Mahindra University Hyderabad

École Centrale School of Engineering

2nd Year B.Tech. (2023 Batch) Fall Semester

Regular Examinations, December – 2024

Program: B.Tech. Branch: Computation & Mathematics Year: Second Semester: First

Subject: Graph Algorithms (MA 2105)

Date: 17/12/2024 Start Time: 10: 00 AM

Time Duration: 3: 0 Hours Max. Marks: 100

Instructions:

1) **All questions are compulsory.**
2) **Start each answer on a new page and number your answers clearly. Answer all parts of the same question together and in sequence.**
3) **An explanation of every step is essential. Correct outcomes without valid explanation will not be considered.**

Q 01: Choose the correct option or provide the appropriate answer for the following questions. A short and valid explanation is required, as correct answers without justification will not be considered. [02 × 10]

A: If a graph G has five vertices with a degree of 4 and four vertices with a degree of 3, then the number of edges in G is _____.

B: Which one of the following contains a lesser number of nodes than the original graph?

a) Proper subgraph b) Improper subgraph

c) Planar graph d) Non-planer graph

C: The minimum number of colors required to color the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same color is

a) 2 b) 3 c) 4 d) $n - 2\left\lfloor\frac{n}{2}\right\rfloor + 2$

D: Let G be a connected planar graph with ten vertices. If the number of edges on each face is three, then the number of edges in G is _____.

E: Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?

a) 7 b) 1 c) $\frac{1}{8}$ d) 8

F: Consider the following statements:

P: There exists no simple, undirected, and connected graph with 80 vertices and 77 edges.

Q: All vertices of the Euler graph are of even degree.

R: Every simple, undirected, connected, and acyclic graph with 50 vertices has at least two vertices of degree one.

S: There exists a bipartite graph with more than ten vertices, which is 2-colorable.

What is the number of correct statements among the above statements?

a) 1 b) 2 c) 3 d) 4

G: Consider the following statements regarding simple undirected graphs:

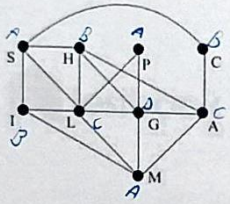
i) If a graph G with n vertices has $n-1$ edges, then G is a tree.

ii) If a graph G with n vertices is acyclic, then G is a tree.

Which of the above statements is correct?

a) Only (ii) b) Only (i) c) Both of these d) None of these

H: What is the chromatic index of the given graph G?



a) 3 b) 4 c) 5 d) 6 e) None of these

I: Let $G = K_n$, where $n \geq 5$. The number of edges in any induced subgraph of G with five vertices is:

a) 10 b) 5 c) 6 d) 8

J: The minimum number of colors needed to properly color the vertices of $K_{8,12}$ is _____.

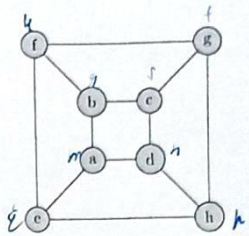
Q 02: Answer any five of the following questions, providing a detailed explanation for each step. [06 × 05]

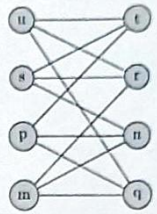
A: At a party, is it possible for an even number of people to speak to an odd number of other people? If yes, explain using graph theory concepts. If no, provide a valid explanation.

B: How many simple graphs are there with exactly n vertices? Additionally, how many of these graphs are non-null? What will be these numbers when $n = 5$?

C: Show that it is impossible for every vertex of a graph to have a different degree. Explain why this is true, considering all possible cases for the degrees of the vertices in the graph.

D: Explain, in your own words, what it means for two graphs to be isomorphic. Examine the two graphs provided below—are they isomorphic? If they are, present an explicit isomorphism that maps one graph to the other.





E: A tree has 100 leaves, 20 vertices of degree 6, and half of the remaining vertices have degree 4. The leftover vertices are degree 2. How many vertices are of degree 2?

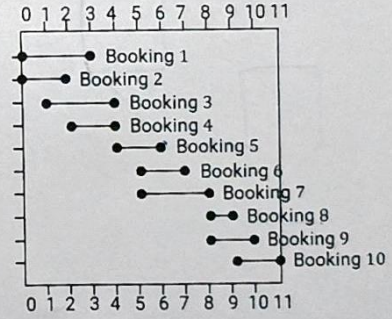
F: Show that a connected, planar graph with order 22 has no more than 60 edges.

Q 03: Answer any three of the following questions, providing a detailed explanation for each step. [10 × 03]

A: A graph G is obtained by adding an extra vertex α to $K_{5,4}$ first and then adding an edge from α to all the vertices of $K_{5,4}$, i.e., by making α adjacent to every vertex of $K_{5,4}$. Then, calculate the minimum number of colors required to edge-color G.

B: Does a plane graph with five regions exist, such that every region is bounded by exactly four edges? Explain.

C: Consider a cab service provider with ten booking requests, each specifying a start time (when the taxi departs from the depot) and a finish time (when the taxi is expected to return). Using the concept of graph coloring, assign these bookings to vehicles in such a way that the number of cars used is minimized. What is the minimum number of cars required?



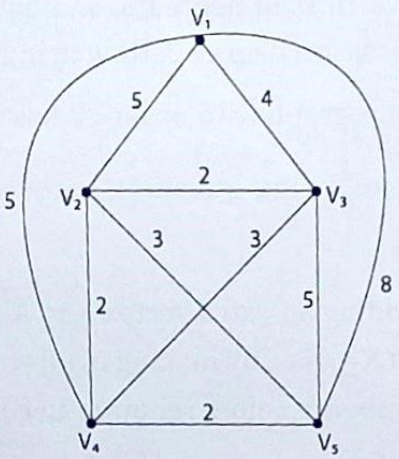
D: We know that a strongly connected tournament T on n vertices contains directed cycles of length 3, 4, ..., n. Show that a tournament T is Hamiltonian if and only if it is strongly connected.

E: What is a regular graph? Explain by taking a suitable example. Does a 3 – regular graph on 14 vertices exist? What can you say about 17 vertices? If possible, do mention the number of edges in both cases.

Q 04: Answer any one of the following questions. Ensure that each step is adequately explained. [20 × 01]

A: What is the traveling salesman problem? Explain the central idea of the closest insertion algorithm. Use the closest insertion algorithm to solve the traveling salesman problem for the graph below. The salesman starts his journey from V_1.

[2 + 4 + 14]



B: What are Breadth-First Search (BFS) and Depth-First Search (DFS) in a graph? Write their mechanism to produce the spanning tree in a connected graph. Point out at least three differences between BFS and DFS. Finally, find the spanning tree obtained by both BFS and DFS algorithms for the following graph by assuming a as the root.

[4 + 6 + 2 + 8]