Quantum Chapter 2

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Example 0.1 (2.16)

Quantum state given in y-direction.

Solution. a. We first switch the basis to the z-direction using the basis switch equation. We have that:

$$|\psi\rangle = \frac{2}{\sqrt{29}}\left(\frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle\right) + \frac{3i}{\sqrt{29}}\left(\frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|-1\rangle\right) - \frac{4}{\sqrt{29}}\left(\frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle\right)$$

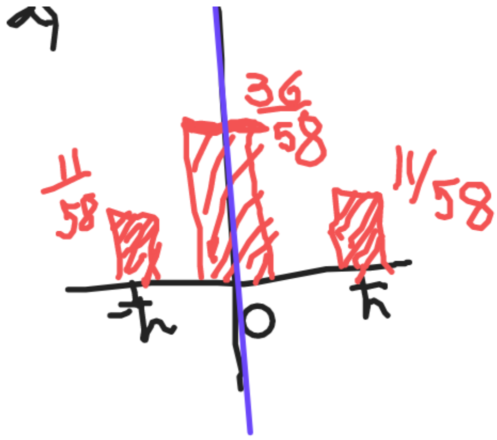
Using the equations on the sheet and the given equation in the problem. This reduces to:

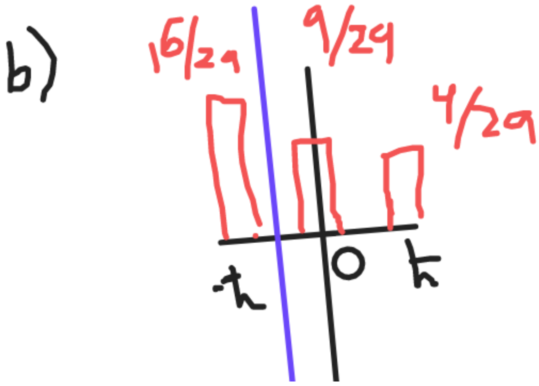
$$|\psi\rangle = \frac{3i-\sqrt{2}}{\sqrt{58}}|1\rangle + \frac{6i}{\sqrt{58}}|0\rangle + \frac{3i+\sqrt{2}}{\sqrt{58}}|-1\rangle$$

Hence we can measure a z-spin of $\hbar$ with probability $\left|\left(\frac{3i-\sqrt{2}}{\sqrt{58}}\right)^2\right| = \frac{11}{58}$. We can measure a z-spin of 0 with probability $\left|\left(\frac{6i}{\sqrt{58}}\right)^2\right| = \frac{36}{58}$. Finally, we can measure a z-spin of $-\hbar$ with probability $\frac{11}{58}$.

b. We simply square the components of the y-spin of $|\psi\rangle$ in the y-basis set. We can measure $\hbar$ with probability $\frac{4}{29}$, 0 with probability $\frac{9}{29}$, and -1 with probability $\frac{16}{29}$.

c. The histograms are as follows:





The expected value for part a) is simply 0 because it's evenly distributed about 0. The expected value for part b) can be calculated as:

$$\frac{4}{29}\hbar - \frac{16}{29}\hbar = -\frac{14}{29}\hbar$$

. $\square$

Example 0.2 (2.17)

Quantum state of spin-1 particle

Solution. a. We can rewrite ψ as:

$$|\psi\rangle = \frac{1}{\sqrt{30}}\,|1\rangle + \frac{2}{\sqrt{30}}\,|0\rangle + \frac{5i}{\sqrt{30}}\,|-1\rangle$$

Hence the probability we get $\hbar$ is $\frac{1}{30}$, the probability we get 0 is $\frac{4}{30} = \frac{2}{15}$, the probability we get $-\hbar$ is $\frac{25}{30} = \frac{5}{6}$. Hence the expected value is $\frac{\hbar}{30} - \frac{25\hbar}{30} = -\frac{24\hbar}{30} = -\frac{2\hbar}{5}$

b. We can use that

$$\langle S_x\rangle = \langle\psi|S_x|\psi\rangle$$

$$= \frac{\hbar}{30\sqrt{2}}\begin{pmatrix}1 & 2 & -5i\end{pmatrix}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 2\\ 5i\end{pmatrix}$$

$$= \frac{\hbar}{30\sqrt{2}}\begin{pmatrix}2 & 1-5i & 2\end{pmatrix}\begin{pmatrix}1\\ 2\\ 5i\end{pmatrix}$$

$$= \frac{4\hbar}{30\sqrt{2}}$$

$$= \frac{\hbar\sqrt{2}}{15}$$

$\square$