Mahindra University

Mahindra University Hyderabad

École Centrale School of Engineering

Minor-II

Program: B. Tech. **Branch: AI, CAM, CE, CSE, ECE, NT** **Year: II** **Semester: II**

Subject: Numerical Methods (MA2208)

Date: 02/05/2023 **Start Time: 10:00 AM**

Time Duration: 1.5 Hours **Max. Marks: 20**

Instructions:

1) Each question carries 5 marks.

2) All questions are compulsory.

3) Use of scientific calculator is allowed.

4) It is strictly prohibited to share calculators during exams.

Question 1 (5 marks)

Construct the Lagrange interpolation polynomial of degree 2 for the function $f(x) = e^x,\ x \in [-1, 1]$, with interpolation points $x_0 = -1, x_1 = 0, x_2 = 1$.

Question 2 (5 marks)

Show that the following boundary value problem (BVP) has a unique solution:

$$\frac{d^2y}{dx^2} = 4y + 3x, \quad 0 < x < 1$$
$$y(0) = 1,\ y(1) = 1$$

Then use finite difference method to solve the above BVP with $h = \frac{1}{4}$.

Question 3 (5 marks)

Find the largest eigenvalue (in magnitude) and the corresponding eigenvector of the following matrix A using the power method. Take the initial approximation of the eigenvector as $x_0 = [1, 1, 1]^T$ and iterate 2 times.

$$A = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{pmatrix}$$

Question 4 (5 marks)

Derive the Newton-Cotes formula for $\int_0^1 f(x)\,dx$ based on the data $\{(0, f(0)), (\frac{1}{2}, f(\frac{1}{2})), (1, f(1))\}$. Then find an error bound.