

Program: B.Tech

Branch: AI/CB/CM/CMD/CSE/ECE/ECM

Subject: Discrete Mathematical Structures (AI/CSE 1202)

Time: 10:00 To 13:00

Year: First

Semester: Second

Date: 31-05-2024

Please read the following instructions carefully before answering questions.

1. Answer **all** questions; there are eleven questions. Maximum marks you can score is 100.
 2. First question carries 30 marks (3 marks for each sub-question). Every subsequent question carries 7 marks.
 3. For each statement that appears in the question paper, you have to either **prove** its correctness or **refute** it with a counter example.
 4. If you feel any question is ambiguous, clearly state your assumption(s) and answer it accordingly.
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1. Define/State the following terms:

- (a) Uncountable Set.
 - (b) Equivalence Class.
 - (c) Maximal element of a *POSET*.
 - (d) Lattice.
 - (e) Multinomial Theorem.
 - (f) Generalized Pigeon-Hole Principle.
 - (g) Adjacency matrix of an undirected graph.
 - (h) Bipartite graph.
 - (i) Handshaking Lemma.
 - (j) Tree.
2. $|X + Y| \geq |X| + |Y| - 1$, where $X + Y = \{x + y | x \in X, y \in Y\}$ for any two non-empty finite subsets X and Y of integers. Here $|A|$ represents the number of elements in the set A .
 3. Every equivalence relation on a set X produces a partition of X .
 4. Let R be a relation on $X = \{1, 3, 5, 30, 45, 60\}$ defined as $a R b$ iff $a | b$ (i.e., a divides b). Then, check if (X, R) is *POSET*; if so, check if it is a Lattice.

5. In a group of 6 people, every two individuals are either friends or enemies. Then, there exists a group of three people such that they are either mutual friends or mutual enemies.
6. Assuming x_i s are non-negative integers, find the number of solutions for $x_1 + x_2 + x_3 \leq 11$.
7. Find a recurrence relation and give initial conditions for the following problem. The number of ways to climb n stairs if the person climbing the stairs can take one, two, or three stairs at a time.
8. Let G be an undirected graph such that degree of every vertex is at least k , then there exists a path in G of length at least k .
9. Let $G(V, E)$ be an undirected graph with \mathbf{n} vertices, and it has \mathbf{k} connected components. Then, G has at least $\mathbf{n} - \mathbf{k}$ edges, and at most $\binom{n-k+1}{2}$ edges.
10. Any Tree with n vertices has exactly $n - 1$ edges.
11. If G is a simple bipartite graph with n vertices and e edges, then $e \leq \frac{n^2}{4}$.

_____ All the best _____