(Working Paper) Illustrated Numerical Reviews: Nonlinear Time Series Models

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Working Papers in this series of Illustrated Numerical Reviews are designed for scientific collaboration, communication and classroom examples with a revision update before modified for publication.

Abstract

Nonlinear Time Series models can be used to forecast stock prices such as ABBV PFE MRK and JNJ from a list of major pharmaceutical stocks on the NYSE. Here NonLinear AutoRegressive (NLAR), LSTAR and SETAR are three possible mathematical descriptions for predicting the n step ahead daily prices necessary for optimal portifolio investment selections. In the context of N units of a stock from ABBV PFE MRK and JNJ within some historical period, portfolio evaluation with the first two moments with constrained OLS estimation can be used and the forecast from the specification of these three nonlinear time series models examined. Here the Beta distribution for historical prices and the SETAR model is used for generating the forecasts of the low JNJ stock price for both portfolio analysis and development of effective and efficient trading strategies.

Introduction

Consider the discrete-time univariate stochastic process $\{X_t\}_{t \in T}$ and suppose X_t is generated by the map: [2] [3]

$$X_{t+s} = f(X_t, X_{t-d}, \ldots, X_{t-(m-1)d}; \theta) + \epsilon_{t+s} \tag{1}$$

with $\{\epsilon_t\}_{t \in T}$ white noise, ϵ_{t+s} indipendent w.r.t. X_{t+s}, and with f a generic function from $\mathbf{R}^m$ to $\mathbf{R}$. [1015]

This NLAR(m), *NonLinear AutoRegressive* of order m. A model is given by

$$X_{t+s} = \begin{cases} \phi_1 + \phi_{10}X_t + \phi_{11}X_{t-d} + \ldots \\ +\phi_{1L}X_{t-(L-1)d} + \epsilon_{t+s} & Z_t \leq \text{th} \\ \\ \phi_2 + \phi_{20}X_t + \phi_{21}X_{t-d} + \ldots \\ +\phi_{2H}X_{t-(H-1)d} + \epsilon_{t+s} & Z_t > \text{th} \end{cases} \tag{2}$$

with Z_t a threshold variable. SETAR models Z_t is a selection from $\{X_t, X_{t-d}, X_{t-(m-1)d}\}$. The threshold variable Z_t *threshold delay* δ such that

$$Z_t = X_{t-\delta d}$$

then the where `thDelay` is δ, and an integer between 0 and $m-1$. The threshold variable as a linear combination of lagged time series values: [1015]

$$Z_t = \beta_1 X_t + \beta_2 X_{t-1} + \ldots + \beta_m X_{t-(m-1)d}$$

The LSTAR model is a generalization of the SETAR model: [1015]

$$X_{t+s} = \begin{cases} (\phi_1 + \phi_{10}X_t + \phi_{11}X_{t-d} + \ldots \\ +\phi_{1L}X_{t-(L-1)d})(1 - G(Z_t, \gamma, \text{th})) \\ +(\phi_2 + \phi_{20}X_t + \phi_{21}X_{t-d} + \ldots \\ +\phi_{2H}X_{t-(H-1)d})G(Z_t, \gamma, \text{th}) + \epsilon_{t+s} \end{cases} \tag{3}$$

with G the logistic function, and Z_t the threshold variable. For Z_t, L and H specification, SETAR models is followed. LSTAR models require starting values for all the parameters to be estimated: $(\phi, \gamma, \text{th})$. Estimation is by ϕ_1 and ϕ_2 (linear regression) and then minimizing residuals sum of squares with respect to th and γ. These two steps are repeated until convergence. [1015]

Consider the movement in the daily low price for a selected stock as shown in Figure 1 with ABBV PFE MRK and JNJ.

Figure 1: Low Prices for ABBV PFE MRK and JNJ with different moving averages.[1002]

The correlation structure of ABBV PFE MRK and JNJ is shown in Figure 2.

