Exercise 2 - Solution

Task 2.1

given : $a = 58000km, \epsilon = 0.7, i = 51°, \mu_E = 3.986 \cdot 10^5 \frac{km^3}{s^2}, J_2 = 1082.7 \cdot 10^{-6}, t_{Earth} = 24h$
to be determined: $T, r_a, r_p, \Delta\Omega, \Delta\omega$

1.
$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} = 139012.2944s = 2316.87min$$

2.
$$r_a = a(1+\epsilon) = 17400km$$
$$r_p = a(1-\epsilon) = 98600km$$

3.
$$\Delta\Omega = -\frac{3\pi J_2 R_E^2}{a^2(1-\epsilon^2)^2} cos(i) \cdot \frac{t_{Earth}}{T} = -1.86 \cdot 10^{-4} \frac{rad}{day} \approx 0.0106 \frac{deg}{day}$$
$$\Delta\omega = \frac{3\pi J_2 R_E^2}{2a^2(1-\epsilon^2)^2}(4 - 5sin^2(i)) \cdot \frac{t_{Earth}}{T} = 1.4451 \cdot 10^{-4} \frac{rad}{day} \approx 0.0083 \frac{deg}{day}$$

Task 2.2

given: $i = 97.76°, \epsilon = 0.0063, \Omega = 352.26°, T = 97.7min, \omega = 213.57°, \nu = 146.03°$
to be determined: a, average angular velocity, $lat_{SSP}, long_{SSP}$
SSP = subsatellite point

1.
$$a = \sqrt[3]{\frac{T^2}{4\pi^2} \cdot \mu_E} = 7026.78km$$

2.
$$\text{average angular velocity} = \frac{2\pi}{T} = \frac{360°}{T} = 3.685 \frac{deg}{min}$$

3.
$$lat_{SSP} = arcsin\left(sin(i) \cdot sin(\nu + \omega)\right) = -0.4°$$
$$long_{SSP} = arccos\left(\frac{cos(\nu + \omega)}{cos(lat_{SSP})}\right) + \Omega + \lambda_{greenwich} = 681.01° = -38.99°$$

Task 2.3

given: $a = 26562km, \epsilon = 0.77, i = 63.4°$
to be determined: $\Delta\Omega, \Delta\Phi$

1. J_2 decreases the value of $\Delta\Omega$

$$\Delta\Omega = -\frac{3\pi J_2 R_E^2}{a^2(1-\epsilon^2)^2} cos(i) = -0.0015895\frac{rad}{rev} = -0.09107\frac{deg}{rev}$$

2.

$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} = 718.044min = \frac{718.044}{23 \cdot 60 + 56 + 4/60} \text{ sidereal days} = 0.5 \text{ sidereal days}$$

After 100 sidereal days: $\frac{100 \text{ sidereal days}}{T} = 200$ revolutions

$$\Rightarrow \Delta\Omega_{100} = 200\ rev \cdot (-0.09107)\frac{deg}{rev} = -18.214deg$$