美国MCM/ICM竞赛指导丛书

Write Right for the American Mathematical Contest in Modeling

正确写作美国大学生数学建模竞赛论文

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内容提要

本书讨论如何正确写作美国大学生数学建模竞赛论文。书中包括两部分内容：一部分是地道的英文内容，另一部分是与之对应的中文介绍，这样的安排有助于读者先了解关键知识点，然后逐步培养用英文写作及思考的习惯。主要内容包括：论文结构、写作规范、英语用法、论文修改示例、符号与图表、数学表达式和句子以及数学编辑软件LaTeX和 MathType等。书中以中国学生提交的竞赛论文为案例，对常见错误进行了点评。

本书可作为指导大学生学习和准备美国大学生数学建模竞赛的主讲教材，也可作为大学生、研究生学习和准备全国大学生、研究生数学建模竞赛的参考书，也可供写作英文科技论文参考使用。

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“美国 MCM/ICM 竞赛指导丛书”
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COMAP 总裁序

美国大学生数学建模竞赛 (the Mathematical Contest in Modeling, MCM) 已经举办近 30 年了，时间真是快得难以置信。在此期间，竞赛从最初参赛的 90 支美国队逐渐发展成为一个国际大赛，今年已有来自世界各地的 25 个国家超过 5 000 支队伍参赛。尤其令人感动和鼓舞的是我的中国同事们对竞赛赋予的极大热情以及中国参赛队伍的快速增长。COMAP 张开双臂欢迎你们的参与。

COMAP 每年举办 3 个建模竞赛，即 MCM, ICM (the Interdisciplinary Contest in Modeling) 和 HiMCM (the High School Mathematical Contest in Modeling) 竞赛。竞赛的目的不仅仅是奖励同学们所作出的努力 —— 无疑这是同样重要的，我们举办各类数学建模竞赛的目的始终是为了推动在世界各国的各级教育体系中增加应用数学及数学建模的比重。建模是人们为了解世间事物的运作规律所做的尝试，数学的使用能够帮助我们建立更好的模型。这不是一个国家的任务，而是所有国家都应该共同关心的问题。COMAP 建模竞赛从孕育到现在已经演变成为实现这一宏伟目标的有力工具。

我热切地希望同学们通过阅读这套优秀的丛书，对 COMAP 举办的竞赛有更多的了解，并且学到更多有关数学建模的方法与过程。我希望同学们尝试自己解决丛书中讨论的所有建模问题，这些都是令人兴奋并且具有实用价值的问题。我希望更多的同学参加 MCM/ICM 竞赛，并参与推广和普及数学建模的活动，这是很有意义的工作。

Sol Garfunkel, 博士
COMAP 总裁
2012 年 11 月

Forward by Sol Garfunkel

While it is hard for me to believe, the Mathematical Contest in Modeling (MCM) is fast approaching its 30$^{\text{th}}$ year. During this time we have grown from 90 US teams to over 5,000 teams representing 25 countries from all across the globe. We have been especially buoyed by the enthusiasm shown by our Chinese colleagues and the rapid growth in Chinese participation. COMAP welcomes your involvement with open arms.

COMAP runs three contests in mathematical modeling; they are MCM, ICM (the Interdisciplinary Contest in Modeling), and HiMCM (the High School Mathematical Contest in Modeling). The purpose of all of these contests has never been simply to reward student efforts — as important as that is. Rather, our objective from the beginning has been to increase the presence of applied mathematics and modeling in education systems at all levels worldwide. Modeling is an attempt to learn how the world works and the use of mathematics can help us produce better models. This is not a job for one country, but for all. The COMAP modeling contests were conceived and evolved to be strong instruments to help achieve this much larger goal.

It is my supreme hope that through this excellent book series the Chinese students will learn more about COMAP contests and more about the process of mathematical modeling. I hope that you will begin to work on the exciting and important problems you see here, and that you will join the MCM/ICM contests and the rewarding work of increasing the awareness of the importance of mathematical modeling.

Sol Garfunkel, PhD
Executive Director
COMAP
November 2012

ICM 竞赛主席序

数学建模的训练与经验能使同学们在解决问题时更有创意，同时也能帮同学们成为更为优秀的研究生。“美国 MCM/ICM 竞赛指导丛书”的出版，将通过数学建模竞赛题目和概念的解析，帮助同学们掌握数学建模的技能，并为同学们在今后的工作中获得成功打下坚实的基础。

数学建模是一种过程，也是一种理念，或者说是一种哲学。作为过程，学生在理解及使用建模过程或框架时需要指导并积累经验。作为经验，学生需要使用不同的数学方法（离散、连续、线性、非线性、随机、几何及分析）构造数学模型，从中体验不同的细节及复杂程度。作为理念，学生需要发现各种相关的、具有挑战性的及有趣的实际问题，从中培养数学建模的兴趣，并认识到数学建模在实际生活中的作用。数学建模的主要目的是指导学生用建模的方法解决实际问题。尽管在实际中，有些问题或许可以使用已有的算法和公式来求解，但数学建模的方法比简单使用已有算法和公式能解决更多的问题，特别是解决新的、没有固定答案及没有被解决过的问题。

为了积累经验，同学们应尽早地接受数学建模的训练，至少应该在大学低年级就开始，这样可以在以后的课程学习中进一步强化数学建模能力。由于数学建模的综合与交叉特性，各个专业的学生都能够从数学建模活动中受益。

本套丛书从将数学模型作为研究工具的角度出发，包括介绍模型的构造，分析建模过程，这些都是帮助学生更好地掌握数学建模技能的重要因素。数学建模是充满挑战的高级技能，更重要的是能够帮助学生更快地成长。当今世界需要解决的问题往往很复杂，所以建立的数学模型也很复杂，通常需要通过精细的计算和模拟才能获得解答或得到对模型结果的分析与检验。由于数据可视化技术的普及，解题方法的增加，所以现在是培养更多数学建模高手的最佳时期。

我希望同学们在数学建模探索中取得进步，也希望指导教师在使用这套丛书提供的例子及方法指导学生时取得很好的效果。尽管学生的层次可能不同，但我对你们的忠告是同样的：树立你的信心，发展你的技能，用你的才能解决社会中最具挑战性及最重要的问题。祝各位建模好运!

Chris Arney，博士
美国西点军校数学系教授
ICM 竞赛主席
2011 年 10 月

Forward by Chris Arney

Undergraduate students who receive instruction and experiences in mathematical modeling become better and more creative problem solvers and graduate students. This book series is being published to prepare and educate students on the topics and concepts of mathematical modeling to help them establish a problem solving foundation for a successful career.

Mathematical modeling is both a process and a mindset or philosophy. As a process, students need instruction and experience in understanding and using the modeling process or framework. As part of their experience, they need to see various levels of sophistication and complexity, along with various types of mathematical structures (discrete, continuous, linear, nonlinear, deterministic, stochastic, geometric, and analytic). As a mindset, students need to see problems that are relevant, challenging, and interesting so they build a passion for the process and its utility in their lives. A major goal in modeling is for students to want to model problems and find their solutions. Recipes for structured or prescribed problem solving (canned algorithms and formulas) do exist in the real world, but mathematical modelers can do much more than execute recipes or formulas. Modelers are empowered to solve new, open, unsolved problems.

In order to build sufficient experience in modeling, student exposure must begin as early as possible – definitely by the early undergraduate years. Then the modeling process can be reinforced and used throughout their undergraduate program. Since modeling is interdisciplinary, students from all areas of undergraduate study benefit from this experience.

The articles and chapters in this series expose the readers to model construction, model analysis, and modeling as a research tool. All these areas are important and build the students' modeling skills. Modeling is a challenging and advanced skill, but one that is empowering and important in student development. In today's world, models are often complex and require sophisticated computation or simulation to provide solutions or insights into model behavior. Now is an exciting time to be a skilled modeler since methodology to provide visualization and find solutions are more prevalent and more powerful than ever before.

I wish the students well in their adventure into modeling and I likewise wish faculty well as they use the examples and techniques in this book series to teach the modeling process to their students. My advice to all levels of modelers is to build your confidence and skills and use your talents to solve society's most challenging and important problems. Good luck in modeling!

Chris Arney, PhD
United States Military Academy at West Point
Professor of Mathematics
Director of the Interdisciplinary Contest in Modeling
October, 2011

丛书简介

美国大学生数学建模竞赛（the Mathematical Contest in Modeling, MCM/the Interdisciplinary Contest in Modeling, ICM），即“数学建模竞赛”和“交叉学科建模竞赛”，是一项国际级的竞赛活动，为现今各类数学建模竞赛的鼻祖。

1985 年，在美国教育部的资助下，在美国针对在校大学生创办了一个名为“数学建模竞赛”的竞赛，其宗旨是鼓励大学师生对不同领域的各种实际问题进行阐明、分析并提出解决方案。它是一种完全公开的竞赛，参赛形式为学生三人组成一队，在三天（72 小时）（近年改为四天，即 96 小时）内任选一题，完成数学建模的全过程，并就问题的重述、简化和假设及其合理性的论述、数学模型的建立和求解（及软件）、检验和改进、模型的优缺点及其可能的应用范围与自我评价等内容写出论文。MCM/ICM 非常重视解决方案的原创性、团队合作与交流以及结果的合理性。由专家组成的评阅组进行评阅，评出优秀论文。除了不允许在竞赛期间与团队外的任何人（包括指导教师）讨论赛题之外，允许使用图书资料、互联网上的资料、任何类型的计算机程序和软件等各种资料和途径，从而为参赛学生提供了广阔的创作空间。第一届竞赛时，只有美国的 158 个队报名参加，其中只有 90 个队提交了解答论文。2012 年 MCM/ICM 共有 5 026 个队参加，其中 MCM 有 3 697 个队，ICM 有 1 329 个队，遍及五大洲。MCM/ICM 已经成为最著名的国际大学生竞赛之一，影响极其广泛。

近年来，已有越来越多的中国学生组队参加美国大学生数学建模竞赛，其中不乏被评为优胜论文（Outstanding Winners）的佼佼者，这充分显示了我国大学生参加 MCM/ICM 的积极性与实力。学生在准备竞赛的时候，除了在指导教师的帮助下阅读和研究以往竞赛的优胜论文以外，普遍希望能有一些专门针对美国大学生数学建模竞赛的书籍，指导和帮助备赛。

“美国 MCM/ICM 竞赛指导丛书”就是为了满足读者的这一需求而出版的，目的是帮助学生学习从全局出发，不受固定模式的限制，用建模的手段解决开放性问题的研究方法，并提高写作能力。丛书的读者对象包括参赛学生及对数学建模与算法感兴趣的研究生、专业人员和业余爱好者。

我们邀请到 COMAP 中国合作总监、美国麻省大学罗威尔分校王杰教授担任丛书主编，他曾为 MCM/ICM 命题，对竞赛具有很多独到的认识。丛书作者

来自各高校，他们都是有经验的指导教师或参加过竞赛的优秀成员。丛书包括一本《正确写作美国大学生数学建模竞赛论文》和若干辑《美国大学生数学建模竞赛题解析与研究》，前者为一本指导学生如何正确写作 MCM/ICM 论文的工具书，后者中的每一辑将讨论若干赛题，包括问题的背景、分析技巧、建模与测试方法及算法设计，并引导读者列出进一步研究的课题。目标是培养学生多方面的能力，如数学、编程、写作及课题研究等的训练，提高学生分析问题、解决问题的水平。

丛书的出版计划得到了美国数学建模专家的广泛支持，COMAP 总裁 Sol Garfunkel 博士及 ICM 主席、美国西点军校数学系教授 Chris Arney 博士受邀担任丛书顾问并为丛书作序。

我们热切希望通过这套丛书的出版，进一步活跃我国大学生参加 MCM/ICM 的积极性，提高他们的自信心，并最终取得满意的成绩。更为重要的是，提高学生的研究和解决实际问题的能力。

前言

MCM/ICM 竞赛不但是数学建模竞赛，也是论文写作竞赛。而且从某种意义上说，写作是一个更重要的环节：一篇写作得体的解答论文，即使模型不是很好，也往往会比一篇模型较好但写作较差的论文获得更好的评审结果。由于解答论文必须使用英语完成，这就使中国学生必须面对双重挑战，即数学建模的挑战以及使用非母语写作的挑战。一些参赛小组由于写作方面的欠缺，使得本来可以进入更高级别的论文没有得到相应的认可，令人惋惜。

建立了合理的数学模型并找到了有效的解题方法以后，写作就是关键的因素。本书假设参加 MCM/ICM 竞赛的大学生都已具备一定的数学基础及英文写作基础，在此基础上针对 MCM/ICM 论文的评审标准，讨论如何写好 MCM/ICM 论文，指出哪些内容是评委期待见到的，以及如何写作才能使评委能够在短时间内充分了解论文的要点。

为了写出高水平的解答论文，首先要从设计论文的结构开始，然后是了解数学论文的写作规范及使用准确无误的英语，并正确书写数学表达式和含有数学表达式的句子。此外，还要有好的写作工具。本书将分 9 章讨论这些内容。

第 1 章介绍 MCM/ICM 竞赛的简史、目的、评审标准、评审过程及论文的等级划分，并讨论写作的重要性；第 2 章讨论 MCM/ICM 论文的结构，包括如何写引言、假设条件、结论及摘要等；第 3 章介绍数学论文的写作规范；第 4 章讨论中国学生在用英文写作数学建模论文时需要注意的一些语法问题，并提供一些在写作数学建模论文时常用的动词；第 5 章以中国学生的 MCM/ICM 竞赛论文为实例，讲解如何修改论文的标题、引言、摘要及假设条件；第 6 章介绍数学符号及图表的使用；第 7 章介绍如何书写数学表达式及含有数学表达式的句子；第 8 和第 9 章针对写作 MCM/ICM 论文的需要，分别介绍数学编辑软件 LaTeX 及 MathType。

本书融汇了作者多年来指导中国学生和青年学者写作数学与计算机科学论文积累的经验，力图以较短的篇幅和在较短的时间内，帮助中国学生提高用英文写作 MCM/ICM 论文的能力。此外，应用数学、计算机科学、计算机工程以及其他相关专业的研究生和专业人员，在科研和软件开发中经常需要用英文写作与数学表达有关的论文和报告，此书对于他们也是一本简单实用的工具书。

本书的初稿仅包含英文部分，为了便于中国学生使用，后来又增加了中文部分，两部分的内容一致。建议英语程度好的读者直接阅读英文版，训练使用英语思考问题的能力。英文部分主要由 Jay Belanger 教授编写，中文部分由王杰教授编写。全书由王杰教授统稿。

麻省大学罗威尔分校计算机科学系李优和数学系盛小波为本书的写作提供了帮助；西安交通大学算法与建模研究所王嘉寅编写了附录“参赛须知”的初稿；高等教育出版社刘英编辑对本书的写作提供了许多建设性意见，使本书质量有了很大提高。作者对他们的帮助表示衷心的感谢。

本书虽经多次修改，缺点和错误仍在所难免，欢迎读者批评指正，并将改进建议发送到作者的电子邮箱：wang@cs.uml.edu。对于读者的支持与帮助，作者谨在此表示由衷的感谢。

Jay Belanger，美国杜鲁门州立大学

王杰 (J. Wang)，美国麻省大学罗威尔分校

2012 年 12 月 16 日

目录

I

正确写作美国大学生数学建模竞赛论文

第 1 章 简介

美国大学生数学建模竞赛(以下简称美国数学建模竞赛)是一年一度的国际大学生数学建模竞赛。竞赛期间,世界各地的大学生以小组形式参赛,在同一时间内解答赛题。为了激发学生的创造热情和培养学生解决实际问题的能力,所有赛题均具有真实背景及应用价值,而且是没有标准答案的开放问题。赛题的解答必须按学术论文的形式和要求完成。所以,美国数学建模竞赛不仅是建模竞赛,而且也是写作竞赛。从某种意义上说,论文写作是竞赛更重要的一个环节,一篇思路清晰、描述得体的解答论文,即使模型不是很好,也通常会比一篇模型较好但写作较差的论文获得更好的评审结果。

1 简史

美国数学建模竞赛的最初设想是举办一个"应用版的普特南数学竞赛"。普特南数学竞赛始于 1938 年,是一年一度的美国大学生数学竞赛。学生个人参赛,独立解答赛题。赛题通常是比较艰深的纯数学问题,导致参赛学生往往只能得到很低的分数:满分 120 分的竞赛,大约一半的学生只能得到 1 分或 2 分,甚至不少学生只得零分。

美国数学建模竞赛的创始人 Ben Fusaro 对普特南竞赛的赛题缺乏实用性,且赛题难度太大导致学生参赛热情不高而深感忧虑。他在 20 世纪 80 年代初提出设立一种新的竞赛机制,鼓励学生以小组的形式参赛,共同解决具有实际应用背景的数学问题。他的设想是,参赛小组在一天内解答两个具有实用背景的建模问题,一个用离散数学的方法,一个用连续数学的方法。这一想法得到了数学界的广泛支持,一种新的竞赛机制由此产生,具体做法是学生以小组形式参赛,共同解答开放型的实用建模问题。比赛时间设在周末,赛题分别涉及离散数学和连续数学。美国数学及应用联合会(the Consortium for Mathematics and Its Applications, COMAP)主席 Solomon Garfunkel 随后建议 Ben Fusaro 向美国教育部申请举办竞赛的经费。1984 年 6 月获得一笔为期三年的基金,数学建模竞赛(the Mathematical Competition in Modeling, MCM[①])在 1985 年开始正式举

① MCM 竞赛后改名为 the Mathematical Contest in Modeling。

办。Fusaro 在基金申请报告[1] 中阐明的竞赛目的如下：

> 该竞赛旨在推动学生和指导教师参加数学建模活动。要求学生首先根据自己的理解，用自己的语言将开放型的赛题重新阐述清楚，然后对赛题进行分析并提出解决方案。竞赛提供新的机制以吸引和鼓励更多的学生参赛，强调数学建模的整体过程，而不是最终的答案。竞赛的主要特点如下：
>
> - 所有赛题均为具有真实背景的开放型建模问题，由工业及政府部门的一线专家命题或在他们的指导下命题。
> - 解答格式有明确的规范，且参赛小组有较长的时间撰写解答论文。
> - 参赛小组在解题过程中允许使用计算机、教科书及其他资源。
> - 论文的表述是否清晰是论文评审的重点，最优论文将在数学期刊上发表。
>
> 在竞赛的深入发展过程中，将会开设各种新课程、研讨会及讨论班，帮助学生和指导教师提高数学建模能力。

1985 年举办的第一届 MCM 竞赛，只有来自美国的 158 个小组参赛（其中只有来自 70 所大学的 90 个参赛小组提交了解答论文）。在国际数学界的支持下，美国数学建模竞赛得到了迅速发展。到了 2010 年，美国数学建模竞赛已发展成为大规模的国际赛事，当年共有 2 254 个小组参赛，除了美国的 358 个参赛小组，其余参赛小组来自世界各地。

竞赛由 COMAP 组织和管理。竞赛题分为两类，一类涉及连续数学，称为 A 题；另一类涉及离散数学，称为 B 题。COMAP 从 2000 年开始增设了交叉学科的建模竞赛，称为 ICM（the Interdisciplinary Contest in Modeling）竞赛，举办时间及参赛规则与 MCM 竞赛一样。ICM 竞赛每年只出一道赛题，称为 C 题，涉及离散数学、连续数学或者二者都涉及。所以从 2000 年开始，美国大学生数学建模竞赛便改称为 MCM/ICM 竞赛①。此外，COMAP 还设有专门面向高中学生的数学建模竞赛，称为 HiMCM（the High School Mathematical Contest in Modeling），不在本书讨论范畴。

参赛小组虽然在比赛开始后才能选择赛题，但应该在开赛前就做好充分准备，精心选择队员。Ben Fusaro 曾经说过，“三名都很优秀的学生不一定能组成最好的参赛小组；最好的参赛小组是由三名配合默契的优秀队员组成的。”换句话说，在组建参赛小组时，应该挑选能够相互配合并且取长补短的小组成员。因此，在每个参赛小组中，除了应该有一名数学非常好的队员外，还应该有一名熟悉计算机应用软件或编程能力强的队员。当然，每名队员都可能肩负多种职能。

① 为了简化称谓，除非特别说明，本书将用 MCM 泛指 MCM 及 ICM 两个竞赛。

2 论文评审

MCM 赛题不设标准答案，解答论文没有及格分数线，也不打分。参赛论文将按评审标准划分等级。

评审标准

评委关心的是参赛小组的解题思路和建模过程，以及是否给出了清晰的描述，并着重检查以下内容：

- 是否给出了令人满意的赛题解读，以及对赛题中可能出现的模糊概念是否给予了必要的澄清；
- 是否明确列出了建模需要用到的所有前提条件及假设，对其合理性是否给出了满意的解释或论证；
- 是否通过对赛题的分析给出了建模的动机或论证了建模的合理性；
- 是否设计出了能有效地解答赛题的模型；
- 是否对模型给出了稳定性测试；
- 是否讨论了模型的优缺点，并给出了清晰的结论；
- 是否给出了符合要求的摘要。

没有全部完成解答的论文是可以被接受的，而且如果在某些方面有创意，仍然有可能获得较好的评审结果。

等级划分

参赛论文如果没有按要求讨论赛题，或违反了竞赛规则，会被评定为**不合格论文**（Unsuccessful Participants）。其余参赛论文根据评审标准分为 5 个级别，由低到高分别为**合格论文**（Successful Participants）、**乙级论文**（Honorable Mention）、**甲级论文**（Meritorious）、**特级提名论文**（Finalist）、**特级论文**（Outstanding Winner）（也称为**优胜论文**）。任何论文只要对赛题进行了适当的讨论，没有违反竞赛规则，就是合格论文。只有建模和写作两方面都最优秀的论文才可能被评为特级论文。各级别的论文所占的百分比如下：

- 合格论文，大约 50% 的论文属于这个级别。
- 乙级论文，大约 30% 的论文属于这个级别。
- 甲级论文，大约 10% 到 15% 的论文属于这个级别。
- 特级提名论文，大约 1% 的论文属于这个级别。
- 特级论文，大约 1% 的论文属于这个级别。

除了论文评级外，MCM 竞赛还设有 INFORMS 奖、SIAM 奖、MAA 奖及 Ben Fusaro 奖等 4 个奖项，奖励优秀论文。

- INFORMS 奖是由美国运筹学和管理学研究协会 (the Institute for Operations Research and the Management Sciences) 设立的。评审委员会在解答 A 题、B 题及 C 题的特级论文中分别选出一篇 (或者不选) 给予奖励。
- SIAM 奖是由美国工业与应用数学学会 (the Society for Industrial and Applied Mathematics) 设立的。评审委员会在解答 A 题、B 题及 C 题的特级论文中分别选出一篇 (或者不选) 给予奖励。
- MAA 奖是由美国数学会 (the Mathematical Association of America) 设立的。评审委员会在解答 A 题或 B 题的特级论文中分别选出一篇论文给予奖励。
- Ben Fusaro 奖是由 COMAP 设立的。评审委员会通常在特级提名论文中选出一篇具有特殊创意和独特见解的论文给予奖励。

评审流程

论文评审的方式是盲评，通常在竞赛结束 3 个星期后的第一个周末进行。所有参赛论文均使用唯一给定的编号统一识别，这个编号称为控制编号 (Control Number)。论文的作者姓名及其所在大学的名称均不得在论文中出现。

评审分为两个阶段。第一阶段也称为鉴别阶段，所有论文在此阶段按质量分别归入以下三类: 第一类是可以进入下一评审阶段的论文，第二类是满足竞赛要求，但却不足以进入下一评审阶段的论文 (这类论文被定为合格论文)，第三类是不符合竞赛要求的论文 (这类论文被定为不合格论文)。

在鉴别阶段中，评审委员会设有一名主审及若干评委，每名评委负责大约 25 篇论文。每篇参赛论文由两名评委独立评审并打分。如果两个分数相差太多，则先由这两名评委协商；如果意见无法统一，则增加第三名评委评审。当论文获得两个比较一致的分数时，这两个分数的和就是该论文在鉴别阶段的得分。主审和竞赛主席商定进入第二阶段评审的分数线，使得略少于二分之一的参赛论文进入第二阶段的评审。

在鉴别阶段中，每位评委平均只有 10 分钟左右的时间评审一篇论文，因此，评委常常只能通过阅读论文的摘要来判断论文水平的高低。所以，从论文能否通过第一阶段评审的角度看，**摘要是论文最重要的部分**。

通过第一阶段的评审后，参赛论文按离散数学和连续数学进行分类，分别进入第二阶段的评审。第二阶段的评审又分成若干轮，通过评审的轮数越多，论文评定的级别将越高。在进入下一轮评审之前，每篇论文都将经过多名评委的评阅。每名评委将会用 15 ~ 30 分钟的时间评审一篇论文。因此，为了能在这样短的时间内给评委留下深刻的印象，论文的写作必须结构严谨、条理清晰、简单易读，同时将主要结果以最明显的方式表达出来。

在第二阶段评审的最后一轮，由所有评委共同讨论评出特级论文，经过竞赛主席和副主席一致同意后，最终确定。

3 写作的重要性

论文的写作在 MCM 竞赛中举足轻重，所以每个参赛小组一定要有擅长写作的队员，有人甚至建议最好能有一名（甚至两名）英语专业的队员。为了准备参赛，在竞赛开始前的几个月，参赛小组应该每周定期集训，在指导教师的帮助下，从往届的 MCM 赛题中选择赛题进行实战演习，练习如何搜集资料、如何建立数学模型解答赛题及如何写好解答论文。开赛以前，通过各项训练，发现、弥补并纠正小组成员在写作方面的欠缺与不良习惯。此外，阅读优秀论文对提高写作能力很有帮助，所以阅读往届 MCM 竞赛的特级论文是学习写出高质量的 MCM 解答论文的有效方法。每年竞赛的特级论文及其讲评都刊登在 UMAP 数学建模期刊上。

竞赛开始后，参赛小组的每名成员应根据预先的分工各司其职。例如，甲负责对赛题做必要的背景分析，乙负责写作，丙负责编写及调试计算机程序。小组成员的分工随着竞赛的深入可进一步调整。

论文的写作应尽早开始。根据以往的经验，许多参赛小组往往低估了论文写作所需的时间，不能及时写出条理清晰的论文。因此，参赛小组可以考虑在竞赛开始后的第二天，约定一个时间结束手头的建模工作，以便全力以赴写好参赛论文。

第 2 章　论文结构

论文的写作应以帮助读者快速阅读和理解论文内容为目的。为了达到这个目的，必须精心设计论文的结构。松散的结构和缺乏条理的书写，将很难把参赛小组的解题思路和方法正确表示出来。因此，论文的组织结构必须条理清晰、主次分明。由于在每一轮评审中，评委只有很短的时间审阅论文，所以为了吸引评委的注意力，论文必须重点突出。

1　小节划分

论文应该按内容划分成小节和子小节，并冠以恰当的标题，使评委无需阅读细节就能把握论文的主线。根据论文的评审标准，MCM 竞赛委员会建议参赛小组按以下结构将论文分节[2]：

- 重述并澄清赛题。
- 列出建模所用的所有前提条件及假设，并给出清晰的解释。
- 分析赛题，给出建模动机或论证建模的合理性。
- 模型设计。
- 讨论如何检验模型，包括误差分析和稳定性测试（如对条件、敏感度等因素进行分析和测试）。
- 讨论模型的优缺点。
- 书写论文摘要。摘要必须按要求写在特定的摘要页上，长度不超过一页，提交时作为参赛论文的首页。

例如，在 1989 年的 MCM 竞赛中，有一篇题为“How to Please Most of the People Most of the Time”的优胜论文[3]，便是一篇结构出众的范文。这篇论文研究飞机起飞时的排队问题。因为机场通常只有一条或两条起飞跑道，所以在相近时间内起飞的飞机通常需要排队等候，如果等候的时间太长，中途需要转机的乘客便有可能会耽误转机。所以需要找出最佳的调度方案，使机内大多数乘客不会因为延迟到达目的地而不满。以下是该论文的小节划分及标题：

Summary

1 Restatement of the Problem

2 Assumptions
3 Justification of Our Approach
4 The Model
  4.1 Dissatisfaction of a passenger needing a connection
  4.2 Dissatisfaction of a passenger not needing a connection
  4.3 Total dissatisfaction on an aircraft
5 Testing the Model
6 Results
7 Strengths and Weaknesses
  References
  （摘要
1 问题重述
2 假设条件
3 建模方法的合理性论证
4 模型设计
  4.1 需要转机乘客的不满程度
  4.2 无需转机乘客的不满程度
  4.3 所有乘客的不满程度
5 模型测试
6 结论
7 模型的优点与弱点
  参考文献）

虽然不是每篇优胜论文都像这篇论文一样与 MCM 建议的小节划分如此吻合，但参赛小组还是应该尽可能按照所建议的内容划分小节。简单明了、条理清晰的小节划分有助于评委迅速掌握论文的要点。

将论文划分小节时，应避免在小节中出现大段的文字叙述，这样的叙述会妨碍评委在浏览论文时掌握论文的要点。重要的句子，包括首次定义的概念，应该用黑体或斜体书写。在快速浏览时，黑体比斜体更醒目，所以更能吸引读者的注意力。然而，如果黑体或斜体出现过多，又会使论文看上去比较杂乱，影响阅读。因此在突出重点的前提下应该尽可能少用黑体或斜体字体。

重要的数学公式应另起新行单独列出。建模所用的假设条件以及所有可以用列表方式表述的内容，为方便阅读，都应该用符号列表（或编号列表）的方式逐条陈列出来，不要将它们淹没在段落里。此外，图表（即图形和表格）也是很好的表现形式，在使用图表的时候要给每个图表加上简单明确的文字说明，使读者很快知道图表所要表达的内容。

2 写好引言

引言（Introduction）是论文的第一节，尽管其标题不一定叫“引言”（例如，也可用“问题重述”作为标题）。引言应该包括以下内容：对赛题的解读、对现有研究成果的综述与评论以及对解题思路和主要方法的简要介绍。

参赛小组应通过引言向评委表明，参赛小组对赛题做了认真仔细的思考与研究。为了写好引言，参赛小组应该在解题时就开始思考如何写引言，并在解题过程中反复修改，直到满意为止。

引言的第一句话是全文最重要的句子，用于激发读者阅读论文的兴趣。同理，引言的第一段也是全文最重要的一个段落，应该写得浅显易懂，尽量少用或者不用数学表达式，使得即使读者对所要解决的数学问题不甚了解，也能看懂第一段话的内容而产生继续阅读的兴趣。所以，引言的第一个句子及第一个段落需要经过反复推敲和修改，不可掉以轻心。

赛题选定后，参赛小组首先要用自己的语言重述赛题，明确解题目标，并澄清原题叙述上可能出现的模糊概念。一个问题可能存在多种解读方式，参赛小组必须明确表述他们是如何解读赛题的。例如，在 2010 年的 MCM 竞赛中，有一道赛题要求参赛小组算出棒球棍上的最佳击球点。可是“最佳击球点”这个概念却有多种解释。在题为“The Sweet Spot: A Wave Model of Baseball Bats”[4] 的优胜论文的引言中，参赛小组是这样解释最佳击球点的：

> “There are at least two notions of where the sweet spot should be – an impact location on the bat that either
>
> - minimizes the discomfort to the hands, or
> - maximizes the outgoing velocity of the ball.
>
> We focus exclusively on the second definition.”
>
> （最佳击球点至少存在两种不同的解释：
>
> - 一种是让击球者的手感觉最舒适的击球位置。
> - 一种是使棒球飞出速度最快的击球位置。
>
> 本文只考虑第 2 种解释。）

即便是已经表述得很精确的概念，仍可以给出更有利于解题的解读方式。例如，1988 年的 MCM 竞赛中有一道赛题，要求参赛小组在已知船只当前位置的前提下，设计一个搜寻方法定位其驶离后的位置，位置要求“精确到 2° 以内”。一篇题为“Locating a Drug Runner: Miami Vice① Style”[5] 的优胜论文是这样解释“精确度”这个概念的:

① “Miami Vice”（即《迈阿密风云》）是美国 20 世纪 80 年代的一部电视连续剧，讲述迈阿密的两名便衣警察破案的故事。

"We interpret the error of $\pm 2°$ as a normal distribution, ... with standard deviation of $1°$."
（我们将精确到 $\pm 2°$ 这个概念理解成一个正态分布，…… 其标准偏差为 $1°$。）

明确题意之后，参赛小组应该立刻开始查阅资料，阅读与赛题相关的文献，了解已有的模型和方法。因此，在引言中，除了重述赛题外，还应对现有文献在相似问题上所做的研究（包括方法和结论）做适当的综述和评论。此外，在引言中还要简明地描述参赛小组的解题方法以及所得到的主要结果。

3 论文主体

论文的主体是指引言之后及结论之前的部分，用于描述模型的设计，包括列出和论证建立数学模型所做的假设，以及对赛题所做的分析。论文主体通常分为若干小节。

合理的数学模型应基于合理的假设，所以在描述模型之前，参赛小组应该将模型设计所用的假设条件一一列出并解释清楚。不要有未经说明的假设，以免读者自行猜测而造成误解。此外，还应该对建模的初衷和动机适当地加以讨论。

本节举例说明如何解释假设条件，并讨论两种建模思路。一种是通过建立一系列的子模型一步一步地构造最终模型，另一种是以赛题为特例构造普遍模型。

假设条件与解释

2010 年的 MCM 竞赛中有一道赛题，要求参赛小组根据以往的作案地点预测连环犯罪的位置。解答这道赛题的重点是分析罪犯的活动方式，一篇题为 "Centroids, Clusters, and Crime: Anchoring the Geographic Profiles of Serial Criminals" [6] 的优胜论文，采用了一种特殊的距离空间来描述犯罪分子在连环作案时的活动区域。参赛小组在"假设条件"这一节中逐条列出了建模所用的假设条件，并给出了这些假设的合理性论证。有一条假设是"罪犯的活动不受限制"，但罪犯在市区的活动，实际上会受到街道布局及街道两旁建筑物的限制。由于街道的布局通常类似于网格，所以参赛小组对这个假设作了以下解释：

"**Criminal's movement is unconstrained.** Because of the difficulty of finding real-world distance data, we invoke the 'Manhattan assumption': There are enough streets and sidewalks in a sufficiently grid-like pattern that movements along real-world movement routes is the same as 'straight-line' movement in a space discretized into city blocks ..."

