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Game Theory (CS4187)

Lecture 2

Date: 12/08/2024

Instructor: Gourav Saha

NOT THE FINAL VERSION!

No additional lecture notes will be provided other than the slides.

Contents of this lecture

1. **What is a game theory?**
2. Breakdown of the Modules (with examples).
3. Course logistics.
4. Mandatory reading.

What is game theory?

- In the course “Optimizing techniques for AI”, you have learned how to **formulate** and **solve** optimization problems:

$$\max_{X} \quad f(X)$$

$$\text{subject to:}$$

$$g(X) \leq \theta$$

where $X \in \mathbb{R}^N$ and $\theta \in \mathbb{R}^M$ (N decision variables and M constraints).

What is game theory?

Let's consider two setups:

Setup 1: Consider an "optimization setup" with two scalar decision variables x_1 and x_2, where x_1 and x_2 can be adjusted by a single individual.

$$\max_{x_1, x_2} \quad f(x_1, x_2)$$

Setup 2: Consider an "optimization setup" with two scalar decision variables x_1 and x_2, where x_1 and x_2 are adjusted by two different individuals each having their own objective function.

$$\max_{x_1} \quad f_1(x_1, x_2)$$

Individual 1

$$\max_{x_2} \quad f_2(x_1, x_2)$$

Individual 2

What is game theory?

Setup 2: Consider an "optimization setup" with two scalar decision variables x_1 and x_2, where x_1 and x_2 are adjusted by two different individuals each having their own objective function.

- In setup 2, the decision variable of individual 1, i.e. x_1, effects the objective function of individual 2 and vice versa.

What is game theory?

EXAMPLE (Relative order preserving marks update):

1. Towards the end of a course the marks of the students are low.

2. Suppose the instructor decided to give an **additional assignment** in the end of the course to increase the marks of the students so that they get better grades.

3. The instructor also decided that since the assignment was not planned from the beginning the marks increase due to this additional assignment **should not change the relative ordering** of the student based on their original marks.

4. Hence, it was decided that everyone's **marks will be increased by the same amount**. One possible way to do so is to **calculate the average score** of all the students **for this additional assignment** and **add it to their original marks**.

What is game theory?

EXAMPLE (Relative order preserving marks update):

- This game-theoretic setup because the marks of one student will effect the marks of the other student (because of the average). Student are the individuals.

- For this setup, the objective function of the $\boldsymbol{i^{th}}$ **student** is

$$f_i\left(x_1, x_2, \ldots, x_N\right) = s_i + \frac{1}{N}\left(\sum_{n=1}^{N} x_n\right) - \rho\left(x_i\right)$$

where s_i is the original marks of the i^{th} student, x_i is the marks of the i^{th} student in the additional assignment, and $\rho(x_i)$ is a monotonic increasing function of x_i that captures the effort put by the i^{th} student for the additional assignment. The second term of the equation is the average marks of the students for the additional assignment.

What is game theory?

Definition: Game theory is the study of decision making in a multi-player setup where:

1. The players are rational (“rational” means that the player will try to maximize their objective).

2. Each player has their own objective.

3. Decision of one players effects the objective of other players and vice-versa.

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2. **Breakdown of the Modules (with examples).**
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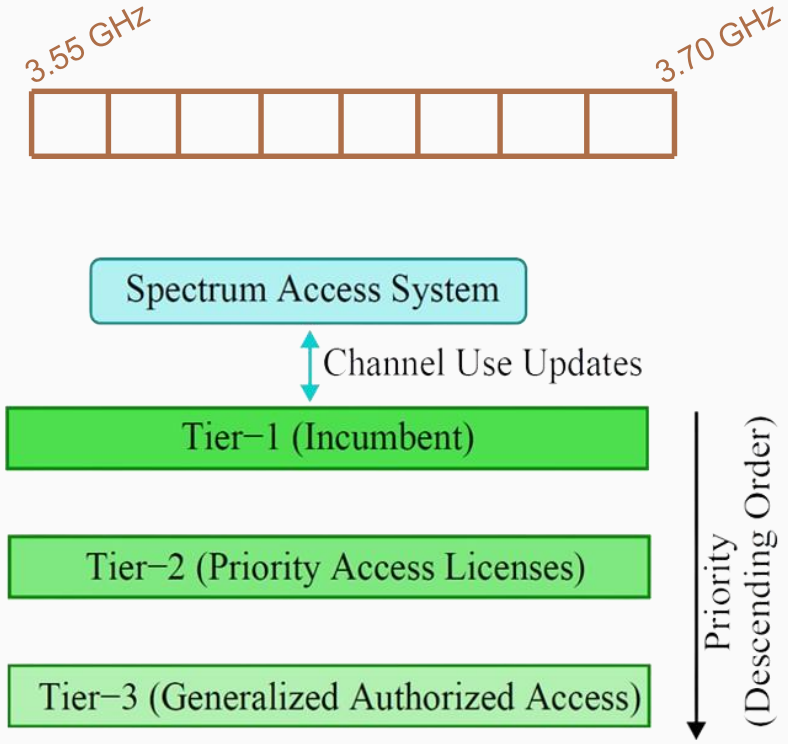
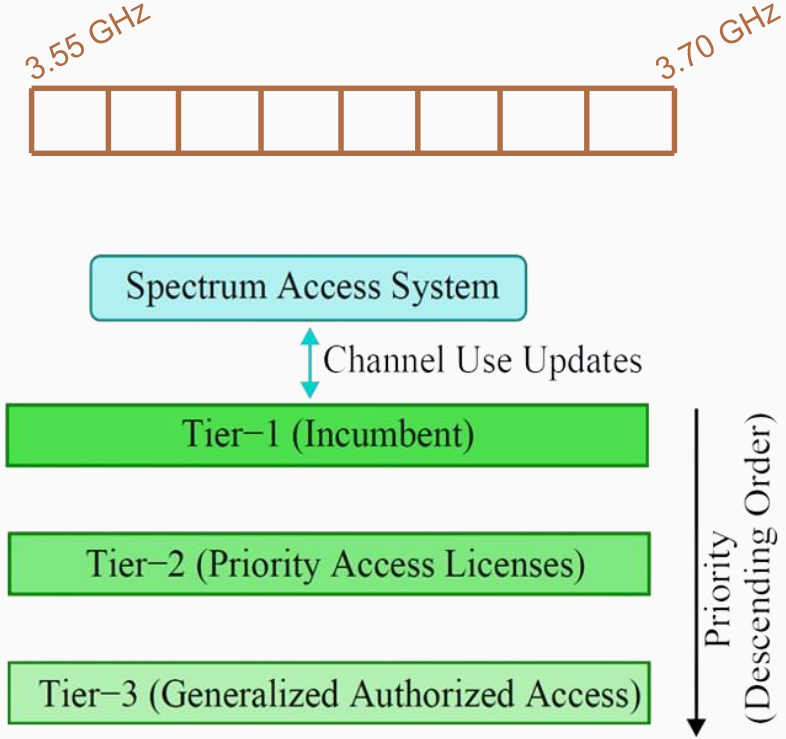
Breakdown of the Modules



Brief overview of my PhD work

- I will be using my **PhD research** as example to explain the contents of every module. So very briefly about the setup of my PhD research.
- My PhD research centered around **Citizens Broadband Radio Service (CBRS) Band**:
  - 150 MHz band from 3.55 GHz to 3.7 GHz divided into N channels.
  - 3-tier spectrum sharing policy (lower tier→higher priority). **Spectrum access system** implement this priority order.

