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# Spacecraft System Design
Tafelanschriebe

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$$m \cdot \ddot{\vec{r}} = F = -G \cdot \frac{m_1 m_2}{r^2}$$

$G$ ist die *universal gravity constant*: $6.674 \cdot 10^{-11} \frac{Nm^2}{kg^2}$

$$\ddot{\vec{r}} = \frac{\mu}{\|r\|^2}$$

| | | |
|---|---|---|
| $\mu$ | gravity constant of the specific gravity attractor. | |
| $\mu_E$ | *gravity constant of earth*: | $398600 \frac{km^3}{s^2}$ |
| $m_E$ | *mass of earth*: | $5.97219 \cdot 10^{24} kg$ |

Die Lösungen dieser Gleichung sind Kegelschnitte. Außerdem gilt: $\mu_E = G \cdot m_E$

$$r = \frac{p}{1 + \varepsilon \cdot \cos\varphi}$$

| | |
|---|---|
| $r$ | radius towards focal point |
| $\varepsilon$ | eccentricity of the orbit |
| $p$ | parameters (spartial extensions of orbit) |
| $\varphi$ | true anomaly |

**Interrelationships between parameters:**
$r_a$ apocenter, $r_p$ pericenter

$$a = \frac{p}{1 - \varepsilon^2} \Rightarrow p = a(1 - \varepsilon^2) = r_p(1 + \varepsilon) = r_a(1 - \varepsilon)$$

$$a = \frac{r_a + r_p}{2},\ r_a = a(1 + \varepsilon),\ r_p = a(1 - \varepsilon)$$

$$\varepsilon = \frac{r_a - r_p}{r_a + r_p}$$

For the specific case of circular orbits:

- gravitational acceleration in distance $a$ to gravitational center: $\frac{\mu}{a^2}$
- centrifugal acceleration: $\omega^2 \cdot r$ with $\omega = \frac{2\pi}{T}$ angular velocity and $T$ the orbital period

$$\left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow \frac{\mu}{a^2} = \left(\frac{2\pi}{T}\right)^2 \cdot a \Rightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

which also holds for general ellipses

$$v_{\text{circle}} = \frac{2\pi a}{T} = \sqrt{\frac{a^3}{\mu}}$$

*Potential energy*: $E_{\text{pot}} = \frac{-\mu m}{r} + C$
*Kinetic energy*: $E_{\text{kin}} = \frac{mv^2}{2}$

$$E = E_{\text{kin}} + E_{\text{pot}} = \frac{mv^2}{2} - \frac{\mu m}{r} \overset{\substack{\varepsilon\to 1,\\ v_a\to 0,\\ r_a\to 2a}}{=} -\frac{\mu m}{2a}(\text{constant})$$

$v_a$ velocity at apocenter

*Binet's equation*, *Vis-Viva equation*:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

**For circles:** $r = a$, $v = \sqrt{\frac{\mu}{a}}$
*v first cosmic velocity*: needed to bring a satellite on closed orbit(without perturbing forces, just gravity considered). In case of earth: $r = 6378km$, $\mu_E = 398600\frac{km^3}{s^2}$
$\Rightarrow 7.905\frac{km}{s}$ required as minimum velocity from the rocket launch to insert into an earth orbit.

A rocket launch in equatorial direction and in rotation direction of earth, we recieve a velocity component $0.463\frac{km}{s}$ for free earth rotation rate.

**For parabola:** $a \to \infty$, $v = \sqrt{\frac{2\mu}{r}}$ *v second cosmic velocity (escape velocity)*: is the minimum velocity applied to leave the gravitational field of the home planet. For earth: $11.179\frac{km}{s}$

Derive position of the satellite in the orbit plane, e.g. we want the true anomaly $\varphi$ as the function of time $\varphi = f(t)$. There is no explicit solution but only an algorithm using the support variable eccentric anomaly $E$ can be derived.