(**罪犯的活动不受限制**：由于实际数据不易获得，本文将采用“曼哈顿假设”来描述罪犯的活动方式，也就是说，假设市区内有足够多的街道及人行道，纵横交错地构成一个与离散方格相似的网状布局，罪犯在此网状布局中的活动路径（即沿着方格的边作运动）与现实世界中的“直线运动”相同 ……)

很多论文往往忽略了这个假设，或者只隐含了这个假设，但没有明确地指出来。无论解答什么样的赛题，参赛小组在论文中都应该明确列出所有用到的假设条件，并解释其合理性。如果对某个假设无法给出满意的解释，则应重新考虑这个假设是否合理，并进行修改，使得修改后的假设能有满意的解释。评委不但会检查论文是否列出了在建模过程中用到的所有假设，而且还会审查这些假设是否合理，以及论文对这些假设的合理性是否给出了满意的解释。

模型设计

建立数学模型是解决问题的手段，而非目的。所以对模型本身不需要做过多的解释，而且在大多数情况下也没有必要。设计的模型能够解决问题才是最重要的。**在所有能够解决问题的模型中，最简单的模型也许就是最好的模型**。

在研究赛题的背景和查阅文献资料时也许会发现，某些现有的模型经过适当的修改或扩充后，就能解决当前的问题。由于在短时间内一切从零开始很难设计出满意的模型，所以只要有可能，应该充分利用现有的研究成果和方法，经过理解和消化后建立自己的模型（不经过理解所建立的模型将很难做到简单明了）。

系列模型

设计模型时，可以尝试从简单模型开始，逐步加工、修改及完善，一次比一次更接近实际，最终得到能完满地解答赛题的模型。例如，在 2008 年的 MCM 竞赛中，有一道赛题要求参赛小组建立一个数学模型，预测在全球温度升高而导致北极冰盖融化的假设下，美国佛罗里达州的沿海地区将会受到怎样的影响。其中一篇题为“The Impending Effects of North Polar Ice Cap Melt”[7]的优胜论文，就是采取建立一系列模型的方法最终给出了满意的解答。其具体做法如下：

(1) 首先建立模型 1：*常温模型*。这个模型假设全球温度不变，冰盖的融化速度不变及海洋水量不变。

(2) 接着建立模型 2: *变温模型*。这个模型假设全球温度在不断变化。

(3) 最后建立模型 3: *气候变暖下的海洋水量模型*。这个模型将前面建立的模型中所忽略的问题考虑进来，包括南北两半球的相对海洋水面面积。值得指出的是，海洋水面的概念是论文作者提出来的，赛题描述中并没有提到这个概念。

建模时，参赛小组应集中精力设计一个模型，或者最终能导出一个较好模

型的一系列子模型。不要分散精力，拼凑出几个很一般的模型，企图用这些一般的模型碰运气。这种做法将会分散评委的注意力，因而不被评委看好。

普遍模型

模型设计另一个要注意的问题是，不要只针对赛题给出的参数值设计模型。高水平的论文通常会把赛题看成是一个普遍问题的特例，首先探讨普遍问题的求解，然后再对赛题这一特例给出具体解答。

例如，在 1988 年的 MCM 竞赛中有一道赛题，要求参赛小组设计一个算法，根据火车货运车厢的长度、宽度、高度及载重量，计算如何在两节车厢装载大小不一的货物，货物的长、宽、高、重量及类型是给定的，使得车厢内的空余空间最小。用变量表示车厢及货物的长、宽、高、车厢的载重量及货物的重量，就得到一个普遍的装载优化问题，而赛题只是这个普遍问题的一个具体特例。由于参数数值已知，因此赛题将比普遍问题简单。例如，一篇题为 "Loading Two Railroad Cars" [8] 的优胜论文就是这样处理的，论文作者明确指出：

> "We have produced a general algorithm to solve this type of problem, but for our problem a relationship exists that greatly simplified the algorithm."
> （本文给出了一个解决此类问题的通用算法。利用赛题给定的参数数值，本算法将大为简化。）

4 如何写结论

论文的结论部分应描述模型的测试方法以及所得到的结果。建模是解答问题的手段，即使赛题没有标准答案，参赛小组也应该在所建立模型的基础上进行分析并得出明确的结论。结论应该在结论部分的开始就明确地写出来（陈述结论的小节通常使用 "Conclusions" 做标题）。写得好的论文不需要读者自己寻找结论，也不需要读者自己将某些句子理解为结论。此外，这一节还应对结论进行适当的讨论，并指明哪些是作者的观点，哪些是得到的结果。

根据 MCM 竞赛的要求，参赛小组应该对所建立的数学模型进行敏感性分析和稳定性测试，使模型更具有说服力。遗憾的是，很多参赛小组对这一要求往往重视不够，甚至完全忽略。模型通常会用到一些参数（例如，交通建模问题可能会用到平均速度这一参数），在结论中应该讨论这些参数值的轻微变化对模型及结论产生的影响。

在描述结论时，应设法使读者认同论文给出的解答，尽管该解答不一定是最好的。有些问题的解答可能是一个计算机程序，在这种情况下，应该对程序进行

多次测试，并且每次的测试数据应稍有不同。测试时应尽可能使用真实的数据，避免因为人造数据而引起读者对结论的怀疑。

在描述结果时一定要给出足够的信息，使得如果有必要，读者自己也能得到相同的结果。如果某些结果是由计算机程序计算出来的，则必须指明所使用的计算机程序。如果参赛小组自己编写了程序代码，则应将程序运行的算法描述清楚。

MCM 赛题没有标准答案，但由于时间的限制，几乎每篇论文给出的模型和解答都会存在这样或那样的缺陷。评委肯定也会发现这些缺陷，所以在写结论时应该明确指出这些缺陷，表明参赛小组不但知道模型所含的缺陷，而且也思考过如何改进和修补。论文缺陷在前面描述模型时可能已经提到过，在这里应该再次指出来。

由于时间和计算资源的限制，参赛小组可能无法完成本来有能力解决的一些问题。所以，写结论时也应该明确指出，假如有充足的时间和计算资源，参赛小组将能够解决的问题。

5 写好摘要

摘要是论文最重要的部分。竞赛要求每篇论文的首页为摘要页，如果摘要写得不好，即使有好的模型和解答，论文也将难以通过鉴别阶段的初审而进入下一阶段。

根据 MCM 的竞赛规则[9]，摘要应该包含以下内容：

- **赛题重述与阐明：** 用自己的语言描述将要解决的问题。
- **解释假设条件及其合理性：** 强调建模所用的假设，而且清楚地列出建模所需的所有变量。
- **模型设计及合理性论证：** 指出所用模型的类型或构造新的模型。
- **描述模型的测试情况及灵敏度分析：** 包括误差分析等检测项目。
- **优缺点讨论：** 包括模型及解题方法的优点与不足。

摘要不应写得太长，长度稍超过半页即可。论文摘要是全文的总结，有些作者简单地通过剪贴论文中的句子拼凑出摘要，这种做法是不可取的。摘要应该重新构思，反复推敲并修改直到满意为止。

摘要应达到吸引读者进一步阅读论文的目的，不要用“First we ...then we ...”这样干巴巴的句子重复论文中的内容，也不要在开篇时使用类似“This paper will solve ...”这样的句子。摘要的第一句话尤其重要，应该以吸引人的语言激发读者的兴趣。例如，在 2008 年的 MCM 竞赛中有一道关于数独游戏的赛题，有一篇题为“Taking the Mystery Out of Sudoku Difficulty: An Oracular Model”[10] 的优胜论文，其摘要的第一句话就达到了这样的效果：

"In the last few years, the 9-by-9 puzzle grid known as Sudoku has gone from being a popular Japanese puzzle to a global craze."
(在过去的几年中，一个在 9 乘 9 方阵上玩的游戏，即数独游戏，在日本流行后风靡全球。)

摘要尽管不能（而且也不应该）包含太多的细节，但必须简明扼要地将解题方法描述清楚，包括全部要点及主要思路，并阐明所得出的结论。如果有数值运算，还应该给出重要的计算结果。因为数学公式在很短的篇幅内很难解释清楚，所以最好不要在摘要中使用数学公式。

第 3 章　写作规范

确定论文结构以后，论文的下一个要素就是符合数学论文的写作规范，包括合理使用人称、时态、语态、句子结构、分段及词汇，使读者将注意力集中在论文的内容上，而不是写作本身。论文应写得生动流畅，避免艰涩的词组或生硬的句子，引导读者顺利地阅读完整篇论文。

不符合写作规范的论文会分散评委的注意力，甚至打断评委的思路。任何写作问题，比如需要分析语法才能读懂的句子，以及需要回忆前文虽然提到、但已经很久没有引用过的概念，都将占用评委用于思考论文解答的时间，使解答本身得不到应有的重视。由于评委评阅论文的时间十分有限，所以解答论文是否符合写作规范会直接影响评审结果。

MCM 论文是数学论文。数学论文的写作有特定的写作规范，评委不但习惯这些规范，而且也希望参赛小组按这些规范写作。通过学习能够帮助学生写出更易于评委阅读的解答论文，本章将提纲挈领地介绍一些常用的写作规范，并举例说明需要注意的问题。

1　使用第一人称复数代词

使用英语撰写数学论文时，在涉及作者的句子中应该使用第一人称复数代词。例如，"我们"看到了事情的真相及"我们"得到这样的结论。("We" see what is going on and "We" draw conclusions.)"我们"在这里不仅包括作者，而且也包括读者，所以即使只有一位作者，也应该用第一人称复数。因为第一人称复数代词可以将读者包括在内，所以读起来会更加亲切，有助于缩短读者和作者之间的距离。此外，在叙述时，用第一人称复数代词做主语也很方便。如果在句子中需要特别指明作者本人时（尽管这种场合很少见），可用"作者"（the authors）或者"作者之一"（one of the authors）做主语。

英文数学论文中使用第一人称复数代词与中国学生熟悉的中文数学教材通常不用第一人称（单数或复数）代词的惯例不同，因此，同学们在撰写 MCM 论文时应注意避免使用第二或第三人称代词，也不要使用第一人称单数代词。

2 使用简单时态

数学论文的写作应尽可能使用简单时态。如何正确使用时态，可参照以下原则：

(1) 在作者当前位置描述正在发生的事，使用一般现在时。例如，“We [now] solve the equation.”

(2) 在作者当前位置描述已经发生过的事，使用一般过去时。例如，“We solved the equation in the previous section.”

(3) 在作者当前位置描述将要发生的事，使用一般将来时。例如，“We will solve the equation in the next section.”

换句话说，如果将阅读论文比做阅读电子文档并跟随光标移动，即光标位置是作者正在写作的位置，描述光标所在位置发生的事件用一般现在时，描述光标以前发生的事件用一般过去时，描述光标以后发生的事件用一般将来时。尽管在某些场合可用现在完成时描述在光标前已经发生的事件，但在写作数学论文时，用一般过去时更简单明了。

在概述他人的工作时，由于是过去所做的工作，故用一般过去时，而对于定理或事实则用一般现在时。例如

They showed that the problem is NP-complete.

3 使用主动语态

叙述一个事件可以用主动语态也可以用被动语态，但主动语态通常比被动语态更生动有力。数学内容逻辑性强，但也相对枯燥，所以在写作数学论文时，使用主动语态通常能将内容表达得更清楚，而且表达方式也更生动。

在主动语态的句子中，主语是动作的执行者。例如

The model produced a desirable conclusion.
（这个模型生成了所需结论。）

在这个句子中，“这个模型”（主语）执行“生成”（谓语）这个动作。

在被动语态的句子中，主语是动作的接受者。例如

A desirable conclusion was produced.（所需结论已生成。）

或者

A desirable conclusion was produced by the model.

（所需结论由此模型产生。）

由以上两个句子可以看出，使用被动语态不如使用主动语态直接有力。

中国学生在写数学论文时习惯使用被动语态，这个习惯可能与早期学习科技英语时常用被动语态有关。当你对是否应该使用主动语态有疑问时，记住以下这点会帮助你做出合理的选择：主动语态的表述形式比被动语态更简洁、更生动、更直接有力。只要有可能，应该尽可能使用主动语态写数学论文。

4 写简单的句子

结构松散或语意模糊的句子会造成阅读障碍。读者如果需要花时间去揣测作者的意图，或需要通过分析句子的语法结构才能读懂句子的内容，阅读速度就会受到影响。为了避免这种情况，写作时应尽可能写简单的句子，越简单越好，一个句子只表达一个意思。此外，谓语应该紧跟在主语之后，中间最好不要再包含副词或状语。如果句子很复杂，比如需要包含几个从句表达多个意思，就应该考虑将它分解成若干个简单的短句子，这样更有利于阅读理解。例如

> 差: The value of the parameter λ, which was used in the previous section to determine the height of the building, can also be used to determine its width.

这个长句子可以分解成以下两个简单的短句：

> 好: In the previous section, we use the value of the parameter λ to determine the height of the building. We can also use λ to determine the width of the building.

5 写简短的段落

段落用来表达一个比较完整的观点或描述一个比较大的事件，通常由多个句子组成。段落的结尾给读者提供了一个自然的停顿机会，用于思考这个段落所表达的思想或所描述的事件。所以应将较长的段落，根据具体内容分成若干较短的段落，使读者更容易掌握段落的内容。分段的时候也要尽量避免只含一两个简单句子的超短段落。如果论文包含很多这样的超短段落，会迫使读者频繁停顿，从而影响阅读的连贯性。

6 使用有具体含义的词汇

使用有具体含义的词汇能够将意思表达得更清楚，应避免使用多义词或抽象词。例如，在下面的两个句子中：

差: Using the previous expression, we can conclude ...

好: Using the previous equation, we can conclude ...

第一个句中 expression 这个词既可以是数学表达式也可以是其他表达方式，第二个句子中的 equation 比第一个句子中的 expression 更具体，意思更清楚。

7 不含琐碎细节

为了帮助读者领会作者的意图，论文应给出充足的信息及必要的细节。但描述细节时要注意把握好尺度，不要将琐碎的细节都写在论文中，比如常规运算中的计算过程就不应该写进来。将这些琐碎的细节写在论文中会使评委感到乏味，甚至产生反感。

然而，哪些细节是琐碎的而哪些不是，却没有精确的定义，必须由作者根据自己的经验做出判断和选择。如果对某些细节是否应该写进论文有疑惑，请牢记：评委都是数学建模方面的专家，他们不希望受到琐碎细节的干扰，以便能在最短的时间内了解作者的解题思路和方法。

8 突出重点

为了引起读者的注意，论文中的主要论点应该以醒目的方式表达出来。比如，将重要的论点用黑体字写在段落之首，然后用段落中的其他句子解释这个论点。不过，这种段落不应太多，否则读者将难以区分哪个是真正的重点。此外，作者定义的新概念应该用黑体字书写。

相互关连的重要句子，比如为建立数学模型所做的一系列假设及其合理性论证，最好将它们以列表的形式按顺序逐一列出来。在许多优秀的 MCM 解答论文中，"Assumptions"（假设条件）这一节通常就只有这样的列表。

9 删掉多余词汇

句子中如果含有多余的词汇，会产生画蛇添足的感觉，应该删去。例如，下面这个句子：

差: The variable x is positive, due to the fact that it is a perfect square.

句中的“due to the fact that”给人感觉很臃肿，而“since”与“due to the fact that”意思是一样的，所以可将这个句子改写为

好: The variable x is positive since it is a perfect square.

又比如，下面这段话摘自一篇关于计算机网络建模论文的摘要，其中有许多话是多余的：

差: In this paper, taking energy consumption into consideration, we rethink the virtual-network embedding problem through (1) the rebuild of the network model by considering the power consumption of both the working and intermediate nodes; and (2) the design of an energy-aware heuristic and optimized virtual-network embedding algorithms under this new network and power model.

此外，作者对其中提到的“working and intermediate nodes”在摘要中没有做任何解释，应该从摘要中删去。可将这段话改写为

好: We present an energy consumption model for the virtual-network embedding problem and devise an energy-aware virtual-network embedding algorithm to reduce power consumption.

如果一个词组能够用一个单词代替，那么就应该选用这个单词。此外，很多以 ion 结尾的短语都可以用相应的动词取代，从而获得更简化的句子。例如

差: We will now find the solutions of the following equation.

可以简化成

好: We will now solve the following equation.

在简化句子时要注意，不要将句子过分简化而失去原意，或产生新的阅读障碍。例如

差: We will assume $x > 0$.

也许会被误认为是下面这个句子：

好: We will assume that $x > 0$.

的一个合理的简化版；that 在这里是必需的，不能省略。在写数学论文时，常

会使用 assume 和 suppose，如果表达式紧随其后，则最好用“assume that”和“suppose that”。

10 使用并列短语强调相似性

在叙述相似的事件或描述相似的对象时，用结构相似的句子或短语有助于强调它们之间的联系。例如，下面这个句子：

差: If a is a root of the function f, then the function g has root b.

其中的主句与从句都提到了函数的根，但由于使用了不同的语言结构来表达，削弱了它们之间的联系。这个句子可以改写如下：

好: If a is a root of the function f, then b is a root of the function g.

或者

好: If the function f has a root a, then the function g has a root b.

11 避免单调重复

上节提到，相似的语句结构有助于强调相似的对象，但是，也要注意避免重复使用单调的语句结构，因为单调的语句结构会使句子显得呆板和生硬。例如

差: First we solved an equation. Then we solved another equation.

这里将两个简单句子简单地罗列在一起，看上去很生硬。为了解决这个问题，可考虑用不同的语句结构改写第二句话。例如，如果第二个方程的解是根据第一个方程的解得到的，则可将这两句话改写为

好: First we solved an equation. The solution of the first equation allowed us to solve the second equation.

改写后的这段话比原文更生动，而且还提供了重要信息，即从第一个方程的解可得到第二个方程的解。

使用不同的连接词可以减少一些单调重复，但只是更换连接词不会改变句子的结构，所以是远远不够的。例如

差: We know ... Hence, ... It follows that ....

这样的单调重复会使读者很快失去阅读的兴趣。比如下面这段话：

差: We know that Equation A has a solution. Hence, Equation B has a solution. It follows that Equation C has a solution.

会使读者感到厌倦。将这段话做如下简单修改后就会更生动有力：

好: We know that Equation A has a solution, which implies that Equation B has a solution. Thus, Equation C also has a solution.

12 不使用同一词汇描述不同的对象

如果用相同的词汇来描述不同的对象，会使读者感到困惑，读者不得不停下阅读，以便搞清楚这个词汇到底指的是哪个对象。例如

差: We ran into several problems while trying to solve the problem.

这句话中 problem 出现了两次，分别代表不同的对象，第一个 problems 指的是困难和障碍，第二个 problem 指的是需要解决的问题。将其改写为

好: We ran into several obstacles while trying to solve the problem.

会使句子的含义更清楚。写作的时候，手头上应该有一本好的同义词词典，以便随时选择同义词。

13 代词所指的名词必须清楚

代词用于指代前面出现过的名词，通常是前面最靠近该代词的名词。例如

好: The function is continuous; it is also nonnegative.

这个句子中的代词 it 显然指的是 function。然而，我们也见过这样模棱两可的代词，例如

差: The function and its derivative are continuous; it is also nonnegative.

句中的第一个代词 it（在 its 中出现）指的是 function，但第二个 it 指的是哪个名词却不清楚，可能是 function 也可能是 its derivative。遇到这种情况时，就不要使用代词。如果第二个代词 it 指的是 function，可以将其改写为

好: The function and its derivative are continuous; the function is also nonnegative.

如果第二个代词 it 指的是 the derivative，则可将其改写为

好: The function and its derivative are continuous; the derivative is also nonnegative.

使用代词 it 或 this 时，一定要检查其所指的名词是否显而易见，避免混淆。

14 不过分渲染

每个参赛小组都希望给出好的解答，但是在描述所得结果的重要性时，一定要注意掌握好尺度。为了吸引评委的注意，有些参赛小组可能会夸大其词。言过其实的渲染会使评委在读到结果时感到失望，从而产生负面的影响。最好的方法是将结果以叙事的方式告诉读者，不作任何评论，由读者自己判断结果的重要性。特别需要指出的是，在描述结果时不要使用感叹号。

第 4 章　英语用法

本章只讨论中国学生在用英语撰写数学论文时常遇到的一些语法问题，并给出一些在写作数学建模论文时常用的动词及例句。语法错误通常都会被发现。虽然评委也许不一定认为这是很严重的问题，但错误只要一经发现，无论大小，就会使评委感到论文有缺陷。如果语法错误反复出现，就会成为阅读障碍，增加评委用于阅读和理解论文的时间，由此产生的负面情绪将会影响论文的评审结果。

自然语言是不断发展变化的有机体。语言的正确用法有些是规定，有些只是建议。虽然几乎对于每一个语法规则，都可能找到某个很有说服力的反例，但 MCM 竞赛不是实验新规则的地方。任何违背规则的用法都会被评委注意到，所违反的规则有可能恰好是某位评委非常在意的规则，从而给此评委留下不好的印象。

写作的时候手头至少应有一本如何正确使用英语的书籍，以便随时翻阅，参考正确的用法。比如，Fowler 编著的 *A Dictionary of Modern English Usage, 2nd Edition* [11] 就是一本很好的英语用法指南。

1　保持主谓一致

主语和谓语不一致是常见的错误，比如主语为单数使用了复数动词。仔细检查是保证主语和谓语一致的有效方法。

如果主语简单并且谓语在位置上很接近主语，则除了个别情况外，保持主谓一致是容易的。但是，如果主语和谓语之间含有一些短语，则保持主谓一致就会困难一些。例如

One of the solutions is positive.

在这个句子中尽管 solutions 是复数，但主语是单数 one，所以应该用单数谓词 is。又例如

The positive solution and the negative solutions are ...

The positive solution or the negative solutions are ...

这两个句子中的主语都是由一个单数主语和一个复数主语复合而成，所以是复数主语。另一个句子：

The positive solution, as well as the negative solutions, is ...

句中的主语是单数 (The positive solution)，因此谓词是 is。

2 正确使用 that 和 which

在什么场合下使用 that 以及在什么场合下使用 which 是许多语法书关心的话题，正确区分 that 及 which 到底有多重要，看法也不尽相同，不过花些时间和精力把这个问题搞清楚是值得的。

通常 that 是指句中某个特定的对象。例如

The car that was blue went through the stop sign.

句中的“that was blue”特指蓝色的那辆轿车。因为可能还会有其他轿车，所以如果没有这个短语，则读者可能不知道是哪辆轿车闯了停车标志牌。

通常 which 是用来给句中的某个对象做补充说明。例如

The car, which was blue, went through the stop sign.

假设读者已经知道是哪辆车闯了停车标志牌，“which was blue”（用逗号隔开）只是给这辆车做补充说明。

如何确定何时使用 that 何时使用 which 的经验是：只要 that 听起来顺耳，就使用 that。

3 避免拼写错误

单词误拼错误是写作中常见的问题。可以使用拼写检查软件及时发现和纠正论文中的拼写错误。由于 MCM 竞赛由美国主办，故应将拼写检查软件设置成美式英语。

然而，拼写检查软件只能检查拼写错误，不能检查用词是否恰当，所以要养成勤查字典的习惯。当对词义有疑问，特别是面对拼写相似但意思不同的词，一定要查字典把词义搞清楚。下面的每个例子中均含有两个拼写相似、但词义不同的单词，也是中国学生常用错的单词。

discrete（形容词）。词义：离散的。例句：One of the MCM problems involves discrete mathematics.
discreet（形容词）。词义：尊重隐私的。例句：You should be discreet when discussing private matters.

principle（名词）。词义：原则。例句：One of the principles of good writing is to be concise.
principal（形容词）。词义：主要的。例句：Poor writing is the principal reason that MCM papers don't make it past the first round.

lose（动词）。词义：失去。例句：You will lose your book if you don't write your name in it.
loose（形容词）。词义：宽松的。例句：You can untie a knot if it is loose enough.

affect（动词）。词义：影响。例句：If you change the value of a parameter, it will affect your model.
effect（名词）。词义：某动作的结果。例句：The effect of not studying for a test is a failing grade.

ensure（动词）。词义：确保。例句：You can use a spellchecker to ensure there are no misspelled words.
insure（动词）。词义：买保险。例句：You should insure any valuable jewelry.

its（名词）。词义："it"的所有格。例句：A dog will chew its toys.
it's（缩写式）。词义："It is"的缩写。例句：If a dog is hungry then it's going to eat.

在正式写作中最好不使用缩写形式，尤其是不使用会引起争议的缩写形式。

单词 cannot 常被误写为"can not"（注：这一点即便母语是英语的学生也常常搞不清楚），其缩写式为 can't。所以，应该写成

好：He cannot complete the task in time.

或者

好：He can't complete the task in time.

而不是

差：He can not complete the task in time.

4 用无争议的代词

英语中没有用于泛指第三人称单数的代词，历史上常使用男性第三人称单数代词代替，如下面这个句子“If an observer touches the experiment, then he will affect the outcome of the experiment.”中的 he。有人反对这一用法，也有人试图引入新的代词体系，但这些尝试都没有被广泛采纳。

使用 they 作为第三人称单数及第三人称复数是一个没有争议且有历史依据的解决方案。比如，下面的写法是正确的：“If an observer touches the experiment, then they will affect the outcome.”又比如，“A student should be careful not to lose their books.”

5 正确使用冠词（the，a 及 an）

由于中文不要求在名词前加冠词，因此中国学生在使用英语名词时经常用错冠词。比如，在该用冠词的地方不用冠词，或者在所有名词前都加定冠词 the。下面是使用冠词的一些简单法则。

在单数可数名词前一定要加冠词。如果所指的对象唯一，应使用定冠词 the。如果这个名词是泛指一般的对象，则应使用不定冠词 a 或者 an。例如，可微函数的微分只有一个，故用“the derivative”。同理，函数的不定积分有多个，故用“an antiderivative”。

对于泛指一般对象的名词，如果名词的发音（注意不是字母）以辅音开始，用不定冠词 a，否则用 an。例如，名词 norm 以辅音“N”开始发音，故用“a norm”，“ℓ_1 norm”以元音“E”开始发音，故用“an ℓ_1 norm”。始发音为“U”的名词，应该把其看成是以辅音（半元音）“Y”为始发音的“you”，所以用 a 而不用 an。例如，用“a European company”，而不是“an European company”；同理，应该用“a US team”，而不是“an US team”。

对于名词前是否需要加冠词，以下是两条规则：

(1) 在表示单个物体的名词前一定要加冠词。比如，不能说“Function is differentiable”，而应该说“The function is differentiable”或“A function is differentiable”。

(2) 在表示某一类物体的名词前不加冠词。比如，如果想表示（在一般情况下）函数的导数不一定是连续的，可以说 "Derivatives are not necessarily continuous"。然而，如果讨论的是特定的导数，比如某类函数的导数，则应该说 "The derivatives are not necessarily continuous"。

6 常用动词

写数学建模论文有一些经常需要用到的动词，掌握这些动词对论文的构思和写作会带来方便。例如

> You may need to *investigate* the problem from different points of view, *survey* what has been done in the literature, *explore* different ideas, *formulate* and *justify* your assumptions, *design* a model, *device* an algorithm, *carry out* numerical simulations, and *compare with* different approaches.

Compare to 与 compare with 将几个对象排列起来做比较，特别是强调同类对象之间的差异时，应该用 compare with。例如

> We compare our results with the existing ones. We show that our method is more robust and our algorithm is faster.

如果希望强调不同类对象的相似性时，则应该使用 compare to。例如

> We can compare her model for loading a railroad car to his method for loading a ship cargo.

Study 与 investigate 区别是 study 可以做动词也可以做名词。可以这样说：

> A 24-hour study to investigate this problem suggests a new direction.

也可以这样说：

> A 24-hour investigation to study this problem suggests a new direction.

Seek 与 explore 这两个动词可用于描述解题思路。例如

> We seek to devise a new model for solving the problem by exploring the new direction suggested by their investigations.

Design 与 devise 这两个动词可用于描述解决方案。例如

Based on our analysis, we design a model for the problem using integral linear programming. We then devise a polynomial-time approximation algorithm to produce near-optimal results.

Tackle 与 solve 这两个动词可用于描述所得结果。例如

We tackle the problem using the new technique we developed in the previous section. While it is difficult to solve the problem completely, we are able to solve a major subproblem.

Approach 与 propose 这两个动词可用于描述作者的计划或打算。例如

We approach the problem using the proposed method.

注意, approach 也可以用做名词, 例如

We propose a new approach to tackling the problem.

或

We propose a new approach to tackle the problem.

第 5 章　论文修改示例

论文初稿完成后，应该回过头来仔细阅读，重点检查论文的内容、格式及语法是否有不妥当的地方，论文的某些部分可能需要做较大的改动，甚至重写。修改论文时，手头最好有一些写作规范参考书及英语用法指南，以便随时翻阅并参考正确的用法。检查和修改的重点是：

- 尽量将被动语态的句子转换成主动语态。
- 尽量将复杂的词汇用较简单的词汇代替。
- 尽量将复杂的句子用较简单的句子代替，将长句分成 2 到 3 个较短的句子。
- 确认对所有假设条件都已给出合理的解释。
- 确认所有信息都已按逻辑顺序表达清楚。
- 确认所有引用他人资源的地方都已标明出处。

修改论文应该从摘要开始，这是评委首先阅读的内容，应逐字逐句仔细修改。除上述检查项目以外，需要检查摘要是否包含了所有要点以及是否在逻辑上表述清楚。如果仍不理想，应该考虑重新组织摘要的逻辑结构，以更易于阅读和理解。

接下来将论文快速浏览一遍，确定所有已经强调的地方（各节标题、数学表达式、表格、图形以及用黑体书写的句子）确实能够快速地帮助读者了解论文所讨论的问题，同时没有强调不该强调的内容。

最后检查论文的全文。由于时间限制，检查全文时可能做不到像检查摘要那样仔细，但检查的重点都是一样的。

本章以往届 MCM 竞赛中中国学生提交的论文为例，详细介绍如何修改论文的标题、摘要、引言及假设条件。

1　示例：如何修改标题

2010 年的 MCM 竞赛中有一道赛题，要求参赛小组找出棒球棍上的最佳击球点。提交的论文中有一篇题为“Science in Sweet Spot”的论文[12]，本节以这篇论文为例介绍如何修改论文的标题。

论文的标题需要认真推敲。这篇论文的标题有以下 3 处错误：

(1) 介词使用不当。在这个标题中，最好使用介词 of 而不是 in。
(2) 单数可数名词 spot 前没有加冠词。这篇论文讨论的最佳击球点是特定的点，所以应该加定冠词 the。
(3) 关于名词 science。这篇论文研究的不是一般性的科学，而是关于最佳击球点的特定的科学，所以需要加定冠词 the。

综上所述，这篇论文标题的正确写法应该是：The Science of the Sweet Spot。

2 示例：如何修改摘要

摘要虽然不长，但却需要用较长的时间思考和写作。写摘要所花的时间与论文其他部分相比往往会长很多，这是正常的。同样，修改摘要的时间相比修改其他部分也要长很多。在保证重要内容前提下，应该尽可能将摘要写得精练一些。

仍以上节提到的题为 “Science in Sweet Spot” 的论文[12] 为例，其摘要如下：

> This paper mainly studies the “sweet spot” on a baseball bat. Firstly, by analyzing the video of a professional hitter, a double-pivot swing (the arm and the finesse) model is established to describe a batter hitting a baseball. Then based on the law of energy conservation, the double-pivot swing model is further transferred into a single-pivot swing model. With the given shape of the bat and the parameters for the single-pivot swing model, the moment of inertia of a wooden bat, corked bat and aluminum bat is calculated, respectively. By the law of kinetic energy conservation, hitting a baseball at different spots of the bat can be seen as an imperfect elastic collision of two balls. Thus given a bat, for different speeds of the coming ball and that of swinging the bat, the speed of the hitting ball can be calculated according to the Newton's collision law.

修改如下：

(1) *This paper mainly studies the “sweet spot” on a baseball bat.*

用 “This paper” 为主语作为摘要的开篇，会把作者和读者都视为论文之外的旁观者。因为论文描述的是作者自己的工作，所以应该用 We 做主语。论文中除了研究最佳击球点之外还会涉及其他内容，所以在这里用 mainly 这个词没有必要，应该删去。最后，“sweet spot” 是棒球棍上最佳击球位置的标准说法，除非论文作者觉得这个名称不合适，否则不应该用引号把它括起来。因此，可将摘要的第一句话修改为

We study the sweet spot on a baseball bat.

改写后的句子不但简洁，而且生动有力。同年的竞赛有一篇题为“The Sweet Spot: A Wave Model of Baseball Bats”的优胜论文，其摘要的第一句话就是

We determine the sweet spot on a baseball bat.

(2) *Firstly, by analyzing the video of a professional hitter, a double-pivot swing (the arm and the finesse) model is established to describe a batter hitting a baseball.*

这个句子是被动语态，应该把它改为主动语态。与第一句话一样，这句话也忽视了作者和读者，只站在第三者的立场上说论文分析了视频并建立了模型。有关职业击球手的视频很多，作者没有指明是哪个视频，所以应该将 video 前的定冠词（至少在第一次出现的时候）改为不定冠词，即将“the video”改成“a video”。最后，firstly 这个词在现代英语中已近很少有人用了，大多数美国人认为这是一个已经过时的词。因此，尽管用“firstly”没有语法错误，但还是不用为好，或将 firstly 改为 first。这个句子可以改写为

We begin by establishing a model of a bat hitting a ball by analyzing a video of a professional hitter.

(3) *Then based on the law of energy conservation, the double-pivot swing model is further transferred into a single-pivot swing model.*

这句话是被动语态，应改写为主动语态。

(4) *With the given shape of the bat and the parameters for the single-pivot swing model, the moment of inertia of a wooden bat, corked bat and aluminum bat is calculated, respectively.*

这是摘要第一次且唯一一次提到棒球棍的形状；同样，也是第一次且唯一一次提到参数。这些内容都不应该在摘要中出现。棒球棍的形状及参数是在建模时使用的，所以可以把这部分的内容改写成“Using our model ...”。在摘要中也没有必要把棒球棍的材料细分为木制的、软木的或是铝制的。

前面曾经说过，在修改论文时，除了逐字逐句地修改外，必要时需要对论文做大幅度的改动。这篇论文的摘要想要表达的意思是，用棒球棍击球与用球击球的效果是相同的，作者稍后将其称为“hitting ball”。可以将摘要改写为

We study the sweet spot on a baseball bat. We begin by establishing a model of a bat hitting a ball by analyzing a video of a professional hitter. Using this model, we determine that hitting a ball with a bat can be seen as an imperfect collision of two balls, where the bat is replaced by a "hitting ball".

3 示例：如何修改引言

在 2008 年的 MCM 竞赛中有一道赛题，要求参赛小组设计一个算法，自动产生不同难度的数独游戏。在提交的参赛论文中有一篇题为 "The Solvers' Sudoku or computers' Sudoku" 的论文[13]。本节以这篇论文为例讲解如何修改引言。可以看到，这篇论文的标题含有两个错误，第一是 "computer" 中的 "c" 应该大写，第二是要加定冠词 "the"，所以这篇文章的题目应为 "The Solver's Sudoku or the Computer's Sudoku"。

这篇论文的引言如下：

> Sudoku (Japanese: sūdoku) is derived from Switzerland, developed in America and carried forward in Japan. It is a simple game of logic and so easy to learn. It is also a fun and addictive game and is puzzling millions of players all over the world. Many people like it. Partial people began to study solving the puzzles by computer. However, generating Sudoku puzzles is even more difficult when difficulty levels and a unique solution both are considered. It is also the task we are required to finish.
>
> The aim is to create a reliable sudoku algorithm which has least complexity and a unique solution should be guaranteed. Difficulty levels are greatly depended on the metrics developed. The basic thought is that we delete digits from a solved puzzle, and then check out if there is a unique solution. Develop a series of metrics to define the the difficulty levels. Judge its difficulty level utilizing metrics.
>
> Metrics are very important in the whole course of constructing puzzles. In our paper, we develop metrics fit for persons' thought and metrics aimed at computers' "thought" respectively. The former is subjective and the latter is impersonal.

修改如下：

(1) *Sudoku (Japanese: sūdoku) is derived from Switzerland, developed in America and carried forward in Japan.*

这句话尽管读起来很上口，但句子里的动词排比却不一致：is derived, developed 及 carried forward 这 3 个动词的句式结构都不相同。而且，"is derived from

Switzerland”的意思是，数独游戏是从瑞士这个国家转变而来的，用“originated in”会更加准确。

如果数独游戏不是众所周知的游戏，则应该在引言中首先简要介绍一下什么是数独游戏。这篇论文的作者假定了数独游戏是读者熟知的游戏，这是正确的。

(2) *It is a simple game of logic and so easy to learn.*

这句话中的“so easy”是习惯用语，与“very easy”同义。作者也许想用 so 来表示后半句话是由前半句话导出的结论，用“hence easy to learn”或“so it is easy to learn”能够更清楚地表达这层逻辑关系。

(3) *It is also a fun and addictive game and is puzzling millions of players all over the world. Many people like it.*

前面的句子和这个句子都以“It is ...”作为句子的开头，读起来很单调。此外，既然这是个很有趣的游戏，并有数以百万计的人玩过这个游戏，因此可以假设很多人都喜欢这个游戏，所以最后一句话是多余的。

(4) *Partial people began to study solving the puzzles by computer.*

这是一个让人感到毛骨悚然的句子，因为“partial people”指的是部分是人的怪物。作者可能想说的是有些人开始用计算机来解答数独游戏。

(5) *However, generating Sudoku puzzles is even more difficult when difficulty levels and a unique solution both are considered.*

这句话和上句话的衔接不连贯。However 是转折词，可以用来指出设计数独游戏的难度与做某件事情的难度相反，但从这句话中看不出数独游戏在与什么做比较。而且这句话也没有说清楚是跟什么进行比较会 even more difficut。还有一点是，两个相似的单词 difficult 和 difficulty 不应该放在如此接近的地方使用。

(6) *It is also the task we are required to finish.*

这句话应改为主动语态。

(7) *The aim is to create a reliable sudoku algorithm which has least complexity and a unique solution should be guaranteed.*

作者应该把自己和读者融入到这句话中，即以“Our aim ...”作主语。此外，“sudoku algorithm”是指设计数独游戏的算法，还是指为数独游戏寻找解答的算法，上下文没有说明。尽管读者或许能猜到它指的是设计数独游戏的算法，但作者应当明确地将这个意思表达出来。短语“has least complexity”可以简写为“is simplest”。最后，应把“a unique solution should be guaranteed”改为主动语态。

(8) *Difficulty levels are greatly depended on the metrics developed.*

应将 “depended on” 改写为 “dependent on”。度量标准 (metrics) 在引言里是首次出现，但是给人的感觉是读者已经非常熟悉它了。所以应该在这句话之前解释什么是 metrics，或者表明是在这里引入 metrics。

(9) *The basic thought is that we delete digits from a solved puzzle, and then check out if there is a unique solution.*

作者没有指出谁在思考，所以这个句子最好以 “Our basic thought ...” 开头。实际上，作者想要表达的是他们正在做什么，因此用 “Our approach ...” 会更合适。

(10) *Develop a series of metrics to define the difficulty level. Judge its difficulty level utilizing metrics.*

这是两个不完整的句子。作者也许是想在第一个句子中列出他们在解题中用到的度量标准。如果是这样的话，应该将这些度量标准用列表的形式写出来。这两个句子不是平行的，所以需要用完整的句子分别加以描述。

(11) *Metrics are very important in the whole course of constructing puzzles.*

如果有些东西很重要，但是它的重要性不容易被读者理解，则作者需要解释为什么这些东西很重要。诸如 “X is important” 这种形式的句子基本上都是不必要的，应该删掉。

(12) *In our paper, we develop metrics fit for persons' thought and metrics aimed at computers' "thought" respectively.*

引言肯定是针对目前这篇论文的，所以 “in our paper” 是多余的，应当删除。因为没有对两个列表做比较，所以这句话不应当以 respectively 结尾。最后，两次出现的单词 thought 都应该用复数形式 thoughts。

根据以上的分析，可将这个引言改写如下：

> Sudoku, originated in Switzerland, was developed in America and popularized in Japan. It is easy to learn, and millions of players enjoy being puzzled by it. While some players have created algorithms to solve sudoku puzzles, our task is to create an algorithm to generate sudoku puzzles of various difficulty levels with unique solutions.
>
> Since our task involves measuring the difficulty of the puzzles our algorithm generates, we develop a series of metrics to define and measure the difficulty. We create metrics to measure the difficulty of a puzzle for a person and metrics to measure the difficulty of a puzzle for a computer.

Our approach is to

- start with an already solved puzzle;
- delete some digits;
- check to see if there is a unique solution;
- measure the difficulty level.

