



Mahindra University Hyderabad
École Centrale School of Engineering
End-semester Regular Examinations, December 2023
Program: B. Tech. Branch: CM Year: 3rd Semester: I
Subject: MA-3116 Financial Mathematics

Date: 22/12/2023
Time Duration: 3 Hours

Start Time: 10:00 AM
Max. Marks: 100

Instruction:

1. All answers that belong to a particular question should be answered at one place in your answer booklet.
2. Use of scientific but non-programmable calculators permitted.
3. Each MCQ has only one correct answer.

Q 1: Each MCQ carries equal marks.

Marks: $10 \times 2 = 20$

1. Consider an American option and an European option on a non-dividend paying stock with same strike price K and initial stock price S_0 . Which of the following statement is true:
 - (a) The price of the American call option is greater than that of the European call option.
 - (b) The price of the American put option is greater than that of the European put option.
 - (c) The price of the American call option is less than that of the European call option.
 - (d) The price of the American put option is less than that of the European put option.
2. A European put option on a stock with strike price 1500 and time to expiry 1 month has delta -0.486 . Then the corresponding call option delta is
 - (a) 0.486
 - (b) 0.614
 - (c) 0.586
 - (d) 0.514
3. Let $V_0^c = 2.6$ be the price of a European call option with strike price $K = 12$, initial stock price $S_0 = 13$, expiration time six months, and continuously compounding risk free interest rate $r = 5\%$ per annum. Then V_0^p , the price of corresponding European put option, equals to

- ~~(a)~~ 1.3037
(b) 1.6574

- (c) 2.7295
(d) 2.0265

4. Let $\{W^{(n)}(t) : t \geq 0\}$ denote the scaled symmetric random walk starting at 0. Consider the following two statements:

- I : $Var(W^{(n)}(t_2) - W^{(n)}(t_1)) = n(t_2 - t_1)$.
II : For any fix t , $W^{(n)}(t) \sim N(0, t)$ normal distribution.

- ~~(a)~~ Both I and II are true
(b) I is true and II is false

- (c) I is false and II is true
(d) Both I and II are false

5. Let $\{W(t) : t \geq 0\}$ denote Brownian motion starting at 0. Consider the following two statements:

- I : For any $t_2 > t_1$, $Var(W(t_2) - W(t_1)) = (t_2 - t_1)^2$.
II : For any positive times t_1 and t_2 , $Cov(W(t_1), W(t_2)) = \min(t_1, t_2)$.

- (a) Both I and II are true
(b) I is true and II is false

- ~~(c)~~ I is false and II is true
(d) Both I and II are false

6. Let $\{S(t) : t \geq 0\}$ denote a stock price process driven by geometric Brownian motion with mean rate of return μ and volatility σ . Then which of the following is true:

- (a) $E[S(t)] = S(0)e^{\mu t}$
~~(b)~~ $E[S(t)] = S(0)e^{(\mu - \sigma^2/2)t}$

- (c) $\log(S(t))$ is normal $N(\log(S(0)), \sigma^2 t)$
(d) None of the above

7. Let $V_N(\omega) = (M_N - S_N)(\omega)$ denote the expiration time pay-off of an option, where $M_N(\omega) = \max\{S_n(\omega) : 0 \leq n \leq N\}$, $S_N(\omega)$ is expiration time stock price, $\omega = \omega_1 \omega_2 \cdots \omega_N$ is sequence of N coin toss outcome. Then the option is

- (a) Non-path dependent option
(b) European type look-back option

- ~~(c)~~ American type look-back option
(d) European call option

8. A European put option with 3 month expiration time and strike price 18500 has delta -0.45 . An option portfolio of the put options with 1000 short position may be hedged with analogous call options. Then the hedge position is

- ~~(a)~~ 450 short call options
(b) 818 short call options

- (c) 550 short call options
(d) 818 long call options

9. Let $\{S(t) : 0 \leq t \leq T\}$ be a geometric Brownian motion and denote the underlying stock price process of a European call option. The Black-Scholes-Merton formula for pricing the European call option with strike price K , expiration time T year(s), and continuously compounding risk free annual interest rate r is obtained from

- (a) $V(S(t), t) = e^{-r(T-t)} \tilde{E}_t[(S(T) - K)^+]$ (c) $V(S(t), t) = e^{-rT} \tilde{E}_t[(S(T) - K)^+]$
 (b) $V(S(t), t) = e^{-r(T-t)} \tilde{E}_t[(K - S(T))^+]$ (d) $V(S(t), t) = e^{-r(T-t)} \tilde{E}_t[(S(T) - K)^+]$

10. Let $X(t) : t \geq 0$ be an adapted stochastic process specified by the stochastic differential equation

$$dX(t) = rX(t)dt + \Delta(t)(\mu - r)S(t)dt + \sigma\Delta(t)S(t)dW(t),$$

where μ, r, σ are positive constants, $S(t), \Delta(t)$ are adapted stochastic processes and $W(t)$ is adapted Brownian motion. Then the quadratic variation $[X, X]_{[0, T]}$ is equal to

- (a) $\sigma^2 \int_0^T \Delta^2(t) S^2(t) dt$ (c) $\sigma^2 \int_0^T \Delta(t) S(t) dt$
 (b) $\frac{\sigma^2}{2} \int_0^T \Delta^2(t) S^2(t) dt$ (d) $\sigma \int_0^T \Delta^2(t) S^2(t) dt$

Q 2:

Marks: 20

- (a) Consider a 3-period binomial model of pricing American option. Let the initial stock price be $S_0 = 10$ per share, $u = 2$ be up factor, $d = 0.5$ be down factor, $r = 0.25$ be rate of interest per time period, $K = 15$ be strike price.
 (a) Find the initial price of the American put option.
 (b) Find the delta (Δ_n) hedging portfolio process.

Q 3:

Marks: 20

- (a) Write the governing equation, terminal condition and boundary conditions for an European put option price process. Assume that the underlying stock price process is driven by a geometric Brownian motion. (8)
 (b) Derive the implicit finite difference scheme for the European put option price process. (12)

Q 4:

Marks: 20

- (a) Using Ito-Doebelin formula solve the CIR interest rate model SDE

$$dX(t) = (\alpha - \beta X(t))dt + \sigma\sqrt{X(t)}dW(t), \quad t > 0, \quad X(0) = X_0.$$

Find $\mathbb{E}[X(t)]$ and $\text{Var}[X(t)]$.

Marks: 20

Q 5:

a) A financial institution has the following portfolio of options on US dollar:

<i>Type</i>	<i>Position</i>	<i>Delta</i>	<i>Gamma</i>	<i>Vega</i>
<i>Call</i>	-1000	0.5	2.2	1.8
<i>Put</i>	+500	-0.8	0.6	0.2
<i>Put</i>	-2000	-0.4	1.3	0.7
<i>Call</i>	-500	0.7	1.8	1.4

A traded option is available with a delta of 0.4, a gamma of 1.5, and a vega of 0.8.

- (a) What position in the traded option and in US dollar would make the portfolio both gamma neutral and delta neutral?
 - (b) What position in the traded option and in US dollar would make the portfolio both vega neutral and delta neutral?
-