Write  $w = \frac{p}{q}$  in simplest form, with p > q. Choose m large such that mq > a. Then

$$\frac{mp}{mq} = \frac{mq+1}{mq} \times \frac{mq+2}{mq+1} \times \dots \times \frac{mp}{mp-1},$$

which we can rewrite as

$$\left(1+\frac{1}{mq}\right)\left(1+\frac{1}{mq+1}\right)\cdots\left(1+\frac{1}{mp-1}\right),$$

as required.