4 示例：如何修改假设条件

2011 年的 MCM 竞赛中有一道赛题，要求参赛小组对汽油驱动的常规汽车和电动汽车的性能做比较。在提交的参赛论文中有一篇题为 "Can Electric Vehicle Be Widely Used" 的论文[14]，本节以这篇论文为例讲解如何修改假设条件。

需要指出的是，这篇论文的标题有错。因为论文讨论的不是一辆汽车，所以标题应该改为"Can Electric Vehicles Be Widely Used"。如果作者考虑的是整类电动汽车，则应该把"electric vehicle"改为"the electric vehicle"，因而标题应该改为"Can the Electric Vehicle Be Widely Used"。

这篇论文讨论了三种类型的汽车，即常规汽车（CV）、电动汽车（EV）及混合型电动汽车（HEV）。论文的第 2 节列出了作者需要用到的所有假设条件，并用编号列表的形式列出如下：

1. We will select one vehicle mode to represent CV, EV and HEV.
2. We assume there is no difference in performance of each vehicle type.
3. We select France, USA, and China to represent European countries, American countries and Asian countries.
4. We assume performance of CV, EV and HEV will not change in the future.
5. Based on common sense, we assume each vehicle travels 10,000 km every year.
6. We will not specify different types of power plants and assume that each plant will generate 10,000,000 kwh annually on average.
7. We do not consider the dissipation during energy conversion.

但这些假设合理与否，论文没有给出任何解释。

修改如下：

(1) *We will select one vehicle mode to represent CV, EV and HEV.*

这里作者其实是想说"vehicle model"，不是"vehicle mode"。这句话读起来的感觉是，作者要用一种车型代表三类不同类型的汽车。但其实作者想表达的是，每种类型的汽车都将用不同的车型来代表。实际上读到论文的最后才会

发现，选择一种车型代表一类汽车是参赛小组解题方法中的一部分，不是假设条件。

(2) *We assume there is no difference in performance of each vehicle type.*

单词 Performance（性能）有很多种解释，所以应该在文中明确指出是何种性能。同时，虽然参赛小组的原意是想说所有常规汽车的性能类似，所有电动汽车的性能类似，以及所有混合型电动汽车的性能也类似。但所写的句子却可以解释为所有常规汽车和电动汽车的性能类似，并且和混合型电动汽车的性能类似。然而这两种解释都是不合理的，任意一种解释都要求作者说明为什么做此假设。其实在这里所做的假设都只是为了将问题简化，应该明确指出这一点。如果作者有证据支持这个假设的合理性，则应该写清楚。

(3) *We select France, USA, and China to represent European countries, American countries and Asian countries.*

参赛小组并不是想表达所选的每一个国家都可以代表所有的洲，而是可以代表其所在的洲，所以这句话的结尾应该加上 respectively 来表明这种关系。此外，这句话其实并不是假设，而是作者建模的方法。如果作者有证据支持这样的选择是合理的，即所选的国家确实能够代表其所在的洲，则应该在列出假设后将这些证据写清楚。

(4) *We assume performance of CV, EV and HEV will not change in the future.*

根据目前科技发展的速度，这个假设很可能是错误的。这个简化假设也许很必要，但作者必须将这点明确交代清楚。

(5) *Based on common sense, we assume each vehicle travels 10,000 km every year.*

读者不能根据常理推断每辆车每年行驶的里程数是 10 000 km，所以应该明确指出这只是一个为简化问题所做的假设，或者提供支持这个里程数的证据。

(6) *We will not specify different types of power plants and assume that each plant will generate 10,000,000 kwh annually on average.*

做这个假设的依据是什么？需要提供证据支持这个假设的合理性。

(7) *We do not consider the dissipation during energy conversion.*

这句话读起来不像是假设。假设的写法应该是“There is no dissipation during energy conversion”。由于这不是一种合理的假设，所以作者必须明确指出这是一个必要的简化条件，并在讨论模型的弱点时应该把这个假设提出来讨论。

根据以上的讨论，可将假设部分改写如下：

We will choose the 2010 Ford Focus 1.4 Duratec to represent conventional vehicles, the Tesla Roadster to represent electric vehicles, and the Chevrolet Volt to represent hybrid-electric vehicles. We will also choose France as a representative European country, the USA as a representative American country, and China as a representative Asian country.

We will make the following simplifying assumptions.

1. Each car we chose is typical of its class.
2. Vehicle performance will not change in the future.
3. Each vehicle will travel 10,000 km per year.
4. Each country we chose is typical of its continent.
5. Each power plant will generate an average of 10,000,000 kwh per year.
6. There is no energy dissipation during energy conversion.

此外，参赛小组应该阐明 performance 在这篇论文中的含义。如果参赛小组可以提供证据支持某些假设的合理性，则应该用新的列表列出这些假设，并给出合理性依据。最后，对所有为了简化问题而提出的假设，都应该在讨论模型缺陷时予以讨论。

第 6 章　符号与图表

MCM 论文的写作将不可避免地要使用数学符号，此外，MCM 论文也会经常使用图形和表格。本章讨论数学符号及图表的常规用法，并给出建议。

1　选择合适的字体与字号

为了使论文更容易阅读，应该选择恰当的字体及字号。数学论文用 11 号或 12 号字比较合适。由于字体的变化会分散读者的注意力，所以文字叙述部分应该固定使用一种字体。同样，数学符号也应该固定使用某种字体。字体的风格和大小可以有一些变化，比如将要强调的文字加黑，将小节的标题字体放大并加黑，等等。依照惯例，数学论文通常采用正体 Times New Roman 作为文字叙述的字体。小节以及再下一级的标题应该与正文的字体有所不同，以便在翻页时很容易看到这些标题。

书写数学论文应该使用专门的数学编辑软件，以保证有足够的数学符号可供使用，否则，某些数学符号只能用相近的字符来代替，看上去很不专业。比如，不要用希腊字母 ϵ 表示集合和元素的隶属关系，即不要写成 $x\epsilon X$，而应该写成 $x \in X$。

2　不在论文标题中使用数学符号

标题一般都很短，所以不可能在标题中定义数学符号。应该避免在标题中使用数学符号，除非数学符号所代表的意思是大家熟知的。

3　符号的常规用法

数学符号有许多习惯用法，采用常规用法有助于阅读和理解。例如，数学变量符号字母的常规用法是斜体，如果不这样书写，就有可能会产生阅读障碍。比如下面这句话

差: It requires that I be a constant for ... to be true.

中的 I 是数学符号，应该用斜体，即应写成

好: It requires that I be a constant for ... to be true.

否则，这个句子会被误读为要求写句子的人（即第一人称单数 “I”）为常数。用大写 I 表示数学符号尽管不是最佳选择，但将其用斜体字书写至少能够避免误解。再举一例，矩阵符号应用黑斜体大写字母书写，而矩阵中的元素则通常用相应的斜体小写字母书写。如果反过来用小写字母 a 表示矩阵，并用大写字母 A_{ij} 表示矩阵元素，则一定会产生阅读障碍。习惯用法是用 $\boldsymbol{A}$ 表示矩阵及 a_{ij} 表示这个矩阵的元素。同样，希腊字母 ϵ 习惯用来表示数值很小的正数，如果用 ϵ 表示数值很大的数就会使读者感到困惑。

数学符号的大小写以及字体是有区别的，它们分别代表不同的对象。比如符号 X、$\mathbf{X}$、x 及 $\mathbf{x}$ 可分别代表四种不同的变量。通常用大写字母表示集合，小写字母表示集合中的元素，黑体字母表示向量和矩阵。函数和变量一般都用斜体的 Times New Roman 字体来表示，比如 $f(x)$，但常用数学函数仍使用正体的 Times New Roman 字体来表示，比如 $\sin(x)$ 和 $\max\{a, b, c\}$。

4 避免一符多用

论文中的每一个数学符号都应该只代表一个对象。不要在论文中有时用 Δ 表示矩阵，有时又用它表示多项式的判别式。

同样要避免使用形状相似的符号。如果用 $\sum$ 表示求和符号，就不要用 Σ 表示变量。

如果论文涉及复数，则不要用 i 表示下标变量。

5 不用符号取代文字

MCM 论文一般不会涉及符号逻辑，所以不要将 MCM 论文写成符号逻辑论文。比如，应该将下面这个句子

差: $x = 2 \Rightarrow x^2 - 4 = 0.$

改写成

好: Since $x = 2$, we know $x^2 - 4 = 0$.

或与此类似的句子。同样，全称量词及存在量词应尽可能用文字书写，而不用逻辑符号 ∀ 及 ∃。

与此原则类似，不要用数学符号取代文字，以免本来可以不含符号的句子含有符号。比如，不要写出下面这样的句子：

差: The two functions are =.

应写成

好: The two functions are equal.

6 用文字书写作为形容词的数字

在阅读时，将阿拉伯数字转换成文字会降低阅读速度，因此，作为形容词的数字应该直接用文字书写。例如，下面的句子

差: There are 3 solutions.

应该改写成

好: There are three solutions.

但如果数字是计量单位的一部分，则此规则不适用。例如，下列句子中的阿拉伯数字应该保留：

好: The rod is 5 feet long.
好: It is a 5-foot-long rod.

7 避免不必要的上下标

如有可能，应尽量用 x 和 y 表示在一起出现的两个变量，而不是带下标的变量 x_1 及 x_2。例如，有人喜欢用 $x_1, x_2 \in X$ 表示集合 X 的两个元素，但除非下标会被直接引用，否则应该写成 $x, y \in X$，这样更便于阅读。

在使用上下标的时候，要保持上下标的用法一致。例如，假如用 x_1 及 x_2 表示两个在一起出现的变量，则它们的线性组合应该写成 $a_1x_1 + a_2x_2$，而不是 $ax_1 + bx_2$。同样，如果用 x 及 y 表示两个在一起出现的变量，则它们的线性组合应该写成 $ax + by$，而不是 $a_1x + a_2y$。

8 保持符号一致

两个句子 "x_i for $i = 1, 2, 3, \ldots$" 及 "x_j for $j = 1, 2, 3, \ldots$" 表示同一个数列，但由于使用了不同的下标，会使人误以为是两个数列。所以，同样的变量应该用同样的符号来书写。

9 保持下标顺序一致

如果若干下标反复出现，则它们出现的顺序应保持一致。例如，当描述一个递减数列时，应当避免以下的书写形式：

差: $x_i < x_j$ when $j < i$.

而应该写成

好: $x_i < x_j$ when $i > j$.

第一种写法在数学上尽管没错，但看上去却很别扭。

10 删除只用过一次的符号

如果某个符号只用过一次，则说明这个符号实际上是不需要的。比如，在下面这个句子中：

差: A differentiable function f is continuous.

如果符号 f 只在这个句子中出现，则没必要使用符号，应该改写成

好: A differentiable function is continuous.

11 图形和表格

图形和表格是显示数据直观而且有效的方法。与文字叙述相比，大多数人都能更容易从图形或表格中获取信息。例如，用曲线图比较不同的模型能够使读者更容易看到参数变化所产生的结果。

数据量较小时，用表格比较合适。数据量较大时，用图形比较合适。在保证提供足够信息的前提下，应注意简化图形。简单的图形有助于阅读理解，而复杂的图形则可能适得其反。

使用了图形或表格后，应在论文中对它们进行讨论，把图表与论文中的相关内容联系起来。对图表不作任何讨论的论文是不被评委看好的。很多 MCM 参赛论文虽然使用了图表，但却没有在正文中对图表加以讨论。读者也许能够自己判断图表与所讨论问题的相关性，但是把讨论交给读者自己去做是不可取的。此外，应该给图表编号，比如“图 2.1”或“表 2.3”，以便在讨论时引用。

对每个图表都应给出简要、恰当的文字说明，使读者即便没有读过论文或只是快速浏览论文，也能明白图表所表示的内容。曲线图的坐标轴必须说明它所代表的变量，并适当地使用图例。表格也应该有标题行或标题列，并清晰标示。

第 7 章　数学表达式和句子

任何包含数学表达式的论文，数学表达式应该与论文的文字内容融合在一起。论文应以句子为单位，包括其中的数学表达式。因此，除了合理使用数学符号和图表，还要将数学表达式与文字叙述有机结合，并通过适当的排版使读者很容易辨认。本章介绍如何书写好含有数学表达式的句子。

1　不用数学符号作为句子的开头

如果用数学符号作为句首，比如

差: f is differentiable.

这个句子看起来很不好。在符号 f 之前应该加一些适当的描述，比如

好: The function f is differentiable.

如果符号 f 已经很长时间没用过了，则应给读者提供一些提示，比如，

好: The exponential function f is differentiable.

2　句中多次出现的同一符号读法要一致

如果同一个数学符号在一个句子中出现多次，则将句子读出来（或者默读）时，这个符号的读法在每次出现的地方应该一致。例如，下面这个句子

差: For $x = 2$ we know that $x^2 = 4$.

的读法是"For x equal to two, we know that x squared is equal to 4"。等号 $=$ 在这个句子中出现两次，第一次出现时读为"equal to"，而第二次出现时读为"is equal to"。为了使两次出现的等号的读法一致，可将这个句子改写为

好: If $x = 2$ then $x^2 = 4$.

改写后的句子的读法是 “If x is equal to 2 then x squared is equal to 4”，等号在两次出现时都读做 “is equal to”。

3 标点符号与阅读的连贯性

任何数学表达式，只要在论文中出现，就应该属于某个句子，因此都必须加上适当的标点符号断句。大声朗读论文，文字部分和数学部分都应该听起来清晰流畅。事实上，作者确实应该朗读刚写完的论文，这是检查论文阅读是否连贯的有效方法。

比如，将 “If $x > 0$ then $f(x)$ is differentiable” 这句话朗读出来就是 “If x is greater than zero, then f of x is differentiable”。在朗读的过程中注意等式（或者不等式）可以作为主语、谓语或宾语。例如，“The equation $x^2 = 1$ has two solutions” 读起来应该是 “The equation x squared equal to one has two solutions”。这句话里的等式 “$x^2 = 1$” 是主语。又比如，“If $x = 1$ then $x^2 = 1$” 这句话读起来是 “If x is equal to one then x squared is equal to one”，此处等式 “$x^2 = 1$” 中的 x^2 是主语，$=$ 是谓语，1 是宾语。

4 用文字分隔相邻的表达式

除了使用标点符号以外，两个相邻的数学表达式应该用文字隔开。比如，下面这句话

差: Let x_i, $i = 1, 2, 3, \ldots$ be a sequence.

只用了逗号做分隔符，可将其改写为

好: Let x_i for $i = 1, 2, 3, \ldots$ be a sequence.

或者

好: Let x_i be a sequence, where $i = 1, 2, 3, \ldots$.

又比如，下面这句话

差: Since $a = 2$, $a^2 = 4$.

看上去是一个不完整的因果关系。为了将因果关系表达清楚，应将其改写为

好: Since $a = 2$, we have $a^2 = 4$.

5 以文字叙述为主

尽管 MCM 论文会包含大量的数学内容，但还是应该以简单的文字叙述为主，尽量少用数学符号和专业术语。没有人会喜欢阅读充满专业术语和符号的文章，数学家也不例外。使用适当的专业术语是必要的，但应尽量少用。

6 不要超负荷使用同一词汇

第 3 章的规则 12 提到不要用同一词汇描述不同的对象，这一规则对数学词汇同样适用。例如，在写数学论文时，element 这个词常常会超负荷使用。在下面这段话 "If $\mathbf{x} = (x, y, z)$ is a vector in $\mathbb{R}^3$, then x is sometimes referred to as an element of $\mathbf{x}$. But then x is called an element of $\mathbf{x}$ and $\mathbf{x}$ is an element of $\mathbb{R}^3$." 中，第 2 个句子中的出现的两个 element 分别代表两个不同的对象，所以应使用不同的词汇来表达。比如可以用 component 取代第一个 element。

7 每一个 if 都应该与一个 then 匹配

在 "if ... then" 因果句型中，then 可以省略，由读者自己推断哪部分是句子的结论。比如，下面这个句子中的结论部分就很容易推断出来：

差: If it rains, the grass will get wet.

好: If it rains, then the grass will get wet.

但在数学论文中最好用 then 明确指出其结论部分。如果省略 "then"，则有时会很难推断哪个部分是结论。比如，下面这个句子

差: If $x = 1$, $y = 2$, $x + y = 3$.

句中表达式之间没有用文字分隔，也没有用文字明确指出哪个是结论、哪个是假设，读起来很费解。应该将其改写成

好: If $x = 1$ and $y = 2$, then $x + y = 3$.

否则这个句子也可以理解成 "If $x = 1$, then $y = 2$ and $x + y = 3$."

在写因果关系句子时，要保证其结论部分显而易见。将每一个 if 配上一个 then 就能很容易地做到这一点。

8 提供必要的提示

写作的时候，作者本人熟知前面已经写的内容，但却不能要求读者也能记住前面的内容。不要试图考验读者的记忆力或要求读者重新阅读前面写过的内容。如论文在开始的时候列出了许多假设，后文在这些假设的基础上展开论述，则一定要记得告诉读者目前的论述是基于哪个假设。例如，下面这句话

差: By our assumptions, we can conclude that f is continuous.

没有告诉读者指的是哪一个假设。根据实际情况，可这样改写这句话：

好: Since we assumed that the density function is continuous, we can conclude that f is continuous.

此外，为了提示符号的含义，可再将句子改写成

好: Since we assumed that the density function is continuous, we can conclude that the function f is continuous.

更好的写法是

好: Since we assumed that the density function is continuous, we can conclude that the mass function f is also continuous.

此外，每次引用已经定义过的变量时，都应该在句子中适当提示该变量的含义。比如，应该将

差: Hence $\boldsymbol{A}$ is invertible.

改写为

好: Hence the matrix $\boldsymbol{A}$ is invertible.

9 术语应该在即将使用时定义

在写作数学论文时，作者可能需要定义一些术语。由于读者很难在短时间内接受大量新名词，所以不要在一个段落里定义很多新术语。通常的做法是，在即将使用某个新术语前再定义它，所以不要这样写

差: A function is **smooth** if it infinitely differentiable. A function is $\mathbf{C}^1$ if it has a continuous derivative. A function is ...

而应该这样写

好: A function is **smooth** if it is infinitely differentiable. Suppose that f is a smooth function.

新术语在第一次定义时应该使用黑体。

只有在需要多次使用某个术语时，才定义这个术语。比如，如果定义 smooth 只是为了说明函数 f 是光滑的，则应该去掉这个定义。这句话可改写成

好: Suppose that the function f is infinitely differentiable.

10 如何排列数学表达式

数学表达式可以与文字内容在同一行中排列，称为同行排列；也可以另起新行，与上下文区分开来，称为单行排列。比如，表达式 x^2-1 是同行排列，而表达式

$$x^2-1=0$$

是单行排列。

单行排列能够使表达式更明显，所以当需要强调表达式的时候，应该令其单行排列。单行排列的表达式还可以有公式编号。如果论文不会引用这个表达式，则没有必要对其进行编号。

数学表达式，无论是同行排列还是单行排列，都是句子的一部分，所以要加标点符号。数学表达式本身是句子的一部分，不要因为表达式而额外添加冒号或其他标点符号。比如，不要写成

差: Thus, a simpler form of double Pareto distribution density function is:

$$f(x)=\frac{\alpha\beta}{\alpha+\beta}\begin{cases}x^{\beta-1}, & 0<x\leqslant 1\\ x^{-\alpha-1}, & x\geqslant 1\end{cases}$$

for some $\alpha>0$ and $\beta>0$.

而应该写成

好: Thus, a simpler form of double Pareto distribution density function is

$$f(x)=\frac{\alpha\beta}{\alpha+\beta}\begin{cases}x^{\beta-1}, & 0<x\leqslant 1\\ x^{-\alpha-1}, & x\geqslant 1\end{cases}$$

for some $\alpha>0$ and $\beta>0$.

但是，如果句子中使用了“following”、“below”或“as follows”，则要使用冒号。例如

好: Thus, a simpler form of double Pareto distribution density function is as follows:

$$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \leqslant 1 \\ x^{-\alpha-1}, & x \geqslant 1 \end{cases}$$

for some $\alpha > 0$ and $\beta > 0$.

11 如何书写分数

分数可用斜线（x/y）或水平短线（$\frac{x}{y}$）两种形式来表示。如果是在同行排列的数学公式或者在垂直方向空间比较小的地方（比如上标和下标）出现分数，用斜线形式较好。如果分数表达式较复杂，斜线形式则不便于阅读，这时用水平分数线的形式较好。

12 数学表达式的断行与对齐

不要将同行排列的表达式断行。如果表达式很长，则应该单行排列。如果需要将数学表达式分成多行排列，应该在优先级低的运算符号处断行，即尽可能在加号或减号处断行。分行之后，通常把运算符号放在新行的前面。例如

$$\begin{gathered} abcdef + ghijkl + mnopqr + stuvwx \\ +yz. \end{gathered}$$

单行排列的数学公式不应该分页表示。如果公式很长需要分页，则应该在分页的地方用合适的文字断开，然后在下一页继续书写公式的剩余部分。

如果一个表达式包含一系列等式，那么这些等式应该在等号处对齐，如下所示

$$\begin{aligned} a+b &= c+d, \\ e &= f+g+h, \\ i+j &= k. \end{aligned}$$

与此类似，连等式也应该在等号处对齐，例如

$$\begin{aligned} a+b &= c+d \\ &= e+f+g \\ &= h. \end{aligned}$$

13 省略号的对齐

在写序列的时候，中间部分可以用省略号代替。省略号由三个小点组成，它应该与所在行的底部对齐，例如

$$x_1, x_2, \ldots, x_n.$$

省略号的两侧都应该用逗号分开。

如果一个连加表达式中间的一部分被省略，如

$$x_1 + x_2 + \cdots + x_n,$$

则省略号应该在该行的中间对齐，省略号的两侧都应该有加号。这个规则同样适用于其他运算符号。

第 8 章　LaTeX

LaTeX 是常用的数学编辑软件。MCM 论文必须同时用纸质和电子版两种形式提交给竞赛委员会，电子版必须是 Adobe PDF 文档或 MS Word 文档。因为 LaTeX 能生成很美观的 PDF 文档，所以很多指导教师都推荐使用 LaTeX 书写 MCM 解答论文。

LaTeX 不是“所见即所得”的文字处理软件，这点与大家熟悉的 MS Word 不同。LaTeX 是一种标识语言，这点与 HTML 类似。LaTeX 的可配置性极高，不过，书写 MCM 论文用基本的配置就足够了，不需要用到很高的配置。本章介绍 LaTeX 的基本配置，读者如果希望对 LaTeX 做更多的了解，可参考 *The Not So Short Introduction to LaTeX 2ε*[15]。

1　编译

LaTeX 文档是含有指令、文字及图表的 ASCII 文档，其文件扩展名为 `.tex`，比如 `mydocument.tex`。建立 LaTeX 文档应使用文档编辑软件，而不是 MS Word 那样的文字处理软件。当然，也可以使用文字处理软件建立 LaTeX 文档，但要将文档存为纯文本格式。有些文档编辑软件含有编辑 LaTeX 文档的功能，有些甚至还提供“所见即所得”的编辑环境。推荐中文读者使用 CTeX 软包（包括文档编辑软件 WinEdt）编辑 LaTeX 文档。CTeX 软包支持中英文两种语言，可从 `http://www.ctex.org` 免费下载。

LaTeX 文档必须经过编译才能生成 PDF 文档，可用以下编译指令

```
> pdflatex mydocument.tex
```

进行编译，或在 WinEdt 工具栏内点击 `TeX` 选项，然后点击 `PDF` 选项，再点击 `PDFLaTeX` 选项。（也可以用指令 `latex` 先将 LaTeX 文档编译生成 DVI 文档，然后从 DVI 文档生成 PDF 文档。）如果文档含有文献索引，则需要将文档编译两次才能获得正确的索引。为方便起见，本章把 LaTeX 文档的内容称为输入，编译后生成的文档称为输出。

LaTeX 指令的格式是：用反斜杠（\）作为标示符，其后是指令，再后面是包

含在花括号（{}）内的参数（注：有些指令不含参数）。

在编译过程中可以在计算机屏幕上看到编译信息。如果文档不含 LaTeX 句法错误，便可得到 mydocument.pdf。如果文档含有句法错误，则需要纠正这些错误后才能完成编译。编译错误往往是由于指令拼写错误或括号不匹配而引起的。例如，本来想输入以下指令

```
\command{arg}
```

结果却输入了

```
\commnd{arg}
```

或者

```
\command{arg
```

则编译时就会出错。

PDF 文档生成后，可以先用 PDF 阅读程序在计算机屏幕上预览，例如

```
> pdfviewer mydocument.pdf
```

打印前可对文档作修改补充，然后再编译和预览，如此反复，直到满意为止。

2 简单示例

以下是一个简单的“Hello World”文档：

```
\documentclass[12pt]{article}
\begin{document}
Hello, World!
\end{document}
```

将其存为 mydocument.tex 并编译，便可得到 PDF 文档，其页面顶端会输出“Hello, World!”，底端会输出页码“1”。

文档的第一行

```
\documentclass[12pt]{article}
```

用来阐明文档的类别，此例是一个“article”类型的文档。文档类型还包括“book”和“report”。撰写 MCM 论文应该用“article”这个类型。这行中的 [12pt] 要求输出以 12 号字显示。这是一个可选项，如果空缺，其默认值是 [10pt]，但用 10 号字打印会过小。MCM 论文应该用 11 号或 12 号字体打印。

位于

```
\documentclass[12pt]{article}
```

与

```
\begin{document}
```

之间的位置称为 LaTeX 文档的引子（注：不是论文的引言），许多指令必须在引子内列出。“Hello World”这个例子中不含引子，本章会在稍后的地方提供包含引子的实例。

位于

```
\begin{document}
```

与

```
\end{document}
```

之间的内容是 LaTeX 文档的正文。

正文部分除了输入论文的内容外，还会插入一些 LaTeX 指令及 LaTeX 环境。比如 `\ldots` 这个指令会在文档中插入一个 3 点省略号：

输入
`a, b, c, \ldots, z.`
输出
a, b, c, …, z.

又比如，`\textbf{text}` 是带参数的指令，将括号中的 `text` 输出为黑体。

输入
`This is \textbf{bold text}.`
输出
This is **bold text**.

LaTeX 环境是如下定义的：

```
\begin{environment}
text
\end{environment}
```

用于指明 text 该如何显示，其中 environment 为具体的 LaTeX 格式。例如，center 会将所包含的内容在该行的中间位置打印出来。

输入

```
The following text
\begin{center}
is centered.
\end{center}
```

输出

The following text

is centered.

输出时，LaTeX 会对文档做格式化处理。LaTeX 能自动处理文档中出现的空白部分，即使在两个词之间连续输入几个空格（LaTeX 不区分空格键及跳格键），LaTeX 仍然会按其认为合适的空白将这两个词分开。例如

输入

```
Hello World!

Hello           World!

Hello
    World!
```

输出

Hello World!
Hello World!
Hello World!

在文档中另起一行不会在输出中产生新的段落。如果需要另起一段，需要插入空白行。如果需要在一个段落内另起一行，需要在前一行的结尾处输入双反斜杠 \\。例如

输入

```
Here is the first sentence of a paragraph.  Here is the
second sentence of a paragraph.\\
Here is the third sentence of a paragraph.

Here is the first sentence of a new paragraph.
```

输出

Here is the first sentence of a paragraph. Here is the second sentence of a paragraph.
Here is the third sentence of a paragraph.

  Here is the first sentence of a new paragraph.

以 % 开始的行是注释行，LaTeX 在编译过程中将忽略注释行，例如

输入

```
Hello world.
% Goodbye World.
Hello again.
```

输出

Hello world. Hello again.

可将论文按不同的行间距打印。比如，在引子中加入以下指令

```
\usepackage[doublespacing]{setspace}
```

就可将论文按双倍行间距打印。又比如，在引子中加入以下指令

```
\renewcommand{\baselinestrech}{1.5}
```

可将论文按 1.5 倍行间距打印。

3 字符

LaTeX 能够识别所有的 ASCII 字符。此外，在引子中加入以下指令

```
\usepackage[utf8]{inputenc}
```

还可使用功能包 inputenc 输入非 ASCII 字符及重音符号。也可不用这个功能包，而是通过普通 ASCII 键盘输入重音符号。比如，在字符前加上 \" 就能在输出时，在这个字符上方显示分音符；\"a 的输出是 ä。表 8.1 列出了 LaTeX 支持的一些常见的重音符。

表 8.1　重音符号

输入	输出	输入	输出	输入	输出
\`a	à	\'a	á	\"a	ä
\~a	ã	\^a	â	\c{c}	ç

大部分 ASCII 字符都会在 PDF 文档中直接显示，但下列字符\、#、$、%、^、&、_、{、} 及 ~ 是 LaTeX 标识符，不能直接打印。如果需要输出这些字符，比如需要打印反斜杠，其指令是 \textbackslash。打印其他标识符也是通过在其前面添加一个反斜杠的前缀来实现，即

输入

```
\textbackslash \# \$ \% \^{} \& \_ \{ \} \~{}
```

输出

\# $ % ^ & _ { } ~

如果某些标示符在输出时出现乱码，可以通过在标示符后添加一对空括号 {} 来解决。

除了省略符指令 \ldots 外，表 8.2 列出了一些常见的特殊字符的指令。如果需要使用其他欧洲文字的字符，则需要在引子中加入以下指令

```
\usepackage{textcomp}
```

表 8.2　特殊字符

输入	输出	输入	输出	输入	输出
\ldots	...	``	“	''	”
\textbackslash	\	\#	#	\$	$
\%	%	\^{}	^	\&	&
_{}	_	\{	{	\}	}
\texteuro	€	\~{}	~	$\sim$	∼

在 LaTeX 文档中，双引号 " 是没有左右之分的，要用 `` 输出左双引号，用 '' 输出右双引号。例如

输入
``Hello'' looks better than "Hello".
输出
“Hello” looks better than ”Hello”.

前面讲过，指令 \textbf{text} 将 text 按黑体字输出，指令 \textit{text} 将 text 以斜体字输出，指令 \texttt{text} 将 text 以等宽的打字机字体输出。如果需要将一整段文字输出成等宽的打字机字体，比如需要打印一段程序代码，则可使用 verbatim。在这里任何标识符及特殊字符均无效，可以直接输入这些字符。例如

输入

```
To use the Euro symbol, you'll want to have
\begin{verbatim}
\usepackage{textcomp}
\end{verbatim}
in the preamble of your document.
```

输出

To use the Euro symbol, you'll want to have
\usepackage{textcomp}
in the preamble of your document.

4 数学表达式

LaTeX 的数学功能包 amsmath 及 amssymb 提供了数学排版的许多功能以及各种数学符号。在 MCM 文档的引子中加入以下指令

```
\usepackage{amsmath, amssymb}
```

后，便可使用这些功能与符号。

简单表达式

LaTeX 文档中的数学表达式（包括数学符号）必须在数学模式下输入。同行排列的数学表达式在两个美元符号之间输入，单行排列的数学表达式模式在 \[及 \] 之间输入，或在两个双美元符号之间输入。例如

输入

```
\[\sum_{k=1}^n k = \frac{n(n+1)}{2}\] and
$$ \sum_{k=1}^n k = \frac{n(n+1)}{2}$$
have the same displaying effect.
```

输出

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

and

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

have the same displaying effect.

复杂一些的单行排列格式将在以后介绍。

在数学模式下，所有字母的输出都是斜体字体，而且任何输入的空格都会在输出时被删去。任何数学表达式，甚至只是一个变量，都应该在数学模式下输入。例如

输入

```
You should refer to the variable $x$,
not the variable x.
```

输出

You should refer to the variable x, not the variable x.

输出黑体数学符号的指令是 \mathbf{math}。例如

输入

```
Consider the vector $\mathbf{x} = (x,y,z)$.
```

输出

Consider the vector $\mathbf{x} = (x, y, z)$.

简单的数学符号可以直接使用计算机键盘上的符号，例如

输入
`If $a+b=c$ and $a>0$, then $b<c$.`
输出
If $a+b=c$ and $a>0$, then $b<c$.

函数名应该用斜体字，但对于 sin 及 log 等，应该用正体字。这些函数名在数学输入模式下均有相应的指令，通常是在反斜杠后面输入函数名，比如 `$\sin$`及 `$\log$`。

输入
`The function $f(x)$ is in math italic,` `but $\sin(x)$ is not.`
输出
The function $f(x)$ is in math italic, but $\sin(x)$ is not.

很多数学符号及二元运算符在键盘上没有相应的字符，需要用指令才能输入这些符号及运算符。表 8.3 列出了一部分这样的指令。指令 `\circ` 除了表示二元运算外，还可以表示角度，例如 90°。

希腊字母需要在数学模式下输入，指令是反斜杠后面跟希腊字母的英文拼写。将英文拼写的第一个字母用大写输入，就得到大写的希腊字母，反之就得到小写的希腊字母。例如

输入
`From $\alpha$ to $\Omega$.`
输出
From α to Ω.

某些大写希腊字母与英文大写字母相同，如大写的 α 及 β 分别是 A 及 B，LaTeX 没有为这些大写希腊字母提供相应的指令，用英语的大写字母输入即可。例如

输入
`Capital alpha is $A$; capital delta is $\Delta$.`
输出
Capital alpha is A; capital delta is Δ.

表 8.3 常用数学符号

输入	输出	输入	输出
`\le`	$\le$	`\ge`	$\ge$
`\subset`	$\subset$	`\supset`	$\supset$
`\subseteq`	$\subseteq$	`\supseteq`	$\supseteq$
`\in`	$\in$	`\notin`	$\notin$
`=`	$=$	`\neq`	$\neq$
`\pm`	$\pm$	`\div`	$\div$
`\cdot`	$\cdot$	`\times`	$\times$
`\to`	$\to$	`\mapsto`	$\mapsto$
`\partial`	∂	`\infty`	∞
`\circ`	$\circ$	`\vdots`	$\vdots$
`\ldots`	$\ldots$	`\cdots`	$\cdots$
`\ddots`	$\ddots$		

LaTeX 的 amssymb 功能包所支持的数学符号，包括希腊字母及希伯来字母，均可在 http://amath.colorado.edu/documentation/LaTeX/Symbols.pdf 中找到。

括号

数学表达式常常需要使用括号，包括花括号（{}）。前面说过，花括号是 LaTeX 的功能符，所以输入时需在花括号前加反斜杠，即 \{ 和 \}。例如

输入
`$\{(a+b)\cdot c\}\cdot d$`
输出
$\{(a+b)\cdot c\}\cdot d$

如果数学表达式在垂直方向上占得位置比较大，则应该使用大一些的括号。LaTeX 可以自动产生大小适中的括号，通过在括号前加入 \left 及 \right 来实

现。例如

输入
`The expression $(1+\frac{2}{3})$ may look okay, but $\left(1+\frac{2}{3}\right)$ looks better.`
输出
The expression $(1+\frac{2}{3})$ may look okay, but $\left(1+\frac{2}{3}\right)$ looks better.

上下标和分数

输出指数或其他上标的指令是字符 ^，它将紧跟着的字符输出为上标。例如

输入
`The expression $a$ squared is written as $a^2$.`
输出
The expression a squared is written as a^2.

如果上标部分包含若干字符，则必须用花括号将这些字符括起来。例如

输入
`Note that $a^23$ is not the same as $a^{23}$.`
输出
Note that a^23 is not the same as a^{23}.

下标和上标的指令类似，使用底线符 _ 即可。例如

输入
`Again, $a_23$ is not the same as $a_{23}.$`
输出
Again, a_23 is not the same as a_{23}.

上标与下标可以嵌套使用，不过，应尽可能避免使用嵌套的上下标，因为嵌套的上下标看起来很累赘。例如

输入

```
The expression $a_{3^2}$ looks awkward,
but isn't as bad as $a_{i^{j_{k^l}}}$.
```

输出

The expression a_{3^2} looks awkward, but isn't as bad as $a_{i^{j_{k^l}}}$.

有些数学运算符号也要用上下标。例如，连加运算需要将下标变量的起始值按下标方式放在连加号（$\sum$）的下方，并将终止值按上标方式放在连加号的上方。输入连加号的指令是 \sum。这样的运算还有连乘运算（\prod）、积分运算（\int）以及极限运算（\lim）等。例如

输入

```
$\lim_{n \to \infty}
\sum_{k=1}^{n} ((b-a)/n) f(a + k(b-a)/n) =
\int_a^b f(x){\rm d}x$
```

输出

$\lim_{n \to \infty} \sum_{k=1}^{n} ((b-a)/n) f(a + k(b-a)/n) = \int_a^b f(x){\rm d}x$

分数可以用一条斜杠表示。例如

输入

```
Two thirds can be typed as $2/3$.
```

输出

Two thirds can be typed as 2/3.

如果希望用水平线表示分数，指令是 \frac{top}{bottom}。例如

输入

```
Two thirds can also be written $\frac{2}{3}$.
```

输出

Two thirds can also be written $\frac{2}{3}$.

二项式系数的输入与分数类似，指令是\binom{top}{bottom}。例如

输入

```
The value of $\binom{6}{4}$ is $15$.
```

输出

The value of $\binom{6}{4}$ is 15.

表达式单行排列

单行排列表达式至少有 3 个好处。第一，LaTeX 会自动给单行排列的表达式分配足够的空间，使排版更美观。第二，单行排列的表达式更引人注目。第三，单行排列的表达式能够编号，方便引用。

除了前面已经提到过的单行排列的简单方法外，单行排列的另一个方法是在 equation 环境下输入表达式。如果表达式不会被引用，则不要编号，不编号的表达式可以在 equation* 环境下输入。对同行排列的表达式适用的指令也适用于单行排列的表达式，后者在垂直方向上的额外空间可使表达式更美观。例如

输入

```
The equations
$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$,
\begin{equation}
\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8},
\end{equation}
and
\begin{equation*}
\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}
\end{equation*}
are all the same expression.
```

输出

The equations $\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$,

$$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}, \tag{8.1}$$

and

$$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$$

are all the same expression.

