Mahindra University
Global Thinkers Engaged Leaders

Mahindra University Hyderabad

École Centrale School of Engineering

Minor - I Exam

Program: B. Tech. **Branch: CM** **Year: II** **Semester: II**

Subject: Functional Analysis (MA2212)

Date: 27/02/2025 **Start Time: 2:00 PM**

Time Duration: 1.5 Hours **Max. Marks: 25**

Instructions:

1) Answer all the questions.
2) All questions are self-explanatory; no clarification will be provided during the exam.

Course outcomes (COs)

CO 1: Explain fundamental concepts of metric and normed spaces, including Banach spaces, with relevant examples.

CO 2: Classify and examine bounded linear operators and functionals, and characterize the dual of common Banach spaces.

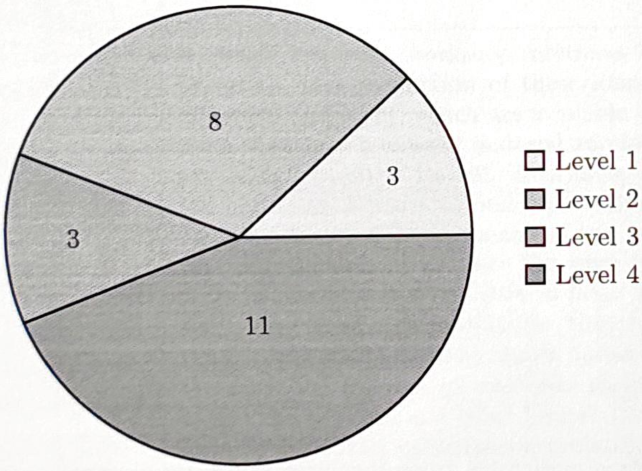
CO 3: Utilize key results such as the Schwarz inequality, projection theorem, and Riesz representation theorem to analyze Hilbert spaces and related operators.

CO 4: Apply the Banach fixed point theorem to solve problems in analysis and understand its significance in applications.

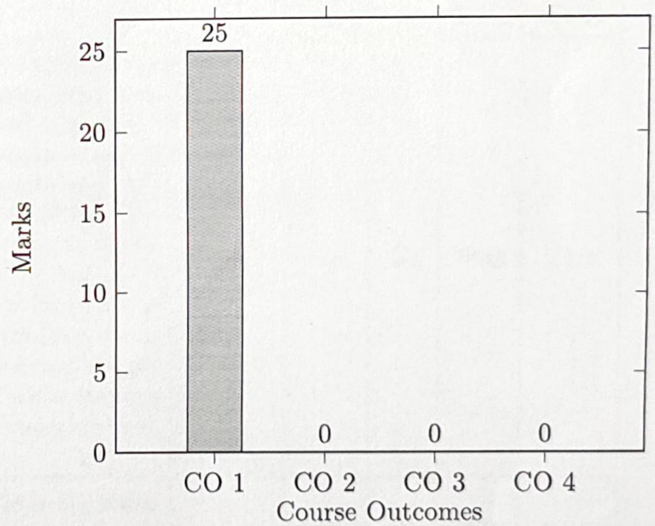
Q.No.	Questions	Marks	CO	BL	PO	PI Code
1	Let $X = R[a,b]$ be the set of all Riemann integrable functions defined on $[a,b]$. For $f, g \in X$, we define $$d(f,g) = \int_a^b \lvert f(x) - g(x) \rvert dx.$$ Check whether d is a metric on X or not.	3	CO1	L2	PO1	1.2.1
2	State Hölder's, Cauchy-Schwarz and Minkowski's inequalities.	3	CO1	L1	PO1	1.2.1
3	Using the theorem "A mapping f of a metric space (X,d) into a metric space $(Y,\tilde{d})$ is continuous if and only if the inverse image of any open subset of $(Y,\tilde{d})$ is an open subset of X," show that every constant function defined from $\mathbb{R}$ to $\mathbb{R}$ is continuous, where $\mathbb{R}$ be the set of all real numbers with the usual metric.	3	CO1	L3	PO1	1.2.1

	Questions	Marks	CO	BL	PO	PI Code
4	Define a Cauchy sequence and completeness. Show that the sequence $\{f_n\}$ in $X = C[0,1]$ given by $f_n(t) = \frac{nt}{n+t}$ for all $t \in [0,1]$, is a Cauchy sequence with respect to metric $d_\infty(f,g) = \sup_{t\in[0,1]} \lvert f(t) - g(t)\rvert$.	5	CO1	L4	PO1	1.2.1
5	Define an isometry and homeomorphism. Show that a bijective isometry is necessarily a homeomorphism, but the converse need not be true.	5	CO1	L2	PO1	1.2.1
6	Define equivalent metrics. Show that the following metrics on $\mathbb{R}^2$ $d_1(x,y) = \lvert x_1 - y_1\rvert + \lvert x_2 - y_2\rvert$, $d_2(x,y) = \left((x_1-y_1)^2 + (x_2-y_2)^2\right)^{\frac{1}{2}}$, $d_\infty(x,y) = \max\{\lvert x_1 - y_1\rvert, \lvert x_2 - y_2\rvert\}$ satisfy the following relation $d_2(x,y) \le d_1(x,y) \le 2d_\infty(x,y) \le 2d_2(x,y)$, where $x = (x_1, x_2), \quad y = (y_1, y_2)$.	6	CO1	L4	PO1	1.2.1

Bloom's Level wise Marks Distribution



Course Outcome wise Marks Distribution



BL – Bloom's Taxonomy Levels:
1 – Remembering, 2 – Understanding, 3 – Applying, 4 – Analysing, 5 – Evaluating, 6 – Creating
CO – Course Outcomes
PO – Program Outcomes
PI Code – Performance Indicator Code