

Mahindra University École Centrale School of Engineering  
Hyderabad

Midterm Examination

Program: B. Tech. Branch: Common to All Branches

Year: I Semester: I

Subject: Mathematics I (MA 1101)

Date: 10/11/2022

Time Duration: 90 Minutes

Time: 10.00 AM to 11.30 AM

Max. Marks: 30

**Instructions:**

1. There are 5 questions, all of which are compulsory.
2. The order of answers should be same as the order of questions.
3. Justify your answer wherever required. Guesswork will not be considered in evaluation.

1. Let  $f$  be a real-valued continuous function on the closed interval  $[a, b]$  such that  $f(a) \leq a^2 + 1$  and that  $f(b) \geq b^2 + 3$ . Prove that there is at least one point  $c \in (a, b)$  for which  $f(c) = c^2 + 2$ .

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2. Obtain the fourth degree Taylor polynomial approximation to  $f(x) = e^{3x}$  about  $x = 1$ . Use this to approximate  $f(0)$ .

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3. Sketch the graph of

$$f(\theta) = |\cos(2\theta)|, \quad 0 \leq \theta \leq 2\pi$$

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in polar coordinates and compute the area of the radial set of  $f$  over the interval specified.

4. Using the Root test, determine whether the series  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$  converges or diverges.

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5. Find a continuous function  $f$  on positive real axis such that

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$$f(x) = 1 + \frac{1}{x} \int_1^x f(t) dt$$

for all  $x > 0$ .