Mahindra University Hyderabad
École Centrale School of Engineering
End-Semester Regular Examination (2021 - Batch)

Program: B. Tech. Branch: AI, CAM, CE, CSE, ECE, NT Year: II Semester: II
Subject: Numerical Methods (MA2208)

Date: 08/06/2023 **Start Time: 10:00 AM**
Time Duration: 3 Hours **Max. Marks: 100**

Instructions:

1) All questions are compulsory.

2) Use of non-programmable calculator is allowed.

Question 1 (20 marks)

(a) Write down a pseudocode to implement the Composite Trapezoidal rule for approximating [10]

$$I = \int_0^{\pi} \sin(x)\, dx$$

by dividing the domain of integration into 10 equal parts.

(b) Write down a pseudocode to implement the Secant method for finding root of the equation [10]

$$\cos x - xe^x = 0.$$

Start the iteration with the initial values $x_0 = 0, x_1 = 1$ and iterate 10 times.

Question 2 (20 marks)

Use the Simpson's 1/3 rule with (a) four, and (b) eight subintervals to approximate the integral

$$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin 2x}{3 - e^{-|x|}}\, dx.$$

Question 3 (20 marks)

(a) Perform three iterations to solve $\frac{dy}{dx} = y - x^2 + 1, 0 \le x \le 2, y(0) = 0.5$ using Taylor's method of order 2 with step size $h = 0.2$. Then find pointwise errors with respect to the exact solution $y(x) = (x+1)^2 - 0.5e^x$. [10]

(b) Find the stability condition to fix step size (h) for solving $\frac{dy}{dx} = -10y, 0 \le x \le 1, y(0) = 1,$ using Runge-Kutta method of order 4. [10]

Question 4 (20 marks)

(a) Perform 4 iterations of Newton-Raphson method to find the root of the equation [10]

$$f(x) = x^3 - 17 = 0$$

taking the initial approximation as $x_0 = 2$.

(b) Perform 4 iterations of bisection method to find root of $f(x) = x^2 - 3$, starting with the interval $[1, 2]$. [10]

P.T.O.

Question 5 (20 marks)

(a) The water level in the sea is mainly determined by tide, whose period is about 12 hours. The height $H(t)$ thus roughly has the form [10]

$$H(t) = a + b\sin(\pi t/6) + c\cos(\pi t/6),$$

where time t is measured in hours. Use the method of least square to the following measurement data to find the constants a, b and c in $H(t)$.

t	0	2	4	6	8	10
H(t)	1.2	1.6	1.4	0.6	0.4	0.2

(b) Obtain the least square approximation for $f(x) = x^2, x \in [0, 1]$ using Legendre polynomial of degree two. Also, find the least square error (also known as mean squared error). [10]