如果需要在表达式内加入文字（包括空格），指令是 \text{text}。例如

输入

```
\begin{equation*}
x^2 \ge 0 \text{ for all real } x.
\end{equation*}
```

输出

$$x^2 \ge 0 \text{ for all real } x.$$

假如在论文中需要引用某个表达式，则需要为这个表达式编号，做法是在这个表达式前插入指令 \label{Label}，其中 Label 是作者任意选取的、作为标签的字符串。然后用指令 \eqref{Label} 就可引用这个表达式，例如

输入

```
If you are working with
\begin{equation}
\label{eqn}
 a^2 + b^2 = c^2
\end{equation}
you can refer to it as Equation \eqref{eqn}
in other parts of the paper.
```

输出

If you are working with

$$a^2 + b^2 = c^2 \tag{8.2}$$

you can refer to it as Equation (8.2) in other parts of the paper.

在输出一组等式时，等式应在等号处对齐，这可在 align 环境下实现。在每个等式的等号（或其他需要对齐的符号）前加一个 & 符号，在每一行的结尾加上符号 \\ 做结束。在这个环境下，每个等式都会显示各自的编号。如果等式不需要编号，应在其结尾处加上 \nonumber 这个指令。如果所有等式都不需要编号，则可直接使用 align* 环境。例如

输入

```
\begin{align}
a + b &= c\\
    d &= e + f \nonumber\\
g + h &= i
\end{align}
\begin{align*}
a + b &= c\\
    d &= e + f \nonumber\\
g + h &= i
\end{align*}
```

输出

$$\begin{aligned} a+b &= c & (1)\\ d &= e+f \\ g+h &= i & (2)\end{aligned}$$

$$\begin{aligned} a+b &= c \\ d &= e+f \\ g+h &= i \end{aligned}$$

这个方法同样适用于不等式组。

用 align 环境也可将拖式在等号处对齐。例如

输入

```
\begin{align*}
a + b &= c + d\\
      &= e + f\\
      &= g
\end{align*}
```

输出

$$\begin{aligned} a+b &= c+d \\ &= e+f \\ &= g \end{aligned}$$

这个方法同样适用于不等式拖式，以及混合的等式与不等式拖式。

矩阵

矩阵应在矩阵环境 matrix 下输入，逐行列出矩阵的元素，每行各元素之间用 & 符号隔开，每行的结尾用 \\ 符号结束。例如

输入

```
\begin{equation*}
  \begin{matrix}
    1 & 2 \\
    345 & 6
  \end{matrix}
\end{equation*}
```

输出

$$\begin{matrix} 1 & 2 \\ 345 & 6 \end{matrix}$$

但是，矩阵环境 matrix 不会自动产生矩阵符号。常见的矩阵符号有三种，即圆括号、方括号及竖线，分别可用 pmatrix、bmatrix 及 vmatrix 环境自动生成。例如

输入

`\begin{equation*}`
`\begin{pmatrix}`
`1 & 2\\`
`3 & 4`
`\end{pmatrix}`
`\begin{bmatrix}`
`a & b\\`
`c & d`
`\end{bmatrix}`
`\begin{vmatrix}`
`A & B\\`
`C & D`
`\end{vmatrix}`
`\end{equation*}`

输出
$$\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{vmatrix}A & B\\C & D\end{vmatrix}$$

5 定义新指令

在 LaTeX 文档的引子加入以下指令

```
\newcommand{command}{definition}
```

就能定义用户自己的新指令，其中 command 的格式是反斜杠后面加指令名称，definition 为新指令的内容。例如，在引子中加入以下指令

```
\newcommand{\infd}{infinitely differentiable}
```

以后每次在文档中输入 \infd，编译时它都会被替换成"infinitely differentiable"。例如

输入
`Suppose the function $f$ is \infd.`

输出

Suppose the function f is infinitely differentiable.

新指令还可带参数，格式如下：

```
\newcommand{command}[n]{definition}
```

其中 n 表示参数的个数， `definition` 可以包含 #1，...，#n 表示 n 个参数。例如，在文档的引子中加入以下指令

```
\newcommand{\boldit}[1]{\textbf{\textit{#1}}}
```

则可用指令 `\boldit{text}` 将 `text` 以黑体和斜体输出。

输入

```
Here is \textbf{bold}.
Here is \textit{italics}.
Here is \boldit{bold italics}.
```

输出

Here is **bold**. Here is *italics*. Here is ***bold italics***.

又比如，如果在文档的引子中加入以下指令

```
\newcommand{\vfrac}[2]{\ensuremath{\frac{#1}{#2}}}
```

则可在非数学模式下用指令 `\vfrac{x}{y}` 输出竖排形式的分数。

输入

```
Let the desired fraction be \vfrac{x}{y}.
```

输出

Let the desired fraction be $\frac{x}{y}$.

除定义新指令外，还可以定义新环境，不过书写 MCM 论文一般不需要定义新环境。

6 页眉

MCM 论文需要指定形式的页眉，即在每页的左上角打印小组的控制编号，在右上角打印本页的页码和论文的总页码。例如

Team # 3211 Page 6 of 13

可以用 fancyhdr 及 lastpage 这两个功能包来实现，即先在文档的引子部分加入以下指令：

```
\usepackage{fancyhdr}
\pagestyle{fancy}
\usepackage{lastpage}
```

然后在引子里加入 \lhead 指令，在每页的左上角输出参赛小组的控制编号：

```
\lhead{Team \# 3211}
```

同样，在引子里加入 \rhead 指令，在每页的右上角输出该页的页码：

```
\rhead{Page \thepage{} of \pageref{Lastpage}}
```

其中 \thepage 用于打印本页的页码，\pageref{Lastpage} 用于打印最后一页的页码。此外，还可以使用指令 \cfoot{} 防止在页脚的位置上出现页码。综上所述，应在文档的引子部分加入以下指令：

```
\usepackage{fancyhdr}
\pagestyle{fancy}
\usepackage{lastpage}
\lhead{Team \# 3211}
\rhead{Page \thepage{} of \pageref{LastPage}}
\cfoot{}
```

编译后便可得到以下形式的页眉，以及一根横线：

Team # 3211 Page 6 of 13

如果不希望出现这根横线，可在引子中加入以下指令：

```
\renewcommand{\headrulewidth}{0pt}
```

7 论文标题与小节

论文标题可用 LaTeX 指令 \title{Title} 自动插入，其中 Title 为论文的标题。但这个指令还会输出一些不需要的信息。还可以用其他方式输入论文标题。在论文的正文开始前，即紧接在 \begin{document} 之后，用以下指令输入论文标题：

```
\begin{center}
\Large  在此输入论文标题
\end{center}
```

MCM 论文需要分节。设立新节的指令是

```
\section{section title}
```

其中 section title 为本小节的实际标题。编译时，LaTeX 将对各小节按顺序自动编号，并将小节标题用大号黑体字体显示。例如

输入
\section{Introduction} 引言的描述。 \section{Conclusion} 结论的描述。
输出
1 Introduction 引言的描述。 **2 Conclusion** 结论的描述。

小节也可以用以下指令

```
\subsection{subsection title}
```

再分成子小节。例如

输入
\section{Introduction} \subsection{Restatement of the problem} 问题重述的内容。 \subsection{Random thoughts} 思路的描述。 \section{Conclusion} 结论的描述。
输出
1 Introduction **1.1 Restatement of the problem** 问题重述的内容。 **1.2 Random thoughts** 思路的描述。 **2 Conclusion** 结论的描述。

子小节还可以进一步划分，其指令是

```
\subsubsection{subsubsection name}
```

通常 MCM 论文没有必要划分这么多层次。

MCM 论文应该有目录。在需要出现目录的地方输入以下指令：

```
\tableofcontents
```

文档在编译后即可自动生成目录。例如

输入
\tableofcontents \section{Introduction} \subsection{Restatement of the problem} 问题重述的内容。 \subsection{Random thoughts} 思路的描述。

```
\section{Conclusion}
结论的描述。
```

输出

Contents

1 Introduction

1.1 Restatement of the problem

问题重述的内容。

1.2 Random thoughts

思路的描述。

2 Conclusion

结论的描述。

8 脚注

论文中如果某个地方需要加脚注，指令是

```
\footnote{footnote text}
```

其中 footnote text 为实际的脚注内容。例如

输入

```
This is part of
a paper\footnote{This is a footnote.}
```

输出

This is part of a paper[a]

[a]This is a footnote.

注意：\footnote 与前面的正文之间应没有空格。顺便指出，书写括号时应在括号外加空格，即在左括号之前及右括号之后加空格。

9 列表

符号列表可用 itemize 环境生成，列表中的每一项均由指令 \item 开头。例如

输入

```
\begin{itemize}
\item First item.
\item Second item.
\end{itemize}
```

输出

- First item.
- Second item.

编号列表可用 enumerate 环境生成。编号列表中的每一项也由 \item 开头。例如

输入

```
\begin{enumerate}
\item First item.
\item Second item.
\end{enumerate}
```

输出

1 First item.
2 Second item.

列表可以嵌套。在双层列表中，子列表的项由字母编号。例如

输入

```
\begin{enumerate}
\item First item.
      \begin{enumerate}
```

```
        \item First subitem.
        \item Second subitem.
    \end{enumerate}
\item Second item.
\end{enumerate}
```

输出

1 First item.

(a) First subitem.

(b) Second subitem.

2 Second item.

10 图形

尽管 LaTeX 有绘图的功能，但使用比较麻烦。通常的做法是，先用其他绘图软件做好图后，再将其嵌入到 LaTeX 文档中。为此目的，需要将以下功能包

```
\usepackage[pdftex]{graphicx}
```

加到文档的引子中。这个功能包可以显示 PNG、JPG 及 PDF 格式的图形。用以下指令

```
\includegraphics{filename}
```

嵌入图形。

如果已有图片的大小不合适，可以通过以下指令在嵌入到 LaTeX 文档时

```
\includegraphics[width=width]{filename}
```

调整图形的宽度 `width`，其高度会按原图比例自动调整。长度单位可以是厘米 (cm) 也可以是英寸 (in)。例如

输入

```
Here is a figure:

\includegraphics[width=2cm]{fig/figure.png}

Here it is again, smaller.
```

```
\includegraphics[width=1cm]{fig/figure.png}
```

输出

Here is a figure:

Here it is again, smaller.

为了美观，图形并不一定要在其输入位置上显示，用 `figure` 环境可设置图形的打印位置，还能给出图形的文字说明，并对图形编号。例如

```
\begin{figure}
\centering
\includegraphics[width=width]{figure}
\caption{Description}
\label{Label}
\end{figure}
```

指令 `\centering` 将图形在纸上居中打印出来。引用图形的指令是 Figure `\ref{Label}`，例如

输入

```
The circle in Figure \ref{face} looks like a face.
\begin{figure}
\centering
\includegraphics[width=1.5cm]{fig/figure.png}
\caption{This looks like a face.}
\label{face}
\end{figure}
```

输出

Figure 1: *This looks like a face.*

The circle in Figure 1 looks like a face.

11　表格

表格可在 tabular 环境下建立，格式如下：

```
\begin{tabular}{format}
item & ... & last item \\
...
\end{tabular}
```

其中 format 是由 l、c 及 r 这 3 个字母组成的字符串，每个字母对应表格中的相应列，指定该列内容的对齐方式：

- 字母 l 表示列中的内容向左对齐；
- 字母 c 表示列中的内容居中对齐；
- 字母 r 表示列中的内容向右对齐。

在两个字母间插入一条竖线（|），打印时可在表格中对应的两列间加一根垂直线；在字符串的左右两旁加入竖线就可在表格的左右两侧各画一根竖线。在每一行中，每一列的内容用 & 隔开，每一行要用双反斜线（\\）作为结尾。如果要在某行下画一条水平线，其指令为 \hline。例如

输入

```
\begin{tabular}{|c|lr|}
\hline
Centered & Left & Right \\
\hline
0 & 1 & 2\\
1 & $x$ & $x^2$\\
\hline
\end{tabular}
```

输出

Centered	Left	Right
0	1	2
1	x	x^2

如果表格中的某一项需要再分成若干行，比如某个标题要分成两行，可以用以下指令

```
\multirow{2}{format}{entry}
```

完成，其中 entry 为具体的标题。如果要分成 n 行，把 2 换为 n 即可。同样，也可用下面的指令

```
\multicolumn{n}{format}{entry}
```

将表中的某一项分成多列显示。例如

输入

```
\begin{tabular}{|c|l|r|}
\hline
Centered & \multicolumn{2}{c|}{Spread out} \\
\hline
0 & 1 & 2\\
\hline
1 & $x$ & $x^2$\\
\hline
\end{tabular}
```

输出

Centered	Spread out	
0	1	2
1	x	x^2

和图形一样，表格也可在文中某个合适的位置显示，不一定要在定义表格的地方显示。这可在 table 环境下实现，这个环境同时还允许对表格进行编号并给出文字说明。例如

```
\begin{table}
```

```
\centering
\begin{tabular}{format}
...

\end{tabular}
\caption{Hello}
\label{label}
\end{table}
```

输入

```
The first four letters of the alphabet
are in Table \ref{alpha}.
\begin{table}
\centering
\begin{tabular}{c|c}
a & b \\
\hline
c & d
\end{tabular}
\caption{Some letters.}
\label{alpha}
\end{table}
```

输出

a	b
c	d

Table 1: *Some letters.*

The first four letters of the alphabet are in Table 1.

12 参考文献

MCM 解答论文必须列出所有引用过的文献资料。最简便的方式是使用 `thebibliography` 环境，这个环境以

```
\begin{thebibliography}{99}
```

开始，并以

`\end{thebibliography}`

结束，每条文献用下面的指令开头：

`\bibitem{label}`

通过 `label` 作者可以在论文中引用这个参考文献，指令如下：

`\cite{label}`

例如

输入

```
It was shown in \cite{Te} that termites eat wood.

\begin{thebibliography}{99}
\bibitem{Carp}
Carpenter, Bob.  \textsl{The Life of Ants.}
Springer-Verlag, Berlin, 1994.

\bibitem{Te}
Terwilliger, Sam.  \textsl{Termites.}
Prentice Hall, New York, 2004.
\end{thebibliography}
```

输出

It was shown in [2] that termites eat wood.

References

[1] Carpenter, Bob. *The Life of Ants.* Springer-Verlag, Berlin, 1994.

[2] Terwilliger, Sam. *Termites.* Prentice Hall, New York, 2004.

13 文档结构样例

以下是构建一篇 MCM 论文的 LaTeX 文档样例（读者可根据 MCM 竞赛委员会的要求做修改或增删）：

```
\documentclass[12pt]{article}
```

```
\usepackage[utf8]{fontenc}
\usepackage[doublespacing]{setspace}
\usepackage{textcomp}
\usepackage{amsmath,amssymb}
\usepackage{fancyhdr}
\usepackage{lastpage}

\pagestyle{fancy}
\lhead{Team \# 控制编号}
\rhead{Page \thepage{} of \pageref{Lastpage}}
\cfoot{}

\begin{document}

\begin{center}
{\Large Title}  论文标题
\end{center}

\tableofcontents

\section{Introduction}

赛题解读、已知研究成果的综述、解题思路

\section{Assumptions and Justifications}

列出所用假设及给出合理性解释

\section{Model Design}

模型设计

\section{Conclusions}

描述模型测试及所得结果，并讨论模型的优缺点
```

```
\begin{thebibliography}{99}

列出所引用的文献

\end{thebibliography}

\end{document}
```

第 9 章　MathType

MathType 是为 MS Word 配置的数学编辑软件，也可在 PPT 幻灯片及其他应用程序中使用。与 LaTeX 文档需要编译不同，MathType 是“所见即所得”的文字处理软件，不需要编译。MathType 将数学表达式称为等式 (equation)，为方便起见，本章将混合使用表达式和等式这两个术语。MathType 可从如下官方网页`http://www.dessci.com/en/products/mathtype/`下载。

用 MathType 输入数学表达式有以下两种方法。

(1) 先用 MathType 输入表达式，然后将表达式复制到 Word 文档。图 9.1 所示是 MathType 的输入界面及内置模板。MathType 内置模板给出了多种表达式格式。输入数学表达式时，使用键盘输入字符，常用的数学格式都可通过选取相应的模板及模板内的选项而得到。比如，点击 分数和根式选项即进入图 9.2 所示模板，然后根据需求选取合适的格式，再用键盘输入数字或字母。

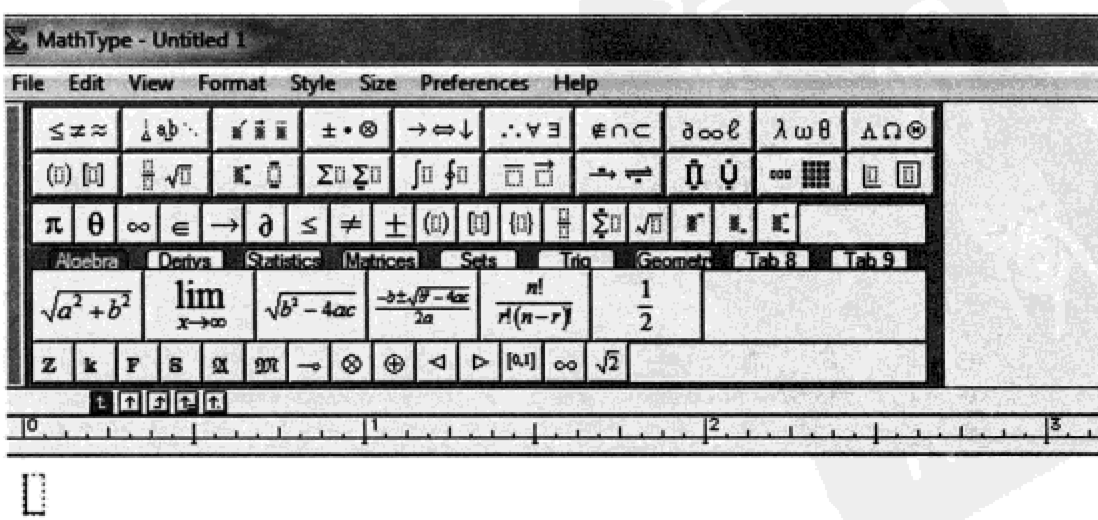


图 9.1　MathType 用户界面

(2) 在打开的 Word 文档中直接使用 MathType，即在 Word 的工具栏中点击 `MathType`，然后在文档需要输入表达式的地方，根据需求选择以下 4 个选

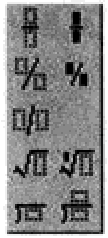


图 9.2 分数和根式选项

项中的一个：Inline（同行显示）、Display（单行显示）、Left-numbered（左编号显示）和 Right-numbered（右编号显示），如图 9.3 所示。点击其中一个选项后会弹出 MathType 的输入界面，在此输入表达式后退出 MathType，则表达式自动在 Word 文档的所需位置上显示出来。

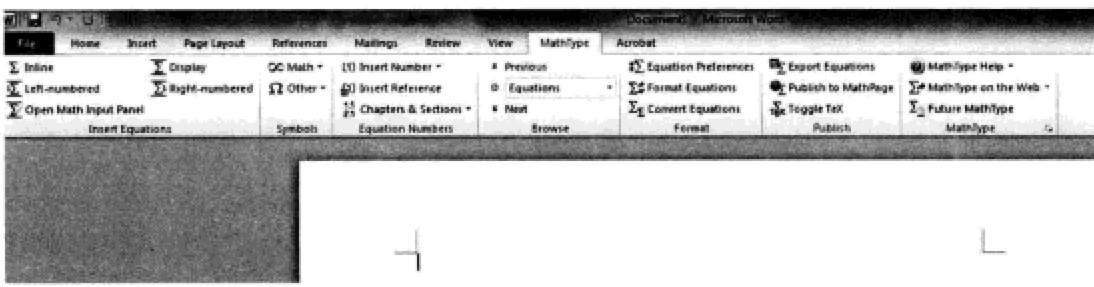


图 9.3 Word用户界面

下面举例说明如何在 MathType 界面输入数学表达式。

1 分数及平方根

假设需要输入下列等式：

$$y=\sqrt{\frac{3}{16}}\sin x-c^2\pm\mu\tan x.$$

步骤如下：

(1) 在 MathType 的输入框内输入“$y=$”。点击 分数和根式选项，然后点击 选取平方根模板，再点击 选取分数和根式模板，然后在分子和分母的位置上分别输入数字“3”和“16”便可得到以下等式：

$$y=\sqrt{\frac{3}{16}}$$

(2) 将光标移到根号外，并输入“$\sin x-$”。MathType 能自动识别正弦函数 sin 并将其以适当的方式显示出来。

(3) 现在输入平方。先输入字母“c”，然后点击 上下标选项，在上下标模板中找到所需的上标格式并输入数字“2”。
(4) 点击 选项选取加减号 $\pm$。
(5) 点击 希腊字母选项并从希腊字母模板中选取希腊字母 μ，然后输入“$\tan x$”，就得到所需等式，如图 9.4 所示。

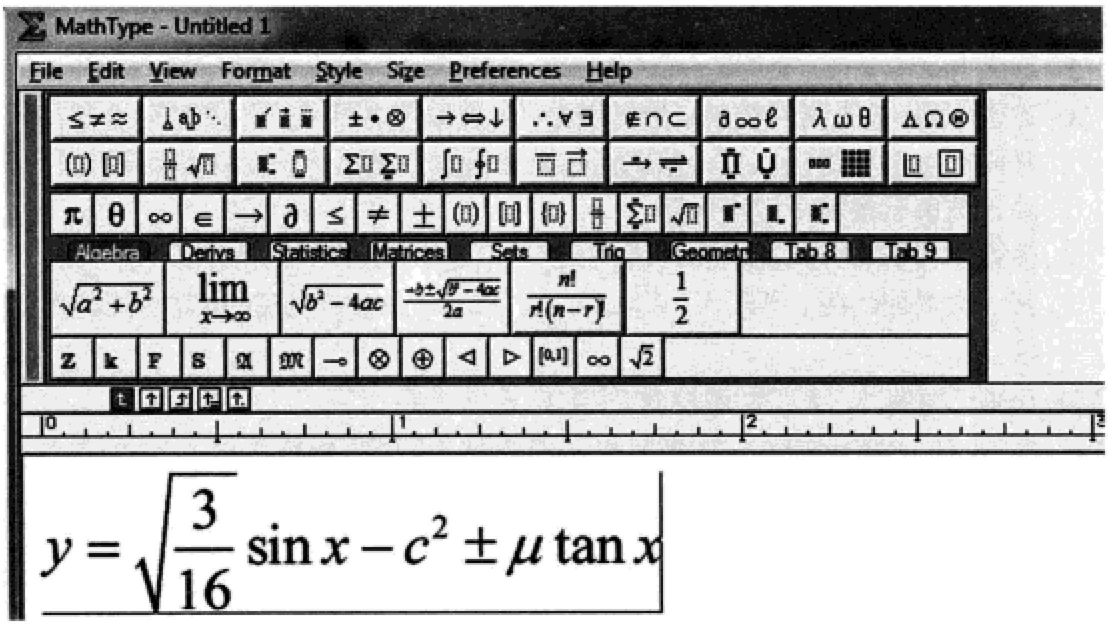


图 9.4 输入分数及平方根

2 积分、下标、求和

假设需要输入下列表达式:

$$\sum_{i=1}^{\infty}\frac{1}{i}\quad\iiint_E \mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

步骤如下：
(1) 点击 求和选项并在其模板中选取所需的求和符号。
(2) 插入分式 $\frac{1}{i}$。
(3) 点击 积分选项并在其模板中选取所需的积分符号。
(4) 输入 $\mathrm{d}x\mathrm{d}y\mathrm{d}z$。

3 多重方程式

假设需要输入以下多重方程式:

$$\sigma^2=\begin{cases}\dfrac{1}{3}\displaystyle\sum_{i=1}^{n}x_i^2 & i\geq 2\\ 2 & i=1\end{cases}$$

步骤如下：

(1) 点击 进入希腊字母模板并选取“σ”。然后点击 进入上下标模板并选取所需上标格式，然后敲入“2”。

(2) 敲入“=”。

(3) 点击 进入括号模板，然后点击 选取左花括号，并按回车键在花括号内产生两行的输入框。左花括号会按行数自动增大。(注：也可在当行行输入结束后按回车键增加新行。)

(4) 在第一行输入分式及求和表达式，然后点击 进入空格模板，并点击 若干次增加间距，再输入“$i\geq 2$”。(注：MathType 不支持用空格键输入空格。)

(5) 在第二行输入“2”、间距以及“$i=1$”。

在 MathType 的输入框内这时可看到如图 9.5 所示表达式。

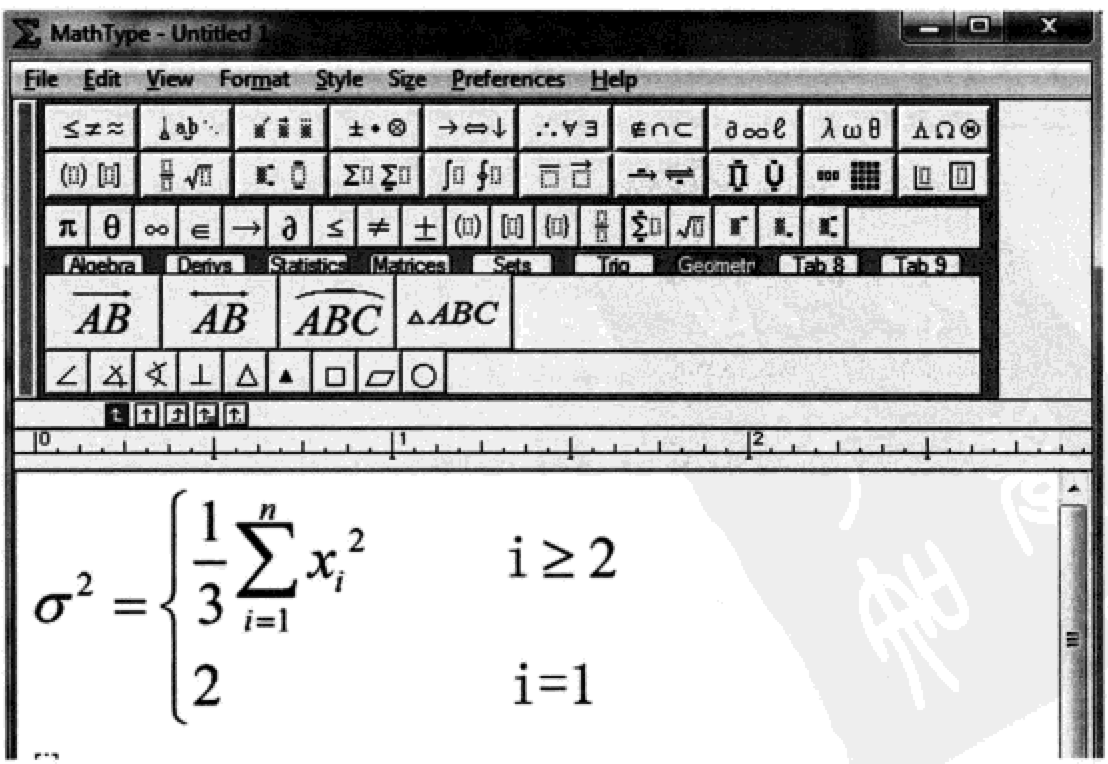


图 9.5 输入多重方程式

4 矩阵与行列式

在主界面的工具栏中可以找到 `Matrices` 选项，它给出了输入矩阵的 5 种格式。但工具栏不含行列式的内置模板，行列式的输入可按下列步骤进行：

(1) 点击 进入括号内置模板，然后点击 选取行列式括号。

(2) 点击 进入矩阵排列模板，然后选择输入格式，并输入行列式元素。

5 在工具栏中存储表达式

MathType 按数学分支学科在主界面的工具栏中提供表达式模板，包括代数、统计、积分、矩阵、集合、三角函数及几何表达式，如图 9.6 所示。

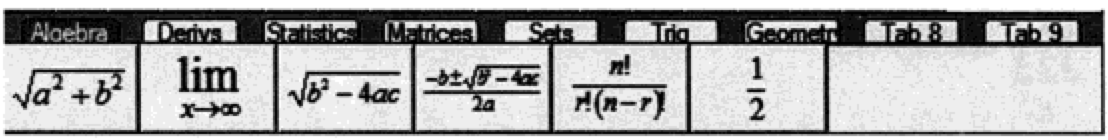


图 9.6 MathType 工具栏

MathType 支持所有常见的数学符号，并且用户可以根据需要编辑表达式。MathType 允许用户将自己编排好的表达式保存在工具栏上以后使用，步骤如下：

(1) 选择需要存储的表达式。例如，选取图 9.7 所示的表达式。

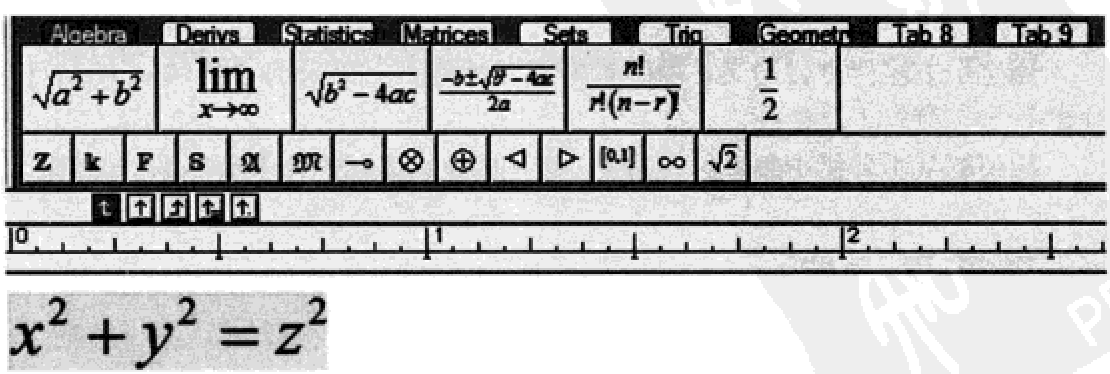


图 9.7 选择需要存储的表达式

(2) 按住鼠标左键，并拖到图中的黑框位置后释放左键，如图 9.8 所示。

在工具栏中便可以看到这个表达式，表明这个表达式已经保存到工具栏中，可随时调用。如果表达式属于某个分支学科，可将其存入这个分支中。否则，可将其存在 `Tab 8` 或 `Tab 9` 作为用户表达式，如图 9.9 所示。

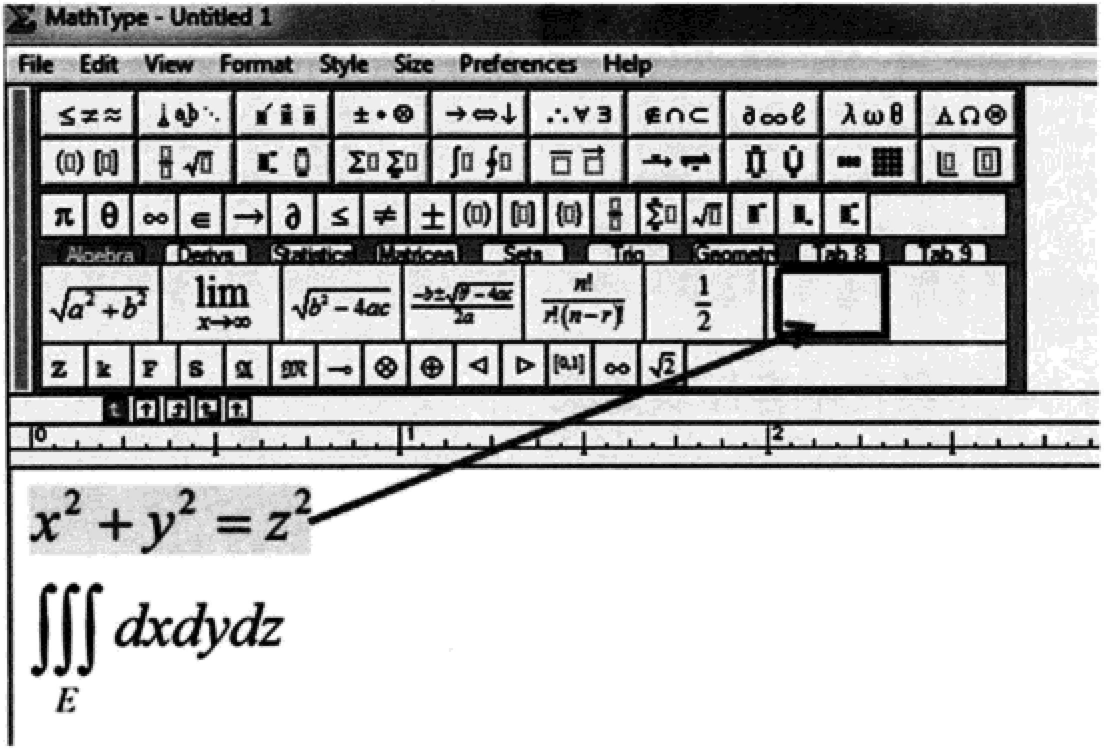


图 9.8 将表达式拖放到工具栏上

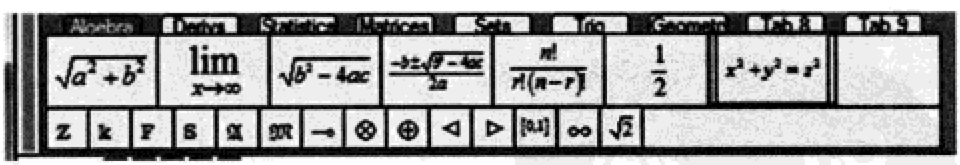


图 9.9 用户表达式

用这个方法也可将论文常用的希腊字母或其他在 Symbol 栏和 Template 栏中的符号保存在工具栏中。

如果要从工具栏中删除这些符号，按住鼠标右键，然后选择 delete 即可。

6 对齐表达式

假设需要输入以下两个不等式并在 $\leq$ 符号处对齐:

$$\int_0^1 a(x)\,\mathrm{d}x \leq \limsup_{n\to\infty} \phi_n(a),$$
$$\int_0^1 a(x)b(x)\,\mathrm{d}x \leq \limsup_{n\to\infty} \varphi_n(a,b).$$

步骤如下：

(1) 首先输入第一个不等式 $\le$ 符号左面的积分，并在 $a(x)$ 与 $\mathrm{d}x$ 之间用 空格模板插入适当间距。
(2) 输入不等式的剩余部分，按回车键换行。
(3) 复制第一个表达式，并稍做修改后完成第二个不等式的输入。(这样比较节省时间。)
(4) 此时可以看到如下的两个左对齐的不等式：

$$\int_0^1 a(x)\,\mathrm{d}x \le \limsup_{n\to\infty} \phi_n(a)$$
$$\int_0^1 a(x)b(x)\,\mathrm{d}x \le \limsup_{n\to\infty} \varphi_n(a,b)$$

(5) 为了将这两个不等式在 $\le$ 符号处对齐，点击工具栏中的 `Format` 选项，然后点击 `Align at =` 选项（如图 9.10 所示）。结果如图 9.11 所示。

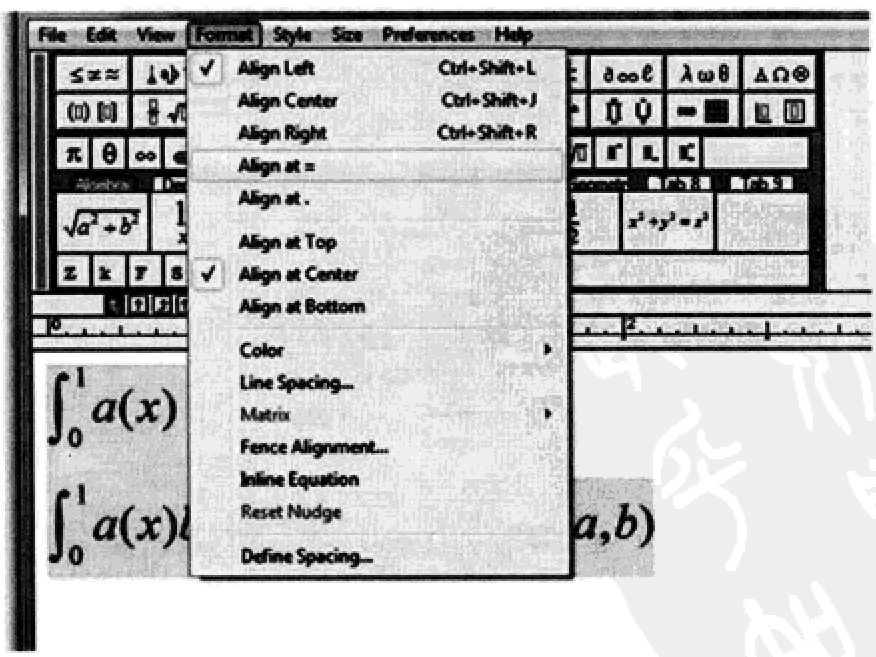


图 9.10 “Align at=”选项

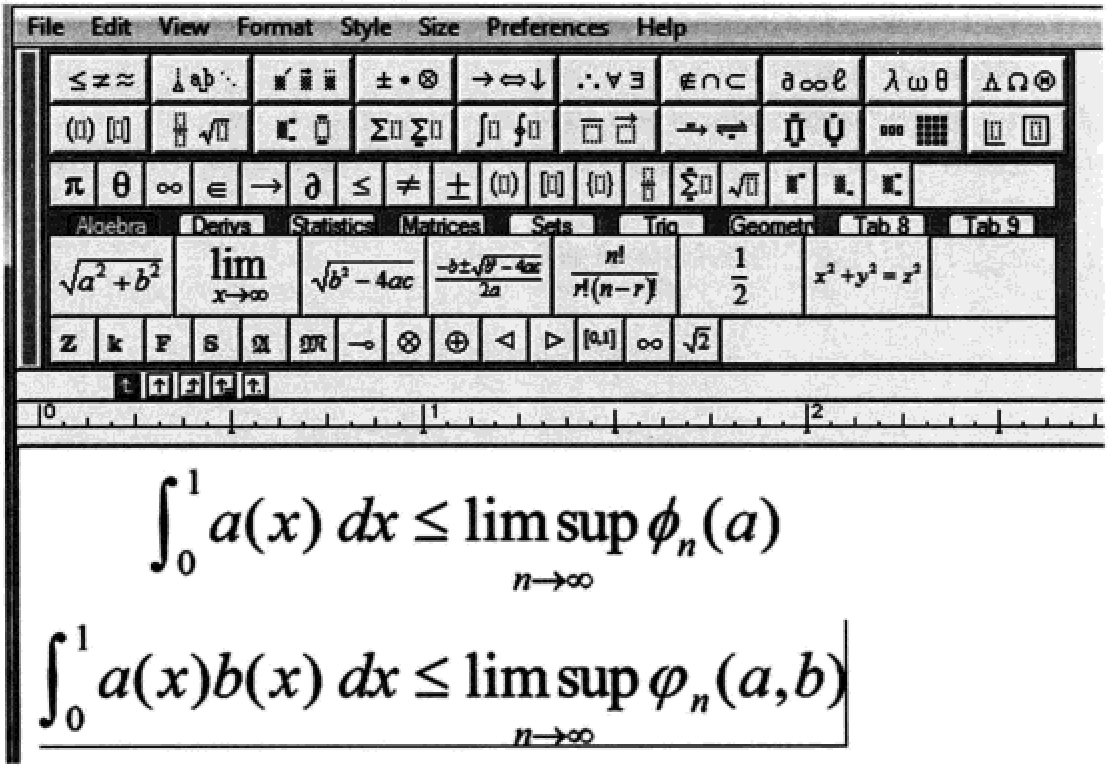


图 9.11 对齐结果

7 更改字体

假设需要将下列等式的默认字体 Math 改换成 Euclid:

$$u = \phi \cdot \exp\{\frac{1}{2}\sigma(x+y)\}.$$

步骤如下:

