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Spacecraft System Design
Tafelanschriebe

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$$m \cdot \ddot{\vec{r}} = F = -G \cdot \frac{m_1 m_2}{r^2}$$

G ist die *universal gravity constant*: $6.674 \cdot 10^{-11} \frac{Nm^2}{kg^2}$

$$\ddot{\vec{r}} = \frac{\mu}{\|r\|^2}$$

μ is the gravity constant of the specific gravity attractor.
μ_E ist die *gravity constant of earth*: $398600 \frac{km^3}{s^2}$, außerdem gilt: $\mu_E = G \cdot m_E$
m_E ist die *mass of earth*: $5.97219 \cdot 10^{24} kg$
Die Lösungen dieser Gleichung sind Kegelschnitte.

$$r = \frac{p}{1 + \varepsilon \cdot \cos\varphi}$$

r ist radius towards focal point
ε eccentricity of the orbit p parameters (spartial extensions of orbit)
φ true anomaly

Interrelationships between parameters:
r_a apocenter, r_p pericenter

$$a = \frac{p}{1 - \varepsilon^2} \Rightarrow p = a(1 - \varepsilon^2) = r_p(1 + \varepsilon) = r_a(1 - \varepsilon)$$

$$a = \frac{r_a + r_p}{2},\ r_a = a(1 + \varepsilon),\ r_p = a(1 - \varepsilon)$$

$$\varepsilon = \frac{r_a - r_p}{r_a + r_p}$$

For the specific case of circular orbits:

- gravitational acceleration in distance a to gravitational center: $\frac{\mu}{a^2}$
- centrifugal acceleration: $\omega^2 \cdot r$ with $\omega = \frac{2\pi}{T}$ angular velocity and T the orbital period

$$\left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow \frac{\mu}{a^2} = \left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

which also holds for general ellipses

$$v_{\text{circle}} = \frac{2\pi a}{T} = \sqrt{\frac{a^3}{\mu}}$$

Potential energy: $E_{\text{pot}} = \frac{-\mu m}{r} + C$
Kinetic energy: $E_{\text{kin}} = \frac{mv^2}{2}$

$$E = E_{\text{kin}} + E_{\text{pot}} = \frac{mv^2}{2} - \frac{\mu m}{r} \overset{\substack{\varepsilon \to 1,\\ v_a \to 0,\\ r_a \to 2a}}{=} -\frac{\mu m}{2a} \text{(constant)}$$

v_a velocity at apocenter

Binet's equation, *Vis-Viva equation*:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

For circles: $r = a$, $v = \sqrt{\frac{\mu}{a}}$
v first cosmic velocity: needed to bring a satellite on closed orbit(without perturbing forces, just gravity considered). In case of earth: $r = 6378km$, $\mu_E = 398600 \frac{km^3}{s^2}$
$\Rightarrow 7.905 \frac{km}{s}$ required as minimum velocity from the rocket launch to insert into an earth orbit.

A rocket launch in equatorial direction and in rotation direction of earth, we recieve a velocity component $0.463 \frac{km}{s}$ for free earth rotation rate.

For parabola: $a \to \infty$, $v = \sqrt{\frac{2\mu}{r}}$ *v second cosmic velocity (escape velocity)*: is the minimum velocity applied to leave the gravitational field of the home planet. For earth: $11.179 \frac{km}{s}$

Derive position of the satellite in the orbit plane, e.g. we want the true anomaly φ as the function of time $\varphi = f(t)$. There is no explicit solution but only an algorithm using the support variable eccentric anomaly E can be derived.

Keplers equation:

$$E - \varepsilon \cdot \sin(E) = \frac{2\pi}{T}(t - t_{\text{perigee}}) = M(t)$$

$M(t)$ average anomaly at time t, can be solved by numerical methods. From E, you can calculate r and φ as follows:

$$r \sin\varphi = a \sin E \cdot \sqrt{1 - \varepsilon^2}$$
$$r \cos\varphi = a(\cos E - \varepsilon)$$
$$\Rightarrow \tan\frac{\varphi}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}} \cdot \tan\frac{E}{2}$$
$$r = a(1 - \varepsilon \cos E)$$

Satellite Ground Tracks:
Analyse the triangle KFB in spherical coordinates.
θ *latitude*: $\sin\theta = \sin i \sin(\omega + \varphi)$
λ *longitude*: $\cos\lambda_1 = \frac{\cos(\omega+\varphi)}{\cos\theta}$

$$\lambda = \lambda_1 + \Omega + \lambda_G$$

$$\lambda_G = \lambda_{G0} - 0.25068448°/\text{min} \cdot t \qquad \text{(Greenwich position versus } \gamma)$$

Draw the line from the satellite to earth center, the intersection with surface is specified by θ and λ (*subsatellite point*).

Perturbations:
gravitational potential of earth

$$u(r, \phi, \Lambda)$$

u gravitational at distance r and latitude ϕ and longitude Λ of the position.

$$u(r,\phi,\Lambda) = \frac{\mu}{r}\left(1+\sum_{n=2}^{\infty}\left[\left(\frac{R_E}{r}\right)^n \cdot J_2 \cdot P_{n0}\cos\phi + \sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \cdot (C_{nm}\cos(m\Lambda) + S_{nm}\sin(m\Lambda)) \cdot P_{nm}(\cos\phi)\right]\right)$$

spherical harmonic expansion with:
r distance from the earth center
P_{nm} Legendre polynomials
R_E radius of earth $6378km$
J_n, C_{nm}, S_{nm} constants of body reflecting the mass distribution:
J_n zonal harmonic coefficient (mass distribution independent from longitude)
C_{nm}, S_{nm} earth's tesseral harminic coefficients (for $n \neq m$) or earth's sectoral harmonic coefficients (for $n = m$)

By far, the largest coefficient is J_n (related to equatorial bulge) by 3 orders of magnitude.
Example: Molniya orbits of $63,4°$ have minimum change in position of perigee and apogee.

Additional gravitational fields:
by other bodies in the solar system

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