Mahindra University Hyderabad
École Centrale School of Engineering,
Minor I

Program: B. Tech. Branch: CE, ECE, ECM, ME, MT, CSE, AI, CM, NT
Year: 2nd   Semester: Fall   Subject: Mathematics III (MA2103)

Date: 15/09/2023   Start Time: 10.00 AM
Time Duration: 90 minutes   Max. Marks: 20

Instructions:

1. Each question carries 5 marks.

2. All questions are compulsory.

3. Please start each answer on a separate page and make sure to clearly number the responses.

4. It is essential to provide an explanation of each step. Correct outcomes without any description will not be evaluated.

Q 1:   5 marks

The chances that Doctor A will diagnose disease X correctly are 60%. The chances that a patient will die from his treatment after a correct diagnosis are 40%, and the chance of death from a wrong diagnosis is 70%. A patient of Doctor A, who has disease X, has died. What is the probability that his disease was diagnosed correctly?

Q 2:   5 marks

(a) A system is composed of four components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector (x_1, x_2, x_3, x_4) where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed. (i) How many outcomes are in the sample space of this experiment? (ii) Suppose that the system will work if components 1 and 2 are both working or if components 3 and 4 are both working. Specify all the outcomes in the event that the system works.

(b) The ShopperTrak device is designed to count the number of customers entering a shopping centre. When two customers enter together, one in front of the other, the following probabilities apply: The probability that the first customer is detected is 0.98. The probability that the second customer is detected is 0.94. The probability that both customers are detected is 0.93. What is the probability that the device will detect at least one of the two customers entering?

Q 3: 5 marks

Answer the following with an explanation.

(a) You have 7 identical balls, and you want to distribute them into 4 distinct boxes. How many different ways can you do this?

(b) A deck of 52 cards contains four aces. If the cards are shuffled and distributed randomly to four players so that each player receives 13 cards, what is the probability that the same player will receive all four aces?

Q 4: 5 marks

A fair die is repeatedly rolled until a number larger than 4 is observed. If N is the total number of times that the die is rolled, then find $P(N = k)$, for $k = 1, 2, 3, \ldots$. Also write its cumulative distribution function.