Mahindra University

Mahindra University Hyderabad

École Centrale School of Engineering

End–semester Regular Examination

Program: B.Tech. **Branch: Computation & Mathematics** **Year: Second** **Semester: Spring**

Subject: Optimization Techniques (MA 2210)

Date: 29/05/2024 **Start Time: 10: 00 AM**

Time Duration: 03: 00 Hours **Max. Marks: 100**

Instructions:

1) **All questions are compulsory.**
2) **Please start each answer on a separate page, and ensure you clearly number the responses. Also, make sure to address all parts of each question together and in the correct order.**
3) **It is essential to provide an explanation of each step. Correct outcomes without any description will not be evaluated.**

Q 01: Select the correct choice for the following questions with a proper explanation. Correct choices without valid justification will not be considered. [02 × 10]

A) A linear programming problem is as follows:

$\max z = 30x - 18y$; subject to the constraints,

$$\left.\begin{matrix} 3x + 4y \leq 60 \\ 5x - 3y \geq 20 \end{matrix}\right\}; x \geq 0 \text{ and } y \geq 0.$$

In the feasible region, the maximum value of z occurs at

i) Two points ii) One point iii) Infinite number of points

iv) More than one of the above v) None of the above

B) The corner points of the feasible region determined by the system of linear constraints are $(0,10), (5,5), (15,15)$, and $(0,20)$. Suppose the objective function is $z = px + 3y$, where $p > 0$. If the maximum of z occurs at both points $(15,15)$ and $(0,20)$, then the value of p is

i) 4 ii) 5 iii) 1

iv) 2 v) None of the above

C) A simplex problem is considered as infeasible when

i) All the variables in the entering column are negative

ii) Variables in the basis are negative iii) Artificial variable is present in the basis

iv) Pivotal value is negative v) None of the above

D) Any solution to a linear programming problem which also satisfies the non-negative restriction of the problem has ________.

i) Solution ii) Basic solution iii) Basic feasible solution

iv) Feasible solution v) None of the above

E) While solving a linear programming problem, given as m equations in n variables $(m < n)$, through the simplex method. In this case, basic solutions are determined by setting $n-m$ variables equal to zero and solving m equations to obtain solutions for the remaining m variables, provided the resulting solutions are unique. This means that the maximum number of basic solutions is:

i) $\frac{n!}{m!(n-m)!}$ ii) $\frac{m!}{n!(n-m)!}$ iii) $\frac{n!}{m!(n+m)!}$

iv) $\frac{m!}{n!(n+m)!}$ iv) None of the above

F) A linear programming problem is defined as $\min z_x = 15x_1 + 12x_2$, subject to the constraints

$$\left.\begin{array}{r} x_1 + 2x_2 \le 3 \\ 2x_1 - 4y_2 \le 5 \end{array}\right\}; x_1 \ge 0 \text{ and } x_2 \ge 0.$$

Then, the objective function of the dual of this linear programming problem is

i) $\max z_y = y_1 + y_2$ ii) $\max z_y = y_1 + 2y_2$ iii) $\max z_y = 2y_1 - 4y_2$

iv) $\max z_y = 3y_1 + 5y_2$ v) None of the above

G) To maintain optimality of the current optimum solution for a change Δc_k in the coefficient c_k of non-basic variables, we must have

i) $\Delta c_k = z_k - c_k$ ii) $\Delta c_k \le z_k - c_k$ iii) $\Delta c_k \ge z_k - c_k$

iv) $\Delta c_k = z_k$ v) None of the above

H) The dummy source or destination in a transportation problem is introduced to

i) Prevent the solution from becoming degenerate

ii) To satisfy RIM conditions iii) Ensure that total cost does not exceed a limit

iv) Solve the balanced transportation problem v) None of the above

I) In a non-linear programming problem,

i) The objective function is non-linear

ii) One or more of the constraints have a non-linear relationship

iii) Both (i) and (ii) iv) None of the above

J) The minimum value of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$ is

i) -128 ii) -126 iii) -120

iv) 0 v) None of the above

Q 02: Solve any two of the following three questions. Explaining each step is highly desirable. [10 × 02]

