Physics Homework With Greenwood, Chapter 4

Anay Aggarwal

October 1, 2021

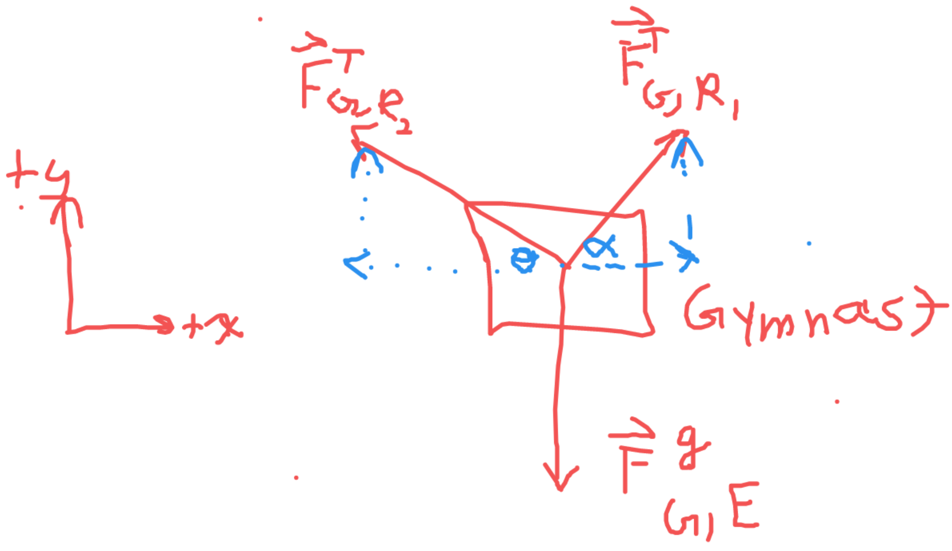
Example 0.1 (Context-Rich Problem)
Gymnast

Solution. To solve this problem, we'll need to know about forces. Specifically, we'll need to know that

$$\sum \overrightarrow{F} = m\overrightarrow{a}$$

We'll also need to know basic trig.

Assume that upwards is positive y, and rightward is positive x. Let G be the gymnast, E be the Earth, T be tension, R_1, R_2 be the ropes, and g be gravity. The freebody diagram is the following:



Since we have forces in various directions, it will likely be helpful to split the force vectors into their components. The components (using the diagram), are the following:

$$\overrightarrow{F^T_{G,R_2}} = -F^T_{G,R_2}\cos\theta\hat{x} + F^T_{G,R_2}\sin\theta\hat{y}$$

$$\overrightarrow{F^T_{G,R_1}} = F^T_{G,R_1}\cos\alpha\hat{x} + F^T_{G,R_1}\sin\alpha\hat{y}$$

Keeping the signs in mind. Notice that there is no total acceleration, so

$$\sum \vec{F} = m\vec{a} = 0$$

Therefore,

$$(F^T_{G,R_1}\cos\alpha - F^T_{G,R_2}\cos\theta)\hat{x} + (F^T_{G,R_1}\sin\alpha + F^T_{G,R_2}\sin\theta - F^g_{G,E})\hat{y} = 0$$

This equation tells us a lot of things, since both the x and y components of the vector are equivalent to 0. This gives us two separate equations:

$$F^T_{G,R_1}\cos\alpha - F^T_{G,R_2}\cos\theta = 0$$

$$F^T_{G,R_1}\sin\alpha + F^T_{G,R_2}\sin\theta - F^g_{G,E} = 0$$

For convenience, denote

$$F^T_{G,R_1} \equiv T_1$$

$$F^T_{G,R_2} \equiv T_2$$

Hence

$$T_1\cos\alpha = T_2\cos\theta \implies T_2 = \frac{T_1\cos\alpha}{\cos\theta}$$

$$T_1\sin\alpha + T_2\sin\theta = F^g_{G,E}$$

We know that $F^g_{G,E} = mg$, Therefore,

$$T_1\sin\alpha + T_2\sin\theta = T_1\sin\alpha + T_1\cos\alpha\tan\theta = T_1(\sin\alpha + \cos\alpha\tan\theta) = mg$$

$$\boxed{T_1 = \frac{mg}{\sin\alpha + \cos\alpha\tan\theta}}$$

$$\boxed{T_2 = \frac{mg\cos\alpha}{\sin\alpha\cos\theta + \cos\alpha\sin\theta}}$$

