MAHINDRA UNIVERSITY, HYDERABAD
Regular Examinations, December-2024, (2022 Batch)
Program: B. Tech (Common to CM and NT)
Year: III Semester: I
Subject: Computational Methods for PDE (MA 3115)

Date: 16/12/2024 **Time: 10:00 AM-01:00 PM**
Time Duration: 3 hours **Max. Marks: 100**

Instructions:

1. Answer all the questions.
2. Marks will not be awarded for guess work.
3. All the answers that belong to a particular question should be answered in one place in your answer booklet.
4. Scientific calculators are permitted.

Q 1:

Marks: 20

Find the half range cosine series representation of $\phi(x) = x,\ 0 < x < \pi$.

Q 2:

Marks: 20

Using central difference approximations obtain the system of equations of the type $AU = b$ to solve the following Laplace equation, with $\Delta x = \Delta y = \frac{1}{3}$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 1, 0 < y < 1,$$

and $u(x, 0) = x,\ u(x, 1) = 0$ on $0 \le x \le 1$ and $u(0, y) = 0,\ u(1, y) = 0$ on $0 \le y \le 1$.

Q 3:

Marks: 20

Perform two iterations using FTCS scheme for solving the following 1D heat equation, with $r = 0.4, \Delta x = \frac{1}{4}$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, t > 0,$$

$$u(0,t) = u(1,t) = 0, \ t > 0 \text{ and } u(x,0) = \sin(\pi x), \ 0 < x < 1.$$

Q 4:

Marks: 20

The following PDE

$$\alpha \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = f(x,t), \ \alpha \in \mathbb{R}$$

is discretized using following finite difference scheme,

$$\frac{\alpha(U_{i,j+1} - 0.5(U_{i+1,j} + U_{i-1,j}))}{\Delta t} + \frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} = f_{i,j},$$

Then investigate the consistency of this scheme for (a) $\Delta t = r(\Delta x)$, and (b) $\Delta t = r(\Delta x)^2$, $r > 0$.

Q 5:

Marks: 20

Perform two iterations using CTCS scheme for solving the following 1D wave equation, with $r = 0.4, \Delta x = \frac{1}{4}$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, t > 0,$$

$u(0,t) = u(1,t) = 0, \ t > 0; \ u(x,0) = x(1-x)/2, \ 0 \le x \le 1$ and $\frac{\partial u}{\partial t} = 0, 0 \le x \le 1$, when $t = 0$.