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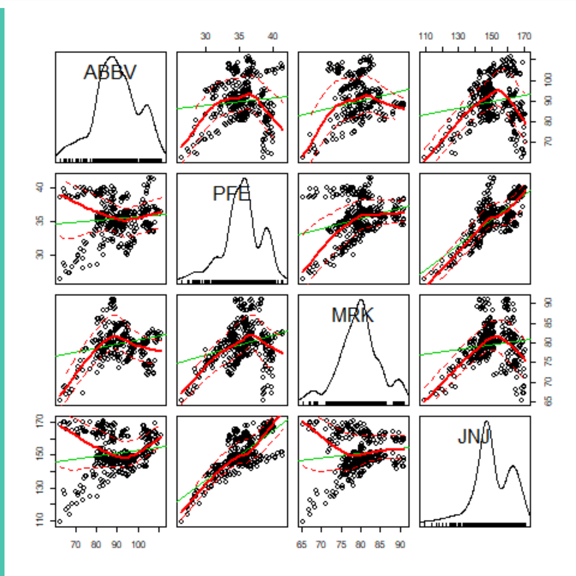

Figure 2: Scatterplot matrix of the daily low prices for ABBV PFE MRK and JNJ

The Complete List of Major Pharmaceutical Stocks on the NYSE presented in Table 1.

	Name	Ticker	Country
1	Abbott Laboratories	ABT	USA
2	AbbVie Inc.	ABBV	USA
3	Actinium Pharmaceuticals Inc.	ATNM	USA
4	ADC Therapeutics SA	ADCT	USA
5	AgeX Therapeutics Inc.	AGE	USA
6	Amneal Pharmaceuticals Inc.	AMRX	USA
7	Ampio Pharmaceuticals Inc.	AMPE	USA
8	Annovis Bio Inc.	ANVS	USA
9	Arcus Biosciences Inc.	RCUS	USA
10	Bausch Health Companies Inc.	BHC	Canada
11	Biohaven Pharmaceutical Holding Company Ltd.	BHVN	USA
12	Bristol-Myers Squibb Company	BMY	USA
13	Can-Fite Biopharma Ltd	CANF	Israel
14	Catalent Inc.	CTLT	USA
15	China Pharma Holdings Inc.	CPHI	China
16	CorMedix Inc.	CRMD	USA
17	Dr. Reddy's Laboratories Ltd	RDY	India
18	Elanco Animal Health Incorporated	ELAN	USA
19	Eli Lilly and Company	LLY	USA
20	Emergent Biosolutions Inc.	EBS	USA
21	GlaxoSmithKline PLC	GSK	United Kingdom
22	iBio Inc.	IBIO	USA
23	Johnson and Johnson	JNJ	USA
24	Lannett Co Inc	LCI	USA
25	Matinas Biopharma Holdings Inc.	MTNB	USA
26	Merck and Company Inc.	MRK	USA
27	Myovant Sciences Ltd.	MYOV	USA
28	NanoViricides Inc.	NNVC	USA
29	NovaBay Pharmaceuticals Inc.	NBY	USA
30	Novartis AG	NVS	Switzerland
31	Novo Nordisk A/S	NVO	Denmark
32	NTN Buzztime Inc.	NTN	USA
33	Oragenics Inc.	OGEN	USA
34	Palatin Technologies Inc.	PTN	USA
35	Panacea Acquisition Corp.	PANA	USA
36	Perrigo Company plc	PRGO	USA
37	Pfizer Inc.	PFE	USA
38	Prestige Consumer Healthcare Inc.	PBH	USA
39	Synthetic Biologics Inc.	SYN	USA
40	Takeda Pharmaceutical Company Limited	TAK	Japan
41	Taro Pharmaceutical Industries Ltd.	TARO	Israel
42	Teva Pharmaceutical Industries Limited	TEVA	Israel
43	Timber Pharmaceuticals Inc.	TMBR	USA
44	Zoetis Inc.	ZTS	USA
45	Zomedica Corp.	ZOM	USA
46	Zymeworks Inc.	ZYME	Canada

Table 1: The Complete List of Major Pharmaceutical Stocks on the NYSE [1]

Results

Numerical computation can be accomplished with many different R APIs in nonlinear time series analysis. [1001] [1002] [1003] [1004] [1005] [1006] [1007] [1008] [1009] [1010] [1011] [1012] [1013][1014] [1015] [1016]. Here the closing price for JNJ is selected over the time period from January 1 2020 for 298 consecutive trading days.