(1) 检查状态栏（在 MathType 界面的底部）是否显示 Style:Math。如果不是，则点击菜单中的 Style 选项，然后点击 Math。

(2) 建立表达式。点击 Style，然后点击 Define，弹出如图 9.12 所示对话框。

(3) 将 Primary font 改为 Euclid，再把 Greek and math fonts 改为 Euclid symbol and Euclid extra。

如果需要恢复默认设置，点击对话框的 Factory settings。如果需要更个性化的设置，点击 Advanced 进入以下菜单（图 9.13）选择其他字体。

用户可将 Text、Function 及 Symbol 等选项设置成不同的字体。

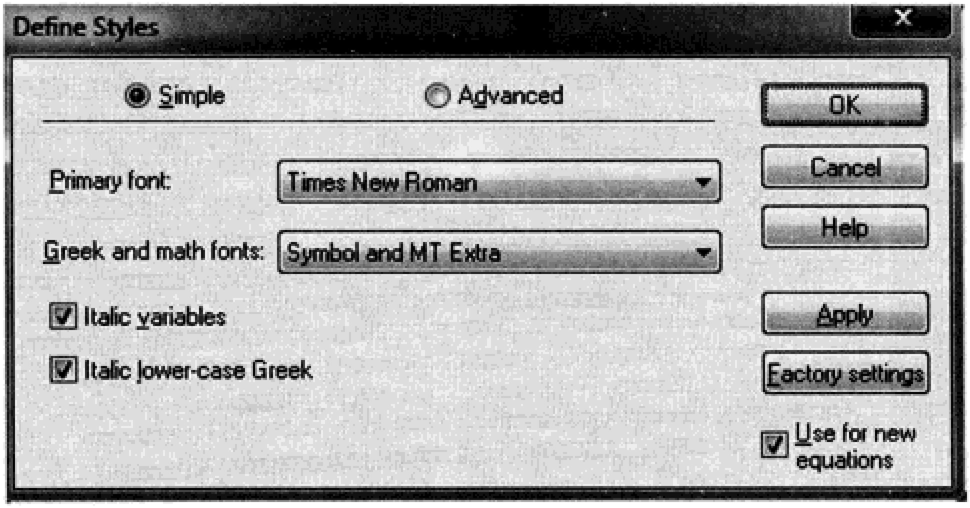


图 9.12 Define Styles 对话框

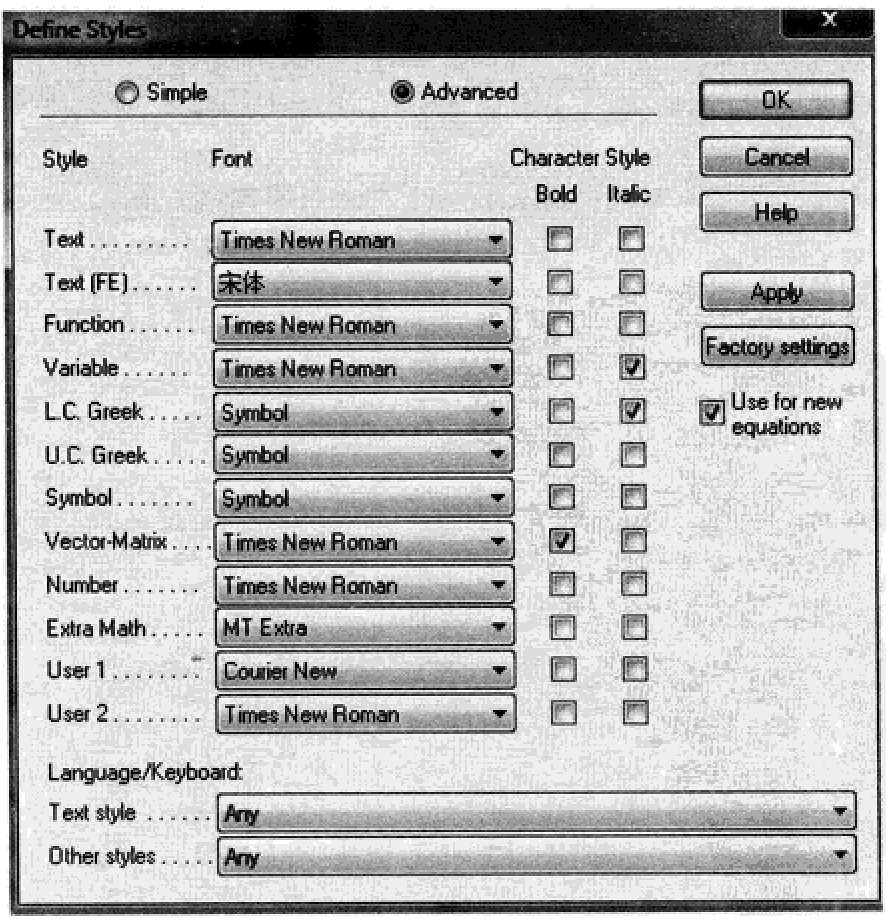


图 9.13 Advanced 菜单

8 表达式编号

假设需要将下列两个表达式分别编号成“(1.1)”及“(1.2)”：

$$\cos^2\theta + \sin^2\theta = 1 \tag{1.1}$$

$$\cos^2\theta - \sin^2\theta = \cos(2\theta) \tag{1.2}$$

步骤如下：

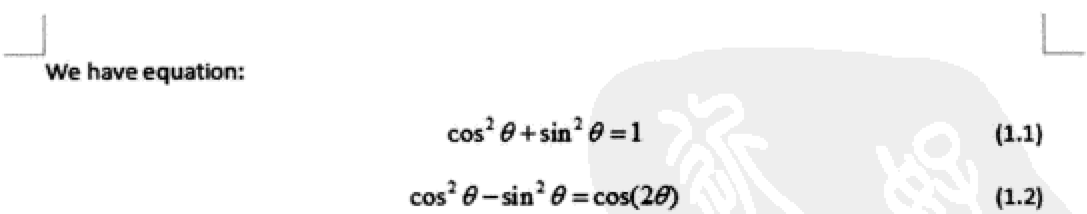
(1) 打开 Word 文档及 MathType 菜单，然后找到

Left-numbered 和 Right-numbered

为表达式编号有两种方式，一种是在表达式的左面编号，一种是在右面编号。

(2) 点击 Right-numbered，其作用是根据表达式所在的小节自动生成编号。此时会弹出 MathType 的输入框。输入第一个表达式并保存，在 Word 中便可看到表达式的右边出现了编号“(1.1)”，小节号码在前，表达式号码在后，中间用小圆点分开。

(3) 用同样的方法输入第二个表达式。完成后便可看到以下效果:



MathType 将自动按出现顺序为表达式编号。如果需要更改某个表达式的编号，可在 MathType 工具栏中，先找到 Chapters & Sections ▾，然后点击 Modify Break 进入如图 9.14 所示对话框，在框内输入期望的编号。

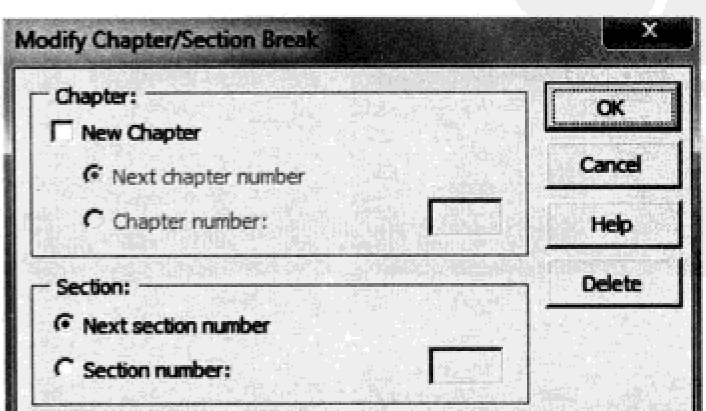


图 9.14 Modify Chapter/Section Break 对话框

9 Word 文档的设置

用 Word 撰写 MCM 论文或其他数学论文时，用 MathType 生成的表达式的字体和大小应该与文档中的文字部分一致。尽管 Word 和 MathType 允许用户直接更改字体及字号，但通过定义新格式会更方便。例如，将“Arial”字体改为“Times New Roman”字体，或将 10 号字体改为 12 号字体，文档会自动按新格式重新排版。

假设需要设置一个 12 号字 Times New Roman 的新格式，步骤如下：

(1) 在 MathType 中，点击 `Style`。在下拉菜单中点击 `Define`，然后在 `Primary Font` 上选择 `Times New Roman`，并选择 `Use for new equations`。

(2) 在 MathType 中，选择 `Size`。在下拉菜单中点击 `Define`，然后在 `Full` 中选择 12 号字，并选择 `Use for new equations`。

(3) 返回 Word，在 MathType 菜单中，点击 `Equation Preferences` 并选择 `MathType's 'New Equation' preferences`。这个新配置将自动应用到所有表达式。

(4) 接下来改变 Word 文档的文字格式。在 Word 中点击 `Home`，在 `Style` 的选择框中，选择一个样式并右击鼠标，然后选择 `Modify`。找到 `Formatting` 后选择 `New Times Roman` 及 12。

10 常用字体

正文中字符与符号的字体称为正文字体。常用的正文字体是“Times New Roman”。表达式应该用正文字体或正文字体的斜体。例如，对数函数（log，ln，lg）、三角函数（sin，cos，tan）、模运算（mod）以及极大值和极小值运算（min，max）应该用正文字体，而变量应该用正文字体的斜体，向量及矩阵用正文字体的斜体和黑体。

以下是 MathType 的常用格式：

Math 数学表达式字体的默认值。

Text 正文字体的默认值。（注：这个字体只对英文有效。）

Function 常用数学函数及运算符字体的默认值，包括对数函数、三角函数、模运算、极大值和极小值。

Variable 数学变量及常数字体的默认值。

Greek-Symbol 希腊字母字体的默认值，也适用于数学运算符。

Vector-Matrix 向量及矩阵字体的默认值，通常与 `Variable` 字体相同，但用黑体。

User 1 及 User 2 用来存储用户自己定义的字体。

MathType 可通过函数识别字符，自动设置合适的格式、字体以及符号间的间距。

在 `Math`、`Variable`、`Function`、`Vector-Matrix` 及 `Greek-Symbol` 格式下，MathType 会自动选择合适的字符。例如，减号在 `Text` 格式下输入时是短横线，在上述格式中会被自动替换成真正的减号。

MathType 能为表达式中出现的符号自动选择合适的大小，包括正文、上下标、上上标及下下标、符号、符号上下标等 5 种字号。

11 使用技巧

除了默认设置外，MathType 还允许用户自己设置格式。不过，默认设置对撰写 MCM 论文应该足够用了。本节总结 MathType 的一些使用技巧。

修改文档中的表达式

修改 Word 文档中的表达式的步骤是，双击表达式进入 MathType 的输入界面，这时表达式会显示在输入框内。用户根据需要修改后退出 MatyType，就可在 Word 文档中看到修改后的表达式。

修改所有表达式的字体和大小

论文中，如果需要修改所有表达式的字体或大小，如果一个一个修改不仅费时费力而且容易出错。以下是解决这个问题的一个小技巧（以修改字号为例）：

(1) 打开 Word 文档，双击一个表达式进入 MathType 输入框，然后依次点击 `Size` 及 `Define`，修改相应的字体及大小。

(2) 然后依次点击 `Preference`、`Equation preference` 及 `Save to file`，将修改后的格式存到一个与默认配置文件不同的文件，然后关闭 MathType 返回 Word 文档。

(3) 点击 Word 工具栏上的 MathType 选项，并依次点击 `Format equation`、`MathType preference File` 及 `Browse`，然后点击上一步保存的配置文件，点击 `Whole document` 及 `OK` 即可。

使用更多的符号

MathType 提供了大量数学符号，在用户界面上可以找到一部分。如果需要的符号没有出现在这里，可依次点击 `Edit` 及 `Insert Symbol` 弹出更多的符号。为了以后的输入方便，建议用户把常用符号存到主界面的工具栏中。

使用现有表达式

MathType 有一个常用数学表达式的分类下拉菜单，比如，一元二次方程的求根表达式存在 `Algebra` 分类里，标准随机变量存在 `Statistics` 分类里。

更改字体、颜色及大小

依次点击 `Style` 及 `Define` 可更改字体，依次点击 `Format` 及 `Color` 可更改颜色，依次点击 `Size` 及 `Define` 可更改字体的大小。

添加常用表达式

首先选择需要添加的表达式，然后用鼠标左键将其拖到工具栏中的适当位置后释放。当不再需要该表达式时，可右击这个表达式，使用删除选项将其删除。

快捷键

MathType 中的很多指令都有快捷键，记住一些常用的快捷键能够提高书写速度。以下是常用的数学运算符、希腊字母以及格式的快捷键。

常用数学运算符

分式

Ctrl+F（分式；即同时按下控制键及 F 键）
Ctrl+/（斜杠分式）

根式

Ctrl+R（根式）
Ctrl+T+N（高次根式；先按 Ctrl+T，释放后再按 N）

上、下标

Ctrl+H（上标）
Ctrl+L（下标）
Ctrl+J（上标+下标）

不等式

Ctrl+K+,（小于等于号 $\leq$；先按Ctrl+K，释放后再按逗号）
Ctrl+K+.（大于等于号 $\geq$；先按Ctrl+K，释放后再按句点）

导数、积分

Ctrl+Alt+'（单撇，导数符号）
Ctrl+Shift+"（双撇，二阶导数符号）
Ctrl+I（定积分符号）
Ctrl+Shift+I（不定积分符号）

括号

Ctrl+9 或 Ctrl+0（小括号）
Ctrl+[或 Ctrl+]（中括号）
Ctrl+{ 或 Ctrl+}（大括号）

其他

Ctrl+Shift+连字符（"-"）（上横线）
Ctrl+Alt+连字符（"-"）（矢量箭头）

希腊字母

先按 Ctrl+G，释放后再输入英文字母就能得到相应的小写希腊字母。
先按 Ctrl+G,放开后，释放后输入"Shift+字母"就会得到相应的大写希腊字母。

格式

改变显示尺寸　这些快捷键仅改变显示尺寸，而不改变实际大小。

Ctrl+1（100%）
Ctrl+2（200%）
Ctrl+4（400%）
Ctrl+8（800%）

微移间距 先选择需要移动的表达式，再按“Ctrl+箭头”键实现上、下、左、右的微移。按“Ctrl+Alt+Space”可添加空格。

自行定义快捷键 MathType 还允许用户对特定的符号及表达式自行定义快捷键。步骤如下：

(1) 在 MathType 的主界面中，右击需要定义的表达式（例如，右击 π），然后点击 `Preference` 弹出如图 9.15 所示对话框。

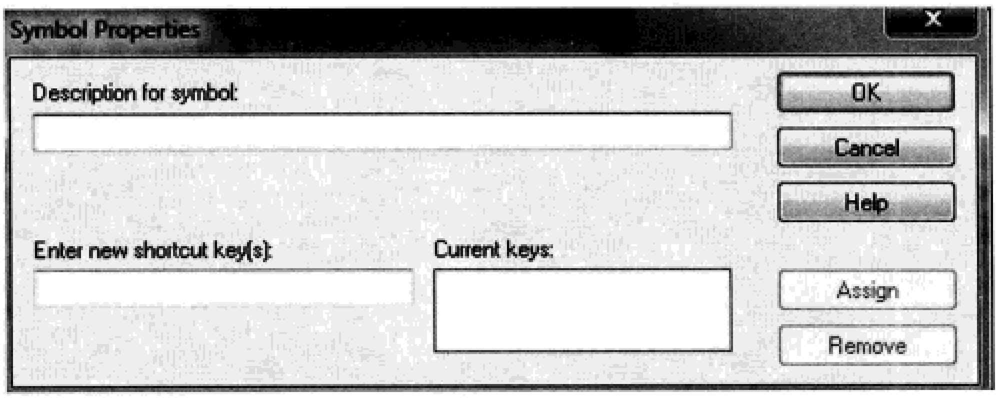


图 9.15 Symbol Properties 对话框

(2) 在 `Enter new shortcut key(s)` 中输入想要使用的快捷键，然后点击 `OK` 即可。

II

Write Right for the American Mathematical Contest in Modeling

Chapter 1 Introduction

The Mathematical Contest in Modeling (MCM) is an annual international mathematical event, where teams of undergraduate students from different countries work out mathematical models around the clock to solve open-ended problems. These are thought-provoking problems with real sense of applications. Their solutions are presented in the form of research papers, and the contest is as much a contest in clear writing as it is in mathematical modeling. In some ways, clear writing is the more important part of the contest: A well-written paper using a poor model will typically do better than a poorly written paper using a good model.

1 A brief history

The MCM contest was originally conceived of as an "Applied Putnam" contest. The Putnam Competition, started in 1938, is an annual American mathematical competition where students are given problems to work on individually. The scores on the Putnam tend to be very low: Out of a possible 120 points, many students get zero and about half get two or fewer points.

In the early 1980's, Ben Fusaro, who founded the MCM, was concerned about the lack of applied problems on the Putnam and realized that students find the low scores discouraging. He thought that a good balance to the Putnam would be a contest where teams of students would work on applied math problems. For example, students would work on two problems in one day, one problem involving discrete math *and* the other involving continuous math. This idea received strong support from his colleagues, with a new format where the teams work on one problem over a weekend, either a problem involving discrete mathematics *or* a problem involving continuous mathematics. Solomon Garfunkel, the director of the Consortium for Mathematics and Its Applications (COMAP) suggested that Ben Fusaro submit a proposal to Fund for the Improvement of Postsecondary Education (FIPSE), a program of the U.S. Department of Education. A three year grant was approved in June 1984, which provided financial support to start the first MCM contest in next year. The final proposal to FISPE describes the purpose of the MCM contest [1]:

> The purpose of this competition is to involve students and faculty in clarifying, analyzing, and proposing solutions to open-ended problems. We propose a structure which will encourage widespread participation and emphasize the entire modeling process. Major features include:
>
> - The selection of realistic open-ended problems chosen with the advice of working mathematicians in industry and government.
> - An extended period for teams to prepare solution papers within clearly defined format.
> - The ability of participants to draw on outside resources including computers and texts.
> - An emphasis on clarity of exposition in determining final awards with the best papers published in professional mathematics journals.
>
> As the contest becomes established in the mathematics community, new courses, workshops, and seminars will be developed to help students and faculty gain increased experience with mathematical modeling.

The first contest, in 1985, was merely a national contest with only 158 US teams entered and no international teams. The contest grew quickly, largely due to international support, which was somewhat unexpected when the idea of the MCM contest was conceived. By 2010 there were 2,254 teams entered, most of them were international, and the number of US teams was also increased to 358.

The MCM contest is organized and administered by COMAP. In each year, the contest consists of two problems, labeled as Problem A and Problem B. Problem A deals with modeling using continuous math, while Problem B deals with modeling using discrete math. COMAP added the Interdisciplinary Contest in Modeling (ICM) in 2000, which takes place at the same time as the MCM contest and follows the same rules. The ICM contest in each year consists of only one problem, labeled as Problem C, which deals with modeling using continuous math, discrete math, or both. Thus, the American mathematical contest in modeling is also referred to as the COMAP MCM/ICM contest. For simplicity, we will use the MCM to refer to both the MCM and the ICM contests, unless otherwise specified. COMAP also organizes the High School Mathematical Contest in Modeling (HiMCM), which is specifically designed for high school students and will not be discussed in this book.

A team taking part in the MCM contest can only work on the problem, and hence the paper, once the contest starts. However, the team should prepare before the contest starts. The team itself should have been chosen carefully. Ben Fusaro once said "It is not the three best that make the team, it is the best three." In other words, the emphasis should be on having a team whose members work well together and whose skills complement each other. As well as a good math student, a team should have someone who is familiar with available computer programs and someone who can write programs. One person can of course fill more than one role.

2 Classifications and judging

MCM contest problems do not have standard solutions, and there are no passing grades. Papers are classified into categories, rather than receiving numerical scores.

Judging criteria

For each MCM paper, the judges will look for the entire modeling process and the clarity of its exposition. Your paper will be judged based on the following criteria:

- Does your paper provide a satisfactory interpretation to the contest problem, with necessary clarifications to vague concepts (if any)?
- Does your paper list all the assumptions used in your modeling, and provide satisfactory explanations or justifications to these assumptions?
- Does your paper provide sufficient analysis that motivates or justifies your modeling approach?
- Does your paper design a model (or a series of models) that can effectively solved the contest problem?
- Does your paper test the stability of the model you constructed?
- Does your paper discuss the strengths and weaknesses of your model and state clearly what results are obtained?
- Does your paper provide a satisfactory summary?

Papers with incomplete solutions are acceptable, and may even obtain a good evaluation if it provides a unique and creative approach to the problem.

Classifications and awards

Papers that do not sufficiently address the MCM problem or somehow violate the contest rules are labeled **Unsuccessful Participant**. The rest of the papers are classified, based on the judging criteria, into five categories. They are **Successful**

Participant, **Honorable Mention**, **Meritorious**, **Finalist**, and **Outstanding Winner**. Any paper that is reasonably well written, addresses the modeling problem, and meets the contest requirements is a Successful Participant. Outstanding Winner papers are the best written papers that present the best models. The categories and the corresponding paper percentages are listed below:

- Successful Participant. About 50% of the papers.
- Honorable Mention. About 30% of the papers.
- Meritorious. Usually 10% to 15% of the papers.
- Finalist. About 1% of the papers.
- Outstanding Winner. About 1% of the papers.

In addition to paper classifications, MCM papers may also receive the following awards:

- The INFORMS Award, created by the Institute for Operations Research and the Management Sciences (INFORMS), may be given to a selected Outstanding Winner paper from solutions to each of the three problems.
- The SIAM Award, created by by the Society for Industrial and Applied Mathematics (SIAM), may be given to a selected Outstanding Winner paper from solutions to each of the three problems.
- The MAA Award, created by the Mathematical Association of America (MAA), is given to a selected Outstanding Winner paper from, respectively, solutions to Problem A and Problem B.
- The Ben Fusaro Award, created by COMAP, is typically given to a finalist paper that is especially creative but contains a flaw that prevents it from attaining the Outstanding Winner designation.

Judging process

The judging occurs three weeks after the contest is over and takes place over a weekend. The judges do not know who wrote the papers; the papers are identified by control numbers assigned by COMAP and the names and institutions of the contestants are removed from the papers before the judges see them.

The judging of the papers occurs in two stages. The first stage of judging is **triage**, where the papers are divided into three groups: (1) papers which are good enough to go on to the next round, (2) papers which are good but not good enough for the next round (these are classified as Successful Participants), and (3) papers which do not meet the standards of the competition (these are labeled Unsuccessful Participants). For this stage, there is a head judge and about one additional judge for each twenty-five papers. Each paper is read by two judges, each of whom gives the paper a score. If the two scores are not very close, the two judges discuss it and, if they cannot agree on the quality of the paper, a third judge is conferred. Once two reasonably close scores are given to a paper, the scores are added to give the paper an overall triage score. The head judge and the contest director determine a cutoff score for the papers. Less than but close to half of the papers will have scores above the cutoff and go to the next round.

The triage judges spend about ten minutes looking over each of their papers, and because of the time limitation they mainly look at the summary to determine the quality of the paper. **This makes the summary the most important part of the paper.**

For the post-triage judging in the second stage, the continuous and the discrete math papers are separated and each have their own sets of judges. There will be several rounds, and papers which make it past one round are read by different judges for the next round. The more rounds a paper passes through, the higher the classification of the paper will be. The judges will spend between fifteen and thirty minutes reading each paper. To impress the judges in so little time, the papers need to be well organized and highlight the important results.

In the final round, where the Outstanding Winners are selected, the papers are read and discussed by all the judges. To be an Outstanding Winner, all of the judges, as well as the contest Director and Associate Director, have to agree that the paper deserves it.

3 Writing is important

Given the importance of writing for the contest, more than one person has suggested that the team include an English major (or two!). At the very least, a team should include very good writers. For a month before the contest the team should meet a few times a week to prepare. The team advisor can assign practice problems for the team to work on; the team members should practice researching the problems, solving them, and writing up solutions. Since the contest will not have started yet, the writing can be critiqued and this is a good time to filter out bad habits. Probably the best way to become a good writer is to read good writing, and so the best way to prepare for writing up an MCM paper is to read previous winners. Each year the Outstanding Winners, along with commentaries, are published in the *UMAP Journal*.

When the contest begins, it is a good idea for each team member to initially take on specific duties. For example, Alice should do any necessary background research on the problem, Bob should begin writing the paper, and Charlie should do any necessary coding.

The writing should begin as soon as the contest starts, for many students tend to underestimate how long it will take to write the paper. Sometime during the second day of the contest the team may want to consider finishing the work on the model and concentrate on the writing.

Chapter 2 Paper Structure

The paper that you submit to the MCM must be designed so that it can be read and understood quickly. For this purpose you must first come up with a good structure for your paper. A poorly organized paper can hide good results, so you need to make sure that your paper is logically organized and carefully laid out. Because the contest judges have little time to spend on your paper each round, your paper should catch the judge's interest and highlight the important points.

1 Good structure goes a long way

Your paper should be organized in sections and subsections, and they should be titled in such a way that merely reading them will provide an outline of the paper. Listed below is the suggested outline for an MCM paper [2] based on the judging criteria:

- A clarification or restatement of the problem, as appropriate.
- A clear exposition of all assumptions and hypotheses.
- An analysis of the problem justifying or motivating the modeling to be used.
- The design of the model.
- A discussion of how the model can be tested, including error analysis and stability (conditioning, sensitivity, etc.).
- A discussion of the strengths and weaknesses of the model.
- A one-page summary of the results, typed on the Summary Sheet, must be attached to the front of each copy of the solution paper.

A textbook example is the paper "How to Please Most of the People Most of the Time" [3] from the 1989 MCM, discussing the best way to line up airplanes for takeoff, to minimize dissatisfactions of passengers because of delays. The followings are the sections and subsections of this paper:

Summary

1 *Restatement of the Problem*

2 *Assumptions*

3 *Justification of Our Approach*

4 *The Model*

  4.1 *Dissatisfaction of a passenger needing a connection*

  4.2 *Dissatisfaction of a passenger not needing a connection*

  4.3 *Total dissatisfaction on an aircraft*

5 *Testing the Model*

6 *Results*

7 *Strengths and Weaknesses*

  References

While not every paper will have sections so clearly lined up with the suggested outline, if you give your paper a simple and standard sectioning it will make it easier for judges to get through the paper quickly and with great comprehension.

When breaking up your paper into sections, you should avoid having large sections of homogeneous text, since it is easy for the reader to gloss over them when skimming a paper. You should highlight important sentences by putting them in bold face. Newly defined words should also be bold. Note that you may also highlight important sentences or words in italics, but bolding them will make them standout better for someone skimming the paper. Overusing bold or italics, however, will make your paper look awkward and so you should keep the use of them to the minimum.

Important equations should be displayed rather than inline. A list of items, such as a list of hypotheses that you will need to make to construct your model, should be presented as a bulleted list rather than a paragraph. When reasonable, information should be given as a figure or table with a short, informative caption.

2 Writing the introduction

The introduction to your paper should be the first part of the paper you write, even though it may not have the title "Introduction" (for example, you may use "Restatement of the Problem"). Your introduction should include your interpretation of the contest problem, your survey and discussions of the existing work related to the problem, and your thoughts and approaches to solving the problem.

The introduction is where you need to show the reader that you have put significant thought into the problem. You should begin writing the introduction as soon as you begin working on the problem, and you should revise it a few times during the course of developing your solution.

The first sentence of your introduction is the most important sentence of your paper and should convince the reader that they want to read the rest. The entire first paragraph should be accessible even to people unfamiliar with the problem at hand and get them interested. This paragraph should not be mathematically heavy and symbols should be kept to a minimum. Thus, composing the first sentence and the first paragraph may often require substantial thought and perhaps a number of revisions.

Once you have decided which contest problem to work on, the first thing you should do is to restate the problem in your own words. You need to make your goal precise and possibly clarify anything ambiguous about the original problem statement. Many problems are subject to interpretation, so you need to let the reader know how you see the problem. For example, in the 2010 contest one of the problems was to explain the sweet spot on a baseball bat. There is more than one meaning of what a "sweet spot" is, and so in the introduction to "The Sweet Spot: A Wave Model of Baseball Bats" [4] (one of the Outstanding Winners), the authors specify:

> *There are at least two notions of where the sweet spot should be – an impact location on the bat that either*
>
> - *minimizes the discomfort to the hands, or*
> - *maximizes the outgoing velocity of the ball.*
>
> *We focus exclusively on the second definition.*

Even apparently precise information may still benefit from a nice interpretation. For example, one of the problems from 1988 was to develop a search method to find a fleeing boat given some information about the boat's current position. The information included angles "accurate to within 2°". One of the Outstanding Winners, titled "Locating a Drug Runner: Miami Vice[1] Style" [5], stated how they interpreted the accuracy:

> *We interpret the error of* $\pm 2^\circ$ *as a normal distribution ... with standard deviation of* 1°.

Once you know exactly what you are working on, you should begin to search the literature for published works on similar problems and the models that were

[1] "Miami Vice" was an American television crime drama series aired from 1984 to 1989, where two police detectives worked undercover in Miami.

used. Your introduction should include your restatement of the problem and a discussion of previous work on similar problems. Your introduction should also include a brief description of your approaches to the problem and your major results.

3 The body of the paper

After the introduction, your paper should begin with a construction of the model (or series of models) that you are going to use. This should include your assumptions, your explanations why these assumptions are reasonable, and your descriptions of your model.

To begin, you need to state very clearly what assumptions you are using to create your model. There should be no unstated assumptions, the reader should never have to wonder what you are doing or why you are doing it.

This section illustrates, using examples from outstanding winners, how to provide explanation to assumptions, how to build a nice model from a series of simpler models, and how to build a general model that contains the contest problem as a special case.

Assumptions and justifications

One of the problems in the 2010 contest was to develop a method to predict the location of a serial criminal based on the location of the crimes. The motions of the criminal needed to be considered. One of the Outstanding Winners, "Centroids, Clusters, and Crime: Anchoring the Geographic Profiles of Serial Criminals" [6], modeled the region of interest as a certain type of metric space. To justify this, the authors, in a section conveniently titled "Assumptions", gave a bulleted list of what they wanted to assume and why they could assume it. One of their assumptions was "Criminal's movements are unconstrained". The movements of a person in a city would normally lie on a grid and be constrained by buildings and walls, so the authors follow their assumption with an explanation:

> ***Criminal's movement is unconstrained.*** *Because of the difficulty of finding real-world distance data, we invoke the 'Manhattan assumption': There are enough streets and sidewalks in a sufficiently grid-like pattern that movements along real-world movement routes is the same as 'straight-line' movement in a space discretized into city blocks ...*

A lesser paper would have left the issue open, and perhaps simply used unrestricted motion of the criminal as an unstated assumption. Always explicitly state any assumptions that you use and provide justification for your assumptions; if you cannot, you should rethink your assumptions. The judges will look at your assumptions, as well as how reasonable and how justified they are.

Model designs

When creating the actual model, keep in mind that the model is a tool used to get your results and not an end in itself. The model itself does not have to be elaborate and in many cases should not be. The important thing is the information you get from the model. As with most things, **the simplest model that does the job is probably the best.**

During your background research for the problem you will probably come across models that you could modify for the current problem. If possible, you should build upon previous results. The model you use should be something that you could have come up with yourself (otherwise it will probably be too complicated).

Model series

One approach to model building is to create a series of models, starting with something simple and making it more realistic along the way. For example, a problem from the 2008 contest was to predict the effects of an increase in global temperatures (and the resulting melting of the North Polar ice cap) on the coast of Florida. One of the Outstanding Winners, "The Impending Effects of North Polar Ice Cap Melt" [7], produced such a series of models as follows:

The authors of the paper first considered **Model 1: Constant Temperature**, where the global temperatures did not increase or decrease and there was no change in the rate of the ice melting and rate of increase in the ocean volume. They then considered **Model 2: Variable-Temperature Model**, where they take into account a changing global temperature. Finally, they get **Model 3: Ocean Volume under Warming**, where they take into account the previously ignored relative surface areas of the Northern and Southern hemisphere oceans (a consideration that was not brought up in the problem statement).

When you create your model, you should focus on a single model (or, as above, a series leading to a single model). Some teams present their attempts of creating several superficial models and hope to score a few points from them. This detracts from the paper and is viewed unfavorably by the judges. On the other hand, you should also try to avoid designing everything from scratch, for it is hard to design a good model from scratch in a short time.

Modeling for general problems

Another approach that you should avoid is creating a model too specifically designed for the problem. While your problem should ultimately be answered, the better papers view the problem as an instance of a more general problem and deal with the larger problem before focusing on the specific problem. As an example, one problem from the 1988 contest was to load two railroad flatcars, each of a given length and capable of holding a certain weight, with as little wasted space as possible. The widths and weights of the crates to be loaded were given, as well as the number of each type of crate. There was clearly a more general problem — you could leave the specific values given in the problem unspecified — but the specific values of the widths and weights in the problem could be used to greatly simplify the problem. A lesser paper would only examine the simpler, specific problem; but one of the Outstanding Winners, "Loading Two Railroad Cars" [8], wrote:

> *We have produced a general algorithm to solve this type of problem, but for our problem a relationship exists that greatly simplified the algorithm.*

They then proceeded to solve the specific MCM problem.

4 Writing a conclusion

You should conclude your paper with a description of model testing and your results. Any models you build are a means to an end. The goal is to answer the question being asked. Even though the problems are open-ended, after creating and analyzing your model (or models) you should reach a definite conclusion. The answer should be clearly stated at the beginning of the section containing your conclusion (and this section is often conveniently titled "Conclusions"); the reader should not have to search for your answer or work at interpreting it. You should discuss your answer, but make it clear what part of the discussion is your opinion and what part is your conclusion.

You are asked to check the sensitivity and stability of your model. This is a stated requirement, meaning that doing it will make your paper look good. Unfortunately, many teams do not put much effort into this part. If your model requires some parameters (for example, a traffic problem might depend on the average speed of the traffic), you should see what happens to your model and solution when the values of the parameters are slightly altered.

You also need to provide evidence to support your solution as a good, if not the best, answer. Depending on the type of problem being asked, your solution might be a procedure of some sort. In this case, you would want to test it multiple times, using slightly different input each time. When possible, use real data to avoid the suspicion of contrived data.

When you write up your results, be sure to include sufficient information for the reader to be able to recreate everything necessary; if part of your result involves the output of a computer program, be sure the reader knows what program was used and present the algorithm you use for the part of the coding your team wrote.

Given the open-ended nature of the MCM problems and the limited time that you have to work on it, your model and solution will almost certainly have deficiencies. The judges will certainly notice problems with your work; you need to show them that you are astute enough to see them yourselves. While the limitations of your model may have been mentioned in your model description, you should still mention them again when you discuss the strengths and weaknesses of your model.

Another consequence of the limited resources you have while working on your problem is that there are many things you were unable to do. Your paper should mention things you could do should more time and resources be available.

5 Writing the summary

The most important part of an MCM paper is the summary sheet. Each paper is required to have a summary sheet attached to the front, and this will basically determine whether or not your paper makes it past the first evaluation stage (triage). A good model and a good solution will not make it past triage if it does not have a good summary.

According to the contest rules [9], the summary sheet should include

- **Restatement and clarification of the problem:** State in your own words what you are going to do.
- **Explain assumptions and rationale/justification:** Emphasize the assumptions that bear on the problem. Clearly list all variables used in your model.
- **Include your model design and justification** for type model

used or developed.

