 & 0 \end{pmatrix}. \end{aligned}$$

Question 5

Given $A = \begin{pmatrix} -4 & -5 & -5 \\ 3 & -1 & 0 \\ -3 & 5 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -1 & 2 \\ -2 & -2 & -4 \\ 0 & -5 & -5 \end{pmatrix}$, and $AB = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}$, find the values of c and e .

Solution: The answer is $c = 37, e = -1$.

In general, to find AB_{ij} (i.e. the entry in the matrix AB that's in the i th row and j th column), we multiply the i th row of A by the j th column of B .

To find c , we multiply the first row of A by the last column of B :

$$-4 \times 2 + -5 \times -4 + -5 \times -5 = 37.$$

To find e we multiply the second row of A by the second column of B :

$$3 \times -1 + -1 \times -2 + 0 \times -5 = -1.$$

Question 6

Given $A = \begin{pmatrix} -3 & 2 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & -1 \\ -3 & 0 \end{pmatrix}$, compute $A + B^T$.

Solution: The answer is $\begin{pmatrix} -8 & -1 \\ 1 & 0 \end{pmatrix}$.

To find B^T , swap the rows and the columns of B . So,

$$\begin{aligned} A + B^T &= \begin{pmatrix} -3 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} -5 & -3 \\ -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -8 & -1 \\ 1 & 0 \end{pmatrix}. \end{aligned}$$

Question 7

Given $A = \begin{pmatrix} 0 & -1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$, solve for X in the equation $AX = B$. You are given that $A^{-1} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ -1 & 0 \end{pmatrix}$.

Solution: The answer is $X = \begin{pmatrix} 7 \\ -9 \end{pmatrix}$.

Multiply both sides of $AX = B$ on the left by A^{-1} to obtain $A^{-1}AX = A^{-1}B$. Now, the inverse of A multiplied by A is just the identity matrix, so $A^{-1}A = I$. Then, we have $IX = X = A^{-1}B$. We now have a formula for finding X . By substituting A^{-1} and B into this,

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 9 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 7 \\ -9 \end{pmatrix}. \end{aligned}$$

(Note: if the question asked to solve for X in the equation $XA = B$, we would multiply both sides of this equation on the right by A^{-1} to obtain $X = BA^{-1}$. Remember, BA^{-1} is different to $A^{-1}B$ – in general, you can't swap the order of matrix multiplication)

Question 9

Suppose that $8 \sin x - 11 \cos x = R \sin(x - a)$. What is the value of R and a ?

Solution: The answer is $R = \sqrt{185}$ and $a = \tan^{-1}\left(\frac{11}{8}\right)$.

To find R , note that $R = \sqrt{8^2 + (-11)^2} = \sqrt{185}$. To find $\tan a$, for $R \sin(x - a)$ disregarding any negative signs, take the coefficient of $\cos x$ and divide by the coefficient of $\sin x$ to get $\frac{11}{8}$. We can then take the inverse tan of both sides to get the answer.

Question 11

Let $f(x)$ be the sinusoidal function $-7 \sin(9\pi x)$. What is the amplitude and period of $f(x)$?

Solution: The answer is: amplitude = 7, period = $\frac{2}{9}$.

The amplitude is simply the coefficient of $\sin(9\pi x)$ disregarding any negative signs. The period is given by $\frac{2\pi}{9\pi} = \frac{2}{9}$.

Question 12

Let the function f be defined by

$$f(x) = \begin{cases} 3x^2 - 2, & \text{for } 0 \leq x < 3 \\ 37 - 4x, & \text{for } 3 \leq x < 5 \\ f(x + 5), & \text{for all } x \end{cases}$$

Find $f(0)$, $f(2)$, $f(4)$, $f(32)$ and the period of the function.

Solution: The answers are: period = 5, $f(0) = -2$, $f(2) = 10$, $f(4) = 21$, $f(32) = 10$.

To find $f(0)$, we notice that it lies in the first case, as 0 certainly satisfies $0 \leq x < 3$. So we substitute 0 into $3x^2 - 2$ to get -2 .