Based on maximum likelihood estimation, the fit of JNJ is a type 1 Pearson distribution with parameters α=5.790304, β=2.329592, location=97.13936 and scale=75.48928. The pearson type is related to the beta distributon where the probability density function (pdf) of the beta distribution, for $0 \le x \le 1$, and shape parameters α, $\beta > 0$, is a power function of the variable x and reflection $(1-x)$ given by

$$\begin{aligned} f(x;\alpha,\beta) &= \mathrm{constant}\cdot x^{\alpha-1}(1-x)^{\beta-1} && (4)\\ &= \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\,du} && (5)\\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,x^{\alpha-1}(1-x)^{\beta-1} && (6)\\ &= \frac{1}{\mathrm{Beta}(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1} && (7)\end{aligned}$$

where $\Gamma(z)$ is the gamma function. The continuous uniform distribution is a limit of type I and the Normal distribution is the result of the limit of type I, III, IV, V, or VI. Table 2 has the model summary based on the SETAR model for JNJ. Figure 3 has Results of the Grid search for stock JNJ. Automatic selection of LSTAR and NNET models hyper parameters with a search of possible combinations of values of specified hyper-parameters is performed while embedding parameters m,d,steps are kept fixed. [1015]

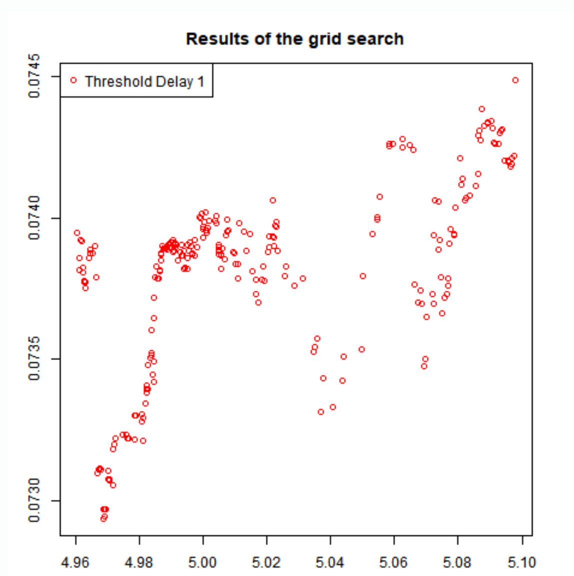

Figure 3: Results of the Grid search for stock JNJ. Automatic selection of LSTAR and NNET models hyper parameters with a search of possible combinations of values of specified hyper-parameters is performed while embedding parameters m,d,steps are kept fixed.

Figure 4 has a Recurrence plot for stock JNJ m=3, d=1, levels=0,0.2, and 1.

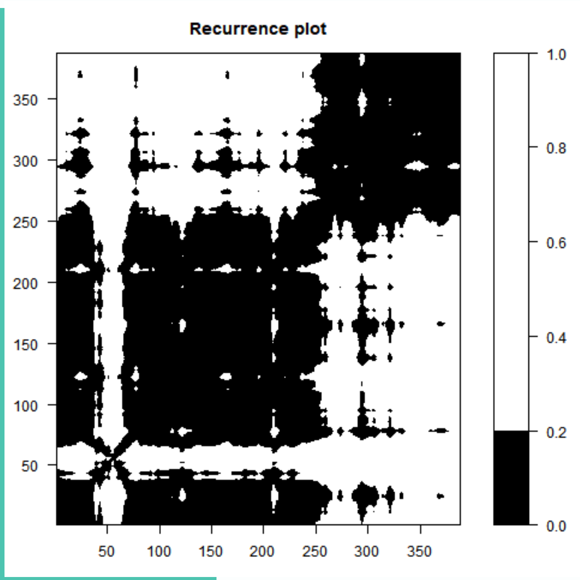

Figure 4: Recurrence plot for stock JNJ m=3, d=1, levels=c(0,0.2,1)

Table 2 has the parameter values and significance level of the SETAR model (2 regimes) for JNJ.

Low regime:			
const.L	phiL.1	phiL.2	phiL.3
6.38150432	1.08032723	-0.04448507	-0.07967492
High regime:			
const.H	phiH.1	phiH.2	phiH.3
14.70032046	0.89347372	0.05968496	-0.04297526

Table 2: Threshold: 154 with the Proportion of points in low regime: 65.12 percent High regime: 34.8 percent.

Figure 5 has the time series plot of the volume traded for stock JNJ. Figure 6 has the four different values for the lags=1,2,3 and 4 for stock JNJ. Figure 7 has four different plots of the four local linear models. Figure 8 has the same for the SETAR model with Figure 9 has the lags 1 and 3 as well as a histogram, autocorrelation, partial correlation function and mutual information for stock JNJ. Figure 6 has 30 day step ahead predictons for the daily low price of stock JNJ.