- **Describe model testing and sensitivity analysis**, including error analysis, etc.
- **Discuss the strengths and weaknesses** of your model or approach.

The summary should only be a little over a half page, summarizing your paper, but it should be written from scratch with considerable efforts and not simply be snippets from the paper. Thus, composing a good summary takes tremendous effort and you should revise it a few times until you are satisfied with it.

The summary should get the reader interested in the paper and want to know the details. Make sure it is not a dry recitation of what's in the paper ("First we ... then we ...") and you should avoid a beginning of the sort "This paper will solve ...". In particular, you should come up with a attractive first sentence to grip the reader's attention. For example, in 2008, one of the problems was to create Sudoku puzzles. The summary of one of the Outstanding Winners, "Taking the Mystery Out of Sudoku Difficulty: An Oracular Model" [10], begins

> *In the last few years, the 9-by-9 puzzle grid known as Sudoku has gone from being a popular Japanese puzzle to a global craze.*

While the summary cannot, and should not, provide too many details, it should still clearly describe your approach to the problem. It should include all of the main points and big ideas, and be specific about the conclusion. It should include numerical results when appropriate, but equations should be avoided if possible.

Chapter 3 Style

You would want anyone reading your paper to notice your mathematical work and your model, not your writing. Thus, your writing should be as unobtrusive as possible. That does not mean it should not be dynamic. The reader should be smoothly led through your work, just without being distracted by awkward phrasing or stilted sentences.

While reading your paper, the reader should be thinking about the problem and your work on it. Any time that the reader needs to pause, such as for parsing your sentences or for recalling the facts brought up in the paper that had not been used for a while, is time taken away from thinking about your solution. Given the little time that the judges have to read your paper, this can prevent your paper from advancing.

MCM papers are math papers, and math papers have certain conventional styles. Judges are used to these styles and expect that your papers are written in such styles. Thus, learning to write in these styles can help you write a better paper. This chapter contains some suggestions of what to do and examples of mistakes to watch out for.

1 Write in the first person, plural

The papers submitted to the MCM, and math papers in general, are typically written in the first person plural; throughout the paper "We" see what is going on and "We" draw conclusions. The number of authors is irrelevant; the "we" does not refer to you and your coauthors but rather to you along with the reader. You are leading the reader through the paper. First person plural is certainly more friendly, which helps shorten the distance between you and your reader. Moreover, when describing things, using "we" as the subject of a sentence is very convenient. In the unlikely event that you need to refer to yourself specifically, you can write "the authors" or "one of the authors".

The convention of using the first person plural in mathematical writings may differ from what you are used to see from mathematics textbooks written in Chinese, and so you should pay special attention to not using the third or the second person in your MCM paper.

2 Use the present tense

Whatever is being done at the moment can reasonably be referred to as the present: "We [now] solve the equation.". The previous parts of the paper would then be the past: "We solved the equation in the previous section." The unread parts of the paper would naturally be the future: "We will solve the equation in the next section".

In other words, if we view writing a paper as writing an electronic file by moving a cursor from the beginning to the end, then while you're writing, to describe what happens at the current location of the cursor is the present, what happened before the cursor is the past, and what will happen after the cursor is the future. While using the present perfect may sometimes make sense for things occurred or done before the current location of the cursor, using the simple past is every bit simpler and cleaner in mathematical writings.

When surveying other people's work, what they did is the past, but the theorems and facts are the present. For example,

They showed that the problem is NP-complete.

3 Use the active voice

When describing the same thing or stating the same fact, one can use the active voice or the passive voice, but active voice is more direct and more powerful. Math has strong logic, but may also be dry, and so using active voice can make math contents clearer and more dynamic.

Recall, the active voice is where the subject of the sentence performs an action:

The model produced a desirable conclusion.

Here, the model does the producing.

The passive voice is where an action is performed on the subject:

A desirable conclusion was produced.

or

A desirable conclusion was produced by the model.

In these examples, the desirable conclusion is the result of the production, not the cause.

Chinese students tend to write math papers in the passive voice. This phenomenon may be an outcome of their early training in technical writings where the passive voice is often used. Whenever you have doubts, please remember: The active voice is more concise and more dynamic than the passive voice. You should write in the active voice whenever possible.

4 Use simple sentences

The reader of your paper should not have to slow down to parse your sentences or guess what you want to say. You should use sentences which are as simple as possible, such that one sentence only describes one idea. The verb should be close to the subject. Complicated sentences can often be broken into simpler sentences. For example, the sentence

Bad: The value of the parameter λ, which was used in the previous section to determine the height of the building, can also be used to determine its width.

can be rewritten

Good: In the previous section, we used the value of the parameter λ to determine the height of the building. We can also use λ to determine the width of the building.

5 Use short paragraphs

A paragraph is used to represent a bigger idea and describe a larger event, often consisting multiple sentences. The end of a paragraph gives the reader a chance to pause and think about the paragraph. If you use short paragraphs, it will be easier on the reader. On the other hand, you should also avoid extremely short paragraphs of one or two simple sentences. Too many extremely short paragraphs would cause the reader to pause too often, which would distract the reading flow.

6 Use words which are as specific as you can

You should avoid vague or abstract words as much as feasible; concrete words are easier on the reader. For example, instead of saying

Bad: Using the previous expression, we can conclude ...

you should probably say

Good: Using the previous equation, we can conclude ...

7 Do not include trivialities

Your paper should include enough details for the reader to follow what you are doing, but if you include too many trivial details then the reader could become bored or annoyed with your paper. Any routine computations, for example, should probably be omitted.

Exactly what constitutes triviality is a judgement call. Whenever you have doubts, please remember: Reviewers of your paper are experts. They want to learn your flow of thoughts as quickly as possible without being bogged down with trivialities.

8 Highlight important statements

You should make important statements stand out so that they will draw the attention of your readers. A single important statement should probably be put in bold font. Putting it at the beginning of a paragraph which explains the statement would probably help the reader the most. A word being defined should also be put in bold font.

If you have several related important statements, such as the assumptions you are using to create your model, they can be highlighted by putting them in a bulleted list. In many outstanding MCM papers, for example, the section "Assumptions" only consists of such a bulleted list.

9 Omit needless words

Many all too common constructions are needlessly complicated. For example,

> *Bad: The variable x is positive, due to the fact that it is a perfect square.*

should be simply

> *Good: The variable x is positive since it is a perfect square.*

As an another example, the following passage is extracted from the abstract of a paper; the abstract does not explain what it means by "working and intermediate nodes":

> *Bad: In this paper, taking energy consumption into consideration, we rethink the virtual-network embedding problem through (1) the rebuild of the network model by considering the power consumption of both the working and intermediate nodes; and (2) the design of an energy-aware heuristic and optimized virtual-network embedding algorithms under this new network and power model.*

It can be rewritten as

> *Good: We present an energy consumption model for the virtual-network embedding problem and devise an energy-aware virtual-network embedding algorithm to reduce power consumption.*

Whenever you see several words taking the place of a single word, you should use the single word instead. Many phrases using a noun ending in "ion" can be shortened. For example, instead of writing

> *Bad: We will now find the solutions of the following equation.*

you should write

> *Good: We will now solve the following equation.*

You will need to be careful of shortening a sentence too much, of course. While the sentence

> *Bad: We will assume $x > 0$.*

may seem like a suitably shortened version of

Good: We will assume that $x > 0$.

the "that" in this case helps the reader through the sentence. In general, if mathematical expressions are followed, the words "assume" and "suppose" should be followed by "that".

10 Use parallel phrasing to emphasize similarities

Similar phrasing should be used when there are similarities between objects; it makes the reading easier and emphasizes connections. For example, instead of writing

Bad: If a *is a root of the function* f*, then the function* g *has root* b.

you should write

Good: If a *is a root of the function* f*, then* b *is a root of the function* g.

or

Good: If the function f *has a root* a*, then the function* g *has a root* b.

11 Avoid pointless repetition

While similar phrasing should be used to emphasize similar objects, it otherwise should not be overused. Monotonous sentence structures will make your writing boring. For example, instead of writing

Bad: First we solved an equation. Then we solved another equation.

you should change the second sentence so it is distinct from the first. If appropriate, something like

Good: First we solved an equation. The solution of the first equation allowed us to solve the second equation.

would be an improvement.

Simply changing key words is not enough to avoid boring repetition. A paragraph that looks like

Bad: We know ... Hence, ... It follows that ....

will make the reader lose interest quickly. For example, the following sentences

Bad: We know that Equation A has a solution. Hence, Equation B has a solution. It follows that Equation C has a solution.

would probably bore or even annoy the reader. The following is an improvement:

Good: We know that Equation A has a solution, which implies that Equation B has a solution. Thus, Equation C also has a solution.

12 Avoid using the same word for different purposes

If you use the same word (or similar words) multiple times in different ways, it will look odd to the reader and they will need to pause to consider it. For example, instead of

Bad: We ran into several problems while trying to solve the problem.

you should write

Good: We ran into several obstacles while trying to solve the problem.

You should make sure you have a thesaurus handy to be able to come up with synonyms as appropriate.

13 Keep the antecedents of any pronouns clear

Any pronoun that you use refers to a previous noun. While a pronoun typically refers to the last noun before the pronoun, you should make sure that it is clear which noun is being referred to. For example, in the sentence

Good: The function is continuous; it is also nonnegative.

the pronoun "it" refers to the function. However, in the sentence

Bad: The function and its derivative are continuous; it is also nonnegative.

the first "it" (in "its") refers to the function, but it is not clear what the second "it" refers to; does it refer to the function or the derivative? In this case, the second pronoun should be avoided and the sentence should be rewritten. For example, if the second "it" refers to the function, then you may want to rewrite the sentence to something like

Good: The function and its derivative are continuous; the function is also nonnegative.

Likewise, if the second "it" refers to the derivative, then you may want to rewrite the sentence to

Good: The function and its derivative are continuous; the derivative is also nonnegative.

Whenever you write "it" or "this", be sure it is obvious what the "it" or "this" refers to.

14 Do not overstate your results

While you want to emphasize important results, you should not need to build them up too much. By inflating their importance, you open the possibility of disappointing the reader. Once the reader has a statement pointed out to them, you should leave it to them to judge how important it is. In particular, exclamation marks should be avoided.

Chapter 4 English Usage

You should be careful to use proper English throughout your paper. Even small mistakes will be noticed by some of the readers and, whether or not they regard it as important, once it is noticed it will be a speedbump. The more speedbumps there are, the longer it will take to read the paper and the less pleasant it will be.

Natural languages are organic, and the difference between a hard and fast rule and a suggestion is never clear. For just about any rule, you can probably find a good author who breaks that rule. However, the MCM contest is not the time to experiment with the language. Ignoring the standard rules will be noticed and it is possible that you will be violating a rule near and dear to the heart of one of the judges.

You should have a book on proper English usage, such as Fowler's *A Dictionary of Modern English Usage* [11], available, and consult it frequently. The rest of this chapter is devoted to mentioning some specific things to watch out for.

1 Make sure your subject and verb agree

If the subject and verb of a sentence disagree (both must be singular or both must be plural) it will stand out to the reader. Be sure to double check agreement whenever there is any doubt.

Making sure the subject and verb agree is usually straightforward when the subject is simple and close to the verb, although there are some cases which can be tricky. Ensuring agreement is less simple when there is a phrase between the subject and verb. In a sentence like

One of the solutions is positive.

even though several solutions are implied, the subject is the singular "One", and so gets a singular verb. In the sentences

The positive solution and the negative solutions are ...

The positive solution or the negative solutions are ...

the subjects are plural, however in the sentence

The positive solution, as well as the negative solutions, is ...

the subject ("The positive solution") is singular.

2 Use "that" and "which" correctly

While the importance of properly distinguishing between "that" and "which" depends on the reader, it is a topic in many books on good style and it is worth your trouble to get it right.

The word "that" is used to specify an object; in the sentence

The car that was blue went through the stop sign.

there may be several cars, but the phrase "that was blue" specifies the car under discussion. Without the phrase, the reader may not know what car went through the stop sign.

The word "which" is used to add information about an object; in the sentence

The car, which was blue, went through the stop sign.

the reader already knows what car is being discussed. The writer is adding information about the color of the car.

As rule of thumb, when deciding whether to use "that" or "which", use "that" whenever it sounds good.

3 Use correct spelling

Be sure to use a spellchecker on your paper. Since the MCM contest is based in the United States, you should have your spellchecker set to check for American English.

A spellchecker can only check if a word is spelled correctly, and it does not check to make sure you are using the correct word. If there is any doubt about the meaning of a word you are using be sure to look it up. Here are some words which are spelled similarly but have different meanings.

discrete *(adjective)*. Separate. "One of the MCM problems involves discrete mathematics."
discreet *(adjective)*. Respectful of privacy. "You should be discreet when discussing private matters."

principle *(noun)*. A fundamental rule. "One of the principles of good writing is to be concise."
principal *(adjective)*. Primary. "Poor writing is the principal reason that MCM papers do not make it past the first round."

lose *(verb)*. Fail to keep. "You will lose your book if you do not write your name in it."
loose *(adjective)*. Not tight. "You can untie a knot if it is loose enough."

affect *(verb)*. Influence. "If you change the value of a parameter, it will affect your model."
effect *(noun)*. Result of an action. "The effect of not studying for a test is a failing grade."

ensure *(verb)*. Make certain. "You can use a spellchecker to ensure there are no misspelled words."
insure *(verb)*. Put an insurance policy on. "You should insure any valuable jewelry."

its *(noun)*. Possessive form of "it". "A dog will chew its toys."
it's *(contraction)*. "It is". "If a dog is hungry then it's going to eat."

However, it is probably best not to use contractions in formal writings, particularly when contractions could cause confusion.

The correct way to write out "can't" is "cannot", not "can not". Thus, you should say

Good: He cannot complete the task in time.

or

Good: He can't complete the task in time.

but not

Bad: He can not complete the task in time.

4 Use unobtrusive pronouns

English does not have a third person singular indefinite pronoun. Historically, the male pronoun has been used, such as "he" in the sentence "If an observer touches the experiment, then he will affect the outcome of the experiment." There are people who object to using the male pronoun as the default and there have been several attempts to introduce a new set of pronouns for third person indefinite. None are commonly used.

One unobtrusive solution to this problem that has historical justification is to use "they" for singular as well as plural. You could write "If an observer touches the experiment, then they will affect the outcome." and "A student should be careful not to lose their books."

5 Use articles ("the", "a" and "an") properly

The proper use of articles is not a simple matter, particularly to the Chinese people, for the Chinese language does not require an article before a noun. As a result, Chinese students tend to omit articles when they should be there, or use "the" for almost any noun. If there is any doubt about whether you are using an article correctly, you should consult a book on English usage or look for a similar construction in a known piece of good writing.

If you are using an article before a noun, you should use the definite article "the" if the noun refers to a unique object, otherwise you should use the indefinite article "a" or "an". For example, if you are talking about a differentiable function, you can refer to "the derivative", since there is only one derivative, and "an antiderivative", since there is more than one antiderivative.

In the case of non-unique objects, you should use "a" if the word *sounds* like it begins with a consonant, otherwise you should use "an". For example, you could refer to "a norm", since "norm" begins with an "N" sound, but you would refer to "an ℓ_1 norm", since "ℓ" begins with an "E" sound. For nouns beginning with the "u" sound, you should think of it as "you" that begins with "y", and so "a" instead of "an" should be used. Thus, we write "a European company" instead of "an European company", and we write "a US team" instead of "an US team".

Whether or not you should use an article before a noun is a bit trickier, but there are a couple things to keep in mind.

You should use an article before a noun which represents a single object.

You would never say "Function is differentiable", you would say "The function is differentiable" or "A function is differentiable".

You should not use an article before a noun which is supposed to represent all objects of its type. If you want to note that (in general) the derivative of a function is not necessarily continuous, you might say "Derivatives are not necessarily continuous." However, if you want to discuss certain derivatives, such as the derivatives of a certain group of functions, you could say "The derivatives are not necessarily continuous."

6 Useful verbs

We provide a list of verbs that could come in handy when writing a paper on mathematical modeling. For example,

> *You may need to investigate the problem from different points of view, survey what has been done in the literature, explore different ideas, formulate and justify your assumptions, design a model, device an algorithm, carry out numerical simulations, and compare one approach with another one.*

Compare to and compare with When you intend to juxtapose two or more items to illustrate differences, similarities, or both, use "compare with". If the differences are important, you should definitely use "compared with". For example,

> *We compare our results with the existing ones. We show that our method is more robust and our algorithm is faster.*

When you intend to assert similarities, use "compare to". For example,

> *We can compare her model for loading a railroad car to his method for loading a ship cargo.*

Study and investigate The word "study" can be used both as a verb and as a noun. Thus, we can say

> *A 24-hour study to investigate this problem suggests a new direction.*

Similarly, we can also say

A 24-hour investigation to study this problem suggests a new direction.

Seek and explore These two verbs are useful when describing your thoughts. For example,

We seek to devise a new model for solving the problem by exploring the new direction suggested by their investigations.

Design and devise These two verbs are useful when describing your solution. For example,

Based on our analysis, we design a model for the problem using integral linear programming. We then devise a polynomial-time approximation algorithm to produce near-optimal results.

Tackle and solve These two verbs are useful when describing your accomplishment. For example,

We tackle the problem using the new technique we developed in the previous section. While it is difficult to solve the problem completely, we are able to solve a major subproblem.

Approach and propose These two verbs are useful when describing a proposal. For example,

We approach the problem using the proposed method.

Note that "approach" can also be a noun.

We propose a new approach to tackling the problem.

We can also say

We propose a new approach to tackle the problem.

Chapter 5 Revising

After you finish writing your paper, you will need to reread it, paying close attention to content and style. You should be prepared to make minor adjustments and also rewrite parts of it. You should probably have a style guide and an English usage guide handy while you do this.

While you're reading, you should

- look for passive sentences; make them active if at all possible.
- look for complicated words that can be replaced by simpler words.
- look for complicated sentences than can be replaced by simpler sentences, possibly by breaking up the original sentence into two or three smaller sentences.
- make sure that all of your assumptions are explicitly justified.
- make sure that you present all of your information in logical order.
- make sure that any parts of your paper that you took from other sources are properly credited.

You should begin with your summary, which is where the judges will begin. Read each word and each sentence carefully. Remove information that is irrelevant, and make sure that everything that is relevant is included in the summary in a logical order. Be prepared to restructure your summary if it helps.

Next, you should skim your paper, and make sure that the parts of your paper that you highlighted (the section titles, the displayed equations, the tables and figures, and sentences put in bold font) give the reader a good idea of what is happening in your paper. Make sure that irrelevant information is not highlighted.

Finally, go through the entire paper. Although you will not be able to reexamine your paper as carefully as you did the summary, you should look for the same things.

This chapter provides examples of changes to actual papers to show what sort of changes could be made.

1 Example: Revising a title

One of the problems in the 2010 contest was to explain the sweet spot on a baseball bat. One of the papers submitted for this problem was "Science in Sweet Spot" [12]. We use this paper to explain how to revise titles and summeriness.

The title of your paper should be carefully chosen. The title of this paper consists of the following three errors:

1. It uses an inappropriate preposition. Prepositions are tricky, and the preposition here should probably be "of", not "in".
2. Since "spot" refers to a Hnique point, it should get an article, and in this case there should be a "the" before "spot".
3. Science in general is not being studied in this paper, but the particular science that is associated with the sweet spot is. So it should be "the science".

The title should probably be "The Science of the Sweet Spot".

2 Example: Revising a summary

Just as the summary should take up a disproportionate amount of your writing time, it should also take up a disproportionate amount of your revising time. As well as stylistic considerations, effort should be made to shorten it without losing any impact.

As an example of revising a summary, consider the summary of the same paper "Science in Sweet Spot" [12], which begins:

> *This paper mainly studies the 'sweet spot' on a baseball bat. Firstly, by analyzing the video of a professional hitter, a double-pivot swing (the arm and the finesse) model is established to describe a batter hitting a baseball. Then based on the law of energy conservation, the double-pivot swing model is further transferred into a single-pivot swing model. With the given shape of the bat and the parameters for the single-pivot swing model, the moment of inertia of a wooden bat, corked bat and aluminum bat is calculated, respectively. By the law of kinetic energy conservation, hitting a baseball at different spots of the bat can be seen as an imperfect elastic collision of two balls. Thus given a bat, for different speeds of the coming ball and that of swinging the bat, the speed of the hitting ball can be calculated according to the Newton's collision law.*

Revising

1. *This paper mainly studies the "sweet spot" on a baseball bat.*
   A summary should not start off describing what "the paper" does; it leaves the authors and readers as outside observers. The paper does not do anything but describe what the authors did, the first sentence should begin with "We". While the authors may do other things besides study the sweet spot, the word "mainly" is superfluous and should be removed. Finally, "sweet spot" is a standard term for a particular part of the bat; unless the authors are suggesting that it is an inappropriate term, it should not be in quotes. The first sentence could be rewritten

   We study the sweet spot on a baseball bat.

   which is not only shorter, but also more dynamic. Compare this to the first sentence of "The Sweet Spot: A Wave Model of Baseball Bats", one of the Outstanding Winners:

   We determine the sweet spot on a baseball bat.

2. *Firstly, by analyzing the video of a professional hitter, a double-pivot swing (the arm and the finesse) model is established to describe a batter hitting a baseball.*
   This sentence is passive and should be made active. As written, the video is analyzed and the model is established, but the author and reader are left out. Notice that there are probably many videos of professional hitters, since no particular one has been established, the summary should refer (at least the first time) to "a video" rather than "the video". Finally, the word "firstly" is considered obsolete by many Americans. While this attitude is not universal, you would probably be better off by replacing "firstly" by "first" whenever it appears.
   This sentence could be replaced by

   We begin by establishing a model of a bat hitting a ball by analyzing a video of a professional hitter.

3. *Then based on the law of energy conservation, the double-pivot swing model is further transferred into a single-pivot swing model.*
   This sentence continues the passive tone of the summary. All of the passive

sentences should be rewritten to be active.

4. *With the given shape of the bat and the parameters for the single-pivot swing model, the moment of inertia of a wooden bat, corked bat and aluminum bat is calculated, respectively.*
This is the first mention of the shape of the bat, nothing about it has so far been given and it is not mentioned again in the summary. Similarly, "the" parameters have not been mentioned before and are not mentioned again in the summary. These should not be mentioned here. Since they are used to create the model, they can be replaced by "Using our model ...". There is also no need yet to divide the types of bats into wooden, corked and aluminum.

Recall, you should not just revise your paper line by line; you also need to be willing to change your paper on a larger scale. The point of the summary is that hitting a ball with a bat is equivalent to hitting a ball with another ball, which the authors later refer to as the "hitting ball", and so the summary could be replaced by

> *We study the sweet spot on a baseball bat. We begin by establishing a model of a bat hitting a ball by analyzing a video of a professional hitter. Using this model, we determine that hitting a ball with a bat can be seen as an imperfect collision of two balls, where the bat is replaced by a "hitting ball".*

3 Example: Revising an introduction

One of the problems in the 2008 contest involved developing "an algorithm to construct Sudoku puzzles of varying difficulty." One of the papers submitted for this problem was "The Solvers' Sudoku or computers' Sudoku" [13]. We first note that there are two errors in the title. First, the "C" in computer should be capitalized. Second, there should be a "the" before "Computer". Thus, the title should be "The Solver's Sudoku or the Computer's Sudoku".

The paper is described in its introduction.

> *Sudoku (Japanese: sūdoku) is derived from Switzerland, developed in America and carried forward in Japan. It is a simple game of logic and so easy to learn. It is also a fun and addictive game and is puzzling millions of players all over the world. Many people like it. Partial people began to study solving the puzzles by computer. However, generating Sudoku puz-*

zles is even more difficult when difficulty levels and a unique solution both are considered. It is also the task we are required to finish.

The aim is to create a reliable sudoku algorithm which has least complexity and a unique solution should be guaranteed. Difficulty levels are greatly depended on the metrics developed. The basic thought is that we delete digits from a solved puzzle, and then check out if there is a unique solution. Develop a series of metrics to define the the difficulty levels. Judge its difficulty level utilizing metrics.

Metrics are very important in the whole course of constructing puzzles. In our paper, we develop metrics fit for persons' thought and metrics aimed at computers' "thought" respectively. The former is subjective and the latter is impersonal.

Revising

1. *Sudoku (Japanese: sūdoku) is derived from Switzerland, developed in America and carried forward in Japan.*
   This is a catchy first sentence. However, the parts of the sentence are not parallel; "is derived", "developed" and "carried forward" do not all have the same structure. Also, saying that sudoku was "is derived from Switzerland" implies that the country itself was transformed into the game, the phrase "originated in" would be more accurate.
   If sudoku puzzles were relatively unknown, then the introduction should start with a sentence briefly stating what they are. The authors of this paper rightfully assume that sudoku puzzles are familiar to the readers.
2. *It is a simple game of logic and so easy to learn.*
   This sentence is simple and straightforward, but "so easy" is an idiom meaning "very easy". The authors probably want "so" to be used to indicate a conclusion; "hence easy to learn" or "so it is easy to learn" would be clearer.
3. *It is also a fun and addictive game and is puzzling millions of players all over the world. Many people like it.*
   Beginning too many sentences with "It is . . . " can be dull, and since the game is fun and played by millions, it is safe to assume that many people like it. The last sentence can be removed.
4. *Partial people began to study solving the puzzles by computer.*
   This is an unfortunate sentence; the thought of partial people doing anything is unsettling. It is probably a misedit from a draft of the paper. The authors

of the paper probably wanted to say that some people began to use computers to solve Sudoku puzzles.

5. *However, generating Sudoku puzzles is even more difficult when difficulty levels and a unique solution both are considered.*
   This does not flow from the previous sentence. "However" indicates that the difficulty of generating the puzzles is being contrasted with something, but it is not clear what that something is. There is also no indication what it is "even more difficult" than. Also, two similar words, such as "difficult" and "difficulty" should not be used close together.
6. *It is also the task we are required to finish.*
   This is passive and should be made active.
7. *The aim is to create a reliable sudoku algorithm which has least complexity and a unique solution should be guaranteed.*
   The authors should bring themselves into this sentence; it should begin "Our aim ...". The phrase "sudoku algorithm" is ambiguous; the reader can infer that the authors mean an algorithm to create the puzzles, but it would not hurt to mention that explicitly. The phrase "has least complexity" can be simplified to "is simplest". Finally, "a unique solution should be guaranteed" is passive and should be active.
8. *Difficulty levels are greatly depended on the metrics developed.*
   The phrase "depended on" should be "dependent on". This sentence is also the first mention of metrics, but it gives the impression that "the metrics" are already familiar to the reader. Either the authors should bring up metrics before this sentence, or make it clear they are introducing metrics here.
9. *The basic thought is that we delete digits from a solved puzzle, and then check out if there is a unique solution.*
   The authors did not indicate who is doing the thinking; the sentence would be better off starting with "Our basic thought". However, since they want to discuss what they are doing, "Our approach ..." would be a better start.
10. *Develop a series of metrics to define the difficulty level. Judge its difficulty level utilizing metrics.*
    These are not sentences. The authors probably intended the previous sentence to begin a list of items in their approach, but in that case the items should be in a bulleted list and they should be parallel. Here the items being listed are not parallel, and so they should be described separately with complete

sentences.

11. *Metrics are very important in the whole course of constructing puzzles.*
If something is important and it is not obvious to the reader, the authors should say why it is important. A statement of the form "X is important" is typically unnecessary and should be removed.

12. *In our paper, we develop metrics fit for persons' thought and metrics aimed at computers' "thought" respectively.*
The introduction is about the paper; the phrase "In our paper" is unnecessary and should be removed. Since two lists are not being compared, the sentence should not end with "respectively". Finally, the metrics should work for "a person's thoughts" and "a computer's 'thoughts'".
Based on the discussions given above, this introduction could be rewritten as

Sudoku, originated in Switzerland, was developed in America and popularized in Japan. It is easy to learn, and millions of players enjoy being puzzled by it. While some players have created algorithms to solve sudoku puzzles, our task is to create an algorithm to generate sudoku puzzles of various difficulty levels with unique solutions.

Since our task involves measuring the difficulty of the puzzles our algorithm generates, we develop a series of metrics to define and measure the difficulty. We create metrics to measure the difficulty of a puzzle for a person and metrics to measure the difficulty of a puzzle for a computer. Our approach is to

- *start with an already solved puzzle;*
- *delete some digits;*
- *check to see if there is a unique solution;*
- *measure the difficulty level.*

4 Example: Revising assumptions

One of the problems in the 2011 contest involved comparing conventional and electric cars. One of the papers submitted for this problem was "Can Electric Vehicle Be Widely Used" [14]. Since the paper is about more than one electric vehicle, the title should probably be "Can Electric Vehicles Be Widely Used". If by "electric vehicle" the authors mean the entire class of such vehicles, then the title should be "Can the Electric Vehicle Be Widely Used."

The paper discusses three types of vehicles: conventional vehicles (CV), electric vehicles (EV) and hybrid-electric vehicles (HEV). The second section of the paper is a list of the assumptions the authors will use, given nicely as an enumerated list.

1. *We will select one vehicle mode to represent CV, EV and HEV.*
2. *We assume there is no difference in performance of each vehicle type.*
3. *We select France, USA, and China to represent European countries, American countries and Asian countries.*
4. *We assume performance of CV, EV and HEV will not change in the future.*
5. *Based on common sense, we assume each vehicle travels 10,000 km every year.*
6. *We will not specify different types of power plants and assume that each plant will generate 10,000,000 kwh annually on average.*
7. *We do not consider the dissipation during energy conversion.*

The assumptions here, whether reasonable or not, are not given explicit justifications.

Revising

1. *We will select one vehicle mode to represent CV, EV and HEV.*
   The authors mean "vehicle model", not "vehicle mode". The way this assumption is phrased, it sounds as if the authors are choosing a single model to represent three different types of vehicles; they mean to say that each type of vehicle will be represented by a different model. Finally, choosing one vehicle to represent a class of cars is not an assumption, but part of the authors' approach to the problem.

2. *We assume there is no difference in performance of each vehicle type.*
   "Performance" could mean a lot of things; what type of performance is being considered should be mentioned explicitly. Also, while the authors mean that all conventional vehicles perform similarly, all electric vehicles perform similarly, and all hybrid-electric vehicles perform similarly, the statement they make could be interpreted as meaning that conventional vehicles perform similarly to electric vehicles, and both of those perform similarly to hybrid-electric vehicles. Neither interpretation of this assumption is reasonable, however, and either one would require an explanation of why the authors are making the assumption. Here, the assumptions are purely for simplifying the problem and that should be stated explicitly; if the authors have evidence that their assumption is reasonable they should provide it.

3. *We select France, USA, and China to represent European countries, American countries and Asian countries.*
   The authors do not mean that all of the countries listed represent all of the continents listed, they mean that each country listed represents the corresponding continent. The second list should end with "respectively" to indicate this. Still, this item is not an assumption but rather the way the authors are approaching the problem. Their assumption would be that these choices are reasonable. If there is evidence that the countries are typical of their continents, that should be stated with the assumption.
4. *We assume performance of CV, EV and HEV will not change in the future.*
   Given the pace of technology, this is probably wrong. It may be a necessary simplifying assumption, but that reason should be stated explicitly.
5. *Based on common sense, we assume each vehicle travels 10,000 km every year.*
   You cannot conclude how far each type of car will travel per year from common sense; the authors should either state this as a simplifying assumption or they should provide data to support this distance.
6. *We will not specify different types of power plants and assume that each plant will generate 10,000,000 kwh annually on average.*
   They should provide data to support this assumption.
7. *We do not consider the dissipation during energy conversion.*
   This is not stated as an assumption; it should be "There is no dissipation during energy conversion." While not a reasonable assumption, it should be stated as a necessary simplifying assumption and should be mentioned when they discuss weaknesses of their model.

The section on assumptions for this paper could be rewritten to something like:

> *We will choose the 2010 Ford Focus 1.4 Duratec to represent conventional vehicles, the Tesla Roadster to represent electric vehicles, and the Chevrolet Volt to represent hybrid-electric vehicles. We will also choose France as a representative European country, the USA as a representative American country, and China as a representative Asian country.*
>
> *We will make the following simplifying assumptions.*
>
> 1. *Each car we chose is typical of its class.*
> 2. *Vehicle performance will not change in the future.*
> 3. *Each vehicle will travel 10,000 km per year.*

4. *Each country we chose is typical of its continent.*
5. *Each power plant will generate an average of 10,000,000 kwh per year.*
6. *There is no energy dissipation during energy conversion.*

The authors should also clarify what they mean by "performance". If the authors could have gotten data to support any simplifying assumptions, that assumption should be mentioned in a different list with the supporting information. Finally, the simplifying assumptions they make should be mentioned as weaknesses when they discuss the strengths and weaknesses of their model.

Chapter 6 Symbols and Figures

Any paper involving a substantial amount of mathematics, such as your MCM paper, will need to use mathematical symbols. Moreover, papers on mathematical modeling may also use figures and tables. Symbols should be used sparingly, however; they should only be used when necessary. This chapter discusses some conventional rules and suggestions for using mathematical symbols, figures and tables.

1 Typography

Good typography can be important in making sure that the paper is easy to read. The font size should be a comfortable size, you should use 11 or 12 point base font. Since font changes can be distracting, you should probably stick with one serifed font for text (as well as a font for math). The style and size of the font can change in limited ways; bold for emphasis, large for section headings, italic for book titles, etc., but otherwise the font should be fixed. For example, it is conventional to use the Times New Roman for text, and its italic form for math. The section and subsection headings should stand out distinctly from the text of the paper; a brief flipping through the pages should allow easy perusing of the section titles.

You should make sure that whatever software you use to typeset your paper has enough math symbols. For example, you should not have to use the Greek letter ϵ for set inclusion; you should be able to write $x \in X$ and not $x \epsilon X$.

2 Avoid using symbols in titles

Since titles are short, there is no way to define symbols in a title. Thus, using symbols in a title may puzzle the reader, unless the meanings of the symbols are well understood by the general audience.

3 Stick with notational and typographical conventions

Mathematics involves a lot of conventions which are not formal rules but still make it easier for the reader to follow what you are doing. For example, mathematical symbols should be typeset in italics. Thus, instead of

> *Bad: It requires that* I *be a constant for ... to be true.*

you should say

> *Good: It requires that I be a constant for ... to be true.*

Otherwise, the sentence could cause confusion that the author who wrote the sentence is required to be a constant. "I" here is a mathematical symbol; although it may not be the best choice for a symbol, writing it in italics will eliminate any possible confusion. For another example, matrices are usually represented by capitals in math bold (or italic capital in bold) and the entries of a matrix are usually represented by the corresponding lower case letter in italics. If you referred to the matrix a and the entry A_{ij}, for example, it would give the reader reason to pause; you should be using the matrix A and the entry a_{ij}. Similarly, the Greek letter ϵ typically represents a small positive number. Using ϵ to represent a large number would make the reader look twice and interfere with the flow of the reading.

Be sure to keep in mind that mathematical symbols are case sensitive, and even emphasis sensitive. The symbols X, $\mathbf{X}$, x and $\mathbf{x}$ are all different. In math mode, standard typographical conventions (such as upper case for sets, lower case for elements, bold for vectors and matrices) should be used to help the reader follow your work. While most functions and variable are typeset in math italics, such as $f(x)$, standard math functions are typeset in upright Roman, such as $\sin(x)$ and $\max\{a, b, c\}$.

4 Do not reuse symbols

Each symbol in your paper should be used for only one purpose. Do not use Δ to represent a matrix in one part of your paper and then use it for the discriminant of a polynomial in another part. Similarly, if you use a sigma notation for a sum, then you should not use Σ as a variable anywhere else in your paper. If you are using complex numbers in your paper, you should avoid using i as an index variable.

5 Do not use symbols for words

Since you are not writing a paper on symbolic logic, you should not write like you are. Instead of writing

Bad: $x = 2 \Rightarrow x^2 - 4 = 0$

you should write something like

Good: Since $x = 2$*, we know* $x^2 - 4 = 0$.

Likewise, you should write out "for all" and "there exists" and not use the symbols $\forall$ and $\exists$.

Similarly, you should not use symbols to take the place of words in an otherwise symbol-free sentence. You should never write something like

Bad: The two functions are $=$.

it should always be

Good: The two functions are equal.

6 Spell out numbers that are used as adjectives

Switching numerals to words can slow down reading, so if a number represents an adjective it should be written out. Instead of

Bad: There are 3 solutions.

you should write

Good: There are three solutions.

This does not apply to numbers being used as parts of units; in the sentences

Good: The rod is 5 feet long.

Good: It is a 5-foot-long rod.

the "5" should be left as a numeral.

7 Avoid unnecessary subscripts

Whenever possible, use x and y instead of x_1 and x_2. For example, it is often tempting to let $x_1, x_2 \in X$ when mentioning two elements of a set X, but unless the actual subscripts are used, you should let $x, y \in X$.

You should also avoid inconsistency in your use of subscripts. For example, if you are using x_1 and x_2, then a linear combination should be written $a_1x_1 + a_2x_2$ rather than $ax_1 + bx_2$. Likewise, if you are using x and y, then a linear combination should be written $ax + by$ rather than $a_1x + a_2y$.

8 Be consistent with your notation and terminology

Even though the expressions x_i for $i = 1, 2, 3, \ldots$ and x_j for $j = 1, 2, 3, \ldots$ are the same sequence, using different letters for the subscripts make them look different. You should consistently use the same letter for the same subscripts.

The use of inconsistent terminology often go unnoticed but can be puzzling to the reader. For example, do not say "Cholesky factorization" in one part and then "Cholesky decomposition" in another part; do not refer to $ker(A)$ as the null-space or $null(A)$ as the kernel.

