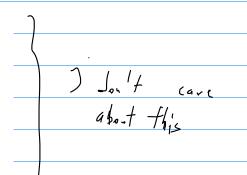
Seninar W9-915

Definition 3.3.1 Let V be a vector space over K, $B = (v_1, \ldots, v_n)$ a basis of V and $X = (u_1, \ldots, u_m)$ a list of vectors in V. Let

$$\begin{cases} u_1 = a_{11}v_1 + a_{12}v_2 + \dots + a_{1n}v_n \\ u_2 = a_{21}v_1 + a_{22}v_2 + \dots + a_{2n}v_n \\ \dots \\ u_m = a_{m1}v_1 + a_{m2}v_2 + \dots + a_{mn}v_n \end{cases}$$

be the unique writings of the vectors in X as linear combinations of vectors of the basis B, for some $a_{ij} \in K$. The matrix of the list of vectors X in the basis B is the matrix having as its rows the coordinates of the vectors in X in the basis B, that is,

$$[X]_B = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}.$$



Compute by applying elementary operations the ranks of the matrices:

1.
$$\begin{pmatrix} 0 & 2 & 3 \\ 2 & 4 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 4 \end{pmatrix}$$
 2.
$$\begin{pmatrix} 1 & -1 & 3 & 2 \\ -2 & 0 & 3 & -1 \\ -1 & 2 & 0 & -1 \end{pmatrix}$$
 3.
$$\begin{pmatrix} \beta & 1 & 3 & 4 \\ 1 & \alpha & 3 & 3 \\ 2 & 3\alpha & 4 & 7 \end{pmatrix} (\alpha, \beta \in \mathbb{R}).$$

vank M = vank (echolor form) = # of nonzero vowa in the echelor form

=)
$$rank | M \in \{43\}$$

 $ranh M = 2 (=)$

$$\begin{cases} 3 - 3B - 2 - \frac{2B^{-1}}{2} = 0 \\ 5 - 3B + \frac{2B^{-1}}{2} = 0 \end{cases}$$

(c)
$$\begin{cases} 34 - 32B - 22B + 2 = 0 \\ 44 - 34B + 4B - 1 = 0 \end{cases}$$
 $\begin{cases} 34 - 54B + 2 = 0 \\ 42 - 24B - 1 = 0 \end{cases}$

4.
$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$
. 5. $\begin{pmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

5.
$$(9)$$
 4 2 1 0 0 $L_2 \in L_2 - 2L_1$ 1 4 2 1 0 0 $L_3 \in L_3 - 3L_3$ 0 -5 -3 -2 1 0 - 2 - 2 - 2 - 3 0 1

7. In the real vector space \mathbb{R}^3 consider the list $X = (v_1, v_2, v_3, v_4)$, where $v_1 = (1, 0, 4)$, $v_2 = (2, 1, 0)$, $v_3 = (1, 5, -36)$ and $v_4 = (2, 10, -72)$. Determine dim < X > and a basis of < X >.

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7. In the real vector space \mathbb{R}^3 consider the list $X = (v_1, v_2, v_3, v_4)$, where $v_1 = (1, 0, 4)$ $v_2 = (2, 1, 0), v_3 = (1, 5, -36)$ and $v_4 = (2, 10, -72)$. Determine dim < X > and a basis o < X >.

9. Determine the dimension of the subspaces S, T, S+T and $S \cap T$ of the real vector space \mathbb{R}^3 and a basis for the first three of them, where

$$S = <(1,0,4), (2,1,0), (1,1,-4)>,$$

$$T = <(-3,-2,4), (5,2,4), (-2,0,-8)>.$$