Mahindra University Hyderabad
École Centrale School of Engineering
Minor I
Program: B. Tech. Branch: CM Year: 2nd Semester: Fall
Subject: Real Analysis (MA2104)

Date: 16/09/2023
Time Duration: 1.5 Hours

Start Time: 10.00 AM
Max. Marks: 20

Instructions:

1. Each question carries 5 marks.
2. All questions are compulsory.
3. Please start each answer on a separate page.

Q 1: 5 marks

Answer by **True** or **False** the following questions:

(a) Suppose S is an ordered set, and $E \subset S$. If there exists a $\beta \in S$ such that $x \geq \beta$ for every $x \in E$, we say that E is bounded above.

(b) If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for every $\epsilon > 0$, then $a = 0$.

(c) For every $n \in \mathbb{N}$, let E_n be a countable set. If $S = \cup_{n=1}^{\infty} E_n$, then S is also uncountable.

(d) Every nonempty set of real numbers that has an upper bound also has a supremum in $\mathbb{R}$.

(e) For any two sets A and B, if $A \sim B$, then either A and B are finite with the same number of elements, or A and B are both countable or A and B are both uncountable.

Q 2: 5 marks

(a) Let S be a set. Define an order on S.

(b) Suppose S is an ordered set, $E \subset S$. Define the least upper bound (or supremum) of E.

(c) Explain the Archimedean property of real numbers.

(d) Define "at most countable" and "uncountable" sets.

Q 3: 5 marks

(a) Find infimum and supremum of the following sets in $\mathbb{R}$. [3 Marks]
(i) $\{-2, -\frac{3}{2}, -\frac{4}{3}, -\frac{5}{4}, \cdots, -\frac{n+1}{n}, \cdots\}$, (ii) $\{1 - \frac{(-1)^n}{n} \mid n \in \mathbb{N}\}$, (iii) $\{x \in \mathbb{R} \mid x+2 \geq x^2\}$.

(b) Let A be the set of all positive rationals p such that $p^2 < 2$ i.e.,

$$A = \{p \in \mathbb{Q} \mid p > 0,\ p^2 < 2\}$$

then show that A contains no largest number. [2 Marks]

(OR)

(b′) If x and y are real numbers with $x < y$, then there exists an irrational number $z \notin \mathbb{Q}$ such that $x < z < y$. [2 Marks]

Q 4: 5 marks

(a) Define a relation $\sim$ as follows: For any two sets A and B, $A \sim B$ if and only if there exists a bijection $f : A \to B$. Show that $\sim$ is an equivalence relation. [3 Marks]

(b) Show that the set of all irrational numbers is uncountable. [2 Marks]

(OR)

(b′) Show that the set of all integers is countable. [2 Marks]
