Mahindra University Hyderabad
École Centrale School of Engineering
Minor-I
Program: B. Tech.
Branch: AI/CE/CM/CSE/ECE/ECM/ME/MEC/NT
Year: I Semester: I
Subject: Mathematics I (MA 1101)

Date: 15/09/2023 Start Time: 10.00 AM
Time Duration: 1.30 Hours Max. Marks: 20

Instructions:

1. There are 4 questions, all of which are compulsory.
2. You should not write anything on the question paper, except your roll number.
3. Work out the given problem in your answer booklet. Then, at the end of each solution, write the complete question along with the **answers underlined**.
4. Justification of your answers is absolutely necessary. Guesswork will not be considered in evaluation.

1. Let $f(x) = e^x + e^{-x}$. The domain of this function is _____. The function is increasing in _____ and decreasing in _____. It is convex in _____. The function has relative minima at _____. **5 M**

2. Let f be continuous on $[0, 1]$, $f(0) = f(1)$ and $f(0) > f(\frac{1}{2})$. Define $g(x) =$ _____. Then g is continuous on _____. Moreover, $g($_____$) > 0$ and $g($_____$) < 0$. Therefore, by Bolzano's theorem there is a point c in _____ such that $f(c) = f(c+\frac{1}{2})$. **5 M**

3. Let $f(x) = e^{-x}$. Using the Taylor polynomial of degree 1 generated by f at **0**, the approximate value of $f(\frac{1}{2})$ is _____. The lower bound on the error in this approximation is _____. The upper bound on the error in this approximation is _____. Using the Taylor polynomial of degree 1 generated by f at **1**, the approximate value of $f(\frac{1}{3})$ is _____. The lower bound on the error in this approximation is _____. **5 M**

4. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by 5 M

$$f(x) = \begin{cases} \dfrac{1}{|x^2 - 4|}, & \text{if } x \neq \pm 2 \\ x, & \text{if } x = \pm 2. \end{cases}$$

(i) The point/points at which f is not continuous is/are _____.

(ii) If the domain of the function is taken as $[-3, 3]$ instead of $\mathbb{R}$ then the infimum of the function is _____.

(iii) If the domain of the function is taken as $[-123, b]$ instead of $\mathbb{R}$, where b is the largest integer such that the supremum of f is attained, then the supremum of f is _____.

(iv) If the domain of the function is taken as $[\sqrt{6}, 3]$ instead of $\mathbb{R}$, then the range of the function is _____.

(v) The largest real number b such that f preserves the sign at every point of $[-4, b)$ is _____.

****