

Program: B.Tech

Branch: CSE/AI/ECM/CM/CB

Subject: Design and Analysis of Algorithms (CS/AI 2102)

Start Time: 02:00 PM

Year: Second/Third

Semester: First

Date: 12-09-2023

Please read the following instructions before answering questions.

1. Answer all seven questions. The exam is worth a total of 20 marks.
 2. Provide concise and focused responses – include only essential details.
 3. Answer all parts of a question in one place, one after the other. If not, you may risk losing marks.
 4. If a question seems unclear, clearly state your assumptions before answering.
-

1. Check if $f = O(g)$, or $f = \Omega(g)$, or both (in which case, $f = \Theta(g)$) for the following two problems.

(a) $f(n) = \log_2(n)$, $g(n) = \log_3(n)$. (1 Mark)

(b) $f(n) = n!$, $g(n) = 2^n$. (1 Mark)

2. Answer the following questions on master theorem.

(a) State the Master Theorem. (2 Marks)

- (b) For each of the following recurrence relations give a bound in big- O notation.

i. $T(n) = 2T(n/3) + n$. (1 Mark)

ii. $T(n) = 17T(n/4) + n^2$. (1 Mark)

iii. $T(n) = 8T(n/2) + n^3$. (1 Mark)

3. Given a sorted array of distinct integers $A[1, \dots, n]$, you want to find out whether there is an index i for which $A[i] = i$. Give a divide-and-conquer algorithm that runs in time $O(\log n)$. (3 Marks)

4. You are given two sorted lists of size m and n . Give an $O(\log m + \log n)$ time algorithm for computing the k -th smallest element in the union of the two lists. (3 Marks)

5. Explain the basic concept of a greedy algorithm. How does it work, and why does it make decisions that seem best at the moment? Provide an example where a greedy algorithm is effective, and discuss how making the locally optimal choice at each step can lead to a globally optimal solution in that problem. (2 Marks).
6. A thief enters a house intending to rob it and can carry a maximum weight of 60kg in his bag. Inside the house, there are five items with the following weights and values: item-1 weighs 5kg and is worth ₹30k, item-2 weighs 10kg and is worth ₹40k, item-3 weighs 15kg and is worth ₹45k, item-4 weighs 22kg and is worth ₹77k, and item-5 weighs 25kg and is worth ₹90k. The thief can take fractions of any item if needed. What items should the thief take to maximize the total value he can carry? Explain the strategy the thief should use to determine which items, or fractions of items, to take in order to achieve the highest possible value within the weight limit. (2 Marks).
7. Under a Huffman encoding of n symbols with frequencies f_1, f_2, \dots, f_n , what is the longest codeword could possibly be? Give an example set of frequencies that would produce this case. (3 Marks).