Mahindra University
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Class Test I, Nov-2023 (2022 – BATCH)
Program: B. Tech. Branch: CM
Year: II Semester: Autumn Subject: Real Analysis (MA2104)

Date: 03/11/2023 **Start Time: 10:00 AM**
Time Duration: 1 hour **Max. Marks: 20**

Instructions:

1. All questions are compulsory.

Q 1: (10 marks)

(a) Define a metric. For any $x \in \mathbb{R}$ and $y \in \mathbb{R}$, define

$$d_1(x,y) = \frac{|x-y|}{1+|x-y|},$$

$$d_2(x,y) = |\sin(x-y)|.$$

Determine, for each of these, whether it is a metric or not. [5 marks]

(b) Define a diameter for a set in a metric space. Let $A = \{\bar{x} = (x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 1\}$ be a set in $\mathbb{R}^2$. Compute the diameter of A with respect to each of the following metrics: (i) usual, (ii) taxicab, (iii) max, and (iv) discrete. [3 marks]

(c) Define the Cantor set. [2 marks]

Q 2: (10 marks)

(a) Define the neighbourhood of a point in a metric space. For each of the following subsets of metric space $\mathbb{R}$, find its closure.

(i) The set of all irrational numbers.

(ii) $\bigcap_{n=1}^{\infty} [-3, \frac{1}{n}]$.

(iii) $(0,1) \cap \mathbb{Q}$. [4 marks]

(b) For any collection $\{F_\alpha\}$ of closed sets in a metric space, show that $\bigcap_\alpha F_\alpha$ is closed. [3 marks]

(c) The intersection of infinitely many open sets in $\mathbb{R}$ need not ~~closed~~ open. Prove or justify with an example. [3 marks]