MAHINDRA UNIVERSITY, HYDERABAD
Regular Examinations, December-2023, (2021 Batch)
Program: B. Tech (Common to CM and NT)
Year: III Semester: I
Subject: Computational Methods for PDE (MA 3115)

Date: 18/12/2023 Time: 10:00 AM-01:00 PM
Time Duration: 3 hours Max. Marks: 100

Instructions:

1. Answer all the questions.
2. Marks will not be awarded for guess work.
3. All the answers that belong to a particular question should be answered in one place in your answer booklet.
4. Scientific calculators are permitted.

Q 1: Marks: 20

Solve the following BVP (Boundary Value Problem) using finite difference method with step size $h = \frac{1}{4}$. Then compare with exact values

$$\frac{d^2y}{dx^2} = e^x, y(0) = -5, y(1) = e.$$

Q 2: Marks: 20

Obtain the system of equations $AU = b$ to solve the following Laplace equation using finite difference method, with $\Delta x = \Delta y = \frac{1}{3}$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 1, 0 < y < 1$$

and $u(x, y) = x^2 - y^2$ on it's boundary.

$\frac{180}{3}$

10:36

Q 3: **Marks: 20**

Find the system of equations $AU = b$ to perform two iterations for solving the following 1D heat equation using Crank–Nicolson scheme, with $r = 0.4, \Delta x = \frac{1}{4}$

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, t > 0.$$

$$u(0,t) = u(1,t) = 0, \; t \geq 0 \text{ and } u(x,0) = x(1-x), \; 0 < x < 1.$$

Q 4: **Marks: 20**

The PDE

$$\alpha\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = f(x,t), \; \alpha \in \mathbb{R}$$

is discretized using following Finite Difference scheme,

$$\frac{\alpha(U_{i,j+1} - 0.5(U_{i+1,j} + U_{i-1,j}))}{\Delta t} + \frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} = f_{i,j}, \; \Delta t = r(\Delta x)^{\alpha}, \; r > 0.$$

Then find the domain for α such that the scheme is consistent.

Q 5: **Marks: 20**

Find CFL (stability) condition for the following numerical scheme to solve 1D wave equation

$$\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta t)^2} = c^2\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{(\Delta x)^2}.$$
