



Mahindra University Hyderabad

École Centrale School of Engineering,

End-semester Regular Examination, January-2023 (2022 batch)

Program: B. Tech. Branch: CE, ECM, ME, MT, NT, CSE, AI, CM, ECE

Year: II Semester: Fall Subject: Mathematics III (MA2103)

Date: 18/12/2023

Start Time: 10.00 AM

Time Duration: 3 Hours

Max. Marks: 100

Instructions:

1. Each question carries 25 marks.
2. All questions are compulsory.
3. It is essential to provide an explanation of each step. Correct outcomes without any description will not be evaluated.

Q 1:

25 marks

(a) Suppose five out of 100 men and 25 out of 10,000 women are color blind. A color-blind person is chosen at random. What is the probability of the person being male, assuming that males and females are in equal numbers? [10 marks]

(b) The following are the scores on IQ tests of a random sample of 18 students at an educational institution.

130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142

Construct a 95% confidence interval estimate of the average IQ score for all students at the institution. You can use the following values to obtain the answer. [15 marks]

Hint: $z_{0.05} = 1.6449$ and $t_{0.025,17} = 2.11$.

Q 2:**25 marks**

- (a) The number of emails received on a weekday follows a Poisson distribution with an average of 0.2 emails per minute. What is the probability of receiving at least one email in a 5-minute interval? [9 marks]

- (b) The Edwards Theater chain has conducted a study on its movie customers to determine the amount of money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37.

- (i) What percentage of customers will spend less than \$3.00 on concessions?
(ii) What spending amount corresponds to the top 87th percentile?

Hint: $z_{0.13} = 1.125$, $z_{0.209} = 0.81$

[16 marks]

Q 3:**25 marks**

- (a) Suppose that a fair coin is tossed 900 times. [16 marks]

- (i) Approximate the probability of obtaining more than 495 heads.
(ii) Approximate the probability of obtaining less than 405 heads.

- (b) Suppose that a random variable X has a uniform distribution on the interval $[0, 1]$. Determine the p.d.f. of $Y = X^2$. [9 marks]

You may require the value $\Phi(3) = 0.9986$.**Q 4:****25 marks**

- (a) Let X and Y be jointly continuous random variables with joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} e^{-x-y} & 0 \leq x < \infty, 0 \leq y < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

- (i) Find $P(X < Y)$.
(ii) Find $P(X < a)$. [12 marks]

- (b) If X and Y are independent standard normal random variables then find the probability density function of $Z = \frac{X}{Y}$. [13 marks]