9 Keep subscripts in the same order

When repeating pairs of subscripts, you should try to keep them in the same order. For example, when describing a decreasing sequence you should avoid

Bad: $x_i < x_j$ *when* $j < i$.

instead you should say

Good: $x_i < x_j$ *when* $i > j$.

The first sentence, while mathematically correct, looks awkward, which may cause the reader to pause.

10 Remove symbols that are used only once

If a symbol appears only once, there is no reason for it to appear. If you write

Bad: A differentiable function f is continuous.

the f is typically superfluous; you should write

Good: A differentiable function is continuous.

11 Figures and tables

Displaying figures and tables is an efficient way to quickly and clearly express data. Most people grasp information visually easier than textually. Many MCM papers use graphs to compare the results of their models; the reader can easily see the effect of changing the parameters.

For a small data set, tables are a better way of showing the information. For larger data sets, some type of graph would be more appropriate, but care should be taken to make the graphs as simple as possible while still getting across the desired information. A simple figure will help the reader through your paper, a complicated figure can slow the reader down.

Many MCM papers include figures but fail to discuss them in the paper itself. The judges view this unfavorably. While the reader may be able to determine the relevance of the figure, you should not make the reader work at it. The figure, as well as the table, should have a label, such as "Figure 2.1" or "Table 2.3", so that you can refer to it when you discuss it in the main text of your paper.

Each figure should have a caption; the caption should describe the figure so that even someone who has not read the paper, or someone who is simply skimming the paper, will understand what the figure represents. The figure itself should also have relevant labels; graphs should have the axes labeled with what information they represent and what information is referred to in the paper, legends should be used when appropriate, and tables should have the rows and columns labeled.

Chapter 7 Math and Sentences

In any paper involving mathematics, the math should flow smoothly through the narrative. Your paper should be written in sentences, including the mathematics. Thus, in addition to using symbols, figures, and tables appropriately, you must also make the math and the narrative coherent, and you must make sure that they are clearly distinguishable using typography. We will discuss how to write coherent sentences that contain math.

1 Do not begin a sentence with a symbol

A sentence beginning with a symbol looks awkward. Instead of writing

> *Bad: f is differentiable.*

you should put a description before the symbol f,

> *Good: The function f is differentiable.*

If the reader has not seen the symbol f in a while, a reminder could be helpful,

> *Good: The exponential function f is differentiable.*

2 Make sure the same symbol is always read the same

If you use a symbol twice in the same sentence, you should be able to read it the same way both time. For example, the sentence

> *Bad: For $x = 2$ we know that $x^2 = 4$.*

is read "For x equal to two, we know that x squared is equal to 4." The symbol $=$ has two different readings in the same sentence; first it is read as "equal to" and then it is read as "is equal to". This sentence should be

Good: If $x = 2$ *then* $x^2 = 4$.

which is read "If x is equal to 2 then x squared is equal to 4", and so the symbol $=$ is consistently read as "is equal to".

3 Punctuation and coherence

Any mathematical expressions that are part of the paper are part of a sentence and should be punctuated appropriately. The paper should sound good when read out loud; in fact, you should probably read it out loud after you have written it. This is an effective method to find out whether your writing is coherent.

A sentence like "If $x > 0$ then $f(x)$ is differentiable" would be read "If x is greater than zero, then f of x is differentiable." Note that an equation or inequality can be a noun in a sentence or the subject, verb, and object in a sentence. The sentence "The equation $x^2 = 1$ has two solutions" is read aloud as "The equation x squared equal to one has two solutions", where the equation $x^2 = 1$ is a noun. The sentence "If $x = 1$ then $x^2 = 1$" is read aloud as "If x is equal to one then x squared is equal to one", where the equation $x^2 = 1$ consists of a subject (x^2), verb ($=$), and object (1).

4 Separate different formulas with words

Two symbols from different expressions should be separated by words, in addition to punctuation. For example, instead of

Bad: Let x_i, $i = 1, 2, 3, \ldots$ *be a sequence.*

you should write

Good: Let x_i *for* $i = 1, 2, 3, \ldots$ *be a sequence.*

or

Good: Let x_i *be a sequence, where* $i = 1, 2, 3, \ldots$.

Instead of

Bad: Since $a = 2$, $a^2 = 4$.

you should write

Good: Since $a = 2$*, we have* $a^2 = 4$.

5 Use simple English

Even though your paper will involve a lot of mathematics, it should primarily be written in simple English. Nobody, including mathematicians, like to read jargon-heavy papers. Some technical terms will be necessary, but they should be kept to a minimum.

6 Do not overload math terms

Rule 12 in Chapter 3 says that we should void using the same word for different purposes. This rule also applies to mathematical terms. For example, the term "element" may often be overloaded in mathematical writing. If $\mathbf{x} = (x, y, z)$ is a vector in $\mathbb{R}^3$, then x is sometimes referred to as an element of $\mathbf{x}$. But then x is called an element of $\mathbf{x}$ and $\mathbf{x}$ is an element of $\mathbb{R}^3$. The word "element" means two different things here; it would be better to use two different words. In this case, x could be called a *component* of $\mathbf{x}$, for example.

7 Every "if" should have a "then"

Too often, the "then" in an "if ... then" statement is left unstated, the conclusion of the statement is implied. The conclusion of such a statement is sometimes easy to imply and sometimes hard. For example, the conclusion of the following statement is easy to imply.

Bad: If it rains, the grass will get wet.

It is better to make sure that your conclusions are explicit, particularly in math papers.

Good: If it rains, then the grass will get wet.

But the following statement

Bad: If $x = 1$*,* $y = 2$*,* $x + y = 3$

is poor not only because the equations are not separated by words, but the hypotheses are not clearly separated from the conclusion. The statement should be

> *Good: If* $x = 1$ *and* $y = 2$*, then* $x + y = 3$.

Otherwise, the sentence may also be interpreted as "If $x = 1$, then $y = 2$ and $x + y = 3$."

You should always make sure that your conclusions are explicit. Simply including a "then" for every "if" will do it.

8 Give the reader useful reminders

Try not to make the reader tax their memory or reread previous parts of your paper. You will base a lot of your work on the assumptions you make at the beginning of your paper; whenever you use one of your assumptions be sure to tell the reader which one you are using. For example, instead of writing

> *Bad: By our assumptions, we can conclude that* f *is continuous.*

you should write

> *Good: Since we assumed that the density function is continuous, we can conclude that* f *is continuous.*

Recalling that symbols should be given reminders themselves, you should actually write

> *Good: Since we assumed that the density function is continuous, we can conclude that the function* f *is continuous.*

or, better yet,

> *Good: Since we assumed that the density function is continuous, we can conclude that the mass function* f *is also continuous.*

When you refer to a variable that has already been defined, you should remind the reader what sort of object it represents. Instead of writing

> *Bad: Hence* A *is invertible.*

you should write

Good: Hence the matrix A is invertible.

9 Define terms right before you use them

The reader of your paper should not have to deal with several new terms at one time. There should not be a paragraph with several definitions in it, instead new terms should be defined right before they are used. For example, instead of writing

> *Bad: A function is* ***smooth*** *if it infinitely differentiable. A function is* $\mathbf{C}^1$ *if it has a continuous derivative. A function is ...*

You should write

> *Good: A function is* ***smooth*** *if it is infinitely differentiable. Suppose that* f *is a smooth function.*

New terms should also be in bold font when they are first defined.

You should only define a term if it is used more than once; if the only reason you define "smooth" is to mention that the function f is smooth, you should skip the definition and write

> *Good: Suppose that the function* f *is infinitely differentiable.*

10 Use displayed equations appropriately

Recall that a mathematical expression in a paper can be inline, where it simply appears on a line with the rest of the text, or displayed, where it appears on its own line, usually separated from the surrounding text by some vertical space. The equation $x^2 - 1 = 0$ is inline and the equation

$$x^2 - 1 = 0$$

is displayed. In both cases the expression is part of a sentence and needs to be treated as such, including appropriate punctuation.

A displayed equation will stand out from the rest of the text, and so should be used when you want to emphasize it. A displayed equation should only be numbered when it will be referred to later in the paper.

Mathematical expressions, inline or displayed, are parts of sentences and should be punctuated appropriately. The text before the expression normally does not get any special punctuation, and you should not put a colon there. Thus, instead of writing

> *Bad: Thus, a simpler form of double Pareto distribution density function is:*
>
> $$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \leq 1 \\ x^{-\alpha-1}, & x \geq 1 \end{cases}$$
>
> *for some* $\alpha > 0$ *and* $\beta > 0$.

You should write

> *Good: Thus, a simpler form of double Pareto distribution density function is*
>
> $$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \leq 1 \\ x^{-\alpha-1}, & x \geq 1 \end{cases}$$
>
> *for some* $\alpha > 0$ *and* $\beta > 0$.

But if you use the word "following", "below", or the phrase "as follows", then do use a colon to separate the text from the expression. For example, write

> *Good: Thus, a simpler form of double Pareto distribution density function is as follows:*
>
> $$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \leq 1 \\ x^{-\alpha-1}, & x \geq 1 \end{cases}$$
>
> *for some* $\alpha > 0$ *and* $\beta > 0$.

11 Make your fractions readable

Fractions can be written with a slash (x/y) or a horizontal bar ($\frac{x}{y}$). For inline mathematics, and other places where vertical space is limited (such as in subscripts and superscripts), writing fractions with a slash is probably better. If a fraction is somewhat complicated, however, using a slash might make the fraction hard to read, in which case a horizontal line would be better.

12 Watch your breaks and alignment in expressions

An inline expression should not be broken across lines. If an expression is too long, then display it. If a displayed expression is broken across lines, the line break should occur at a low precedence binary operation; break the expression at a plus or minus if possible. Often the binary operation appears on the second line when this is done, such as

$$\begin{gathered} abcdef + ghijkl + mnopqr + stuvwx \\ +yz. \end{gathered}$$

There should not be a page break in the middle of a displayed formula. If a math expression is too long and needs to go over to the next page, then you should use appropriate words to separate the expression at the end of the current page, and continue to write the rest of the expression in the next page.

If a displayed expression consists of a series of equalities, then the equalities should line up at the equal signs; the result should look like

$$\begin{aligned} a + b &= c + d, \\ e &= f + g + h, \\ i + j &= k. \end{aligned}$$

Similarly, a string of equalities should line up at the equal signs, such as

$$\begin{aligned} a + b &= c + d \\ &= e + f + g \\ &= h. \end{aligned}$$

13 Align your ellipsis properly

If the middle part of a sequence is left out, it is replaced by a symbol consisting of three dots, called an ellipsis. The ellipsis should be aligned along the baseline, as in

$$x_1, x_2, \ldots, x_n.$$

There should be commas on either side of the ellipsis. If the middle part of a sum is left off, as in

$$x_1 + x_2 + \cdots + x_n,$$

then the ellipsis should be centered, and there should be a plus sign on both sides. The same rule apples to other operations.

Chapter 8 LaTeX

LaTeX is a commonly used math formatting program. Your paper needs to be submitted both in paper form and electronic form. The electronic version is required to be in either Adobe PDF format or MS Word. LaTeX can produce nice-looking output and many MCM coaches recommend using LaTeX to produce PDF.

LaTeX is not a WYSIWYG (What You See Is What You Get) word processor, like MS Word; it's more of a markup language, like HTML. LaTeX is very configurable, but you should not worry about that while you are working on your MCM paper. You should just use the basics of LaTeX on your MCM paper and save any fancier typography for some other occasion. This chapter provides you with a very short introduction to LaTeX. It covers just enough of the mechanics to allow you to write an MCM paper but omitting some commands and options you probably will not need. If you want to learn more about LaTeX, a good place to start is *The Not So Short Introduction to LaTeX* 2_ε [15].

1 Compiling

When typing a LaTeX document, you should use a text editor, not a word processor like MS Word. If you insist on using a word processor, be sure to save your work as a plain text file. There are editors with extra features which will help you work with LaTeX and there are programs which allow you to work with LaTeX in a WYSIWYG manner, but this chapter will discuss how to work with LaTeX without these extra features. The authors recommend CTeX to the Chinese users, which can be downloaded free at `http://www.ctex.org` and includes a text editor WinEdt.

Once you have a document typed in, you should have it as a file with an extension of `.tex`, such as `mydocument.tex`. Then you need to compile it. If you have LaTeX installed properly you should be able to compile your file into a PDF document with the command

```
> pdflatex mydocument.tex
```

or on the tool bar of WinEdt, select option `TeX`, then `PDF`, and then `PDFLaTeX`. Compiling a LaTeX file is often called **latexing** it. (You may also use the `latex` command to compile `mydocument.tex` and generate a DVI file, and then produce a PDF file from it.) If your document uses any of LaTeX's automatic referencing features, you will need to latex it twice to get the references correct.

LaTeX commands typically start with a backslash (\) and are often followed by arguments enclosed in braces ({}).

When compiling your LaTeX file, you will probably see messages on the screen, and if there are no syntactic errors you will end up with a file `mydocument.pdf`. To view the result, you will need to use a PDF viewer; make sure you know the name of the viewer on your computer. You can then view your formatted document with a command like

```
> pdfviewer mydocument.pdf
```

If there are typesetting errors, they will need to be fixed before you get a PDF file. Errors are usually misspelled command names or unbalanced braces. For example, if you are supposed to type

```
\command{arg}
```

and you accidentally type

```
\commnd{arg}
```

or

```
\command{arg
```

you will get an error.

Working on a LaTeX document involves a typing, latexing, and previewing cycle. Repeat this cycle a few times to produce a satisfactory PDF file.

2 A very basic document

A very simple "Hello World" document would look like

```
\documentclass[12pt]{article}
\begin{document}
Hello, World!
\end{document}
```

If you save this as a file `mydocument.tex` and latex it, you will get a PDF file with the line "Hello, World!" near the top and the page number "1" at the bottom.

The first line,

```
\documentclass[12pt]{article}
```

states what kind of document is being created. In the above example, an article is being written; there are other options, such as "book", but for the MCM you will want to write an article. The `[12pt]` between the `\documentclass` and `{article}` is optional; without it the PDF file will have 10 point font size, with this option the file will have 12 point font size. For an MCM paper, you should use 11 point or 12 point font.

The space between

```
\documentclass[12pt]{article}
```

and

```
\begin{document}
```

is called the **preamble**. In this example the preamble is empty, but later you will see some useful commands to put in the preamble.

The actual text of the paper goes between the lines

```
\begin{document}
```

and

```
\end{document}
```

Ordinary text gets typed here, augmented by commands and environments. A command is typically given by a backslash followed by some text, such as `\ldots`. This command will insert a three dot ellipsis which is used to indicated that part of a sequence is omitted.

Input
`a, b, c, \ldots , z.`
Output
a, b, c, $\ldots$, z.

Some commands require an argument, which will be given between two braces ({}) following the command, such as `\textbf{text}`. This command will cause **`text`** to be typeset in bold face font.

Input
`This is \textbf{bold text}.`
Output
This is **bold text**.

An environment is the text between

`\begin{environment}`

and

`\end{environment}`

An environment will affect the enclosed text in some way. For example, the `center` environment will center the text.

Input

```
The following text
\begin{center}
is centered.
\end{center}
```

Output

The following text

is centered.

When you type your document, LaTeX will do the formatting. The spacing is taken care of by LaTeX, if you type several spaces (and in LaTeX, spaces and tabs are treated the same) between words, LaTeX will still space the words as it deems fit.

Input

```
Hello World!


Hello           World!
```

```
Hello
    World!
```

Output

Hello World!
Hello World!
Hello World!

Note that starting a new line in the input file does not start a new paragraph in the output file. To start a new paragraph, you will need to insert a blank line. If you want to start a new line without a new paragraph, you can end a line with \\.

Input

```
Here is the first sentence of a paragraph. Here is the
second sentence of a paragraph.\\
Here is the third sentence of a paragraph.

Here is the first sentence of a new paragraph.
```

Output

Here is the first sentence of a paragraph. Here is the second sentence of a paragraph.
Here is the third sentence of a paragraph.

Here is the first sentence of a new paragraph.

Any line beginning with a percent sign (%) is a *comment* line and is ignored by LaTeX.

Input

```
Hello world.
% Goodbye World.
Hello again.
```

Output

Hello world. Hello again.

You can doublespace your paper, if needed, by putting

`\usepackage[doublespacing]{setspace}`

in the preamble of your document.

You can also set different line space. For example, if you want space of 1.5 lines, you may do so by putting

`\renewcommand{\baselinestrech}{1.5}`

in the preamble of your document

3 Characters

LaTeX can read any of the 128 ASCII characters. The number of characters you can type into a LaTeX document can be increased to include non-ASCII characters by using the `inputenc` package; you can use the package by inserting the line
`\usepackage[utf8]{inputenc}`
into the preamble of your document.

With the `inputenc` package, you can insert accented characters into your paper; LaTeX also provides a way to do that with the usual keyboard characters. For example, you can insert an umlaut over a character by preceding it with `\"`; `\"a` will result in "ä". Table 8.1 gives some more of the accents that LaTeX can provide.

Table 8.1 Accents.

Input	Output	Input	Output	Input	Output
`` \`a ``	à	`\'a`	á	`\"a`	ä
`\~a`	ã	`\^a`	â	`\c{c}`	ç

Most characters that you put into a LaTeX document will be typeset, but the characters `\`, `#`, `$`, `%`, `^`, `&`, `_`, `{`, `}` and `~` have special meanings. If you want to have these characters show up in your document you will have to use special commands. The backslash can be entered with the command `\textbackslash`, the rest can be entered by using a backslash prefix.

Input
`\textbackslash \# \$ \% \^{} \& \_ \{ \} \~{}`
Output
\ # $ % ^ & _ { } ~

The empty braces ({}) are necessary in some cases to prevent unwanted effects after the character.

You have already seen that the command `\ldots` can be used to insert an ellipsis into your document. Commands for some other special characters you can insert are given in Table 8.2. If you want to use the euro character, you need to have

```
\usepackage{textcomp}
```

in the preamble of your document. Note that the quote character " does not distinguish between left and right; LaTeX uses `` for left quotation marks and '' for right quotation marks.

Input

```
``Hello'' looks better than "Hello".
```

Output

“Hello” looks better than ”Hello”.

You have seen that the command `\textbf{text}` will typeset `text` in bold face. Similarly, `\textit{text}` will typeset `text` in italics and `\texttt{text}` will typeset `text` in a monospace typewriter font. If you want to insert a large block of text in monospace typewriter font, such as when you are inserting some computer code, you can use the `verbatim` environment. This will also inhibit the meanings of any special characters, so they can be inserted without commands.

Table 8.2 Characters.

Input	Output	Input	Output	Input	Output
\ldots	...	``	“	''	”
\textbackslash	\	\#	#	\$	$
\%	%	\^{}	^	\&	&
_{}	_	\{	{	\}	}
\texteuro	€	\~{}	~	$\sim$	∼

Input

```
To use the Euro symbol, you will want to have
\begin{verbatim}
\usepackage{textcomp}
\end{verbatim} in the preamble of your document.
```

Output

To use the Euro symbol, you will want to have
`\usepackage{textcomp}`
in the preamble of your document.

4 Mathematics

LaTeX was created to typeset mathematics well, and your MCM paper will necessarily include a lot of mathematics. The **amsmath** and **amssymb** additions to LaTeX help with the mathematics, so you should add
`\usepackage{amsmath, amssymb}`
to the preamble of your document.

Simple mathematics

Any mathematics in your document must be entered in *math mode.* For inline mathematics, math mode takes place between dollar signs. For displayed mathematics, math mode takes place between `\[` and `\]` , or between $$ and $$.

Input

```
\[\sum_{k=1}^n k = \frac{n(n+1)}{2}\] and
$$ \sum_{k=1}^n k = \frac{n(n+1)}{2}$$
have the same displaying effect.
```

Output

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

and

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

have the same displaying effect.

More complex displaying methods will be mentioned later.

Letters in math mode are typeset in math italics, and so any mathematics, even a single variable, should be entered in math mode.

Input
`You should refer to the variable $x$,` `not the variable x.`
Output
You should refer to the variable x, not the variable x.

If you want to use a bold math symbol, you should use the `\mathbf{math}` command.

Input
`Consider the vector $\mathbf{x} = (x,y,z)$.`
Output
Consider the vector $\mathbf{x} = (x, y, z)$.

Simple formulas and equations can be entered using keyboard characters:

Input
`If $a+b=c$ and $a>0$, then $b<c$.`
Output
If $a + b = c$ and $a > 0$, then $b < c$.

While basic functions are also typeset in math italics, named functions, such as sin and log, are typeset in upright Roman font. These functions have commands which are used to enter them in math mode; the commands are typically a backslash followed by the function name.

Input
`The function $f(x)$ is in math italic,` `but $\sin(x)$ is not.`
Output
The function $f(x)$ is in math italic, but $\sin(x)$ is not.

There are many characters used in math that are not available on a typical keyboard. LaTeX provides commands to enter these characters. A partial list of such commands and binary operators is given in Table 8.3. The `\circ` symbol can be used as a superscript to indicate degrees, 90°.

To insert a Greek letter in math mode, you can use the command beginning with a backslash followed by the spelling of the Greek letter. If the spelling is capitalized, then the result will be the capital Greek letter.

Input
`From $\alpha$ to $\Omega$.`
Output
From α to Ω.

Some capital Greek letters, like capital alpha and capital beta, are identical to English letters. In this case, LaTeX does not have a command for this Greek letter, the English equivalent must be used.

Input
`Capital alpha is $A$; capital delta is $\Delta$.`
Output
Capital alpha is A; capital delta is Δ.

Table 8.3 Common math symbols.

Input	Output	Input	Output
`\le`	$\le$	`\ge`	$\ge$
`\subset`	$\subset$	`\supset`	$\supset$
`\subseteq`	$\subseteq$	`\supseteq`	$\supseteq$
`\in`	$\in$	`\notin`	$\notin$
`=`	$=$	`\neq`	$\neq$
`\pm`	$\pm$	`\div`	$\div$
`\cdot`	$\cdot$	`\times`	$\times$
`\to`	$\to$	`\mapsto`	$\mapsto$
`\partial`	∂	`\infty`	∞
`\circ`	$\circ$	`\vdots`	$\vdots$
`\ldots`	$\ldots$	`\cdots`	$\cdots$
`\ddots`	$\ddots$		

Mathematical symbols (including Greek and Hebrew letters) supported by the **amssymb** package can be found at the following web page: **http://amath.colorado.edu/documentation/LaTeX/Symbols.pdf**.

Delimiters

You can group expressions together in math mode using parentheses. When you nest groupings, you may want to use different delimiters; you can also use brackets ([]) and braces ({}). Since braces are special characters, you will need to enter them with \{ and \}.

Input
`$\{(a+b)\cdot c\}\cdot d$`
Output
$\{(a+b)\cdot c\}\cdot d$

When you are putting delimiters around a tall expression, you may wish to use larger delimiters to make it look better. LaTeX can automatically create appropriately sized delimiters. To enter left and right parentheses, you precede then with **left** and **right**.

Input

```
The expression $(1+\frac{2}{3})$ may look okay,
but $\left(1+\frac{2}{3}\right)$ looks better.
```

Output

The expression $(1+\frac{2}{3})$ may look okay, but $\left(1+\frac{2}{3}\right)$ looks better.

Basic math constructions

Superscripts To create exponents and other superscripts, you can use the caret (^). In math mode, the caret will turn the next object into an exponent.

Input

```
The expression $a$ squared is written as $a^2$.
```

Output

The expression a squared is written as a^2.

If the exponent should include more than the next character, you will need to group the characters you want within braces.

Input

```
Note that $a^23$ is not the same as $a^{23}$.
```

Output

Note that a^23 is not the same as a^{23}.

Subscripts Subscripts are like superscripts, except they use the underscore (_) instead of the caret.

Input

```
Again, $a_23$ is not the same as $a_{23}.$
```

Output

Again, a_23 is not the same as a_{23}.

Superscripts and subscripts can be nested, but you should avoid doing that if at all possible.

Input

```
The expression $a_{3^2}$ looks awkward,
but is not as bad as $a_{i^{j_{k^l}}}$.
```

Output

The expression a_{3^2} looks awkward, but is not as bad as $a_{i^{j_{k^l}}}$.

Certain operators have parts of their expression raised and lowered; for example, summation sums usually have their index and beginning index value lowered and the end index value raised. Sigma sums are typed in with the command `\sum`; other such operators are products (`\prod`), integrals (`\int`), and limits (`\lim`). The parts being lowered are entered as subscripts and the parts being raised are entered as superscripts.

Input
`$\lim_{n \to \infty} \sum_{k=1}^{n} ((b-a)/n) f(a + k(b-a)/n) = \int_a^b f(x){\rm d}x$`
Output
$\lim_{n \to \infty} \sum_{k=1}^{n} ((b-a)/n) f(a + k(b-a)/n) = \int_a^b f(x){\rm d}x$

Fractions Fractions can be simply typed in with a slash.

Input
`Two thirds can be typed as $2/3$.`
Output
Two thirds can be typed as $2/3$.

If you want a fraction with a horizontal line, you can use the `\frac{top}{bottom}` command.

Input
`Two thirds can also be written $\frac{2}{3}$.`
Output
Two thirds can also be written $\frac{2}{3}$.

Binomial coefficients Binomial coefficients are similar to fractions, but use the command `\binom{top}{bottom}`.

Input
`The value of $\binom{6}{4}$ is $15$.`

Output

The value of $\binom{6}{4}$ is 15.

Displayed Equations

There are three reasons you might want to display an equation. The extra room given to displayed equations allows LaTeX to typeset a nicer looking equation. The extra room makes a displayed equation more noticeable, and so provides emphasis. Finally, displayed equations can be given labels that allow you to refer to them in other parts of the document.

An equation in an `equation` environment will be displayed and labeled. If you are not going to refer to it elsewhere you should not label it, for this you can use the `equation*` environment. The same commands that work for inline mathematics also work for displayed equations; the extra vertical space in displayed equations can be used to improve the appearance of the expression.

Input

```
The equations
$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$,
\begin{equation}
\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8},
\end{equation}
and
\begin{equation*}
\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}
\end{equation*}
are all the same expression.
```

Output

The equations $\textstyle\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$,

$$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}, \tag{8.1}$$

and

$$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$$

are all the same expression.

If you want to include ordinary text in a displayed equation, you can use the `\text{text}` command.

Input

```
\begin{equation*}
x^2 \ge 0 \text{ for all real } x.
\end{equation*}
```

Output

$$x^2 \geq 0 \text{ for all real } x.$$

If you want to refer to a displayed equation at a different part of the document, you need to give it a label. To label an equation, put `\label{Label}` before the equation. You can then refer to the equation in other parts of your paper with `Equation\eqref{Label}`.

Input

```
If you are working with
\begin{equation}
\label{eqn1}
a^2 + b^2 = c^2
\end{equation}
you can refer to it as Equation \eqref{eqn1}
in other parts of the paper.
```

Output

If you are working with

$$a^2 + b^2 = c^2 \tag{8.2}$$

you can refer to it as Equation (8.2) in other parts of the paper.

If you want to discuss a system of equations (or inequalities), you will probably want to align them at the equal signs (or other signs). You can do this with the `align` environment. Each equation will need to have an ampersand (`&`) before the equal sign (or wherever else you choose to align the expressions) and each equation will need to end with `\\`. Each equation will get its own label; if you do not want an equation labeled you can put `\nonumber` before the end of the line. If you do not want any of the equations labeled you can use the `align*` environment.

Input

```
\begin{align}
a + b &= c\\
    d &= e + f \nonumber\\
g + h &= i
\end{align}
\begin{align*}
a + b &= c\\
    d &= e + f \nonumber\\
g + h &= i
\end{align*}
```

Output

$$\begin{align} a+b &= c \\ d &= e+f \nonumber \\ g+h &= i \end{align}$$

$$\begin{align*} a+b &= c \\ d &= e+f \\ g+h &= i \end{align*}$$

The `align` environment can also be used to enter a string of equalities (or inequalities):

Input

```
\begin{align*}
a + b &= c + d\\
      &= e + f\\
      &= g
\end{align*}
```

Output

$$\begin{aligned} a+b &= c+d \\ &= e+f \\ &= g \end{aligned}$$

Matrices

You can create a matrix in math mode by using the `matrix` environment. The row elements of the matrix are separated by ampersands and each row ends with `\\`.

Input

```
\begin{equation*}
  \begin{matrix}
    1 & 2 \\
    345 & 6
  \end{matrix}
\end{equation*}
```

Output

$$\begin{matrix} 1 & 2 \\ 345 & 6 \end{matrix}$$

Notice a matrix does not come with delimiters; if you want a matrix with delimiters you can use the `pmatrix` environment to get parenthesis, `bmatrix` for brackets, and `vmatrix` for vertical lines.

Input

```
\begin{equation*}
\begin{pmatrix}
1 & 2\\
3 & 4
\end{pmatrix}
\begin{bmatrix}
a & b\\
```

```
c & d
\end{bmatrix}
\begin{vmatrix}
A & B\\
C & D
\end{vmatrix}
\end{equation*}
```

Output

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

5 New commands

You can create your own commands by putting the line

`\newcommand{command}{definition}`

in the preamble of your document, where `command` consists of a backslash followed by letters and `definition` is what the command should be replaced with. For example, if you have

`\newcommand{\infd}{infinitely differentiable}`

in the preamble, then every time you enter `\infd` in your document the entire "infinitely differentiable" will be typeset.

Input
`Suppose the function $f$ is \infd.`
Output
Suppose the function f is infinitely differentiable.

You can create a command with arguments by putting

`\newcommand{command}[`n`]{definition}`

where n is the number of arguments, and `definition` can contain "#1", . . . , "#n" to represent the n arguments. For example, if you have

```
\newcommand{\boldit}[1]{\textbf{\textit{#1}}}
```

in the preamble, then `\boldit{text}` will typeset `text` in bold italics.

Input

`Here is \textbf{bold}.`
`Here is \textit{italics}.`
`Here is \boldit{bold italics}.`

Output
Here is **bold**. Here is *italics*. Here is ***bold italics***.

For another example, if you add the following command

```
\newcommand{\vfrac}[2]{\ensuremath{\frac{#1}{#2}}}
```

in the preamble, then `\vfrac{x,y}` in the regular text mode will display a vertical fraction.

Input
`Let the desired fraction be \vfrac{x}{y}.`
Output
Let the desired fraction be $\frac{x}{y}$.

It is also possible to create your own environments, but it is unlikely you will need to do that for your MCM paper.

6 The header

While the header of a LaTeX document is configurable, a paper submitted to the MCM needs to have a header of the form

Team # 3211 Page 6 of 13

That is, you need to print the team's control number in the upper left corner, and the page number of the current page with the total number of pages in the upper right corner. You can do this using the `fancyhdr` and `lastpage` packages; you will need to have

```
\usepackage{fancyhdr}
\pagestyle{fancy}
\usepackage{lastpage}
```

in the preamble. Also in the preamble, you can specify that you want the team number in the upper left of every page with the `\lhead` command:

```
\lhead{Team \# 3211}
```

You can put the appropriate page number in the upper right of every page with the `\rhead` command:

```
\rhead{Page \thepage{} of \pageref{Lastpage}}.
```

The command `\thepage` will output the page number, and `\pageref{Lastpage}` will output the number of the last page. You will also want to use `\cfoot{}` to prevent the page number from being at the bottom of every page. Putting

```
\usepackage{fancyhdr}
\pagestyle{fancy}
\usepackage{lastpage}
\lhead{Team \# 3211}
\rhead{Page \thepage{} of \pageref{LastPage}}
\cfoot{}
```

in the preamble will result in the appropriate

Team # 3211 Page 6 of 13

---

at the top of the page and the line under the header. If you do not want this line to appear, you can add

```
\renewcommand{\headrulewidth}{0pt}
```

to the preamble.

7 Title and sections

LaTeX has a mechanism (i.e., `\title{Title}`, where `Title` is the actual title of the paper) for automatically inserting a title, however it is designed to insert more information than you want for an MCM paper. You should probably not use this mechanism, instead, put the title at the beginning of the paper by putting

```
\begin{center}
\Large  The Title of the Paper
\end{center}
```

right after `\begin{document}`.

Your paper will need to be divided into sections. To begin a section, you simply use the command `\section{section title}`. The section will be numbered automatically, and the section title will be in a larger, bolder, font.

Input

```
\section{Introduction}

Begin.

\section{Conclusion}

End.
```

Output

1 Introduction

Begin.

2 Conclusion

End.

Subsections can be created with the `\subsection{subsection title}` command.

Input

```
\section{Introduction}

\subsection{Restatement of the problem}

Problem.

\subsection{Random thoughts}

Thoughts.
```

```
\section{Conclusion}

End.
```

Output

1 Introduction

1.1 Restatement of the problem

Problem.

1.2 Random thoughts

Thoughts.

2 Conclusion

End.

You can similarly create subsubsections, but it is unlikely that you will need to use them.

You should create a table of contents for your paper. You can do this by inserting the command `\tableofcontents` where you want the contents to appear.

Input

```
\tableofcontents

\section{Introduction}

\subsection{Restatement of the problem}

Problem.

\subsection{Random thoughts}

Thoughts.

\section{Conclusion}
```

```
End.
```

Output

Contents

1 Introduction

1.1 Restatement of the problem

Problem.

1.2 Random thoughts

Thoughts.

2 Conclusion

End.

8 Footnotes

You can insert a footnote with the command

`\footnote{footnote text}`

Input

```
This is part of
a paper\footnote{This is a footnote.}
```

Output

This is part of a paper[a]

[a]This is a footnote.

Note that there should not be a space between `\footnote` and the previous word.

On a separate note, you must insert a space between the left parenthesis and the word preceding it; and a space between the right parenthesis and the word succeeding it.

9 Lists

LaTeX can automatically create various types of lists. A basic, bulleted list is created with the `itemize` environment. Each item in the list is preceded with an `\item`.

Input

```
\begin{itemize}
\item First item.
\item Second item.
\end{itemize}
```

Output

- First item.
- Second item.

A numbered list is created with the `enumerate` environment. Again, each item is preceded with an `\item` command.

Input

```
\begin{enumerate}
\item First item.
\item Second item.
\end{enumerate}
```

Output

1. First item.
2. Second item.

Lists can be nested; if enumerated lists are nested then the sublists are labeled with letters instead of numbers.

Input

```
\begin{enumerate}
```

```
\item First item.
      \begin{enumerate}
        \item First subitem.
        \item Second subitem.
      \end{enumerate}
\item Second item.
\end{enumerate}
```

Output

1 First item.
  (a) First subitem.
  (b) Second subitem.
2 Second item.

10 Figures

While LaTeX has some picture drawing capabilities, you will probably want to create graphics with another program and include them in your document. To do this, you will need to put the line

`\usepackage[pdftex]{graphicx}`

in the preamble. This will allow you to include pictures in the PNG, JPG or PDF formats. To do this, you use the line

`\includegraphics{filename}`

Since you may not want to worry about creating a picture that is already the correct size, you can scale the included graphic to the proper width using an optional argument: `\includegraphics[width=width]{filename}`.
The width can be specified in a variety of units, including `cm` and `in`.

Input

```
Here is a figure:

\includegraphics[width=2cm]{fig/figure.png}
```

```
Here it is again, smaller.

\includegraphics[width=1cm]{fig/figure.png}
```

Output

Here is a figure:

Here it is again, smaller.

To make the typesetting look better, you typically will not want the figure to appear right where you refer to it. A `figure` environment will tell LaTeX to place the figure in an appropriate spot and will allow you to give it a caption and a label.

```
\begin{figure}
\centering
\includegraphics[width=width]{figure}
\caption{Description}
\label{label}
\end{figure}
```

The `\centering` will make sure that the figure is centered between the margins, and LaTeX will place the figure at an appropriate place in the document. You can refer to it with

```
Figure \ref{label}
```

Input

```
The circle in Figure \ref{face} looks like a face.
\begin{figure}
\centering
\includegraphics[width=1.5cm]{figure.png}
\caption{This looks like a face.}
\label{face}
\end{figure}
```

Output

Figure 1: *This looks like a face.*

The circle in Figure 1 looks like a face.

11 Tables

Tables can be created in LaTeX with the `tabular` environment. The environment should look like

```
\begin{tabular}{format}
item & ... & last item \\
...
\end{tabular}
```

The `format` should consist of a letter `l`, `c` or `r` for each column;

- `l` for left justified columns;
- `c` for centered justified columns;
- `r` for right justified columns.

A vertical bar (|) in the format will create a vertical line between the appropriate columns. In each row, the row entries should be separated by an ampersand (`&`) and the row should end with a double backslash (\\). An `\hline` at the beginning of a row will put a horizontal line above the row in the typeset document.

Input

```
\begin{tabular}{|c|lr|}
\hline
Centered & Left & Right \\
\hline
0 & 1 & 2\\
1 & $x$ & $x^2$\\
\hline
\end{tabular}
```

Output

Centered	Left	Right
0	1	2
1	x	x^2

If you want an entry to extend across more than one row, for example if you want a title to apply to more than one row, you can use the following command:

`\multirow{n}{format}{entry}`

where the number n represents how many rows should be used, `format` is the formatting for a single row, and `entry` is the actual table entry. Likewise, you can also extend an entry across more than one column using the following command:

`\multicolumn{n}{format}{entry}`

Input

```
\begin{tabular}{|c|l|r|}
\hline
Centered & \multicolumn{2}{c|}{Spread out} \\
\hline
0 & 1 & 2\\
\hline
1 & $x$ & $x^2$\\
\hline
\end{tabular}
```

Output

Centered	Spread out	
0	1	2
1	x	x^2

As with figures, you typically will want a table to appear right when you refer to it. You can put it in a table environment, where you can give it a caption and label.

```
\begin{table}
```

```
\centering
\begin{tabular}{format}
...

\end{tabular}
\caption{Hello}
\label{label}
\end{table}
```

Input

```
The first four letters of the alphabet
are in Table \ref{alpha}.
\begin{table}
\centering
\begin{tabular}{c|c}
a & b \\
\hline
c & d
\end{tabular}
\caption{Some letters.}
\label{alpha}
\end{table}
```

Output

a	b
c	d

Table 1: *Some letters.*

The first four letters of the alphabet are in Table 1.

