SSD – Exercise 2

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Task 2.1

given : $a = 58000km, \epsilon = 0.7, i = 51°, \mu_E = 3.986 \cdot 10^5 \frac{km^3}{s^2}, J_2 = 1082.7 \cdot 10^{-6}, t_{Earth} = 24h$
to be determined: $T, r_a, r_p, \Delta\Omega, \Delta\omega$

1. $$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} = 139012.2944s = 2316.87min$$

2. $$r_a = a(1+\epsilon) = 17400km$$
$$r_p = a(1-\epsilon) = 98600km$$

3. $$\Delta\Omega = -\frac{3\pi J_2 R_E^2}{a^2(1-\epsilon^2)^2} cos(i) \cdot \frac{t_{Earth}}{T} = -0.00022\frac{rad}{day} = -0.01273\frac{deg}{day} \approx -1.27 \cdot 10^{-2}\frac{deg}{day}$$

$$\Delta\omega = \frac{3\pi J_2 R_E^2}{2a^2(1-\epsilon^2)^2}(4-5sin^2(i)) \cdot \frac{t_{Earth}}{T} = 0.0000865\frac{rad}{day} = 0.004957\frac{deg}{day} \approx 4.96 \cdot 10^{-3}\frac{deg}{day}$$

Task 2.2

given: $i = 97.76°, \epsilon = 0.0063, \Omega = 352.26°, T = 97.7min, \omega = 213.57°, \nu = 146.03°$
to be determined: a, average angular velocity, $lat_{SSP}, long_{SSP}$
SSP = subsatellite point

1. $$a = \sqrt[3]{\frac{T^2}{4\pi^2} \cdot \mu_E} = 7026.78km$$

2. $$\text{average angular velocity} = \frac{2\pi}{T} = \frac{360°}{T} = 3.685\frac{deg}{min}$$

3. $$lat_{SSP} = arcsin\left(sin\left(i \cdot \frac{2\pi}{360°}\right) \cdot sin\left(\nu \cdot \frac{2\pi}{360°}\right)\right) = 33.62°$$
$$long_{SSP} = arctan\left(cos\left(i \cdot \frac{2\pi}{360°}\right) \cdot tan\left(\nu \cdot \frac{2\pi}{360°}\right)\right) = 5.198°$$

Task 2.3

given: $a = 26562km, \epsilon = 0.77, i = 63.4°$
to be determined: $\Delta\Omega, \Delta\Phi$

1. J_2 decreases the value of $\Delta\Omega$

$$\Delta\Omega = -\frac{3\pi J_2 R_E^2}{a^2(1-\epsilon^2)^2} cos(i) = -0.0015895 \frac{rad}{rev} = -0.09107 \frac{deg}{rev}$$

2.

$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} = 718.044min = \frac{718.044}{23 \cdot 60 + 56 + 4/60} \text{ sidereal days} = 0.5 \text{ sidereal days}$$

After 100 sidereal days: $\frac{100 \text{ sidereal days}}{T} = 200$ revolutions

$$\Rightarrow \Delta\Omega_{100} = 200\ rev \cdot (-0.09107) \frac{deg}{rev} = -18.214deg$$