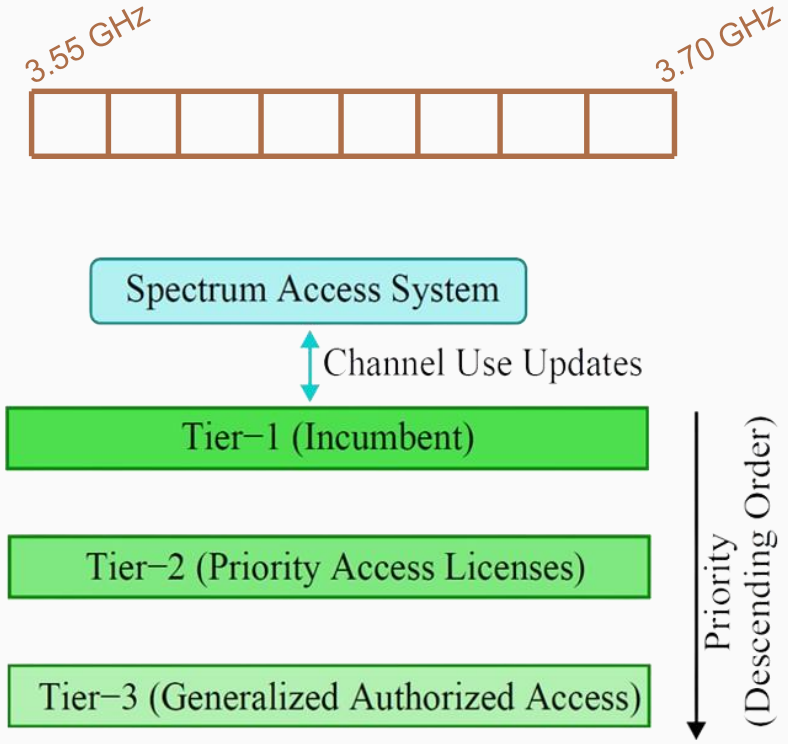
Breakdown of the Modules



Brief overview of my PhD work

- Tier-1 (Federal users):
  - Military radars and satellite service earth stations.
- Tier-2 (PAL users):
  - PAL → Priority Access Licences.
  - Leases channels through periodic auctions. Lease duration in T years.
  - Can use a channel if Tier-1 user is not using it.

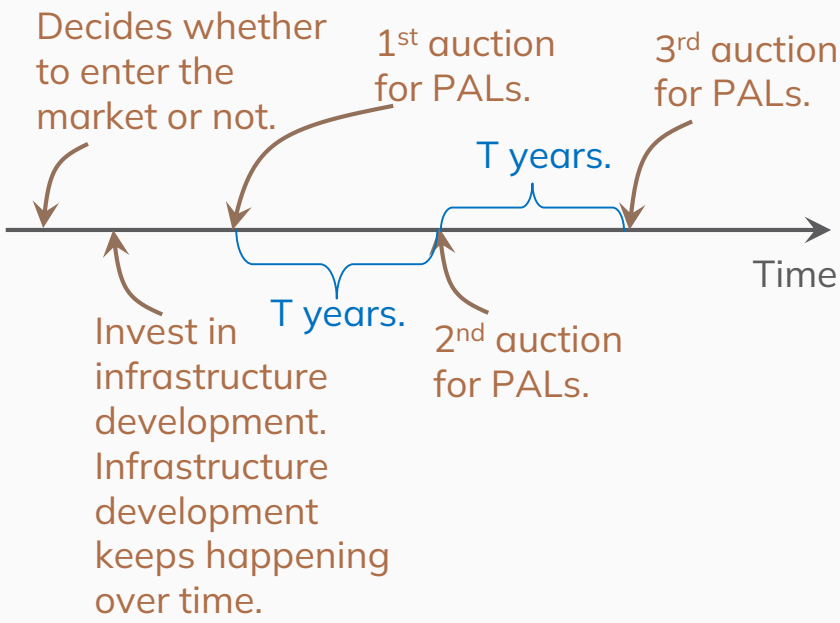
Breakdown of the Modules



Brief overview of my PhD work

- Tier-3 (GAA users):
  - GAA → General Authorized Access.
  - M of the N channels reserved solely for GAA users.
  - Opportunistic spectrum access for free.
  - Can use a channel if Tier-1 and Tier-2 users are not using it.

