Mahindra University Hyderabad
École Centrale School of Engineering
Minor I
(2023-Batch)

Program: B. Tech Branch: Computation & Mathematics
Year: II Semester: Fall
Subject: Real Analysis (MA 2104)

Date: 12/09/2024 Start Time: 10.00 AM
Time Duration: 1 hr 30 mins Max. Marks: 20

Instructions

1. All questions are compulsory.
2. In all questions, justifications are required.

Q.1

a) Define a metric space. **2M**
b) Show that for $x, y \in \mathbb{R}$, $d(x, y) = |x - y|^3$ does not define a metric on R. **3M**

Q.2

a) Prove that for any finite collection $G_1, G_2, \ldots, G_n$ of open sets, $\bigcap_{i=1}^{n} G_i$ is open. **3M**

b) Give an example to show that intersection of an infinite collection of open sets need not be always open. **2M**

Q.3

a) Prove that set of real numbers is uncountable. **4M**
b) Give an example to show that not every uniformly continuous function defined on a set A is a Lipschitz function. **1M**

Q.4

a) If $(x_n) = \left(\sin \frac{n\pi}{2}\right)$, find $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$. Does the sequence (x_n) converge? **3M**
b) Determine whether the function $f(x) = 2x^2 - 3x + 5$ is uniformly continuous on $[-2, 2]$. **2M**