Mahindra University Hyderabad
École Centrale School of Engineering
Minor – I

Program: B.Tech.   Branch: Computation & Mathematics   Year: Second   Semester: Fall
Subject: Graph Algorithms (MA 2105)

Date: 13/09/2024   Start Time: 10:00 AM
Time Duration: 1.5 Hours   Max. Marks: 20

Instructions:

1) **Start each answer on a new page and number your answers clearly. Answer all parts of the same question together and in sequence.**
2) **Explanation of every step is highly desirable.**

Q 01: Select the correct choice for the following questions with proper explanation. The right choice without valid justification will not be considered. $[01 \times 05]$

A) If G is the forest with 54 vertices and 17 connected components. Then, what is the total number of edges G can have?

     a) 37   b) 71   c) 17   d) 54   e) None of these

B) The number of edges in a regular graph of degree 46 and 8 vertices are:

     a) 347   b) 186   c) 184   d) 230   e) None of these

C) Radius of a graph, denoted by rad(G) is defined as:

     a) $\max\{e(v): v \text{ belongs to } V\}$ b) $\min\{e(v): v \text{ belongs to } V\}$

     c) $\max\{d(u,v): u,v \in V \text{ and } u \neq v\}$ d) $\min\{d(u,v): u,v \in V \text{ and } u \neq v\}$

D) If a simple graph G contains n vertices and m edges, then the number of edges in the complement graph of G, i.e., G' is:

     a) $\dfrac{n^2+n-2m}{2}$ b) $\dfrac{n^2+2n-2m}{2}$ c) $\dfrac{n^2-n-2m}{2}$ d) $\dfrac{n^2-2n-2m}{2}$ e) None of these

E) What is the maximum number of possible non-zero values in an adjacency matrix of a simple graph with n vertices?

     a) $\dfrac{n(n-1)}{2}$ b) $\dfrac{n(n+1)}{2}$ c) $n(n-1)$ d) $n(n+1)$ e) None of these

Q 02: Each of the following questions consists of two marks. Each step must be discussed in detail. $[02 \times 05]$

A) A tree has five vertices of degree 2, three of degree 3, and four of degree 4. How many vertices of degree 1 does it have?

B) Does a simple graph with seven vertices having degrees (1, 3, 3, 4, 5, 6, 6) exist? Explain the answer in detail.

C) There are 25 telephones in the office of Mahindra University. Is it possible to connect them with wires to connect each phone with exactly seven others?

D) What is a hypercube (or n-cube) graph? How many vertices and edges are there in an n-cube graph? Explain your answer in detail.

E) Consider the following graphs, G and H:
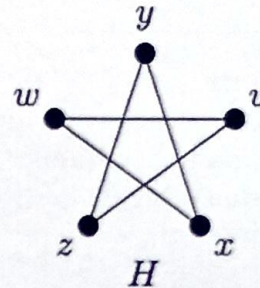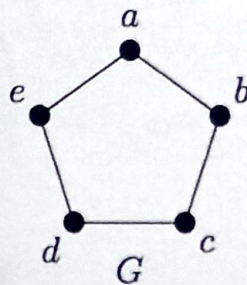
Are the graphs G and H isomorphic? If they are, provide an isomorphism $f: G \to H$ that maps the vertices of G to the vertices of H.

Q 03: Prove the following theorem: [05]

Let u and v be distinct vertices of a tree T. Then there is precisely one path from u to v.