Mahindra University Hyderabad
École Centrale School of Engineering
End-semester Regular Examination
(2021-Batch)

Program: B. Tech **Branch: Common to all Branches**

Year: I **Semester: Spring**

Subject: Mathematics - II (MA 1202)

Date: 06/06/2022 **Start Time: 08.30 AM**

Time Duration: 3 Hours **Max. Marks: 100**

Instructions

1. Attempt all the questions.
2. In Q1, your answer should be either True or False. Explanations are not required in Q1.
3. For Q2, Q3, Q4 and Q5 justifications are compulsory.

Q. 1 Marks: $10 \times 2 = 20$

State whether the following statements are True or False.

a) The Laplace transform of te^{-3t} is $\dfrac{1}{(s-3)^2}$.

b) If x is a nontrivial solution of $Ax = 0$, then every entry in x is zero.

c) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation. Let $T(1,1) = (3,1)$ and $T(1,2) = (5,0)$. Then $T(0,-2)$ is $(-4,2)$.

d) The inverse Laplace transform of $\dfrac{1}{(s+1)(s+2)}$ is $e^{-t} + e^{-2t}$.

e) There exists a linearly independent set in $\mathbb{R}^5$ which contains 5 elements but does not span $\mathbb{R}^5$.

f) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x,y) = (0, x-y)$. Then the rank of T is 2.

g) If A is a 4×3 matrix of rank 2 then the dimension of the null space of A is 2.

h) The residue of the function $f(z) = \dfrac{\sin 2z}{z-1}$ at $z = 1$ is 0.

i) There exists a number $b \in \mathbb{R}$, $b > 0$, such that $\langle x, y\rangle = bxy$ is not an inner product on $\mathbb{R}$.

j) The function $f(z) = e^{2/z^2}$ has a pole of order two at $z = 0$.

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Q. 2 **Marks: 20**

Using Laplace transform method, solve the following initial value problem

$$y'' + 2y' + y = 1, \quad y(0) = 2, \quad y'(0) = -2.$$

Q. 3 **Marks: 20**

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation such that

$$T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}, \ T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}.$$

Then find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x} \in \mathbf{R}^2$, and find the rank and nullity of T.

Q. 4 **Marks: 20**

Let W be a subspace of $\mathbb{R}^4$ spanned by the set $S = \{(1,1,2,-1),(3,4,3,-1),(1,5,3,-2)\}$.

(i) Is S a basis for W ? Justify your answer. [8]

(ii) Using Gram-Schmidt process find an orthogonal basis for W. [12]

Q. 5 **Marks: 20**

Let A be a 2×2 matrix such that $\begin{bmatrix}1\\1\end{bmatrix}$ is an eigenvector for A with eigenvalue 2 and $\begin{bmatrix}2\\3\end{bmatrix}$ is an eigenvector for A with eigenvalue 1. If $v = \begin{bmatrix}3\\4\end{bmatrix}$, compute A^3v. Also, Find the matrix A.