MAHINDRA UNIVERSITY HYDERABAD

École Centrale School of Engineering

Minor-II Examinations, November 2023

Program: B.Tech. Branch: CM Year: 3^{rd} Semester: I

Subject: Financial Mathematics (MA3116)

Date: 10/11/2023 **Start Time:** 10:00 AM

Time Duration: 1.5 Hours **Max. Marks:** 30

Instructions:

1. Answer all questions.
2. All questions carry equal marks.

Q 1: Marks : 10

a) Consider the stock price process $S_n = e^{\sigma M_n}\left(\frac{2}{e^{\sigma}+e^{-\sigma}}\right)^n$, $n = 0, 1, 2, \cdots$ driven by the symmetric random walk M_n starting at 0 and volatility $\sigma > 0$. Show that the stock price process is a martingale.

b) Compute $E[\log(S_n)]$ and $Var[\log(S_n)]$.

Q 2: Marks : 10

a) Consider a 3-period binomial model of pricing American option. Let the initial stock price be $S_0 = 10$ per share, $u = 2$ be up factor, $d = 0.5$ be down factor, $r = 0.25$ be rate of interest per time period, $K = 12$ be strike price.

(i) Find the initial price of the American put option.

(ii) Let τ denote a exercise policy and defined as follows: $\tau(HHH) = \infty, \tau(HHT) = \infty, \tau(HTH) = 2, \tau(HTT) = 2, \tau(THH) = 1, \tau(THT) = 1, \tau(TTH) = 1, \tau(TTT) = 1$. Check whether τ is a stopping time and compute the stopped option price process.

Q 3: **Marks : 10**

a) Consider the scaled random walk $\{W^{(100)}(t),\ t \geq 0\}$.

(i) Determine the probability distribution of $W^{(100)}(0.3)$.

(ii) Compute $Cov(W^{(100)}(0.6), W^{(100)}(0.4))$.

b) Let $W^{(n)}(t)$ denote a scaled random walk. Let $S_n(t)$ denote the stock price process driven by $W^{(n)}(t)$ with up factor $u_n = e^{\sigma/\sqrt{n}}$ and down factor $d_n = e^{-\sigma/\sqrt{n}}$. If the stock price process $S_n(t)$ converges in distribution to the process

$$S(t) = S_0 \exp\left(\sigma W(t) + (\mu - \frac{\sigma^2}{2})t\right)$$

then find the probabilities of Head (H) and Tail (T) in the underlying random walk process. Also compute $\mathbb{E}[S(t)]$ and $\mathbb{E}[\log(S(t))]$.
