CS 4187: Game Theory

Practice Problems Set 1 (till lecture 10)

Release date: 10th September, 2024

Book 1: Harrington, J.E., Games, Strategies, and Decision Making, 2nd edition, Worth Publishers: Macmillan Ed., 2015.

Book 2: Y. Narahari, "Game Theory and Mechanism Design", volume 4, World Scientific, 2014.

Q1: The first set of problems are reading assignments:

(a) Chapter 4 (Strategic form games) of Book 2.
(b) Chapter 5 (Dominant strategy equilibira) of Book 2.
(c) Chapter 2 of Book 1. Specifically, the following examples (i) Baseball, (ii) Haggling at auto dealership, (iii) iraq war, and (iv) competition for elected office. Also, section 2.8 that deals with common knowledge.
(d) Chapter 3 of Book 1. You **don't have to read** sections 3.7 and 3.8.

Q2: There are 61 students in the Game Theory class of MU in 2024. They play the following game. Each student guesses a **natural number** between 0 to 100. The student whose guess is closest to 2/3rd of the average of all the guesses wins the game and receives a cash prize of $P > 0$. If there are ties, the prize money gets shared equally between the students who tied. Answer the following questions:

(a) Is this a simultaneous move game or a sequential move game?
(b) Dependent on your answer to part (a), formulate the problem either as an SFG or an EFG.

Q3: Problems 8 to 11 in the exercise of chapter 2 of book 1.

Q4: Three players, P1, P2 and P3, play the following game. Two cards, one red and the other black, are shuffled well and put face down on the table. P2 picks the top card, looks at it without showing it to the other players and puts it back face down. Then P2 whispers either Black or Red in P1's ear, making sure that P3 doesn't hear. However, it is possible that P1 didn't hear correctly, i.e. heard the opposite of what P2 said. This happens with a probability of 0.2. P1 then tells P3 either "Black" or "Red". Finally, P3 announces either Black or Red and the game ends. If P3's final announcement matches the true color of the card P2 looked at, then P2 and P1 give \$2 each to P3. In every other case P3 gives \$2 each to P2 and P1. Formulate this game as an EFG and then convert it to an equivalent SFG.

Q5: Read the problem description of problem 15 in the exercise of chapter 2 of book 1. We will call the version of the game in book 1 as the **original version**. Also consider the following **modified version**:

Everything remains except that the contributors don't contribute simultaneously. One of the two contributors pledge first. One approximate way to model this is to allow nature to select the contributor who will pledge first. Natures chooses each contributor with 0.5 probability. After the first contributor pledges, the other contributor gets to know this pledge based on which it decides its pledge.

Answer the following questions:

(a) For both the versions, draw the tree corresponding to this EFG.
(b) Find the strategy set of the players for both the versions.
(c) Convert the original version to an equivalent SFG.
(d) **(Coding question)** Find dominant strategy equilibrium and PSNE (if they exist) of the SFG of both the versions. Try to interpret them logically.

Q6: Consider the two EFGs shown in Figs. 1 and 2.

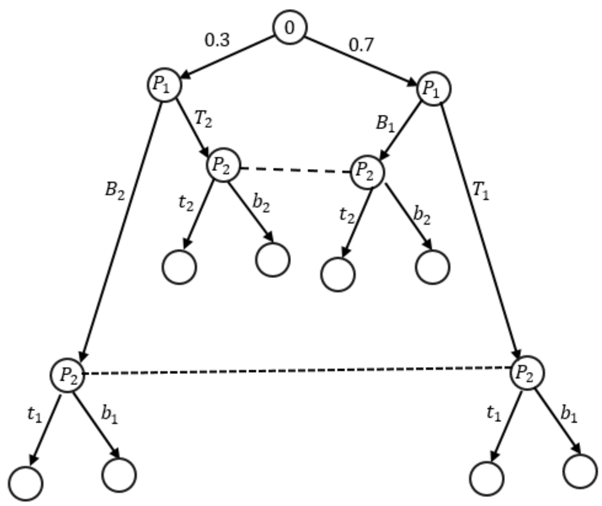

Fig. 1

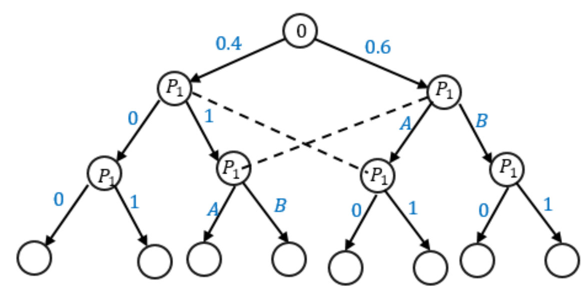

Fig. 2

Answer the following question:

(a) For the EFG in Fig. 2, what does the player P_1 know, and does not know, at each of its information sets?
(b) For the EFG in Fig. 2, how many strategies does P_1 have?
(c) For the EFG in Fig. 1, what does the player P_2 know, and does not know, at each of its information sets?
(d) For the EFG in Fig. 1, pepict the same game as a game in extensive form in which player P_2 makes his move prior to the chance move, and player P_1 makes his move after the chance move.

Q7: Consider the pollution game discussed in lectures 3 and 4 notes. Prove that doing no climate control is a dominant strategy for all the players.

Q8: Consider a SFG with the following payoff matrix,

$a, -a$	$-b, b$
$b, -b$	$c, -c$

where $a, b, c > 0$. What are the conditions on a, b, c that are required for this game to be **dominance solvable**? (Just FYI, this is a symmetric zero sum game. This information is not going help you solve this problem. But, I am hoping it will motivate you to learn what symmetric games and zero sum games are.)