A) A salesman estimates that the following would be the cost of his route, visiting the six cities as shown in the following table.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	∞	20	23	27	29	34
C_2	21	∞	19	26	31	24
C_3	26	28	∞	15	36	26
C_4	25	16	25	∞	23	18
C_5	23	40	23	31	∞	10
C_6	27	18	12	35	16	∞

The salesman can visit each of the cities once and only once. Determine the optimum sequence the salesman should follow to minimize the total distance traveled. What is the total distance traveled?

B) Use any discussed methods to find the basic feasible solution to the following transportation problem, consisting of four origins and five destinations. Check whether the obtained solution is optimal. If not, then find the optimal transportation plan and cost.

	D_1	D_2	D_3	D_4	D_5	Available
O_1	4	3	1	2	6	80
O_2	5	2	3	4	5	60
O_3	3	5	6	3	2	40
O_4	2	4	4	5	3	20
Required	60	60	30	40	10	200

C) Find the optimum integer solution to the following linear programming problem:

$\max z = x_1 + 4x_2$; subject to the constraints

$$\left.\begin{array}{l} 2x_1 + 4x_2 \leq 7 \\ 5x_1 + 3x_2 \leq 15 \end{array}\right\}; x_1 \geq 0 \text{ and } x_2 \geq 0\text{, and are integers.}$$

Q 03: Explaining each step in the following questions is highly desirable. Each question carries a weightage of 15 marks. [15 × 04]

A) Apply the Kuhn–Tucker conditions to determine x_1, x_2, and x_3 to

$\min f(x_1, x_2, x_3) = x_1^2 = 2x_2^2 + 3x_3^2$; subject to the constraints

$g_1(x_1, x_2, x_3) = x_1 - x_2 - 2x_3 \leq 12$ and $g_2(x_1, x_2, x_3) = x_1 + 2x_2 - 3x_3 \leq 8$.

B) Use the dual simplex method to solve the following,

$\min z = 2x_1 + x_2$; subject to the constraints

$$\left.\begin{array}{l} 3x_1 + x_2 \geq 3 \\ 4x_1 + 3x_2 \geq 6 \\ x_1 + 2x_2 \leq 3 \end{array}\right\}; x_1 \geq 0 \text{ and } x_2 \geq 0.$$

C) A company wants to produce three products: A, B, and C. The unit profits on these products are ₹ 4, ₹ 6, and ₹ 2, respectively. These products require two types of resources: manpower and raw material. The linear programming model formulated for determining the optimal product mix is as follows:

$$\max z = 4x_1 + 6x_2 + 2x_3; \text{ subject to the constraints}$$

$$\left.\begin{array}{ll} x_1 + x_2 + x_3 \le 3 & \text{manpower constraints} \\ x_1 + 4x_2 + 7x_3 \le 9 & \text{raw material constraints} \end{array}\right\}; \; x_1 \ge 0,\; x_2 \ge 0, \text{ and } x_3 \ge 0.$$

Here x_1, x_2, and x_3 denote the number of units of product A, B, and C to be produced.

1) Find the optimal product mix and the company's corresponding profit.

2) Find the range of the profit contribution of products A and C in the objective function so the optimal product mix remains unchanged.

3) What shall be the new optimal product mix when profit per unit of product C is ₹ 8 and not ₹ 2?

D) Find the minimum value of the function $f(x) = x^2 + \frac{54}{x}$ with corresponding x value. Next, minimize the function $f(x) = x^2 + \frac{54}{x}$ using the Fibonacci Search Method by taking lower bound $a = 0$ and upper bound $b = 5$. Also, perform the procedure with the desired number of function evaluations $n = 3$.