Breakdown of the Modules



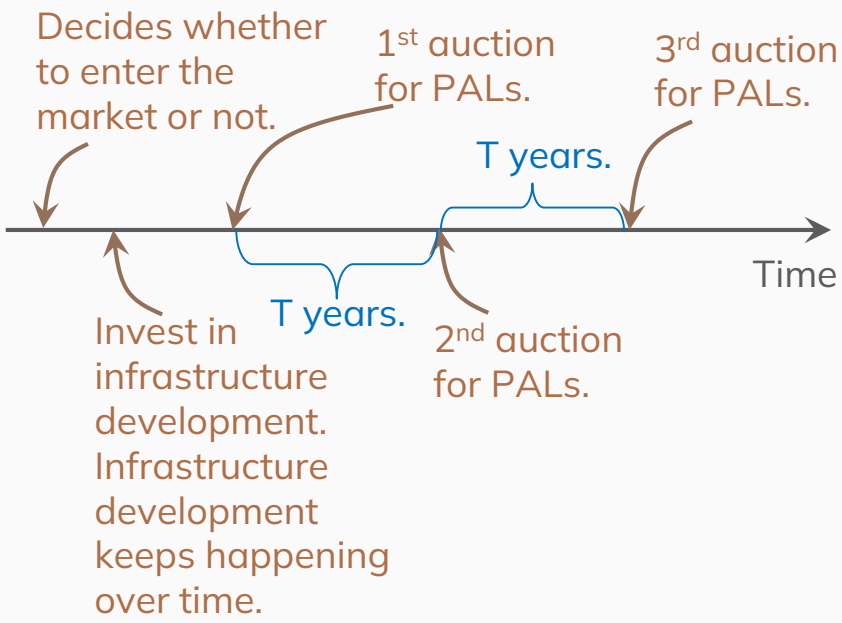
Brief overview of my PhD work

There are two stakeholders*:

- **Government:** Who decides the values for N, T, and M.
- **Wireless service providers:** They decide whether to invest in CBRS band. This is a tough decision because:
  - There is a lot of **upfront investment** (building transmission towers, buying land contracts, maintenance).
  - WSPs have to analyze whether they will make enough **return on their investment** in **presence of competition**.

* Actually three, 3rd being the end-user but they don't arise in my PhD work.

Breakdown of the Modules

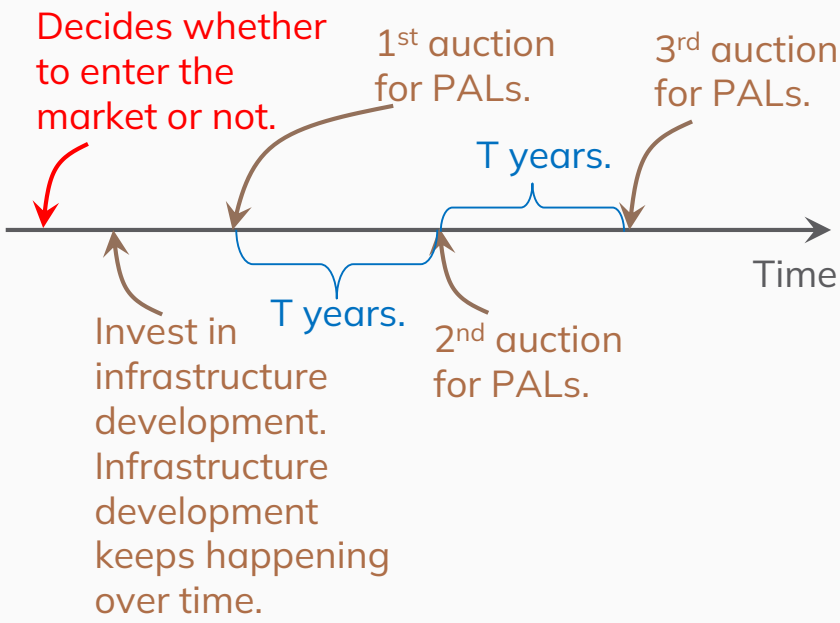


Module 1 (Different kind of Games)

We assume that N, T, and M are decided by the government.

- This is a game-theoretic setup because:
  - A WSP's return on invest depends on whether other WSPs are entering the market or not (that decides the competition during auctions).
  - Also the kind of WSPs entering the market decides the competition. Big WSPs can win auctions because they have a lot of capital to invest.

