Mahindra University Hyderabad
École Centrale School of Engineering
Minor Examinations-2021 Batch
Program: B. Tech Branch: Common to all Branches
Year: I Semester: Spring
Subject: Mathematics - II (MA 1202)

Date: 18/04/2022 Start Time: 9:00 AM
Time Duration: 1 Hour 30 mins Max. Marks: 30

Instructions

1. Attempt all the questions.
2. No marks without justification.

Q. 1 Marks: 6

Write the function $f(z) = \cos z$ in the form of $f = u + iv$, then show that u and v satisfy Cauchy-Riemann equations.

Q. 2 Marks: 6

Show that $u(x, y) = e^x \sin y$ is a harmonic function. Find the harmonic conjugate $v(x, y)$ so that $f(z) = u(x, y) + iv(x, y)$ is an analytic function. Also, find the function $f(z)$ in terms of z.

Q. 3: Marks: 6

Find the Taylor series of $f(z) = \frac{1}{2-z}$ centered at $z = 3i$. Then find its radius of convergence.

Q. 4 Marks: 6

Using Cauchy integral formula, find the value of the integral $\int_\gamma \frac{(z^2+4)}{z^2+2z+5}\, dz$, where γ is given by $|z+1+i| = 2$.

Q. 5 Marks: 6

Let $V = \{(x_1, x_2) | x_1 + x_2 = 1, 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$ with the operations $(x_1, x_2) \bigoplus (y_1, y_2) = \left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$ and $a \bigodot (x_1, x_2) = (x_1, x_2)$. Are the closure axioms satisfied? Is V a vector space? Justify your answer.