Chapter 11 Homework (Waves)

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December 3, 2021

Example 0.1 (Context-Rich Problem)
Vibrator

Solution. To solve this problem we need to know basic oscillation equations:

$$v_{string} = \sqrt{\frac{F_T}{\sigma}}$$

$$v_{string} = \lambda f$$

We also know that the length of a standing wave with n anti-nodes is $\frac{n\lambda}{2}$. We are going to compute v_{string}, then use this to compute λ. After this, since we know L, we can compute the number of antinodes in terms of m. Firstly, we can balance forces on the mass m. There is tension F_T upward and mg downward, so $F_T - mg = 0 \implies F_T = mg$. Thus

$$v_{string} = \sqrt{\frac{mg}{\sigma}} = \lambda f$$

$$\lambda = \frac{1}{f}\sqrt{\frac{mg}{\sigma}}$$

Thus, if there are n anti-nodes,

$$L = \frac{n\lambda}{2} = \frac{n}{2f}\sqrt{\frac{mg}{\sigma}}$$

$$\frac{2Lf}{n} = \sqrt{\frac{mg}{\sigma}}$$

$$\frac{4L^2 f^2}{n^2} = \frac{mg}{\sigma}$$

$$m = \frac{4L^2 f^2 \sigma}{gn^2}$$

Which is m in terms of known quantities. A quick symbolic check: As n increases, m decreases which makes sense as more anti-nodes means a shorter wavelength which means a smaller mass. As L, f, σ increase, m increases, which also makes sense. The units of m check out, since n is a constant, g is $\frac{m}{s^2}$, L is m, f is $\frac{1}{s}$, and σ is $\frac{kg}{m}$. Now, plugging, we get:

$$n = 1 \implies m \approx 1.26kg$$

$$n = 2 \implies m \approx 0.316kg$$

$$n = 5 \implies m \approx 0.0505kg$$

These answers sort of make logical sense, although its hard to logically assess the situation (because its not common). □

Example 0.2 (Explanation Task)
Spring Vibrator

Solution. To solve this problem, we need to know that a spring equation is

$$x(t) = A\cos(\omega t)$$

Where A, ω. Also, the spring potential energy is $\frac{1}{2}kx^2$.

Hence in our case, the amplitude is simply 0.650. If the frequency is f, then $f = \frac{\omega}{2\pi} = \frac{7.4}{2\pi} \approx 1.18\frac{1}{s}$.

The total energy remains constant (by conservation of energy). This means that the total energy is equal to the total energy when the spring has velocity 0 (when its fully stretched). This x_{max} value is just A, or 0.65. Then,

$$E = \frac{1}{2}kx_{max}^2$$

And since $\omega = \sqrt{\frac{k}{m}}$, $k = \omega^2 m$. Thus

$$E_{total} = \frac{1}{2}(7.4)^2(2)(0.65)^2 \approx 23.14J$$

When $x = 0.26m$, the potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(7.4)^2(2)(0.26)^2 \approx 3.7J$$

Thus the kinetic energy is

$$K = E - U = 23.14 - 3.7 = 19.44J$$

□