Suppose three vectors $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$ are linearly dependent.
Then $\alpha\mathbf{x} + \beta\mathbf{y} + \gamma\mathbf{z} = \mathbf{0}$ has more than one solution for $\alpha, \beta, \gamma \in \mathbb{R}$.

i.e.

$$\left(\begin{array}{ccc|c} x_1 & y_1 & z_1 & 0 \\ x_2 & y_2 & z_2 & 0 \\ x_3 & y_3 & z_3 & 0 \end{array}\right)$$

has multiple solutions.

$$\implies \det \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = 0$$

But

$$\mathbf{x} \cdot (\mathbf{y} \times \mathbf{z}) = \det \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \det \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = 0$$

where we use the fact that $\det(A) = \det(A^T)$.

Hence, if $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$ are linearly dependent, then $\mathbf{x} \cdot (\mathbf{y} \times \mathbf{z}) = 0$. The converse can be proven by simply reversing the steps here.

$\therefore$ $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$ are linearly dependent if and only if their scalar triple product is zero.

aside:
linearly dependent
$\Leftrightarrow$ one of the vectors lies in the same plane as the other two
$\Leftrightarrow$ volume of the parallelepiped contained by the three vectors is zero
$\Leftrightarrow$ scalar triple product is zero