***Keplers equation***:

$$E - \varepsilon \cdot \sin(E) = \frac{2\pi}{T}(t - t_{\text{perigee}}) = M(t)$$

$M(t)$ average anomaly at time $t$, can be solved by numerical methods. From $E$, you can calculate $r$ and $\varphi$ as follows:

$$r\sin\varphi = a\sin E \cdot \sqrt{1-\varepsilon^2}$$
$$r\cos\varphi = a(\cos E - \varepsilon)$$
$$\Rightarrow \tan\frac{\varphi}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}} \cdot \tan\frac{E}{2}$$
$$r = a(1 - \varepsilon\cos E)$$

**Satellite Ground Tracks:**
Analyse the triangle KFB in spherical coordinates.

| | | |
|---|---|---|
| $\theta$ | *latitude*: | $\sin\theta = \sin i \sin(\omega + \varphi)$ |
| $\lambda$ | *longitude*: | $\cos\lambda_1 = \frac{\cos(\omega+\varphi)}{\cos\theta}$ |

$$\lambda = \lambda_1 + \Omega + \lambda_G$$
$$\lambda_G = \lambda_{G0} - 0.25068448°/\text{min} \cdot t \qquad \text{(Greenwich position versus } \gamma)$$

Draw the line from the satellite to earth center, the intersection with surface is specified by $\theta$ and $\lambda$ (*subsatellite point*).

***Perturbations***:
**gravitational potential of earth**

$$u(r, \phi, \Lambda)$$

$u$ gravitational at distance $r$ and latitude $\phi$ and longitude $\Lambda$ of the position.

$$u(r, \phi, \Lambda) = \frac{\mu}{r}\left(1+\sum_{n=2}^{\infty}\left[\left(\frac{R_E}{r}\right)^n \cdot J_2 \cdot P_{n0}\cos\phi + \sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \cdot (C_{nm}\cos(m\Lambda) + S_{nm}\sin(m\Lambda)) \cdot P_{nm}(\cos\phi)\right]\right)$$

*spherical harmonic expansion* with:

| | | |
|---|---|---|
| $r$ | distance from the earth center | |
| $P_{nm}$ | Legendre polynomials | |
| $R_E$ | radius of earth | $6378km$ |

$J_n$, $C_{nm}$, $S_{nm}$ are constants of body reflecting the mass distribution:
$J_n$ zonal harmonic coefficient (mass distribution independent from longitude)
$C_{nm}$, $S_{nm}$ earth's tesseral harminic coefficients (for $n \neq m$) or earth's sectoral harmonic coefficients (for $n = m$)

By far, the largest coefficient is $J_n$ (related to equatorial bulge) by 3 orders of magnitude.
Example: Molniya orbits of $63,4°$ have minimum change in position of perigee and apogee.

**Additional gravitational fields:**
by other bodies in the solar system

$$a_d = \mu_d\sqrt{\vec{R}\cdot\vec{R}}$$

with $\vec{R} = \frac{\vec{r}_{sd}}{\|\vec{r}_d\|}$

$$\frac{a_d}{a_c} = \frac{M_d}{M_c}\left(\frac{\|\vec{r}_s\|}{\|\vec{r}_{sd}\|}\right)^3\sqrt{1+3\cos^2\beta} = 2\frac{M_d}{M_c}\left(\frac{\|\vec{r}_s\|}{\|\vec{r}_d\|}\right)^3$$

| | | |
|---|---|---|
| $a_c$ | actual acceleration | |

***atmospheric drag***:
Drag: $\vec{F}_0 = -\frac{1}{2}\rho S c_D \|\vec{v}_R\|^2 \frac{\vec{v}_r}{\|\vec{v}_r\|}$

| | | |
|---|---|---|
| $v_r$ | velocity vector of the spacecraft relative to the atmosphere | |
| $\rho$ | atmospheric density | |
| $S$ | reference area of the spacecraft | |
| $c_D$ | drag coefficient of the spacecraft (determined in wind channel) | default: $c_d \approx 2.5$ |

Main effect of altitude changes and changes of orbital period.

***solar pressure***:

| | | |
|---|---|---|
| $1AU$ | *distance earth-sun* | $149.6 \cdot 10^6 km$ |
| | energy density | $1372\frac{W}{m^2}$ |

from impulse conservation law:

$$p = \frac{1372W}{c \cdot m^2} = \frac{1372W\ s}{299793km\ m^2} = 0.46 \cdot 10^{-5}\frac{N}{m^2}$$

solar pressure is increased by reflection

$$p_{\text{eff}} = (1 + r)p$$

| | |
|---|---|
| $p_{\text{eff}}$ | effective pressure |
| $r$ | reflectivity factor |

At equatorial orbits the solar pressure affects a change of eccentricity $\varepsilon$.

