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# Game Theory (CS4187)

# Lectures 21, 22, and 23

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Instructor: Gourav Saha

# Games with Incomplete Information: Example

Student 1: Dedicated
Student 2: Dedicated

| Student 1 \ Student 2 | Work | Chill |
|---|---|---|
| Work | 5, 5 | 3, 3 |
| Chill | 3, 3 | 1, 1 |

Student 1: Chiller party
Student 2: Chiller party

| Student 1 \ Student 2 | Work | Chill |
|---|---|---|
| Work | 3, 3 | 1, 5 |
| Chill | 5, 1 | 3, 3 |

Student 1: Dedicated
Student 2: Chiller party
(or vice-versa in which case the number in the payoff matrix flips)

| Student 1 \ Student 2 | Work | Chill |
|---|---|---|
| Work | 5, 3 | 3, 5 |
| Chill | 3, 1 | 1, 3 |

- Consider the game of extra assignment that we discussed during lecture 1 and again mentioned briefly while discussing SFGs.
- Let’s consider a variant of this game where students 1 and 2 can be either dedicated or “chiller party”. Each student don’t know whether he/she is dedicated or not but does not know what kind of student the other student is.

# Bayesian Games

- Bayesian Games is **one of the models** of incomplete information games.
  - Players are characterized by their types.
  - Type of a player determines it's payoff.
  - The types of a the players are sampled from a joint probability distribution. This joint distribution is also called the **common prior distribution**.
    - The common prior distribution is a **common knowledge**.
  - We assume that nature samples types of all the players from this common prior.
  - The **nature tells individual player it's type**, but **does not tell the types of other players**.
    - This ensures that players don't know each other's payoff matrix.
    - But has some idea of the payoff matrix of the other players.

**Mention that this is just a model. There is nothing like nature actually telling player anything.**

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    - But has some idea of the payoff matrix of the other players.

- Bayesian Games was proposed by **John Harsanyi** (won Nobel prize along with John Nash; same year).

- We will be first taking about **Simultaneous move** Bayesian games.

# Bayesian Games: Definition

**Definition (Simultaneous Move Bayesian Games):** A strategic form game (SFG) $\Gamma$ is a tuple, $\langle N, (S_i)_{i\in N}, (\Theta_i)_{i\in N}, P, (u_i)_{i\in N}\rangle$ where,

1. $\mathrm{N} = \{1, 2, \cdots, \mathrm{n}\}$ is the **set of players**.

2. $\mathrm{S_i}$ is the **set of actions** of the $\mathrm{i^{th}}$ player. The action of the $\mathrm{i^{th}}$ player, denoted by $s_i$ belongs to set $\mathrm{S_i}$, i.e. $s_i \in \mathrm{S_i}$.

3. $\Theta_i$ is the **set of types** of the $\mathrm{i^{th}}$ player. The type of the $\mathrm{i^{th}}$ player, denoted by $\theta_i$ belongs to set $\Theta_\mathrm{i}$, i.e. $\theta_i \in \Theta_\mathrm{i}$. We have, $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_\mathrm{n}$. We have $\theta = (\theta_1, \theta_2, \cdots, \theta_n)$ be the type of all the players. Also, $\theta = (\theta_i, \theta_{-i})$.

4. $P: \Theta_1 \times \Theta_2 \times \cdots \times \Theta_\mathrm{n} \to \mathbb{R}$ is the common prior.

5. $\mathrm{u_i} : \Theta_1 \times \Theta_2 \times \cdots \times \Theta_\mathrm{n} \times \mathrm{S_1} \times \mathrm{S_2} \times \cdots \times \mathrm{S_n} \to \mathbb{R}$ is the utility function of the $\mathrm{i^{th}}$ player. To elaborate, the payoff the $\mathrm{i^{th}}$ player is $u_i(\theta_1, \theta_2, \cdots, \theta_n, s_1, s_2, \cdots, s_n)$.

# Bayesian Games: Strategy

- So in Bayesian games, a player knows its type but it may or may not know types of the other players.

