Mahindra University Hyderabad
École Centrale School of Engineering
End-semester Regular Examination, May 2024

Program: B. Tech. Branch: All Year: I Semester: II
Subject: Physics (PH1201)

Date: 28-05-2024 **Start Time: 10.00 am**
Time Duration: 3 Hours **Max. Marks: 100**

Instructions:
1) All the questions are compulsory
2) Only non-programmable scientific calculator is allowed
3) Values of useful constants are given at the end of the question paper.

Q1. Marks (10 + 10)

(a) What are the requirements for a well-behaved wavefunction?
Indicate whether each of the functions given below is a solution of Schrodinger's equation over the given range. If it is not, write down the reason thereof in brief.

(i) $\Phi = A\,e^{-x^2}$ $\quad -\infty < x < \infty$

(ii) $\Phi = A\sec x$ $\quad -\infty < x < \infty$

(iii) $\Phi = A\sin^{-1}x$ $\quad -1 \le x \le 1$

(iv) $\Phi = A\,\mathrm{i}\left(e^{ix} + e^{-ix}\right)$ $\quad -\pi \le x \le \pi$

Note A is a non-zero constant.

(b) A particle limited to the x-axis has the wave function

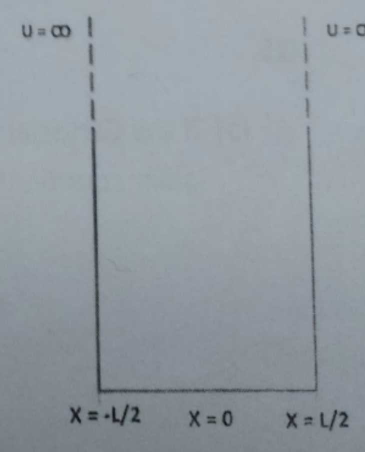
$$\emptyset(x) = Ax\,; \qquad 0 < x < 1;$$

$$= 0; \qquad elsewhere$$

What is the probability of finding the particle in the region $0.4 < x < 0.6$?

Q2. Marks (10 + 10)

(a) A particle with mass m is in an infinite square well potential with walls at $x = -L/2$ and $x = L/2$. Write the energy eigenvalues and eigenfunctions for the states $n = 1, n = 2$.



(b) The tunneling probability for a free particle of mass m and energy E, through a potential barrier of height V_0 and width L is 0.05. How will this change **qualitatively** if only

i. the barrier height is reduced

ii. the barrier width is reduced

iii. the energy of the particle is reduced.

iv. Find the new tunneling probability if $(V_0 - E)$ is changed to 0.49 times the original value, and the new width is 0.5 of its original value.

Q3. Marks (10 + 10)

(a) Write down the Heisenberg position-momentum uncertainty relation. Use Heisenberg's uncertainty principle to obtain an order of magnitude estimate for the minimum kinetic energy of an electron in a hydrogen atom.

(b) Find the kinetic energy of a proton whose de Broglie wavelength is 1 pm. (1 pm = 10^{-12} m)

Q4. Marks (10 + 6 + 4)

(a) Lithium, beryllium, and mercury have work functions of 2.30 eV, 3.90 eV, and 4.50 eV, respectively. Light with a wavelength of 270 nm is incident on each of these metals.
Determine which of these metals exhibit the photoelectric effect for this incident light. Explain your reasoning.

(b) Calculate the wavelength (in nm) of light associated with energy 7.83×10^{-19} J per photon.

(c) Find the phase and group velocity of the de Broglie waves of an electron whose speed is 0.85 c. Here c is the velocity of light.

Q5. Marks (4 + 6 +10)

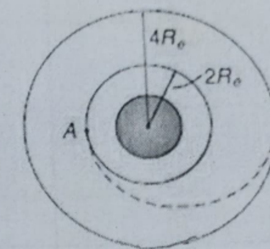
(a) If the Cartesian coordinates (x, y) of a point are $(1,1)$, then what are the corresponding polar coordinates (r, θ)?

(b) The effective potential corresponding to a pair of particles interacting through a central force is given by the expression

$$U_{\mathrm{eff}}(r) = U(r) + \frac{L^2}{2\mu r^2} = Cr^3 + \frac{L^2}{2\mu r^2}$$

where L is the angular momentum, μ is the reduced mass and C is a constant. What is the radius r_0 of the circular orbit allowed in this potential? Express your answer in terms of L, C, and μ.

(c) An efficient way to accomplish the transfer of satellite from one orbit to another orbit is to use a semi-elliptical orbit (known as a *Hohmann transfer orbit*), shown in the figure. What velocity changes are required at the point of intersection, point A?



---Useful constants and unit conversions ---------------------

$c = 3 \times 10^8$ m/s
1 eV = 1.6×10^{-19} J

$h = 6.6 \times 10^{-34}$ J.s
Or
$h = 4.1 \times 10^{-15}$ eV.s
$\hbar = 1.1 \times 10^{-34}$ J.s
Mass of an electron $= 9 \times 10^{-31}$ kg
Mass of a proton $= 1.67 \times 10^{-27}$ kg
Charge of an electron $= 1.6 \times 10^{-19}$ C
Mass of one hydrogen atom ~ mass of a proton $= 1.67 \times 10^{-27}$ kg