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Spacecraft System Design

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$$m \cdot \ddot{\vec{r}} = F = -G \cdot \frac{m_1 m_2}{r^2}$$

G ist die *universal gravity constant*: $6.674 \cdot 10^{-11} \frac{Nm^2}{kg^2}$

$$\ddot{\vec{r}} = \frac{\mu}{\|r\|^2}$$

μ	gravity constant of the specific gravity attractor.	
μ_E	*gravity constant of earth*:	$398600 \frac{km^3}{s^2}$
m_E	*mass of earth*:	$5.97219 \cdot 10^{24} kg$

Die Lösungen dieser Gleichung sind Kegelschnitte. Außerdem gilt: $\mu_E = G \cdot m_E$

$$r = \frac{p}{1 + \varepsilon \cdot \cos\varphi}$$

r	radius towards focal point
ε	eccentricity of the orbit
p	parameters (spartial extensions of orbit)
φ	true anomaly

Interrelationships between parameters:
r_a apocenter, r_p pericenter

$$a = \frac{p}{1-\varepsilon^2} \Rightarrow p = a(1-\varepsilon^2) = r_p(1+\varepsilon) = r_a(1-\varepsilon)$$
$$a = \frac{r_a + r_p}{2},\ r_a = a(1+\varepsilon),\ r_p = a(1-\varepsilon)$$
$$\varepsilon = \frac{r_a - r_p}{r_a + r_p}$$

For the specific case of circular orbits:

- gravitational acceleration in distance a to gravitational center: $\frac{\mu}{a^2}$
- centrifugal acceleration: $\omega^2 \cdot r$ with $\omega = \frac{2\pi}{T}$ angular velocity and T the orbital period

$$\left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow \frac{\mu}{a^2} = \left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

which also holds for general ellipses

$$v_{\text{circle}} = \frac{2\pi a}{T} = \sqrt{\frac{a^3}{\mu}}$$

Potential energy: $E_{\text{pot}} = \frac{-\mu m}{r} + C$
Kinetic energy: $E_{\text{kin}} = \frac{mv^2}{2}$

$$E = E_{\text{kin}} + E_{\text{pot}} = \frac{mv^2}{2} - \frac{\mu m}{r} \overset{\substack{\varepsilon\to 1,\\ v_a\to 0,\\ r_a\to 2a}}{=} -\frac{\mu m}{2a}(\text{constant})$$

v_a velocity at apocenter

Binet's equation, *Vis-Viva equation*:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

For circles: $r = a$, $v = \sqrt{\frac{\mu}{a}}$
v first cosmic velocity: needed to bring a satellite on closed orbit(without perturbing forces, just gravity considered). In case of earth: $r = 6378km$, $\mu_E = 398600\frac{km^3}{s^2}$
$\Rightarrow 7.905\frac{km}{s}$ required as minimum velocity from the rocket launch to insert into an earth orbit.

A rocket launch in equatorial direction and in rotation direction of earth, we recieve a velocity component $0.463\frac{km}{s}$ for free earth rotation rate.

For parabola: $a \to \infty$, $v = \sqrt{\frac{2\mu}{r}}$ *v second cosmic velocity (escape velocity)*: is the minimum velocity applied to leave the gravitational field of the home planet. For earth: $11.179\frac{km}{s}$

Derive position of the satellite in the orbit plane, e.g. we want the true anomaly φ as the function of time $\varphi = f(t)$. There is no explicit solution but only an algorithm using the support variable eccentric anomaly E can be derived.

Keplers equation:

$$E - \varepsilon \cdot \sin(E) = \frac{2\pi}{T}(t - t_{\text{perigee}}) = M(t)$$

$M(t)$ average anomaly at time t, can be solved by numerical methods. From E, you can calculate r and φ as follows:

$$r\sin\varphi = a\sin E \cdot \sqrt{1-\varepsilon^2}$$
$$r\cos\varphi = a(\cos E - \varepsilon)$$
$$\Rightarrow \tan\frac{\varphi}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}} \cdot \tan\frac{E}{2}$$
$$r = a(1 - \varepsilon\cos E)$$

Satellite Ground Tracks:
Analyse the triangle KFB in spherical coordinates.

θ	*latitude*:	$\sin\theta = \sin i \sin(\omega + \varphi)$
λ	*longitude*:	$\cos\lambda_1 = \frac{\cos(\omega+\varphi)}{\cos\theta}$

$$\lambda = \lambda_1 + \Omega + \lambda_G$$
$$\lambda_G = \lambda_{G0} - 0.25068448°/\text{min} \cdot t \qquad \text{(Greenwich position versus } \gamma)$$

Draw the line from the satellite to earth center, the intersection with surface is specified by θ and λ (*subsatellite point*).

