

## Mahindra University Hyderabad

École Centrale School of Engineering

2<sup>nd</sup> Year B.Tech. (2022 Batch) Fall Semester

Regular Examinations, December – 2023

Program: B.Tech.    Branch: Computation & Mathematics    Year: Second    Semester: First  
Subject: Graph Algorithms (MA 2105)

Date: 26/12/2023

Time Duration: 3: 0 Hours

Start Time: 10: 00 AM

Max. Marks: 100

### Instructions:

- 1) All questions are compulsory.
- 2) Start each answer on a new page and number your answers clearly. Answer all parts of the same question together and in sequence.
- 3) An explanation of every step is essential. Correct outcomes without description will not be evaluated.

Q 01: Select the correct choice for the following questions with a proper explanation.  
Correct choices without valid justification will not be considered. [02 × 10]

A) The graph with a closed trail that includes every edge of the graph is known as:

- a) Planar graph
- b) Euler graph
- c) Directed graph
- d) Hamiltonian graph

B) If  $G$  is the forest with 54 vertices and 17 connected components. Then, what is the total number of edges  $G$  can have?

- a) 37
- b) 71
- c) 17
- d) 54
- e) None of these

C) How many regions are there in a connected planar graph with six vertices and seven edges?

- a) 1
- b) 3
- c) 5
- d) 2
- e) None of these

D) A connected undirected graph containing  $n$  vertices and  $n - 1$  edges:

- a) Cannot have cycles
- b) Must contain at least one cycle
- c) Can contain at most two cycles
- d) Must contain at least two cycles

E) Which of the following statements is/are TRUE?

P: A cycle is a walk with the same end vertices.

Q: A cycle is a path with the same end vertices.

- a) Both P and Q
- b) Q only
- c) P only
- d) Neither P nor Q

F) The number of edges in a regular graph of degree 46 and 8 vertices are:

- a) 347
- b) 186
- c) 184
- d) 230
- e) None of these



G) A graph with  $n$  vertices will have a parallel edge or self-loop if the total number of edges is:

a) Greater than  $n-1$

b) Less than  $n(n-1)$

c) Greater than  $\frac{n(n-1)}{2}$

d) Less than  $\frac{n^2}{2}$

H) A path is a particular subgraph consisting of an ordered sequence of branches having which of the following properties?

i) At all but two of its nodes, called internal nodes, there are incidents in exactly two branches of the subgraph.

ii) At each of the remaining two nodes, called terminal nodes, there is incident exactly one branch of the subgraph.

Select the correct answer using the code given below.

a) i only

b) Both i and ii

c) Neither i nor ii

d) ii only

I) Which of the following statements for a simple graph is correct?

a) Every trail is a path

b) Every path is a trail

c) Path and trail are the same

d) Path and trail have no relation

J) If the origin and terminus of a walk are the same, the walk is known as:

a) Open

b) Closed

c) Path

d) None of these

Q 02: Answer the following questions: Each step needs to be discussed in detail. Each question is worth a maximum of five marks. [05 × 08]

A) Show that the degree of a vertex of a simple graph  $G$  on  $n$  vertices cannot exceed  $n-1$ .

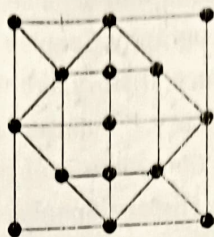
B) Does a 3-regular graph on 14 vertices exist? What can you say about 17 vertices?

C) Explain the order and size of a graph by providing relevant examples. Additionally, explain the concept of a subgraph of a graph. Is there a proper subgraph, denoted as  $S$ , of graph  $G$  that exists with a size equal to the size of the entire graph?

