Game Theory (CS4187)

Lecture 34 (19/11/2024)

> Notes are not complete. Refer chapter 17 of the book. You don't need to read the entire chapter. Just read till Theorem 17.1 (including it).

Recap and few additional topics related to lecture 33

- Consider set of outcome X.

- Utility function of player i is $U_i(x, \theta_i)$ where $x \in X$ and $\theta_i \in \Theta_i$.

  utility of player i depends only on its type and not others.

- We discussed that a social choice function maps a type profile θ to outcome x,

$$f : \Theta \rightarrow X$$

Recap and few additional topics related to lecture 33

- We discussed the concept of preference relation which is nothing but an ordering of outcomes in X for player i as induced by its utility function $u_i(x, \theta_i)$.
For $x, y \in X$,

$$x \succsim y \iff u_i(x, \theta_i) \geq u_i(y, \theta_i)$$

- Strict prefence relation,

$$\underbrace{x \succ y}_{\text{Notation not discussed in prev. lecture.}} \iff u_i(x, \theta_i) > u_i(y, \theta_i)$$

Recap and few additional topics related to lecture 33

- Set of all preference relation for player i is R_i.
- Set of all strict preferce relation over outcome set X, P.

Example:

- $X = \{x_1, x_2, x_3, x_4\}$

$|X|!$ $\quad$ $4!$

$x_1 \succ x_2 \succ x_3 \succ x_4$

$x_4 \succ x_3 \succ x_2 \succ x_1$

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Recap and few additional topics related to lecture 33

- Set of all preference relation for player i is R_i.
- Set of all strict preferce relation over outcome set X, P.

Example:

$R_i = ?$

- $X = \{x_1, x_2, x_3, x_4\}$

	x_1	x_2	x_3	x_4
$\theta_i = A$	20	10	15	80
$\theta_i = B$	5	1	0	-1
$\theta_i = C$	5	8	2	16

R_i

$= \{x_4 \succ x_1 \succ x_3 \succ x_2,$

$x_1 \succ x_2 \succ x_3 \succ x_4,$

$x_4 \succ x_2 \succ x_1 \succ x_3\}$

Recap and few additional topics related to lecture 33

- Set of all preference relation for player i is $\mathcal{R}_i$.
- Set of all strict preferce relation over outcome set X, $\mathcal{P}$.

Example:

- $X = \{x_1, x_2, x_3, x_4\}$

$R_i = ?$

-

	x_1	x_2	x_3	x_4
$\theta_i = A$	20	10	15	80
$\theta_i = B$	5	1	0	-1
$\theta_i = C$	5	8	2	8

$\mathcal{R}_i$

$= \{ x_4 \succ x_1 \succ x_3 \succ x_2,$

$x_1 \succ x_2 \succ x_3 \succ x_4,$

$x_4 \succsim x_2 \succ x_1 \succ x_3 \}$

$x_2 \succsim x_4 \succ x_1 \succ x_3$

Recap and few additional topics related to lecture 33

- So θ_i of player i defines its preference relation, r_i where $r_i \in R_i$.
  - r_i is an ordering, e.g. $x_2 \succsim x_1 \succsim x_3 \succsim x_4$

- We learnt that SCF $f : \Theta \to X$.

We can alternatively or equivalently think that SCF $f : R \to X$ where,

Preference profile

$$R = R_1 \times R_2 \times \cdots \times R_n$$

Recap and few additional topics related to lecture 33

Gibbard Satterthwaite Theorem

- GS theorem: If SCF f satisfies
  - $|X| \geq 3$,
  - $R_i = P$; $\forall i \in N$ (The set of preferernce relation of player i is equal to the set of all strict prefence relation over X, $\forall i$)
  - f is an onto mapping.

Then f is DSIC if and only if f is dictatorial.

Strategyproof

Recap and few additional topics related to lecture 33

Gibbard Satterthwaite Theorem

$|P| = |X|$!

