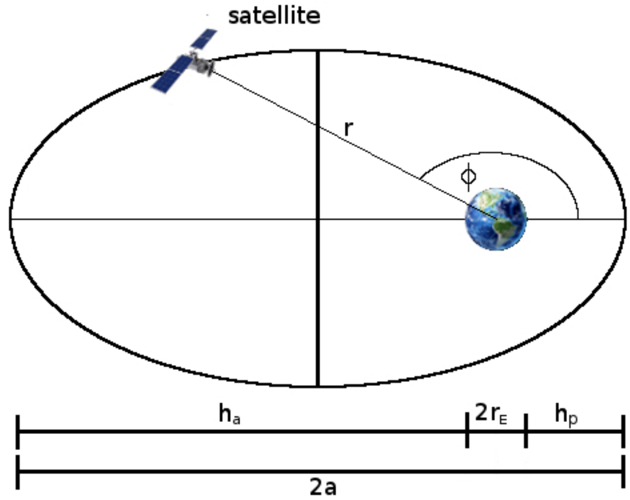
Formeln SSD

Bahn- und Orbitdinge

3. Keplersches Gesetz (Quadrate der Umlaufdauern proportional zu den Kuben der großen Halbachsen)

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Ellipsendinge:



$$2a = h_a + h_p + 2r_E$$

$$r_p = h_p + r_E \qquad r_a = h_a + r_E$$

$$r_p = a(1-\varepsilon) \qquad r_a = a(1+\varepsilon)$$

$$\varepsilon = \frac{r_p - r_a}{r_p + r_a}$$

Vis-Viva-Gleichung/Binet-Gleichung:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Vis-Viva-Gleichung für Kreis:

$$r = a \Rightarrow v = \sqrt{\frac{\mu}{r}}$$

für Erde:

$$v = \sqrt{\frac{\mu}{r_E}} = 7.905\frac{km}{s}$$

(= erste kosmische Geschwindigkeit = minimale Geschwindigkeit, die ein Objekt haben muss, um in einen Erdorbit zu gelangen)

Vis-Viva-Gleichung für Parabel:

$$a \to \infty \Rightarrow v = \sqrt{\frac{2\mu}{r}}$$

für Erde:

$$v = \sqrt{\frac{2\mu}{r_E}} = 11.179\frac{km}{s}$$

(= zweite kosmische Geschwindigkeit = minimale Geschwindigkeit, die ein Objekt haben muss, um das Gravitationsfeld der Erde zu verlassen)

Geschwindigkeitsdinge für Ellipse:

maximale Geschwindigkeit (Periapsis):

$$v_{max} = \sqrt{\mu\left(\frac{2}{h_p + r} - \frac{1}{a}\right)}$$

minimale Geschwindigkeit (Apoapsis):

$$v_{min} = \sqrt{\mu\left(\frac{2}{h_a + r} - \frac{1}{a}\right)}$$

wahre Anomalie (Φ, ν, ...)

$$r = a\frac{1-\varepsilon^2}{r \cdot \varepsilon \cdot cos(\nu)} \Rightarrow \nu = arccos\left(a\frac{1-\varepsilon^2}{r^2 \cdot \varepsilon}\right)$$

wahre Anomalie ist auch über die exzentrische Anomalie berechenbar:

$$tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}} \cdot tan\left(\frac{E}{2}\right)$$

$$E - \varepsilon \cdot sin(E) = \frac{2\pi}{T}(t - t_{perigee})$$

right ascension of the ascending node:

$$\Delta\Omega_{rev} = -\frac{3\pi J_2 r_E^2}{a^2(1-\varepsilon^2)^2}cos(i) \qquad \text{in} \left[\frac{\text{rad}}{\text{rev}}\right]$$

$$\Delta\Omega_{rev} = -\frac{3\pi J_2 r_E^2}{a^2(1-\varepsilon^2)^2}cos(i) \cdot \frac{360°}{2\pi} \qquad \text{in} \left[\frac{\text{deg}}{\text{rev}}\right]$$

$$\Delta\Omega_{day} = -\frac{3\pi J_2 r_E^2}{a^2(1-\varepsilon^2)^2}cos(i) \cdot \frac{\tau_E}{T} \qquad \text{in} \left[\frac{\text{rad}}{\text{day}}\right]$$

$$\Delta\Omega_{rev} = -\frac{3\pi J_2 r_E^2}{a^2(1-\varepsilon^2)^2}cos(i) \cdot \frac{360°}{2\pi} \cdot \frac{\tau_E}{T} \qquad \text{in} \left[\frac{\text{deg}}{\text{day}}\right]$$

argument of perigee:

$$\Delta\omega = \frac{3\pi J_2 r_E^2}{2a^2(1-\varepsilon^2)^2}(4 - 5sin^2(i)) \qquad \text{in} \left[\frac{\text{rad}}{\text{rev}}\right]$$