Breakdown of the Modules

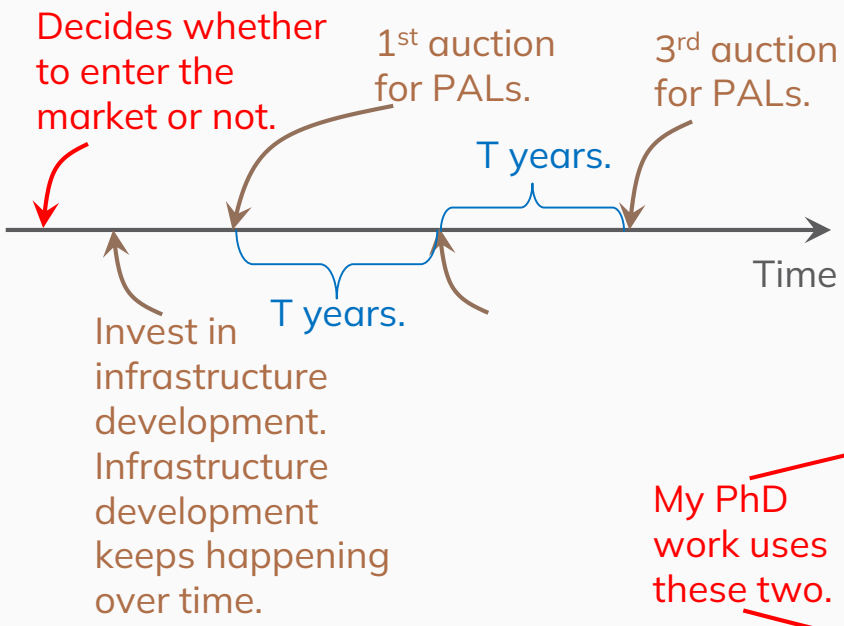


Module 1 (Different kind of Games)

We assume that N, T, and M are decided by the government.

- **Simultaneous move games**: All the WSPs together decides whether to enter the market or not.
- **Sequential move games:** Words travel! Somehow a WSP can get to know (maybe with certain probability) if another WSP has decided to enter the market or not. It then uses this knowledge to make its decision.
  - This leak of this knowledge can be helpful for some WSP, bad for others.

Breakdown of the Modules



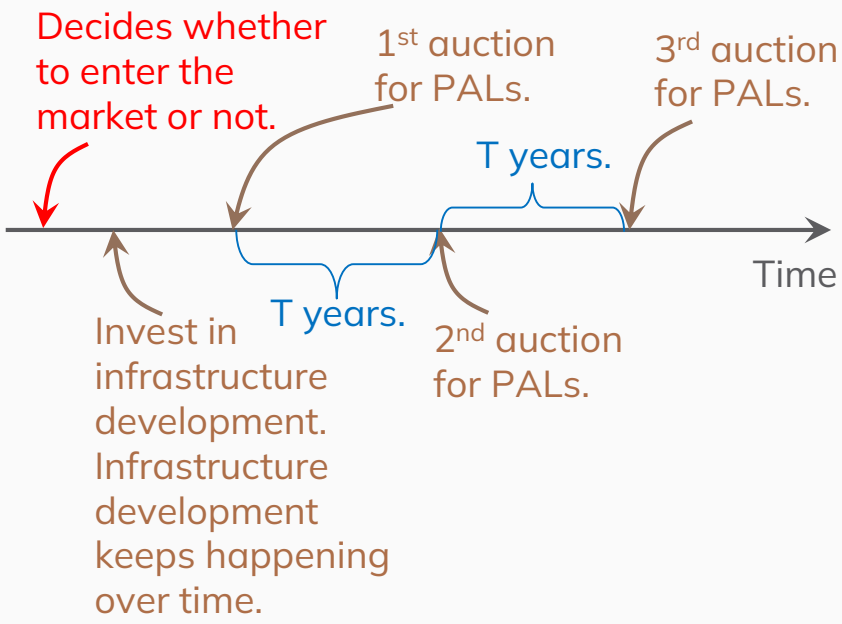
Module 2 (Solution concepts of games)

We assume that N, T, and M are decided by the government.

- So, module 1 deals with formulating a problem using game-theoretic modeling techniques.
- Module 2 then discusses various types of decisions and algorithms to find those decisions.
  - Dominant strategy, Iterated removal of dominant strategy, Nash equilibria (pure and mixed), Correlated equilibria, max-min equilibria.

My PhD work uses these two.

