Chapter 8 Homework

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November 17, 2021

Example 0.1

Clay problem, part a.

Solution. To solve this problem, we need to know about conservation of angular momentum. We need to know that $L = I\omega$, and we will need to know that the moment of inertia of a solid disk is $\frac{1}{2}MR^2$, where M is the mass and R is the radius (I proved this in the last homework).

Let M be the mass of the potters wheel, R be the radius of the potters wheel, m be the mass of the disk of clay, r be the radius of the disk of clay. Let ω_i be the angular velocity of the wheel initially, and ω_f be the angular velocity of the wheel finally.

The only force applied to the wheel is vertically downwards, so it doesn't effect the disks spinning about the vertical axis, so there is no net torque. Hence, be conservation of angular momentum,

$$\Delta L = \tau \cdot t = 0$$
$$L_i = L_f$$
$$I_i \omega_i = I_f \omega_f$$
$$\frac{1}{2}MR^2 \omega_i = I_f \omega_f$$

In order to compute ω_f, it remains to compute I_f. Note that the final moment of inertia is the initial moment plus the moment added on. Since the clay added is a disk, the added moment of inertia is $\frac{1}{2}mr^2$. Thus

$$\frac{1}{2}MR^2 \omega_i = \left(\frac{1}{2}MR^2 + \frac{1}{2}mr^2\right)\omega_f$$

$$\omega_f = \omega_i \left(\frac{MR^2}{MR^2 + mr^2}\right)$$

Which how fast the wheel is now spinning!

Some symbolic analysis: The units are correct since $\frac{MR^2}{MR^2+mr^2}$ is a scalar (the numerator and the denominator are both kgm^2). If the clay disk is almost nothing to the potters wheel, the final angular velocity will almost be the same as the initial angular velocity, which makes sense.

Let's plug in the numbers:

$$\omega_f = (1.5rev/s)\left(\frac{5(0.2)^2 kgm^2}{5(0.2)^2 + 3.1(0.04)^2 kgm^2}\right)$$

$$\omega_f \approx (1.5rev/s)(0.98) = 1.47rev/s$$

This makes sense, it's just a little bit under the original velocity since the clay is very little. □

Example 0.2

Clay problem, part b.

Solution. This problem is nearly analogous to the previous problem, except that the final moment of inertia is different because the clay is a point mass and it is dropped at the edge of the disk. Hence we will use everything the same in the previous solution, up until we computed I_f. We have that

$$\frac{1}{2}MR^2\omega_i = I_f\omega_f$$

Now, I_f is the sum of the initial moment and the added moment, as before. This time, the added moment is of a point mass, so it's moment is mR^2. This is because it has a mass m and is a distance R (notice, big R not small r) away from the axis that it's rotating around. Thus

$$\frac{1}{2}MR^2\omega_i = \left(\frac{1}{2}MR^2 + mR^2\right)\omega_f$$

$$M\omega_i = (M + 2m)\omega_f$$

$$\omega_f = \omega_i\left(\frac{M}{M + 2m}\right)$$

Which is the answer!

Some symbolic check: the units are correct because $\frac{M}{M+2m}$ is a scalar, since the denominator and numerator both have units kg. As M is large compared to m, ω_f becomes closer to ω_i as expected. As m is large compared to M, ω_f becomes far less than ω_i, which makes sense. R doesn't matter, which makes sense because the clay is just a point mass.

Now to plug in the numbers:

$$\omega_f = (1.5rev/s)\left(\frac{5kg}{5kg + 6.2kg}\right) \approx 0.67rev/s$$

This makes sense because ω_f is significantly less than ω_i, because the clay is heavy. □

Example 0.3

Clay problem, part c.

Solution. To solve this, we need to know that rotational energy is $\frac{1}{2}I\omega^2$. Hence the change in rotational energy is

$$\frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$$

We can simply plug in $I_f, I_i, \omega_f, \omega_i$ for each case.

Case 1: Part a. We can use the values from part a to get:

$$\Delta\mathrm{RE} = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$$

$$\Delta\mathrm{RE} = 0.5((0.102kgm^2)(1.47rev/s)^2 - (0.1kgm^2)(1.5rev/s)^2)$$

$$\Delta\mathrm{RE} = -0.002kgm^2rev^2/s^2$$

Notice that we lost some energy! This is probably due to heat and sound. The clay falling onto the wheel makes a sound and exerts a bit of heat, which is where the energy is lost.

Case 2: Part b. We can use the values from part b to get:

$$\Delta\mathrm{RE} = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$$

$$\Delta\mathrm{RE} = \frac{1}{2}(0.162kgm^2)(0.67rev/s)^2 - \frac{1}{2}(0.1kgm^2)(1.5rev/s)^2$$

$$\Delta\mathrm{RE} = -0.076kgm^2rev^2/s^2$$

We lost some energy here too, due to the same reasons as before. We lost more energy here because the blob is now a point-mass so we lost more velocity (as shown by the results in parts a and b). □