Mahindra University Hyderabad

École Centrale School of Engineering

Minor - I (2022 Batch), Feb 2024

Program: B. Tech. **Branch: CM** **Year: II** **Semester: Spring**

Subject: Functional Analysis (MA2212)

Date: 29/02/2024 **Start Time: 2:00 PM**

Time Duration: 1:30 Hours **Max. Marks: 15**

Instructions:

1) All questions are compulsory.

Q 1: **5 marks**

(a) Define the following terms with examples: 3 marks

(i) Complete metric space

(ii) Separable metric space

(iii) Isometry and homeomorphism

(b) State the following inequalities: 2 marks

(i) Hölder's

(ii) Minkowski's

Q 2: **6 marks**

(a) Let (X, d) be a metric space. Prove that the function $\bar{d} : X \times X \to \mathbb{R}$ defined by

$$\bar{d}(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$

satisfies the following triangle inequality

$$\bar{d}(x, y) \leq \bar{d}(x, z) + \bar{d}(z, y), \text{ for all } x,\ y,\ z \in X.$$

3 marks

(b) Let $C[0, 1]$ be a space of all continuous functions defined on $[0, 1]$. Consider d_∞, d_1 defined on $C[0, 1]$ by

$$d_\infty(f, g) = \max_{x \in [0,1]} |f(x) - g(x)|,$$

$$d_1(f, g) = \int_0^1 |f(x) - g(x)|\, dx$$

for all $f,\ g \in C[0, 1]$. Show that d_∞ and d_1 are not equivalent. 3 marks

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Q 3: **4 marks**

(a) Let $P[0,1]$ be the set of all polynomials defined on $[0,1]$. Define a metric d_∞ on $P[0,1]$ by

$$d_\infty(f,g) = \max_{t\in[0,1]} |f(t) - g(t)|.$$

Show that the following sequence

$$f_n(t) = 1 + \frac{t}{2} + \frac{t^2}{2^2} + \cdots + \frac{t^n}{2^n} \text{ for all } t \in [0,1]$$

is Cauchy in $P[0,1]$. 2 marks

(b) Let $M = \{y = (y_1, y_2, \ldots, y_n, 0, 0, \ldots) | y_n \in \mathbb{Q},\ n \in \mathbb{N}\}$. Show that M is a countable subset of l^p for $1 \leq p < \infty$. 2 marks