Mahindra University Hyderabad
École Centrale School of Engineering
Minor I Examinations

Program: B. Tech. Branch: CM Year: II Semester: I
Subject: Algebra (MA 2106)

Date: 14/09/2024 **Start Time: 10.00 AM**
Time Duration: 90 Minutes **Max. Marks: 30**

Instructions:

1. There are 6 questions, all of which are compulsory.
2. Justify your answer wherever required.

1. State "True" or "False". No justification is required. Negative marking (-1) will apply in case of each incorrect answer. **[5] M**

   (i) $\{\bar{0}, \bar{5}, \overline{10}, \overline{15}\}$ is a subgroup of $\mathbb{Z}_{20}$.
   (ii) If A and B are subgroups of a group G, then $A \cup B$ is also a subgroup of G.
   (iii) Let G be a group and H be a subset of G. Then H is a subgroup of G if and only if H is closed under the operation of G.
   (iv) If H is a subgroup of $\mathbb{Z}$ then $H = m\mathbb{Z}$ for some $m \geq 1$.
   (v) Let G be a group of order 24. There exists a subgroup H, of G, of order 7.

2. If A and B are subgroups of a group G. Show that $A \cap B$ is also a subgroup of G. **[5] M**

3. Let H be a subgroup of $\mathbb{Z}$ such that H contains $7\mathbb{Z}$ and $11\mathbb{Z}$. Show that $H = \mathbb{Z}$. **[4] M**

4. Consider the group Z_{15} and its subgroups $H_1 = \{\bar{0}, \bar{5}, \bar{10}\}$ and $H_2 = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \bar{12}.\}$ Find all the left cosets of H_1 and H_2 in G. **[5] M**

5. Fill in the blanks: **[6] M**

   (i) Order of $\bar{4}$ in $\mathbb{Z}_9$ is —. **[1]**
   (ii) Order of $\bar{3}$ in $U(13)$ is —. **[2]**
   (iii) Order of the group $U(24)$ is —. **[1]**
   (iv) Inverse of $\bar{7}$ in $U(15)$ is —. **[2]**

6. Does the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ belong to the center of the group $GL(2, R)$? Justify your answer. **[5] M**