Mahindra University Hyderabad
École Centrale School of Engineering,
Minor-1 Examination

Program: B.Tech **Branch:** Computation & Mathematics **Year:** III
Semester: II

Subject: Mathematical Foundations for Machine Learning (MA3219)

Date: 28/02/2024 **Start Time:** 10.00 AM
Time Duration: 1.5 Hours **Max. Marks:** 20

Instructions:

1. All questions are compulsory.

Q 1: 5 marks

State Jensen's Inequality and prove that for any $x_1, x_2, \cdots, x_n \in \mathbb{R}$, .

$$\left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right)^2 \leq \frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}.$$

Q 2: 5 marks

For the following random sample, find the likelihood function:
$X_i \sim \text{Binomial}(3, \theta)$, and we have observed $(x_1, x_2, x_3, x_4) = (1, 3, 2, 2)$.

Hint: $P_{X_i}(x, \theta) = \binom{3}{x} \theta^x (1-\theta)^{3-x}$

Q 3 : 5 marks

a) Define Vapnik-Chervonenkis(VC) dimension. [3]

b) Let $X = \mathbb{R}$ and let $C = \{c_{a,b} | a, b \in \mathbb{R}, a < b\}$ be the concept class of intervals, where $c_{a,b} : \mathbb{R} \to \{0, 1\}$ is defined as $c_{a,b}(x) = 1$ if $x \in [a, b]$ and 0 otherwise. Find the VC dimension of C. [2]

Q 4: 5 marks

In a two-class problem with a single feature x the pdfs are Gaussians with variance $\sigma^2 = 1/2$ for both classes and mean values 0 and 1, respectively, that is,

$$p(x|\omega_1) = \frac{1}{\sqrt{\pi}} \exp(-x^2)$$

$$p(x|\omega_2) = \frac{1}{\sqrt{\pi}} \exp(-(x-1)^2)$$

If $P(\omega_1) = P(\omega_2) = 1/2$, compute the threshold value x_0 for minimum error probability.