Mahindra University

Mahindra University Hyderabad

École Centrale School of Engineering
End–semester Regular Examination

Program: B.Tech. **Branch: Computation & Mathematics** **Year: Second** **Semester: Spring**
Subject: Optimization Techniques (MA 2210)

Date: 07/06/2023 **Start Time: 10: 00 AM**
Time Duration: 3: 0 Hours **Max. Marks: 100**

Instructions:

1) **All questions are compulsory.**
2) **Please start each answer on a separate page, and ensure you clearly number the responses. Also, make sure to address all parts of each question together and in the correct order.**
3) **It is essential to provide an explanation of each step. Correct outcomes without any description will not be evaluated.**

Question 01: Select the correct choice for the following questions with a proper explanation. Correct choices without valid justification will not be considered. [02 × 10]

A) In a maximization linear programming problem (LPP), if at least one artificial variable is in the basis but not at zero level and the net evaluation $(z_j - c_j)$ for each variable is non-negative, then we have

a) A feasible solution b) No feasible solution

c) An unbounded solution d) An optimum solution

B) To maintain optimality of the current optimum solution for a change Δc_k in the coefficient c_k of non-basic variables, we must have

a) $\Delta c_k = z_k - c_k$ b) $\Delta c_k \leq z_k - c_k$

c) $\Delta c_k \geq z_k - c_k$ d) $\Delta c_k = z_k$

C) The dual of the primal maximization LPP having m constraints and n non-negative variable should

a) Be a minimization LPP b) Have n constraints and m non-negative variables

c) Both (a) and (b) d) None of the above

D) A constraint in an LPP restricts

a) Value of the objective function b) Use of available resource

c) Value of a decision variable d) Uncertainty of optimum value

E) A feasible solution to an LPP

a) Must satisfy all of the problem's constraints simultaneously

b) Must be a corner point of the feasible region

c) Need not satisfy all of the constraints; only some of them

d) Must optimize the value of the objective function

F) The role of artificial variables in the simplex method is

a) To aid in finding an initial basic feasible solution

b) To start phases of the simplex method

c) To find shadow prices from the final simplex table

d) None of the above

G) In an integer programming problem (IPP), rounding off solution values of decision variables in an LPP may not be acceptable because

a) It may violate non-negative conditions

b) It does not satisfy the constraints

c) Objective function value of the IPP is more than the objective function value of the LPP

d) None of the above

H) The dummy source or destination in a transportation problem is introduced to

a) Prevent the solution from becoming degenerate

b) To satisfy RIM conditions

c) Ensure that total cost does not exceed a limit

d) Solve the balanced transportation problem

I) An assignment problem can be

a) Designed and solved as a transportation problem

b) Of maximization type

c) Solved only if the number of rows equals the number of columns

d) All of the above

J) Let x and y be two variables and $x > 0, xy = 1$, then the minimum value of $x + y$ is

a) 1 b) 2 c) $2\frac{1}{2}$ d) $3\frac{1}{3}$

Question 02: Explaining each step in the following questions is highly desirable. Each question carries a weightage of ten marks. [10 × 04]

A) Consider the linear programming problem:

$$\max z = \alpha x_1 + x_2; \text{ subject to the constraints}$$

$$\left.\begin{aligned} 2x_1 + x_2 &\le 6 \\ -x_1 + x_2 &\le 1 \\ x_1 + x_2 &\le 4 \end{aligned}\right\}; x_1 \ge 0 \text{ and } x_2 \ge 0, \text{ where } \alpha \text{ is a constant.}$$

If $(3, 0)$ is the only optimal solution, then what will be the possible range of α?

$\frac{a+b}{2}$

$b - \frac{1}{4}$

$a + \frac{1}{4}$

B) Explain the interval halving method. Consider the function $f(x) = 0.1x^4 + 0.5x^3 - 3.3x^2 + 11.3x + 49$ for minimization, where $x \in [-1, 5]$. Find the desired interval and interval length after two iterations by using $\epsilon = 0.001$.

C) A milk plant manufactures two types of products A and B and sells them at a profit of ₹ 5 on type A and ₹ 3 on type B. Each product is processed on two machines, G and H. Type A requires one minute of processing time on G and two minutes on H. Type B requires one minute on G and one minute on H. Machine G is available for not more than 6 hours 40 minutes, while machine H is available for 8 hours 20 minutes during any working day. Formulate and solve the LPP to maximize the profit.

D) Tata steel company manufactures three steel products: A, B, and C, which are processed on three machines: X, Y, and Z. The following table demonstrates the complete technical specifications and input constraints.

$+x_3 \le 100$

$+2x_3 \le 77$

$+x_3 \le 80$

Product	Machine			Profit per unit
	M_1	M_2	M_3	
A	10	7	2	12
B	2	3	4	3
C	1	2	1	1
Available Time	100	77	80	

$12x_1 + 3x_2 + x_3$

Formulate the given linear programming problem and find the optimal product mix of the company and the corresponding profit. Also, find the range of the profit contribution of products A and B in the objective function so that the current optimal product mix remains unchanged.

Question 03: Solve any two of the following three questions. Explaining each step is highly desirable.

[20 × 02]

A) A salesman plans to tour cities B, C, D, and E, starting from his home city A. The inter-city distances are shown in the following table:

City	A	B	C	D	E
A	∞	103	188	136	38
B	103	∞	262	176	52
C	188	262	∞	85	275
D	136	176	85	∞	162
E	38	52	275	162	∞

How should the salesman plan his tour so that (i) he visits each of the cities only once and (ii) travels the minimum distance?

B) Find the optimum integer solution to the following linear programming problem:

$\max z = 7x_1 + 9x_2$; subject to the constraints

$\left.\begin{matrix} -x_1 + 3x_2 \le 6 \\ 7x_1 + x_2 \le 35 \end{matrix}\right\}$; and $x_1 \ge 0,\ x_2 \ge 0$ are integers.

C) The following table shows all the necessary information on the available supply to each warehouse, the requirement of each market, and the unit transportation cost from each warehouse to each market.

		Market				Supply
		I	II	III	IV	
Warehouse	A	5	2	4	3	22
	B	4	8	1	6	15
	C	4	6	7	5	8
Requirement		7	12	17	9	

The shipping clerk has worked out the following schedule from experience:

$$x_{12} = 12, x_{13} = 1, x_{14} = 9, x_{23} = 15, x_{31} = 7, \text{ and } x_{33} = 1.$$

a) Check and see if the clerk has the optimal schedule.

b) Find the optimal schedule and minimum total shipping cost.