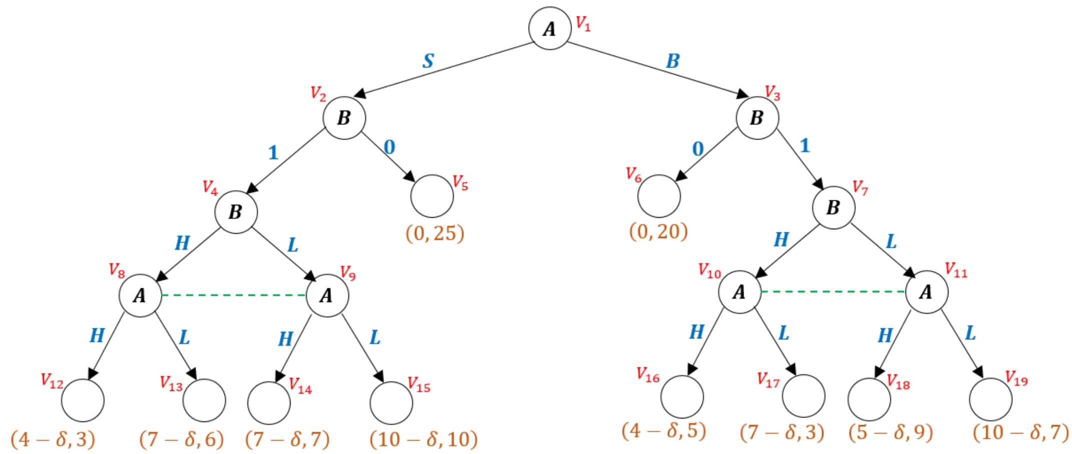
CS 4187: Game Theory

Practice Problems Set 3 (lectures 11 to 25)

Release date: 8th November, 2024

Q1: Consider the imperfect information EFG shown below (this the SAME as minor 1):



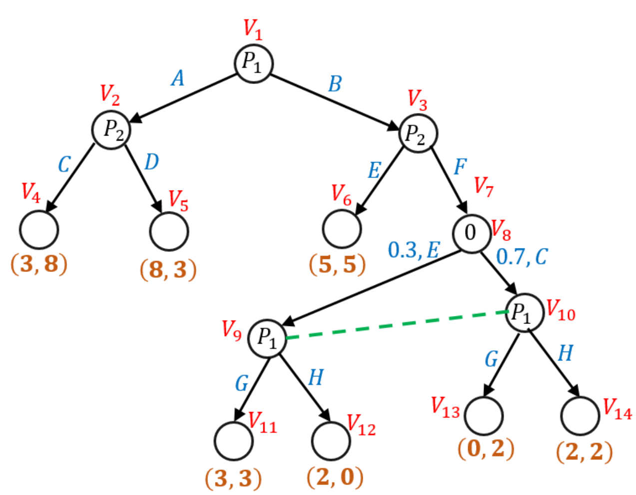
Answer the following questions:

- **(a)** Find **ONE** SPNE of the above IIEFG.
- **(b)** Find **ALL** SPNEs of the above IIEFG.

Importance of Q1:

1. When asked to find ONE SPNE (and for that matter one NE), don't directly try to solve for MSNE using definition 6. This is because the game may have a PSNE (or even better dominant strategy). Finding PSNE is much simpler.
2. When asked to find ALL SPNE, you have to bite the bullet, i.e. you have to find MSNEs are well. That said, sometimes you can reduce the game by removing strictly dominated strategy or IRSDS.

Q2: Fine a SPNE of the IIEFG shown below where the player at vertex V_8 is nature. HINT: It melts down to writing the payoff matrix corresponding to the subgame at vertex V_3. The tricky part is the chance move and the information set. The process to write the SFG at vertex V_3 is same as that in slides 60 to 64 of lectures 5 to 8.



Q3: Consider a two-player Bayesian game with the following payoff matrix,

	L	R
a	$\theta, \theta + \gamma$	$0, 0$
b	$0, \gamma$	θ, θ

where $\theta \in \{1,3\}$ is privately known by Player 1 and $\gamma \in \{-2,2\}$ is privately known by player 2. The prior probabilities are as follows,

$$P[\theta = 3, \gamma = 2] = P[\theta = 1, \gamma = -2] = {}^{1}/_{3}$$

$$P[\theta = 3, \gamma = -2] = P[\theta = 1, \gamma = 2] = {}^{1}/_{6}$$

Compute a Bayesian Nash equilibrium of the above game. NOTE: I will try to send the solution of this tomorrow morning based on how free I am. It is not difficult at all. Just something that I did not do in lectures.

Q4: Two siblings have inherited a property from their parents. The value of the property is v_i for sibling $i \in \{1,2\}$. v_i's are **private information** of the siblings. v_i's are identically and uniformly distributed between $[0,1]$. Each of the siblings submit their bids b_i **simultaneously**. The one who bids higher wins the property and **pays his own bid to the other sibling**.

The strategy of sibling i is his bid b_i. Definitely, b_i is a function of v_i, the sibling's value of the property. Find a Bayesian Nash equilibrium of this game where the function that maps v_i to b_i satisfies three conditions:

(a) Condition 1: b_i is strictly monotonic increasing in v_i.
(b) Condition 2: The function is differentiable with respect to v_i.
(c) Condition 3: The function is symmetric for both the siblings, i.e. bid of both the siblings will be same if their value is the same.

HINT: I told in class that when you have to find Nash equilibrium, you can make some assumption of the structure of the Nash equilibrium. Based on your assumption, you may or may not be successful. What a function that satisfies both conditions 1 and 2.