Mathematical Olympiads Discord Server

2019 June Advanced Contest

Time: 4 hours   *Each problem is worth 7 points*

Calculators and protractors are not allowed. Do not write your name on your working. At the end of the contest, please scan your solutions and working and send them to `Tony Wang#6285` *via direct message. Do not discuss the contents of this paper outside the text channel* `#finished-contestants` *and the voice channel* `Post-Contest Banter` *until notified by staff.*

Problem 1. Let n be a given positive integer. Find the minimum m such that for all real sequences $x_1, x_2, \ldots, x_n$ there exists a real number y such that

$$\langle y - x_1 \rangle + \langle y - x_2 \rangle + \cdots + \langle y - x_n \rangle \leq m,$$

where $\langle x \rangle = x - \lfloor x \rfloor$ is the difference between x and the greatest integer less than or equal to x.

Problem 2. Sharky has a collection of 2^n strips of $n \times 1$ strips of paper, with each strip divided into n unit squares. Each square on a strip is coloured black or white such that every strip is unique. Find the smallest m such that for any m strips, Sharky can choose n of these strips and arrange them (without flipping any of the strips) into a $n \times n$ square grid with the property that a main diagonal is monochromatic.

Problem 3. Let ABC be a triangle with circumcentre O, and let P be a point on BC distinct from B and C. Construct X and Y on AB and AC respectively such that $XB = XP$ and $YP = YC$. Prove that $AXOY$ is cyclic.

Problem 4. Prove that for all Pythagorean triples A and B there exists a finite sequence of Pythagorean triples starting with A and ending with B such that any two consecutive triples share at least one number.

    (A *Pythagorean triple* is a triple of positive integers (a, b, c) such that a, b, and c are the side lengths of a right-angled triangle.)