- GS theorem: If SCF f satisfies
  - $|X| \geq 3$.
  - $R_i = P$; $\forall i \in N$ } Minimum Cardinality of Θ_i ?
  - f is an onto mapping.

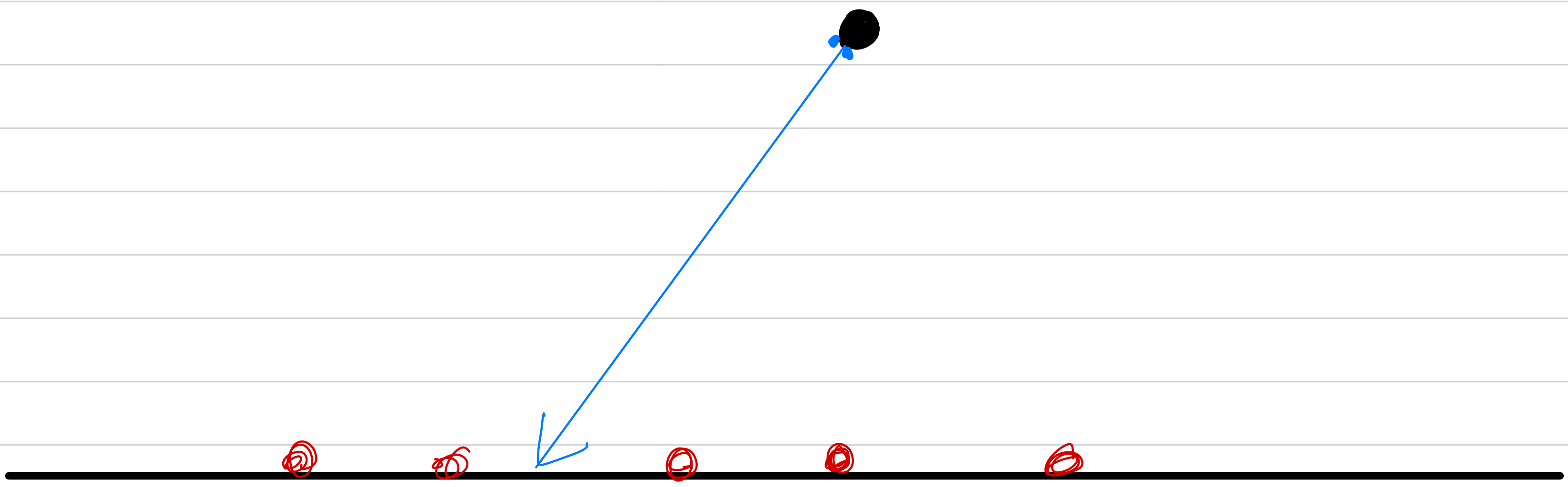
Then f is DSIC if and only if f is dictatorial.

