Mahindra University
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Class Test I, April-2023 (2021 – BATCH)
Program: B. Tech. Branch: CM
Year: II Semester: Spring Subject: Functional Analysis (MA2212)

Date: 12/04/2023
Time Duration: 55 mins

Start Time: 4:30 PM
Max. Marks: 10

Instructions:

1. All questions are compulsory.

Q 1: (4 × 0.5) marks

Answer by **True** or **False** the following questions:

(a) In l^∞, let $Y = c_{00}$ be the subset of all sequences with only finitely many nonzero terms. Then Y is a closed subspace of l^∞.

(b) The sequence $\{\frac{1}{n^{1/3}}\}$, $n \in \mathbb{N}$, is an element of l^4 space but not in l^1 space.

(c) If $||\cdot||$ and $||\cdot||_0$ are equivalent norms on a vector space X, then the Cauchy sequences in $(X, ||\cdot||)$ and $(X, ||\cdot||_0)$ are the same.

(d) Every infinite dimensional subspace Y of a normed space X is closed in X.

Q 2: 3 marks

Let $\{x, y\}$ be a linearly independent set of vectors in the normed space $\mathbb{R}^2$. Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1,\ \alpha_2$, we have

$$||\alpha_1 x + \alpha_2 y||_1 \geq c(|\alpha_1| + |\alpha_2|), \tag{1}$$

where for $x = (x_1, x_2) \in \mathbb{R}^2$, $||x||_1 = |x_1| + |x_2|$.

(a) What is the largest possible c in inequality (1) if $x = (1, 3)$, $y = (3, 1)$?

(b) Is it possible to apply inequality (1) if $x = (1, 2)$, $y = (1/2, 1)$? Explain.

Q 3: e_1 e_n — 2 marks

Show that $e_n = (\delta_{nj})$, is a Schauder basis for l^p, where $1 \leq p < +\infty$.

Q 4: — 3 marks

(a) In a normed linear space X, show that $x_n \to x$ and $y_n \to y$ implies $x_n + y_n \to x + y$. Show that $\alpha_n \to \alpha$ and $x_n \to x$ implies $\alpha_n x_n \to \alpha x$.

OR

(a) Show that l^1 (as a vector space) is a subspace of l^2. Is this subspace closed in l^2 with $l^2 - norm$?