- Since a player knows it's type, it is only logical that the player uses it's type while deciding it's strategy. As before, there are two kind of strategies:
  - **Pure strategy:** Pure strategy of player $i$ mapping from the set of types of player $i$, $\Theta_i$, to the set of pure strategies of player $i$, $S_i$. Mathematically,

$$s_i : \Theta_i \rightarrow S_i$$

    Let $s_i(\theta_i)$ denote the pure strategy of player $i$ when it's type is $\theta_i$.

  - **Mixed strategy:** Mixed strategy of player $i$ mapping from the set of types of player $i$, $\Theta_i$, to the set of pure strategies of player $i$, $\Delta(S_i)$. Mathematically,

$$\sigma_i : \Theta_i \rightarrow \Delta(S_i)$$

    Let $\sigma_i(\theta_i, s_i)$ denote the probability that player $i$ plays pure strategy $s_i$ if it's type is $\theta_i$.

    Let $\sigma_i(\theta_i)$ denote the mixed strategy of player $i$ when it's type is $\theta_i$. $\boldsymbol{\sigma_i(\theta_i)}$ **is a vector** whose elements are $\sigma_i(\theta_i, s_i)$.

# Bayesian Games: Utility function

- We now discuss utility function of a player in a Bayesian game separately for pure strategy and mixed strategy.

<u>PURE STRATEGY</u>

- Let $U_i(\theta, s)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $s = (s_i, s_{-i})$.
  - For all $i \in N$, $s_i = \left(s_i(\theta_{i,1}), s_i(\theta_{i,2}), \cdots, s_i(\theta_{i,|\Theta_i|})\right)$ where $\theta_{i,j}$, $j = 1,2,\cdots,|\Theta_i|$, is the $j^{th}$ type of player $i$ (this highlighted component is salient to incomplete information games because strategy of a player is complete plan of what it will do if it's type is so and so.).

# Bayesian Games: Utility function

- We now discuss utility function of a player in a Bayesian game separately for pure strategy and mixed strategy.

PURE STRATEGY

- Let $U_i(\theta, s)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $s = (s_i, s_{-i})$.

- Now player $i$ knows it's type, $\theta_i$. But it does not know the type of other players, $\theta_{-i}$. Hence, **for player $\boldsymbol{i}$, $\boldsymbol{\theta_{-i}}$ is a random variable** and consequently the function $U_i(\theta, s) = U_i(\theta_i, \theta_{-i}, s)$ is **function of random variable $\boldsymbol{\theta_{-i}}$**. But player $i$ can find the expected value of $U_i(\theta, s)$ conditioned on it's type being $\theta_i$; this is exactly the utility function of player $i$. Mathematically, **the utility of player $\boldsymbol{i}$ is**,

$$U_i\left(s_i, s_{-i} \middle| \theta_i\right) = E\left[U_i\left(\theta, s\right) \middle| \theta_i\right] \quad (1)$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} E\left[U_i\left(\theta, s\right) \middle| \theta_i, \theta_{-i}\right] P\left[\theta_{-i} \middle| \theta_i\right] \quad (2)$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} P\left[\theta_{-i} \middle| \theta_i\right] u_i\left(\theta_i, \theta_{-i}, s_i\left(\theta_i\right), s_{-i}\left(\theta_{-i}\right)\right) \quad (3)$$

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- Let $U_i(\theta, s)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $s = (s_i, s_{-i})$.

- Now player $i$ knows it's type, $\theta_i$. But it does not know the type of other players, $\theta_{-i}$. Hence, **for player $i$, $\theta_{-i}$ is a random variable** and consequently the function $U_i(\theta, s) = U_i(\theta_i, \theta_{-i}, s)$ is **function of random variable $\theta_{-i}$**. But player $i$ can find the expected value of $U_i(\theta, s)$ conditioned on it's type being $\theta_i$; this is exactly the utility function of player $i$. Mathematically, **the utility of player $i$ is**,

$$U_i\left(s_i, s_{-i} \middle| \theta_i\right) = E\left[U_i\left(\theta, s\right) \middle| \theta_i\right] \quad (1)$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} E\left[U_i\left(\theta, s\right) \middle| \theta_i, \theta_{-i}\right] P\left[\theta_{-i} \middle| \theta_i\right] \quad (2)$$

**$\theta_i$ fixed because it is known to a player $i$.**

$$= \sum_{\theta_{-i} \in \Theta_{-i}} P\left[\theta_{-i} \middle| \theta_i\right] u_i\left(\theta_i, \theta_{-i}, s_i\left(\theta_i\right), s_{-i}\left(\theta_{-i}\right)\right) \quad (3)$$

# Bayesian Games: Utility function

- We now discuss utility function of a player in a Bayesian game separately for pure strategy and mixed strategy.

