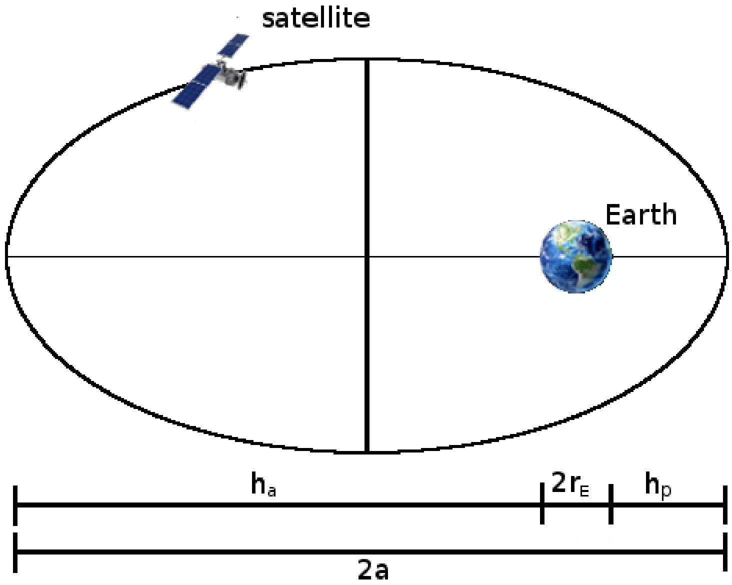
Exercise 1 - Solution



Task 1.1

given : $T = 106min = 6360s, h_p = 200km, \mu_E = 3.986 \cdot 10^5 \frac{km^3}{s^2}, r_E = 6378km$
to be determined: h_a

$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} \Rightarrow a = \sqrt[3]{\frac{T^2\mu_E}{4\pi^2}}$$

$$h_a + h_p + 2r_E = 2a \Rightarrow h_a = 2a - h_p - 2r_E = 1882.634...km = 1883km$$

Task 1.2

1. The satellite has an orbital period of one sideral day, i.e. 23 hours 56 min 4s = 1436.07 min = 1436 min. Since the orbit is circular, the eccentricity is 0.

2. $$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} \Rightarrow a = \sqrt[3]{\frac{T^2\mu_E}{4\pi^2}} = 42164.1897...km = 42164km$$

$$h = a - r_E = 35786km$$

Task 1.3

given : $h_a = 320km, h_p = 270km, \mu_E = 3.986 \cdot 10^5 \frac{km^3}{s^2}, r_E = 6378km$
to be determined: a, ε

$$2a = h_a + h_p + 2r_E \Rightarrow a = \frac{h_a + h_p + 2r_E}{2} = 6673km$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} = 5424.91..s = 90.415min = 90min$$

$$h_p + r_E = a(1-\varepsilon) \Rightarrow \varepsilon = 1 - \frac{h_p + r_E}{a} = 0.0037 \Rightarrow \text{almost circular}$$

Task 1.4

given : $T = 225$ earth days $= 225 \cdot 86400s, \mu_S = 1.327 \cdot 10^{11} \frac{km^3}{s^2}$
to be determined: a

$$a = \sqrt[3]{\frac{T^2\mu_S}{4\pi^2}} = 1.083 \cdot 10^8 km = 108 \text{ million km}$$

Task 1.5

given : $T = 6.5$ years $= 6.5 \cdot 365 \cdot 24 \cdot 3600s, \mu_S = 1.327 \cdot 10^{11} \frac{km^3}{s^2}, h_p = 185 \cdot 10^6 km, r_S = 695,800km$
to be determined: $a, \varepsilon, v_{min}, v_{max}, \Phi$

1.
$$a = \sqrt[3]{\frac{T^2\mu_S}{4\pi^2}} = 5.20775... \cdot 10^8 km = 521 \text{ million km}$$

2.
$$h_p + r_E = a(1-\varepsilon) \Rightarrow \varepsilon = 1 - \frac{h_p + r_S}{a} = 0.643578 = 0.64$$

3.
$$v_{max} = \sqrt{\mu_S\left(\frac{2}{h_p + r_S} - \frac{1}{a}\right)} = 34.2640...\frac{km}{s} = 34.26\frac{km}{s}$$
$$h_a = 2a - 2r_S - h_p = 853,608,400 = 854 \text{ million km}$$
$$v_{min} = \sqrt{\mu_S\left(\frac{2}{h_a + r_S} - \frac{1}{a}\right)} = 7.4383\frac{km}{s} = 7.44\frac{km}{s}$$

4. formula from lecture:
$$r = a\frac{1-\varepsilon^2}{r\varepsilon cos(\Phi)}$$
$$\Rightarrow \text{true anomaly } \Phi = arccos(a\frac{1-\varepsilon^2}{r^2\varepsilon})$$