Mahindra University Hyderabad

École Centrale School of Engineering
End-semester Examination

SE23UCAM020

Program: B. Tech. Branch: AI, CM, CE, CSE, ECM, NT, BTCM Year: II Semester: II
Subject: Numerical Methods (MA2208)

Date: 26/05/2025 Start Time: 10:00 AM
Time Duration: 3 Hours Max. Marks: 100

Instructions:

1) Answer all the questions.
2) All questions are self-explanatory; no clarification will be provided during the exam.
3) Use of one non-programmable scientific calculator is allowed.

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1. (a) (10 points) A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval $(0, 1)$. Perform four iterations of the secant method to approximate the root.

   (b) (10 points) Perform three iterations of the power method to find the dominant eigenvalue and the corresponding eigenvector of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -3 \\ 2 & 4 & 4 \end{bmatrix},$$

   using the initial guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

2. (a) (10 points) Consider the composite trapezoidal rule to approximate the integral

$$\int_0^{\pi} \sin(x)\, dx.$$

   Determine the step size h such that the absolute error in the approximation is less than 2×10^{-5}.

   (b) (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -1 \\ 2 & 1 & 0 \end{bmatrix}.$$

   Compute the following norms of A and list them in increasing order:

   (i) Frobenius norm $\|A\|_F$
   (ii) 1-norm $\|A\|_1$
   (iii) Infinity norm $\|A\|_\infty$

P.T.O.

(iv) 2-norm $\|A\|_2$

3. (a) (10 points) Using the method of undetermined coefficients find the constants A, B, and C in the following quadrature formula

$$\int_a^b f(x)\,dx \approx Af(a) + Bf\left(\frac{a+b}{2}\right) + Cf(b).$$

(b) (10 points) Apply the quadrature rule derived above in 3(a) to approximate the integral

$$\int_{-\pi}^{\pi} \cos(x)\,dx,$$

and compute the approximation error.

4. (a) (10 points) Solve the following system of equations using Doolittle's method:

$$\begin{aligned} 2x + 3y + z &= 9 \\ 4x + 7y + 5z &= 24 \\ 6x + 18y + 10z &= 54 \end{aligned}$$

(b) (10 points) Answer the following based on the given data:

x	$f(x)$
1	1
2	8
3	27
4	64

(i) Construct the divided difference table for the given data.
(ii) Use Newton's divided difference method to find the interpolating polynomial $P(x)$.
(iii) Expand and simplify the polynomial $P(x)$.
(iv) Verify that the resulting polynomial matches all the given data points.

5. (a) (10 points) Solve the initial value problem

$$y' = -2xy^2, \quad y(0) = 1,$$

at $x = 0.6$ using the Taylor series method of order two with step size $h = 0.2$.

(b) (10 points) Consider the initial value problem

$$y' = -5y, \quad y(0) = 1.$$

Discuss the stability of the following second-order Runge–Kutta method when applied to the above problem.

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right) \\ y_{n+1} &= y_n + k_2 \end{aligned}$$

Comment specifically on whether the step size $h = 0.5$ is appropriate from a stability point of view.

End of Question Paper