**orbit change manouvers:**
Based on impule conservation law.
Propulsion systems:

| | | |
|---|---|---|
| Hydrazin | $2.2\frac{km}{2}$ | particle velocity |
| 2-component | $3\frac{km}{2}$ | particle velocity |
| ion thrusters | $30\frac{km}{2}$ | particle velocity |

$$\Delta v_s m_s = -\Delta m_s v_p$$

| | |
|---|---|
| $\Delta m_s$ | reduction of satellite mass due to ejected propellant |
| $v_p$ | velocity of propellant ejection |

$$\Delta v_s = -\frac{\Delta m_s}{m_s} v_p$$

$$\int_{v_0}^{v} dv = -v_p \int_{m_s}^{m_s - m_p} \frac{dm}{m}$$

$$\Delta v_s = v_p(\log_2 m_s - \log_2(m_s - m_p))$$

*Rocket equation*, *Ziolkowski equation*:

$$m_{\text{after}} = m_{\text{before}} \cdot e^{\frac{-\Delta v_s}{v_p}}$$

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**Keplerian Orbit Transfer**
Calculate a transfer between two circular orbits with radius $r_A$ to $r_B$. The velocity at pericenter of the transfer ellipse:

$$v_p^2 = 2\mu\left(\frac{1}{r_A} - \frac{1}{r_A + r_B}\right) = 2\mu\frac{r_B}{r_A(r_A + r_B)}$$

The required $\Delta v_A$ to inject from the transfer orbit:

$$\Delta v_A = v_P - v_A = \sqrt{\frac{\mu}{r_A}}\left(\sqrt{\frac{2r_B}{r_A + r_B}} - 1\right)$$

The required $\Delta v$ to inject from the transfer orbit into orbit with $r_B$:

$$\Delta v_B = v_B - v_{\text{apo}} = \sqrt{\frac{\mu}{r_B}}\left(1 - \sqrt{\frac{2r_A}{r_A + r_B}}\right)$$

where $v_B$ is the circular velocity at $r_B$.

The *Hohmann transfer* is the most energy-efficient transfer between two circular orbits.

$$\Delta v_{\text{total}} = \Delta v_A + \Delta v_B = \sqrt{\mu}\left[\sqrt{\left(\frac{2}{r_A} - \frac{2}{r_A + r_B}\right)} - \sqrt{\frac{1}{r_A}} + \sqrt{\frac{2}{r_B} - \frac{2}{r_A + r_B}} - \sqrt{\frac{1}{r_B}}\right]$$

## 2.3 Mission Analysis

Selection of orbit parameters for most appropriate orbits for the given task.

***Earth synchronous orbits***: after a given period of time the subsatellite's ground track repeats. *Rotation of Earth*: $\tau_E$ *sidereal day* ($= 86161.10555 + 0.015C[s]$ where $C$ is the number of centuries after 2000). As Earth rotation is in eastward direction, the satellite orbit with respect to Earch seems to have west direction.

$$\Delta\Phi_R = \frac{\tau}{\tau_E} - 2\pi \quad [\text{rad/rev}]$$

| | |
|---|---|
| $\tau$ | satellite orbit period |

$J_2$ theory taking into account Earth oblateness, we can calculate the satellite orbit plane rotation:

$$\Delta\Omega = \frac{3\pi J_2 R_E^2 \cos i}{a^2(1 - \varepsilon^2)^2} \quad [\text{rad/rev}]$$

total displacement at equator: $\Delta\Phi = \Delta\Phi_R - \Delta\Omega$.
For Earth synchronous orbits, the following property needs to be satisfied:

$$n\Delta\Phi = m2\pi$$

| | |
|---|---|
| $n$ | number of orbits before identical ground track repeats |
| $m$ | number of Earth revolutions (day) before identical ground track repeats |

***Sun synchronous orbits***:
the sunlight incidents repats
take into account the motion of Earth around Sun. Due to the Earth's orbit around Sun, the Earth/Sun-line intersect at points $P$ moving westward.