The units check out since T_1, T_2 are forces and their values are a scalar multiplied by force, which is a force.
To check limiting cases, let's fix θ and play with α. When $\alpha \to \frac{\pi}{2}$, $T_1 \to mg$ and $T_2 \to \frac{mg}{\cos\theta}$, which makes sense (more force required). When $\alpha \to 0$, $T_1 \to \frac{mg}{\tan\theta}$ and $T_2 \to \frac{mg}{\sin\theta}$. This also makes sense since $\tan\theta$ can get large or small. When $\alpha \to \theta$, we get $T_1 \to T_2 \to \frac{mg}{2\sin\theta}$ which is what we got in class. □

Example 0.2 (Example Problem)
Block on ramp

Solution. To solve this, we need to know about free-body diagrams and Newton's second law:

$$\sum \vec{F} = m\vec{a}$$

For the first part, we can simply apply Newton's second law, which can be rewritten as

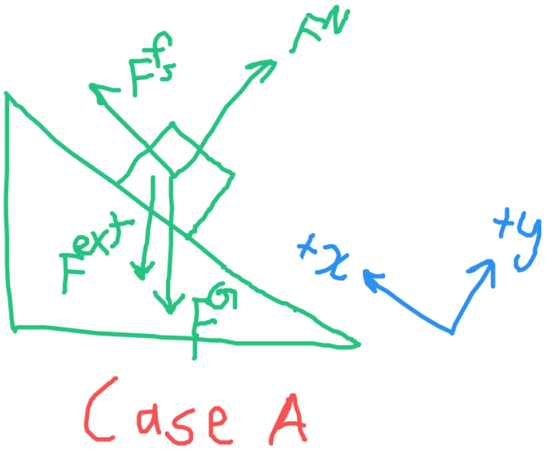
$$\overrightarrow{F}_{net} = m\overrightarrow{a}$$

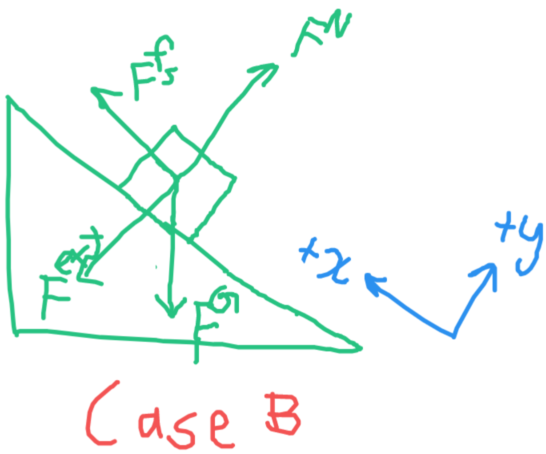
Now, the blocks are at rest, hence

$$\overrightarrow{F}_{net,both} = m \cdot 0 = 0$$

Hence both net forces are zero, so the answer is $\boxed{\text{equal to}}$.

For the second part, we will need to draw some free-body diagrams. N denotes normal force, f_s denotes static frictional force, ext denotes external force, and G denotes gravitational force.





Notice that the axes are along the plane (for convenience). Now, I claim that the frictional force in Case A is greater than in Case B. To show this, notice that by Newton's second law, the forces sum to 0. In Case A, the only forces in the x-direction are:

- Positive static frictional force
- Negative x-component of gravity

- Negative x-component of external force

And for Case B:

- Positive static frictional force
- Negative x-component of gravity

Noting that the x-components of gravity are equal since the angles and masses are equal, the frictional force in Case A has to make up for gravity *and more*, whereas Case B only has to make up for gravity (since the external force in this case is in the y-direction). $\square$