Perturbations:
gravitational potential of earth

$$u(r, \phi, \Lambda)$$

u gravitational at distance r and latitude ϕ and longitude Λ of the position.

$$u(r, \phi, \Lambda) = \frac{\mu}{r}\left(1+\sum_{n=2}^{\infty}\left[\left(\frac{R_E}{r}\right)^n \cdot J_2 \cdot P_{n0}\cos\phi + \sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \cdot (C_{nm}\cos(m\Lambda) + S_{nm}\sin(m\Lambda)) \cdot P_{nm}(\cos\phi)\right]\right)$$

spherical harmonic expansion with:

r	distance from the earth center	
P_{nm}	Legendre polynomials	
R_E	radius of earth	$6378km$

J_n, C_{nm}, S_{nm} are constants of body reflecting the mass distribution:
J_n zonal harmonic coefficient (mass distribution independent from longitude)
C_{nm}, S_{nm} earth's tesseral harminic coefficients (for $n \neq m$) or earth's sectoral harmonic coefficients (for $n = m$)

By far, the largest coefficient is J_n (related to equatorial bulge) by 3 orders of magnitude.
Example: Molniya orbits of $63,4°$ have minimum change in position of perigee and apogee.

Additional gravitational fields:
by other bodies in the solar system

$$a_d = \mu_d\sqrt{\vec{R}\cdot\vec{R}}$$

with $\vec{R} = \frac{\vec{r}_{sd}}{\|\vec{r}_d\|}$

$$\frac{a_d}{a_c} = \frac{M_d}{M_c}\left(\frac{\|\vec{r}_s\|}{\|\vec{r}_{sd}\|}\right)^3\sqrt{1+3\cos^2\beta} = 2\frac{M_d}{M_c}\left(\frac{\|\vec{r}_s\|}{\|\vec{r}_d\|}\right)^3$$

a_c	actual acceleration	

atmospheric drag:
Drag: $\vec{F}_0 = -\frac{1}{2}\rho S c_D \|\vec{v}_R\|^2 \frac{\vec{v}_r}{\|\vec{v}_r\|}$

v_r	velocity vector of the spacecraft relative to the atmosphere	
ρ	atmospheric density	
S	reference area of the spacecraft	
c_D	drag coefficient of the spacecraft (determined in wind channel)	default: $c_d \approx 2.5$

Main effect of altitude changes and changes of orbital period.

solar pressure:

$1AU$	*distance earth-sun*	$149.6 \cdot 10^6 km$
	energy density	$1372\frac{W}{m^2}$

from impulse conservation law:

$$p = \frac{1372W}{c \cdot m^2} = \frac{1372W\ s}{299793km\ m^2} = 0.46 \cdot 10^{-5}\frac{N}{m^2}$$

solar pressure is increased by reflection

$$p_{\text{eff}} = (1+r)p$$

p_{eff}	effective pressure
r	reflectivity factor

At equatorial orbits the solar pressure affects a change of eccentricity ε.

orbit change manouvers:
Based on impule conservation law.
Propulsion systems:

Hydrazin	$2.2\frac{km}{2}$	particle velocity
2-component	$3\frac{km}{2}$	particle velocity
ion thrusters	$30\frac{km}{2}$	particle velocity

$$\Delta v_s m_s = -\Delta m_s v_p$$

Δm_s	reduction of satellite mass due to ejected propellant
v_p	velocity of propellant ejection

$$\Delta v_s = -\frac{\Delta m_s}{m_s} v_p$$

$$\int_{v_0}^{v} dv = -v_p \int_{m_s}^{m_s - m_p} \frac{dm}{m}$$
$$\Delta v_s = v_p(\log_2 m_s - \log_2(m_s - m_p))$$

Rocket equation, *Ziolkowski equation*:

$$m_{\text{after}} = m_{\text{before}} \cdot e^{\frac{-\Delta v_s}{v_p}}$$

Sun and earth synchronous satellites :

m	n	corresponding altitude
1	14	$894km$
1	13	$567km$
1	12	$275km$
2	22	

minimum drift orbits: $\frac{n\pm1}{m} = k$, $k \in \mathbb{N}$, integer. orbits of successive days to fill the displacements between two successive orbits.

