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Mahindra University Hyderabad

École Centrale School of Engineering

2nd year B.Tech.(2022 batch) Fall Semester Regular Examinations, December-2023

Program: B. Tech. Branch: Computation & Mathematics Year: II Semester: I

Subject: Algebra (MA 2106)

Date: 23/12/2023 **Start Time: 10.00 AM**

Time Duration: 3 Hours **Max. Marks: 100**

Instructions:

1. All questions are compulsory. The credit for each question is mentioned at the end of the question.
2. Justify your answer wherever required.
3. All the notations are standard as used in class lectures.

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1. Find the following. [3 × 13]

(i) Inverse of $\bar{6}$ in $U(7)$.

(ii) Representation of (1326845)(389415) as a product of disjoint cycles.

(iii) The order of the field $\frac{\mathbb{Z}_3[x]}{\langle x^3+2x+1\rangle}$.

(iv) The order of the element (4231)(6571) in S_7.

(v) The number of cosets of $\{(1),(12)\}$ in S_6.

(vi) The number of elements of order 15 in $\mathbb{Z}_5 \times \mathbb{Z}_3$.

(vii) The number of abelian groups of order 225.

(viii) The number of elements in the ring $\frac{\mathbb{Z}[i]}{\langle 7+i\rangle}$.

(ix) The remainder upon dividing x^5+2x^3+3x+1 by x^2+x+1 in $\mathbb{Z}_5[x]$.

(x) The order of the element [5] in $U(8)$.

(xi) The number of units in the ring $\mathbb{Z}_{17}$.

(xii) The order of the factor group $\frac{\mathbb{Z}_4\times\mathbb{Z}_{12}}{\langle(1,1)\rangle}$.

(xiii) The list of zeros of x^4+5x^2+2x+2 in $\mathbb{Z}_7$.

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2. Let

$$R = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}.$$

(i) Show that R is a subring of $M_2(\mathbb{Z})$. [06]

(ii) Let $\phi : R \to \mathbb{Z}$ be the map defined by

$$\phi\left(\begin{pmatrix} a & b \\ b & a \end{pmatrix} \right) = a - b.$$

Prove or disprove that ϕ is a ring homomorphism. [09]

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3. (i) Show that $x^5 + 5x^2 + 1$ is irreducible over $\mathbb{Q}$. [11]

(ii) Show that $x^5 + 9x^2 + 12x + 6$ is irreducible over $\mathbb{Q}$. [05]

(iii) Construct a field of order 121. [05]

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4. (i) Determine the number of elements of order 7 in S_7. [6]

(ii) Let f be a group homomorphism from $\mathbb{Z}_{36}$ to $\mathbb{Z}_9$ such that $f(\bar{1}) = \bar{1}$. Find $\ker f$. [4]

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5. (i) Let $I = \{f(x) \in \mathbb{Z}[x] \mid f(0) = 0\}$. Show that I is a prime ideal of $\mathbb{Z}[x]$ but not a maximal ideal of $\mathbb{Z}[x]$. [10]

(ii) Let G be a group such that $xy = zx$ implies that $y = z$ for every $x, y, z \in G$. Show that G is abelian. [5]
