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Mahindra University Hyderabad
École Centrale School of Engineering
End-semester Regular Examination
(2023-Batch)

Program: B. Tech Branch: CSE/ICE, ECM, ECE, AI, CE, ME, CM, MT, NT, COM

Year: I Semester: Spring

Subject: Mathematics - II (MA 1202)

Date: 27/05/2024 Start Time: 10.00 AM

Time Duration: 3 Hours Max. Marks: 100

Instructions

1. Attempt all the questions. The order of answers should be same as the order of questions.
2. In Q1, your answer should be either True or False. Explanations are not required in Q1.
3. In Q2, Q3, Q4 and Q5 justifications are compulsory. Guesswork will not be considered in evaluation.

Q. 1 Marks: $10 \times 2 = 20$

State whether the following statements are True or False.

a) If Q is a 2×2 symmetric matrix with real entries, then eigen vectors corresponding to different eigen values will be orthogonal.

b) $\mathcal{L}^{-1}\Big(\dfrac{1}{(s-2)(s-3)}\Big) = e^{2t} - e^{3t}$.

c) $\int_C \frac{e^z}{z} dz = 2\pi i$, where C is the positively oriented circle $|z| = 1$.

d) If a function f is entire, then the radius of convergence(R) of a Taylor series of f centered at any point z_0 is necessarily, $R = \infty$.

e) The Laplace transform of $e^{2t}\sin 3t$ is $\dfrac{3}{(s-2)^2+4}$.

f) If A=$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, then the eigen values of A are $-1, 3$.

g) For any 2×2 real matrix A, if one of the eigen value is zero, then det(A)$\neq$0.

h) 1 is a removable singularity of the function $f(z) = \sin\left(\frac{1}{z-1}\right)$.

i) The sum of the two eigenvalues of a 2×2 real matrix equals the sum of its diagonal entries.

j) Let $f(z) = \frac{g(z)}{h(z)}$. If g and h are analytic functions at $z = z_0$ and h has a zero of order n at $z = z_0$. Then f has a pole of order n at $z = z_0$.

Q. 2 **Marks: 20**

If $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$, then find the matrix A^{23}.

Q. 3 **Marks: 20**

Expand the function $f(z) = \frac{1}{3+z}$ in a Taylor series centered at $z_0 = 3i$. Then find its radius of convergence.

Q. 4 **Marks: 20**

Evaluate the following integral using Residue Theorem:

$$\int_\gamma \frac{2z+1}{(z+3)(z-2)(z+1)} dz,$$

where γ is the positively oriented circle $|z - i| = 5/2$.

Q. 5 **Marks: 20**

Use the Laplace transform method to solve the initial-value problem

$$y'' + y = e^{-t}, \quad y(0) = \frac{3}{2}, y'(0) = \frac{1}{2}$$