MATH1231/1241 Lab Test 2
Calculus Sample Solutions

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We cannot guarantee that our answers are correct - please notify us of any errors or typos at unswmathsoc@gmail.com, or on our Facebook page. There are sometimes multiple methods of solving the same question. Remember that in the real class test, you will be expected to explain your steps and working out.

Note: Any text presented `like this` is presented as required in Maple syntax.

Question 1

Solve the initial value problem given by the differential equation

$$\frac{dy}{dx} = x^6 y^2$$

together with the initial condition: If $x = -2$ then $y = -3$.
To solve this, rearrange and integrate to get

$$\int f(y)dy = \int g(x)dx.$$

If $\dfrac{dy}{dx} = x^6 y^2$ then we have $\dfrac{1}{y^2}\dfrac{dy}{dx} = x^6$. So,

$$\int \frac{1}{y^2} dy = \int x^6 dx.$$

Hence $f(y) = \dfrac{1}{y^2}$ and $g(x) = x^6$.

By integrating, we have

$$-\frac{1}{y} = \frac{1}{7}x^7 + C.$$

So $y = \dfrac{-7}{x^7 + 7C}$. Applying the initial condition, then $C = \dfrac{391}{21}$. Hence $y = \dfrac{-21}{3x^7 + 391}$.

Question 3

Determine which of the following differential equations are exact.

- $\Big(3x^2y^2 - 2\Big)dx + \Big(2xy^3 + 1\Big)dy = 0.$
- $(3\sin(3x)(2x + y) + 2\cos(3x))dx + \cos(3x)dy = 0.$
- $\Big(3x^2y^2 - 2\Big)dx + \Big(2x^3y + 1\Big)dy = 0.$
- $(3\cos(3x)(2x + y) + 2\sin(3x))dx + \sin(3x)dy = 0.$

The first ODE is not exact since

$$\begin{aligned}\frac{\partial}{\partial y}(3x^2y^2 - 2) &= 6x^2y,\\ \frac{\partial}{\partial x}(2xy^3 + 1) &= 2y^3\end{aligned}$$

and so $\dfrac{\partial}{\partial y}(3x^2y^2 - 2) \neq \dfrac{\partial}{\partial x}(2xy^3 + 1)$.

The second ODE is not exact since

$$\begin{aligned}\frac{\partial}{\partial y}(3\sin(3x)(2x + y) + 2\cos(3x)) &= 3\sin 3x,\\ \frac{\partial}{\partial x}(\cos 3x) &= -3\sin 3x\end{aligned}$$

and so $\dfrac{\partial}{\partial y}(3\sin(3x)(2x + y) + 2\cos(3x)) \neq \dfrac{\partial}{\partial x}(\cos 3x)$.

The third ODE is exact, since

$$\frac{\partial}{\partial y}(3x^2y^2 - 2) = 6x^2y,$$
$$\frac{\partial}{\partial x}(2x^3y + 1) = 6x^2y$$

and so $\dfrac{\partial}{\partial y}(3x^2y^2 - 2) = \dfrac{\partial}{\partial x}(2x^3y + 1)$.

The fourth ODE is exact, since

$$\frac{\partial}{\partial y}(3\cos(3x)(2x + y) + 2\sin(3x)) = 3\cos 3x,$$
$$\frac{\partial}{\partial x}(\sin 3x) = 3\cos 3x$$

and so $\dfrac{\partial}{\partial y}(3\cos(3x)(2x + y) + 2\sin(3x)) = \dfrac{\partial}{\partial x}(\sin 3x)$.

Question 4

Find a particular solution, $y_p(x)$, of the non-homogeneous differential equation

$$\frac{d^2}{dx^2}y(x) + 2\left(\frac{d}{dx}y(x)\right) + y(x) = -3x - 2,$$

given that $y_h(x) = Ae^{-x} + Bxe^{-x}$ is the general solution of the corresponding homogenous ODE.

The RHS of our ODE is a first order polynomial, so we should try a first order polynomial for our particular solution: $y_p(x) = ax + b$.

By substituting $y = y_p(x)$, then $y'(x) = a$ and $y''(x) = 0$. Hence

$$ax + (2a + b) = -3x - 2.$$

So $a = -3$ and $b = 4$, and $y_p = -3x + 4$.

Question 5

Use Maple to find the solution of the initial value problem

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

with initial conditions $y(0) = 5$ *and* $y'(0) = 5$.

To compute the solution to the differential equation via Maple, enter the command `dsolve({y(x)*diff(y(x),x,x)+(diff(y(x),x))^2=0,y(0)=5,D(y)(0)=5},y(x));`. Maple gives us the solution $y(x) = 5\sqrt{1+2x}$.

Question 9

Suppose that a function f *has derivatives of all orders at* a*. Then the series*

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

is called the Taylor series for f *about* a*, where* $f^{(n)}$ *is the nth order derivative of* f*.*
Suppose that the Taylor series for $\frac{e^{4x}}{1-x}$ *about* 0 *is*

$$a_0 + a_1x + a_2x^2 + ... + a_6x^6 + ...$$

Find the exact values of a_0 *and* a_6*.*

We can use the Maple command `taylor(exp(4*x)/(1-x),x=0,7);` to find the Taylor series of this function around 0, up to the 6th term. We find that

$$\frac{e^{4x}}{1-x} = 1 + 5x + 13x^2 + \frac{71}{3}x^3 + \frac{103}{3}x^4 + \frac{643}{15}x^5 + \frac{437}{9}x^6 + O(x^7).$$

Hence we have $a_0 = 1$ and $a_6 = \frac{437}{9}$.