Breakdown of the Modules



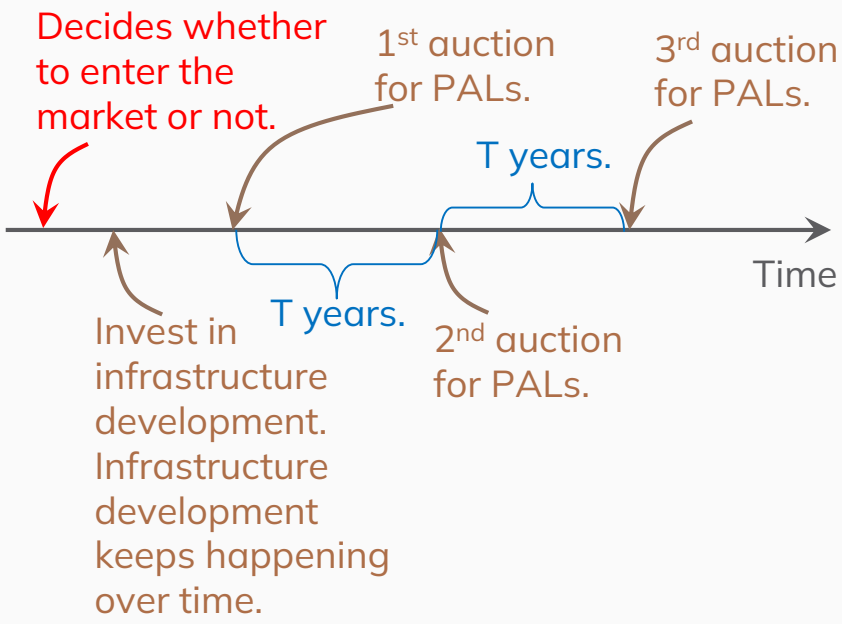
Module 3 (Games with incomplete information)

We assume that N, T, and M are decided by the government.

- In order to make decision about entering the market or not, a WSP needs to knows the objective function (also called payoff) of other WSPs.
  - Payoff of a WSP is characterized my its revenue requirement, its investment, statistics of its revenue from users etc.

- In Modules 1 and 2, we assume that these information and hence the payoff of other WSPs are not known.

Breakdown of the Modules



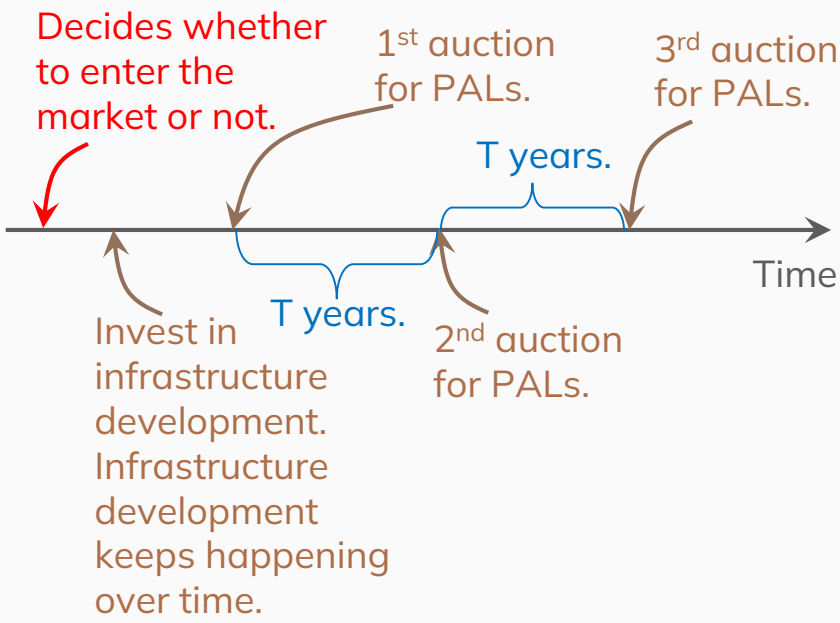
Module 3 (Games with incomplete information)

We assume that N, T, and M are decided by the government.

- In order to make decision about entering the market or not, a WSP needs to knows the objective function (also called payoff) of other WSPs.

- In Modules 3, we remove this assumption. A WSP may have some idea (captured by a probability distribution) about these parameters. Hence, also called **Bayesian games**.

Breakdown of the Modules



Module 4 (Mechanism Design)

- Mechanism design deals with **designing the rules of the game** in order to make the players (WSPs here) behave in a **desired way**. In our example, it involves **tuning N, T, and M.**

To be completed (more about module 4, and then 6).

