Mahindra University Hyderabad
École Centrale School of Engineering,
Minor-1 Examination
Program: B.Tech Branch: Computation & Mathematics Year: III
Semester: I
Subject: Advanced Linear Algebra (MA3117)

Date: 19/09/2023 Start Time: 02.00 PM
Time Duration: 1.5 Hours Max. Marks: 20

Instructions:

1. All questions are compulsory.
2. In the questions 1 and 2, for a vector $x \in \mathbb{R}^{n\times 1}$, x^* is the transpose of x.

Q 1: 5 marks

Find the vectors $u, v \in \mathbb{R}^{4\times 1}$ such that the following relation holds true, where I_4 denotes the Identity matrix of order 4.

$$\begin{bmatrix} 5 & 4 & 4 & 4 \\ 6 & 7 & 6 & 6 \\ 9 & 9 & 10 & 9 \\ 3 & 3 & 3 & 4 \end{bmatrix} = I_4 + uv^*.$$

Q 2: 5 marks

Let $< x, y >= x^* y$ be an inner product defined for any two vectors $x, y \in \mathbb{R}^{3\times 1}$. Then

(a) For any matrix $A \in \mathbb{R}^{3\times 3}$, show that $< x, Ay >=< A^* x, y >$, where A^* is the transpose of A. [1]

(b) Using part(a) prove that if A is a real symmetric matrix then its eigenvalues are real. Also prove that if A is a real skew-symmetric matrix then its eigenvalues are either zero or purely imaginary. [4]

Q 3 : 5 marks

Construct an LU decomposition for the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 4 \\ 1 & -1 & 0 \end{bmatrix}.$$

Q 4: 5 marks

Let $r_1, r_2, r_3, \ldots, r_n$ be a set of vectors in $\mathbb{R}^n$ and forms a basis for the row space of $A \in \mathbb{R}^{n \times n}$. Then show that the n vectors $Ar_1, Ar_2, \ldots, Ar_n$ in the column space of A are linearly independent and form a basis for the columns space of A.