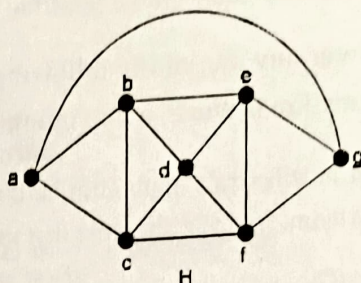
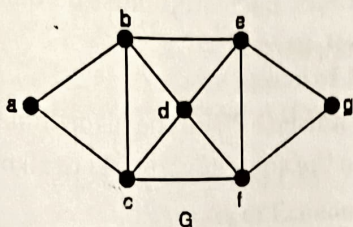
D) If a graph  $G$  of  $n$  vertices is isomorphic to its complement  $\bar{G}$ , show that  $n$  or  $(n-1)$  must be a multiple of 4.

E) Determine the number of vertices, the number of edges, and the number of regions in the graphs shown below. Then, show that your answer satisfies Euler's formula for connected planar graphs.





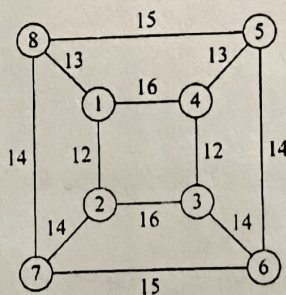
f) Explain the chromatic number of a graph. What are the chromatic numbers of the following graphs, G and H?



- g) A graph G is obtained by adding an extra vertex  $\alpha$  to  $K_{5,4}$  first and then adding an edge from  $\alpha$  to all the vertices of  $K_{5,4}$ , i.e., by making  $\alpha$  adjacent to every vertex of  $K_{5,4}$ . Then, calculate the minimum number of colors required to edge-colour G.
- h) A tree has five vertices of degree 2, three of degree 3, and four of degree 4. How many vertices of degree 1 does it have?

Q 03: Answer the following questions. Each step must be discussed in detail. Each question is worth a maximum of ten marks. [10 × 02]

A) Explain the spanning and minimum spanning trees (MST) in a graph G. Write Prim's algorithm for finding the minimum spanning tree. Apply Prim's algorithm to obtain the minimum spanning tree in the following graph. Also, point out the list of edges in the MST in the order they are received. Start the algorithm from vertex 1.



B) Suppose that in one particular semester, students are taking each of the following combinations of courses.

- Mathematics, English, Biology, Chemistry

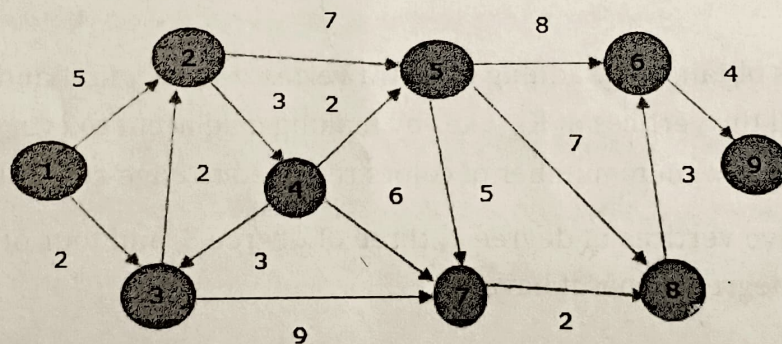


- Mathematics, English, Computer Science, Geography
- Biology, Psychology, Geography, Spanish
- Biology, Computer Science, History, French
- English, Psychology, History, Computer Science
- Psychology, Chemistry, Computer Science, French
- Psychology, Geography, History, Spanish.

What is the minimum number of examination periods required for exams in the ten courses specified so that students taking any of the given combinations of courses have no conflicts? Find a possible schedule that uses this minimum number of periods.

Q 04: Answer any one of the following questions. Each step must be discussed in detail.  
Each question is worth a maximum of 20 marks. {20 × 01}

A) What is Dijkstra's algorithm, and why is it used? Write the algorithm. Using Dijkstra's algorithm, find the shortest distance from the source (vertex 1) to all other vertices.



B) Let  $G$  be a nonempty graph. Then  $\chi(G) = 2$  if and only if  $G$  is bipartite.