Exercise 5 - Solution

Task 5.1

given: $h = 23222km$, $\epsilon_{min} = 10°, \tau_S = 3.155815 \cdot 10^7 s, \tau_E = 86164.10555 + 0.015 \cdot C, \mu_E = 398600 \frac{km^3}{s^2}$

1.
$$\rho = arcsin(\frac{R_E}{R_E + h}) = 12.44°$$
$$\lambda_{max} = 90° - \rho = 77.5567°$$
$$D_{foot} = 2 \cdot \lambda_{max} = 155.1104°$$

2.
$$T = 2\pi \sqrt{\frac{(r_E + h)^3}{\mu_E}} = 5.0681 \cdot 10^4 s$$
$$T_{view} = \frac{T}{360°} \cdot 2\lambda_{max} = \frac{T}{180°} \cdot \lambda_{max} = 363.95 min$$

3. one satellite every 40° in one orbit
   $\Rightarrow$ number of satellites in $2\alpha = 155.11°$ is four (one at 0°, one at 40°, one at 80°, one at 120°)

4.
$$cos(\lambda_{street}) = \frac{cos(\lambda_{max})}{cos(\frac{\Delta\nu}{2})} = \frac{cos(90° - \eta_{max} - \epsilon_{min})}{cos(\frac{\frac{360°}{9}}{2})} = \frac{cos(77.56°)}{cos(20°)}$$
$$\Rightarrow \lambda_{street} = arccos(\frac{cos(77.56°)}{cos(20°)}) = 76.74°$$

5.
$$S = \frac{360°}{T_{earth}} \cdot T = 211.76°$$

6. perturbations, launch sites, lighting conditions, ground sites, ...
   less propellant to maintain, coverage pattern, ...

Task 5.2

$$\Delta\Omega = \Theta \text{ (motion regression equals motion of earth around sun)}$$

$$-\frac{3\pi J_2 R_E^2}{a^2} cos(i) = \frac{2\pi}{\tau_{ES}} \cdot T = 4\pi^2 \frac{\sqrt{\frac{a^3}{\mu_E}}}{\tau_{ES}}$$

$$a = \left(\frac{3}{4\pi} J_2 R_E^2 \tau_{ES} \sqrt{\mu_E}\right)^{\frac{2}{7}} \cdot (-cos(i))^{\frac{2}{7}}$$

$$\Rightarrow a = 12352.7km \cdot (-cos(i))^{\frac{2}{7}}$$

a ist maximal für $cos(i) = -1 \Rightarrow i = \pi \Rightarrow h_{max} = 12352.7 - R_E = 5974.7km$

Task 5.3

given: $r_p = 9.048AU = 1.354 \cdot 10^9 km$, $r_a = 10.116AU = 1.513 \cdot 10^9 km$, $T = 29 years = 9.145 \cdot 10^8 s$, $\mu_{Sun} = 132.7 \cdot 10^9 \frac{km^3}{s^2}$

1.

$$a = \frac{r_p + r_a}{2} = 1.43 \cdot 10^9 km$$

$$\varepsilon = \frac{r_p - r_a}{r_p + r_a} = 0.0557$$

2.

$$T = 2\pi \sqrt{\frac{a^3}{\mu_{Sat} + \mu_{Sun}}}$$

$$\Leftrightarrow \mu_{Sat} = a^3 \cdot \frac{4\pi^2}{T^2} - \mu_{Sun} \Rightarrow \mu_{Sat} \approx 6.3317 \cdot 10^9 \frac{km^3}{s^2}$$