

Mahindra University Hyderabad**École Centrale School of Engineering****Minor – II****Program: B.Tech.****Branch: Computation & Mathematics****Year: Second****Semester: Fall****Subject: Graph Algorithms (MA 2105)****Date: 25/10/2024****Time Duration: 1.5 Hours****Start Time: 10:00 AM****Max. Marks: 20****Instructions:**

- 1) Start each answer on a new page and number your answers clearly. Answer all parts of the same question together and in sequence.
- 2) Explanation of every step is highly desirable.

Q 01: Select the correct choice for the following questions with proper explanation. The right choice without valid justification will not be considered. [01 × 04]

A) Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$, which one of the following is TRUE?

- i) In any planar representation, the number of faces is at least $\frac{n}{2} + 2$
- ii) In any planar representation, the number of faces is less than $\frac{n}{2} + 2$
- iii) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta+1}$
- iv) None of the above

B) Select the correct alternatives (more than one option may be correct):

A non-planar graph with the minimum number of vertices has:

- i) 9 edges and 6 vertices
- ii) 6 edges and 4 vertices
- iii) 10 edges and 5 vertices
- iv) 9 edges and 5 vertices

C) Consider a complete graph G with six vertices. The graph G has _____ spanning trees.

- i) 1296
- ii) 4096
- iii) 30
- iv) None of these

D) For which of the following graphs is an Euler circuit never possible if the number of vertices is greater than 2?

- i) K_n
- ii) C_n
- iii) W_n
- iv) Q_n

Q 02: Each of the following questions consists of two marks. Each step must be discussed in detail. [02 × 03]

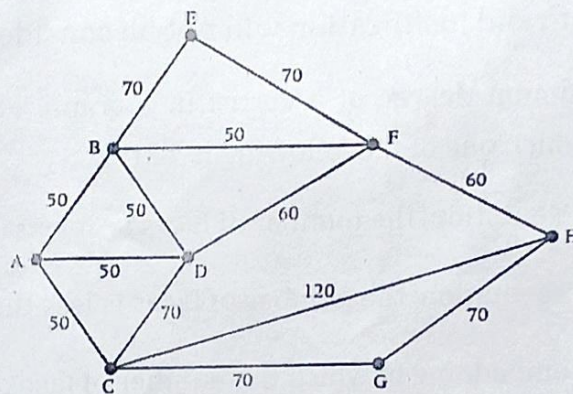
A) What is the Eulerian graph? Is it possible to have an Euler tour for a regular graph G with six vertices and nine edges? If not, what should be the conditions on the graph (not necessarily regular) so that G will be Eulerian? Also, give the degree sequence of G .

B) What is the Hamiltonian cycle? As we know, a complete graph with n vertices, i.e., K_n always has a Hamiltonian cycle. What is the minimum number of edges that can be removed from K_n to make it non-Hamiltonian?

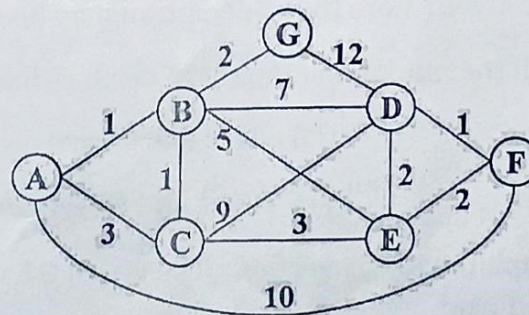
C) The graph G has six vertices with degrees 2, 2, 3, 4, 4, and 5. How many edges does G have? Could G be planar? If so, how many bounded faces would it have? If not, explain.

Q 03: Each of the following questions consists of five marks. Explain every step in detail, and discuss any relevant concepts as necessary. [05 × 02]

A) What is the Chinese postman problem? Solve the Chinese postman problem for the following graph consisting of 13 streets, and the postman has to start at A. What will be the optimal route and length of the Chinese postman problem?



B) Explain the Dijkstra's algorithm. Consider the following undirected, weighted graph:



Step through Dijkstra's algorithm to calculate the single-source shortest paths from the source vertex A to every other vertex. Finally, indicate the lowest-cost path from node A to node F.