Exercise 6 - Solution

Task 6.1

formula to convert W to dBW:

$$P[dBW] = 10 \cdot log_{10}\left(\frac{P[W]}{1W}\right)$$

formula to convert dBW to W:

$$P[W] = 1W \cdot 10^{\frac{P[dBW]}{10}}$$

formula to calculate the gain G:

$$G = 10 \cdot log_{10}\left(\frac{P_1}{P_2}\right) \Rightarrow \frac{P_1}{P_2} = 10^{\frac{G}{10}}$$

1a)

- $P[W] = 10W$

$$P[dBW] = 10 \cdot log_{10}\left(\frac{10W}{1W}\right) = 10dBW$$

- $P[W] = 5 \cdot 10^{-12}$

$$P[dBW] = 10 \cdot log_{10}\left(\frac{10^{-12}W}{1W}\right) = -120dBW$$

- $P[W] = 20kW$

$$P[dBW] = 10 \cdot log_{10}\left(\frac{2 \cdot 10^{4}W}{1W}\right) = 43.01dBW$$

- $P[dBW] = -178dBW$

$$P[W] = 1W \cdot 10^{\frac{-178dBW}{10}} = 1.58 \cdot 10^{-18}W$$

1b)

- $G = 20dB$

$$\frac{P_1}{P_2} = 10^{\frac{20}{10}} = 100$$

- $G = 0dB$

$$\frac{P_1}{P_2} = 10^{\frac{0}{10}} = 1$$

- $G = 6dB$

$$\frac{P_1}{P_2} = 10^{\frac{6}{10}} = 3.988$$

- $G = -15dB$

$$\frac{P_1}{P_2} = 10^{\frac{-15}{10}} = 0.0316$$

1b)

$\frac{G}{T} = \frac{13dB}{K}$

$$\frac{P_1}{P_2} = 10^{\frac{13}{10}} = 19.95$$

$$\Rightarrow \frac{P_1}{P_2} \cdot \frac{1}{K} = \frac{19.95}{K}$$

2a)

$G = -5dB + 20dB - 2.5dB$

$$G_{log} = -5dB + 20dB - 2.5dB = 12.5dB$$

$$G_{lin} = 10^{\frac{12.5dB}{10}} = 17.783$$

2b)

$EIRP = P + G = 10dBW + 30dBi$

$$EIRP_{log} = 10dBW + 30dBi = 40dBW$$

$$EIRP_{lin} = 1W \cdot 10^{\frac{40dBW}{10}} = 10,000W$$

3.

$G = 30dBm$

$$P[W] = \frac{1}{1000}W \cdot 10^{\frac{30}{10}} = 1W$$

$$P[dBW] = 10 \cdot log_{10}\left(\frac{1W}{1W}\right) = 0dBW$$

4.

$P = -178dBW$

$$P[W] = 1W \cdot 10^{\frac{-178dBW}{10}} = 1.58 \cdot 10^{-18}W$$

$$P[dBm] = 10 \cdot log_{10}(1000 \cdot 1.58 \cdot 10^{-18}) = -148.02dBm$$

Task 6.2

$$Q = A_r \varepsilon_{IR} \sigma T_r^4$$

$$T_{r_1}[Q = 210W] = \left[\frac{210W}{0.5625m^2 \cdot 0.85 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4}}\right]^{\frac{1}{4}} = 296.74K$$

$$T_{r_2}[Q = 100W] = 246.44K$$

$$Q = A_r \varepsilon_{IR} \sigma T_r^4 = 0.5625m^2 \cdot 0.85 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot (270K)^4 = 144.1W$$

$$\Rightarrow P = 144.1W - 100W = 44.1W \text{ (just heater)}$$

$$A_r = \frac{Q}{\varepsilon_{IR} \sigma T_r^4} = \frac{210W}{0.85 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot (310K)^4} = 0.4718m^2$$

$$Q = A_r \varepsilon_{IR} \sigma T_r^4 = 0.4718m^2 \cdot 0.5625m^2 \cdot 0.85 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot (220K)^4 = 120.8W$$

$$\Rightarrow P = 120.8W - 100W = 20.8W \text{ (partially isolated heater)}$$

louver:

$$Q = A_r \varepsilon_{IR} \sigma T_r^4 = 0.8 \cdot 0.5625m^2 \cdot 0.85 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot (310K)^4 = 200.3W$$

optimum: insulate $0.5625m^2 - 0.4718m^2 = 0.0907m^2$ and use a heater $20.8W$ (ok as well: $44.1W$)