Mahindra University Hyderabad

École Centrale School of Engineering

Minor–II

Program: B. Tech. Branch: CSE, AI, ECE and CAM Year: III Semester: I

Subject: Design and Analysis of Algorithm (CS/AI 3101)

Date: 09/11/2023 **Start Time: 10:00 AM**

Time Duration: 1.5 Hours **Max. Marks: 30**

Instructions:

1) **Answers written with pencil will not be evaluated.**
2) **Answer each question precisely and to the point.**

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1. Answer the following question with respect to the problem of Optimal Course Registration described as follows **(12 Marks)**

   You are given a list of $'n'$ courses, where each course $'i'$ is represented by a tuple (c_i, t_i, cap_i):

   c_i: The credit value of the course.
   t_i: The time slot when the course is offered (e.g., A, B, C, D).
   cap_i: The maximum capacity (number of students that can enroll) for the course.

   Additionally, you have information about the available time slots (all time slots are available at the beginning of the algorithm) for your schedule and the maximum number of credits you can achieve in the semester.

   Your task is to design an algorithm to select a combination of courses that maximizes your total credit value while ensuring that you do not exceed the available time slots, course capacities or maximum credit.

   a) What approach (Greedy/Dynamic Programming) will you choose? Justify your answer based on the characteristics of the problem. (3 Marks)
   b) Write the algorithm for solving the problem. (6 Marks)
   c) Justify the correctness of the algorithm by executing an example (2 Marks)
   d) What is the time complexity of your algorithm? Justify. (1 Marks)

2. A salesperson needs to visit five different cities (A, B, C, D, E) and return to the starting city (A). The distances between these cities are as follows: **(4 Marks)**

   Distance from A to B: 10 units
   Distance from A to C: 15 units
   Distance from A to D: 20 units

Distance from A to E: 12 units
Distance from B to C: 18 units
Distance from B to D: 25 units
Distance from B to E: 20 units
Distance from C to D: 30 units
Distance from C to E: 22 units
Distance from D to E: 28 units

Using equation $Cost(i,S) = \min_{j \in S}\{c_{ij} + Cost(j, S - \{j\})\}$, where symbols have their usual meaning, compute the path which the salesperson should choose to reach back to the source (A) by traversing minimum distance.

3. Suppose you are designing a dictionary lookup system and need to implement an Optimal Binary Search Tree (OBST) to minimize the expected search time. You have a set of words with associated probabilities of search, and you want to construct an OBST for efficient retrieval *(For only successful search cases).* **(8 Marks)**

   a) Define what is an Optimal Binary Search Tree (OBST)? (2 Mark)
   b) Given the following list of words and their associated search probabilities, construct an optimal binary search tree: (3 Marks)
      Words: "A," "B," "C," "D," "E"
      Probabilities: 0.2, 0.1, 0.3, 0.2, 0.2
   c) Calculate the expected search time for the word "D" in the constructed OBST. (2 Mark)
   d) Explain the time complexity of constructing an OBST. (1 Mark)

4. Answer the following with respect to flow network **(6 Marks)**
   a) What is Flow network? What is cut in a flow network?
   b) What is max flow -min cut theorem. Prove the correctness of the theorem
   c) Prove the correctness of the theorem: "If there is no augmenting path relative to flow f, then there exists a cut whose capacity equals the value of f"

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