Harmonic Sound Homework

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December 13, 2021

Example 0.1 (Context-Rich Problem)
Singing in shower

Solution. To solve this problem, we need to know about standing waves and the speed of sound:

$$v_{sound} \approx (331 + 0.6T)m/s$$

We also need to know that

$$v = \lambda f$$

Vertical Waves: The shower is an open-closed instrument over vertical waves. In an open-closed situation, we have

$$L = \frac{n_{odd}}{4}\lambda_n$$

We know that $\lambda = \frac{v_{sound}}{f} = \frac{331+0.6T}{f}m$, where T is the temperature in Celsius (35 in this case). Thus

$$L = \frac{n_{odd}}{4}\frac{331 + 0.6T}{f}$$
$$\frac{331 + 0.6T}{f} = \frac{4L}{n_{odd}}$$
$$\frac{f}{331 + 0.6T} = \frac{n_{odd}}{4L}$$
$$f = \frac{(331 + 0.6T)n_{odd}}{4L}$$

Note that L is a known quantity ($2.2m$ in our case). Plugging in the numbers,

$$f \approx 38n_{odd}\text{Hz}$$

Since

$$20\text{Hz} \leq f \leq 20,000\text{Hz}$$

We can have $n_{odd} = 1, 3, 5, ..., 525$ Thus there are 263 possible frequencies.

Horizontal Waves: The shower is a closed-closed instrument over horizontal waves. In a closed-closed situation, we have

$$L = \frac{n}{2}\lambda_n$$

Using that $\lambda_n = \frac{v_{sound}}{f} = \frac{331+0.6T}{f}m$,

$$f = \frac{(331 + 0.6T)n}{2L}$$

Since $T = 35\text{C}$, $L = 1.1\text{m}$, we have

$$f \approx 151.5n\text{Hz}$$

Since

$$20\text{Hz} \leq f \leq 20000\text{Hz}$$

We can have $n = 1, 2, ..., 131$. Here, there are 131 possible frequencies. □