Mahindra University Hyderabad
École Centrale School of Engineering
Spring Semester Regular Examinations (2022 Batch), June-2024
Program: B. Tech. **Branch: CM** **Year: 2^{nd}** **Semester: II**
Subject: MA-2213 Stochastic Processes

Date: 04/06/2024 | **Start Time: 10:00 AM**
Time Duration: 3 Hours | **Max. Marks: 100**

Instruction:

1. All answers that belong to a particular question should be answered at one place.
2. Use of scientific but non-programmable calculators permitted.

Q 1: Each MCQ carries equal marks. Marks: $10 \times 2 = 20$

1. Auto-correlation function of a zero-mean Weiner process is:
   (a) $\sigma^2|t_1 - t_2|$
   (b) $\sigma^2 \min(t_1, t_2)$
   (c) $\sigma^2 \max(t_1, t_2)$
   (d) $\sigma^2|t_1 + t_2|$

2. Consider the following statements:
   I. Weiner process is WSS random process.
   II. White noise process is mean square derivative of a Weiner process.
   (a) Both I and II are true.
   (b) I is true and II is false.
   (c) I is false and II is true.
   (d) Both I and II are false.

3. Let $Z(t) = X(t) + N(t)$, where $X(t)$ and $N(t)$ are independent zero-mean WSS random process. Then power spectral density $S_Z(\omega)$ and $S_{Z,X}(\omega)$ are:
   (a) $S_X(\omega) + S_N(\omega)$ and $S_X(\omega) + S_{Z,N}(\omega)$
   (b) $S_X(\omega) - S_N(\omega)$ and $S_X(\omega) - S_{Z,N}(\omega)$
   (c) $S_X(\omega) + S_N(\omega)$ and $S_X(\omega)$
   (d) $S_X(\omega) + S_N(\omega)$ and $S_{Z,N}(\omega)$

4. Let $Y(t) = X(t - d)$, where d is a constant delay and $X(t)$ is WSS. Then correlation function $R_Y(\tau)$ is
   (a) $R_X(\tau - d)$
   (b) $R_X(\tau)$
   (c) $R_X(\tau) - d$
   (d) $R_X(\tau + d)$

5. Let $Y(t) = X(t)\cos(2\pi\omega_0 t + \Theta)$, where $\omega_0 > 0$ is constant, Θ is uniform random variable on $(0, 2\pi)$, $X(t)$ is WSS and independent of Θ for all t. Then $R_Y(\tau)$ is

(a) $(1/2)R_X(\tau)\cos(2\pi\omega_0\tau)$
(b) $(1/2)R_X(\tau)\sin(2\pi\omega_0\tau)$
(c) $(1/4)R_X(\tau)\cos(2\pi\omega_0\tau)$
(d) $(1/4)R_X(\tau)\sin(2\pi\omega_0\tau)$

6. Let $X(t)$ be WSS and mean square differentiable for all $t \geq 0$. Then auto-correlation function $R_{X'}(\tau)$ is

(a) $\frac{d^2}{d\tau^2}R_X(\tau)$
(b) $\frac{d}{d\tau}R_X(\tau)$
(c) $-\frac{d}{d\tau}R_X(\tau)$
(d) $-\frac{d^2}{d\tau^2}R_X(\tau)$

7. Let $X'(t)$ denote the mean square derivative of a random process $X(t)$. Then the cross-correlation function $R_{X',X}(t_1, t_2)$ is

(a) $\frac{\partial^2}{\partial t_1^2}R_X(t_1, t_2)$
(b) $\frac{\partial^2}{\partial t_2^2}R_X(t_1, t_2)$
(c) $\frac{\partial}{\partial t_1}R_X(t_1, t_2)$
(d) $\frac{\partial}{\partial t_2}R_X(t_1, t_2)$

8. Let X_n denote a Binomial counting process. Then consider the following statements:
I. X_n is WSS.
II. X_n satisfy Markov property.

(a) Both I and II are true.
(b) I is true and II is false.
(c) I is false and II is true.
(d) Both I and II are false.

9. Let $Y(t) = \int_0^t h(s)X(t-s)ds$ where $X(t)$ is WSS random process. Then consider the following:
I. $Y(t)$ is WSS.
II. $R_{X,Y}(\tau) = R_{X,Y}(-\tau)$.

(a) Both I and II are true.
(b) I is true and II is false.
(c) I is false and II is true.
(d) Both I and II are false.

10. Consider the autoregressive process $Y_n = 0.5Y_{n-1} + X_n$, where $Y_0 = 0$ and X_n is iid Gaussian random process $N(0, \sigma^2)$. Then

(a) Y_n is iid random process.
(b) Y_n is non-Markov process.
(c) Y_n is Markov process.
(d) None of these.

Q 2: **Marks: 20**

a) Let $Z(t) = X(t) - aX(t-s)$, where $X(t)$ is the Wiener process.
(i) Find the pdf of $Z(t)$.
(ii) Find $m_Z(t)$ and $C_Z(t_1, t_2)$.

Marks: 20

Q3:

a) Let $X(t)$ be a zero-mean random process with autocovariance $R_X(\tau) = \sigma^2 e^{-\alpha|\tau|}$.

(i) Write the eigenvalue integral equation for the Karhunen-Loeve expansion of $X(t)$ on the interval $[-T, T]$.

(ii) Differentiate the above integral equation to obtain the differential equation

$$\frac{d^2}{dt^2}\phi(t) = \frac{\alpha^2}{\lambda}\left(\lambda - 2\sigma^2/\alpha\right)\phi(t).$$

(iii) Show that the solutions to the above differential equation are of the form $\phi(t) = A\cos bt$ and $\phi(t) = B\sin bt$. Find an expression for b.

Marks: 20

Q4:

a) Let $X_\alpha = Z_\alpha + N_\alpha$, $\alpha \in \{n-p, n-p+1, \cdots, n\}$. Here Z_α has $R_Z(k) = \sigma_Z^2 (r_1)^{|k|}$ and N_α has $R_N(k) = \sigma_N^2 (r_2)^{|k|}$, where r_1 and r_2 are less than one in magnitude.

(i) Find the equation for the optimum filter for estimating Z_α.

(ii) Write the matrix equation for the optimum filter coefficients.

(iii) Find the mean square error for the optimum filter.

Marks: 20

Q5:

a) A machine consists of two parts that fail and are repaired independently. A working part fails during any given day with probability α. A part that is not working is repaired by the next day with probability β. Let X_n be the number of working parts in day n.

(i) Show that X_n is a three-state Markov chain and give its one-step transition probability matrix.

(ii) Show that the steady state pmf π is binomial with parameter $p = \beta/(\alpha+\beta)$.
