Putnam NT??

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July 12, 2021

Example 0.1

Let n be a positive integer such that $24|(n+1)$. Prove that the sum of all divisors of n is also divisible by 24

Solution. Note that $n \equiv -1 \pmod{24}$, n can't be a square. So any $d|n$ satisfies $d \equiv 1, 2 \pmod{3}$ and $d \equiv 1, 3, 5, 7 \pmod{8}$. In $d, \frac{n}{d}$ one is 1 and the other is 2 mod 3. Hence the possibilities are

$$d \equiv 1, \frac{n}{d} \equiv 2 \pmod{3}$$

$$d \equiv 1, \frac{n}{d} \equiv 7 \pmod{8}$$

$$d \equiv 3, \frac{n}{d} \equiv 5 \pmod{8}$$

Hence the sum is always $0 \pmod{3}$ and $0 \pmod{8}$, i.e. $0 \pmod{24}$. □