



Mahindra University Hyderabad  
École Centrale School of Engineering  
End Semester Regular Exam  
(2023-Batch)

Program: B. Tech Branch: Computation & Mathematics

Year: II Semester: Fall

Subject: Real Analysis (MA 2104)

Date: 14/12/2024

Time Duration: 3 Hrs

Start Time: 10.00 AM

Max. Marks: 100

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**Instructions**

1. All questions are compulsory.
  2. In all questions, justifications are required.
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**Q.1**

- a) Let  $f_n(x) = n^2 x(1-x^2)^n$ ,  $x \in [0, 1]$ , prove that  $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \neq \int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$ . Does  $\{f_n\}$  converge uniformly on  $[0, 1]$ ? 14M
- b) Define the following with examples: i) Limit point of a set, ii) Complete metric space. 6M

**Q.2**

- a) Prove that discrete metric space is complete. 12M
- b) Draw open balls  $B((0, 0), 1)$  in  $\mathbb{R}^2$  with respect to the following metrics:
- i)  $d_1(\mathbf{x}, \mathbf{y}) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ ,
- ii)  $d_\infty(\mathbf{x}, \mathbf{y}) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$ , where  $\mathbf{x} = (x_1, y_1)$ ,  $\mathbf{y} = (x_2, y_2)$ . 8M

**Q.3**

- a) Let

$$f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational.} \end{cases}$$

Prove that  $f$  is not continuous at any point of  $\mathbb{R}$ .

12M

- b) Define the following with examples :

- i) Uniform continuity of a function, ii) Limit superior of a sequence.

8M



#### Q.4

- a) Evaluate  $\int_0^3 x[x]d(e^x)$ , where  $[x]$  denotes the greatest integer function. 12M
- b) Define the following with examples : i) Connected set, ii) Compact set. 8M

#### Q.5

- a) Define measure zero set in  $\mathbb{R}$ . Prove that a countable set in  $\mathbb{R}$  is a set of measure zero. 10M
- b) Prove that outer measure is translation invariant. 10M