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Mahindra University Hyderabad

École Centrale School of Engineering

Minor - II (2022 Batch), April 2024

Program: B. Tech. **Branch: CM** **Year: II** **Semester: Spring**

Subject: Functional Analysis (MA2212)

Date: 18/04/2024 **Start Time: 2:00 PM**

Time Duration: 1:30 Hours **Max. Marks: 20**

Instructions:

1) All questions are compulsory.

Q 1: 6 marks

(a) Define a convex set in a normed linear space and show that the closed unit ball

$$\tilde{B}_1(0) = \{x \in X \mid ||x|| \leq 1\}$$

in a normed space X is convex. [2 marks]

(b) In l^∞, let $Y = c_{00}$ be the subset of all sequences with only finitely many nonzero terms. Show that Y is a subspace of l^∞, but not a closed subspace with respect to the norm $||x||_\infty = \sup_n |x_n|$. [4 marks]

(OR)

(b) Define the Schauder basis in a normed linear space. Show that (e_n), where $e_n = (\delta_{nj})$, is a Schauder basis for the space l^1 with respect to the norm $||x||_1 = \sum_{n=1}^{\infty} |x_n|$. [4 marks]

Q 2: 7 marks

(a) Define the equivalent norms on a normed linear space X. Verify that the following norms are equivalent on the space $\mathbb{R}^2$:

$$||x||_1 = |x_1| + |x_2|,$$
$$||x||_2 = (|x_1|^2 + |x_2|^2)^{1/2},$$

where $x = (x_1, x_2) \in \mathbb{R}^2$. [4 marks]

(b) Define a sequentially compact set in a normed linear space. Show that the closed unit ball in l^2 space with respect to the norm $||x||_2 = (\sum_{n=1}^{\infty} |x_n|^2)^{1/2}$ is not compact. [3 marks]

Q 3: **7 marks**

(a) Define a linear and bounded operator. Consider an operator $T : l^1 \to l^1$ defined by

$$T(x) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots\right)$$

for all $x = (x_1, x_2, x_3, \ldots) \in l^1$. Show that T is linear and bounded with respect to the norm $||x||_1 = \sum_{n=1}^{\infty} |x_n|$. [4 marks]

~~(b)~~ For a matrix

$$A = \begin{bmatrix} -3 & 5 & 7 \\ 2 & 6 & 4 \\ 0 & 2 & 8 \end{bmatrix},$$

find $||A||_1$, $||A||_\infty$ and $||A||_F$ and for the matrix

$$B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix},$$

find $||B||_2$. [3 marks]