To find $f(2)$, we notice that it also lies in the first case, so substituting 2 into $3x^2 - 2$ gives 10.

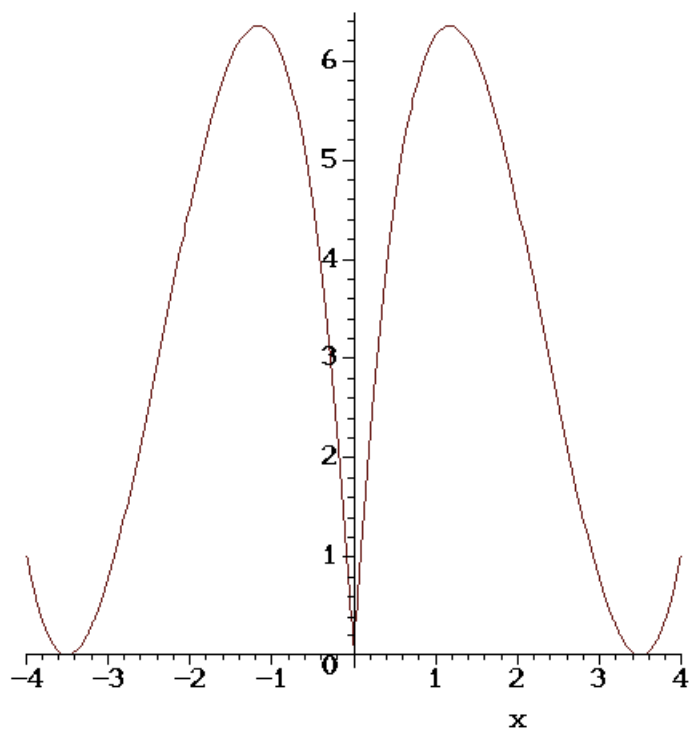
To find $f(4)$, we notice that it lies in the second case, so substituting 4 into $37 - 4x$ gives 21.

To find $f(32)$, we use the last case which is $f(x) = f(x + 5)$. This means the function repeats every 5 units, so $f(32) = f(27) = f(22) = \dots = f(2)$ which now lies in the first case, and equals 10.

Question 13

The function $f(x)$, defined on $[-4, 4]$ except 0, has the following property $f(x) = x(x - 3.5)^2$ when $0 < x \leq 4$, and the function is even. What is the graph of the function?

Solution: The answer is:



Since $f(x)$ is an even function defined on $0 < x \leq 4$, we can plot the function on the domain $0 < x \leq 4$, then reflect it along the y axis. To plot the graph, we could either use a table of values or differentiate the function to find the stationary points.

Question 15

Calculate the limit

$$\lim_{x \rightarrow -2} \frac{x^2 + 4x + 4}{x + 2}.$$

Solution: The answer is: 0.

Factorise the numerator to get $\frac{(x+2)^2}{x+2}$. Now the $(x+2)$ cancels, so we are left with $\lim_{x \rightarrow -2} x + 2$, which is 0.

Question 16

Calculate the limit

$$\lim_{x \rightarrow \infty} \frac{4x^3 - x + 8}{\sqrt{4x^6 - 6x^3 - 11}}.$$

Solution: The answer is: 2.

When we are taking the limit as x goes to infinity, we look at the largest terms to see what the function "behaves like".

In this case, the function behaves like $\frac{4x^3}{\sqrt{4x^6}}$, or $\frac{4x^3}{2x^3}$, for large x . In this case, we can simply take the leading coefficient of the numerator and divide by the leading coefficient of the denominator to find the limit. Therefore the limit is $\frac{4}{2} = 2$.