## MIXED STRATEGY

- Let $U_i(\theta, \sigma)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $\sigma = (\sigma_i, \sigma_{-i})$.
  - For all $i \in N$, $\sigma_i = \left(\sigma_i(\theta_{i,1}), \sigma_i(\theta_{i,2}), \cdots, \sigma_i(\theta_{i,|\Theta_i|})\right)$ where $\theta_{i,j}$, $j = 1,2, \cdots, |\Theta_i|$, is the $j^{th}$ type of player $i$ (this highlighted component is salient to incomplete information games because strategy of a player is complete plan of what it will do if it's type is so and so.).
  - And, $\sigma_i(\theta_{i,j}) = \left(\sigma_i(\theta_{i,j}, s_{i,1}), \sigma_i(\theta_{i,j}, s_{i,2}), \cdots, \sigma_i(\theta_{i,j}, s_{i,|S_i|})\right)$ where $s_{i,k}$, $k = 1,2, \cdots, |S_i|$, is the $k^{th}$ pure strategy of player $i$ and hence $\sigma_i(\theta_{i,j}, s_{i,k})$ is the probability that player $i$ will choose pure strategy $s_{i,k}$ if it's type is $\theta_{i,j}$.

# Bayesian Games: Utility function

- We now discuss utility function of a player in a Bayesian game separately for pure strategy and mixed strategy.

<u>MIXED STRATEGY</u>

- Let $U_i(\theta, \sigma)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $\sigma$.

- Now player $i$ knows it's type, $\theta_i$. But it does not know the type of other players, $\theta_{-i}$. Hence, **for player $i$, $\theta_{-i}$ is a random variable** and consequently the function $U_i(\theta, \sigma) = U_i(\theta_i, \theta_{-i}, \sigma)$ is **function of random variable $\theta_{-i}$**. But player $i$ can find the expected value of $U_i(\theta, \sigma)$ conditioned on it's type being $\theta_i$; this is exactly the utility function of player $i$. Mathematically, **the utility of player $i$ is**,

$$U_i\left(\sigma_i, \sigma_{-i} \middle| \theta_i\right) = E\left[U_i\left(\theta, \sigma\right) \middle| \theta_i\right] \tag{4}$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} E\left[U_i\left(\theta, \sigma\right) \middle| \theta_i, \theta_{-i}\right] P\left[\theta_{-i} \middle| \theta_i\right] \tag{5}$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} P\left[\theta_{-i} \middle| \theta_i\right] U_i\left(\theta_i, \theta_{-i}, \sigma_i\left(\theta_i\right), \sigma_{-i}\left(\theta_{-i}\right)\right) \tag{6}$$

# Bayesian Games: Utility function

- We now discuss utility function of a player in a Bayesian game separately for pure strategy and mixed strategy.

MIXED STRATEGY

- Let $U_i(\theta, \sigma)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $\sigma$.

- Now player $i$ knows it's type, $\theta_i$. But it does not know the type of other players, $\theta_{-i}$. Hence, **for player $i$, $\theta_{-i}$ is a random variable** and consequently the function $U_i(\theta, \sigma) = U_i(\theta_i, \theta_{-i}, \sigma)$ is **function of random variable $\theta_{-i}$**. But player $i$ can find the expected value of $U_i(\theta, \sigma)$ conditioned on it's type being $\theta_i$; this is exactly the utility function of player $i$. Mathematically, **the utility of player $i$ is**,

$$U_i\left(\sigma_i, \sigma_{-i} \middle| \theta_i\right) = E\left[U_i\left(\theta, \sigma\right) \middle| \theta_i\right] \tag{4}$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} E\left[U_i\left(\theta, \sigma\right) \middle| \theta_i, \theta_{-i}\right] P\left[\theta_{-i} \middle| \theta_i\right] \tag{5}$$