$$\theta = \frac{360°}{365.25 \text{ days}} \approx 1 \quad [°/\text{day}]$$

orbital period of Earth around Sun $\tau_{ES}$: $3.155815 \cdot 10^7$ $s \approx 365.25$ days.
angular velocity $\theta = 2\pi \frac{\tau_E}{\tau_{ES}}$ [rad/day] $= 2\pi \frac{\tau_E}{\tau_{ES}} \cdot \frac{\tau}{\tau_E}$ [rad/rev]
and for Sun synchronous orbits $\Leftrightarrow \Delta\Omega = \theta$.
Sun and Earth synchronous orbit:

$$n(\Delta\Phi_R - \Delta\Omega) = m2\pi \Rightarrow n\tau\left(1 - \frac{\tau_E}{\tau_{ES}}\right) = m\tau_E$$

displacement between subsequent orbits ($\rightarrow$ gap in observation areas):

$$\Delta\Phi = 2\pi\tau\left(\frac{1}{\tau_E} - \frac{1}{\tau_{ES}}\right) = 7.27 \cdot 10^{-5} \cdot \tau \quad [\text{rad/rev}]$$

typical Earth observation orbits: $550 - 750$ $km$ above surface $\Rightarrow \Delta\Phi \approx 4.3 \cdot 10^{-1}$ rads and offset $\approx 2875$ $km$.

Sun and earth synchronous satellites :

| m | n | corresponding altitude |
|---|---|---|
| 1 | 14 | $894km$ |
| 1 | 13 | $567km$ |
| 1 | 12 | $275km$ |
| 2 | 22 | |

minimum drift orbits: $\frac{n\pm1}{m} = k$, $k \in \mathbb{N}$, integer. orbits of successive days to fill the displacements between two successive orbits.

| | |
|---|---|
| $k$ | orbits of successive days to fill the displacements between two successive orbits |

Example: Landsat 1/2
18 day repeat period minimum drift orbit

$$\tau \approx 103.3\text{min},\ i = 99°,\ \varepsilon = 0.002,\ \text{apogee} = 920km,\ \text{descending node: } 9:38\text{local time}$$

$\Rightarrow$ orbit seperation between successive orbits $\approx 2875km$.
$m = 18 \Rightarrow$ distance between adjacent ground tracks $= 160km$.

***Eclipse calculation***:
Angle between earth-sun direction $\vec{s}$ and the orbit plane $= \beta$.

$$\beta = \sin^{-1}(\vec{s}\vec{n})$$

| | |
|---|---|
| $\vec{n}$ | normal to orbit plane |

$$\sin\beta^* = \frac{r_E}{r_E + h} \Rightarrow \beta^* = \sin^{-1}\left(\frac{r_E}{r_E + h}\right)$$

$$E_1E_2 = 2\Delta v \text{ eclipse orbital arc}$$

$$\Delta v = \cos^{-1}\left(\frac{\cos\beta^*}{\cos\beta}\right) \text{ in triangle } ACE_1$$

eclipse fraction of a circular orbit:

$$F_e = \frac{2\Delta v}{2\pi} = \frac{1}{\pi}\frac{\sqrt{h^2 + 2r_E h}}{(r_E + h)\cos\beta}\ \text{[rad]}$$

| | |
|---|---|
| $h$ | altitude |
| $\beta$ | angle sun-orbit-plane |
| $r_E$ | earth radius |

*launch energy*: $\varepsilon = \cos^{-1}(\cos^2 i + \sin^2 i - \cos\Delta\Omega)$
change angle related to plane (proportional):
$\Delta\Omega$: longitudal seperation
launch window: $2\frac{\Delta\Omega}{w_c} \Rightarrow \Delta\Omega = w_c t \quad w_c$ | earth rotation rate |

**ground contact and coverage analysis**

$$\sin\rho = \cos\lambda_0 = \frac{r_E}{r_E + h}$$

from known $\lambda$ (from $\Lambda_t, \theta_t / \Lambda_s, \theta_s$) we derive $\eta$.

$$\tan\eta = \frac{\sin\rho\sin\lambda}{1 - \sin\rho\sin\lambda}$$

$$\cos\varepsilon = \frac{\sin\eta}{\sin\rho} \Rightarrow \sin\eta = \cos\varepsilon\sin\rho$$

$$\eta + \varepsilon + \lambda = 90^\circ$$

$$D = r_E\frac{\sin\lambda}{\sin\eta}$$

1) angular Radius of Earth $\varphi$: $\sin\varphi = \frac{R_E}{R_E + h}$

2) compute spacecraft viewing angles from subsatellite point $(\Lambda_s, \theta_s)$ and target point $(\Lambda_t, \theta_t)$:

$$\cos\lambda = \sin\theta_s\sin\theta_t + \cos\theta_s\cos\theta_t\cos|\Lambda_s - \Lambda_t|$$

