Suppose that $\lim_{n\to\infty} x_n = x$ and $x > 0$.

Then, by definition, for all $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $n \geq N \Rightarrow |x_n - x| < \varepsilon$.
Now,

$$|x_n - x| < \varepsilon \implies x_n - x > -\epsilon$$

i.e.

$$x_n > x - \varepsilon$$

Putting $\varepsilon = \frac{x}{2}$ ($\varepsilon > 0$ as $x > 0$) and $N = M$, we find that:

$$\begin{aligned} x_n &> x - \frac{x}{2} \\ &= \frac{x}{2} \\ &> 0, \text{ for } n \geq M \end{aligned}$$

where we use the fact that $x > 0$ to get the last inequality.

$\therefore \exists M \in \mathbb{N}$ such that $x_n > 0$ for all $n \geq M$.