Qualifiers Round 1

Write ONLY your final answers on this sheet. You may omit the constant of integration.

$$1. \boxed{3} \int \frac{1}{x \log_{10} x} \, \mathrm{d}x$$

2.
$$2 \int_0^{2\pi} (x-\pi)^3 \cos x \, dx$$

3.
$$2 \int_{-2}^{4} e^{|x|} dx$$

4.
$$\boxed{2} \int \left(\tan^{-1} x + \frac{x}{1+x^2} \right) dx$$

5. 3
$$\int \sec^2(\sqrt{x}) dx$$

6. 3
$$\int_0^{\pi/2} (\log(\sec x - 1) + \log(\sec x + 1)) dx$$

7.
$$\boxed{4} \quad \int \frac{\sin x - x}{1 - \cos x} \, \mathrm{d}x$$

8. 4
$$\int \frac{1}{1+\cos^2 x} dx$$

9.
$$\boxed{4} \quad \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} \, \mathrm{d}x$$

10. 3
$$\int_{\sqrt[3]{2}}^{\sqrt[3]{3}} (3x^{14} - 12x^{11} + 18x^8 - 12x^5 + 3x^2) dx$$

Qualifiers Round 2

Write ONLY your final answers on this sheet. You may omit the constant of integration.

$$1. \int \frac{1}{\sqrt{1+e^x}} \, \mathrm{d}x$$

$$2. \int e^{\sin x} (\cos^2 x - \sin x) \, \mathrm{d}x$$

$$3. \int_{-\pi}^{\pi} \lfloor \sin(\sin 2x) \rfloor \, \mathrm{d}x$$

$$4. \int \frac{1}{x^{101} + x} \, \mathrm{d}x$$

$$5. \int_0^{\frac{\pi}{4}} \frac{\sec x}{\sqrt{1 + \sin 2x}} \, \mathrm{d}x$$

FINALS QUESTIONS

1.
$$\int \frac{x^2 + 1}{x\sqrt{x^4 - 3x^2 + 1}} \, \mathrm{d}x$$

$$2. \int e^{\sin x} (\tan x \sec x + 1) \, \mathrm{d}x$$

3.
$$\int_{-\pi/2}^{\pi/2} \frac{x \sin x \cos x}{e^x + 1} \, \mathrm{d}x$$

4.
$$\int \frac{1 - x^2}{x^4 + 3x^2 + 1} \, \mathrm{d}x$$

5.
$$\int_0^1 \frac{\tan^{-1} x}{1+x} \, \mathrm{d}x$$

6.
$$\int_{-\pi/4}^{\pi/4} \tan^{-1}(e^{\tan x}) \, \mathrm{d}x$$

7.
$$\int \frac{1}{\prod_{k=1}^{2021} (x+k)} \, \mathrm{d}x$$

8.
$$\int \frac{1}{\sin^4 x + \cos^4 x} \, \mathrm{d}x$$

9.
$$\int_0^1 \frac{x^2 - 1}{x^3 + (1 - x^2)^{\frac{3}{2}}} \, \mathrm{d}x$$

10.
$$\int e^x x^{e^x - 1} (x \log x + 1) dx$$

Qualifiers Round 1 Solutions

- 1. $\log 10 \log (|\log x|)$
- 2. 0
- 3. $e^4 + e^2 2$
- 4. $x \tan^{-1} x$
- 5. $2\sqrt{x}\tan(\sqrt{x}) + 2\log(|\cos(\sqrt{x})|)$
- 6. 0
- 7. $x \cot(\frac{x}{2})$
- $8. \ \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right)$
- 9. $\frac{\pi^2}{4}$
- 10. $\frac{2^5-1}{5} = \frac{31}{5}$

Qualifiers Round 2 Solutions

1.
$$-\log(e^{-x} + \frac{1}{2} + \sqrt{e^{-x} + e^{-2x}})$$

- 2. $e^{\sin x} \cos x$
- $3. -\pi$
- 4. $\frac{1}{100} \log \left(\left| \frac{x^{100}}{x^{100} + 1} \right| \right)$
- $5. \log 2$

FINALS SOLUTIONS

1.
$$\log \left| x - \frac{1}{x} + \frac{\sqrt{x^4 - 3x^2 + 1}}{x} \right|$$

2.
$$e^{\sin x} \sec x$$

3.
$$\frac{\pi}{8}$$

4.
$$-\tan^{-1}\left(x + \frac{1}{x}\right)$$

$$5. \ \frac{\pi}{8} \log 2$$

6.
$$\frac{\pi^2}{8}$$

7.
$$\frac{1}{2020!} \sum_{k=1}^{2021} (-1)^{k+1} {2020 \choose k-1} \log(|x+k|)$$

8.
$$\frac{1}{\sqrt{2}}(\tan^{-1}(\sqrt{2}\tan x + 1) + \tan^{-1}(\sqrt{2}\tan x - 1))$$

9.
$$-\frac{\pi}{4}$$

10.
$$x^{e^x}$$