Question 18

Calculate the limit

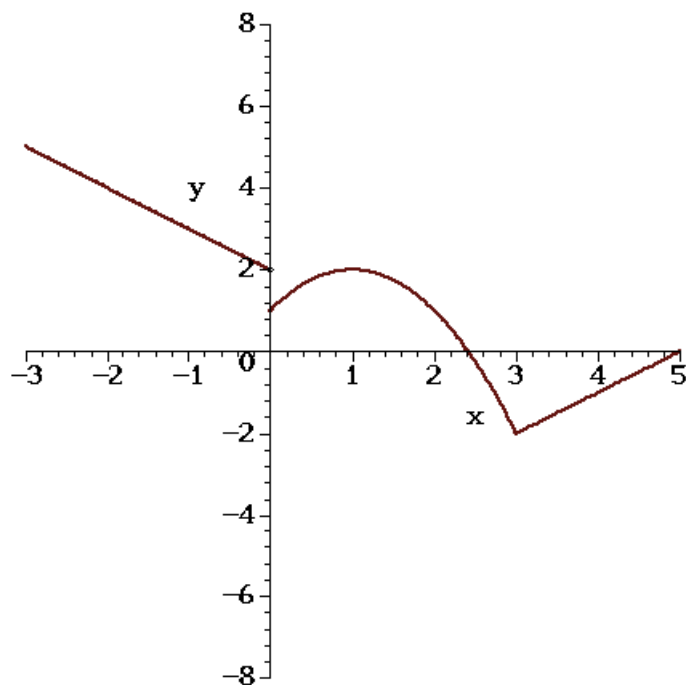
$$\lim_{x \rightarrow 4} \frac{x - 4}{x^2 - 7x + 12}.$$

Solution: The answer is: 1.

The denominator can be factorised to $(x - 3)(x - 4)$. Hence, the fraction can be simplified to $\frac{1}{x - 3}$, and substituting 4 into this gives 1.

Question 20

Given the graph of $y = f(x)$ below, state all the values of x where $f(x)$ is not continuous, and all the values where $f(x)$ is not differentiable.



Solution: The answer is: not continuous at $x = 0$, not differentiable at $x = 0, 3$.

There is a jump discontinuity at $x = 0$, so it is not continuous or differentiable here. It is also not differentiable at $x = 3$ since there is a sharp point.

Question 21

Find $\frac{dy}{dx}$ if $y = \frac{2x^7 + 2}{x}$.

Solution: The answer is: $12x^5 - \frac{2}{x^2}$.

We can split up the fraction and differentiate:

$$y = 2x^6 + \frac{2}{x}$$

$$\frac{dy}{dx} = 12x^5 - \frac{2}{x^2}.$$

Alternatively, we could use the quotient rule which gives:

$$\begin{aligned}\frac{dy}{dx} &= \frac{14x^6(x) - (2x^7 + 2)}{x^2} \\ &= \frac{14x^7 - 2x^7 - 2}{x^2} \\ &= \frac{12x^7 - 2}{x^2} \\ &= 12x^5 - \frac{2}{x^2}.\end{aligned}$$

Question 22

Find the exact value of the gradient of $y = x^3 - 2x^2 - 6 + \frac{2}{x^2}$ at $x = -1$.

Solution: The answer is: 11.

Differentiate y to get $y' = 3x^2 - 4x - 4x^{-3}$ then substitute in $x = -1$ to find that the gradient is 11.

Question 24

Find $\frac{dy}{dx}$ if $y = (3x - 1)(\sin x)$.

Solution: The answer is: $3 \sin x + (3x - 1) \cos x$

The product rule gives:

$$y' = 3 \sin x + (3x - 1) \cos x.$$

Question 25

Find $\frac{dy}{dx}$ if $y = (\ln x)^4$.

Solution: The answer is: $4(\ln x)^3(\frac{1}{x})$.

We can find the answer by using the chain rule, and the fact that the derivative of $\ln x$ is $\frac{1}{x}$.

Question 26

Find $\frac{dy}{dx}$ if $y = \sin(4x^3 + 1)$

Solution: The answer is: $12x^2 \cos(4x^3 + 1)$.

We apply chain rule, by changing the sin to cos and then multiplying by the derivative of $4x^3 + 1$ to get our answer.