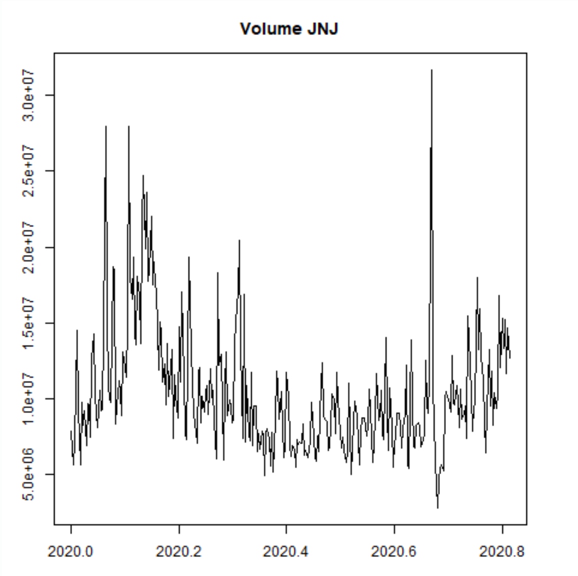

Figure 5: Time series plot of the volume traded for stock JNJ

Figure 6 has Four different scatterplots, directed lines and kernel density estimations for the lags=1,2,3 and 4 for the daily low stock price for JNJ.

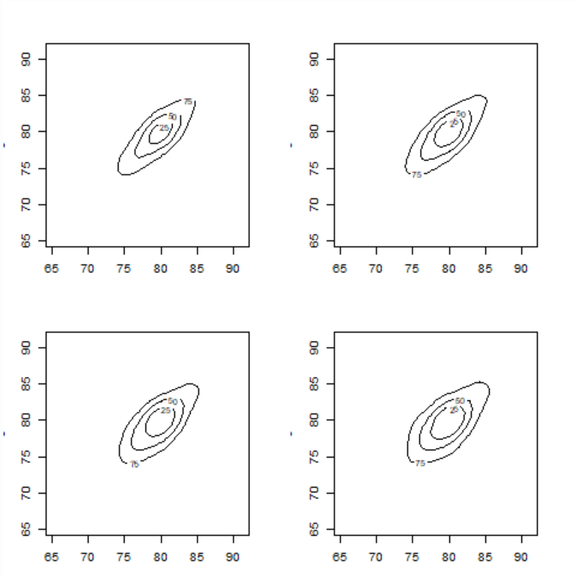

Figure 6: Four different scatterplots, directed lines and kernel density estimations for the lags=1,2,3 and 4 for the daily low stock price for JNJ.

Figure 7 has the Local linear autogression models with m=1,2,3,4 for stock JNJ.

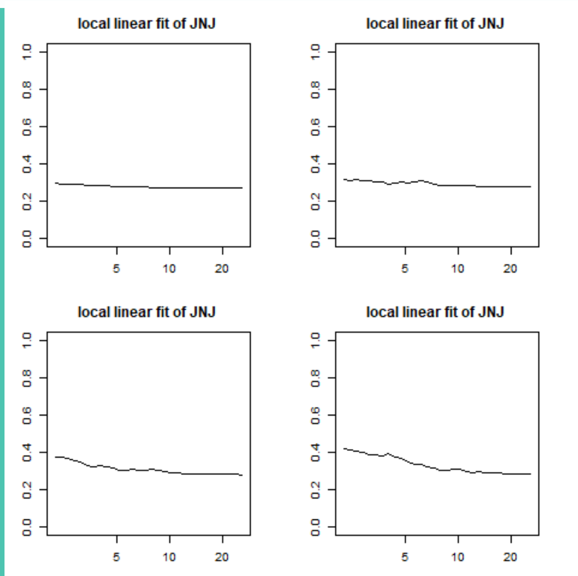

Figure 7: Local linear autogression models with m=1,2,3,4 for stock JNJ.

Figure 8 has (a) Local linear autogression with m=3 and (b) SETAR model search with m=3, mL=1:3, mH=1:3, thSteps = 5, thDelay=0:2 for JNJ.

