MAHINDRA UNIVERSITY HYDERABAD
École Centrale School of Engineering
Minor-II Examinations, October 2024

Program: B.Tech. **Branch: CM** **Year: 3^{rd}** **Semester: I**

Subject: Financial Mathematics (MA3116)

Date: 25/10/2024 **Start Time: 10:00 AM**
Time Duration: 1.5 Hours **Max. Marks: 30**

Q 1:

Marks : 10

a) Consider a 3-period binomial model of pricing American option. Let the initial stock price be $S_0 = 10$ per share, $u = 2$ be up factor, $d = 0.5$ be down factor, $r = 0.25$ be rate of interest per time period, $K = 12$ be strike price.

(i) Find the initial price of the American put option.

(ii) Let τ denote a exercise policy and defined as follows: $\tau(HHH) = \infty, \tau(HHT) = \infty, \tau(HTH) = 2, \tau(HTT) = 2, \tau(THH) = 1, \tau(THT) = 1, \tau(TTH) = 1, \tau(TTT) = 1$. Check whether τ is a stopping time and compute the stopped option price process.

Q 2:

Marks : 10

a) Let $W^{(n)}(t)$ denote scaled symmetric random walk. Let $S_n(t)$ denote the stock price process driven by $W^{(n)}(t)$ with up factor $u_n = e^{\sigma/\sqrt{n}}$ and down factor $d_n = e^{-\sigma/\sqrt{n}}$. If the stock price process $S_n(t)$ converges in distribution to the process

$$S(t) = S_0 \exp\left(\sigma W(t) + (\mu - \frac{\sigma^2}{2})t\right)$$

then find the probabilities of Head (H) and Tail (T) in the underlying random walk process. Also compute $\mathbb{E}[S(t)]$ and $\mathbb{E}[\log(S(t))]$.

Q 3:

Marks : 10

a) Solve the following SDE using Itô-Doeblin formula

$$dX(t) = (\mu - \lambda X(t))dt + \sigma dW(t), \quad t \geq 0, \; X(0) = X_0.$$

Compute $E[X(t)]$, $Var[X(t)]$. Determine the probability distribution of $X(t)$.
