MATH1231/1241 Lab Test 2
Calculus Sample Solutions

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We cannot guarantee that our answers are correct - please notify us of any errors or typos at unswmathsoc@gmail.com, or on our Facebook page. There are sometimes multiple methods of solving the same question.

Note: Any text presented `like this` is presented as required in Maple syntax.

Question 1

Solve the initial value problem given by the differential equation

$$\frac{dy}{dx} = x^6 y^2$$

together with the initial condition: If $x = -2$ then $y = -3$.
To solve this, rearrange and integrate to get

$$\int f(y)dy = \int g(x)dx.$$

If $\frac{dy}{dx} = x^6 y^2$ then we have $\frac{1}{y^2}\frac{dy}{dx} = x^6$. So,

$$\int \frac{1}{y^2}dy = \int x^6 dx.$$

Hence $f(y) = \dfrac{1}{y^2}$ and $g(x) = x^6$.

By integrating, we have

$$-\frac{1}{y} = \frac{1}{7}x^7 + C.$$

So $y = \dfrac{-7}{x^7 + 7C}$. Applying the initial condition, then $C = \dfrac{391}{21}$. Hence $y = \dfrac{-21}{3x^7 + 391}$.

Question 3

Determine which of the following differential equations are exact.

- $\Big(3x^2y^2 - 2\Big)dx + \Big(2xy^3 + 1\Big)dy = 0.$
- $(3\sin(3x)(2x + y) + 2\cos(3x))dx + \cos(3x)dy = 0.$
- $\Big(3x^2y^2 - 2\Big)dx + \Big(2x^3y + 1\Big)dy = 0.$
- $(3\cos(3x)(2x + y) + 2\sin(3x))dx + \sin(3x)dy = 0.$

The first ODE is not exact since

$$\begin{aligned}\frac{\partial}{\partial y}(3x^2y^2 - 2) &= 6x^2y,\\ \frac{\partial}{\partial x}(2xy^3 + 1) &= 2y^3\end{aligned}$$

and so $\dfrac{\partial}{\partial y}(3x^2y^2 - 2) \neq \dfrac{\partial}{\partial x}(2xy^3 + 1)$.

The second ODE is not exact since

$$\begin{aligned}\frac{\partial}{\partial y}(3\sin(3x)(2x + y) + 2\cos(3x)) &= 3\sin 3x,\\ \frac{\partial}{\partial x}(\cos 3x) &= -3\sin 3x\end{aligned}$$

and so $\dfrac{\partial}{\partial y}(3\sin(3x)(2x + y) + 2\cos(3x)) \neq \dfrac{\partial}{\partial x}(\cos 3x)$.

The third ODE is exact, since

$$\frac{\partial}{\partial y}(3x^2y^2 - 2) = 6x^2y,$$
$$\frac{\partial}{\partial x}(2x^3y + 1) = 6x^2y$$

and so $\frac{\partial}{\partial y}(3x^2y^2 - 2) = \frac{\partial}{\partial x}(2x^3y + 1)$.

The fourth ODE is exact, since

$$\frac{\partial}{\partial y}(3\cos(3x)(2x + y) + 2\sin(3x)) = 3\cos 3x,$$
$$\frac{\partial}{\partial x}(\sin 3x) = 3\cos 3x$$

and so $\frac{\partial}{\partial y}(3\cos(3x)(2x + y) + 2\sin(3x)) = \frac{\partial}{\partial x}(\sin 3x)$.

Question 4

Find a particular solution, $y_p(x)$, of the non-homogeneous differential equation

$$\frac{d^2}{dx^2}y(x) + 2\left(\frac{d}{dx}y(x)\right) + y(x) = -3x - 2,$$

given that $y_h(x) = Ae^{-x} + Bxe^{-x}$ is the general solution of the corresponding homogenous ODE.

The RHS of our ODE is a first order polynomial, so we should try a first order polynomial for our particular solution: $y_p(x) = ax + b$.

By substituting $y = y_p(x)$, then $y'(x) = a$ and $y''(x) = 0$. Hence

$$ax + (2a + b) = -3x - 2.$$

So $a = -3$ and $b = 4$, and $y_p = -3x + 4$.

Question 5

Use Maple to find the solution of the initial value problem

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

with initial conditions $y(0) = 5$ *and* $y'(0) = 5$.

To compute the solution to the differential equation via Maple, enter the command `dsolve({y(x)*diff(y(x),x,x)+(diff(y(x),x))^2=0,y(0)=5,D(y)(0)=5},y(x));`. Maple gives us the solution $y(x) = 5\sqrt{1+2x}$.

Question 9

Suppose that a function f *has derivatives of all orders at* a. *Then the series*

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

is called the Taylor series for f *about* a, *where* $f^{(n)}$ *is the nth order derivative of* f.
Suppose that the Taylor series for $\frac{e^{4x}}{1-x}$ *about* 0 *is*

$$a_0 + a_1x + a_2x^2 + ... + a_6x^6 + ...$$

Find the exact values of a_0 *and* a_6.

We can use the Maple command `taylor(exp(4*x)/(1-x),x=0,7);` to find the Taylor series of this function around 0, up to the 6th term. We find that

$$\frac{e^{4x}}{1-x} = 1 + 5x + 13x^2 + \frac{71}{3}x^3 + \frac{103}{3}x^4 + \frac{643}{15}x^5 + \frac{437}{9}x^6 + O(x^7).$$

Hence we have $a_0 = 1$ and $a_6 = \frac{437}{9}$.