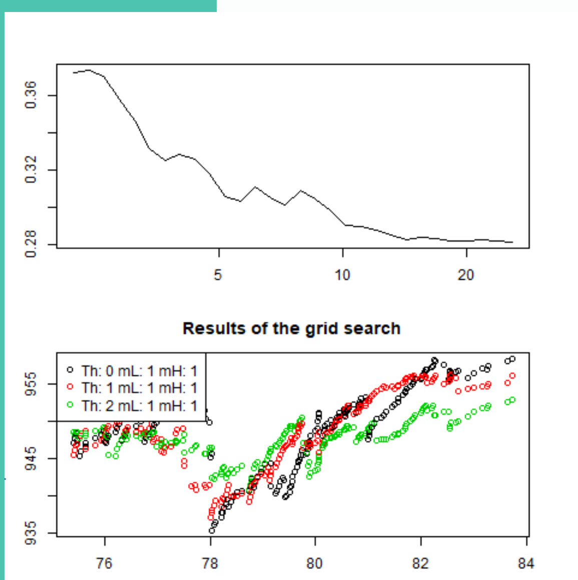

Figure 8: (a) Local linear autogression with m=3 and (b) SETAR model search with m=3, mL=1:3, mH=1:3, thSteps = 5, thDelay=0:2 for JNJ

Figure 9 has Lags 1 and 3 as well as a histogram, autocorrelation, partial correlation function and mutual information for stock JNJ.

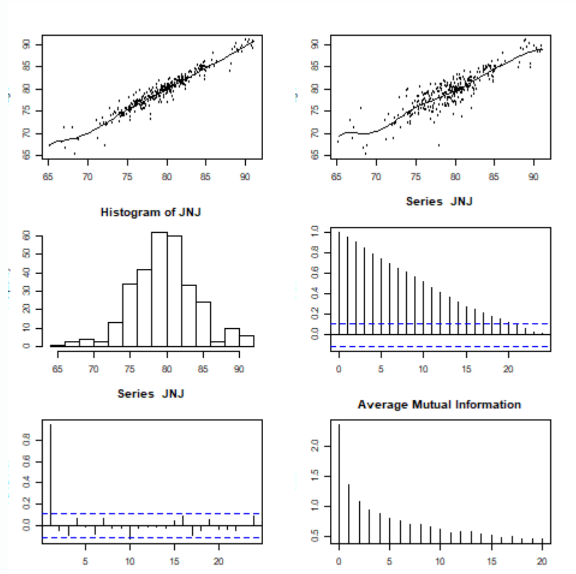

Figure 9: Lags 1 and 3 as well as a histogram, autocorrelation, partial correlation function and mutual information for stock JNJ.

Figure 10 has 30 day 1 step ahead predictons for stock JNJ with the SETAR model.

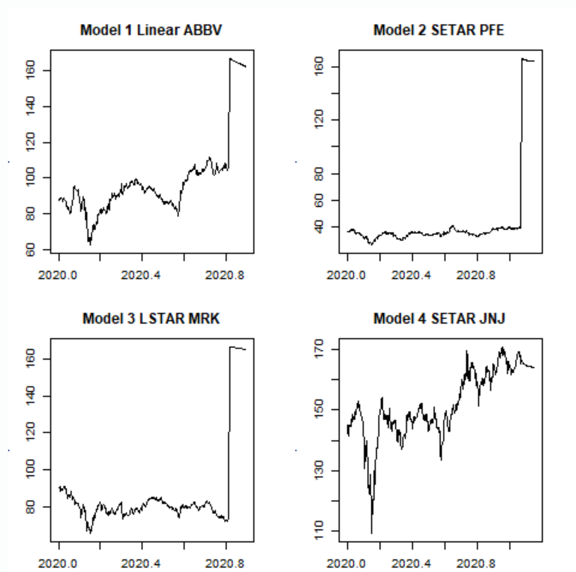

Figure 10: 30 day 1 step ahead predictons for (a) ABBV (b) PFE (c) MRK and (d) stock JNJ with prices [1] 166.3401 166.1744 165.9179 165.7095 165.5179 165.3491 165.1990 165.0658 164.9476 164.8427 164.7496 164.6669 164.5936 164.5285 164.4708 164.4195 164.3740 164.3337 164.2978 164.2660 164.2378 164.2128 164.1906 164.1708 164.1533 164.1378 164.1240 164.1118 164.1009 164.0913 from the SETAR Model m=2 delay=1

Conclusions

In this study, nonlinear time series for daily low stock prices such as ABBV PFE MRK and JNJ in pharmaceutical industry were examined .

Here NonLinear AutoRegressive (NLAR), LSTAR and SETAR are three possible mathematical descriptions for predicting the n step ahead daily values necessary for optimal portifolio investment selections. Here the Beta distribution and the SETAR model is used for generating the forecasts of JNJ stock price for portfolio analysis and trading strategies as an example for the classroom and illustrated here with an illustrated numerical review for nonlinear time series models and the financial instruments of the pharmaecutical industry.

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