12 Bibliography

Your paper will need to have a comprehensive bibliography; it should contain all references that you've used. The simplest way to include a bibliography is to use the `thebibliography` environment, which begins with

```
\begin{thebibliography}{99}
```

and ends with

```
\end{thebibliography}
```

Each bibliography item begins with the command

```
\bibitem{label},
```

the label can be used to refer to it in the paper with the command

```
\cite{label}
```

Input

```
It was shown in \cite{Te} that termites eat wood.

\begin{thebibliography}{99}
\bibitem{Carp}
Carpenter, Bob. \textsl{The Life of Ants.}
Springer-Verlag, Berlin,
1994.

\bibitem{Te}
Terwilliger, Sam. \textsl{Termites.}
Prentice Hall, New York, 2004.
\end{thebibliography}
```

Output

It was shown in [2] that termites eat wood.

References

[1] Carpenter, Bob. *The Life of Ants.* Springer-Verlag, Berlin, 1994.

[2] Terwilliger, Sam. *Termites.* Prentice Hall, New York, 2004.

13 Sample file structure

The following is a sample skeleton of an MCM paper (You may modify or expand your sections according to the suggestions of MCM):

```
\documentclass[12pt]{article}

\usepackage[utf8]{fontenc}
\usepackage[doublespacing]{setspace}
\usepackage{textcomp}
\usepackage{amsmath,amssymb}
\usepackage{fancyhdr}
\usepackage{lastpage}

\pagestyle{fancy}
\lhead{Team \# control number}
\rhead{Page \thepage{} of \pageref{Lastpage}}
\cfoot{}

\begin{document}

\begin{center}
{\Large Title}  Title of the Paper
\end{center}

\tableofcontents

\section{Introduction}

A restatement of the context problem, a survey of existing
results, and a brief description of your approach to the problem

\section{Assumptions and Justifications}

A list of your assumptions and justifications

\section{Model Design}

A description of your model

\section{Conclusions}
```

```
A description of model testing, your results, and the strengths
and weaknesses of your approach

\begin{thebibliography}{99}

A list of references

\end{thebibliography}

\end{document}
```

Chapter 9 MathType

MathType is an equation editor for typesetting math in MS Word documents, powerpoint slides, and other applications. Unlike LaTeX that needs compiling, MathType is a WYSIWYG application. MathType refers to mathematical expressions as equations. We will use equations and expressions interchangeably. You can download MathType from its official web site at `http://www.dessci.com/en/products/mathtype/`.

You may enter equations using the following two methods.

1. Open MathType and enter the desired equations in the input area provided by MathType. Then copy the equations to the desired locations in the Word document. MathType provides a window with an input area for entering equations (see Fig. 9.1). The built-in modules provide common math templates. When entering an equation, you would need your keyboard to type in characters and select appropriate templates from the built-in modules. For example, suppose that you want to enter a fraction. Click (for fractions and roots) to enter the following module (see Fig. 9.2).

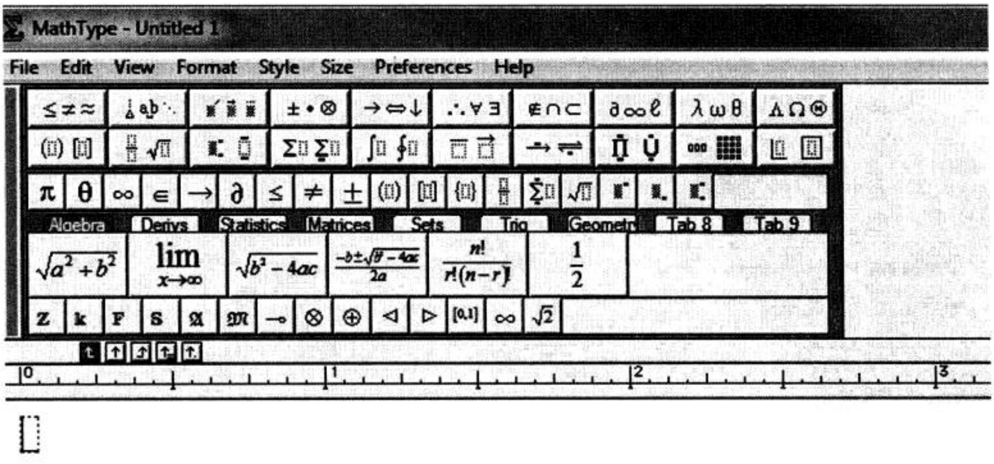


Fig. 9.1 MathType window

Then select a particular template based on your need, and enter variables or numbers to complete the fraction.

Fig. 9.2 Fractions and roots

2. Use MathType inside your Word document. That is, on the tabbed bar of the Word window, click `MathType`. At the location of the document where you want to enter your equation, select one of the following displaying options: `Inline`, `Display`, `Left-numbered` and `Right-numbered` (see Fig. 9.3) Clicking on one of these options will pop up a MathType window for you to enter equations. When you are done entering, exit MathType and you will see that the equations you just created are included in your word document at the location you selected.

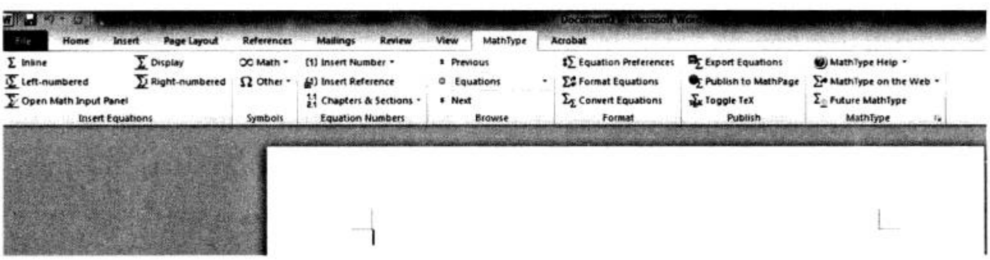


Fig. 9.3 Word window with MathType

We will use examples to illustrate how to typeset math using MathType.

1 Fractions and square root

Suppose that you want to create the following equation:

$$y = \sqrt{\frac{3}{16}} \sin x - c^2 \pm \mu \tan x.$$

Follow these steps:

1. Type "$y =$" in the typing area of the MathType window. Click . Click to select the square root template, and then click to select the fraction

template. Type "3" in the numerator and "16" in the denominator to get

$$y = \sqrt{\frac{3}{16}}$$

2. Type "$\sin x-$" outside of the square root. MathType automatically identifies function sin and displays it appropriately.
3. To enter the superscript for squaring, first type a letter "c", then click and select an appropriate superscript template.
4. On the tabbed bar of the MathType window, select the plus and minus sign $\pm$ at $\boxed{\pm}$.
5. Click $\lambda\omega\theta$ (the Greek symbol module), and select letter μ. Then type "$\tan x$". You now have the desired equation as follows, see Fig. 9.4.

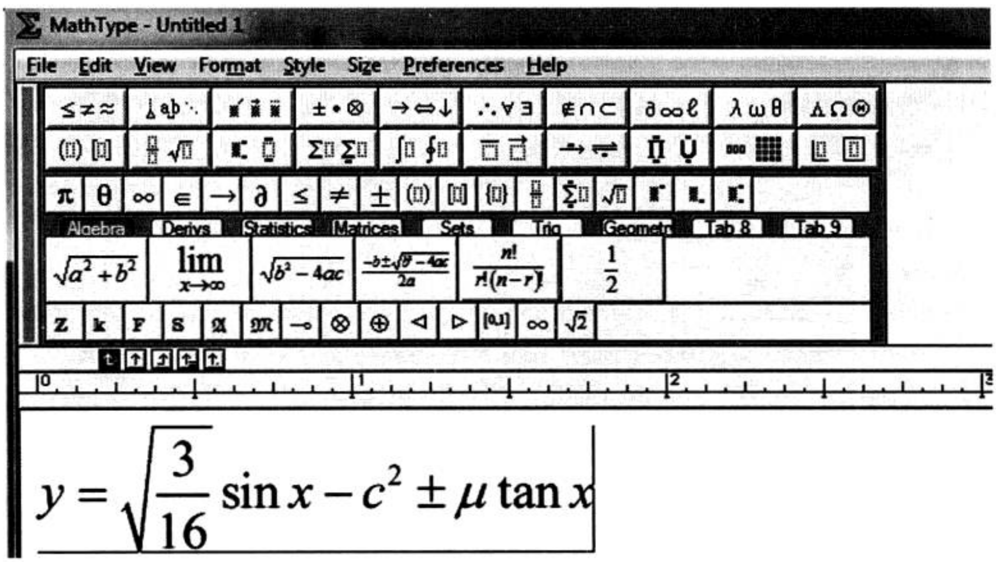


Fig. 9.4 Type fractions and square root

2 Integrals, subscripts, and summations

Suppose that you want to typeset the following expression:

$$\sum_{i=1}^{\infty} \frac{1}{i} \quad \iiint_E \mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

Follow these steps:

1. Click to enter the summation module and select an appropriate summation template.
2. Insert `Fraction` $\frac{1}{i}$.
3. Click to enter the integral module and select an appropriate integral template.
4. Type dxdydz.

3 Multiple equations

Suppose that you want to typeset the following multiple equations:

$$\sigma^2 = \begin{cases} \dfrac{1}{3}\displaystyle\sum_{i=1}^{n} x_i^2 & i \geq 2 \\ 2 & i = 1 \end{cases}$$

Follow these steps:

1. Click to enter the Greek-symbol module and select "σ". Then click to enter the superscript and subscript module, select the desired superscript template, and type "2"。
2. Type "="。
3. Click to enter the delimeter module, and then click to select left brace. Press the enter key to generate two input boxes, one in each row. The left brace will automatically expand vertically to cover these two rows. (Note that you may also press the enter key at the end of the current expression to create a new row.)
4. In the first input box enter the desired fraction and summation. Click to enter the module of speical symbols, and then click a few times to create sufficiently large blank space. Enter "$i \geq 2$". (Note that MathType does not allow you to use the space bar to enter blank space.)
5. In the second input box enter "2", blank space (in the same way as in the first input box), and "$i = 1$".

   In the MathType typing area you will now see Fig. 9.5.

4 Matrices and determinants

On the tabbed bar of the MathType window you can find the `Matrices` option, which provides five templates for entering matrices. However, there is no option

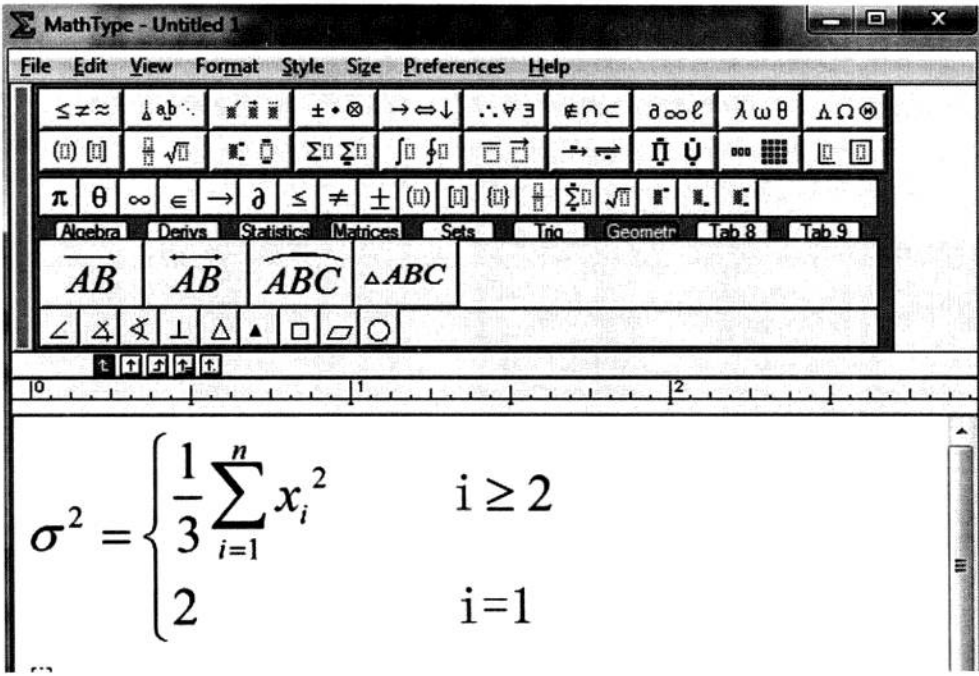


Fig. 9.5 Typeset multiple equations

for entering determinants on the tabbed bar. Follow the following steps to enter a determinant.

1. Click to enter the delimeter module. Then click to select the delimiters for determinants.
2. Click to enter the matrix module, from which select a suitable template to enter the needed determinant.

5 Saving user-created expressions in the tabbed bar

MathType provides a special tabbed bar (shown in Fig. 9.6) for typesetting equations in common math subjects, including algebra, statistics, integrals, matrices, sets, trigonometry, and geometry.

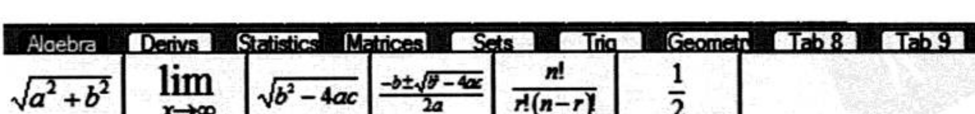


Fig. 9.6 MathType tab bar

MathType supports all math symbols commonly used in the literature and allows users to personalize expressions. It also allows users to move the expressions they created to the tabbed bar for easy access later. Follow these steps to do so:

1. Select an expression that you want to move to the tabbed bar. For example, select the following expression (see Fig. 9.7).

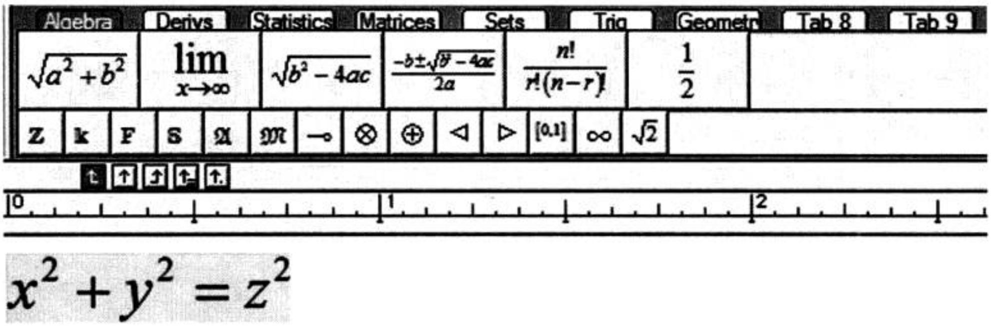


Fig. 9.7 Select an expression

2. Hold the left button of your mouse and drag it into the black rectangle position as shown in Fig. 9.8.

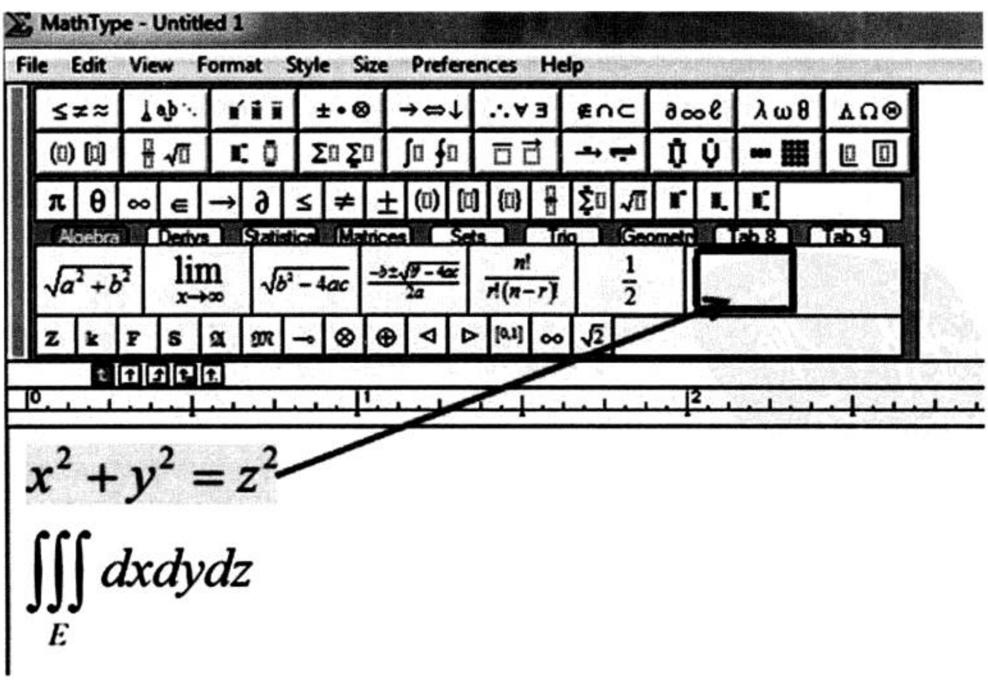


Fig. 9.8 Drag the expression into the black rectangle

Your expression will now appear in the tabbed bar, indicating that you have successfully added your expression. If you know which existing category this expression belongs to, you could put it under that category. Otherwise, you could simply move it to the tabbed bar under Tab 8 or Tab 9 as a self-created expression, as shown in Fig. 9.9.

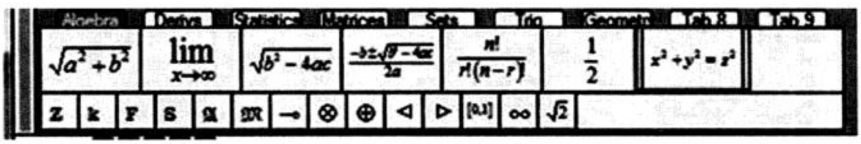


Fig. 9.9 Add your expression under Tab 8 or Tab 9

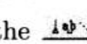
This method also applies to Greek symbols, or any other symbols in the `Symbol` palettes and the `Template` palettes.

To remove an expression from the tabbed bar, hold the right button of your mouse, and select `delete`.

6 Alignment

Suppose that we would like to create the following two expressions aligned at $\leq$:

$$\begin{aligned} \int_0^1 a(x)\,dx &\leq \limsup_{n\to\infty} \phi_n(a), \\ \int_0^1 a(x)b(x)\,dx &\leq \limsup_{n\to\infty} \varphi_n(a,b). \end{aligned}$$

1. For the first equation, enter the desired integral on the left-hand side of the $\leq$ sign. Enter appropriate blank space between $a(x)$ and dx using the  module.
2. Complete the rest of the equation and press the `Enter` key.
3. Copy and paste the first equation and then modify it to create the second equation. (This will save time.) You will now see that the two equations are aligned at the left-hand side as follows:

$$\int_0^1 a(x)\,dx \leq \limsup_{n\to\infty} \phi_n(a)$$
$$\int_0^1 a(x)b(x)\,dx \leq \limsup_{n\to\infty} \varphi_n(a,b)$$

4. To align them at $\leq$, click `Format` in the menu see Fig. 9.10, and then click

`Align at =` to get the desired alignment, shown in Fig. 9.11.

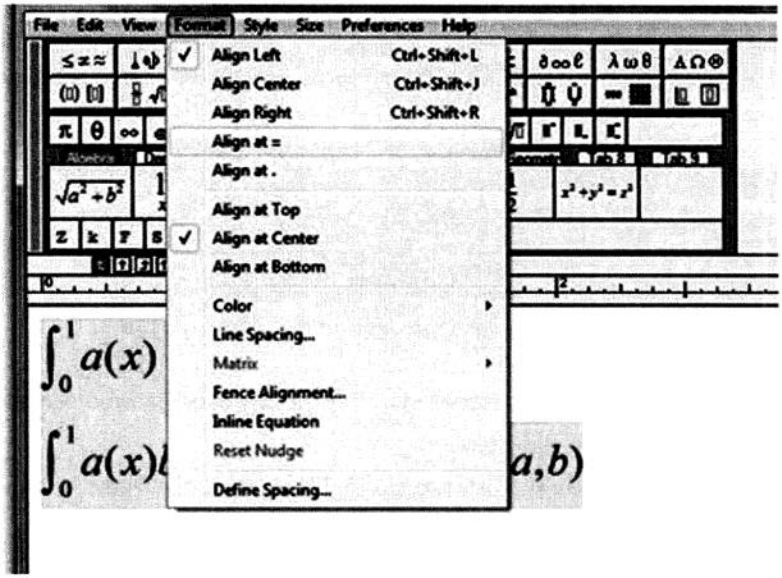


Fig. 9.10 Format menu

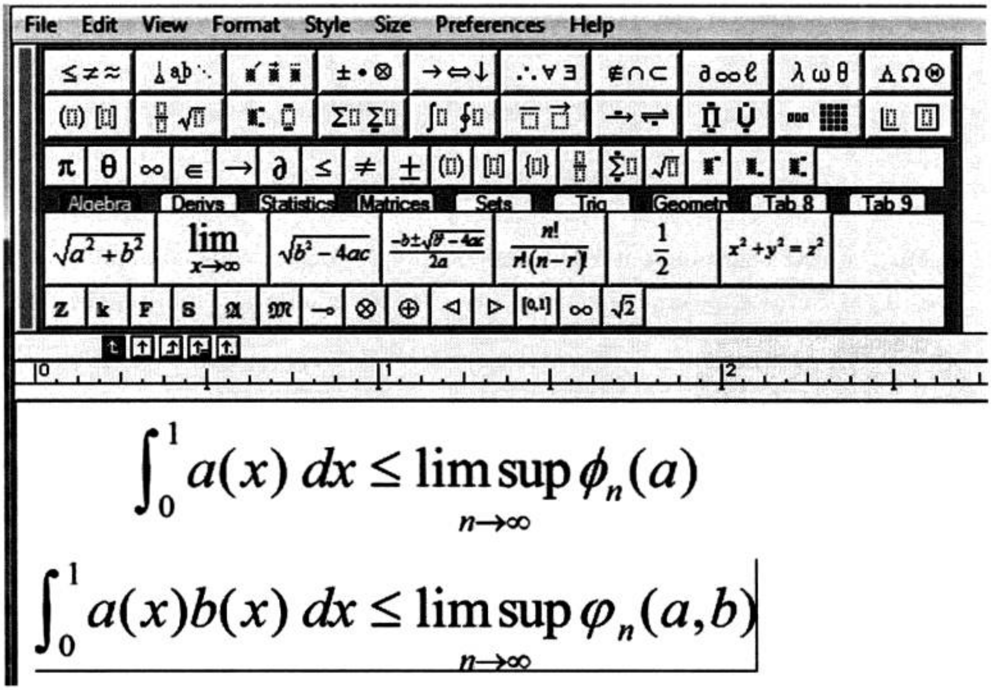


Fig. 9.11 Get the desired alignment

7 Change fonts and styles

Suppose that we want to change the following equation from the default `Math` style to the `Euclid` style:

$$u = \phi \cdot exp\{\frac{1}{2}\sigma(x + y)\}.$$

1. On the status bar at the bottom of the typing area of the MathType window you should be able to see `Style:Math`. If not, click `Style` on the menu and then click `Math`.
2. Create the equation. Click `Style` and then `Define` to pop up the following dialog (see Fig. 9.12).

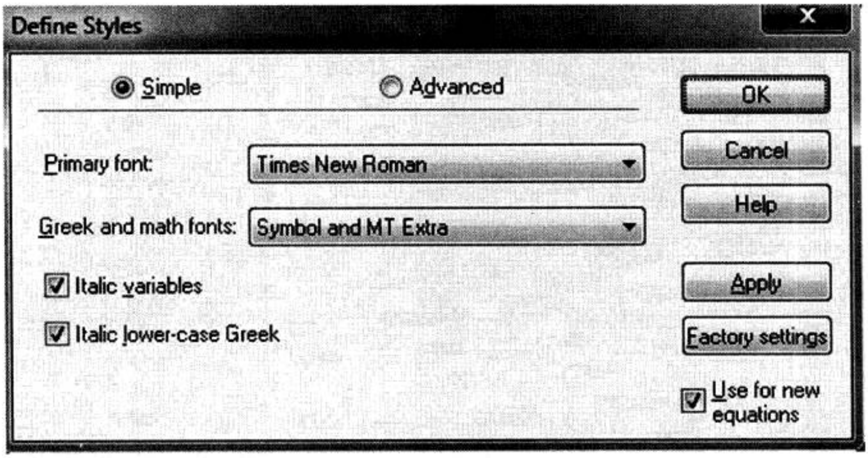


Fig. 9.12 Define Styles dialog

3. Change `Primary font` to `Euclid` and change `Greek and math fonts` to `Euclid Symbol and Euclid Extra`.

   Note that you may reset the settings by selecting `Factory settings`. Click `Advanced` for more options, see Fig. 9.13.

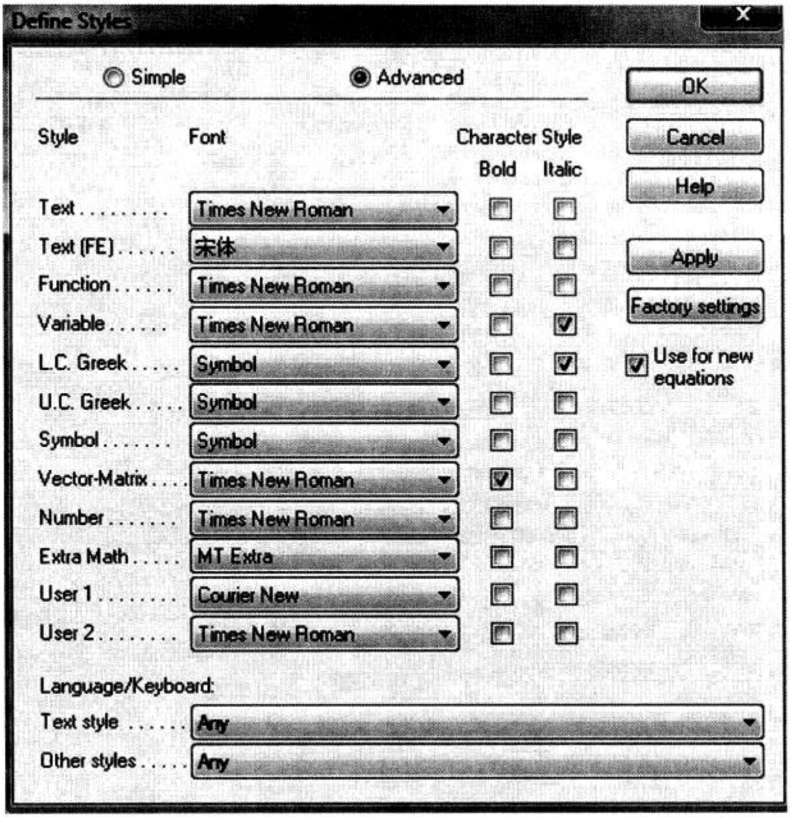


Fig. 9.13 Advanced menu

You may set `Text`, `Function`, and `Symbol` with different styles according to your needs.

8 Numbering equations

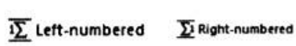
Suppose that we want to number the following two equations as "(1.1)" and "(1.2)":

$$\cos^2\theta + \sin^2\theta = 1 \tag{1.1}$$

$$\cos^2\theta - \sin^2\theta = \cos(2\theta) \tag{1.2}$$

Follow these steps:

1. Open your Word document and click the MathType option. You can find the following



on the tabbed bar, which provides two methods to number equations. The first method places a numbering on the left-hand side of an equation, and the second places that on the right-hand side.

2. Click Right-numbered to automatically generate numberings according to which chapter and which section the equations are located, and a MathType window will pup up. Create the first equation in the Mathtype typing area. Exit Mathtype and you will see the numbering "(1.1)" at the right-hand side of the equation in the Word document, where the section number appears before the expression number, separated by a dot.
3. Create and number the second equation in the same way. You'll see the following in your Word document:

We have equation:

$$\cos^2\theta+\sin^2\theta=1 \tag{1.1}$$

$$\cos^2\theta-\sin^2\theta=\cos(2\theta) \tag{1.2}$$

MathType automatically numbers each equation in the correct order. Should you want to change the numbering of a certain equation, click on Chapters & Sections, and then click `Modify Break` to get the dialog, shown in Fig. 9.14.

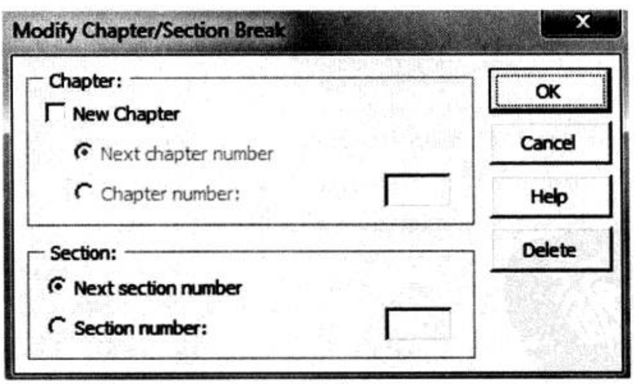


Fig. 9.14 Modify Chapter/Section Break dialog

Enter the desired numbering there.

9 Word document settings

When creating a Word document containing equations, you probably want them to match with the font and size of the text body, and maintain a consistent format.

While Word and MathType allow you to change fonts and sizes directly, it is better to define a new style to specify them, for doing so will make it easy to modify the format of your equations and text. This is done by changing the definition of style, e.g., from "Arial" to "Times New Roman", or from "10 pt italic" to "12pt plain". After the new style is defined, your document will be reformatted automatically with the new setting.

As an example, let us consider how to produce a document with "12pt Times New Roman". Define a MathType setting to match with the Word document as follows:

1. In MathType, select `Style`. Then select `Define` and choose `Times New Roman` as the primary font. Font `Use for new equations`.
2. Repeat the above steps. Then choose `12 pt` and select `Use for new equations`.
3. Back to Word, and select the MathType option, from there select `Equation Preferences` and choose `MathType's 'New Equation' preferences`. This new setting will automatically apply to any new equation.
4. Now change the style of the text body. Click `Home`. In the `Styles` box, choose a style and right click your mouse, and then choose `Modify`. Find `Formatting`, and select `Times New Roman` and `12`.

10 Default fonts and styles

The font chosen to typeset characters and symbols for the main body of the text is referred to as the primary font. The default primary font is "Times New Roman". Equations should use the primary font or different forms of the primary font. For instance, the logarithmic functions (log, ln, lg), the trigonometry functions (sin, cos, tan), the modular operation (mod), and the minimum and maximum operations (min, max) should use the primary font. Variables should use the primary font italic. Vectors and matrices should use the primary font italic and bold.

Listed below are the common styles in MathType:

Math The `Math` style is the default style for characters and symbols in mathematical expressions.

Text The `Text` style is the default style for entering text.

Function The `Function` style is used to display the logarithmic functions, the trigonometry functions, the modular operation, the minimum and maximum operations, and other functions and operations with a common fixed name.

Variable The `Variable` style is used to display characters in mathematical expressions.

Greek-Symbol The `Greek-Symbol` style is used to display Greek characters. It also applies to mathematical operators.

Vector-Matrix The `Vector-Matrix` style is used to display vectors and matrices. It is typically the same as the Variable style but in bold.

User 1 and User 2 These two tabs are for users to define their own styles.

MathType automatically assigns certain styles and sizes to certain types of characters with appropriate spacing, based on function identification and character substitution.

Under the styles of `Math`, `Variable`, `Function`, `Vector-Matrix`, and `Greek-Symbol`, MathType may automatically substitute certain characters appropriately. For example, the "minus" sign that is entered as a hyphen under the `Text` style will be replaced with a real "minus" sign, which is longer than a hyphen.

MathType automatically assigns an appropriate size for characters and symbols in equations, selected from the following types: full, subscript (superscript), two-layer subscript (superscript), symbol, and sub-symbol.

11 Tips

In addition to the default settings, MathType also allows the user to define their own styles, but for writing an MCM paper, the default settings would be sufficient. To use MathType more efficiently, we provide a few tips in this section.

Modify equations in the document

To fix a typo in an existing expression or make a minor amendment, double click on the expression you want to modify. The MathType interface will appear with the selected expression. Perform the modification you desire, save it, and close MathType. The modification will appear in the Word document.

Change the font and size for multiple expressions at the same time

You may need to change the font and size for the entire paper. Follow these steps:

1. Open the Word document, double click an expression. In the MathType popup window, click `Size` then `Define`, and change the size according to your need.
2. Click `Preference`, `Equation preference`, and `Save to file` in order. Then save this preference to a file with a file name different from the default name. Close MathType and go back to Word.
3. Select the MathType option on the Word tabbed bar. Click `Format equation`, `MathType preference File`, and `Browse` in order. Then click the file you just saved, select `Whole document`, and press `Ok`.

More symbols

MathType offers thousands of symbols for typesetting mathematical expressions. You may find them from the tabbed bar, the `Symbol` option, and the `Template` option. If you still cannot find what you need, click `Edit` and then `Insert Symbol` to popup a window with more symbols. For your convenience, we recommend that you save commonly used symbols in the tabbed bar.

Use existing standard expressions

MathType has a category-breakdown tabbed bar, where you can find templates existing standard equations in the subject area. For example, the formula of roots for quadratic equations is saved under `Algebra`, and standardized random variables are saved under `Statistics`.

Change font, color, and size

Font, color, and size can be changed by clicking, respectively, `Style` then `Define`, `Format` then `Color`, and `Size` then `Define`.

Save favorite equations

Drag the selected expression into the blank area on the large tabbed bar or the small tabbed bar to save it. When it is no longer needed, right click on it and use the delete option to delete it.

Shortcut keys

Most commands in MathType can be executed using shortcut keys to save time. It is useful to memorize a few shortcut keys for the items you would use often.

Listed below are common keystrokes in the categories of mathematical operators, Greek symbols, and formatting.

Math operators

Fraction

Ctrl+F (regular fraction; i.e., hold the Ctrl key while pressing F)
Ctrl+/ (forward slash fraction)

Square root

Ctrl+R (square root)
Ctrl+T-N (root of high dimension; Ctrl-T-N means to press Ctrl+T, release both keys, then press the N key)

Subscript and superscript

Ctrl+H (superscript)
Ctrl+L (subscript)
Ctrl+J (superscript and subscript)

Inequalities

Ctrl+K-, ($\leq$)
Ctrl+K-. ($\geq$)

Differentials and integrals

Ctrl+Alt+' (first derivative)
Ctrl+Shift+" (second derivative)
Ctrl+I (definite integral)
Ctrl+Shift+I (indefinite integral)

Brackets

Ctrl+9 or Ctrl+0 (parentheses)
Ctrl+[or Ctrl+] (brackets)
Ctrl+{ or Ctrl+} (curly brackets)

Others

Ctrl+Shift+- (bar)
Ctrl+Alt+- (vector arrow)

Greek Symbols

Press Ctrl+G, release both keys, then enter a lower-case letter. You'll get the relevant lower-case Greek symbol.
Press Ctrl+G, release both keys, then enter an upper-case letter. You'll get the relevant upper-case Greek symbol.

Formatting

Change visual size These shortcut keys only change visual size, which do not affect the real size.

Ctrl+1 (100%)
Ctrl+2 (200%)
Ctrl+4 (400%)
Ctrl+8 (800%)

Tweak and spacing

Ctrl+Arrow (select an item you want to move, then press Ctrl+Arrow to move the item in the direction of the arrow)
Ctrl+Alt+Space (insert space between characters)

Define your own shortcut keys

You may define your own shortcut keys for a specific symbol or equation as follows:

1. In the MathType window, right click on a symbol (e.g., π), then click `Properties` to pop up the following window, see Fig. 9.15.

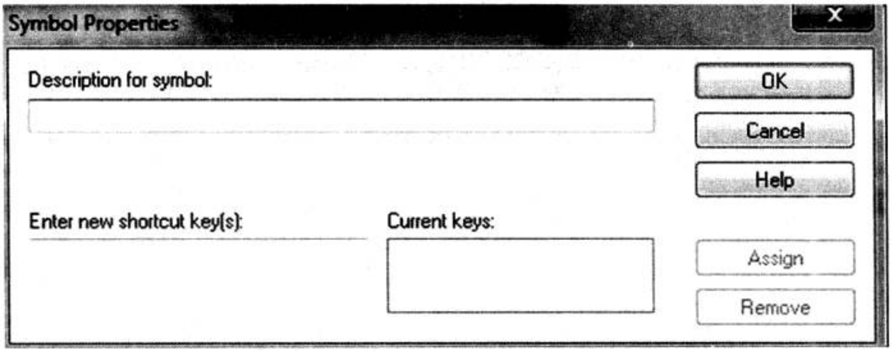


Fig. 9.15 Symbol Properties dialog

2. In the `Enter new shortcut key(s):` box, type the shortcut key you selected, and click `OK`.

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建立了合理的数学模型并找到了有效的解题方法以后，写作就是关键的因素。本书针对 MCM/ICM 论文的评审标准，讨论如何写好 MCM/ICM 论文，指出哪些内容是评委期待见到的，以及如何写作才能使评委能够在短时间内掌握论文的要点。具体内容包括论文结构、写作规范、英语用法、论文修改示例、符号与图表、数学表达式和句子以及数学编辑软件 LATEX 和 MathType 的使用等。

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