Mahindra University Hyderabad

École Centrale School of Engineering

End Semester Regular Examination, December-2024

Program: B. Tech. Branch: CM Year: IV Semester: I

Subject: Dynamical Systems (MA4125)

Date: 12/12/2024 Start Time: 10:00 AM

Time Duration: 180 Minutes Max. Marks: 100

Instructions:

1) All questions are compulsory.
2) Everything you write (including any notes and rough work) must be in the answer booklet.
3) Give proper justification for your answers. Marks will not be awarded for guess work.

Q 1: 20 Marks

Consider the nonlinear system $\dot{X} = F(X)$, where $X = [x_1\ x_2]^T \in \mathbb{R}^2$ and $F(X) = \begin{bmatrix} x_1 - x_2 - x_1^3 \\ x_1 + x_2 - x_2^3 \end{bmatrix}$

i) Show that the origin is an unstable focus for this system. **10 Marks**

ii) Determine the region where the Poincare-Bendixson theorem can be applied to show the existence of a periodic orbit. **10 Marks**

Q 2: 25 Marks

i) Consider the function $V = \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2 + \frac{1}{3}x_3^3$. Show that this is a possible candidate for Lyapunov function. Study stability of the origin of the system $\dot{x_1} = x_2,\ \dot{x_2} = -x_1^3 - x_2^3 - x_3^3,\ \dot{x_3} = x_2 - x_3$. **10 Marks**

ii) Consider the system $\dot{x_1} = -x_1 + x_1x_2,\ \dot{x_2} = \frac{x_1}{1+x_1^2}$. Find the function $\phi(x_1)$ in Lyapunov function of the form $V = \phi(x_1) + x_2^2$ so that the origin is asymptotically stable. Is the origin globally asymptotically stable **15 Marks**

Q 3: 20 Marks

Consider the system $\dot{x_1} = ax_1 - x_1x_2,\quad \dot{x_2} = bx_1^2 - cx_2$, where a and c are positive constants with $c > a$. Let $D = \left\{(x_1, x_2)^T \in \mathbb{R}^2 | x_2 \geq 0\right\}$.

i) Find the value of b for which the system is a gradient system and find potential function U. **10 Marks**

ii) Show, using Bendixson's criterion, that there are no periodic orbits through any point in D. **10 Marks**

Q 4: 20 Marks

Find the limit cycle of the system $\dot{x_1} = x_2 + x_1(x_1^2 + x_2^2 - 1)$, $\dot{x_2} = -x_1 + x_2(x_1^2 + x_2^2 - 1)$ and investigate its stability. Also find the complete solution of the system.

Q 5: 15 Marks

Consider the one dimensional system $\dot{x} = \alpha - x - \exp(-x)$, $\alpha \in \mathbb{R}$. Show that the system exhibits saddle node bifurcation and determine the critical points and the bifurcation value for this differential equation. Draw the bifurcation diagram.