$$\cos A_z = \frac{\sin\theta_t - \cos\lambda\sin\theta_s}{\sin\lambda\cos\theta_s} \quad \textit{Azimuth}$$

$$\tan\eta = \frac{\sin\varphi\sin\lambda}{1 - \sin\varphi\cos\lambda}$$

3) compute coordinates on Earth:

$$\cos\varepsilon = \frac{\sin\eta}{\sin\varphi}$$

$$\lambda + \eta + \varepsilon = 90^\circ$$

# 3 20.10.2014

**ground station contacts**
typically a minimum elevation angle is specified for each ground station $\varepsilon_{\text{min}}$ below contacts are limited. ($\varepsilon_{\text{min}} \approx 5^\circ - 10^\circ$)
given a $\varepsilon_{\text{min}}$, derive the maximum Earth control angle $\lambda_{\text{max}}$ and the max. nadir angle $\eta_{\text{max}}$, and maximum range $D_{\text{max}}$ at which the satellite is still in the FOV of ground station.

$$\sin \eta_{\text{max}} = \sin \varphi \cos \varepsilon_{\text{min}}$$

$$\lambda_{\text{max}} = 90^\circ - \varepsilon_{\text{min}} - \eta_{\text{max}}$$

$$D_{\text{max}} = R_E \frac{\sin \lambda_{\text{max}}}{\sin \eta_{\text{max}}}$$

to calculate the timing of a lfyover, we need $\lambda_{\text{min}}$ minimum Earth central angle between the satellite's ground track and the ground station.

$$\sin \lambda_{\text{min}} = \sin(90^\circ - i) \sin \sigma_{\text{gs}} + \cos(90^\circ - i) \cos \sigma_{\text{gs}} \cos \Delta\text{long}$$

where $\Delta$long is the longitude difference between ground station and the orbit pole
($\sigma_{\text{pole}} = 90^\circ - i$,$\Lambda_{\text{pole}} = \Lambda_{\text{node}} - 90^\circ$).

$$\tan \eta_{\text{min}} = \frac{\sin \varphi \sin \lambda_{\text{min}}}{1 - \sin \varphi \cos \lambda_{\text{min}}}$$

$$\varepsilon_{\text{max}} = 90^\circ - \lambda_{\text{min}} - \eta_{\text{min}}$$

$$D_{\text{min}} = R_E \frac{\sin \lambda_{\text{min}}}{\sin \eta_{\text{min}}}$$

parameters at the satellites closest approach to ground station.

the maximum angular rate of the satellite, seen from the ground station

$$\sigma_{\text{max}} = \frac{v_{\text{sat}}}{D_{\text{min}}} = \frac{2\pi(R_E + h)}{T D_{\text{min}}}$$

where $T$ is the orbital period

$$AT = \frac{T}{180^\circ} \arccos \left( \frac{\cos \lambda_{\text{max}}}{\cos \lambda_{\text{min}}} \right)$$

# 4 14.11.2014

## 4.1 Onboard Data Handling (OBDH)

OBDH functions:

- to receive, validate, decode and distribute commands
- to gather, process and format the data onboard, spacecraft housekeeping and mission data
- spacecraft time synchronisation, health monitoring of computer itself (watchdogs), security interfaces
- autonomous reaction capabilities, redundancy

standards for command message formats CCSDS:

- synchronisation code
- adress bit
- command message bits
- error check bits

Output: discrete commands (fixed amplitude, fixed pulse derration)
Data handling combines telemetry from multiple sources and provides it for downlink to earth.

- analog telemetry data (conversion needed)

## 4.2 Command & Data Handling System Sizing

Command processing

- number of command output channels
- requirements for stored commands
- requirements for computer commands of ACS functions

Telemetry processing

- spacecraft housekeeping data

- feedback for onboard control of spacecraft functions
- routing of onboard and subsystem data
  - to and from receivers and transmitters
  - to storage systems
  - to affected system controllers

Timing

- time granularity
- stability requirement
- acceptable uncertainty

Computer watchdog
**constraints:**
Spacecraft Bus constraints

- single unit systems
- multiple units, distributing system
- integrated systems

Reliability ($\rightarrow$ redundancy/parts quality)
Radiation
**Budgets** (Size, Weight, Power)
$\Rightarrow$ OBDH is a tool to configure, control or program the payload and the spacecraft subsystems.

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