Mahindra University Hyderabad

École Centrale School of Engineering

Minor-I

Program: B. Tech. Branch: CM Year: IV Semester: I

Subject: Dynamical Systems (MA4125)

Date: 11/09/2024 Start Time: 10:00 AM

Time Duration: 90 Minutes Max. Marks: 15

Instructions:

1) All questions are compulsory.

2) Everything you write (including any notes and rough work) must be in the answer booklet.

3) Give proper justification for your answers. Marks will not be awarded for guess work.

Q 1: 5 Marks

Find the evolution operator for the system $\dot{x} = x \ln x,\ x > 0,\ \ x(0) = x_0,\ x_0,\ x \in \mathbb{R}$. Verify that $\phi_t(\phi_s(x)) = \phi_{t+s}(x),\ \forall\ x,\ t, s \in \mathbb{R}$.

Q 2: 5 Marks

Reduce the differential equation $\ddot{x} + 3\dot{x} + 2x = 0$ with $x(0) = 1,\ \dot{x}(0) = -1$ to a system of first order differential equations and find its solution using fundamental matrix.

Q 3: 5 Marks

Find the general solution of the system $\dot{X} = AX,\ X(0) = [c_1, c_2]^T$, using matrix exponential, where the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $a + d \neq 0,\ ad - bc = 0$. Also find the general solution if $a + d = 0,\ ad - bc = 0,\ X \in \mathbb{R}^2,\ a, b, c, d, c_1, c_2 \in \mathbb{R}$.
