Mahindra University Hyderabad

École Centrale School of Engineering

End-Semester Exam (2023 - Batch)

Program: B. Tech. Branch: All

Year: II Semester: I

Subject: Mathematics - III (MA2103)

Date: 12/12/2024 Start Time: 10:00 AM

Time Duration: 3 Hours Max. Marks: 100

Instructions:

1) All questions are compulsory.
2) Each question carries 20 marks.
3) **Please start each answer on a separate page and make sure to clearly number the responses.**
4) It is essential to provide a valid explanation of each step. The correct outcomes without a valid explanation will not be considered for evaluation.

Q 1: 20 marks

(a) Two boxes containing marbles are placed on a table. The boxes are labeled B_1 and B_2. Box B_1 contains 7 green marbles and 4 white marbles, whereas the box B_2 contains 3 green marbles and 10 yellow marbles. The boxes are arranged so that the probability of selecting the box B_1 is $\frac{1}{3}$ and the probability of choosing the box B_2 is $\frac{2}{3}$. Mr. Kumar is blindfolded and asked to choose a marble. He will win a color TV if he selects a green marble. (i) What is the probability that Kumar will win the TV (i.e., he will choose a green marble)? (ii) If Kumar wins the color TV, what is the probability that the green marble was selected from the first box? (14 mark)

(b) Let A and B be events in a sample space S such that $P(A) = \frac{1}{2} = P(B)$ and $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$. Find $P(A \cap B)$ first and then $P(A \cup \overline{B})$. (6 mark)

Q 2: 20 marks

The joint density function of X and Y is given by

$$f_{XY}(x,y) = \begin{cases} 2e^{-x}e^{-2y} & \text{if } 0 < x < \infty,\ 0 < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Compute:

(a) $P(X > 1, Y < 1)$, (7 marks)

(b) $P(X < Y)$, (7 marks)

(c) $P(X < a)$, where $a > 0$ is some constant. (6 marks)

Q 3: 20 marks

(a) State Central Limit Theorem. (4 mark)

(b) Suppose that, on the average, $\frac{1}{3}$ of the graduating seniors at a certain college have two parents attend the graduation ceremony, another third of these seniors have one parent attend the ceremony, and the remaining third of these seniors have no parents attend. If there are 600 graduating seniors in a particular class, use the Central Limit Theorem to find the probability that not more than 650 parents will attend the graduation ceremony. (**Use:** $\Phi(2.5) = 0.9938$)

(16 mark)

Q 4: 20 marks

(a) Let X and Y be jointly Gaussian random variables with $E[Y] = 0$, $\sigma_X = 1$, $\sigma_Y = 2$, and $E[X|Y = y] = 1 + \frac{y}{4}$. Find the joint PDF of X and Y. (10 marks)

(b) Let $W = X + Y$ and $Z = X - Y$.

(i) Find an expression for the joint PDF of W and Z.

(ii) Find $f_{WZ}(z, w)$ if X and Y are independent exponential random variables with parameter $\lambda = 1$.

(10 mark)

Q 5: 20 marks

(a) Let $X_1, X_2, \ldots, X_n$ be a random sample from an Exponential(θ) distribution with probability density function (PDF) given by:

$$f_{X_i}(x_i; \theta) = \theta e^{-\theta x_i}, \quad x_i \geq 0,\ \theta > 0.$$

(i) Derive the likelihood function $L(\theta)$.

(ii) Find the maximum likelihood estimator (MLE) for θ.

(ii) Suppose the observed sample is $(x_1, x_2, x_3, x_4) = (2.35, 1.55, 3.25, 2.65)$. Compute the maximum likelihood estimate of θ based on this data.

(10 mark)

(b) Let X be a Gaussian random variable with unknown mean and unknown variance. A set of 20 independent measurements of X yields a sample mean of 57.3 and a sample variance of 23.2.

(i) Find the 99% confidence intervals for the population mean.

(ii) Find the 99% confidence intervals for the population variance.

[**Use:** $t_{0.005,19} = 2.861$, $\chi^2_{0.005,19} = 38.58$, $\chi^2_{0.995,19} = 6.843$]

(10 mark)
