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Spacecraft System Design
Tafelanschriebe

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14. November 2014

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$$m \cdot \ddot{\vec{r}} = F = -G \cdot \frac{m_1 m_2}{r^2}$$

G ist die *universal gravity constant*: $6.674 \cdot 10^{-11} \frac{Nm^2}{kg^2}$

$$\ddot{\vec{r}} = \frac{\mu}{\|r\|^2}$$

μ	gravity constant of the specific gravity attractor.	
μ_E	*gravity constant of earth*:	$398600 \frac{km^3}{s^2}$
m_E	*mass of earth*:	$5.97219 \cdot 10^{24} kg$

Die Lösungen dieser Gleichung sind Kegelschnitte. Außerdem gilt: $\mu_E = G \cdot m_E$

$$r = \frac{p}{1 + \varepsilon \cdot \cos \varphi}$$

r	radius towards focal point
ε	eccentricity of the orbit
p	parameters (spartial extensions of orbit)
φ	true anomaly

Interrelationships between parameters:
r_a apocenter, r_p pericenter

$$a = \frac{p}{1 - \varepsilon^2} \Rightarrow p = a(1 - \varepsilon^2) = r_p(1 + \varepsilon) = r_a(1 - \varepsilon)$$

$$a = \frac{r_a + r_p}{2},\ r_a = a(1 + \varepsilon),\ r_p = a(1 - \varepsilon)$$

$$\varepsilon = \frac{r_a - r_p}{r_a + r_p}$$

For the specific case of circular orbits:

- gravitational acceleration in distance a to gravitational center: $\frac{\mu}{a^2}$
- centrifugal acceleration: $\omega^2 \cdot r$ with $\omega = \frac{2\pi}{T}$ angular velocity and T the orbital period

  $$\left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow \frac{\mu}{a^2} = \left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

  which also holds for general ellipses

  $$v_{\text{circle}} = \frac{2\pi a}{T} = \sqrt{\frac{a^3}{\mu}}$$

Potential energy: $E_{\text{pot}} = \frac{-\mu m}{r} + C$
Kinetic energy: $E_{\text{kin}} = \frac{mv^2}{2}$

$$E = E_{\text{kin}} + E_{\text{pot}} = \frac{mv^2}{2} - \frac{\mu m}{r} \overset{\substack{\varepsilon\to 1,\\ v_a\to 0,\\ r_a\to 2a}}{=} -\frac{\mu m}{2a}(\text{constant})$$

v_a velocity at apocenter

Binet's equation, *Vis-Viva equation*:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

For circles: $r = a$, $v = \sqrt{\frac{\mu}{a}}$
v *first cosmic velocity*: needed to bring a satellite on closed orbit(without perturbing forces, just gravity considered). In case of earth: $r = 6378km$, $\mu_E = 398600\frac{km^3}{s^2}$
$\Rightarrow 7.905\frac{km}{s}$ required as minimum velocity from the rocket launch to insert into an earth orbit.

A rocket launch in equatorial direction and in rotation direction of earth, we recieve a velocity component $0.463\frac{km}{s}$ for free earth rotation rate.

For parabola: $a \to \infty$, $v = \sqrt{\frac{2\mu}{r}}$ v *second cosmic velocity (escape velocity)*: is the minimum velocity applied to leave the gravitational field of the home planet. For earth: $11.179\frac{km}{s}$

Derive position of the satellite in the orbit plane, e.g. we want the true anomaly φ as the function of time $\varphi = f(t)$. There is no explicit solution but only an algorithm using the support variable eccentric anomaly E can be derived.

Keplers equation:

$$E - \varepsilon \cdot \sin(E) = \frac{2\pi}{T}(t - t_{\text{perigee}}) = M(t)$$

$M(t)$ average anomaly at time t, can be solved by numerical methods. From E, you can calculate r and φ as follows:

$$r \sin\varphi = a \sin E \cdot \sqrt{1 - \varepsilon^2}$$
$$r \cos\varphi = a(\cos E - \varepsilon)$$
$$\Rightarrow \tan\frac{\varphi}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}} \cdot \tan\frac{E}{2}$$
$$r = a(1 - \varepsilon \cos E)$$

Satellite Ground Tracks:
Analyse the triangle KFB in spherical coordinates.