Subsatellitenpunkt

latitude of SSP:

$$\Theta = arcsin(sin(i) \cdot sin(\omega + \nu))$$

longitude of SSP:

$$\lambda = arccos\left(\frac{cos(\omega+\nu)}{cos(\Theta)} + \Omega + \lambda_{greenwich}\right)$$

Hohmann-Transfer:

große Halbachse der Transferellipse:

$$a_{transfer} = \frac{r_A + r_B}{2}$$

Transferdauer:

$$T_{transfer} = \pi\sqrt{\frac{a_{transfer}^3}{\mu}}$$

Geschwindigkeitsänderung, um ersten Orbit zu verlassen (beschleunigen):

$$\Delta v_1 = \sqrt{\frac{\mu}{r_A}}\left(\sqrt{\frac{2r_B}{r_A + r_B}} - 1\right)$$

Geschwindigkeitsänderung, um auf zweiten Orbit zu kommen (abbremsen):

$$\Delta v_2 = \sqrt{\frac{\mu}{r_B}}\left(1 - \sqrt{\frac{2r_A}{r_A + r_B}}\right)$$

noch mehr Bahnparameterdinge:

earth angular radius:

$$\rho = arcsin\left(\frac{r_E}{r_E + h}\right)$$

maximum nadir angle:

$$\eta_{max} = arcsin(sin(\rho) \cdot cos(\varepsilon_{min}))$$

maximum earth central angle:

$$\lambda_{max} = 90^\circ - \varepsilon_{min} - \eta_{max}$$

maximum distance:

$$D_{max} = r_E\left(\frac{sin(\lambda_{max})}{sin(\eta_{max})}\right)$$

minimum earth central angle:

$$\lambda_{min} = arcsin(sin(90^\circ - i) \cdot sin(\mathrm{lat}_{gs}) + cos(90^\circ - i) \cdot cos(\mathrm{lat}_{gs}) \cdot cos(\mathrm{long}_{gs} - \mathrm{L}_{node} + 90^\circ))$$

minimum nadir angle:

$$\eta_{min} = arctan\left(\frac{sin(\rho) \cdot sin(\lambda_{min})}{1 - sin(\rho) \cdot cos(\lambda_{min})}\right)$$

maximum elevation angle:

$$\varepsilon_{max} = 90^\circ - \lambda_{min} - \eta_{min}$$

minimum distance:

$$D_{min} = r_E\left(\frac{sin(\lambda_{min})}{sin(\eta_{min})}\right)$$

maximum angular rate:

$$\dot{\Theta}_{max} = \frac{2\pi(r_E + h)}{T \cdot D_{min}}$$

azimuth range:

$$\Delta\Phi = 2 \cdot arccos\left(\frac{tan(\lambda_{min})}{tan(\lambda_{max})}\right)$$

time in view:

$$T_{view} = \frac{T}{180^\circ} \cdot arccos\left(\frac{cos(\lambda_{max})}{cos(\lambda_{min})}\right)$$

Eclipsedinge:

$$\beta = arcsin\left(\frac{r_E}{r_E + h}\right)$$

eclipsed fraction of a circular orbit:

$$F_e = \frac{1}{\pi}\left(\frac{\sqrt{h^2 + 2r_E \cdot h}}{(r_E + h) \cdot cos(\beta)}\right)$$

