



Mahindra University Hyderabad
École Centrale School of Engineering,
Minor-2 Examination

Program: B.Tech Branch: Computation & Mathematics Year: III
Semester: II

Subject: Mathematical Foundations for Machine Learning (MA3219)

Date: 16/04/2024
Time Duration: 1.5 Hours

Start Time: 10.00 AM
Max. Marks: 20

Instructions:

1. All questions are compulsory.

Q 1:

5 marks

Compute the matrices S_b and S_w in Linear Discriminant Analysis(LDA) method for the following two dimensional dataset.

Samples for class ω_1 : $X_1 = (x_1, x_2) = (4, 2), (2, 4), (2, 3), (3, 6), (4, 4)$

Samples for class ω_2 : $X_2 = (x_1, x_2) = (9, 10), (6, 8), (9, 5), (8, 7), (10, 8)$

Q 2:

5 marks

- a) Consider a regression problem in which we want to predict variable y from a single feature x . We have $n \geq 3$ data points, $(y_i, x_i)_{i=1}^n$. Consider two possible models to be estimated by ordinary linear regression,

$$y_i = w_0 + w_1 x_i + \epsilon_i \quad (1)$$

$$y_i = w_0 + w_1 x_i + w_2 x_i^2 + \epsilon_i \quad (2)$$

where, the error terms ϵ_i are independent and identically distributed from a normal distribution with zero mean. Derive a formula for estimating w_2 in Model 2. Assume w_0 and w_1 are known.

[3]

- b) Will one model fit the training data better than the other, will they fit equally well, or is it impossible to say? Explain your reasoning.

[2]

Q 3 :

5 marks

Find the loss function in Logistic regression to fit the following two-dimensional binary classification data:

$$X1 = (x1, x2) = (4, 1), (2, 4), (2, 3), (3, 6), (4, 4),$$

$$X2 = (x1, x2) = (9, 10), (6, 8), (9, 5), (8, 7), (10, 8).$$

Q 4:

5 marks

Prove that the following functions are convex on the given domain.

(i) $f(x) = -\ln\left(\frac{e^x}{1+e^x}\right)$, $x \in \mathbb{R}$. [2]

(ii) $f(x) = x^p$, $x \in [0, \infty)$ and $p \geq 1$ is a constant. [1]

(iii) $f(x) = e^{bx}$, $x \in \mathbb{R}$, where b is a constant. [1]

(iv) $f(x) = -\ln(1 - x)$, $x \in (-\infty, 1)$. [1]