Mahindra University

Global Thinkers. Engaged Leaders

Mahindra University Hyderabad
École Centrale School of Engineering
Minor I Examinations,
September-2023

Program: B. Tech. Branch: CM Year: II Semester: I
Subject: Algebra (MA 2106)

Date: 19/09/2023 **Start Time: 10.00 AM**
Time Duration: 90 Minutes **Max. Marks: 30**

Instructions:

1. There are 6 questions, all of which are compulsory.
2. Justify your answer wherever required.

1. State "True" or "False". Negative marking (-1) will apply in case of each incorrect answer. **[5] M**

   (i) If A and B are subgroups of a group G, then $A \cup B$ is also a subgroup of G.

   (ii) In every group G there exists a non-identity element of finite order.

   (iii) $\{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}\}$ is a subgroup of $\mathbb{Z}_{14}$.

   (iv) If A and B are subgroups of a group G, then $A \cap B$ is also a subgroup of G.

   (v) If H is a finite subset of an infinite group G such that it is closed under the operation of G, then H is a subgroup of G.

---

2. Determine if the following sets G, with the operation indicated, form a group. If not, point out which of the group axioms fail? **5 M**

   (i) $G = \mathbb{Z}, \quad a * b = a + b + ab.$ **[2]**

   (ii) $G = \mathbb{Q} - \{-1\}, \quad a * b = a + b + ab.$ **[3]**

---

3. Let G be a group. The centralizer of an element $x \in G$, denoted by $C_G(x)$ is defined as follows: **[4] M**

$$C_G(x) = \{y \in G \mid xy = yx.\}$$

   Show that $C_G(x)$ is a subgroup of G.

---

4. Let G be group and $a, b \in G$ such that $|ab| = |ba| = 2$. Show that $ab = ba$. [5] M

5. Fill in the blanks: [6] M

(i) Order of $\overline{6}$ in $\mathbb{Z}_{11}$ is —. [1]

(ii) Order of $\overline{5}$ in $U(18)$ is —. [2]

(iii) Order of the group $U(20)$ is —. [1]

(iv) Inverse of $\overline{7}$ in $U(11)$ is —. [2]

6. Does the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ belong to the center of the group $GL(2, R)$? Justify your answer. [5] M

****