**$\theta_i$ fixed because it is known to a player $i$.**

$$= \sum_{\theta_{-i} \in \Theta_{-i}} P\left[\theta_{-i} \middle| \theta_i\right] U_i\left(\theta_i, \theta_{-i}, \sigma_i\left(\theta_i\right), \sigma_{-i}\left(\theta_{-i}\right)\right) \tag{6}$$

# Bayesian Games: Utility function

- We now discuss utility function of a player in a Bayesian game separately for pure strategy and mixed strategy.

<u>MIXED STRATEGY</u>

- Let $U_i(\theta, \sigma)$ denote the utility of player $i$ if the type of the players is $\theta$ and strategy profile is $\sigma$.

- Now player $i$ knows it's type, $\theta_i$. But it does not know the type of other players, $\theta_{-i}$. Hence, **for player $i$, $\theta_{-i}$ is a random variable** and consequently the function $U_i(\theta, \sigma) = U_i(\theta_i, \theta_{-i}, \sigma)$ is **function of random variable $\theta_{-i}$**. But player $i$ can find the expected value of $U_i(\theta, \sigma)$ conditioned on it's type being $\theta_i$; this is exactly the utility function of player $i$. Mathematically, **the utility of player $i$ is**,

$$U_i\left(\sigma_i, \sigma_{-i} \middle| \theta_i\right) = E\left[U_i\left(\theta, \sigma\right) \middle| \theta_i\right] \quad (4)$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} E\left[U_i\left(\theta, \sigma\right) \middle| \theta_i, \theta_{-i}\right] P\left[\theta_{-i} \middle| \theta_i\right] \quad (5)$$

Function overloading

$$= \sum_{\theta_{-i} \in \Theta_{-i}} P\left[\theta_{-i} \middle| \theta_i\right] U_i\left(\theta_i, \theta_{-i}, \boxed{\sigma_i\left(\theta_i\right), \sigma_{-i}\left(\theta_{-i}\right)}\right) \quad (6)$$

# Bayesian Games: Utility function

- In equations (2), (3), (5), 6), $\boldsymbol{P[\theta_{-i}|\theta_i]}$ **is the posterior probability of $\boldsymbol{\theta_{-i}}$ given $\boldsymbol{\theta_i}$**. Using, Bayes' rule,

$$P\left[\theta_{-i}\middle|\theta_i\right] = \frac{P\left[\theta_{-i},\theta_i\right]}{P\left[\theta_i\right]} \tag{7}$$

$$= \frac{P\left[\theta\right]}{P\left[\theta_i\right]} \tag{8}$$

$$= \frac{P\left[\theta\right]}{\sum_{\theta_{-i}\in\Theta_{-i}} P\left[\theta_{-i},\theta_i\right]} \tag{9}$$

# Bayesian Games: Utility function

- For equation (6), $U_i(\theta_i, \theta_{-i}, \sigma_i(\theta_i), \sigma_{-i}(\theta_{-i}))$ is the utility of player $i$ in mixed strategy $\sigma(\theta) = (\sigma_i(\theta_i), \sigma_{-i}(\theta_{-i}))$. We have,

$$U_i\left(\theta_i, \theta_{-i}, \sigma_i\left(\theta_i\right), \sigma_{-i}\left(\theta_{-i}\right)\right) = \sum_{s \in S} \sigma_i\left(\theta_i, s_i\right) \sigma_{-i}\left(\theta_{-i}, s_{-i}\right) u_i\left(\theta_i, \theta_{-i}, s_i, s_{-i}\right)$$

# Bayesian Games: Bayesian NE

- Just like Nash equilibrium (NE) in complete information games, we can define Bayesian Nash equilibrium in (BNE) in Bayesian games.