↳ Strategyproof

Domain Restriction

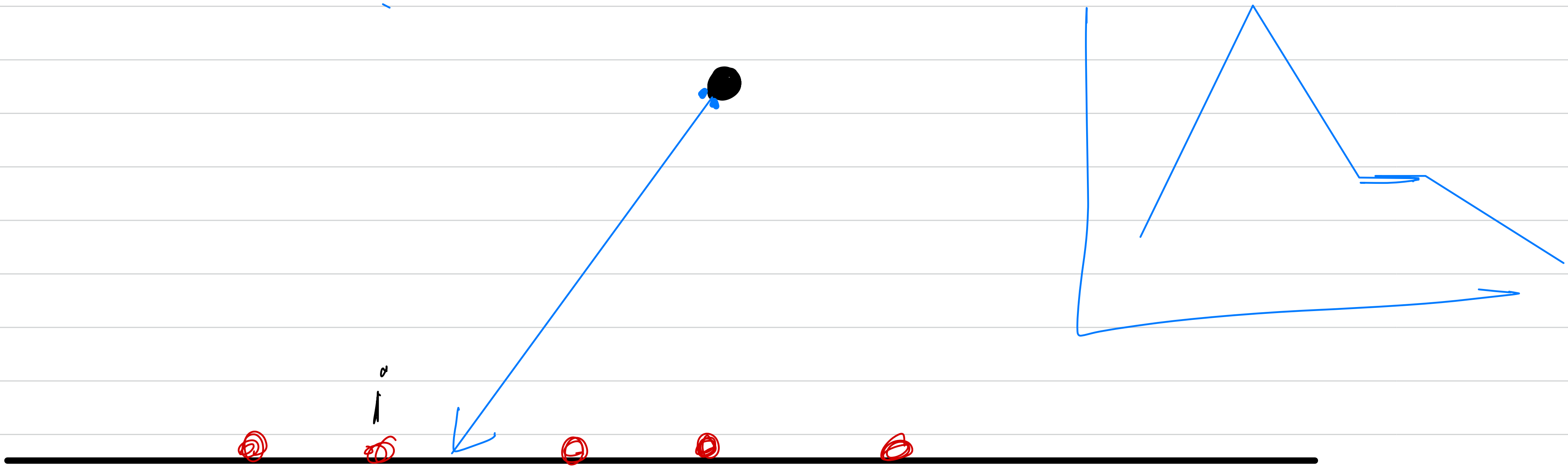
- GS theorem gives a negative result.
- But many real world utility functions U_i and hence set of prefence relation R_i of player i does not satisfy $R_i = P$; $\forall i \in N$.
  - The other two conditions can also get violated.
  - In such cases, GS theorem is NOT aplicable.
  - NOTE: Just because GS theorem is NOT applicable does not mean that $\exists$ a SCF which is DSIC and non-dictatorial.

Example-1 (Single Peaked Preference)

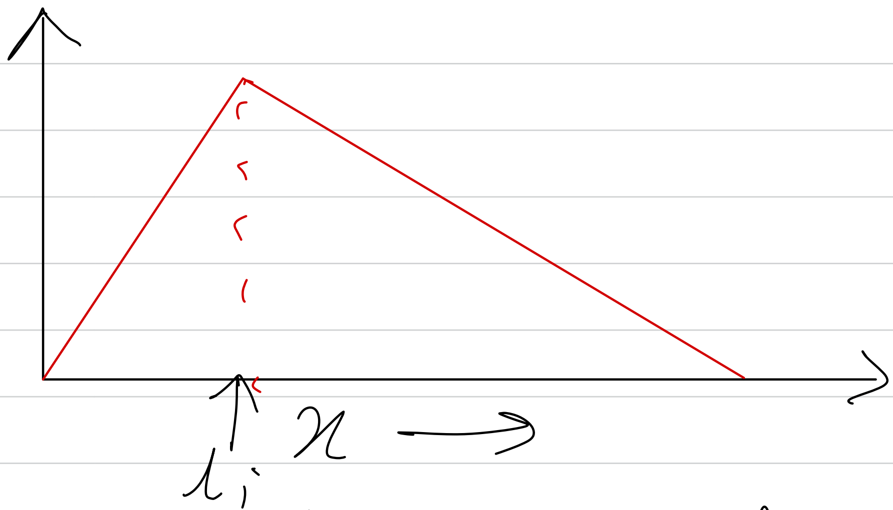


$$f(\overbrace{l_1, l_2, \dots, l_n}^{\theta}) = \frac{l_1 + l_2 + \dots + l_n}{n}$$

Example-1 (Single Peaked Preference)



