Mahindra University Hyderabad

École Centrale School of Engineering

Minor-II

Program: B. Tech. Branch: CM Year: IV Semester: I

Subject: Dynamical Systems (MA4125)

Date: 23/10/2024 Start Time: 10:00 AM

Time Duration: 90 Minutes Max. Marks: 15

Instructions:

1) All questions are compulsory.

2) Everything you write (including any notes and rough work) must be in the answer booklet.

3) Give proper justification for your answers. Marks will not be awarded for guess work.

Q 1: 5 Marks

Find the value(s) of $k \in \mathbb{R}$ such that zero equilibrium will be a stable focus and saddle for the damped linear pendulum $\ddot{x} + \dot{x} + kx = 0$, respectively. Also draw the phase portrait for stable focus and saddle for a particular value of k.

Q 2: 4 Marks

Classify the equilibrium points (as sinks, sources or saddles) of the nonlinear system

$$\dot{X} = F(X),$$

where $X = [x_1 \ x_2]^T \in \mathbb{R}^2$ and $F(X) = \begin{bmatrix} x_1 - x_1 x_2 \\ x_2 - x_1^2 \end{bmatrix}$.

Q 3: 6 Marks

Determine the flow $\phi_t : \mathbb{R}^3 \to \mathbb{R}^3$ for the nonlinear system $\dot{X} = F(X)$, where $X = [x_1 \ x_2 \ x_3]^T \in \mathbb{R}^3$ and $F(X) = \begin{bmatrix} -x_1 \\ x_2 \\ x_3 + x_1^2 + x_1 x_2 \end{bmatrix}$ with $X(0) = [c_1 \ c_2 \ c_3]^T$. Find the stable and unstable manifolds S and U of the system, respectively and show that S is invariant under the flow ϕ_t. Furthermore, find the homeomorphism $H(X)$ that shows the topological equivalence between the nonlinear system and the corresponding linear system $\dot{Y} = AY$, where A is the Jacobian matrix evaluated at the equilibrium point and $Y = [y_1 \ y_2 \ y_3]^T \in \mathbb{R}^3$.
