

An Exploration of "Gauss's Principle With Inequality Constraints for Multiagent Navigation and Control"

by Boyang Zhang and Henri P. Gavin

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1 Section II - Preliminaries

Here, we review Section II of "Gauss's Principle With Inequality Constraints for Multiagent Navigation and Control" by Boyang Zhang and Henri P. Gavin.

Next, I define variables and what their meanings are relating to multiagent navigation and control as if I were to define variables for a MATLAB or Octave program. Please keep in mind that the intention, first of all, is to build a GPLC control framework based pendulum, so the variables set here and their explanations are meant both to cover their generality and to provide understanding required for simulation of a simple pendulum.

N is the number of agents:

$$N = 1 \tag{1}$$

It equals one in this case, due to the requirements of a pendulum (an ideally single mass) and for defining the size of required matrices / vectors.

It is worth noting that the following matrix and vector sizes are dependent on N , but also the degree of freedom (DOF) of each mass particle (or agent). For the rest of this summary DOF is understood to be two, and is not included as a variable in matrix and vector dimensionality specifications, it will be included as "2".

M is a Symmetric Positive Definite (SPD) Mass matrix, of size $2N \times 2N$. A symmetric matrix requires that it be equal to its transpose. So, $M = M^T$. A symmetric matrix M is a positive definite matrix if the real number $x^T M x$ is always positive for $x \in \mathbb{R}^{2N}$.

$$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \tag{2}$$

So, in this case, M obviously satisfies the requirement that it be equal to its transpose. As for the proof that M , in its current configuration, can also be positive definite, we must consider what m would result in the real number $x^T M x$ as always positive for $x \in \mathbb{R}^{2N}$.

For $N = 1$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Evaluating the real number expression $x^T M x$ symbolically results in:

$$m(x_1^2 + x_2^2) \tag{3}$$

showing that for $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ Expression 3 > 0 when $m > 0$.

a is the unconstrained accelerations related to the net forces due to internal and external actions through Newton's Second Law: $a \in \mathbb{R}^{2N}$

f is the net forces to the internal and external actions through Newton's Second Law:
 $f = Ma$

$q(t)$ is the coordinate position vector that varies with time: $q(t) \in \mathbb{R}^{2N}$

$\ddot{q}(t)$ is the second derivative of q with respect to time. $\ddot{q}(t) \in \mathbb{R}^{2N}$

Now, switching contexts to quadratic programming (QP) methods. The objective of QP is to find an n -dimensional vector $\ddot{q}(t)$ that will minimize Z :

$$Z = \frac{1}{2} \ddot{q}^\top M \ddot{q} - f^\top \ddot{q} \quad (4)$$

subject to constraints:

$$A \ddot{q} \preceq b \quad (5)$$

Generally, the constraints to be imposed on the above minimization would enforce geometric relationships among coordinate positions $q(t)$ and velocities \dot{q} . Their form would be:

$$g(q, \dot{q}, t) = 0 \quad (6)$$

2 Scratch

$$\frac{d}{dt} \left(\frac{\delta g(q, \dot{q}, t)}{\delta \dot{q}} \right) \quad (7)$$