Mahindra University Hyderabad
École Centrale School of Engineering
Spring Semester Regular Examinations (2023 Batch), May-2025
Program: B. Tech. Branch: CM Year: 2nd Semester: II
Subject: MA-2213 Stochastic Processes

Date: 29/05/2025 Start Time: 10:00 AM
Time Duration: 3 Hours Max. Marks: 100

Instruction:

1. All answers that belong to a particular question should be answered at one place.
2. Use of scientific but non-programmable calculators permitted.

Q 1: Each MCQ carries equal marks.

Marks: 10 × 2=20

1. Let $X(t)$ denote a WSS real-valued random process and $S_X(\omega)$ its power spectral density. Then $S_X(\omega)$ must be

   (a) non-negative odd function
   (b) non-negative even function
   (c) non-negative periodic function
   (d) non-negative bandlimited function.

2. Let $Y(t) = X(t-d)$, where d is a constant delay and $X(t)$ is WSS. Then cross-correlation $R_{Y,X}(\tau)$ is

   (a) $R_X(t+d)$
   (b) $R_X(t) - d$
   (c) $R_X(t-d)$
   (d) $R_X(t) + d$

3. Consider the following statements:
   I. Auto correlation function of a white noise process is $\sigma^2\delta(t_1 - t_2)$.
   II. Weiner process is a WSS random process.

   (a) Both I and II are true.
   (b) I is true and II is false.
   (c) I is false and II is true.
   (d) Both I and II are false.

4. The auto-correlation function of a zero-mean Gaussian random process $X(t)$ is known to be $R_X(t,s) = e^{-2|t-s|}$. Then for $t_1 < t_2 < t_3$, consider the following statements:
   I. $X(t_1)$ is strongly correlated with $X(t_3)$ as compared with $X(t_2)$.
   II. $X(t_1)$ is strongly correlated with $X(t_2)$ as compared with $X(t_3)$.

(a) If I is true then II is false.

(b) I is true and II is false.

(c) I is false and II is true.

(d) None of these.

5. Consider the following statements:
I. $X(t)$ is mean square continuous for all $t \geq 0$.
II. Mean function $m_X(t)$ is continuous for all $t \geq 0$.

(a) I imply II and II imply I.

(b) I imply II.

(c) II imply I,

(d) I does not imply II.

6. Let $Z(t) = X(t) + N(t)$, where $X(t)$ and $N(t)$ are independent zero-mean WSS random process. Then auto-correlation $R_Z(\tau)$ and cross-correlation $R_{Z,X}(\tau)$ are:

(a) $R_X(\tau) + R_N(\tau)$ and $R_X(\tau) + R_{Z,N}(\tau)$

(b) $R_X(\tau) - R_N(\tau)$ and $R_X(\tau) - R_{Z,N}(\tau)$

(c) $R_X(\tau) + R_N(\tau)$ and $R_{Z,N}(\tau)$

(d) $R_X(\tau) + R_N(\tau)$ and $R_X(\tau)$

7. Let $Y(t) = \int_0^t h(s)X(t-s)ds$ where $X(t)$ is WSS random process. Then power spectral density $S_Y(\omega)$ is

(a) $H(\omega)S_X(\omega)$.

(b) $H^*(\omega)S_X(\omega)$.

(c) $|H(\omega)|^2 S_{X,Y}(\omega)$.

(d) $|H(\omega)|^2 S_X(\omega)$.

8. Let $X'(t)$ denote the mean square derivative of a random process $X(t)$. Then the cross-correlation function $R_{X,X'}(t_1, t_2)$ is

(a) $\frac{\partial^2}{\partial t_1^2} R_X(t_1, t_2)$

(b) $\frac{\partial^2}{\partial t_2^2} R_X(t_1, t_2)$

(c) $\frac{\partial}{\partial t_1} R_X(t_1, t_2)$

(d) $\frac{\partial}{\partial t_2} R_X(t_1, t_2)$

9. Let X_n denote a symmetric Random Walk process. Then consider the following statements:
I. X_n is WSS.
II. X_n satisfy stationary increment property.

(a) Both I and II are true.

(b) I is true and II is false.

(c) I is false and II is true.

(d) Both I and II are false.

10. Consider the moving average random process $Y_n = 0.5(X_n + X_{n-1})$, where X_n is an iid random process. Then

(a) Y_n is Markov process.

(b) Y_n is not Markov process.

(c) Y_n is iid process.

(d) None of these.

Q 2:

Marks: 20

a) Let $Y(t) = X(t+s) - \beta X(t)$, where $X(t)$ is a wide-sense stationary random process.
(i) Determine whether $Y(t)$ is also a wide-sense stationary random process.
(ii) Find the cross-covariance function of $Y(t)$ and $X(t)$. Are the processes jointly wide-sense stationary?

Q 3:

Marks: 20

a) Consider the second-order autoregressive process defined by $Y_n = (3/4)Y_{n-1} - (1/8)Y_{n-2} + W_n$ where W_n is a zero-mean white noise process.

(i) Verify that the unit-sample response is $h_n = 2(1/2)^n - (1/4)^n$ for $n \geq 0$, and 0 otherwise.

(ii) Find the transfer function.

(iii) Find $S_Y(\omega)$ and $R_Y(k)$.

Q4:

Marks: 20

a) Let $X_\alpha = Z_\alpha + N_\alpha$, $\alpha \in \{n-p, n-p+1, \cdots, n\}$. Here Z_α is a first-order autoregressive (AR) process with $R_Z(k) = 4(3/4)^{|k|}$ and N_α is white noise with $\sigma_N^2 = 1$. Z_α and N_α are independent random variables.

(i) Find the optimum $p = 1$ filter for estimating Z_α.

(ii) Find the mean square error of the resulting filter.

Q 5:

Marks: 20

a) A machine consists of two parts that fail and are repaired independently. A working part fails during any given day with probability a. A part that is not working is repaired by the next day with probability b. Let X_n be the number of working parts in day n.

(i) Show that X_n is a three-state Markov chain and give its one-step transition probability matrix.

(ii) Show that the steady state pmf π is binomial with parameter $p = b/(a+b)$.