Contents of this lecture

1. What is a game theory?

2. Breakdown of the Modules (with examples).

3. **Course logistics.**
   - **Prerequisites.**
   - **Marks distribution.**
   - **Lecture notes, Books, and Mandatory Reading.**
   - **Office hours.**
   - **Practice problems and mock exams.**
   - **MU Email Id.**

4. Mandatory reading.

Prerequisites

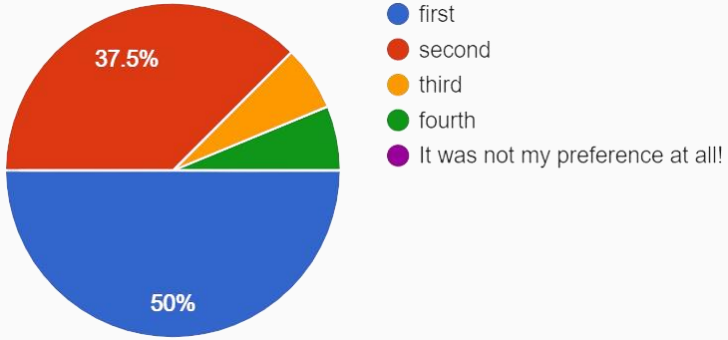
- Probability (basics).
  - Random variables, joint and conditional probability distribution, Baye's rule, expectation, linearity of expectation, independent and disjoint events.

- Any algorithm course that has some amount of **mathematical proofs**.

Marks Distribution

- Minor 1 (10%).
- Minor 2 (20%).
- Class project (30%).
  - Group project. Group size will be decided after the total head-count is known in a week or two.
- End-Sem (40%).

Lecture notes, books, and mandatory reading



- Around **87%** of you choose this course as either your **first or second choice**! This along with the fact that this is not such a hyped subject (like ML, DL, Comp. Seq. Modeling, RL) implies that you are genuinely interested in this subject.

- As someone who is interested in the subject, I expect you to take notes and read books.
  - **You will get the lecture slides**.
  - I know that few other faculties follow similar teaching technic.
  - I got criticized for giving elaborate notes for control systems course! So, trying something different.

Lecture notes, books, and mandatory reading

Book 1: Harrington, J.E., Games, Strategies, and Decision Making, 2nd edition, Worth Publishers: Macmillan Ed., 2015

- Less mathematical jargon.
- Does not contain topics from Module 4 (Mechanism Design).

Book 2: Y. Narahari, "Game Theory and Mechanism Design", volume 4, World Scientific, 2014

- Mathematically involved.
- Topics related to sequential games is not there but I prefer book 3 for this.

Book 3: D. Fudenberg, J. Tirole, "Game Theory", MIT Press, 1991

- Mathematically involved.
- Module 4 (Mechanism design is better book 2).

You only to MAINLY follow these two books.

Book 4: N. Noam, T. Roughgarden, E. Tardos, and V.V. Vazirani, "Algorithmic Game Theory", Cambridge University Press, 2007

- Really mathematically involved.
- We will use it for Module 2 and some times for a few examples.

Lecture notes, books, and mandatory reading

- The **last few slides** of every lecture will contain **mandatory reading**. Here I will **EXPLICITLY tell you which sections of the book to read**.
- Some of these mandatory readings will be **topics that I have covered during lectures** or **something extra to supplement the lecture**.
- I will give lectures notes (typeset in Latex) only when I feel that books are not enough or difficult to follow.

Office Hours

- No weekly office hours! But, I will have office hours before:
  - Exams.
  - A week before project deadline.

  (Based on my previous experience, students don't come for office hours unless it is exam time.)

- You can also come to my cubicle by booking **prior appointments** mentioning the **topic of discussion** in brief. (so that I have some time to prepare).

Practice problems and mock exams

- I will release practice problems **biweekly** (6-8 question per exercise set)
  - 50% of the questions will be straightforward to check your understanding of the lecture.
  - 50% will resemble the questions that will be asked during exams.
  - NOT GRADED.

- I will give you mock exams before all the exams.

MU Email Id

- I will be creating a **group email**. If you did not receive any email from me about this course yesterday (especially **Masters/PhD students**) please email me **your Roll Number and MU Email Id**.

My email: gourav.saha@mahindrauniversity.edu.in

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Mandatory Reading

- To be completed.

Thank You!