θ	*latitude*:	$\sin\theta = \sin i \sin(\omega + \varphi)$
λ	*longitude*:	$\cos\lambda_1 = \frac{\cos(\omega+\varphi)}{\cos\theta}$

$$\lambda = \lambda_1 + \Omega + \lambda_G$$
$$\lambda_G = \lambda_{G0} - 0.25068448°/\text{min} \cdot t \qquad \text{(Greenwich position versus } \gamma)$$

Draw the line from the satellite to earth center, the intersection with surface is specified by θ and λ (*subsatellite point*).

Perturbations:
gravitational potential of earth

$$u(r, \phi, \Lambda)$$

u gravitational at distance r and latitude ϕ and longitude Λ of the position.

$$u(r, \phi, \Lambda) = \frac{\mu}{r}\left(1+\sum_{n=2}^{\infty}\left[\left(\frac{R_E}{r}\right)^n \cdot J_2 \cdot P_{n0}\cos\phi + \sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \cdot (C_{nm}\cos(m\Lambda) + S_{nm}\sin(m\Lambda)) \cdot P_{nm}(\cos\phi)\right]\right)$$

spherical harmonic expansion with:

r	distance from the earth center	
P_{nm}	Legendre polynomials	
R_E	radius of earth	$6378km$

J_n, C_{nm}, S_{nm} are constants of body reflecting the mass distribution:
J_n zonal harmonic coefficient (mass distribution independent from longitude)
C_{nm}, S_{nm} earth's tesseral harminic coefficients (for $n \neq m$) or earth's sectoral harmonic coefficients (for $n = m$)

By far, the largest coefficient is J_n (related to equatorial bulge) by 3 orders of magnitude.
Example: Molniya orbits of $63,4°$ have minimum change in position of perigee and apogee.

Additional gravitational fields:
by other bodies in the solar system

$$a_d = \mu_d\sqrt{\vec{R}\cdot\vec{R}}$$

with $\vec{R} = \frac{\vec{r}_{sd}}{\|\vec{r}_d\|}$

$$\frac{a_d}{a_c} = \frac{M_d}{M_c}\left(\frac{\|\vec{r}_s\|}{\|\vec{r}_{sd}\|}\right)^3\sqrt{1+3\cos^2\beta} = 2\frac{M_d}{M_c}\left(\frac{\|\vec{r}_s\|}{\|\vec{r}_d\|}\right)^3$$

a_c	actual acceleration

atmospheric drag:
Drag: $\vec{F}_0 = -\frac{1}{2}\rho S c_D \|\vec{v}_R\|^2 \frac{\vec{v}_r}{\|\vec{v}_r\|}$

v_r	velocity vector of the spacecraft relative to the atmosphere	
ρ	atmospheric density	
S	reference area of the spacecraft	
c_D	drag coefficient of the spacecraft (determined in wind channel)	default: $c_d \approx 2.5$

Main effect of altitude changes and changes of orbital period.

solar pressure:

$1AU$	*distance earth-sun*	$149.6 \cdot 10^6 km$
	energy density	$1372\frac{W}{m^2}$

from impulse conservation law:

$$p = \frac{1372W}{c \cdot m^2} = \frac{1372W\ s}{299793km\ m^2} = 0.46 \cdot 10^{-5}\frac{N}{m^2}$$

solar pressure is increased by reflection

$$p_{eff} = (1 + r)p$$

p_{eff}	effective pressure
r	reflectivity factor

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2.1 Onboard Data Handling (OBDH)

OBDH functions:

- to receive, validate, decode and distribute commands
- to gather, process and format the data onboard, spacecraft housekeeping and mission data
- spacecraft time synchronisation, health monitoring of computer itself (watchdogs), security interfaces
- autonomous reaction capabilities, redundancy

standards for command message formats CCSDS:

- synchronisation code
- adress bit
- command message bits
- error check bits

Output: discrete commands (fixed amplitude, fixed pulse derration)
Data handling combines telemetry from multiple sources and provides it for downlink to earth.

- analog telemetry data (conversion needed)

2.2 Command & Data Handling System Sizing

Command processing

- number of command output channels
- requirements for stored commands
- requirements for computer commands of ACS functions

Telemetry processing

- spacecraft housekeeping data

- feedback for onboard control of spacecraft functions
- routing of onboard and subsystem data
  - to and from receivers and transmitters
  - to storage systems
  - to affected system controllers

Timing

- time granularity
- stability requirement
- acceptable uncertainty

Computer watchdog
constraints:
Spacecraft Bus constraints

- single unit systems
- multiple units, distributing system
- integrated systems

Reliability ($\rightarrow$ redundancy/parts quality)

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