Mahindra University Hyderabad
École Centrale School of Engineering
End Semester-Regular Examination, December 2023
Program: B.Tech Branch: Computation & Mathematics Year: III
Semester: I
Subject: Advanced Linear Algebra (MA3117)

Date: 23/12/2023 Start Time: 10.00 AM
Time Duration: 3 Hours Max. Marks: 100

Instructions:

1. All questions are compulsory.

Q 1: 20 marks

Given the non-orthogonal basis $a_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$, $a_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, and $a_3 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$, use the modified Gram-Schmidt orthogonalization process to find an equivalent orthonormal basis.

Q 2: 20 marks

Find a spectral decomposition of the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -3 \\ 3 & 1 & -2 \end{bmatrix}$.

Q 3: 20 marks

Find a solution of the least-squares problem $Ax \cong b$, where the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$, and $b = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

Q 4: **20 marks**

Compute the geometric multiplicities of each of the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

Q 5: **20 marks**

Find the singular value decomposition of the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$.
