Mahindra University Hyderabad

École Centrale School of Engineering

End-sem exam

Program: B. Tech. Branch: AI, CAM, CE, CSE, ECM, NT Year: II Semester: II

Subject: Numerical Methods (MA2208)

Date: 27/05/2024 **Start Time: 10:00 AM**

Time Duration: 3 Hours **Max. Marks: 100**

Instructions:

1) Answer all the questions.

2) All questions are self-explanatory; no clarification will be provided during the exam.

3) Use of non-programmable scientific calculator is allowed. However, sharing calculators during exams is strictly prohibited.

Question 1 (20 marks)

(a) Write a MATLAB function named 'GL3' to evaluate a finite integral of the form [10]

$$\int_a^b f(x)\,dx,$$

using the three-point Gauss-Legendre method. All necessary values can be assigned by you.

(b) Find the Crout's LU decomposition for the following matrix: [10]

$$\begin{bmatrix} 2 & -1 & 1 \\ 4 & 1 & 1 \\ -2 & 4 & 0 \end{bmatrix}.$$

Question 2 (20 marks)

(a) Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)\,dx$. [5]

(b) Consider the method: [15]

$$\int_a^b f(x)\,dx \approx c_1 f(x_1). \tag{1}$$

Derive the values of c_1 and x_1 so that the formula in (1) gives exact values for the linear polynomial of the form $f(x) = a_0 + a_1 x$.

P.T.O.

Question 3 (20 marks)

(a) Find an iterative scheme using secant method for computing $\sqrt{a}$. Iterate the obtained scheme five times to find the square root of 3 up to three decimal places with initial data $x_0 = 1.65$ and $x_1 = 1.7$. [10]

(b) The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the method [10]

i) $x_{k+1} = -\frac{1}{x_k}(ax_k + b)$ converges to α if $|\alpha| > |\beta|$.

ii) $x_{k+1} = -\frac{b}{x_k + a}$ converges to α if $|\alpha| < |\beta|$.

iii) $x_{k+1} = -\frac{1}{a}(x_k^2 + a)$ converges to α if $2|\alpha| < |\alpha + \beta|$.

Question 4 (20 marks)

Consider the initial value problem

$$\frac{dy}{dx} = 1 + \frac{y}{x}, \quad y(1) = 2.$$

Compute $y(1.6)$ using Euler's method and Runge-Kutta method of order 2. Take $h = 0.2$.

Question 5 (20 marks)

(a) Fit $p(x) = a + bx$ for the following data: [10]

x:	-2	-1	0	1	2
$f(x)$:	15	1	1	3	19.

(b) Find the interpolating polynomial by Lagrange's formula for the following data: [10]

x:	0	1	2	4	5	6
$f(x)$:	1	14	15	5	6	19.

Then estimate $f(3)$ and $f(6.5)$.

End of exam