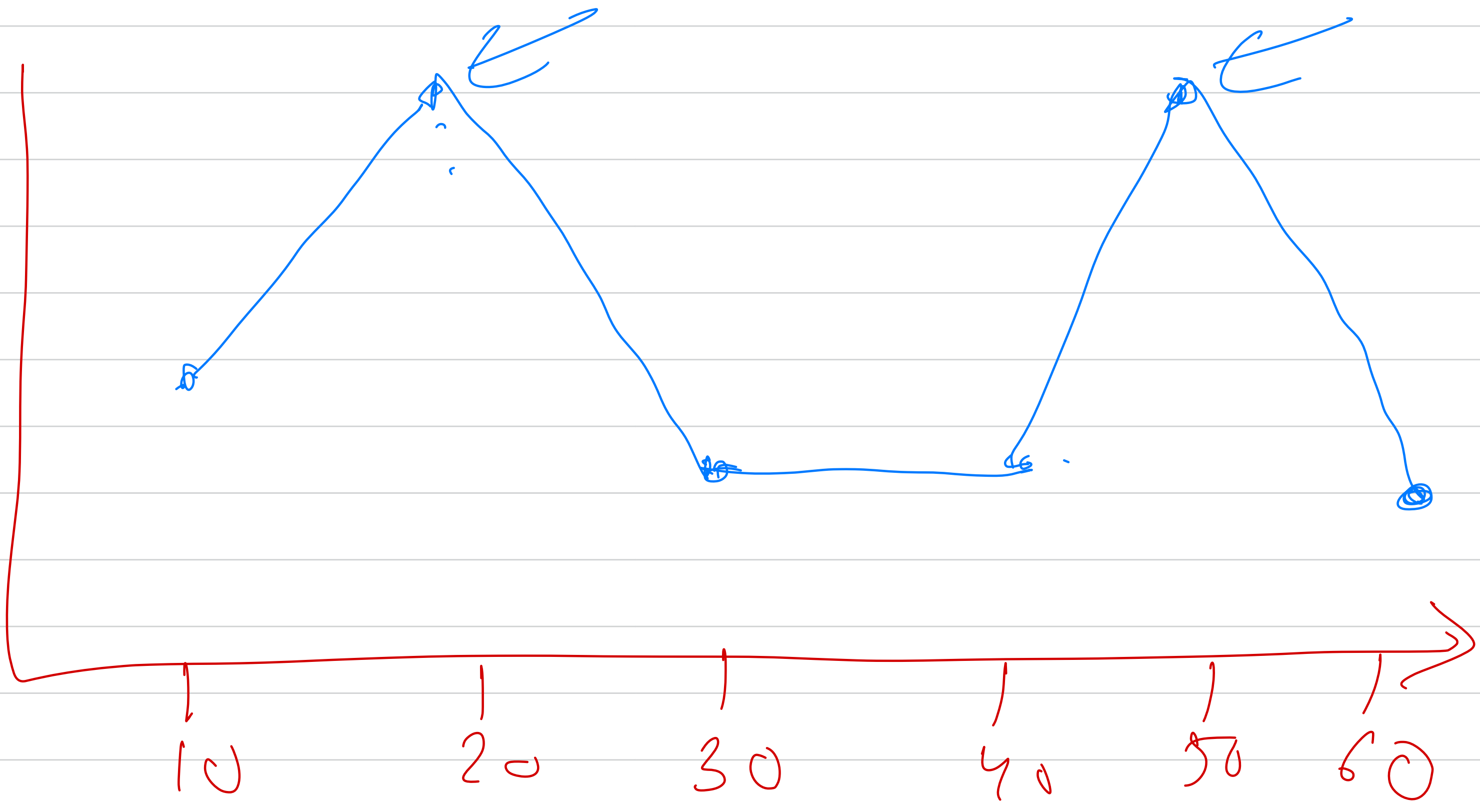
$u_i(x, l_i) =$