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Keplerian Orbit Transfer
Calculate a transfer between two circular orbits with radius r_A to r_B. The velocity at pericenter of the transfer ellipse:

$$v_p^2 = 2\mu\left(\frac{1}{r_A} - \frac{1}{r_A + r_B}\right) = 2\mu\frac{r_B}{r_A(r_A + r_B)}$$

The required Δv_A to inject from the transfer orbit:

$$\Delta v_A = v_P - v_A = \sqrt{\frac{\mu}{r_A}}\left(\sqrt{\frac{2r_B}{r_A + r_B}} - 1\right)$$

The required Δv to inject from the transfer orbit into orbit with r_B:

$$\Delta v_B = v_B - v_{\text{apo}} = \sqrt{\frac{\mu}{r_B}}\left(1 - \sqrt{\frac{2r_A}{r_A + r_B}}\right)$$

where v_B is the circular velocity at r_B.

The *Hohmann transfer* is the most energy-efficient transfer between two circular orbits.

$$\Delta v_{\text{total}} = \Delta v_A + \Delta v_B = \sqrt{\mu}\left[\sqrt{\left(\frac{2}{r_A} - \frac{2}{r_A + r_B}\right)} - \sqrt{\frac{1}{r_A}} + \sqrt{\frac{2}{r_B} - \frac{2}{r_A + r_B}} - \sqrt{\frac{1}{r_B}}\right]$$

2.3 Mission Analysis

Selection of orbit parameters for most appropriate orbits for the given task.

Earth synchronous orbits: after a given period of time the subsatellite's ground track repeats. *Rotation of Earth*: τ_E *sidereal day* ($= 86161.10555 + 0.015C[s]$ where C is the number of centuries after 2000). As Earth rotation is in eastward direction, the satellite orbit with respect to Earch seems to have west direction.

$$\Delta\Phi_R = \frac{\tau}{\tau_E} - 2\pi \quad \text{[rad/rev]}$$

τ	satellite orbit period

J_2 theory taking into account Earth oblateness, we can calculate the satellite orbit plane rotation:

$$\Delta\Omega = \frac{3\pi J_2 R_E^2 \cos i}{a^2(1-\varepsilon^2)^2} \quad \text{[rad/rev]}$$

total displacement at equator: $\Delta\Phi = \Delta\Phi_R - \Delta\Omega$.
For Earth synchronous orbits, the following property needs to be satisfied:

$$n\Delta\Phi = m2\pi$$

n	number of orbits before identical ground track repeats
m	number of Earth revolutions (day) before identical ground track repeats

Sun synchronous orbits:
the sunlight incidents repats
take into account the motion of Earth around Sun. Due to the Earth's orbit around Sun, the Earth/Sun-line intersect at points P moving westward.

$$\sigma = \frac{360°}{365.25 \text{ days}} \approx 1 \quad [°/\text{day}]$$

orbital period of Earth around Sun τ_{ES}: $3.155815 \cdot 10^7\ s \approx 365.25$ days.
angular velocity $\sigma = 2\pi \frac{\tau_E}{\tau_{ES}} \quad [\text{rad/day}] = 2\pi \frac{\tau_E}{\tau_{ES}} \cdot \frac{\tau}{\tau_E} \quad [\text{rad/rev}]$
and for Sun synchronous orbits $\Leftrightarrow \Delta\Omega = \sigma$.
Sun and Earth synchronous orbit:

$$n(\Delta\Phi_R - \Delta\Omega) = m2\pi \Rightarrow n\tau \left(1 - \frac{\tau_E}{\tau_{ES}}\right) = m\tau_E$$

displacement between subsequent orbits ($\rightarrow$ gap in observation areas):

$$\Delta\Phi = 2\pi\tau \left(\frac{1}{\tau_E} - \frac{1}{\tau_{ES}}\right) = 7.27 \cdot 10^{-5} \cdot \tau \quad [\text{rad/rev}]$$

typical Earth observation orbits: $550 - 750\ km$ above surface $\Rightarrow \Delta\Phi \approx 4.3 \cdot 10^{-1}$ rads and offset $\approx 2875\ km$.

orbit change maneuvers
based on impulse conservation law
propulsion systems:

Hydrazin	$2.2\frac{km}{s}$ particle velocity
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$$\Delta v_s = v_p(\ln m_s - \ln(m_s - m_p))$$

Ziolkowski rocket equation:

$$m_{\text{after}} = m_{\text{before}} \exp\left(\frac{-\Delta v_s}{v_p}\right)$$