Dezibeldinge:

$$P[W] = 1W \cdot 10^{\frac{P[dBW]}{10}}$$

$$P[dBW] = 10 \cdot log_{10}\left(\frac{P[W]}{1W}\right)$$

gain:

$$G = 10 \cdot log_{10}\left(\frac{P_1}{P_2}\right)$$

EIRP:

$$\text{EIRP} = G + P$$

Thermaldinge:

$$Q = A_r \cdot \varepsilon_{IR} \cdot \sigma \cdot T^4$$

$$\dot{Q} = \lambda \cdot a \cdot b \cdot \frac{T_1 - T_2}{c} \qquad \text{oder auch} \qquad \dot{Q} = K \cdot A \cdot \frac{T_1 - T_2}{L}$$

series conduction:

$$\sigma_{tot} = \frac{1}{\frac{1}{\sigma_1} + \frac{1}{\sigma_2} + \frac{1}{\sigma_3} + \dots}$$

$$\sigma = \frac{L}{K \cdot A} \qquad \text{oder} \qquad \sigma = \frac{1}{\alpha \cdot A}$$

Free Space Path Loss:

$$L = \left(\frac{4\pi \cdot d}{\lambda}\right)^2 = \left(\frac{4\pi \cdot d \cdot f}{c}\right)^2$$

oder auch (ungenauer)

$$L = 92.4dB + 20 \cdot log_{10}(F[GHz]) + 20 \cdot log_{10}(d[km])$$

thermal noise:
B... bandwidth, R ... resistor, T ... temperature, k ... Boltzmann constant ($1.38 \cdot 10^{-23} \frac{J}{K}$)

$$V_n = \sqrt{4kTBR}$$

Noise Power:
T_e = system receiver noise temperature

$$N[dBW] = 10 \cdot log_{10}(kT_eB)$$

Noise Figure:

$$NF[dB] = 10 \cdot log_{10}\left(1 + \frac{T_e}{290°}\right) = 10 \cdot log_{10}(F)$$

Noise Factor:

$$F = \frac{\left(\frac{S}{N}\right)_{in}}{\left(\frac{S}{N}\right)_{out}} = 1 + \left(\frac{T_e}{290°}\right)$$

Fermienergie:

Wahrscheinlichkeit, dass ein Teilchen mit Energie E

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{k \cdot T}}}$$

Power System Dinge:

solar array power during daylight:

$$P_{sa} = \frac{\frac{P_e \cdot T_e}{X_e} + \frac{P_d \cdot T_d}{X_d}}{T_d}$$

P_e... power in eclipse, T_e ... time in eclipse, P_d ... power in daylight, T_d ... time in daylight, X_e... path efficiency during eclipse, X_d ... path efficiency during daylight

spacecraft power budget:

$$P_{sg} = \frac{\frac{P_e \cdot T_e}{\eta_L \cdot \eta_b} + \frac{P_s \cdot T_s}{\eta_L}}{T_s}$$

P_{sg}... total power from solar array, η_L ... losses between solar array and loads, η_b ... losses of battery, P_S ... available power in sunlight, P_e... power demand in eclipse

wichtige Parameter für BOL/EOL-Berechnungen:

- η ... production efficiency of solar cells (14-22%)
- X ... path efficiency from solar array through batteries to loads (eclipse and daytime): $X_e = 0.65, X_d = 0.85$ (direct energy transfer), $X_e = 0.60, X_d = 0.80$ (peak power tracking)
- I_d ... inherent degradation (≈ 0.77, ranges from $0.49 - 0.88$)
- Θ ... angle between array normal and sun vector; typically use worst-case sun-angle
- L_d ... life degradation: micrometeorites, radiation, etc. (2-4% per year)

$$L_d = (1 - \text{degradation per year})^{\text{satellite life}}$$

From Begin of Life to End of Life

$$P_o = \eta \cdot 1358 \frac{W}{m^2} \qquad \text{(output power)}$$

$$P_{BOL} = P_o \cdot I_d \cdot cos(\Theta)$$

$$P_{EOL} = L_d \cdot P_{BOL}$$

solar array size to meet power requirement:

$$A_{sa} = \frac{P_{sa}}{P_{EOL}}$$

mass of solar array ranges from 14 to $47 \frac{W}{kg}$:

$$M_{sa} = 0.04 \cdot P_{sa} \qquad (\text{for } 25 \frac{W}{kg})$$

Battery Sizing

Power need:

$$P_{avg} = V_{bus} \cdot I$$

$$Ah_{avg} = \frac{T_e}{1h} \cdot I$$

$$Ah_{total} = \frac{Ah_{avg}}{DoD}$$

Capacity:

$$C_r = \frac{P_{avg} \cdot T_e}{DoD \cdot N_{bat} \cdot \eta}$$