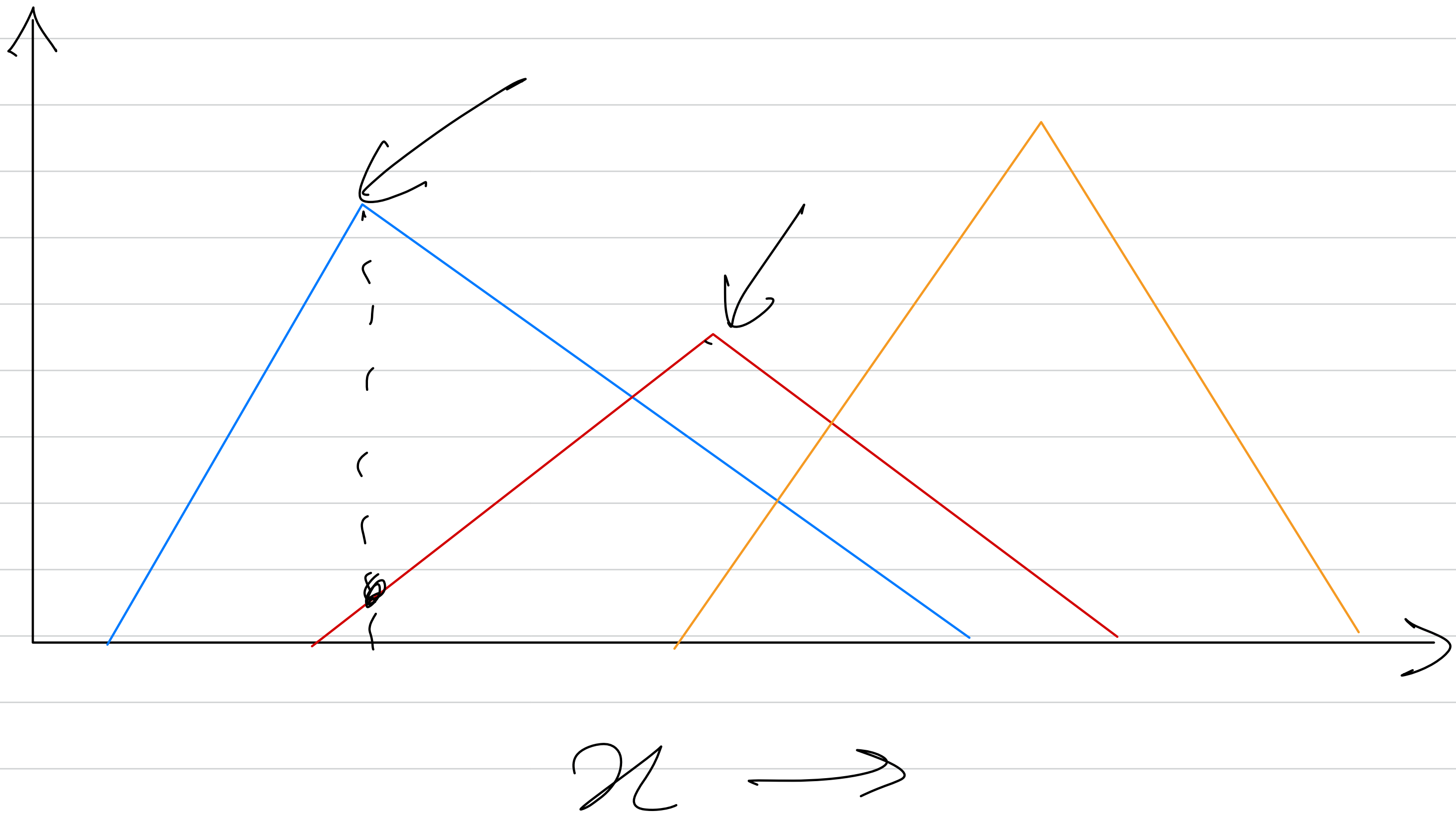
x : Dir. of the mid-point of the bear.

