Mahindra University Hyderabad
École Centrale School of Engineering
End-semester Regular Examinations, December 2023
Program: B. Tech. Branch: CM Year: 3^{rd} Semester: I
Subject: MA-3116 Financial Mathematics

Date: 22/12/2023
Time Duration: 3 Hours

Start Time: 10:00 AM
Max. Marks: 100

Instruction:

1. All answers that belong to a particular question should be answered at one place in your answer booklet.
2. Use of scientific but non-programmable calculators permitted.
3. Each MCQ has only one correct answer.

Q 1: Each MCQ carries equal marks. Marks: 10×2=20

1. Consider an American option and an European option on a non-dividend paying stock with same strike price K and initial stock price S_0. Which of the following statement is true:

   (a) The price of the American call option is greater than that of the European call option.

   (b) The price of the American put option is greater than that of the European put option.

   (c) The price of the American call option is less than that of the European call option.

   (d) The price of the American put option is less than that of the European put option.

2. A European put option on a stock with strike price 1500 and time to expiry 1 month has delta -0.486. Then the corresponding call option delta is

   (a) 0.486

   (b) 0.614

   (c) 0.586

   (d) 0.514

3. Let $V_0^c = 2.6$ be the price of a European call option with strike price $K = 12$, initial stock price $S_0 = 13$, expiration time six months, and continuously compounding risk free interest rate $r = 5\%$ per annum. Then V_0^p, the price of corresponding European put option, equals to

(a) 1.3037

(b) 1.6574

(c) 2.7295

(d) 2.0265

4. Let $\{W^{(n)}(t) : t \geq 0\}$ denote the scaled symmetric random walk starting at 0. Consider the following two statements:
I : $Var(W^{(n)}(t_2) - W^{(n)}(t_1)) = n(t_2 - t_1)$.
II : For any fix t, $W^{(n)}(t) \sim N(0, t)$ normal distribution.

(a) Both I and II are true

(b) I is true and II is false

(c) I is false and II is true

(d) Both I and II are false

5. Let $\{W(t) : t \geq 0\}$ denote Brownian motion starting at 0. Consider the following two statements:
I : For any $t_2 > t_1$, $Var(W(t_2) - W(t_1)) = (t_2 - t_1)^2$.
II : For any positive times t_1 and t_2, $Cov(W(t_1), W(t_2)) = min(t_1, t_2)$.

(a) Both I and II are true

(b) I is true and II is false

(c) I is false and II is true

(d) Both I and II are false

6. Let $\{S(t) : t \geq 0\}$ denote a stock price process driven by geometric Brownian motion with mean rate of return μ and volatility σ. Then which of the following is true:

(a) $E[S(t)] = S(0)e^{\mu t}$

(b) $E[S(t)] = S(0)e^{(\mu-\sigma^2/2)t}$

(c) $log(S(t))$ is normal $N(log(S(0)), \sigma^2 t)$

(d) None of the above

7. Let $V_N(\omega) = (M_N - S_N)(\omega)$ denote the expiration time pay-off of an option, where $M_N(\omega) = max\{S_n(\omega) : 0 \leq n \leq N\}$, $S_N(\omega)$ is expiration time stock price, $\omega = \omega_1\omega_2 \cdots \omega_N$ is sequence of N coin toss outcome. Then the option is

(a) Non-path dependent option

(b) European type look-back option

(c) American type look-back option

(d) European call option

8. A European put option with 3 month expiration time and strike price 18500 has delta -0.45. An option portfolio of the put options with 1000 short position may be hedged with analogous call options. Then the hedge position is

(a) 450 short call options

(b) 818 short call options

(c) 550 short call options

(d) 818 long call options

9. Let $\{S(t) : 0 \leq t \leq T\}$ be a geometric Brownian motion and denote the underlying stock price process of a European call option. The Black-Scholes-Merton formula for pricing the European call option with strike price K, expiration time T year(s), and continuously compounding risk free annual interest rate r is obtained from

(a) $V(S(t),t) = e^{-r(T-t)}\tilde{E}_t[(S(T)-K)^+]$
(b) $V(S(t),t) = e^{-r(T-t)}\tilde{E}_t[(K-S(T))^+]$
(c) $V(S(t),t) = e^{-rT}\tilde{E}_t[(S(T)-K)^+]$
(d) $V(S(t),t) = e^{-r(T-t)}\tilde{E}[(S(T)-K)^+]$

10. Let $X(t) : t \geq 0\}$ be an adapted stochastic process specified by the stochastic differential equation

$$dX(t) = rX(t)dt + \Delta(t)(\mu - r)S(t)dt + \sigma\Delta(t)S(t)dW(t),$$

where μ, r, σ are positive constants, $S(t)$, $\Delta(t)$ are adapted stochastic processes and $W(t)$ is adapted Brownian motion. Then the quadratic variation $[X,X]_{[0,T]}$ is equal to

(a) $\sigma^2\int_0^T \Delta^2(t)S^2(t)dt$
(b) $\frac{\sigma^2}{2}\int_0^T \Delta^2(t)S^2(t)dt$
(c) $\sigma^2\int_0^T \Delta(t)S(t)dt$
(d) $\sigma\int_0^T \Delta^2(t)S^2(t)dt$

Q 2:

Marks: 20

a) Consider a 3-period binomial model of pricing American option. Let the initial stock price be $S_0 = 10$ per share, $u = 2$ be up factor, $d = 0.5$ be down factor, $r = 0.25$ be rate of interest per time period, $K = 15$ be strike price.

(a) Find the initial price of the American put option.
(b) Find the delta (Δ_n) hedging portfolio process.

Q 3:

Marks: 20

a) Write the governing equation, terminal condition and boundary conditions for an European put option price process. Assume that the underlying stock price process is driven by a geometric Brownian motion. (8)

b) Derive the implicit finite difference scheme for the European put option price process. (12)

Q 4:

Marks: 20

a) Using Ito-Doeblin formula solve the CIR interest rate model SDE

$$dX(t) = (\alpha - \beta X(t))dt + \sigma\sqrt{X(t)}dW(t), \quad t > 0, \ X(0) = X_0.$$

Find $\mathbb{E}[X(t)]$ and $Var[X(t)]$.