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1) angular Radius of Earth φ: $\sin\varphi = \frac{R_E}{R_E+h}$

2) compute spacecraft viewing angles from subsatellite point (Λ_s, σ_s) and target point (Λ_t, σ_t):

$$\cos\lambda = \sin\sigma_s \sin\sigma_t + \cos\sigma_s \cos\sigma_t \cos|\Lambda_s - \Lambda_t|$$

$$\cos A_z = \frac{\sin\sigma_t - \cos\lambda \sin\sigma_s}{\sin\lambda\cos\sigma_s}$$

$$\tan\eta = \frac{\sin\varphi\sin\lambda}{1-\sin\varphi\cos\lambda}$$

3) compute coordinates on Earth:

$$\cos\varepsilon = \frac{\sin\eta}{\sin\varphi}$$

$$\lambda + \eta + \varepsilon = 90^\circ$$

ground station contacts

typically a minimum elevation angle is specified for each ground station $\varepsilon_{\min}$ below contacts are limited. ($\varepsilon_{\min} \approx 5^\circ - 10^\circ$)

given a $\varepsilon_{\min}$, derive the maximum Earth control angle $\lambda_{\max}$ and the max. nadir angle $\eta_{\max}$, and maximum range $D_{\max}$ at which the satellite is still in the FOV of ground station.

$$\sin\eta_{\max} = \sin\varphi\cos\varepsilon_{\min}$$

$$\lambda_{\max} = 90^\circ - \varepsilon_{\min} - \eta_{\max}$$

$$D_{\max} = R_E \frac{\sin\lambda_{\max}}{\sin\eta_{\max}}$$

to calculate the timing of a lfyover, we need $\lambda_{\min}$ minimum Earth central angle between the satellite's ground track and the ground station.

$$\sin\lambda_{\min} = \sin(90^\circ - i)\sin\sigma_{\mathrm{gs}} + \cos(90^\circ - i)\cos\sigma_{\mathrm{gs}}\cos\Delta\mathrm{long}$$

where Δlong is the longitude difference between ground station and the orbit pole ($\sigma_{\mathrm{pole}} = 90^\circ - i$,$\Lambda_{\mathrm{pole}} = \Lambda_{\mathrm{node}} - 90^\circ$).

$$\tan\eta_{\min} = \frac{\sin\varphi\sin\lambda_{\min}}{1-\sin\varphi\cos\lambda_{\min}}$$

$$\varepsilon_{\max} = 90^\circ - \lambda_{\min} - \eta_{\min}$$

$$D_{\min} = R_E \frac{\sin\lambda_{\min}}{\sin\eta_{\min}}$$

parameters at the satellites closest approach to ground station.

the maximum angular rate of the satellite, seen from the ground station

$$\sigma_{\max} = \frac{v_{\mathrm{sat}}}{D_{\min}} = \frac{2\pi(R_E+h)}{TD_{\min}}$$

where T is the orbital period

$$AT = \frac{T}{180^\circ} \arccos\left(\frac{\cos\lambda_{\mathrm{max}}}{\cos\lambda_{\mathrm{min}}}\right)$$

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4.1 Onboard Data Handling (OBDH)

OBDH functions:

- to receive, validate, decode and distribute commands
- to gather, process and format the data onboard, spacecraft housekeeping and mission data
- spacecraft time synchronisation, health monitoring of computer itself (watchdogs), security interfaces
- autonomous reaction capabilities, redundancy

standards for command message formats CCSDS:

- synchronisation code
- adress bit
- command message bits
- error check bits

Output: discrete commands (fixed amplitude, fixed pulse derration)
Data handling combines telemetry from multiple sources and provides it for downlink to earth.

- analog telemetry data (conversion needed)

4.2 Command & Data Handling System Sizing

Command processing

- number of command output channels
- requirements for stored commands
- requirements for computer commands of ACS functions

Telemetry processing

- spacecraft housekeeping data

- feedback for onboard control of spacecraft functions
- routing of onboard and subsystem data
  - to and from receivers and transmitters
  - to storage systems
  - to affected system controllers

Timing

- time granularity
- stability requirement
- acceptable uncertainty

Computer watchdog
constraints:
Spacecraft Bus constraints

- single unit systems
- multiple units, distributing system
- integrated systems

Reliability ($\rightarrow$ redundancy/parts quality)
Radiation
Budgets (Size, Weight, Power)
$\Rightarrow$ OBDH is a tool to configure, control or program the payload and the spacecraft subsystems.

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