MATH1231/1241 Calculus Revision
Additional Solutions to Part 1

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We cannot guarantee that our answers are correct - please notify us of any errors or typos at unswmathsoc@gmail.com, or on our Facebook page. There are sometimes multiple methods of solving the same question. Remember that in the real class test, you will be expected to explain your steps and working out.

Question 1 $\int x^n e^{x^2} dx$

$$\begin{aligned} &\int x^n e^{x^2} dx \\ &= \int x^{n-1} x e^{x^2} dx \end{aligned}$$

By considering integration by parts, we have:

$$u = x^{n-1} \qquad dv = xe^{x^2}\,dx$$
$$du = (n-1)x^{n-2}\,dx \qquad v = \frac{1}{2}e^{x^2}$$

Hence we get

$$\begin{aligned} I_n &= uv - \int v du \\ &= \frac{1}{2}e^{x^2}x^{n-1} - \frac{n-1}{2}x^{n-2}e^{x^2} \\ &= \frac{1}{2}e^{x^2}x^{n-1} - \frac{n-1}{2}I_{n-2} \end{aligned}$$

Therefore our reduction formula is:

$$I_n = \frac{1}{2}e^{x^2}x^{n-1} - \frac{n-1}{2}I_{n-2}$$

Question 2 $\int \sqrt{1-x^2}dx$

Using the substitution $x = \sin\theta$

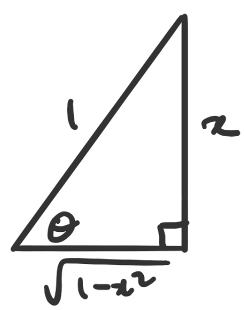
$$dx = \cos\theta d\theta$$

Hence,

$$\begin{aligned}
\int \sqrt{1-x^2}dx &= \int \sqrt{1-\sin^2\theta}\cos\theta d\theta \\
&= \int \cos^2\theta d\theta \\
&= \frac{1}{2}\int 1+\cos 2\theta d\theta \\
&= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C \\
&= \frac{1}{2}\left(\theta + \sin\theta\cos\theta\right) + C
\end{aligned}$$

Now, bring back into terms of x, we consider trig ratios,

$$\sin\theta = x \implies x = \sin^{-1}\theta$$



From the diagram we compute,

$$\cos\theta = \sqrt{1-x^2}$$

Therefore we have,

$$\int \sqrt{1-x^2}dx = \frac{1}{2}(\sin^{-1} x + x\sqrt{1-x^2}) + C$$

Question 3 $\int \sin^2 x \cos^2 x dx$

Using the identities we learned, we can rewrite the integral in linear terms,

$$\begin{aligned}
\int \sin^2 x \cos^2 x dx &= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx \\
&= \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x) dx \\
&= \frac{1}{4}\int 1-\cos^2 2x dx \\
&= \frac{1}{4}\int \sin^2 2x dx \\
&= \frac{1}{8}\int 1-\cos 4x dx \\
&= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C
\end{aligned}$$