Mahindra University Hyderabad

École Centrale School of Engineering

Minor II (2023 - Batch)

Program: B. Tech. Branch: All

Year: II Semester: I

Subject: Mathematics - III (MA2103)

Date: 23/10/2024 Start Time: 10:00 AM

Time Duration: 1.5 Hours Max. Marks: 20

Instructions:

1) Each question carries 5 marks.
2) All questions are compulsory.
3) **Please start each answer on a separate page and make sure to clearly number the responses.**
4) It is essential to provide an explanation of each step. Correct outcomes without any description will not be evaluated.

Q 1: 5 marks

A person draws two balls from a bag containing three white and four red balls. The person earns \$10 for each white ball drawn and \$20 for each red ball. Find the expected value of the total earnings.

Q 2: 5 marks

Let X and Y be two continuous random variables with the joint pdf:

$$f_{X,Y}(x,y) = \begin{cases} 12xy, & 0 \le x \le 1,\ 0 \le y \le x \\ 0, & \text{otherwise} \end{cases}$$

Answer the following questions:

(a) Find the marginal pdfs $f_X(x)$ and $f_Y(y)$.

(b) Are X and Y independent? Provide a mathematical justification.

(c) Compute the covariance $\mathrm{COV}(X,Y)$.

Q 3: 5 marks

The waiting time, in minutes, for a bus is uniformly distributed between 0 and 15 minutes, inclusive.

(a) What is the probability that a person waits less than 12.5 minutes?

(b) On average, how long must a person wait? Calculate the mean and standard deviation of the waiting time.

Q 4: 5 marks

Aptitude test scores of a population of high school students follow a normal distribution with a mean of 500 and a standard deviation of 100. In a random sample of five students, find the probability of:

(a) All five students scoring below 600.

(b) Exactly three of them scoring above 640.

[Use: $z_{0.1587} = 1$, $z_{0.0808} = 1.4$]