**Definition (BNE in pure strategy):** A strategy profile $s^* = (s_i^*, s_{-i}^*)$ is called a BNE in pure strategy if

$$U_i(s_i^*(\theta_i), s_{-i}^* \mid \theta_i) \geq U_i(s_i(\theta_i), s_{-i}^* \mid \theta_i), \forall s_i(\theta_i) \in S_i, \forall \theta_i \in \Theta_i, \forall i \in N$$

$$U_i(s_i^*, s_{-i}^* \mid \theta_i) \geq U_i(s_i, s_{-i}^* \mid \theta_i), \forall s_i \in S_i, \forall \theta_i \in \Theta_i, \forall i \in N$$

Same meaning…
The top one explicitly shows that $\boldsymbol{\theta_i}$ is known for player $\boldsymbol{i}$.

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$$U_i(s_i^*, s_{-i}^* \mid \theta_i) \geq U_i(s_i, s_{-i}^* \mid \theta_i),\ \forall s_i \in S_i, \forall \theta_i \in \Theta_i, \forall i \in N$$

The top one does function overloading.

$\boldsymbol{s_i^*(\theta_i)}$ is the strategy for player $\boldsymbol{i}$ for only type $\boldsymbol{\theta_i}$.

While $s_{-i}^*$ is the strategy of the other players for all their possible types.

# Bayesian Games: Bayesian NE

- Just like Nash equilibrium (NE) in complete information games, we can define Bayesian Nash equilibrium in (BNE) in Bayesian games.

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$$U_i(s_i^*, s_{-i}^* \mid \theta_i) \geq U_i(s_i, s_{-i}^* \mid \theta_i), \forall s_i \in S_i, \forall \theta_i \in \Theta_i, \forall i \in N$$

**Definition (BNE in pure strategy, Best response version):** A strategy profile $s^* = (s_i^*, s_{-i}^*)$ is called a BNE in pure strategy if

$$s_{i,\theta_i}^* \in B_i(s_{-i}^*, \theta_i), \forall \theta_i \in \Theta_i, \forall i \in N$$

where,

$$B_i(s_{-i}, \theta_i) = \arg\max_{s_{i,\theta_i} \in S_i} U_i(s_i, s_{-i} \mid \theta_i)$$

# Bayesian Games: Bayesian NE

- Just like Nash equilibrium (NE) in complete information games, we can define Bayesian Nash equilibrium in (BNE) in Bayesian games.

**Definition (BNE in mixed strategy):** A strategy profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ is called a BNE in mixed strategy if

$$U_i(\sigma_i^*(\theta_i), \sigma_{-i}^* \mid \theta_i) \geq U_i(\sigma_i(\theta_i), \sigma_{-i}^* \mid \theta_i), \forall \sigma_i(\theta_i) \in \Delta(S_i), \forall \theta_i \in \Theta_i, \forall i \in N$$

$$U_i(\sigma_i^*, \sigma_{-i}^* \mid \theta_i) \geq U_i(\sigma_i, \sigma_{-i}^* \mid \theta_i), \forall \sigma_i(\theta_i) \in \Delta(S_i), \forall \theta_i \in \Theta_i, \forall i \in N$$

**Definition (BNE in mixed strategy, Best response version):** A strategy profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ is called a BNE in mixed strategy if

$$\sigma_{i,\theta_i}^* \in B_i(\sigma_{-i}^*, \theta_i), \forall \theta_i \in \Theta_i, \forall i \in N$$

where,

$$B_i(\sigma_{-i}, \theta_i) = \underset{\sigma_{i,\theta_i} \in \Delta(S_i)}{\arg\max}\ U_i(\sigma_i, \sigma_{-i} \mid \theta_i)$$

# Relation between Bayesian NE and NE

- For a strategy profile $\sigma = (\sigma_i, \sigma_{-i})$ define,

$$U_i(\sigma_i, \sigma_{-i}) = \sum_{\theta_i \in \Theta_i} P[\theta_i] U_i(\sigma_i, \sigma_{-i} | \theta_i)$$

$U_i(\sigma_i, \sigma_{-i})$ : Expected utility of player $i$ for strategy profile $\sigma = (\sigma_i, \sigma_{-i})$ **before** player $i$ learns it's type. This is also called **ex-ante utility**.

