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Mahindra University Hyderabad
École Centrale School of Engineering
Minor II Examinations,
October-2024

Program: B. Tech. Branch: CM Year: II Semester: I
Subject: Algebra (MA 2106)

Date: 26/10/2024 Start Time: 10.00 AM
Time Duration: 90 Minutes Max. Marks: 30

Instructions:

1. There are 6 questions, all of which are compulsory.
2. Justify your answer wherever required.

1. State "True" or "False". Negative marking (-1) will apply in case of each incorrect answer. [5] M
   - (i) Let G be a group. If $G = Z(G)$, then G is abelian.
   - (ii) There exists a homomorphism from $\mathbb{Z}_9$ to $\mathbb{Z}_{16}$ such that $f(\bar{2}) = \bar{2}$ and $f(\bar{5}) = \bar{5}$.
   - (iii) The number of units in the ring $\mathbb{Z}_{10}$ is 4.
   - (iv) $SL(2, \mathbb{R})$ is a normal subgroup of $GL(2, \mathbb{R})$.
   - (v) Every permutation in S_n can be written as a product of disjoint transpositions.

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2. (i) Find the number of abelian groups of order 500. [3.5] [5] M
   (ii) How many of them have an element of order 125? [1.5]

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3. Let f be a homomorphism from a group G to a group G'. Define $\ker f$. Show that it is a subgroup of G. Is it normal in G? Justify your answer. [5] M

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4. Define an integral domain. Show that $\mathbb{Z}_5[i]$ is not an integral domain. [1+5] M

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5. Fill in the blanks: [6] M
   - (i) Order of $(\underset{6}{125863})(\underset{5}{24517\cancel{1}})(\underset{2}{38})$ in S_8 is —. [2]
   - (ii) If $\alpha(1357) = (2463)$, and $\alpha\beta = (24)$ then $\beta =$ —. [2]
   - (iii) The maximum order of an element in S_{11} is —. [2]

6. Consider the following two results: [3] M

- If G is a group such that $G/Z(G)$ is cyclic, then G is abelian.
- If G is a group of prime order, then G is cyclic.

Using these two results show that If G is a non-abelian group of order 15, then its center $Z(G) = \{e\}$.

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