CS 4187: Game Theory

Practice Problems Set 2 (lectures 11 to 25)

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I WILL BE ADDING MORE QUESTIONS TODAY

Q1: The first set of problems are reading assignments:

  (a) Go through example 13.5 (first price sealed bid auction) of chapter 13 (Bayesian Games) of the book by Narahari.
  (b) Go through section 11.3 of the chapter 11 (Computation of Nash Equilibria) of the book by Narahari.
  (c) In lecture 12 to 15 notes, go through slides 57 to 63. It discusses a systematic approach to compute MSNE using definition 6. It also shows that number of variable and number of equalities will always be the same (at least some good news)!

Q2: Conceptual questions:

  (a) Give a proof why definition 2 of MSNE is equivalent to definition 4 of MSNE.
  (b) Consider a three-player game where all the players have strategy set $\{1, 2, 3\}$ and we know that:
  - For player 1, action 2 is strictly dominated.
  - For player 2, action 2 is weakly dominated.
  - For player 3, actions 2 and 3 are strictly dominated.

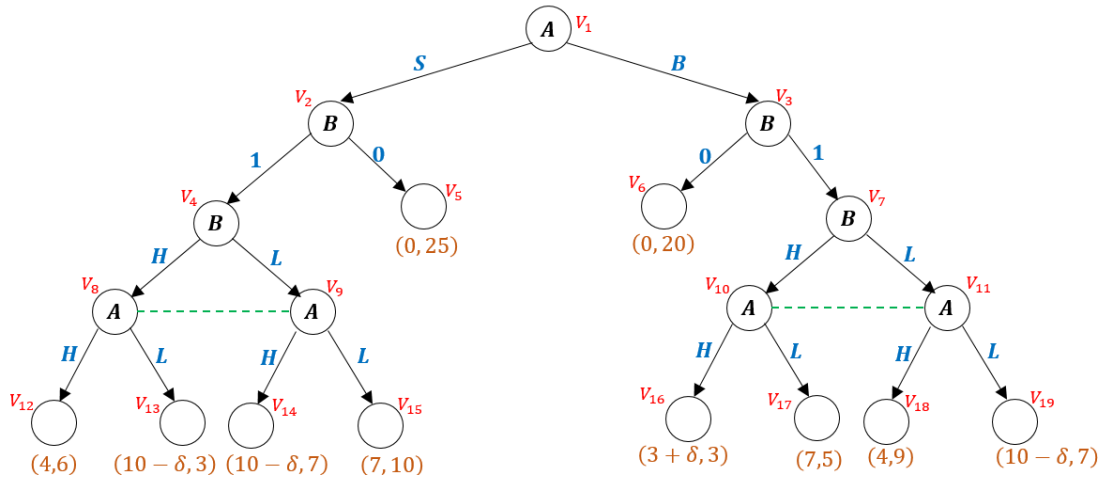 Write a **generic** expression for the MSNE of this game.

Q3: In lecture 16 and 17 notes, we formulated an optimization problem to compute the correlated equilibrium (CE) of network congestion game that also maximizes the expected utility of all the players (nodes s_1 and s_2). Write a similar optimization problem for the following problems:

  (a) Computing MSNE that maximizes the expected utility of all the players. HINT: Use definition 4 of MSNE.
  (b) Computing PSNE that maximizes the expected utility of all the players. HINT: It will be 90% same as the previous one; just one constraint will change.

Q4: Consider a two-player game shown using the payoff matrix before where the row player is player 1 and the column player is player 2. Write a set of linear equalities and inequalities that must be satisfied if action c of player 2 is **strictly dominated by a mixed strategy**. The total number of linear equalities and inequalities **must be finite**.

	a	b	c	d
w	2,0	0,5	1,0	0,4
x	4,1	2,1	0,2	1,0
y	2,1	5,0	0,0	0,3
z	0,0	1,0	4,1	0,0

Q5: Consider the imperfect information EFG shown below (this is NOT the same as minor 1):

Answer the following questions:

(a) What is the range of δ such that there exists at-least one Nash equilibrium of the sub-games starting at nodes V_4 and V_7 in mixed strategy? NOTE: A pure strategy can also be considered as a mixed strategy but that's not what we are asking for; we want mixed strategy where all actions have non-zero probability. **HINT:** What should be support? Use definition 6 of MSNE.

For all the following questions, assume that δ is such that MSNE exists for the sub-games starting at nodes V_4 and V_7 in mixed strategy. We will only consider these MSNEs in the following questions:

(b) Compute an expression for the MSNEs for the sub-games starting at nodes V_4 and V_7 in terms of δ?

(c) Compute sub-game perfect equilibrium of the game in terms of δ?