$U_i(\sigma_i, \sigma_{-i} | \theta_i)$ : Expected utility of player $i$ for strategy profile $\sigma = (\sigma_i, \sigma_{-i})$ **after** player $i$ learns it's type. This is also called **ex-interim utility**.

$P[\theta_i]$ : The probability that type of player $i$ is $\theta_i$. $P[\theta_i]$ can be computed from prior probability as follows,

$$P[\theta_i] = \sum_{\theta_{-i} \in \Theta_{-i}} P[\theta_{-i}, \theta_i]$$

Obviously $\boldsymbol{P[\theta_i] > 0}$ for all $\theta_i \in \Theta_i$. Otherwise, if $P[\theta_i] = 0$, $\theta_i$ is not even a type of player.

# Relation between Bayesian NE and NE

- There is a concept of NE (hence PSNE and MSNE) for Bayesian game too.

**Definition (NE in mixed strategy):** A strategy profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ is called a mixed strategy if

$$U_i(\sigma_i^*, \sigma_{-i}^*) \geq U_i(\sigma_i, \sigma_{-i}^*) \ \ \forall \sigma_i \in \Delta(S_i), \forall i \in N$$

**Physical meaning:** NE in Bayesian game is to check if a strategy profile is a NE if the nature did not tell the player their own types.

# Relation between Bayesian NE and NE

- There is a concept of NE (hence PSNE and MSNE) for Bayesian game too.

**Theorem 1:** If the set of types of all the players are finite, a strategy profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ is a BNE **if and only if** it is a NE.

**Very non-intuitive**: It essentially means:

No player has a profitable unilateral deviation **after he/she knows which type** he/she is **if and only** if he/she has no profitable unilateral deviation **before knowing his/her type**.

**Intuition?**

**Importance:** Theorem 1 implies that computing BNE is same as computing NE in complete information games but **each player has a vector of strategies** depending on it's type. **This is however true only if set of types of all the players are finite**. If the set of types is NOT finite, we can **directly compute the BNE**.

# Bayesian Games: Sequential Move Games

- Bayesian Games with sequential moves can be captured using the same approach which we used to capture complete information sequential move games but with nature node making the FIRST decision about the type of the players and then constructing information sets to ensure that a player knows its type but not the type of other players.

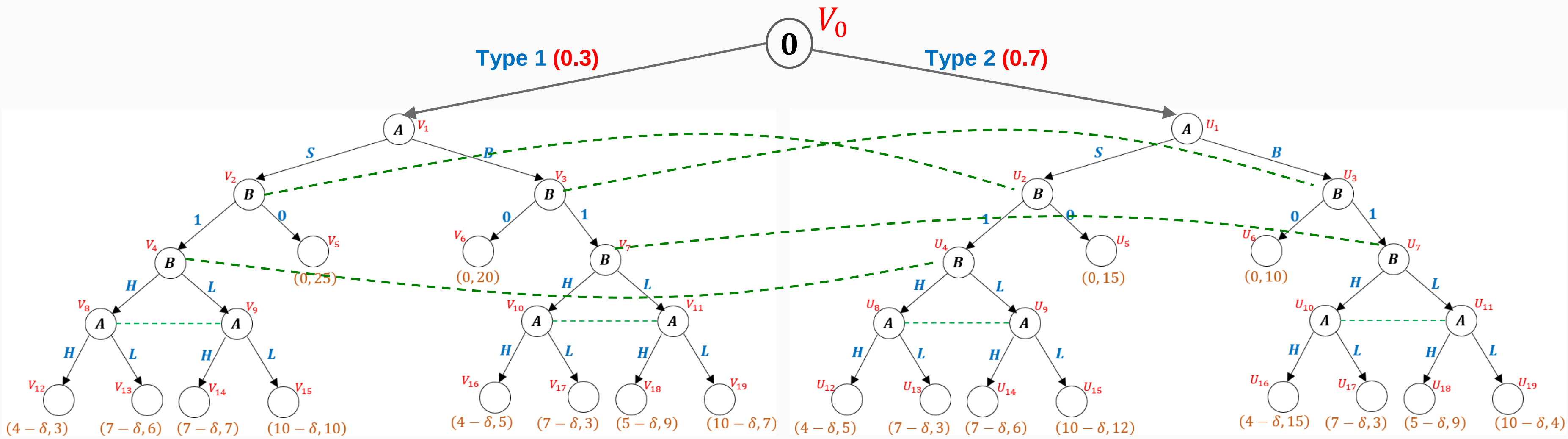
- In the next two slides, we do two examples to demonstrate the above idea. The example that we use is derived from question 1 of minor 1 (so check that first in the exam folder to refresh your idea). The difference is:
  - Example 1: Company A has two types. Company B has one type.
  - Example 2: Company B has two types. Company A has one type.

