Mahindra University Hyderabad
École Centrale School of Engineering
End Semester-Regular Examination, May 2025
Program: B.Tech Branch: Computation & Mathematics Year: III
Semester: II
Subject: Mathematical Foundations for Machine Learning(MA3219)

Date: 26/05/2025 Start Time: 10.00 AM
Time Duration: 3 Hours Max. Marks: 100

Instructions:

1. Justify your answer wherever required. Guesswork will not be considered in evaluation.

Q 1: 20 marks

Consider the set of training data in Table 1. Find the corresponding dual cost function in SVM. (as a function of α_i's only).

Feature	Label	Feature	Label
$x_1 = (0,0,0)$	$d_1 = 1$	$x_5 = (1,0,0)$	$d_5 = -1$
$x_2 = (0,0,1)$	$d_2 = -1$	$x_6 = (1,0,1)$	$d_6 = 1$
$x_3 = (0,1,0)$	$d_3 = -1$	$x_7 = (1,1,0)$	$d_7 = 1$
$x_4 = (0,1,1)$	$d_4 = 1$	$x_8 = (1,1,1)$	$d_8 = -1$

Table 1:

Q 2: 20 marks

Consider the optimization problem

$$\begin{aligned} &\text{minimize} && x^2 + 1 \\ &\text{subject to} && (x-2)(x-4) \leq 0, \end{aligned}$$

with variable $x \in \mathbb{R}$. Derive the Lagrange dual function g and write the dual problem.

Q 3:

20 marks

Calculate the information gain of the dataset for each attribute (altitude, wind, temperature, and humidity) and determine which attribute is the best choice for the root node of the decision tree.

Altitude	Wind	Temperature	Humidity	Outcome
High	Low	Hot	High	Crash
Low	High	Cold	Low	Safe
Low	Low	Mild	High	Safe
Medium	High	Hot	Low	Crash
High	Low	Mild	Low	Safe
Medium	High	Mild	High	Crash
High	Low	Cold	High	Crash
Low	Low	Cold	Low	Safe
Medium	Low	Mild	Low	Safe
Low	High	Hot	High	Crash

Q 4:

20 marks

Five measurements were taken on each of 49 female sparrows. These measurements (in millimeters) were: X_1 = total length, X_2 = alar extent, X_3 = length of beak and head, X_4 = length of humerus and X_5 = length of keel of sternum.
The eigenvalues and eigenvectors of the sample correlation matrix of these 49 observation vectors are given by

Eigenvalues	3.616	0.532	0.402	0.285	0.165
Eigenvectors:					
	−0.451	−0.385	−0.573	−0.471	−0.398
	−0.243	−0.625	0.198	−0.071	−0.721
	−0.495	−1.000	−1.000	−1,000	−1.000

Draw a scree plot. Hence or otherwise, suggest the number, k, of principal components that should be retained. Explain your reasoning. Also, calculate the proportion of variability explained by these k components.

Q 5:

20 marks

Given two normal distributions $p(x|C_1) \sim N(\mu_1, \sigma_1^2)$ and $p(x|C_2) \sim N(\mu_2, \sigma_2^2)$ and $P(C_1)$ and $P(C_2)$, calculate the Bayes' discriminant points analytically.