Program: B. Tech./Pre-PhD   **Branch: CAM**   **Year: —**   **Semester: Spring**
Subject: Functional Analysis (MA2212/MA 6011)

Date: 23/02/2024   **Start Time: 12:35 PM**
Time Duration: 30 minutes   **Max. Marks: 10**

Instructions:

1) All questions are compulsory.

Q 1:   5 marks

Define a metric. Let 's' be the space of all sequences of real numbers, define a function $d : s \times s \to \mathbb{R}$

$$d(x,y) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{|x_n - y_n|}{1 + |x_n - y_n|}$$

for any $x = (x_n)$ and $y = (y_n) \in s$. Show that 'd' satisfies the following triangle inequality

$$d(x,y) \leq d(x,z) + d(z,y) \text{ for all } x, y, z \in s.$$

Q 2:   5 marks

Define a Cauchy sequence. Let $C[0,1]$ be a space of all continuous functions defined on $[0,1]$. Consider d_1 defined on $C[0,1]$ by

$$d_1(f,g) =) = \int_0^1 |f(x) - g(x)| \, dx \text{ for all } f, g \in C[0,1].$$

Consider a sequence $(f_n)_{n \geq 2}$ in $C[0,1]$ defined as

$$f_n(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq \frac{1}{2} - \frac{1}{n} \\ nt - \frac{n}{2} + 1 & \text{if } \frac{1}{2} - \frac{1}{n} < t \leq \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < t \leq 1 \end{cases}$$

Show that $(f_n)_{n \geq 2}$ is Cauchy in $C[0,1]$.