  I did not do the case where both the company has two types because it is going to be messy! **Also, note that in the leaf nodes, of the EFGs drawn in the next two slides, company B's payoff comes first.**

- After we can model the Bayesian Games with sequential moves as discussed above, we can use the same solution approach as we discussed in lectures 19 and 20.

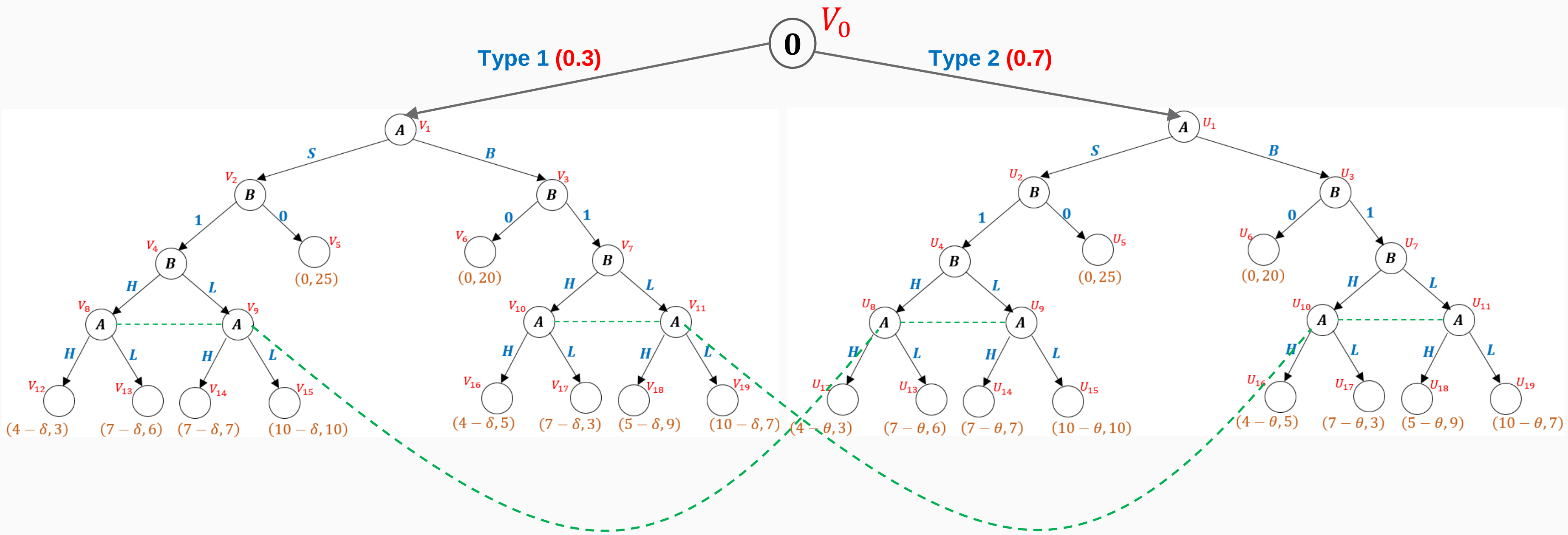
# Bayesian Games: Sequential Move Games

**Company A has two types**



# Bayesian Games: Sequential Move Games

**Company B has two types**



# Thank You!