Exercise 4 - Solution

Task 4.1

1.

$$\lambda = \frac{90°}{2} = 45°$$
$$\eta = 90° - \lambda - \varepsilon_{min} = 42°$$
$$\rho = arcsin\left(\frac{sin(\eta)}{cos(\varepsilon_{min})}\right) = 42.07°$$
$$h = r_E\left(\frac{1}{sin(\rho)} - 1\right) = 3141km$$

2.

$$\lambda = \frac{90°}{3} = 30°$$
$$\rho = 57.12°$$
$$h = 1217km$$

Task 4.2

given: $R_E = 6378km$, $\mu_E = 398600\frac{km^3}{s^2}$, $h = 900km$, $J_2 = 1082.63 \cdot 10^{-6}$, $\tau_e = 86164.10555 + 0.015Cs$, $\tau_s = 3.155815 \cdot 10^7 s$

1.

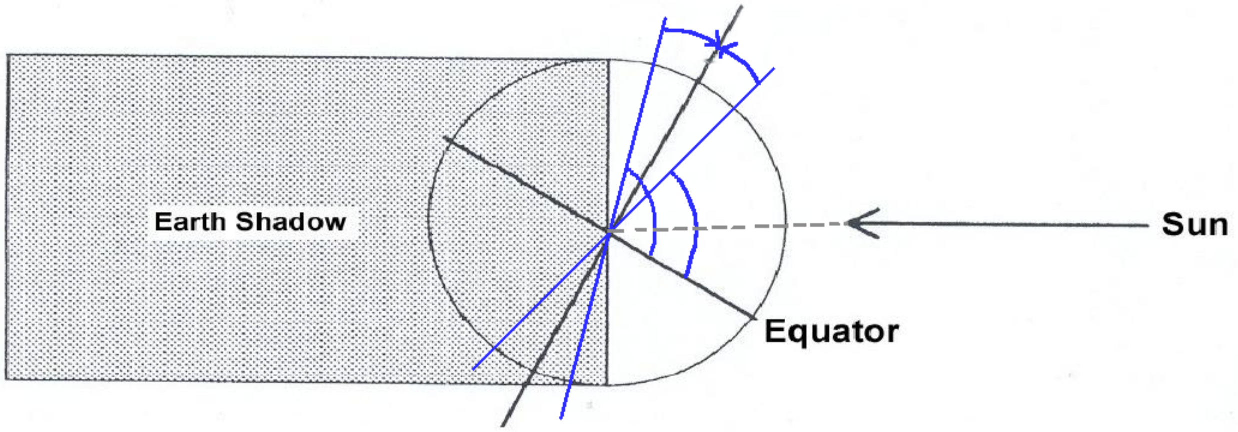
$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} = 2\pi\sqrt{\frac{(h+R_E)^3}{\mu_E}} = 6179.16s$$
$$v = \sqrt{\frac{\mu_E}{a}} = \sqrt{\frac{\mu_E}{R_E + h}} = 7.40\frac{km}{s}$$

2.

$$\Delta\Omega_{day} = 360°\frac{\tau_e}{\tau_s} = 0.983\frac{deg}{day}$$
$$\Delta\Omega_{rev} = 360°\frac{\tau_e}{\tau_s} \cdot \frac{T}{\tau_e} = 0.0705\frac{deg}{rev}$$
$$\Delta\Omega_{rev} = -\frac{3\pi J_2 R_E^2}{a^2(1-\epsilon^2)^2} \cdot cos(i)$$

circular orbit $\Rightarrow$ ϵ=0

$$\Rightarrow i = arccos\left(-\frac{\Delta\Omega_{rev} a^2}{3\pi J_2 R_E^2}\right) = 99.03°$$



$$\beta^* = arcsin(\frac{R_E}{R_E + h}) = 61.20°$$

- orbit 1:

$$\beta_1 = 90° - 23.5° + 9.03° = 75.53°$$
$$\Rightarrow \beta_1 > \beta^* \Rightarrow \text{no eclipse}$$

- orbit 2:

$$\beta_2 = 90° - 23.5° - 9.03° = 57.53°$$
$$\Rightarrow \beta_2 < \beta^* \Rightarrow \text{eclipse}$$

$$F_e = \frac{1}{\pi} arccos\left(\frac{\sqrt{h^2 + 2R_E h}}{(R_E + h)cos(\beta_2)}\right) = 0.1456 \approx 15\%$$
$$T_{ecl} = T \cdot F_e = 14.99min$$

Both orbits will have an eclipse since the earth rotation axis stays fixed. The shadow periods of orbit 2 occur half a year later for orbit 1 which was originally eclipse free.

Task 4.3

given: $R_E = 6378km$, $\mu_E = 398600\frac{km^3}{s^2}$, $J_2 = 1082.63 \cdot 10^{-6}$, $i_e = 0°$,$i = 53°$

1.

$$T = 2\pi\sqrt{\frac{(R_E + h)^3}{\mu_E}} = 5801.07s$$
$$v = \sqrt{\frac{\mu_E}{(R_E + h)}} = 7.55\frac{km}{s}$$

2.

$$\Delta\Omega = -\frac{3\pi J_2 R_E^2}{(R_E + h)^2(1 - \epsilon^2)^2} \cdot cos(i)$$

a) circular orbit $\Rightarrow \epsilon$=0

$$\Delta\Omega = -\frac{3\pi J_2 R_E^2}{(R_E+h)^2} \cdot cos(i) = -0.00513\frac{rad}{rev} = -0.076197\frac{rad}{day}$$

b)

$$t_{ecl} = \frac{2\cdot\beta^*}{360^\circ} = \frac{2\cdot arcsin\left(\frac{r_E}{r_E+h}\right)}{360^\circ} = 2129.19s = 35.49min$$

c) given: $r_a = (600 + 6378)km$, $r_b = (900 + 6378)km$

$$\Delta v_1 = \sqrt{\frac{\mu_E}{r_a}}\left(\sqrt{\frac{2r_b}{r_a+r_b}} - 1\right) = 0.07911\frac{km}{s}$$

$$\Delta v_2 = \sqrt{\frac{\mu_E}{r_b}}\left(1 - \sqrt{\frac{2r_a}{r_a+r_b}}\right) = 0.07828\frac{km}{s}$$