Game Theory (CS4187)

Lectures 11

Date: 09/09/2024

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Broad Idea of Today's Lecture

- The topics for today's lectures are:
  1. Iterated removal of dominated strategies.
  2. Pure Strategy Nash Equilibrium.
  3. Maxmin Strategy.

Recap

Till now we learned three kind of **solution concepts**:

- Strictly dominant strategy equilibrium **(SDSE)**.
- Weakly dominant strategy equilibrium **(WDSE)**.
- Iterated Removal of Strictly Dominated Strategy (IRSDS).
  - IRSDS is **NOT really a solution concept**; it is a **procedure to remove strategy profiles** under **common knowledge** of rationality assumption.
  - IRSDS leads to a solution concept only when the game is **dominance solvable**, i.e. when the strategy set of all the players after IRSDS is a singleton set.

Recap

- Let S_{SDSE} be the set of strategy profile of a game that is SDSE.

- Let S_{WDSE} be the set of strategy profile of a game that is WDSE.

- Let S_{IRSDS} be the set of strategy profile of a game that is obtained after IRSDS.

Which one of the following choices is correct?

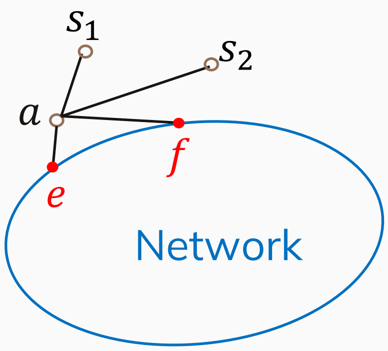
A. $S_{IRSDS} \subseteq S_{WDSE} \subseteq S_{SDSE}$.
B. $S_{IRSDS} \subseteq S_{SDSE} \subseteq S_{WDSE}$.
C. $S_{WDSE} \subseteq S_{SDSE} \subseteq S_{IRSDS}$.
D. $S_{WDSE} \subseteq S_{IRSDS} \subseteq S_{SDSE}$.
E. $S_{SDSE} \subseteq S_{WDSE} \subseteq S_{IRSDS}$.
F. $S_{SDSE} \subseteq S_{IRSDS} \subseteq S_{WDSE}$.

IRDS

		Player 2	
Player 1	L	C	R
T	1, 2	2, 3	0, 3
M	2, 2	2, 1	3, 2
B	2, 1	0, 0	1, 0

- **Order of elimination matters.**
- Elimination T, R, B, C leads to (M,L).
- Elimination B, L, C, T leads to (M,R).

Examples



		s_2 e	f
s_1	e	-5 ms, -5 ms	-1 ms, -2 ms
	f	-2 ms, -1 ms	-6 ms, -6 ms

- There are some games that does not admit any of the following solution concepts:
  - SDSE.
  - WDSE.
  - Dominance solvable using IRSDS.

- Example 1: Network congestion games discussed in lectures 3 and 4 slides.

Examples

		Student 2	
		Work	Chill
Student 1	Work	2, 2	-1, 1
	Chill	1, -1	0, 0

- There are some games that does not admit any of the following solution concepts:
  - SDSE.
  - WDSE.
  - Dominance solvable using IRSDS.
- Example 2: Extra assignment game.

Pure Strategy Nash Equilibrium

- Since it is not possible to solve all games using the concept of **dominance** (SDSE, WDSE, and dominance solvability), we have to necessitates that we look for other solution concepts.

- Another reason to look for other solution concepts is because **dominance is too stringent** a requirement. From a logical point of view, in a multi-player setup, it is unlikely that a player's **"best action" does not depend on what other players' actions are** (that is what dominance essentially means).
  - One may argue that unlike SDSE and WDSE, **dominance solvability** does not assume that the best action of a player depend upon other players' actions.
  - It actually does because were are removing strategies that are strictly dominated just like in SDSE and WDSE. Just that SDSE and WDSE **ONLY assumes that players are rational** while dominance solvability requires **common knowledge of rationality**.

Pure Strategy Nash Equilibrium

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- Rather than **dominance**, Nash equilibrium (both pure strategy and mixed strategy) works on the principle of **stability**.

Pure Strategy Nash Equilibrium

- Before describing pure strategy Nash equilibrium (PSNE), it is useful to define the concept of **best response** as it leads to a more intuitive definition of PSNE.

Definition (Best Response): Best response of player i against strategy s_{-i} played by other players is a set of strategies for player i that will maximize its payoff provided that other players played s_{-i},

$$B(s_{-i}) = \arg\max_{s_i \in S_i} u_i(s_i, s_{-i})$$

NOTE: $argmax$ can return multiple values of s_i because "best response" is a set.

Pure Strategy Nash Equilibrium

Definition 1 (PSNE): A strategy profile $s^* = (s_i^*, s_{-i}^*)$ is called a PSNE if

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \;\; \forall s_i \in S_i, \forall i = 1,2,\dots,n$$

What is the logical interpretation of definition 1?

- The above inequality essentially means that if **all other players "stands their post" and keep playing s_{-i}^***, then player i can't benefit by **unilaterally changing** it's strategy s_i.
- Hence stability because even if player i changes to any other strategy by s_i^*, it will return to s_i^* (this intuition assumes that when player i tries any other strategy but s_i^*, **another player $j \neq i$ does not try any other strategy but s_j^***).
- IMPORTANT: The first point has to be satisfied for ALL the players. Otherwise, at least one player will have incentive to change it's strategy, which will lead to other players changing its strategy and eventually there is not equilibrium.

Pure Strategy Nash Equilibrium

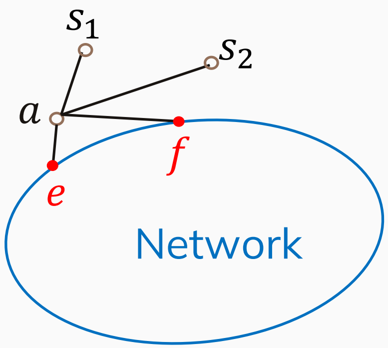
Definition 2 (PSNE): A strategy profile $s^* = (s_i^*, s_{-i}^*)$ is called a PSNE if

$$s_i^* \in B(s_{-i}^*), \forall i = 1,2, \dots, n$$

What is the logical interpretation of definition 2?

- s_i^* is the best action for player i if others are playing s_{-i}^*. **This MUST hold for all the players.**

Find PSNE: Example 1



		s_2: e	s_2: f
s_1	e	-5, -5	-1, -2
	f	-2, -1	-6, -6

Find PSNE: Example 2

Student 2 (columns); **Student 1** (rows)

	Work	Chill
Work	2, 2	-1, 1
Chill	1, -1	0, 0

Relation Between SDSE, WDSE, IRSDS, PSNE

Relation Between SDSE, WDSE, IRSDS, PSNE

Relation Between SDSE, WDSE, IRSDS, PSNE

Relation Between SDSE, WDSE, IRSDS, PSNE

Relation Between SDSE, WDSE, IRSDS, PSNE

Thank You!