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Mahindra University Hyderabad
École Centrale School of Engineering
Minor II Examinations,
November-2023

Program: B. Tech. Branch: CM Year: II Semester: I
Subject: Algebra (MA 2106)

Date: 10/11/2023 **Start Time: 02.00 PM**
Time Duration: 90 Minutes **Max. Marks: 30**

Instructions:

1. There are 6 questions, all of which are compulsory.
2. Justify your answer wherever required.

1. State "True" or "False". Negative marking (-1) will apply in case of each incorrect answer. [5] M
   (i) There exists a homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{12}$ such that $f(\bar{1}) = 0$ and $f(\bar{0}) = 1$.
   (ii) If a group has an element of order 24, then it also has an element of order 6.
   (iii) The element $\bar{7}$ is a unit in the ring $\mathbb{Z}_{28}$.
   (iv) Let G be a group. There exists normal subgroups A and B of G such that $A \cap B$ is not a normal subgroup of G.
   (v) The map $f : \mathbb{Z}_{14} \to \mathbb{Z}_{28}$ defined by $f(\bar{x}) = \overline{5x}$ is a homomorphism.

2. (i) Let F be a homomorphism from a group G to a group G'. Show that the image $f(G)$ is a subgroup of G'. [6] M [2]
   (ii) State first isomorphism theorem. Using it prove that $|f(G)|$ divides $|G|$ and $|G'|$. [2+2]

3. Consider the following theorem: [3] M

   If G is a group such that $G/Z(G)$ is cyclic, then G is abelian.

   Fill in the bianks:

   Let G be a non-abelian group of order pq for some prime p and q. Then $|Z(G)|$ can not be of order pq because ———. We shall now prove that $Z(G) = \{e\}$. On contrary suppose that $Z(G) \neq \{e\}$. Then $|Z(G)|$ is either — or — because ———. Hence $|G/Z(G)|$ is either —or—. It follows that $G/Z(G)$ is cyclic because ———. Hence by the given theorem G is abelian. Which is a contradiction of the assumption that ———.

4. (i) Find the number of abelian groups of order 300. [3.5] [5] M

(ii) How many of them do not have an element of order 25? [1.5]

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5. Fill in the blanks: [5] M

(i) Order of $(13564)(2475)$ in S_{11} is —. [1]

(ii) If $\alpha(328514) = (158342)$, then $\alpha =$ —. [2]

(iii) The number of elements of order 20 in S_5 is —. [2]

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6. (i) Show that $\mathbb{Z}_5[i]$ is not an integral domain. [3] [6] M

(ii) Prove that the characteristic of an integral domain is either zero or a prime. [3]

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