Example-1 (Single Peaked Preference)

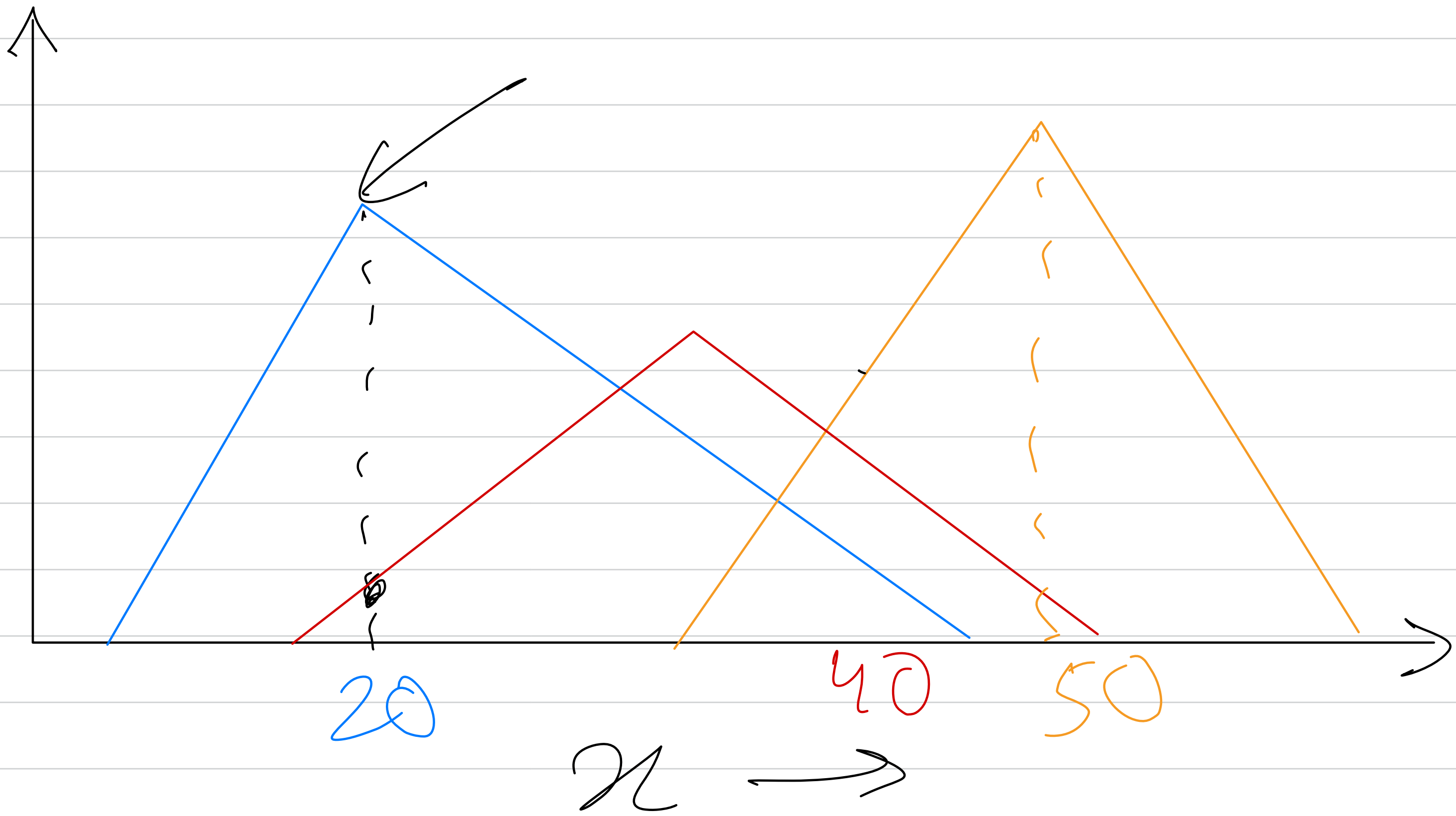
$X = \{10, 20, 30, 40, 50, 60\}$ } Set of outcomes



$20 \succ 50 \succ 10 \succ 30 \succ 40 \succ 60$



$x \rightarrow$



20
40
50
x →